Panel Stationarity Tests with Cross-sectional Dependence

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ABSTRACT

We present a test of the null hypothesis of stationarity against unit root alternatives for panel data that allows for arbitrary cross-sectional dependence. We treat the short run time series dynamics non-parametrically and thus avoid the need to fit separate models for the individual series. The statistic is simple to compute and is asymptotically normally distributed, even in the presence of a wide range of deterministic components. Taken together, these features provide a generally applicable solution to the problem of testing for stationarity versus unit roots in macro-panel based data. The test is applied to assess the validity of the purchasing power parity hypothesis and finds significant evidence against the hypothesis being true.

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1 Introduction

Relatively long time series of many core macroeconomic variables are now available for the majority of developed economies and the use of panel data unit root, or stationarity, tests as a means of empirically validating various important macroeconomic theories has become a rapid growth area of applied econometric research in recent years. For example, panel tests have been used to assess the evidence for the hypotheses of purchasing power parity, for convergence of growth rates, for mean reversion of inflation rates and for the real interest rate parity hypothesis.

These tests attempt to exploit the potential power gains that are offered by analyzing a time series panel as opposed to individual series and, as such, they have the potential to provide more compelling evidence for, or against, certain models of economic behaviour. Recent tests have been proposed by, *inter alia*, O'Connell (1998), Maddala and Wu (1999), Hadri (2000), Choi (2001, 2002), Chang and Song (2002), Levin, Lin and Chu (2002), Chang (2003) and Im, Pesaran and Shin (2003).

The two major factors that any panel test needs to be able to address are cross-sectional dependence and time series dynamics, if reliable inference is to be made in practical situations. Cross-sectional dependencies are likely to be the rule rather than the exception in many empirical settings. For example, in studying cross-country data, dependence is very likely to arise due to the existence of strong inter-economy linkages. The tests of Hadri (2000), Choi (2001), Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) all assume independence across the panel and their size properties are uncertain outside of this rather unrealistic assumption.[2] The test of O'Connell (1998) allows for cross-sectional dependence, but this is restricted to the innovation term driving an assumed finite order AR process in their models. Choi (2002) permits cross-sectional dependence but only after imposing a common additive error component across the panel. The testing approach adopted by Chang and Song (2002) provides, at least in theory, the most general treatment of the problem of cross-sectional dependence up until now, but their procedure relies on user-supplied parameters, whose values are a function of the dependence structure itself, which rather limits its practical appeal. Maddala and Wu (1999) and Chang (2003) approach the problem indirectly, relying on bootstrap procedures but the underlying tests are not pivotal. Regarding time series dynamics, with the exception of the test of Hadri (2000), all of these tests rely on fitting an appropriately specified time series regression model to each individual series in the panel (a tedious and error prone undertaking unless the cross-sectional dimension is relatively small). For tests that allow cross-sectional dependence, this is a doubly vital requirement, as any notion of these tests' robustness to cross-sectional dependence is intimately reliant on the correct modelling of the time series dynamics.

It would seem, then, that none of the extant tests offers a totally satisfactory solution to the problem of testing for unit roots, or stationarity, when the cross-sectional dependence structure and time series dynamics are both unknown. In contrast, the new stationarity test statistic we suggest in this paper is constructed so as to overcome both these problems. We allow for arbitrary unknown cross-sectional dependence between the series in the panel, where the series may be contemporaneously or cross-serially dependent. We also permit a wide range of heterogeneous stationary time series dynamics, which includes the conventional $ARMA$ class.

Our statistic is based on a vector version of the stationarity test of Harris, McCabe and Leybourne (2003) (rather than a $KPSS$-type stationarity test as in Hadri (2000)). The statistic is, in essence, the sum of the lag-k sample autocovariances across the panel, suitably studentized, where we allow k to be a simple increasing function of the time dimension. By controlling k in such a way, we remove any need to explicitly model the time series dynamics of each series in the

[2] O'Connell (1998) shows that the test of Levin, Lin and Chu (2002) can suffer severe size distortions if applied to panels where independence does not hold.

panel, even though their time series dynamics may be quite heterogeneous. At the same time, the studentization automatically robustifies the statistic to the presence of any form of cross-sectional dependence. Our statistic is simple to construct and, conveniently, possesses a limiting null distribution which is standard normal under quite general linear process assumptions.[3] Asymptotic normality also holds when the statistic is calculated using residuals from deterministic regression models fitted to each series. These may include polynomial trends or even structural breaks and there is no requirement that the same deterministic model be fitted to each series. As such, the test can be applied across a range of empirically relevant modelling situations without reference to model-dependent null critical values, or the need to compute bootstrap critical values.

The plan of the paper is as follows. In the next section we motivate the statistic, showing how it can be used to distinguish between stationarity and unit roots in the panel context. In Section 3, we demonstrate asymptotic standard normality of the test under the stationary null hypothesis, and show consistency under the unit root alternative. Section 4 reports the results of a number of Monte Carlo experiments to gauge the empirical size and power of the test. The results are very encouraging. In particular, the robustness of the test's size to different patterns of cross-sectional dependence and time series dynamics stands out as a prominent characteristic. Finally, Section 5 demonstrates an empirical application of our test in the context of testing for the null hypothesis of purchasing power parity in a panel of U.S. Dollar real exchange rates.

2 A Panel Test of Stationarity

By way of motivation of our statistic, consider for the moment a single series, y_t, of T observations, generated by an $AR(1)$ process

$$y_t = \phi y_{t-1} + \varepsilon_t, \qquad \phi \leq 1$$

where the disturbance term ε_t is white noise with variance σ^2. Suppose that we wish to assess if this series is stationary or possesses a unit root. An obvious test statistic is the lag-1 sample covariance (suitably studentized):

$$C_1 = T^{-1/2} \sum_{t=2}^{T} y_t y_{t-1}.$$

In the case of testing the null of a unit root, the (studentized) lag-1 sample covariance is equivalent to the usual Dickey-Fuller procedure.

In many applications it is more natural to test the composite null hypothesis of stationarity, $\phi < 1$, against the unit root alternative $\phi = 1$.[4] The problem with using C_1 under the stationary null is that since

$$E\left[C_1\right] \simeq T^{1/2} \sigma^2 \phi / (1 - \phi^2),$$

its distribution depends on ϕ. An alternative is to consider instead the lag-k autocovariance

$$C_k = T^{-1/2} \sum_{t=k+1}^{T} y_t y_{t-k}$$

[3] Our asymptotics are based on a fixed cross-section dimension, and passing the time series dimension to infinity. For many macroeconomic applications, the assumption of a fixed cross-section dimension would appear reasonable, however.

[4] For example, the purchasing power parity hypothesis would imply stationarity of real exchange rates.

where

$$E\left[C_k\right] \simeq T^{1/2}\sigma^2\phi^k/(1-\phi^2).$$

Now suppose that we set $k = k(T) = o(T)$ so that k increases simultaneously with T. It is then clear that $E\left[C_k\right]$ will converge to zero, eliminating any dependence of $E\left[C_k\right]$ on the parameter ϕ. Suitably studentized then, C_k will have an asymptotic normal distribution free of unwanted parameters (see Harris, McCabe and Leybourne (2003)). Thus C_k may be used to test the null of stationarity and, since $E\left[C_k\right] \simeq \sigma^2 T^{3/2}$ when $\phi = 1$, the test may be expected to be consistent under the unit root alternative.

Consider now a panel of N time series y_{it}, each of T observations, generated by $AR(1)$ processes

$$\begin{aligned} y_{it} &= \phi_i y_{i,t-1} + \varepsilon_{it}, \qquad \phi_i \leq 1 \\ i &= 1, 2, ..., N \ , \quad t = 1, 2, ..., T \end{aligned} \tag{1}$$

where the disturbance term ε_{it} is (temporarily) white noise with variance σ_i^2. Throughout, we consider N to be fixed and we shall let T grow in our limit theory.[5] We wish to test the null hypothesis of joint stationarity

$$H_0 : \phi_i < 1 \;\; \text{for all } i$$

against the unit root alternative

$$H_1 : \phi_i = 1 \text{ for at least one } i.$$

The above analysis, together with the corresponding literature on panel Dickey-Fuller tests, suggests that

$$S_k = \sum_{i=1}^{N} C_{i,k}, \quad C_{i,k} = T^{-1/2} \sum_{t=k+1}^{T} y_{it} y_{i,t-k}$$

could be used as a test statistic. Under H_0, it is easily seen that

$$E\left[S_k\right] \simeq T^{1/2} \sum_{i=1}^{N} \sigma_i^2 \phi_i^k/(1-\phi_i^2)$$

and, setting k as before, $E\left[S_k\right] \to 0$ as $T \to \infty$ since N is fixed. This eliminates the dependence of $E\left[S_k\right]$ on *all* of the ϕ_i simultaneously. As shown below, when suitably studentized S_k will have an asymptotic normal distribution free of unwanted parameters under H_0. Thus S_k may be used to test the null of stationarity in the panel. Under H_1 where, without loss of generality, we suppose only the first $M \leq N$ of the $\phi_i = 1$,

$$E\left[S_k\right] \simeq T^{3/2} \sum_{i=1}^{M} \sigma_i^2 + T^{1/2} \sum_{i=M+1}^{N} \sigma_i^2 \phi_i^k/(1-\phi_i^2),$$

so that the leading right hand side term once more suggests that the test should be consistent.

It proves convenient, for what follows later, to define the generic long-run variance estimator of a sequence of variables $a_1, ..., a_T$ as

$$\hat{\omega}\{a_t\}^2 = \hat{\gamma}_0\{a_t\} + 2\sum_{j=1}^{l}\left(1 - \frac{j}{l}\right)\hat{\gamma}_j\{a_t\}, \quad \hat{\gamma}_j\{a_t\} = T^{-1}\sum_{t=j+1}^{T} a_t a_{t-j}.$$

[5] It is possible to allows the number of observations to vary with the individual time series involved but we use a single T for notational convenience.

In practical situations it is generally the case that some deterministic function, such as a constant or linear trend, will be fitted to the y_{it}. Thus, in place of (1), we will consider the model given by

$$\begin{aligned} y_{it} &= \beta_i' x_{it} + e_{it}. \\ e_{it} &= \phi_i e_{i,t-1} + \varepsilon_{it}, \qquad \phi_i \le 1 \\ i &= 1, 2, ..., N \ , \quad t = 1, 2, ..., T \end{aligned} \tag{2}$$

and let $\hat{e}_{it}$ denote an OLS residual from the regression (2). In addition, it is usually considered a desirable property that a statistic be invariant to relative rescaling and rebasing of indices. Hence, in what follows, in place of y_{it} in the definition of S_k we will use instead the standardized residuals $\tilde{e}_{it} = \hat{e}_{it}/\hat{\gamma}_0\{\hat{e}_{it}\}^{1/2}$. The next section derives the limiting distribution of (studentized) S_k under H_0, when ε_{it} is a linear process. We also demonstrate test consistency under H_1.

3 Distribution Theory

We make the following assumptions regarding time series dynamics of $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}, ..., \varepsilon_{Nt}]'$.[6]

Assumption 1
Let $\boldsymbol{\varepsilon}_t$ be an $N \times 1$ vector of fixed dimension generated by the linear process

$$\boldsymbol{\varepsilon}_t = \mathbf{A}(L)\,\boldsymbol{\xi}_t,$$

where $\mathbf{A}(L) = \sum_{j=0}^{\infty} \mathbf{A}_j L^j$ and $\mathbf{A}_j$ $(N \times N)$ and $\boldsymbol{\xi}_t$ $(N \times 1)$ satisfy
(i) $\mathbf{A}_0 = \mathbf{I}_N$,
(ii) $\sum_{j=0}^{\infty} j^2 \left\|\mathbf{A}_j\right\|^2 < \infty$,
(iii) $\mathbf{A}(1)$ *has full rank,*
(iv) $\{\boldsymbol{\xi}_t, \Im_t\}$ *is a martingale difference sequence where* $\Im_t = \sigma\left\{\boldsymbol{\xi}_{t-j}, \, j \ge 0\right\}$,
(v) $E\left(\boldsymbol{\xi}_t \boldsymbol{\xi}_t' \middle| \Im_{t-1}\right) = \Sigma$ *a.s., for all* t,
(vi) $\left\| E\left(\boldsymbol{\xi}_t \boldsymbol{\xi}_t' \otimes \boldsymbol{\xi}_t \boldsymbol{\xi}_t' \middle| \Im_{t-1}\right)\right\| < \kappa < \infty$ a.s. for all t and some fixed constant κ.

This assumption permits arbitrary cross-sectional dependence between the series in the panel and the series may also be contemporaneously or cross-serially dependent. In addition, it allows for heterogeneity across the panel. The series may exhibit a range of individual temporal dependence structures, including those of stationary $ARMA$ processes. The next assumption defines the class of regression deterministics that are catered for in x_{it}.

Assumption 2
Suppose that $\hat{e}_{it}$ are the OLS residuals

$$\hat{e}_{it} = y_{it} - \left(\hat{\beta}_i - \beta_i\right)' x_{it},$$

and let x_{it} denote a vector of deterministic regressors with the properties

$$\mathbf{D}_{iT}^{-1} x_{i[T\tau]} \to X_i(\tau) < \infty,$$

uniformly in τ, for some $\mathbf{D}_{iT}$, and

$$T^{-1} \sum_{t=1}^{T} \mathbf{D}_{iT}^{-1} x_{it} x_{it}' \mathbf{D}_{iT}^{-1\prime} \to \int_0^1 X_i(\tau)\, X_i(\tau)'\, d\tau > 0.$$

[6]For any matrix A let $\|A\| = \sqrt{\operatorname{tr}(A'A)}$.

Assumption 2 is quite general, allowing for a wide range of deterministic regression functions including polynomial trends and dummy variable/structural break models. In the current context, leading examples are a constant term: $x_{it} = 1$, $\mathbf{D}_{iT} = 1$ and $X_i(\tau) = 1$, or a constant and linear trend: $x_{it} = [1, t']$, then $\mathbf{D}_{iT} = diag[1, T]$ and $X_i(\tau) = [1, \tau]'$. The next condition controls the rate of increase of k and the lag truncation parameter l.

Condition 3.
Let $k \to \infty$ and $l \to \infty$ as $T \to \infty$ such that $k = O(T^{1/2})$ and $l = o(k)$.

We have the following Theorem, the proof of which is given in the Appendix.

Theorem 1 *If Assumption 1, 2 and Condition 3 hold then, under H_0,*

$$\begin{aligned} \hat{S} &= \hat{\omega}\{a_t\}^{-1} S_k \\ &\Rightarrow N[0,1] \end{aligned}$$

where

$$S_k = \sum_{i=1}^{N} C_{i,k}, \quad C_{i,k} = T^{-1/2} \sum_{t=k+1}^{T} \tilde{e}_{it}\tilde{e}_{i,t-k},$$

and $a_t = \sum_{i=1}^{N} \tilde{e}_{it}\tilde{e}_{i,t-k}$.

The role of Condition 3 is to remove the effects of temporal dependence in individual series from the asymptotic distribution of $\hat{S}$. Hence, there is no need to individually model the temporal dependence structure of each series. The long-run variance estimator $\hat{\omega}\{a_t\}^2$ essentially removes the effects of cross-sectional dependence between series as it correctly estimates the variance of S_k. Again, there is no need to model the cross-sectional dependence structure.

The next Theorem establishes consistency of $\hat{S}$.

Theorem 2 *If Assumption 1, 2 and Condition 3 hold then, under H_1, $\hat{S}$ diverges to $+\infty$.*

This last result shows that an upper tail test is appropriate for testing H_0 against H_1.

When dealing with a small number of series, the $N(0, 1)$ asymptotic null distribution of $\hat{S}$ often proves to be an adequate approximation for its finite sample distribution. However, if the panel dimension is not relatively small, individual finite sample biases occurring in the distributions of the $C_{i,k}$, that arise from the estimation of regression models, combine in the construction of the aggregate numerator $S_k = \sum_{i=1}^{N} C_{i,k}$ and can significantly effect the finite sample null distribution of $\hat{S}$.

To illustrate the source of the bias, we consider the leading case of a constant and a single deterministic regressor, $x_{2,it}$ fitted to y_{it}, denoting the OLS residual $\hat{e}_{it} = y_{it} - \hat{\beta}_{1,i} - \hat{\beta}_{2,i}x_{2,it}$. Let $\hat{y}_{it}$ represent a fitted value from this regression. Also, purely for transparency, we temporarily assume that the $C_{i,k}$ are calculated from unstandardized $\hat{e}_{it}$. So, assume H_0 is true and write

$$\begin{aligned} C_{i,k} &= T^{-1/2} \sum_{t=k+1}^{T} \hat{e}_{it}\hat{e}_{i,t-k} \\ &= T^{-1/2} \sum_{t=k+1}^{T} \hat{e}_{it}\left(y_{i,t-k} - \hat{y}_{it-k}\right) \\ &= T^{-1/2} \sum_{t=k+1}^{T} \hat{e}_{it} y_{i,t-k}. \end{aligned}$$

After a little manipulation, and approximating the sums in $y_{i,t-k}$ by those in y_{it}, we find

$$C_{i,k} \approx T^{-1/2} \sum_{t=k+1}^{T} y_{it} y_{i,t-k} \tag{3}$$

$$-T^{-1/2} \left[T^{-1} \left(\sum_{t=k+1}^{T} y_{it} \right)^2 + q_T^{-1} \left(\sum_{t=k+1}^{T} (x_{2,it} - \bar{x}_{2,i}) \, y_{it} \right)^2 \right] \tag{4}$$

where $q_T = \sum_{t=k+1}^{T} (x_{2,it} - \bar{x}_{2,i})^2$. Since the $\hat{e}_{it}$ are invariant to $\beta_{1,i}$ and $\beta_{2,i}$ and we may, for the purposes of the present argument, assume they are zero without loss of generality. Hence, the ideal statistic to use is (3) above; in practice though we must compute the full expression. Under H_0, the two terms in square brackets in (4) are both $O_p(1)$ and so the whole term is $O_p(T^{-1/2})$. It is the term (4) that induces a negative finite sample bias into each individual statistic, and the amplification of this problem is obvious when we subsequently consider $S_k = \sum_{i=1}^{N} C_{i,k}$. Since we are conducting upper tail tests, *ceteris paribus*, we would conjecture that the effect of this is to *reduce* the size of the test.

It is, however, possible to produce a finite sample correction for $C_{i,k}$ by using the expected value of (4) term and subtracting it from $C_{i,k}$. Notice that $E\left[T^{-1}\left(\sum_{t=k+1}^{T} y_{it}\right)^2\right]$ is the long run variance of $T^{-1/2}\sum_{t=k+1}^{T} y_{it}$ and that this expectation may be estimated by $\hat{\omega}\{y_{it}\}^2$. Similarly, the expectation of the second term in (4) may be estimated using $\hat{\psi}\{y_{it}\,(x_{2,it} - \bar{x}_{2,i})\}^2 = T q_T^{-1} \hat{\omega}\{y_{it}\,(x_{2,it} - \bar{x}_{2,i})\}^2$.

In practice, then, we suggest that, in the case of a constant and a single deterministic regressor, when constructing the numerator term of $\hat{S}$, we replace S_k, with S_k^* defined by

$$S_k^* = \sum_{i=1}^{N} C_{i,k}^*$$

$$C_{i,k}^* = T^{-1/2} \sum_{t=k+1}^{T} \tilde{e}_{it} \tilde{e}_{i,t-k} + T^{-1/2} \left[\hat{\omega}\{\tilde{e}_{it}\}^2 + \hat{\psi}\{\tilde{e}_{it}\,(x_{2,it} - \bar{x}_{2,i})\}^2 \right].$$

The following corollary demonstrates that the use of the bias correction in the case of a constant and deterministic linear trend, and shows that it has no asymptotic effect on the null distribution or consistency property of the test. Note in this case $q_T = \sum_{t=k+1}^{T} (t - \bar{t})^2 = O(T^3)$.

Corollary 1 *If Assumption 1, Assumption 2 with $x_{it} = (1,t)'$ and Condition 3 hold then i) under H_0, $\hat{S} = \hat{\omega}\{a_t\}^{-1} S_k^* \Rightarrow N[0,1]$ where $a_t = \sum_{i=1}^{N} \tilde{e}_{it}\tilde{e}_{i,t-k}$ and ii) under H_1, $\hat{S}$ diverges to $+\infty$*

If a constant term alone is fitted, we omit the term $\hat{\psi}\{\tilde{e}_{it}\,(x_{2,it} - \bar{x}_{2,i})\}^2$ from the bias correction. Part of the analysis of the next section is to assess the effectiveness of such a correction under H_0. In a similar fashion, we may also produce bespoke finite sample bias corrections for other, more complicated, deterministic regression models.

4 Finite Sample Simulations

In this section we investigate, via Monte Carlo simulation, the finite sample size and power properties of the test $\hat{S}$. We consider a range of differing scenarios for cross-sectional dependence and time series dynamics for $N = 3, 5, 10, 20, 30$ and $T = 75, 150, 300$. Throughout, we calculate $\hat{S}$ fitting a constant term (i.e. $\hat{e}_{it}$ is a deviation from a mean) and our default approach employs the corresponding finite sample bias correction of the previous section. As regards the user-supplied parameters, based on experimentation we set $k = (3T)^{1/2}$ and $l = 12(T/100)^{1/4}$, both rounded to the lowest integer, and all simulations are conducted using 10,000 replications. The data are generated from the DGP (2) for y_{it}, with $\beta_i = 0$. The disturbance term ε_{it} is generated by an $MA(1)$ process $\varepsilon_{it} = v_{it} - \theta_i v_{it-1}$, with each v_{it} being standard normal white noise. We define $\rho_{ij} = E[v_{it}v_{jt}]$, the contemporaneous correlation between v_{it} and v_{jt}, $i, j = 1, ..., N$, $i \neq j$.

Our first simulations concern empirical size properties. These are reported in Table 1, where table entries represent empirical rejection frequencies at asymptotic 0.05-level null critical values of an upper tail test (i.e. for $\hat{S} > 1.65$). As a benchmark, in Table 1(a) we set $\rho_{ij} = 0$. That is, the y_{it} are uncorrelated processes. In the first column of this table, we set $\phi_i = \theta_i = 0$, such that the y_{it} are uncorrelated white noise processes. The test $\hat{S}$ is seen to have close to nominal 0.05 size for each of the values of N and T we consider. The next four columns generates the y_{it} variously as uncorrelated stationary $AR(1)$ or $MA(1)$ processes, but where each of the N series has identical time series dynamics. The final column of Table 1(a) generates the y_{it} as stationary $ARMA(1, 1)$ processes. Here, the ϕ_i and θ_i are drawn from independent $U[0, 0.8]$ distributions (fixed over replications), thus introducing a degree of heterogeneity into the time series dynamics. Once more, however, $\hat{S}$ generally displays very close to nominal size. Only when $T = 75$ and $N = 20, 30$ do we see observe some noticeable departures from nominal size, and even these are really quite modest in nature.

Table 1(b) repeats the same set of experiments of Table 1(a) except that now we generate a moderate degree of equicorrelation, setting $\rho_{ij} = 0.5$. The story is very much the same as in Table 1(a). The overall sizes of $\hat{S}$ are a little closer to the nominal 0.05 value than was the case previously, and particularly so for $T = 75$ and $N = 20, 30$. In Table 1(c) the analysis is repeated for $\rho_{ij} = 0.9$, representing the situation of a high degree of equicorrelation. It is clear that this has no discernible effect on the size of $\hat{S}$.

We also examine the behaviour of the test outside of the equicorrelated case. Whilst there are numerous such ways in which this more general behaviour could be modelled, here we simply assume that $\rho_{ij} = 0.9^{|i-j|}$. For example, such a correlation structure might be considered appropriate to mimic the effects of spatial separation of economies, whereby neighbouring economies are more highly correlated than ones which lie geographically further apart. Since the test $\hat{S}$ is invariant under i-orderings the distance interpretation is meaningful. The results, shown in Table 1(d), indicate that such a correlation pattern has no untoward effect on the size of $\hat{S}$.

Our final size simulations assess the impact of the finite sample bias correction. Table 1(e) replicates the analysis of Table 1(a), this time *without* this bias correction. It is immediately clear that $\hat{S}$ is generally quite badly undersized and that this problem becomes more severe as N increases (particularly in the cases where $\phi_i > 0$), which supports the conjecture of Section 3. As we would expect, increasing the sample size does improve the situation, but only rather marginally, implying that large sample sizes are required before our asymptotic results apply. In view of the comparable results in Table 1(a), we conclude that our finite sample bias correction is extremely effective in alleviating these undersizing problems.

To examine the power of our test, we consider the DGP (2), where we set $\theta_i = 0$, $i = 1, ..., N$, $\phi_i = 1$, $i = 1, ..., M$ and $\phi_i = 0$, $i = M + 1, ..., N$. That is, the first M of the y_{it} are generated as random walks and the last $N - M$ are white noise processes. The results, for various choices of M and using the same values of N, T and cross-correlation structures as adopted in Table 1, are presented in Table 2. Since size distortions under the null hypothesis do not appear to be an issue, we report size-unadjusted empirical powers based on the asymptotic 0.05-level null critical value.

Considering first Table 2(a), the base case of no cross-sectional correlation, we can draw a number of conclusions. It is clear that for a fixed N, the power of $\hat{S}$ increases as M, the number of random walks, becomes a larger proportion of the total number of series, N. By the same token, we see that for a fixed M, the power of $\hat{S}$ generally decreases as N increases. The power advantages to be gained from the panel approach are also evident from Table 2(a). This is perhaps most clearly illustrated by examining the case of $N = M$ for $T = 75$, where we see a steady growth in power as N increases. For example for $N = M = 3$, the power is 0.67, risings to 1.00 for $N = M = 20$. Alternatively, when $T = 75$, we might compare $N = M = 3$ (0.67) with $N = 20$, $M = 10$ (0.90), which demonstrates an increase in power, even though the proportion of random walks has halved.

Table 2(b) repeats the analysis of Table 1(a) for moderate equicorrelation, $\rho_{ij} = 0.5$. The power of all tests are uniformly lower (apart for some entries where the power is still 1.00). However, perhaps the main feature of this table is the power drop for a fixed M as N increases. The power advantages from the panel approach are still evident, however; for $T = 75$ for example, compare $N = M = 3$ (0.61) with $N = M = 10$ (0.75). Table 2(c) demonstrates the effects of a high degree of equicorrelation, $\rho_{ij} = 0.9$. The power drops off even more rapidly or a fixed M as N increases. Moreover, by this point the power advantages of the panel approach have all but evaporated; for $T = 75$ compare $N = M = 3$ (0.49), $N = M = 10$ (0.48) and $N = M = 30$ (0.49). The obvious reason for this phenomenon is that we are approaching the limit case in which all the N series are perfectly correlated which is akin to having only a single series replicated N times. In such a situation there is no new information to exploit across the i dimension to increase test power above that obtainable from a test on a single series, and the panel approach therefore becomes redundant. Of course, this argument is not unique to panel stationarity tests; it applies equally to the power of panel unit root tests if, under the stationary alternative, all the series are (near) perfectly correlated. Finally, Table 1(d) reports the results of the spatial correlation model, $\rho_{ij} = 0.9^{|i-j|}$. The power advantages of the panel approach are once more clear to see.

A plausible criticism of our statistic is that it only involves sample autocovariances at a single lag, k, and that, potentially, power gains might be obtained by also incorporating sample autocovariances at higher lags. To this end, prompted by Tanaka (1999), we considered a panel analog of our statistic which uses a weighted sum of all sample autocovariances from lag k onwards; that is, $\sum_{i=1}^{N}\sum_{j=k}^{T-1}\frac{1}{j-k+1}C_{i,j}$. We constructed an appropriately studentized version of this statistic[7] and repeated the power analysis of Table 2(a) for both this statistic and $\hat{S}$ but without fitting constant terms and making finite bias corrections. In every case the empirical power of $\hat{S}$ was found to be at least as large as the test based on the weighted sum.[8] Moreover, when constant terms *were* fitted to both tests (without bias corrections), undersizing problems were found to be much more severe for the test based on the weighted sum than for $\hat{S}$. For example, in Table 1(e) whereas, for $T = 150$, the first three sizes for $\hat{S}$ in the leading column are 0.04, 0.03, 0.03, the corresponding sizes for the test based on the weighted sum are 0.01, 0.01, 0.00. Intuitively, this occurs because each of the $C_{i,j}$ terms in the weighted sum over j contributes towards a negative finite sample bias. In addition, there does not seem to be a readily computable bias correction for this statistic. Hence, because the weighted sum test appears to add nothing in terms of extra power, and also suffers more from undersizing problems, we feel that there is considerable justification for using our simple variant.[9]

[7]It can be shown that such a statistic also has a limiting standard normal distribution under H_0.

[8]We do not report the actual results here. They are available upon request.

[9]Of course, statistics which are other functions of higher order sample autocovariances might also be considered. We conjecture, however, that these will always prove rather more susceptible to bias (and hence size) problems than the test based on a single autocovariance, and will not yield substantial compensatory improvements in power.

5 Testing the Purchasing Power Parity Hypothesis

In this section, we empirically test the purchasing power parity (PPP) hypothesis, which is a fundamental ingredient of macroeconomic models of bilateral exchange rate behaviour. The validity of the PPP hypothesis has been an issue that has attracted a vast amount of attention in recent times and has been tested extensively using different panel unit root tests. In general, little evidence in support of PPP has been uncovered. For example, Papell (1997), O'Connell (1998), Cheung and Lai (2000), Wu and Wu (2001) and Chang and Song (2002) are unable to provide strong evidence against the unit root null.[10] A failure to reject this null does not, however, provide compelling evidence against the PPP hypothesis, not least because low test power may be an issue here since real exchange rates tend to be highly correlated as they are typically constructed using a common numeraire currency and price index.

In view of this, it makes some sense to apply our panel stationarity test in this context. Here, the PPP hypothesis is represented by the stationary null and a rejection can, *ceteris paribus*, fairly unambiguously be interpreted as evidence against the PPP hypothesis being true.

We consider monthly real exchange rates against the US Dollar for the following countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the UK. The real exchange rate data was constructed from raw nominal exchange rate and consumer price index data taken from the IMF International Financial Statistics database. It covers the period of the recent float 1973.01 to 1998.12. We have $N = 17$ and $T = 312$. In our notation we take y_{it} to be the natural log of the real exchange rate. The statistic $\hat{S}$ is calculated using $\tilde{e}_{it} = \hat{e}_{it}/\hat{\gamma}_0\{\hat{e}_{it}\}^{1/2}$ where $\hat{e}_{it}$ is a residual from a regression of y_{it} on a constant (a linear trend not being consistent with the PPP hypothesis). The user supplied parameters k and l are chosen as in the previous section (yielding $k = 31$ and $l = 15$). We first applied the test $\hat{S}$ to each of the 17 series individually (i.e. assuming $N = 1$). The test results are given in Table 3, together with the p-value of each the test. We see that there is evidence against the null hypothesis of stationarity being true in 4 of the 17 cases, assuming such evidence is indicated by a p-value of 0.05 or smaller. We also calculate the mean of the individual p-values. This yields a value of 0.126. Hence, at least informally, individual tests would not appear to provide much evidence against the PPP hypothesis.

In Table 4, the column labelled $N = 17$ gives summary information from the sample cross-correlation matrices of $\tilde{e}_{it}$ and $\Delta\tilde{e}_{it}$. The results indicate the presence of a substantial degree of positive cross-correlation - in either case over 80% of cross-correlations exceed 0.5. Thus, regardless of whether testing the stationary or unit root null, the need to employ a test whose behaviour is robust to cross-correlation is clear if reliable inference is to be made. When the test $\hat{S}$ is applied to the panel of all $N = 17$ series, we obtain a value of 1.95, which has approximate p-value of 0.026. This, it would appear, provides rather substantial evidence against the null hypothesis of joint stationarity, and hence the PPP hypothesis - notwithstanding the fact that, in the light of the results of the last section, we might expect the relatively high degrees of cross-correlation present here to have an adverse effect on the power of the test $\hat{S}$.

We also applied the test to the subset of European economies that excludes Canada and Japan. Now, according to the individual tests the null hypothesis of stationarity is true in 2 of the 15 cases (the average p-value is 0.142). The cross-correlation structure is summarized in the column labelled $N = 15$ of Table 4. As may be expected, this subset of geographically closely related economies exhibits an even more pronounced degree of positive cross-correlation - over 95% exceed 0.5. The statistic $\hat{S}$ now yields a value of 1.65, which has a p-value of 0.049. Thus, although the strength of rejection is lower than for the full panel, we are still able to reject the null

[10] In fact, what little empirical evidence there is in support of PPP has mainly arisen from application of tests that do not account for cross-sectional dependence at all; see Oh (1996) and Wu (1996).

of stationarity at the 0.05-level. We consider, therefore, that, in contrast to previous analyses, our present analysis provides rather compelling evidence against the validity of the PPP hypothesis.

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6 Appendix

6.1 Proof of Theorem 1

Let $\hat{\mathbf{e}}_t = (\hat{e}_{1t}, ..., \hat{e}_{Nt})'$ and $\tilde{\mathbf{e}}_t = (\tilde{e}_{1t}, ..., \tilde{e}_{Nt})'$. Then the numerator term of $\hat{S}$ can be written as

$$S_k = \mathbf{d}' vec\left[T^{-1/2}\sum_{t=k+1}^{T}\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_{t-k}'\right]$$

for a selector vector $\mathbf{d}$ defined as $\mathbf{d} = vec[I_{N^2}]$. Now,

$$\begin{aligned} vec\left[T^{-1/2}\sum_{t=k+1}^{T}\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_{t-k}'\right] &= vec\left[T^{-1/2}\hat{\mathbf{G}}_0^{-1}\sum_{t=k+1}^{T}\hat{\mathbf{e}}_t\tilde{\mathbf{e}}_{t-k}'\hat{\mathbf{G}}_0^{-1}\right] \\ &= (\hat{\mathbf{G}}_0^{-1}\otimes\hat{\mathbf{G}}_0^{-1})vec\left[T^{-1/2}\sum_{t=k+1}^{T}\hat{\mathbf{e}}_t\hat{\mathbf{e}}_{t-k}'\right] \end{aligned}$$

where $\hat{\mathbf{G}}_0 = diag[\hat{\gamma}_0\{\hat{e}_{1t}\}^{1/2}, ..., \hat{\gamma}_0\{\hat{e}_{Nt}\}^{1/2}]$. It follows from Harris, McCabe and Leybourne (2003), Theorem 8, that

$$vec\left[T^{-1/2}\sum_{t=k+1}^{T}\hat{\mathbf{e}}_t\hat{\mathbf{e}}_{t-k}'\right] \Rightarrow N[0, \Omega]$$

on noting that $\eta_t = diag[(1-\phi_1 L)^{-1}, .., (1-\phi_N L)^{-1}]\boldsymbol{\varepsilon}_t$ also satisfies the conditions of Assumption 1. Moreover, since $\hat{\mathbf{G}}_0 \Rightarrow \mathbf{G}_0 = diag[E(\eta_t\eta_t')^{1/2}]$,

$$S_k \Rightarrow N[0, \mathbf{d}'(\mathbf{G}_0^{-1}\otimes\mathbf{G}_0^{-1})\Omega(\mathbf{G}_0^{-1}\otimes\mathbf{G}_0^{-1})\mathbf{d}]$$

by the continuous mapping theorem (CMT). Next, with $a_t = \sum_{i=1}^{N}\tilde{e}_{it}\tilde{e}_{i,t-k}$ and $\mathbf{b}_t = vec[\tilde{\mathbf{e}}_t\tilde{\mathbf{e}}_{t-k}']$ and $\mathbf{c}_t = vec[\hat{\mathbf{e}}_t\hat{\mathbf{e}}_{t-k}']$, we may write

$$\begin{aligned} \hat{\omega}\{a_t\}^2 &= \mathbf{d}'\hat{\Omega}\{\mathbf{b}_t\}\mathbf{d} \\ &= \mathbf{d}'(\hat{\mathbf{G}}_0^{-1}\otimes\hat{\mathbf{G}}_0^{-1})\hat{\Omega}\{\mathbf{c}_t\}(\hat{\mathbf{G}}_0^{-1}\otimes\hat{\mathbf{G}}_0^{-1})\mathbf{d} \end{aligned}$$

where, for any vector sequence $\mathbf{f}_1, ..., \mathbf{f}_T$,

$$\hat{\Omega}\{\mathbf{f}_t\} = \hat{\Gamma}_0\{\mathbf{f}_t\} + \sum_{j=1}^{l}\left(1-\frac{j}{l}\right)\left(\hat{\Gamma}_j\{\mathbf{f}_t\} + \hat{\Gamma}_j\{\mathbf{f}_t\}'\right), \quad \hat{\Gamma}_j\{\mathbf{f}_t\} = T^{-1}\sum_{t=j+1}^{T}\mathbf{f}_t\mathbf{f}_{t-j}'.$$

From Harris, McCabe and Leybourne (2003), Theorem 8, for a specified matrix Ω,

$$\hat{\Omega}\{\mathbf{c}_t\} \Rightarrow \Omega$$

and hence, by the CMT,

$$\hat{\omega}\{a_t\}^2 \Rightarrow \mathbf{d}'(\mathbf{G}_0^{-1}\otimes\mathbf{G}_0^{-1})\Omega(\mathbf{G}_0^{-1}\otimes\mathbf{G}_0^{-1})\mathbf{d}$$

so that $\hat{S} = \hat{\omega}\{a_t\}^{-1}S_k \Rightarrow N[0, 1]$.

6.2 Proof of Theorem 2

Suppose, without loss of generality, that $\phi_i = 1$ for $i = 1, ..., M$, $0 < M \leq N$ and $\phi_i < 1$ for $i = M+1, ..., N$ (with the obvious modification for $M = N$). Now

$$T^{-1/2} S_k = \sum_{i=1}^{M} T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it} \tilde{e}_{i,t-k} + \sum_{i=M+1}^{N} T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it} \tilde{e}_{i,t-k},$$

and the second term is $O_p\left(T^{-1/2}\right)$ from the proof of Theorem 1. Noting that $T^{-1}\hat{\gamma}_0\{\hat{e}_{it}\} = O_p(1)$ for $i = 1, \ldots, M$, the first term satisfies

$$\begin{aligned}
\sum_{i=1}^{M} T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it} \tilde{e}_{i,t-k} &= \sum_{i=1}^{M} \frac{1}{T^{-1}\hat{\gamma}_0\{\hat{e}_{it}\}} T^{-2} \sum_{t=k+1}^{T} \hat{e}_{it} \hat{e}_{i,t-k} \\
&= \sum_{i=1}^{M} \frac{1}{T^{-1}\hat{\gamma}_0\{\hat{e}_{it}\}} T^{-2} \sum_{t=1}^{T} \hat{e}_{it}^2 + o_p(1) \\
&= M + o_p(1).
\end{aligned}$$

Thus,

$$S_k = T^{1/2} M + o_p\left(T^{1/2}\right). \tag{5}$$

Next, with $a_t = \sum_{i=1}^{N} \tilde{e}_{it} \tilde{e}_{i,t-k}$,

$$l^{-1} \hat{\omega}\{a_t\}^2 = l^{-1} \left(\hat{\gamma}_0\{a_t\} + 2 \sum_{j=1}^{l} \left(1 - \frac{j}{l}\right) \hat{\gamma}_j\{a_t\} \right) \leq 3\hat{\gamma}_0\{a_t\} \tag{6}$$

where

$$\hat{\gamma}_0\{a_t\} = T^{-1} \sum_{t=k+1}^{T} \left(\sum_{i=1}^{N} \tilde{e}_{it} \tilde{e}_{i,t-k} \right)^2.$$

Thus

$$\begin{aligned}
\hat{\gamma}_0\{a_t\} &\leq \sum_{i=1}^{N} \sum_{j=1}^{N} T^{-1} \sum_{t=1}^{T} \tilde{e}_{it}^2 \tilde{e}_{jt}^2 \\
&= O_p(1)
\end{aligned} \tag{7}$$

since for $i, j = M+1, \ldots, N$,

$$T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it}^2 \tilde{e}_{jt}^2 = \frac{1}{\hat{\gamma}_0\{\hat{e}_{it}\}.\hat{\gamma}_0\{\hat{e}_{jt}\}} T^{-1} \sum_{t=k+1}^{T} \hat{e}_{it}^2 \hat{e}_{jt}^2 = O_p(1)$$

for $i, j = 1, \ldots, M$,

$$T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it}^2 \tilde{e}_{jt}^2 = \frac{1}{T^{-1}\hat{\gamma}_0\{\hat{e}_{it}\}.T^{-1}\hat{\gamma}_0\{\hat{e}_{jt}\}} T^{-3} \sum_{t=k+1}^{T} \hat{e}_{it}^2 \hat{e}_{jt}^2 = O_p(1)$$

and for $i = M+1, \dots, N$ and $j = 1, \dots, M$,

$$T^{-1} \sum_{t=k+1}^{T} \tilde{e}_{it}^2 \tilde{e}_{jt}^2 = \frac{1}{\hat{\gamma}_0\{\hat{e}_{it}\}.T^{-1}\hat{\gamma}_0\{\hat{e}_{jt}\}} T^{-2} \sum_{t=k+1}^{T} \hat{e}_{it}^2 \hat{e}_{jt}^2 = O_p(1).$$

(and similarly for $j = M+1, \dots, N$ and $i = 1, \dots, M$). Combining (5), (6) and (7) we find, as $T \to \infty$,

$$P[\hat{S} > c] = P\left[\frac{M + o_p(1)}{\frac{l}{T^{1/2}} O_p(1)} > c\right] \to 1$$

since $l = o(T^{1/2})$.

6.3 Proof of Corollary 1

i) Under H_0, we have

$$\begin{aligned} l^{-1}\hat{\omega}\{\tilde{e}_{it}\}^2 &= l^{-1}\left(\hat{\gamma}_0\{\tilde{e}_{it}\} + 2\sum_{j=1}^{l}\left(1 - \frac{j}{l}\right)\hat{\gamma}_j\{\tilde{e}_{it}\}\right) \\ &\leq 3\hat{\gamma}_0\{\tilde{e}_{it}\} \\ &= 3 \end{aligned}$$

since $\hat{\gamma}_0\{\tilde{e}_{it}\} = 1$. Hence $T^{-1/2}\hat{\omega}\{\tilde{e}_{it}\}^2 = o_p(1)$ as $l = o(T^{1/2})$. Similarly,

$$\begin{aligned} l^{-1}\hat{\psi}\{\tilde{e}_{it}\left(t - \bar{t}\right)\}^2 &= Tq_T^{-1}l^{-1}\hat{\omega}\{\tilde{e}_{it}\left(t - \bar{t}\right)\}^2 \\ &\leq Tq_T^{-1}3\hat{\gamma}_0\{\tilde{e}_{it}\left(t - \bar{t}\right)\} \\ &= T^3 q_T^{-1} 3\hat{\gamma}_0\{\tilde{e}_{it}[\left(t - \bar{t}\right)/T]\} \\ &= O_p(1) \end{aligned}$$

since $T^3 q_T^{-1} = O(1)$ and $\hat{\gamma}_0\{\tilde{e}_{it}[\left(t - \bar{t}\right)/T]\} = O_p(1)$. So $T^{-1/2}\hat{\psi}\{\tilde{e}_{it}\left(t - \bar{t}\right)\}^2 = o_p(1)$ as $l = o(T^{1/2})$. Because $\hat{\omega}\{a_t\}$ converges in probability to a positive constant, $\hat{\omega}\{a_t\}^{-1}S_k^* = \hat{\omega}\{a_t\}^{-1}S_k + o_p(1)$ and so the corrected statistic has the same limiting null distribution as the original.

ii) Under H_1, the result follows trivially since $T^{-1/2}(\hat{\omega}\{\tilde{e}_{it}\}^2 + \hat{\psi}\{\tilde{e}_{it}\left(t - \bar{t}\right)\}^2)$ is non-negative.

Table 1. Empirical size of $\hat{S}$ based on a fitted constant at asymptotic 0.05-level critical values.

(a) $\rho_{ij} = 0.0$.

N	T	$\phi_i = 0.0$ $\theta_i = 0.0$	$\phi_i = 0.4$ $\theta_i = 0.0$	$\phi_i = 0.8$ $\theta_i = 0.0$	$\phi_i = 0.0$ $\theta_i = 0.4$	$\phi_i = 0.0$ $\theta_i = 0.8$	$\phi_i = U[0, 0.8]$ $\theta_i = U[0, 0.8]$
3	75	0.05	0.06	0.06	0.04	0.04	0.05
5	75	0.05	0.06	0.06	0.05	0.04	0.05
10	75	0.05	0.07	0.05	0.05	0.04	0.05
20	75	0.07	0.08	0.04	0.05	0.05	0.05
30	75	0.07	0.07	0.02	0.04	0.04	0.05
3	150	0.05	0.05	0.05	0.05	0.05	0.05
5	150	0.05	0.05	0.06	0.05	0.05	0.05
10	150	0.05	0.06	0.05	0.05	0.05	0.05
20	150	0.06	0.07	0.05	0.05	0.05	0.05
30	150	0.06	0.07	0.05	0.04	0.04	0.05
3	300	0.05	0.05	0.05	0.05	0.05	0.05
6	300	0.05	0.05	0.05	0.05	0.05	0.05
10	300	0.06	0.06	0.05	0.05	0.05	0.05
20	300	0.05	0.05	0.04	0.05	0.05	0.05
30	300	0.05	0.05	0.04	0.05	0.05	0.05

(b) $\rho_{ij} = 0.5$.

N	T	$\phi_i = 0.0$ $\theta_i = 0.0$	$\phi_i = 0.4$ $\theta_i = 0.0$	$\phi_i = 0.8$ $\theta_i = 0.0$	$\phi_i = 0.0$ $\theta_i = 0.4$	$\phi_i = 0.0$ $\theta_i = 0.8$	$\phi_i = U[0, 0.8]$ $\theta_i = U[0, 0.8]$
3	75	0.04	0.05	0.07	0.04	0.04	0.04
5	75	0.05	0.06	0.06	0.04	0.04	0.05
10	75	0.05	0.05	0.06	0.04	0.04	0.04
20	75	0.05	0.06	0.06	0.04	0.04	0.05
30	75	0.05	0.05	0.05	0.05	0.04	0.05
3	150	0.05	0.06	0.06	0.05	0.05	0.05
5	150	0.05	0.05	0.05	0.05	0.05	0.05
10	150	0.05	0.05	0.06	0.05	0.05	0.05
20	150	0.05	0.05	0.06	0.05	0.05	0.04
30	150	0.05	0.05	0.06	0.04	0.04	0.05
3	300	0.05	0.05	0.06	0.05	0.05	0.05
5	300	0.05	0.05	0.06	0.05	0.05	0.05
10	300	0.05	0.05	0.05	0.05	0.05	0.05
20	300	0.05	0.06	0.05	0.05	0.05	0.05
30	300	0.05	0.05	0.05	0.05	0.05	0.05

(c) $\rho_{ij} = 0.9$.

N	T	$\phi_i = 0.0$ $\theta_i = 0.0$	$\phi_i = 0.4$ $\theta_i = 0.0$	$\phi_i = 0.8$ $\theta_i = 0.0$	$\phi_i = 0.0$ $\theta_i = 0.4$	$\phi_i = 0.0$ $\theta_i = 0.8$	$\phi_i = U[0, 0.8]$ $\theta_i = U[0, 0.8]$
3	75	0.04	0.05	0.07	0.04	0.04	0.04
5	75	0.04	0.05	0.07	0.04	0.04	0.04
10	75	0.05	0.05	0.06	0.04	0.05	0.05
20	75	0.05	0.05	0.06	0.05	0.05	0.05
30	75	0.04	0.05	0.6	0.04	0.04	0.05
3	150	0.05	0.05	0.06	0.05	0.05	0.05
5	150	0.05	0.05	0.05	0.05	0.05	0.04
10	150	0.05	0.05	0.06	0.05	0.04	0.05
20	150	0.05	0.05	0.06	0.05	0.05	0.05
30	150	0.05	0.05	0.05	0.05	0.05	0.05
3	300	0.05	0.05	0.05	0.05	0.05	0.05
5	300	0.05	0.05	0.06	0.05	0.05	0.05
10	300	0.05	0.05	0.06	0.05	0.05	0.05
20	300	0.05	0.05	0.05	0.05	0.05	0.05
30	300	0.05	0.05	0.05	0.05	0.05	0.05

(d) $\rho_{ij} = 0.9^{|i-j|}$.

N	T	$\phi_i = 0.0$ $\theta_i = 0.0$	$\phi_i = 0.4$ $\theta_i = 0.0$	$\phi_i = 0.8$ $\theta_i = 0.0$	$\phi_i = 0.0$ $\theta_i = 0.4$	$\phi_i = 0.0$ $\theta_i = 0.8$	$\phi_i = U[0, 0.8]$ $\theta_i = U[0, 0.8]$
3	75	0.04	0.05	0.07	0.04	0.04	0.04
5	75	0.04	0.05	0.07	0.04	0.04	0.04
10	75	0.05	0.05	0.06	0.04	0.04	0.04
20	75	0.05	0.05	0.06	0.04	0.04	0.05
30	75	0.05	0.05	0.06	0.04	0.04	0.05
3	150	0.05	0.05	0.06	0.05	0.05	0.05
5	150	0.05	0.06	0.05	0.04	0.05	0.05
10	150	0.05	0.05	0.06	0.05	0.05	0.05
20	150	0.05	0.05	0.05	0.05	0.05	0.04
30	150	0.05	0.05	0.06	0.05	0.05	0.05
3	300	0.05	0.05	0.05	0.05	0.05	0.05
5	300	0.05	0.05	0.05	0.05	0.05	0.05
10	300	0.05	0.05	0.05	0.05	0.05	0.05
20	300	0.05	0.05	0.05	0.05	0.05	0.05
30	300	0.05	0.05	0.05	0.05	0.05	0.05

(e) $\rho_{ij} = 0.0$, no bias correction.

N	T	$\phi_i = 0.0$ $\theta_i = 0.0$	$\phi_i = 0.4$ $\theta_i = 0.0$	$\phi_i = 0.8$ $\theta_i = 0.0$	$\phi_i = 0.0$ $\theta_i = 0.4$	$\phi_i = 0.0$ $\theta_i = 0.8$	$\phi_i = U[0, 0.8]$ $\theta_i = U[0, 0.8]$
3	75	0.03	0.02	0.01	0.03	0.04	0.03
5	75	0.02	0.02	0.01	0.03	0.03	0.02
10	75	0.02	0.01	0.00	0.03	0.03	0.02
20	75	0.02	0.01	0.00	0.03	0.03	0.03
30	75	0.01	0.00	0.00	0.03	0.03	0.03
3	150	0.04	0.03	0.02	0.05	0.05	0.04
5	150	0.03	0.02	0.01	0.04	0.04	0.03
10	150	0.03	0.02	0.00	0.04	0.04	0.03
20	150	0.02	0.01	0.00	0.04	0.04	0.03
30	150	0.02	0.01	0.00	0.03	0.04	0.03
3	300	0.04	0.04	0.02	0.05	0.05	0.04
5	300	0.04	0.03	0.02	0.05	0.05	0.04
10	300	0.04	0.02	0.01	0.05	0.04	0.04
20	300	0.03	0.02	0.00	0.04	0.04	0.03
30	300	0.03	0.02	0.00	0.04	0.05	0.03

Table 2. Empirical power of $\hat{S}$ based on a fitted constant at asymptotic 0.05-level critical values.

(a) $\rho_{ij} = 0.0$.

N	T	$M=1$	$M=3$	$M=5$	$M=10$	$M=20$	$M=30$
3	75	0.34	0.67	-	-	-	-
3	150	0.57	0.92	-	-	-	-
3	300	0.81	0.99	-	-	-	-
5	75	0.28	0.64	0.81	-	-	-
5	150	0.50	0.90	0.98	-	-	-
5	300	0.74	0.99	1.00	-	-	-
10	75	0.20	0.52	0.74	0.95	-	-
10	150	0.40	0.86	0.97	1.00	-	-
10	300	0.66	0.98	1.00	1.00	-	-
20	75	0.16	0.40	0.62	0.90	1.00	-
20	150	0.29	0.77	0.95	1.00	1.00	-
20	300	0.55	0.97	1.00	1.00	1.00	-
30	75	0.15	0.35	0.54	0.87	0.99	1.00
30	150	0.25	0.69	0.92	1.00	1.00	1.00
30	300	0.43	0.95	1.00	1.00	1.00	1.00

(b) $\rho_{ij} = 0.5$.

N	T	$M=1$	$M=3$	$M=5$	$M=10$	$M=20$	$M=30$
3	75	0.34	0.61	-	-	-	-
3	150	0.55	0.87	-	-	-	-
3	300	0.79	0.98	-	-	-	-
5	75	0.21	0.55	0.69	-	-	-
5	150	0.41	0.83	0.92	-	-	-
5	300	0.68	0.97	0.99	-	-	-
10	75	0.14	0.37	0.57	0.75	-	-
10	150	0.23	0.71	0.89	0.98	-	-
10	300	0.47	0.95	0.99	1.00	-	-
20	75	0.09	0.21	0.33	0.63	0.81	-
20	150	0.12	0.41	0.68	0.95	0.99	-
20	300	0.24	0.74	0.95	1.00	1.00	-
30	75	0.07	0.14	0.23	0.49	0.76	0.83
30	150	0.09	0.26	0.47	0.84	0.99	1.00
30	300	0.14	0.50	0.83	1.00	1.00	1.00

(c) $\rho_{ij} = 0.9$.

N	T	$M = 1$	$M = 3$	$M = 5$	$M = 10$	$M = 20$	$M = 30$
3	75	0.29	0.49	-	-	-	-
3	150	0.50	0.71	-	-	-	-
3	300	0.75	0.90	-	-	-	-
5	75	0.16	0.44	0.49	-	-	-
5	150	0.30	0.66	0.71	-	-	-
5	300	0.58	0.88	0.91	-	-	-
10	75	0.09	0.25	0.40	0.48	-	-
10	150	0.14	0.49	0.65	0.73	-	-
10	300	0.29	0.78	0.89	0.93	-	-
20	75	0.07	0.14	0.22	0.40	0.49	-
20	150	0.09	0.23	0.43	0.68	0.74	-
20	300	0.14	0.46	0.73	0.90	0.94	-
30	75	0.06	0.09	0.14	0.28	0.45	0.49
30	150	0.08	0.16	0.27	0.54	0.71	0.74
30	300	0.09	0.26	0.50	0.82	0.92	0.94

(d)$\rho_{ij} = 0.9^{|i-j|}$.

N	T	$M = 1$	$M = 3$	$M = 5$	$M = 10$	$M = 20$	$M = 30$
3	75	0.30	0.49	-	-	-	-
3	150	0.51	0.73	-	-	-	-
3	300	0.76	0.91	-	-	-	-
5	75	0.17	0.46	0.52	-	-	-
5	150	0.31	0.68	0.77	-	-	-
5	300	0.59	0.89	0.94	-	-	-
10	75	0.10	0.27	0.43	0.58	-	-
10	150	0.17	0.54	0.71	0.88	-	-
10	300	0.35	0.82	0.93	0.98	-	-
20	75	0.08	0.17	0.28	0.49	0.73	-
20	150	0.10	0.32	0.55	0.81	0.96	-
20	300	0.19	0.63	0.84	0.98	1.00	-
30	75	0.07	0.14	0.21	0.40	0.67	0.81
30	150	0.10	0.25	0.41	0.73	0.94	0.99
30	300	0.14	0.48	0.75	0.96	1.00	1.00

Table 3. Values of $\hat{S}$ for individual countries.

	$\hat{S}$	p-value
Austria	1.49	0.068
Belgium	1.37	0.085
Canada	2.02	0.022
Denmark	1.27	0.102
Finland	0.33	0.371
France	1.01	0.157
Germany	1.33	0.092
Greece	2.40	0.008
Italy	1.08	0.139
Japan	2.77	0.003
Netherland	0.09	0.463
Norway	1.09	0.137
Portugal	2.62	0.004
Spain	1.60	0.055
Sweden	1.36	0.086
Switzerland	1.47	0.070
U.K.	0.57	0.286

Table 4. Summary of cross–correlation matrix.

		$N = 17$		$N = 15$	
		$\tilde{e}_{it}$	$\Delta\tilde{e}_{it}$	$\tilde{e}_{it}$	$\Delta\tilde{e}_{it}$
% of cross-correlations >	0.0	97	100	100	100
	0.3	89	88	100	100
	0.5	81	84	97	100
	0.7	57	48	73	62
	0.9	12	7	15	10