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CYCLING WITH RULES OF THUMB:  
AN EXPERIMENTAL TEST FOR A NEW FORM OF  
NON-TRANSITIVE BEHAVIOUR

**ABSTRACT.** This paper tests a novel implication of the original version of prospect theory (Kahneman and Tversky, 1979): that choices may systematically violate transitivity. Some have interpreted this implication as a weakness, viewing it as an anomaly generated by the 'editing phase' of prospect theory which can be rendered redundant by an appropriate re-specification of the preference function. Although there is some existing evidence that transitivity fails descriptively, the particular form of non-transitivity implied by prospect theory is quite distinctive and hence presents an ideal opportunity to expose that theory to test. An experiment is reported which reveals strong evidence of the predicted intransitivity. It is argued that the existence of this new form of non-transitive behaviour presents a fresh theoretical challenge to those seeking descriptively adequate theories of choice behaviour, and a particular challenge to those who seek explanations within the conventional economic paradigm of utility maximisation.

**KEY WORDS:** Intransitivity, Experimental economics, Prospect theory

1. INTRODUCTION

This paper is concerned with the descriptive theory of decision making under risk and uncertainty. The traditional approach to modelling choice behaviour in economics assumes that individuals behave as if maximising some preference function. Theories of this form typically assume that individuals have complete preferences over prospects (where a prospect is any probability distribution of consequences) and that those preferences satisfy certain properties of consistency. Amongst the properties most commonly built into such theories are *monotonicity* (the property that stochastically dominating prospects are preferred to prospects which they dominate) and *transitivity* (the property that for any three prospects  $P$ ,  $Q$ ,  $R$  if  $P \succeq Q$  and  $Q \succeq R$  then  $P \succeq R$ , where  $\succeq$  is the relation of weak preference). I will call any theory of complete preferences which embodies both of these properties a *conventional preference theory*



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(CPT). Among the best known CPTs in economics are expected utility theory (von Neumann and Morgenstern, 1946), generalised expected utility theory (Machina, 1982) and the theory of anticipated utility (Quiggin, 1982).

A different approach to modelling choice behaviour has its foundations in the psychological literature where emphasis has been placed on decision *processes* as opposed to preferences. Theories in this tradition model behaviour in terms of decision heuristics examples of which include the satisficing heuristic (Simon, 1955), the elimination by aspects heuristic (Tversky, 1972) and the equal weight heuristic (Dawes, 1979). I will refer to such theories as *decision process theories* (DPTs).<sup>1</sup>

Although these two literatures have developed along largely separate lines, one theory of choice under uncertainty incorporating decision heuristics has been widely discussed in the economics literature: Kahneman and Tversky's (1979) prospect theory. I shall refer to this theory as the original form of prospect theory or OPT for short. OPT consists of two main components: a preference function plus an 'editing phase' involving the application of a set of decision heuristics which 'simplify' the choice set prior to application of the preference function.

The preference function in OPT is not generally monotonic and one role of the editing phase is to impose monotonicity: Kahneman and Tversky propose an editing heuristic which eliminates dominated options, so long as they are detected (I will call this the *dominance heuristic*). This strategy for inducing monotonicity has attracted criticism from some economists. Quiggin (1982, 1993), for example, has criticised the approach on two counts. First, he argues that by appropriate specification of the preference function the dominance heuristic can be rendered redundant. Second, he criticises the Kahneman and Tversky strategy for imposing monotonicity in OPT because it has the spin-off effect that the theory then admits violations of transitivity in pairwise choice. Quiggin (1982, p. 327) describes this an 'undesirable result'.

Quiggin (1982) proposes an alternative model (anticipated utility). This retains a central feature of OPT – that individuals maximise a preference function in which objective probabilities are transformed into subjective decision weights – but in Quiggin's theory,

weights are assigned by a cumulative weighting function which ensures preferences satisfy both monotonicity and transitivity. More recently, Tversky and Kahneman (1992) have proposed a revised version of prospect theory, cumulative prospect theory, with similar properties. Like anticipated utility theory, the probability transformation function is cumulative, the distinctive feature of cumulative prospect theory is that weights are constructed separately for gains and losses. Both of these new theories are CPTs and make no reference to decision processes.

If we assume that monotonicity and transitivity are both desirable properties for any satisfactory theory of choice under uncertainty, these later theories would appear superior to OPT: they embody both properties, and in the case of monotonicity, the property is generated in a more economical fashion. But is it legitimate to assume that monotonicity and transitivity are both desirable properties? To the extent that our concern lies in developing models capable of explaining *actual* (as opposed to say, normatively justifiable) behaviour, we must seek models consistent with observed behaviour (though, this is not to deny an important, if separate, role for normative theorising).

There is some evidence to suggest that individuals may be quite prone to violations monotonicity in contexts where the relation of dominance is 'non-transparent' or difficult to detect (see, for example, Tversky and Kahneman, 1986). This is consistent with the two-phase theory proposed in OPT but inconsistent with any generally monotonic theory of preference. Although there is also well documented evidence that individuals may make non-transitive choices among prospects in some contexts (see, for example, Edwards, 1954; Tversky, 1969), the particular violation predicted by OPT, as I shall argue below, is a very specific form of intransitivity involving an 'indirect' violation of monotonicity and, to my knowledge, there is no existing evidence that it occurs in actual choices. This is a novel implication of OPT in the sense that the prediction is a by-product of the procedure designed to eliminate dominated options rather than a feature of the theory specifically developed to account for some previously recognised violation of transitivity. As such, it presents an ideal opportunity for testing OPT.

This paper presents a simple experimental test for the form of intransitivity consistent with OPT. Although the test conducted is quite specific, the results may have some important general implications. The two-stage theory envisaged in OPT has been criticised by some economists on the grounds that it is unnecessarily complicated and because it generates ‘undesirable’ properties. But were we to find that distinctive implications of this two-phase theory incorporating editing were reflected in actual choice behaviour that would provide a counter to such criticism and, for reasons set out in the conclusion, set a fresh challenge to those who seek to provide descriptive accounts of decision making under uncertainty based on utility maximisation alone, without any reference to decision processes.

## 2. PROSPECT THEORY AND INTRANSITIVITY

In this section I outline the relevant features of OPT and demonstrate that this theory implies the possibility of systematically non-transitive choices. In the first stage of OPT, individuals ‘edit’ prospects in various ways to convert them into a form ‘suitable’ for evaluation in the second (utility maximising) stage. Among the editing operations proposed by Kahneman and Tversky are *combination* (probabilities of identical outcomes are added prior to evaluation); and *cancellation* (the elimination of components common to all prospects under consideration). The editing phase also includes the dominance heuristic (henceforth DH) which involves the “scanning of offered prospects to detect dominated alternatives, which are rejected without further evaluation” (Kahneman and Tversky, 1979, p. 275).

In the second stage of OPT, individuals choose among edited prospects by maximising a preference function. For the class of prospect considered in this paper the preference function may be represented as follows.<sup>2</sup>

$$V(P) = \sum_{i=1}^n \pi(p_i)v(x_i) \quad (2.1)$$

In expression 2.1,  $V(P)$  is the overall value of any edited prospect  $P$  which has  $n$  outcomes and  $p_i$  is the (objective) probability of

outcome  $x_i$ .  $\pi(\cdot)$  is a function assigning subjective decision weights to probabilities and  $v(\cdot)$  assigns subjective values to outcomes. Following Machina (1983) I will refer to the function in Expression 2.1 as the *subjective expected utility* form (SEU for short). When  $\pi(\cdot)$  is allowed to be non-linear, SEU implies violations of monotonicity. This is generally viewed as a fatal flaw in SEU (see, for example, Machina, 1982; Quiggin, 1982) and recent theories which retain the central idea behind SEU – that subjective decision weights are assigned to objective probabilities – avoid this implication.

Kahneman and Tversky assume that DH will be applied whenever individuals recognise the presence of a dominated prospect. But, this strategy for avoiding general non-monotonicity implies that choices may be non-transitive. To illustrate this, consider the following three prospects labelled Options A–C:

Option A :  $(p, x; 1 - p, 0)$

Option B :  $(q, y; 1 - q, 0)$

Option C :  $(r, y; s, y - \epsilon; 1 - r - s, 0)$

The letters  $p, q, r$  and  $s$  represent probabilities such that  $p < q, q = r + s; x, y$  and  $\epsilon$  are amounts of money where  $x > y > \epsilon > 0$ . Thus Option A is a gamble involving a  $p$  chance of winning  $x$ , otherwise nothing. Option B offers a larger chance ( $q$ ) of winning a smaller prize ( $y$ ). Finally, Option C is dominated by Option B (in C, the chances of ‘winning’ are the same as in B but the prize is not always as good).

Consider the implications of OPT for pairwise choices between these three options. Given the simplicity of the above options, there is not much scope for the application of editing routines specified in OPT apart from the application of DH to the choice between B and C. I will assume that preferences between A and B or A and C can be determined directly by the application of SEU.<sup>3</sup> Now, suppose that, for some individual,  $A \succ B$  where  $\succ$  is the relation ‘is preferred to’. If  $\pi(\cdot)$  is sub-additive over some range such that  $\pi(r) + \pi(s) > \pi(q)$ , then SEU implies that an individual may also prefer C to A. To see why, first suppose that  $A \succ B$ . It follows from application of

Expression 2.1 above that:

$$\pi(p)v(x) > \pi(q)v(y) \quad \text{or} \quad \pi(p)v(x) - \pi(q)v(y) > 0. \quad (2.2)$$

If  $C \succ A$  then:

$$\pi(r)v(y) + \pi(s)v(y - \epsilon) > \pi(p)v(x) \quad (2.3)$$

or

$$\pi(r)v(y) + \pi(s)v(y - \epsilon) - \pi(p)v(x) > 0. \quad (2.4)$$

Summing across expressions 2.2 and 2.4 gives:

$$\pi(r)v(y) + \pi(s)v(y - \epsilon) - \pi(q)v(y) > 0. \quad (2.5)$$

Hence, a necessary condition for 2.5 is that  $\pi(r) + \pi(s) > \pi(q)$ . Since sub-additivity for some  $r$ ,  $s$  and  $q$  is a direct consequence of the functional form for  $\pi(\cdot)$  assumed by Kahneman and Tversky (1979, p. 283) the conjunction of preference  $A \succ B$  and  $C \succ A$  would be consistent with OPT for some  $\epsilon$ . Since DH requires  $B \succ C$ , OPT implies that a violation of transitivity of the form  $A \succ B$ ,  $B \succ C$  and  $C \succ A$  (call this the *predicted cycle*) may occur. The reverse cycle (call this the *counter cycle*) would not be consistent with OPT as it involves behaviour contrary to DH ( $C \succ B$ ). Thus, OPT implies that we may observe *systematic* violations of transitivity: if cycles occur, they will be predicted cycles and not counter cycles. Although this implication of OPT was recognised by Kahneman and Tversky (1979, p. 284), to my knowledge it has not been tested. The remainder of this paper reports an experimental test of this property of OPT.

### 2.1. *The experimental design*

Two hundred and four subjects took part in the experiment. Each subject was required to respond to twenty questions contained in a single booklet. Each question involved a choice between a pair of prospects. Subjects were motivated to give honest and considered responses to these questions by using a *random-lottery procedure*: they were told that one of the twenty options they chose would be selected at random (by rolling a twenty sided die) and played out for

real.<sup>4</sup> Among the twenty questions were the three possible pairwise choices among the three options labelled *A–C*. below:

A	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20%; border-right: 1px solid black; padding: 2px;">1</td> <td style="width: 60%; padding: 2px;">20, 21</td> <td style="width: 20%; padding: 2px;">100</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">14.00</td> <td colspan="2" style="padding: 2px;">0.00</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">20</td> <td colspan="2" style="padding: 2px;">80</td> </tr> </table>	1	20, 21	100	14.00	0.00		20	80	
1	20, 21	100								
14.00	0.00									
20	80									

B	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%; border-right: 1px solid black; padding: 2px;">1</td> <td style="width: 70%; padding: 2px;">30, 31</td> <td style="width: 20%; padding: 2px;">100</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">8.00</td> <td colspan="2" style="padding: 2px;">0.00</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">30</td> <td colspan="2" style="padding: 2px;">70</td> </tr> </table>	1	30, 31	100	8.00	0.00		30	70	
1	30, 31	100								
8.00	0.00									
30	70									

C	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 15%; border-right: 1px solid black; padding: 2px;">1</td> <td style="width: 15%; padding: 2px;">15, 16</td> <td style="width: 30%; padding: 2px;">30, 31</td> <td style="width: 40%; padding: 2px;">100</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">8.00</td> <td style="padding: 2px;">7.75</td> <td colspan="2" style="padding: 2px;">0.00</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">15</td> <td style="padding: 2px;">15</td> <td colspan="2" style="padding: 2px;">70</td> </tr> </table>	1	15, 16	30, 31	100	8.00	7.75	0.00		15	15	70	
1	15, 16	30, 31	100										
8.00	7.75	0.00											
15	15	70											

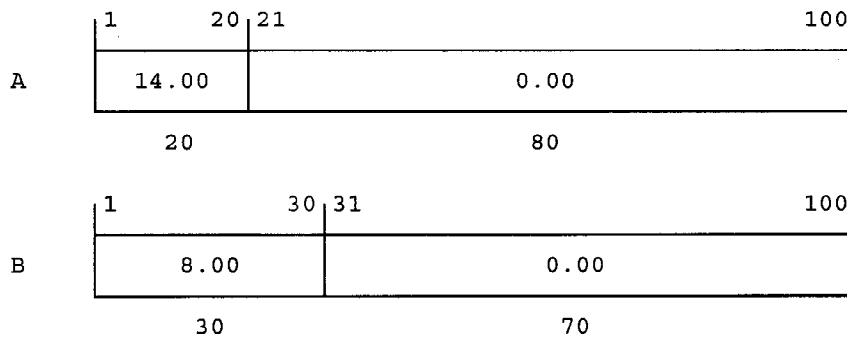
Each of the three boxes represents a prospect: the numbers inside the boxes represent amounts of money (UK £) that could be won, the numbers underneath the boxes show the chances out of one hundred of winning each sum of money.

Figure 1 illustrates how the problems were presented to the subjects using the choice between *A* and *B* which, for the purposes of discussion here, has been labelled ‘Question 1’. Subjects were instructed to interpret the questions in the following way. At the start of the experiment, each subject was required to draw a sealed envelope from a box containing 100 such envelopes. Each envelope contained one from a series of cloakroom tickets numbered from 1–100. The numbers along the top of the boxes refer to these ticket numbers. So, if a subject were playing option *A* for real, and if their envelope contained a ticket numbered from 1–20 they would win £14, otherwise nothing. With option *B*, a ticket numbered from 1–30 would win £8.

The choice is displayed exactly as it was presented to subjects except for the question number and the labelling of the options: these varied according to the location of the question in the subject’s question booklet. For ease of discussion the labelling (*A–C*) of the options is preserved here and I shall refer to the choices {*A* vs *B*}, {*B* vs *C*} and {*A* vs *C*} as Questions 1, 2 and 3 respectively. Each

**QUESTION 1:**

Choose A or B

*Figure 1.*

question appeared on a separate page of the booklet and the order of questions was randomised across six sub-groups of subjects.<sup>5</sup>

Notice that three boxes describe prospects with the same structure as Options A–C discussed in Section 2 with  $p = 0.2$ ,  $q = 0.3$ ,  $r = s = 0.15$ ,  $x = £14$ ,  $y = £8$  and  $\epsilon = £0.25$ , hence OPT implies that systematic violations of transitivity of the form  $A \succ B$ ,  $B \succ C$  and  $C \succ A$  may be observed across Questions 1–3.

## 2.2. *The hypothesis to be tested*

The general hypothesis to be investigated is whether subjects' choices are consistent with transitivity or whether there is evidence of the systematic cycle predicted by OPT. Researchers wishing to experimentally test economic theories face the common problem that economic theories are deterministic but to test them it is necessary to allow for a stochastic element in choice (otherwise a single observation of either cyclical pattern would be sufficient to reject a transitive theory).

Although numerous models of stochastic preferences have been discussed by both economists and psychologists there is no generally agreed theory of stochastic preference. This can create difficulties in the interpretation of test results: since one is inevitably testing a joint hypothesis (e.g. economic hypothesis + stochastic specification) rejection of the null may be open to interpretation as either a failure of the relevant economic hypothesis, or an invalid stochastic specification. One way to respond to this difficulty would be to ap-



ply a test which has a clear interpretation for a range of alternative stochastic specifications. This is the strategy adopted here.

To this end, consider two of the questions used in this experiment, Questions 1 and 3. Let  $\succ_c$  represent the relation ‘is chosen over’, then consider two patterns of choice that could occur across this pair of questions:  $A \succ_c B$  and  $C \succ_c A$  (call this pattern I);  $B \succ_c A$  and  $A \succ_c C$  (call this pattern II). Notice that pattern I (which is part of the predicted cycle) must imply either a violation of monotonicity or transitivity (or both). Hence, for any *deterministic* CPT, the probability of observing pattern I, denoted  $P(I)$ , must be zero.

But now suppose that preferences have some stochastic element. One way to model this would be to assume that individuals have ‘true’ preferences but may make ‘errors’ which could lead them to make choices inconsistent with their true preferences. In the presence of errors,  $P(I)$  may be positive even if true preferences are consistent with some CPT. However, it may be still reasonable to expect pattern I to occur no more frequently than pattern II (i.e.,  $P(I) \leq P(II)$ ) since the reverse could only be true if errors generated some bias towards pattern I. Given the symmetry of the two patterns – recording either by mistake requires an error on one or both of Questions 1 and 3 – it is not obvious why errors would generate such a bias. I shall have more to say about this below, but for the present let us simply *assume* that there is no such bias.

Given this assumption we may conduct an *indirect* test for the violation of transitivity implied by OPT by testing the null hypothesis  $P(I) = P(II)$  against the alternative hypothesis  $P(I) > P(II)$ . Rejection of the null hypothesis implies either a violation of monotonicity, or transitivity, or both. It may then be possible to discriminate between monotonicity and transitivity as the source of the violation by observing responses to Question 2. If most subjects obey monotonicity (as required by DH), then rejection of the null can be interpreted as evidence of intransitivity (high levels of direct monotonicity violation could, of course, lead to rejection of the null in the absence of the cycle).

The main rationale for this indirect test is that the null hypothesis captures a variety of alternative stochastic specifications for CPT. Three models of stochastic preference have recently appeared

in the economics literature. These are the models of Harless and Camerer (1994), Hey and Orme (1994) and Loomes and Sugden (1995).<sup>6</sup> Each model provides a general framework for developing stochastic versions of alternative (deterministic) ‘core’ theories of preference such as expected utility theory or anticipated utility theory, for example. In the first and second of these models, the core theories can be interpreted as ‘true’ preferences with the stochastic element reflecting ‘errors’ arising from mis-calculation, carelessness and so on. The Loomes and Sugden specification has a different interpretation: for any given choice, the individual acts on preferences satisfying the restrictions of the core theory, but the parameters of the core theory to be applied to any given choice are determined by a random process. In this case, the stochastic element is inherent in preferences as opposed to random deviation about true preferences.

The three models are discussed in detail in Loomes and Sugden (*op. cit.*), but for our purposes it is sufficient to note that given any CPT as a core theory each of the three specifications has the following property: for any three prospects,  $A$ ,  $B$ ,  $C$ , where  $B$  dominates  $C$ ,  $P(A \succ_c C) \geq P(A \succ_c B)$ . This property can be interpreted as a stochastic formulation of the principle of monotonicity and I will refer to it as *weak stochastic monotonicity* (WSM). It follows from WSM that  $P(B \succ_c A) \geq P(C \succ_c A)$  and since  $P(I) = P(A \succ_c B)$  and  $C \succ_c A$ ,  $P(II) \equiv P(B \succ_c A \text{ and } A \succ_c C)$ , if choices are independent, any theory of stochastic preferences which embodies WSM implies  $P(I) \leq P(II)$ . Since rejection of the null in the present experiment requires that  $P(I) > P(II)$ , it also implies rejection of any CPT under each of the three stochastic specifications. To the extent that WSM provides a natural way to extend the (deterministic) principle of monotonicity to a stochastic setting, one might expect this to be a property of a wider class of the stochastic models which seek to retain a principle of monotonicity. Consequently, the proposed statistical test has a high degree of generality since it allows us to test the null hypothesis that preferences are consistent with some CPT under a broad range of alternative (possible) stochastic specifications.

One further property of the proposed test procedure is worth highlighting. Given a transitive model of preference which assumes that

subjects may make errors, it seems plausible to imagine that errors might be relatively unlikely in situations where one option (transparently) dominates another. If correct, this would provide a reason for expecting the predicted cycle to occur more frequently than the counter cycle *purely as a result of random error* (recall that the counter cycle necessitates an error on the question involving the dominance relation, whereas the predicted does not). Without necessarily endorsing such a theory of errors it seems desirable, if possible, to use a test which does not allow this interpretation. The test developed above has exactly this property. Since the indirect test relies only on data from Questions 1 and 3, the rate of monotonicity violation on Question 2 plays no role in determining the outcome of the formal statistical test. The role of the data from Question 2 is purely diagnostic.

T A B L E	ORDERINGS (Consistent with Transitivity)						CYCLES	
	1	2	3	4	5	6	Counter	Predicted
	1	A>B B>C A>C	A>C C>B A>B	C>A A>B C>B	C>B B>A C>A	B>C C>A B>A	B>A A>C B>C	B>A C>B A>C
No	75	6	1	5	49	17	1	50
%	36.8	2.9	0.5	2.5	24.0	8.3	0.5	24.5

TABLE 2	RESPONSES TO INDIVIDUAL QUESTIONS		
Ques\choice	Ques 1: A > B	Ques 2: B > C	Ques 3: A > C
No.	132	191	99
%	64.7	93.6	48.5

### 2.3. Results

Tables 1 and 2 summarise the data. Table I shows how subjects' choices were distributed across the eight possible patterns of response to the three questions (i.e. six orderings plus two cyclical patterns). Table 2 reports the frequency of subjects choosing the option with the higher expected value for each question. Corresponding percentages appear below the raw frequencies.

First consider Table 1 and note that 51 of the 204 subjects (25%) made non-transitive responses. This suggests that if these cycles occur as a result of errors, then errors are quite frequent. As a benchmark, if subjects chose entirely at random then we would expect around 25% of the recorded patterns to be cycles. The patterns of choice are, however, clearly non-random. Notice, in particular, the striking difference between the frequency of predicted- and counter-cycles: 50 of the 51 observed cycles are in the predicted direction. Indeed the predicted cycle is the second most frequent pattern. But although this ‘eyeballing’ of the data is suggestive of a systematic cycle, as noted above, this could, in principle, be explained by a CPT plus a theory of errors which, say, assumed that mistakes are less likely in questions involving dominance. We therefore apply the indirect test developed above which compares the relative frequencies of patterns I and II.

There are 51 observations of pattern I compared with only 18 observations of pattern II.<sup>7</sup> On the basis of these frequencies, the null is confidently rejected ( $p < 0.0001$ , test based on the binomial distribution). Hence, we may conclude that either a violation of monotonicity, or transitivity (or both) has occurred. Inspection of Table II indicates that the vast majority of subjects (93.6%) obeyed monotonicity (i.e., chose the dominating option, *B*, in Question 2).<sup>8</sup> This provides a clear indication that the principle being violated by subjects in this experiment is transitivity.

### 3. DISCUSSION AND CONCLUSION

Modelling behaviour in terms of decision heuristics has a long tradition in psychology, but the approach has made relatively little impact within the economics profession. OPT is one of the few models, deriving from this tradition, to have been widely discussed by economists. The intransitivity implied by this two-phase model of choice had been interpreted by some as a weakness of that theory. The experiment reported above reveals exactly the kind of intransitive choice predicted by OPT and the results are inconsistent with any CPT allowing for a general class of stochastic specifications.

There is growing evidence that the choices of ordinary individuals do not generally satisfy transitivity. Some of the most recent

contributions to this evidence include Loomes, Starmer and Sugden (1989, 1991) and Tversky, Slovic and Kahneman (1990). The findings reported above constitute an addition to this body of evidence but also add some new dimensions. For instance, relative to other recent empirical work, the frequency of intransitivity found here is particularly high and the pattern very clearly skewed. This provides one reason for thinking that it would be rash to dismiss intransitivities, in general, as quantitatively minor anomalies. The results also suggest a new question: can the observed cycle be accommodated in *any* theory of utility maximisation without reference to decision processes?

This new form of intransitivity is inconsistent with the one theory of non-transitive utility maximisation which has been widely discussed in the economics literature: the theory of regret due to Loomes and Sugden (1982) and Fishburn (1982). While this theory is consistent with some of the evidence relating to intransitive choice (see Loomes, Starmer and Sugden, 1989, 1991), it is easy to show that it does not permit the specific form of intransitivity observed here (see Appendix for a demonstration of this). Other non-transitive theories of preference have been discussed in the psychological literature. Among them are a number of models which have been developed for application to multi-attribute choice settings including the additive-difference model (Tversky, 1969) and the multi-attribute random weights model (Schoemaker and Waid, 1988). It is not always clear how these models apply to choices among prospects, in particular whether the attribute space should include probabilities, or consequences, or both.<sup>9</sup> However, as far as I am aware, none of the theories in this class has the properties necessary to explain the predicted cycle.

Any theoretical account of the cycle observed in this experiment would have to have two quite specific properties. First, any such theory must rank  $A$  above  $B$ , but below  $C$ , in pairwise choice. But, recall that the only difference between prospects  $B$  and  $C$  (as described in Section 2.1) is that ‘tickets’ 16–30 result in a lower prize for prospect  $C$  (compared with  $B$ ). On the face of it, this difference renders  $C$  worse than  $B$ , so if  $A \succ B$ , what feature of the problems could account for  $C$  being ranked above  $A$ ? There seems to be only one possibility: any theory allowing the pattern  $A \succ B$  and  $C \succ A$

must have the property that two events with probabilities of 0.15 each carry 'more weight' than a single event with probability 0.3. A theory which had this property would also imply a violation of monotonicity ( $C \succ B$ ) unless it embodied some additional feature, like the dominance heuristic, to rule this out. OPT has been criticised by some economists precisely because it had these properties; these results imply that such criticisms may be misplaced. On the other hand, OPT cannot claim to be a generally satisfactory descriptive model, for example, it cannot explain the evidence of cycles consistent with regret theory. Thus, an obvious question to tackle is how to produce a theory capable of explaining this new phenomenon, together with pre-existing evidence of choice behaviour?

Although I know of no other theory which explicitly predicts the cycle implied by OPT, a number of theories which involve decision heuristics allow *some* violations of transitivity (see for example Rubinstein, 1988), and one can readily see how a theory in this tradition could, in principle, explain it. For example, without necessarily endorsing such a theory, a DPT which combined an equal weight heuristic (i.e., a rule assigning an overall value to each prospect by summing the values of outcomes regardless of their probability) with a dominance heuristic could generate the predicted cycle. While this could be thought of as a special case of OPT in which the weights attached to all probabilities are unity, a choice rule involving such extreme weighting might be regarded as reflecting a certain degree of naivety as its application implies that subjects effectively disregard probability information. While it is *possible* that the observed cycle results from subjects applying some relatively naive rule like the equal weight heuristic, the data reported above do not imply such extreme weighting and further investigation would be necessary to discriminate between alternative accounts of the process generating the observed cycle. Such speculation, however, perhaps begs two further questions: are the results robust and if they are, would it be possible to construct a utility maximising theory to account for the phenomenon?

Given that this is the first reported instance of this particular form of non-transitive behaviour, there is an obvious case for exploring whether these results can be replicated, whether they are robust to variants in experimental design and generalise to other contexts. As

yet there is no generally satisfactory descriptive model of choice under uncertainty and if further research suggests that the behaviour reported above is robust, there would be a clear case for arguing that explanation of this new phenomenon should be one objective of future theoretical efforts. Since it is relatively easy to see how this new form of non-transitive behaviour might be explained in terms of decision heuristics, this new phenomenon perhaps presents a particular challenge to those committed to explaining choice under uncertainty within the conventional economic paradigm of utility maximisation.

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#### APPENDIX

Regret theory applies to pairwise choices among *actions* where an action is a mapping from states of the world to consequences. Following Loomes and Sugden (1987) let  $A_i$  and  $A_j$  be any two actions which result in outcomes  $x_{is}$  and  $x_{js}$ , respectively, in state of the world  $s$ . Preferences between  $A_i$  and  $A_j$  are then determined by Expression A.1:

$$A_i \succsim A_j \Leftrightarrow \sum_s P_s \Psi(x_{is}, x_{js}) \geq 0. \quad (\text{A.1})$$

The function  $\psi(., .)$  is skew symmetric (by definition), that is  $\psi(x_{js}, x_{is}) = -\psi(x_{is}, x_{js})$  which implies  $\psi(x_{js}, x_{is}) = 0$ , for  $x_{is} = x_{js}$ . Recall that the predicted cycle requires  $A \succ B$  and  $C \succ A$  (pattern

I). From application of Expression A.1 to Questions 1 and 3 we may derive:

$$A \succ B \Leftrightarrow 0.2\psi(14, 8) + 0.1\psi(0, 8) > 0$$

and

$$C \succ A \Leftrightarrow 0.15\psi(8, 14) + 0.05\psi(7.75, 14) \\ + 0.1\psi(7.75, 0) > 0.$$

Summing across these two conditions, and using the property of skew symmetry, the necessary condition for the occurrence of pattern I is given by:

$$0.05[\psi(14, 8) - \psi(14, 7.75)] + 0.1[\psi(7.75, 0) \\ - \psi(8, 0)] > 0.$$

Since  $\psi(., .)$  is assumed to be increasing in its first argument and non-increasing in its second, both expressions in square brackets are non-positive, hence pattern I is inconsistent with regret theory.

#### NOTES

1. For a review of theories in this tradition see Payne, Bettman and Johnson (1993).
2. Expression 1 applies to what Kahneman and Tversky (1979, pp. 275–276) call ‘regular prospects’. A regular prospect has no more than three consequences. Let  $P = (p_1, p_2, p_3)$  represent any prospect defined over three (monetary) consequences  $x_1 > x_2 > x_3$ .  $P$  is a regular prospect if either ( $x_1 = 0$  and  $p_2 + p_3 < 1$ ) or ( $x_3 = 0$  and  $p_1 + p_2 < 1$ ) or  $x_2 = 0$ . In this paper discussion is confined to regular prospects and so, for convenience, we may work with the special case of Expression 2.1.
3. Kahneman and Tversky (1979) suggest that individuals may sometimes ‘round’ payoffs up or down. In principle such rounding might have the effect of  $\epsilon$  being treated as equal to zero. Also, OPT allows for the possibility that DH might not be applied to the choice between options  $B$  and  $C$  if individuals fail to recognize the dominance relation. In either case, the prediction of transitivity derived below collapses. However, this need only concern us if the intransitivity prediction fails.
4. Holt (1986) raised an objection to the use of the random lottery procedure (RLP) pointing out that, theoretically, the RLP might induce biased responses (relative to ‘true’ preferences) if the independence axiom of expected utility



theory fails. It is easy to show, however, that Holt's argument would not explain the presence of systematic cycles in pairwise choices elicited using the RLP (for a demonstration of this see Loomes, Starmer and Sugden, 1991). The design of this experiment is therefore immune to the Holt criticism in the sense that, if a systematic cycle is found, it cannot be attributed to the bias conjectured by Holt.

5. A full description of the experiment, the question booklets and the instructions given to subjects are available from the author on request.
6. Similar models of stochastic preference have also been discussed by psychologists. See, for example, Hershey and Schoemaker (1985), Eliashberg and Hauser (1985).
7. The figure for pattern I is obtained by adding together the totals for Ordering 3 plus the predicted cycle from Table 1. Similarly, the figure for pattern II is obtained by adding together the totals for Ordering 6 plus the counter-cycle.
8. Given that it is very hard to construct a plausible argument to suggest that *C* is the 'right' choice in Question 2, one might interpret the degree of conformity with monotonicity as some measure of the competence of subjects and/or the care with which they answered the questions. A 'failure' rate of approximately 6% is, arguably, quite reassuring.
9. For a discussion of this point, see Machina (1983, p. 123).

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