Demand commitment in legislative bargaining*

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Abstract

Morelli (1999) provides a model of government formation in which the parties make payoff demands and the order of moves is chosen by the leading party. Morelli's main proposition states that the ex post distribution of payoffs inside the coalition that forms is proportional to the distribution of relative ex ante bargaining power. We provide a counterexample in which the leading party is able to obtain the entire payoff; furthermore, there are coalitions for which proportional payoff division does not occur for any order of moves.

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In a parliamentary democracy, many important decisions including government formation are the outcome of bargaining between the parties in Parliament. The most influential model of legislative bargaining is the closed rule model of Baron and Ferejohn (1989). In this model, a party is randomly recognized to propose a distribution of ministerial payoffs and the remaining parties can accept or reject the proposal. This model has led to many applications and extensions (see Ansolabehere et al. 2005 for a comprehensive list). However, it has some properties that may be perceived as drawbacks: the proposer has a large advantage (he receives more than half of the total payoff under simple majority), and there is a multiplicity of subgame perfect equilibria so that equilibrium refinements need to be used in order to obtain a unique prediction.

An alternative model of legislative bargaining by Morelli (1999) is based not on complete proposals but on demands. Parties make individual demands for ministerial payoffs and a coalition emerges between parties making compatible demands. The Head of State chooses the first mover, and the latter chooses the order in which the parties formulate demands. Morelli's main result (Proposition 2) is that the coalition that forms divides payoffs proportionally to the homogeneous representation of the game regardless of which party is chosen to be the first mover. In this paper we provide a counterexample to Morelli's Proposition 2.

The model

Weighted majority games

Consider a legislature in which a set $N = \{1, 2, ..., n\}$ of parties is represented. There is a budget of size 1 to be divided by majority rule. Each party i has ω_i votes, and a quota of q is needed for a majority. The pair $[q; (\omega_i)_{i \in N}]$ is a *weighted majority game*. Notice that the game is not affected if weights and quota are multiplied by the same positive constant.

Given a vector $x \in \mathbb{R}^N$ and a coalition $S \subset N$, we denote as x_S the sum of the coordinates of the members of S, $x_S := \sum_{i \in S} x_i$.

A coalition $S \subset N$ is *winning* if $\omega_S \geq q$; it is *minimal winning* if it is winning and no $T \subsetneq S$ is winning. We denote as $\Omega(\omega)$ the set of all winning coalitions, and as $\Omega^m(\omega)$ the set of all minimal winning coalitions.

A *dummy* is a party that does not belong to any coalition in $\Omega^m(\omega)$.

A weighted majority game is *proper* if $S \in \Omega(\omega) \Rightarrow N \backslash S \notin \Omega(\omega)$ for all S. It is *strong* if $S \notin \Omega(\omega) \Rightarrow N \backslash S \in \Omega(\omega)$ for all S. It admits an equivalent *homogeneous representation* if there exists a vector of votes $\left(\omega_1^h, ..., \omega_n^h\right)$ and a quota q^h such that $\Omega^m(\omega) = \Omega^m\left(\omega^h\right) = \{S \subset N : \omega_S^h = q^h\}$. A weighted majority game with an equivalent homogeneous representation is called a *homogeneous game*.

Homogeneous representations do not always exist and when they exist they may not be unique. For example, $[5; 3, 2, 2, 1]$ and $[7; 4, 3, 3, 1]$ are two homogeneous representations of the same game. Peleg (1968) shows that proper and strong homogeneous games have a unique homogeneous representation (up to multiplication by a positive constant and assuming that dummies are assigned a weight of 0).

Morelli's bargaining procedure

There are n parties, 1 unit of private benefits to be distributed, and a policy to be chosen from the policy space $[0, 1]$. Party i has utility function $u_i = x_i + 1 - \beta|\theta - \theta_i^*|$, where x_i denotes the share of private benefits accruing to i, θ is the policy implemented and θ_i^* is party i's ideal policy. Bargaining proceeds as follows: First, the Head of State chooses a party i. Second, i chooses an order of play $\rho : N \to \{1, 2, ..., n\}$ such that i is the first one in the order, *i.e.* $\rho(i) = 1$. Third, each party j demands a pair (d_j, θ_j) following the order of play, where $d_j \in [0, 1]$ is the share of the private benefit j claims and $\theta_j \in [0, 1]$ is a policy. If, after party j makes its demand, there exists a winning coalition $S \subset \{k : \rho(k) \leq \rho(j)\}$ such

that $d_S \leq 1$ and $\theta_k = \theta_l$ for every $k, l \in S$, then j has the additional choice of forming S, in which case the policy is implemented and the demands of parties in S are granted. In case of more than one possible coalition, party j decides which one is formed. If all parties have moved and no winning coalition has been formed, the Head of State chooses a first mover again. If after T rounds no agreement is reached, no private benefits are distributed and the policy outcome is the one preferred by the median voter.
Proposition 2 in Morelli (1999) states that, if parties care only about private benefits ($\beta = 0$) and there exists a unique homogeneous representation (ω, q), then a winning coalition $S^* (\rho)$ is formed and each party in $S^* (\rho)$ receives $\frac{\omega_i}{q}$. This is not the case in general, as the following counterexample involving a proper and strong homogeneous game (and thus a game with a unique homogeneous representation) illustrates.[1]

Proposition 1 *Suppose there are five parties with* $3, 2, 2, 1$ *and* 1 *votes respectively, and the quota is* 5*. If* $\beta = 0$ *and* $T = 1$*, the first mover gets the whole surplus in any subgame perfect equilibrium.*

Proof. See appendix. ■
The intuition for this result is as follows. If party 1 (3 votes) is chosen to move first, it can choose the order (14523), associated to votes (31122), and get all the surplus. To see this, suppose party 1 demands the whole surplus. Party 4 (with 1 vote) can either go along with party 1 and demand 0, or make a positive demand and try to form an alternative coalition with parties 2 and 3. However, given the order of moves, any positive demand can be undercut by party 5 and will result in coalition $\{2, 3, 5\}$. Thus, party 4 may as well demand 0 after observing a demand of 1 by party 1 (indeed, it *must* demand 0 in order for party 1 to have a best response). Given that party 4 demands 0, party 5 is helpless as well: a positive demand would result in parties 2 and 3 forming a coalition with 4. If party 2 (2 votes) is chosen to move first, it can choose the order (23451), associated to votes (22113), and get all the surplus. Suppose party 2

demands the whole surplus. This prevents party 3 from getting a positive payoff in any coalition that includes party 2. The only other alternative, a coalition with party 1, would always be sabotaged by the two small parties. Thus, party 3 may as well demand 0 after observing a demand of 1 by party 2. But this in turn prevents party 4 from getting a positive payoff in a coalition with party 1: party 1 will always prefer to form a coalition with party 3. Thus, party 4 may as well form a coalition and get 0.

If party 4 (1 vote) moves first, it can choose the order (42315), associated to votes (12231), and get all the surplus. Suppose party 4 demands the whole surplus. Then party 2 may as well demand 0: any positive demand can be undercut by party 3 and would lead to a coalition of parties 1 and 3. Given that party 2 demands 0, party 3 cannot get a positive payoff: a positive demand would result in party 1 forming coalition $\{1, 2\}$.

The cases for parties 3 and 5 are identical to 2 and 4, respectively.

By committing itself to a demand and sequencing the order of moves of the other parties in a suitable way, the first mover exploits the demand competition between the other parties in its favor. This is the case even though the first mover has no monopoly proposal power and the rules of the game allow the first mover to be excluded from the government.

Morelli's argument for proportionality was that a higher than proportional demand would trigger the reaction of an alternative minimal winning coalition that can divide payoffs proportionally: any party that deviates can be replaced without changing the payoff shares for the others (see Morelli 1999 p. 818). Indeed, such a minimal winning coalition always exists, but by choosing the order of moves the first mover can ensure that the members of the coalition cannot coordinate on forming an alternative government. For example, in the order (14523), after party 1 demands the whole surplus there exists one minimal winning coalition that could exclude 1 and divide payoffs proportionally: coalition $\{2, 3, 4\}$. However, the members of this coalition do not move consecutively and party 4 knows that any attempt to induce coalition $\{2,3,4\}$ will be sabotaged by party 5. There is an alternative minimal winning coalition, $\{2,3,5\}$, whose members move consecutively, but they do not move immediately after party 1: any attempt of party 5 to form $\{2,3,5\}$ will be forestalled by party 4 setting a sufficiently low demand.

Concluding remarks

Proposition 1 shows that proportionality may not hold if the leading party is allowed to choose the order of moves. However, in a companion paper (Montero and Vidal-Puga 2006) we prove the following result: if the voting game is proper, strong and homogeneous and parties must move by decreasing weight as assumed by Austen-Smith and Banks (1988), then there is a unique equilibrium payoff distribution, where the minimal winning coalition S^* comprised of the parties that move first is formed and payoffs for members of S^* are proportional to their number of votes in the homogeneous representation. In the game $[5;3,2,2,1,1]$, coalition $\{1,2\}$ is formed with party 1 receiving $\frac{3}{5}$ and party 2 receiving $\frac{2}{5}$. The proposer has no disproportionate advantage and the result is obtained without resorting to equilibrium refinements.

One may ask whether the Head of State can achieve proportional payoffs for an arbitrary minimal winning coalition by choosing the order of moves appropriately. The answer is negative: for the game $[5;3,2,2,1,1]$, there is no order of moves for which coalition $\{1,4,5\}$ forms with a proportional payoff division. There are three types of possible orders for which the parties in this coalition move first: (31122), (13122) and (11322). It can be shown that the first mover gets the whole budget in order (31122), whereas in the other two orders the first mover gets half of the budget.

From a normative point of view, proportional payoffs are intuitive in the absence of policy preferences and are predicted by many solution concepts in cooperative game theory (see Morelli and Montero 2003 for a discussion). The empirical evidence is consistent with this prediction, at least for parties other than the formateur (see Ansolabehere et al. 2005 and the references therein). We have shown that Morelli's appealing results regarding proportionality in demand bargaining do not hold generally. However, one should keep in mind that they hold for some important types of games, including symmetric and apex games.

Appendix: Proof of Proposition 1

We will denote $\min(a,b)$ by $a \wedge b$ and $\max(a,b)$ by $a \vee b$. The party chosen to move first can be of three different types depending on whether it controls 1, 2 or 3 votes. We will examine each case in turn. Given the order chosen by the first mover, we divide the game in stages (each stage corresponding to one party moving) and, starting by the last stage, construct one equilibrium in which parties use certain tie-breaking rules. We then show that the equilibrium outcome is unique.

CASE 1: Party 1 (3 votes) is the first mover. It can choose the order (14523), associated with votes (31122), and get the whole surplus.

Stage 5. Party 3 (2 votes) faces a vector of demands (d_1, d_2, d_4, d_5) and a vector of policies $(\theta_1, \theta_2, \theta_4, \theta_5)$. It has four choices[2]:

a) Form coalition $\{1,3\}$ and get $1 - d_1$.

b) If $\theta_2 = \theta_4$, it can also form $\{2,3,4\}$ and get $1 - d_2 - d_4$.

c) If $\theta_2 = \theta_5$, it can also form $\{2,3,5\}$ and get $1 - d_2 - d_5$.

d) Form no coalition and get 0.

Denote the cheapest of the two parties with one vote by m (formally, $m \in \arg\min_{i \in \{4,5\}} d_i$). Suppose $\theta_2 = \theta_m$ and forming a coalition is optimal. Then 3 will form $\{1,3\}$ if $1 - d_1 > 1 - d_2 - d_m$, and $\{2,3,m\}$ in the reverse case. Ties are solved in favor of $\{2,3,m\}$, and, if $d_4 = d_5$, of $\{2,3,5\}$.

Stage 4. Party 2 has two options: to form coalition $\{1,2\}$, or to set $\theta_2 = \theta_m$ and make a demand that will induce party 3 to form $\{2,3,m\}$.

The maximum demand 2 can make and still induce $\{2, 3, m\}$ is $d_2 = d_1 - d_m$. If $d_1 - d_m \geq 1 - d_1$, party 2 sets $\theta_2 = \theta_m$ and makes this demand; otherwise it sets $\theta_2 = \theta_1$ and forms $\{1, 2\}$. Ties are solved in favor of inducing $\{2, 3, m\}$, and, if $d_4 = d_5$, of inducing $\{2, 3, 5\}$.
If party 5 wants to induce $\{2, 3, 5\}$ it must set $d_5 \leq d_4$, so that $m = 5$. If $d_1 - d_4 \geq 1 - d_1$ setting $d_5 = d_4$ will do; otherwise $d_5 = 2d_1 - 1$. Thus, the maximum d_5 that induces $\{2, 3, 5\}$ is $d_5 = d_4 \wedge (2d_1 - 1)$. Note that, if $d_1 < \frac{1}{2}$, party 5 cannot induce $\{2, 3, 5\}$: 2 will form $\{1, 2\}$ for any $d_5 \geq 0$.
Stage 3. Party 5 faces (d_1, d_4) and (θ_1, θ_4). If $\theta_1 = \theta_4$, party 5 compares $1 - d_1 - d_4$ and $d_4 \wedge (2d_1 - 1)$. Then party 5 forms $\{1, 4, 5\}$ if $1 - d_1 - d_4 \geq d_4 \wedge (2d_1 - 1)$. If $\theta_4 \neq \theta_1$, the only possible coalition for party 5 is $\{2, 3, 5\}$, or no coalition if $\{2, 3, 5\}$ cannot be induced by any $d_5 \geq 0$. In either case party 4 is excluded, so there is no reason for 4 to set $\theta_4 \neq \theta_1$. The maximum value of d_4 that still induces $\{1, 4, 5\}$ depends on the size of d_1. For a relatively large d_1 ($d_1 \geq \frac{3}{5}$) the critical value is $d_4 = \frac{1-d_1}{2}$.
Stage 2. The only alternative for party 4 is to induce coalition $\{1, 4, 5\}$. If $\frac{3}{5} < d_1 < 1$, party 4 sets $\theta_4 = \theta_1$ and $d_4 = \frac{1-d_1}{2}$; a higher demand would result in party 5 inducing coalition $\{2, 3, 5\}$. If $d_1 = 1$ any demand is optimal, and ties are solved in favor of $\theta_4 = \theta_1$ and $d_4 = \frac{1-d_1}{2}$.
Stage 1. Party 1 sets $d_1 = 1$ together with an arbitrary θ_1.
CASE 2: Party 2 (2 votes) is the first mover. It can choose the order (23451), associated with votes (22113), and get all the surplus.
Stage 5. Party 1 (3 votes) faces a vector of demands (d_2, d_3, d_4, d_5) and a vector of policies $(\theta_2, \theta_3, \theta_4, \theta_5)$. It has four choices:
a) Form coalition $\{1, 2\}$ and get $1 - d_2$.
b) Form coalition $\{1, 3\}$ and get $1 - d_3$.
c) If $\theta_4 = \theta_5$, it can also form $\{1, 4, 5\}$ and get $1 - d_4 - d_5$.
d) Form no coalition and get 0.
Suppose $\theta_4 = \theta_5$ and forming some coalition is optimal. Party 1 forms $\{1, 4, 5\}$ if $1 - d_4 - d_5 \geq 1 - (d_2 \wedge d_3)$. Then the maximum demand party 5

can make and still induce coalition $\{1,4,5\}$ is $d_5 = (d_2 \wedge d_3) - d_4$.

Stage 4. If $\theta_2 = \theta_3$, party 5 can form $\{2,3,5\}$ and get $1 - d_2 - d_3$. Alternatively, it can induce $\{1,4,5\}$ by setting $\theta_5 = \theta_4$ and $d_5 = (d_2 \wedge d_3) - d_4$. It does so if $(d_2 \wedge d_3) - d_4 \geq 1 - d_2 - d_3$, or equivalently $d_4 \leq d_2 + d_3 + (d_2 \wedge d_3) - 1$.

Stage 3. Party 4 can induce coalition $\{1,4,5\}$ by setting $d_4 = d_2 + d_3 + (d_2 \wedge d_3) - 1$. If $\theta_2 = \theta_3$, it can also form coalition $\{2,3,4\}$. It forms $\{2,3,4\}$ if $1 - d_2 - d_3 \geq d_2 + d_3 + (d_2 \wedge d_3) - 1$.

Stage 2. If $d_2 \geq \frac{2}{5}$, party 3 can induce coalition $\{2,3,4\}$ by setting $\theta_3 = \theta_2$ and $d_3 = \frac{2-2d_2}{3}$. A larger demand or/and setting $\theta_3 \neq \theta_2$ would result in party 4 inducing $\{1,4,5\}$.

Stage 1. Party 2 sets $d_2 = 1$ together with an arbitrary value of θ_2.

CASE 3: Party 4 **(**1 **vote) is the first mover.** It can choose the order (42315), associated with votes (12231), and get all the surplus.

Stage 5. If $\theta_1 = \theta_4$, party 5 can form $\{1,4,5\}$ and get $1 - d_1 - d_4$. If $\theta_2 = \theta_3$, it can form $\{2,3,5\}$ and get $1 - d_2 - d_3$. It can also form no coalition and get 0.

Suppose $\theta_1 = \theta_4$ and $\theta_2 = \theta_3$. If $1 - d_1 - d_4 \geq (1 - d_2 - d_3) \vee 0$, party 5 forms $\{1,4,5\}$. If $\theta_2 \neq \theta_3$, the relevant condition is $1 - d_1 - d_4 \geq 0$. Thus the critical value of d_1 is (weakly) higher if $\theta_2 \neq \theta_3$: because party 5 cannot form $\{2,3,5\}$, party 1 can get a better deal in coalition $\{1,4,5\}$.

Stage 4. Party 1 can form the cheapest of coalitions $\{1,2\}$ and $\{1,3\}$ and get $1 - (d_2 \wedge d_3)$, or induce $\{1,4,5\}$ by setting $\theta_1 = \theta_4$ and $d_1 = 1 - d_4$ (if $d_2 + d_3 > 1$ or $\theta_2 \neq \theta_3$) or $d_1 = d_2 + d_3 - d_4$ (if $d_2 + d_3 \leq 1$ and $\theta_2 = \theta_3$). Thus for $\theta_2 = \theta_3$, party 1 forms a two-party coalition if $1 - (d_2 \wedge d_3) \geq (d_2 + d_3 - d_4) \wedge (1 - d_4)$.

Stage 3. Party 3 can form $\{2,3,4\}$ (provided $\theta_2 = \theta_4$) or induce $\{1,3\}$. In order to induce $\{1,3\}$, party 3 must set $d_3 \leq d_2$, and can do no better than setting $\theta_2 = \theta_3$. What is the highest value of d_3 that still induces coalition $\{1,3\}$? If $d_4 > \frac{1}{2}$ there are two possible cases:

If $d_2 \leq \frac{1}{2}$, $d_3 = d_2$ will induce $\{1,3\}$. If party 3 sets $d_3 = d_2$, we have $d_3 + d_2 \leq 1$, thus the relevant inequality for party 1 is $1 - d_2 \geq 2d_2 - d_4$, or equivalently $d_2 \leq \frac{1+d_4}{3}$. This is satisfied for $d_4 > \frac{1}{2}$. Since party 3 can induce $\{1,3\}$ by setting $d_3 = d_2$, party 3 will form $\{2,3,4\}$ if $1 - d_2 - d_4 \geq d_2$, or equivalently $d_2 \leq \frac{1-d_4}{2}$.

If $d_2 > \frac{1}{2}$, $\{2,3,4\}$ leads to a negative payoff. Thus, party 3 always induces $\{1,3\}$. This is achieved by setting $d_3 = d_2 \wedge d_4$.

Stage 2. If $\frac{1}{2} < d_4 \leq 1$, it is a best response for party 2 to set $\theta_2 = \theta_4$ (setting $\theta_2 \neq \theta_4$ would result in coalition $\{1,3\}$) and $d_2 = \frac{1-d_4}{2}$.

Stage 1. Party 4 sets $d_4 = 1$ and an arbitrary θ_4.

UNIQUENESS. We now show the uniqueness of equilibrium payoffs for case 1 (cases 2 and 3 are analogous). Essentially we will show that ties *must* be solved in favor of coalition $\{1,4,5\}$ when $d_1 = 1$ in order for an equilibrium to exist. First we will show that $d_1 = 1 - \epsilon$ leads to coalition $\{1,4,5\}$ for any small $\epsilon > 0$ in any subgame perfect equilibrium. Having established this, it follows that parties 4 and 5 must solve ties in favor of party 1 if $d_1 = 1$; otherwise party 1 would have no best response.

Let $d_1 = 1 - \epsilon$. If party 4 sets $d_4 < \frac{1-d_1}{2}$, party 5's unique best response is to form coalition $\{1,4,5\}$ regardless of the tie-breaking rules used by 2 and 3. It follows that 5 must form $\{1,4,5\}$ for $d_4 = \frac{1-d_1}{2}$ as well: otherwise 4 would not have a best response after observing $d_1 = 1 - \epsilon$.

Since $\{1,4,5\}$ must form for $d_1 = 1 - \epsilon$, it must also form for $d_1 = 1$. Notice that in this case any value of d_4 is optimal regardless of 5's tie-breaking rule; however, 4 and 5 must solve ties in favor of 1.

Notes

[1]Morelli's proposition 2 holds for symmetric and apex games. A game is symmetric if there is an equivalent representation in which each party has one vote. An apex game is equivalent to $[n-1; n-2, 1, ..., 1]$.

[2]In fact, it may have more choices (e.g. forming coalition $\{2, 3, 4, 5\}$) but all of them are dominated by at least one of these four. Without loss of generality we will not consider dominated choices. We will also exclude some situations that do not arise in equilibrium (e.g. demands so high that all coalitions are unfeasible).

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