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The behaviour of water-immersed glass-bronze particulate systems under vertical vibration

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Abstract We report a detailed investigation of the behaviour of water-immersed binary mixtures of glass and bronze spheres under sinusoidal vertical vibration. Excellent separation with a bronze layer above a glass layer is observed for a range of particle sizes and size ratios, with very sharp boundaries between the bronze and glass regions. Convection cells exist within each separated region but convection does not cause global mixing. Separation is maintained in the presence of convection due to gaps which appear between the bronze and glass beds for part of the vibration cycle. We vary the size of the spheres and the size ratio, investigating the separation behaviour as a function of frequency and amplitude of vibration. We also discuss a number of meta-stable granular configurations. The Faraday tilting observed at low frequencies enhances the speed of separation. For very fine particles, the amplitude of vibration necessary for separation becomes very large and the time scale for separation becomes very long. For large particle diameters, the fluid damping of the particle motion weakens and separation may fail through global convective mixing.

1 Introduction

This paper concerns the behaviour of binary granular mixtures when vibrated vertically in a fluid. The dynamic behaviour of fine granular systems is strongly influenced by the presence of a fluid such as air or water. Interactions between fluid and grains provide the basis for many processes of industrial relevance including classification, the separation of mixtures of mineral particulates into two or more categories. In mineral processing the particle sorting may be principally on the basis of particle density or size or some combination of density and size depending upon the design of the separator [1].

Vibration techniques have been used for many centuries for the concentration and separation of minerals. Wet "panning" has been used to separate a heavy constituent such as gold from lighter components and in industrial processing "jigging" is often used to separate minerals of different specific gravity. Vibration is applied to the bed by pulsing the material into and out of water, the fluid motion causing stratification of different classes of particle [2].

In recent years, the physics community has shown an increasing interest in understanding the dynamics of granular systems under vibration, including the behaviour of beds containing a single "intruder". Under vertical vibration, a large and heavy object may move to the top of a bed of smaller grains, the "Brazil nut effect" (BNE) [3,4]. Some types of intruder may pass to the bottom of the bed, the "Reverse Brazil nut effect" (RBNE) [5]. However, the conditions for the BNE and the RBNE have not been completely identified [6,7]. Recently, it has been noted that, for fine beds, the presence of a fluid such as air greatly enhances the BNE [8,9].

The behaviour of multiple intruders in a granular bed, the case of binary mixtures, is also the subject of active debate. It might be expected that the BNE (RBNE) would cause segregation of the intruders to the upper (lower) part of the bed. However, multiple intruder behaviour is influenced by intruder-intruder interactions mediated by the other granular species [10]. The behaviours of binary mixtures under vibration have been studied theoretically [11,12] and by simulation and experiments [13–17] in an attempt to delineate the conditions for separation. In real systems the relatively weak BNE and RBNE effects are often dominated by granular convection which causes mixing. Except for mixtures in which the two components have extremely different properties, separation is often far from complete.

Recently Burtally et al. have studied the behaviour of fine binary mixtures of glass and bronze spheres under vertical vibration [18,19]. They have shown that the presence of air

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provides a strong separation mechanism in mixtures including those in which the two species have the same size. The mechanism requires the fluid to be forced through the bed during vibration. At low frequencies, a bronze-rich region forms above an almost pure region of the less-dense glass component. At higher frequencies, a bronze-rich layer forms between upper and lower glass-rich layers in a "sandwich" configuration. In each case, the boundaries between the glass-rich and bronze-rich regions are extremely sharp. Distinct convection cells are established in the individual separated regions. However, a number of the phenomena require further investigation. For example, it is not yet clear why the high granular shear present at the sharp separation boundaries does not cause mixing and whether the "sandwich" configuration found at higher frequencies is the true equilibrium configuration.

The computer simulations of Biswas et al. reproduce a number of the principal experimental features observed by Burtally et al. and clarify the air-driven separation mechanism [20]. The simulations confirm that it is fluid driven through the granular bed by the vibration which produces segregation. Grains of different types are treated differently by the fluid and move with respect to one another. Grains of the same type experience very similar forces and groups that come together tend to stay together. Over many cycles these processes can lead to segregation into almost completely pure glass and bronze phases.

This explanation for the separation mechanism suggests that, with suitable scaling of the particle sizes, many of the same effects should be observed when the air is replaced by a liquid such as water. Water offers a number of advantages for experimental investigations of the separation behaviour.

Firstly, it enables considerably larger particles to be used, making the observation of both the fluid and granular dynamics considerably simpler than for air. For non-turbulent fluid flow, the effects of fluid drag on the granular dynamics scale approximately as $\rho d^2/\eta$, where ρ is the density of the granular material, d is the grain diameter and η is the dynamic viscosity of the surrounding fluid [19]. At 20^{o}C, water is about 50 times more viscous than air, suggesting that the separation effects may be observed in water for particles ~ 7 times larger in diameter than for the observation of similar effects in air. Effects due to the non-negligible density of water, such as buoyancy, somewhat modify this prediction. Larger particles greatly aid both the observation of the movement of individual grains and the direct observation of the associated fluid flow through the use of small mica fragments which, when added to the water, closely follow the flow [21].

Secondly, the use of water eliminates the effect of static electricity, which often slows and otherwise modifies the dynamics of dry granular systems when they are shaken vigorously for long periods, particularly within an insulating box [19].

Here we report a detailed investigation of the behaviour, under sinusoidal vertical vibration, of a number of water-immersed bronze-glass mixtures. We report the various modes of separation, varying the size and the size ratio of the two components and investigating the behaviour as a function of frequency and amplitude of vibration. Excellent separation is observed with the bronze-rich component uppermost for a range of particle sizes and vibratory conditions. At higher frequencies we observe long-lived meta-stable "sandwich" configurations. For very fine particles, the amplitude of vibration necessary for separation becomes very large and the time scale for separation becomes extremely long. For large particles, this fluid-driven separation fails through convective mixing except over a range of low frequencies which becomes more and more restricted as the particle size is increased. We use high speed photography to study the gaps which open between separated beds over part of the vibration cycle. It is these gaps which enable the establishment of separate convection cells and the avoidance of convective mixing.

In our observations of separation, we note the important influence of Faraday tilting. Under vertical vibration a granular bed may spontaneously tilt; the tilt then influences the separation dynamics. Faraday tilting is well known for fine granular beds in air [22,23] and is due to the influence of the horizontal component of fluid flow, set up once any deviation from symmetry occurs [24,25]. In a recent study of Faraday tilting on a single component water-immersed bed, a similar mechanism was found [26]. The tendency to Faraday tilting is stronger at lower frequencies and at lower amplitudes of vibration, extending to higher frequencies for finer and deeper beds [26].

2 Experimental techniques

The test mixtures consisted of spherical soda glass and bronze particles of densities 2500 kgm^{-3} and 8900 kgm^{-3} respectively. Each sample of bronze and of glass was sieved to provide a variation in particle diameters of about $\pm 10\%$ about the mean, thus avoiding the formation of crystalline blocks of particles during vibration. To determine the effect of size, concentration and size ratio on the dynamics of separation, three sets of experiments were conducted. Firstly, 50% bronze:50% glass mixtures were investigated; the particle diameter ranges chosen were 90–125 μm, 150–200 μm, 300–355 μm, 600–710μm and 1000–1180 μm. Secondly, 25% bronze:75% glass and 75% bronze:25% bronze systems were examined for the 300–355 μm size range. Thirdly, we investigated three 50% : 50% bronze-glass mixtures in which the particles were of different sizes; the combinations chosen were 200–300 μm bronze with 500–600 μm glass, 300–355 μm bronze with 1000–1180 μm glass and 425–500 μm bronze with 1000–1180 μm glass. For each of these mixtures a bed depth of 20 mm was used. All proportions are by volume.

Immersed granular samples were vibrated in sealed rectangular boxes. The majority of the measurements were performed in boxes 40mm in height and 40mm by 10mm in cross-section, a horizontal aspect ratio which ensures that much of the mean granular behaviour is close to two-dimensional. This is discussed further below. The boxes consist of a duralumin frame to which front and back soda glass panels have been glued. Further soda glass panels were glued to the internal duralumin bottom and side surfaces to ensure that all the surfaces in contact with the particles have the same properties. The top of the frame had two holes, which could be sealed with bungs.

Before each experiment, a box was filled to the required depth with dry grains. The box was then filled with water and shaken to free any air bubbles from the grains and box walls. The box was then topped up with water and sealed so that, internally, it was completely free of bubbles. Small amounts of a surfactant, "Decon", were found to aid the wetting of particles and the removal of air during filling, particularly for the smaller particles. The presence of surfactant, or the use of air-free water rather than air-saturated water, was found to have no discernible effect on the granular dynamics studied.

Usually we wished to start an experiment with well-mixed particles. Complete mixing could not be achieved merely by shaking the box, since sedimentation causes a weak separation in which the component having the lower value of ρd^2, in most experiments the glass, is found preferentially in the upper regions of the bed. A weak initial separation usually has some influence on the progress of the subsequent separation under vibration, but not the final outcome, as will be reported. A more homogeneous initial state was achieved by stirring with a spatula through one of the upper holes, which was then resealed.

The box was vibrated by bolting it to the frame of an assembly consisting of two electro-magnetic transducers connected by rigid rods in such a way that one dimensional movement aligned to the vertical to within 0.2^0 was assured [19]. In the present work sinusoidal excitation of the box in the range 5–200Hz. was used. The maximum acceleration of the box was measured using a pair of cantilever capacitance accelerometers covering the ranges 0–8g and 0–80g. The electronic output directly registers the dimensionless acceleration $\Gamma = a\omega^2/g$ where a is the amplitude of vibration, ω is the angular frequency and g is the acceleration due to gravity; $\omega = 2\pi f$, where f is the frequency of vibration. Most of the present work was conducted with Γ in the range 1–10.

3 Observation of separation

3.1 Separation of glass and bronze grains of the same mean diameter

For all of the equal-sized mixtures listed above we observe conditions of frequency and Γ for separation into distinct glass-rich and bronze-rich beds with very sharp interface boundaries. In some cases, the separation is very good indeed with an essentially pure glass phase and a bronze-rich phase containing less than 0.1% glass. At the upper extreme of the size range reported here (1000–1180 μm) the separation is very poor above 100Hz due to the onset of global convection which causes mixing. As the size is further increased, global convection causes mixing at progressively lower frequencies. At the lower limit of our size range (90–125 μm) the dynamics are very slow indeed and separation occurs on a time scale of many hours. Between these two limits lies a size range within which bronze and glass may be effectively separated in times of order tens of seconds to hours. Within this size range, separation usually follows one of the routes shown schematically in figures 1a, 1b and 1c.

At lower frequencies, an initially well-mixed granular bed undergoes Faraday tilting under vibration. Regions rich in bronze form and coarsen in size, moving upwards within the bed, separation passing directly into the "tilted bronze on top" configuration of Figure 1a. The dominant single-cell granular convection associated with a Faraday tilted bed aids separation by effectively flushing residual bronze from the lower glass regions as separation develops. After some time the glass-rich region frequently consists almost entirely of glass grains (< 0.01% bronze).

At somewhat higher frequencies, an initially well-mixed bed may separate by the route shown in figure 1b. The bed breaks symmetry by tilting into the configuration in which part of the separated glass lies above the bronze-rich region. This we describe as the "tilted sandwich" configuration. On a longer time scale, the upper glass passes through the bronze-rich layer and the system eventually adopts the "tilted bronze on top" configuration. This route is encouraged by poor initial mixing.

At high frequencies, the route shown in figure 1c may be observed. The separation process produces a "sandwich" in which the bronze-rich layer is found between almost pure upper and lower glass layers. As vibration continues, glass from the upper layer diffuses through the bronze-rich layer which moves slowly upwards in the bed until eventually it reaches the top, to form a "symmetric bronze on top" config-

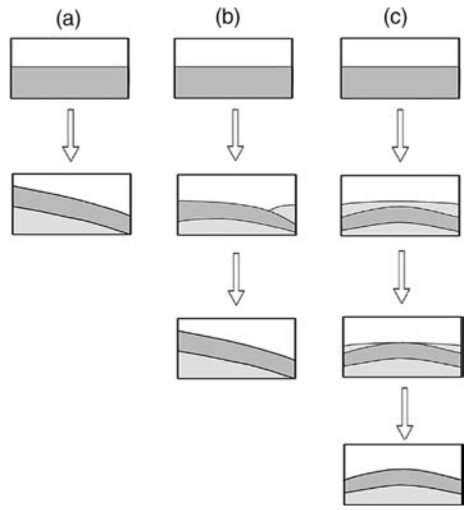

Fig. 1 Modes of separation: a) directly to the "tilted bronze on top" configuration; b) to the "tilted bronze on top" via a "tilted sandwich" configuration; c) to the "symmetric bronze on top" configuration via a "symmetric sandwich" configuration

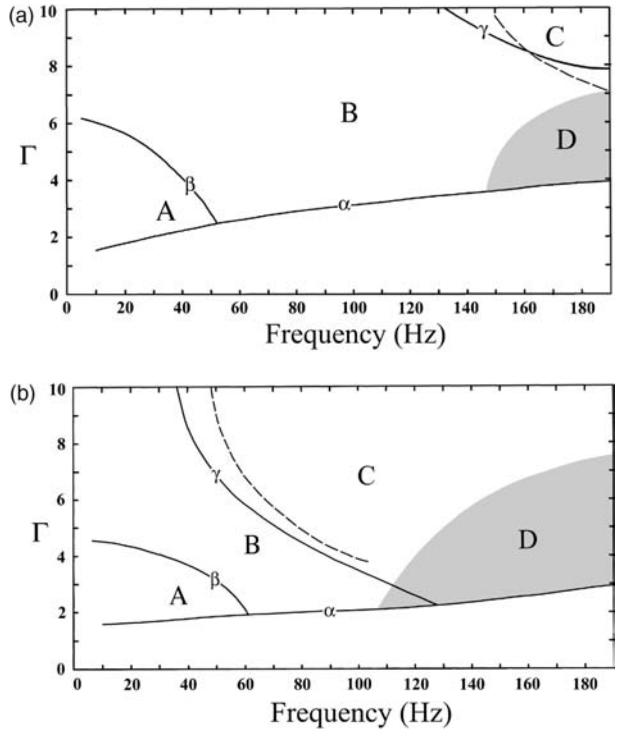

Fig. 2 Upper diagram (a) The behaviour of a 50% : 50% mixture of glass and bronze, each of mean size $300 - 355\mu m$, as function of frequency and Γ. Lower diagram (b) The behaviour of a 50% : 50% mixture of glass and bronze, each of mean size 1000–1180 μm, as function of frequency and Γ. The broken lines are estimates of the boundaries between throwing and settling within each cycle, and continuous dynamics

uration. If the separation is almost complete once a sandwich is formed, the bronze-rich region contains very little glass, diffusion is very slow indeed and the bronze region may take many hours to reach the upper surface. The "sandwich" configuration is then a long-lived metastable state.

Figure 2a indicates, schematically, the observed behaviour as a function of frequency and Γ when vibration is applied to an initially well mixed sample of the 300–355 μm 50% : 50% bronze-glass composition. Throughout the frequency range indicated, separation effects occur for sufficiently large values of Γ. Initially, clusters of glass and bronze grains form. These clusters then merge to form larger regions, eventually leading to one of the forms indicated in figure 1. The continuous line "α" indicates the values of Γ for which clusters have formed, on the length scale of the box, in two minutes. Below "α" the approach to separation is very slow and separation is never as complete as it is above the "α" line. The line "β" separates region A in which the behaviour of figure 1a is found and region B in which the behaviour of figure 1b is observed. Within A, separation proceeds directly to the "tilted bronze on top" configuration. An example of such a separation is shown in figure 3 for 20Hz and $\Gamma = 5$. Here glass regions form and enlarge through glass passing out of the concentrating bronze regions. The later stages of development towards almost complete separation involve the remaining bronze within the glass joining the upper bronze region, aided by convection.

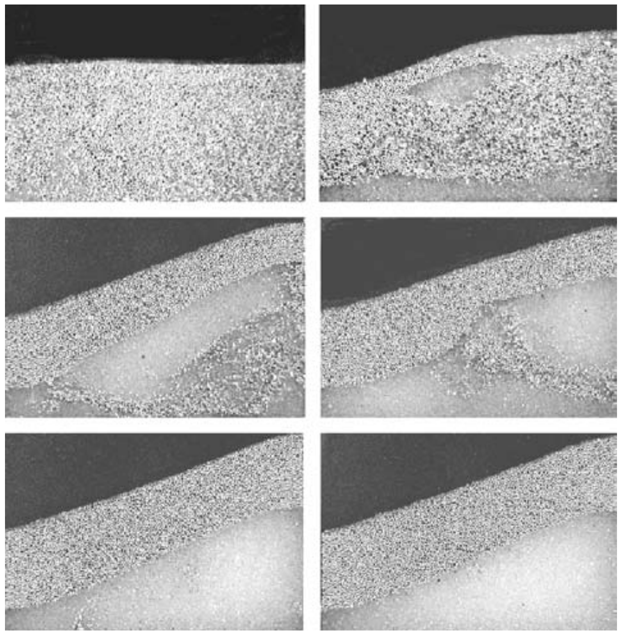

Fig. 3 Stages in the direct separation to the "tilted bronze on top" configuration. The images show the behaviour of a 50% : 50% glass-bronze mixture of $300 - 355\mu m$ diameter grains vibrated at 20Hz at $\Gamma = 5$, after times of 0, 0.5, 1, 2, 3 and 4 minutes from top left to bottom right

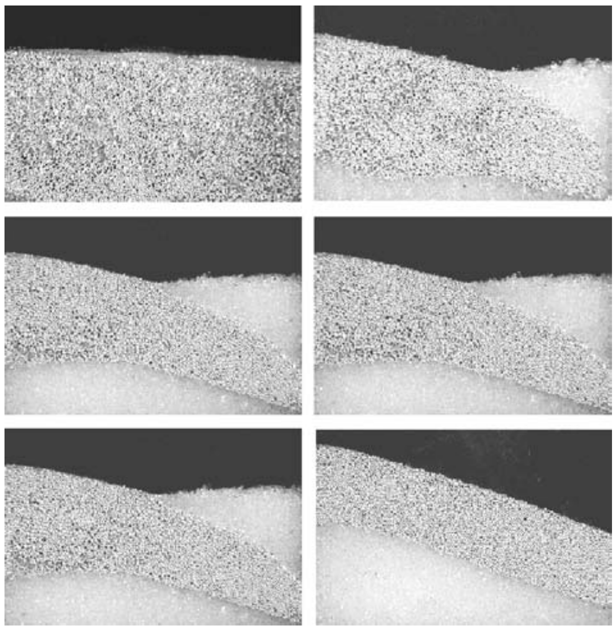

Fig. 4 Stages in the separation to the "tilted bronze on top" configuration via a tilted sandwich configuration. The images show the behaviour of a 50% : 50% glass-bronze mixture of $300 - 355\mu m$ diameter grains vibrated at 80Hz at $\Gamma = 5$, after times of 0, 4, 15, 28, 34 and 40 minutes from top left to bottom right

Throughout the region B, the behaviour of figure 1b is found. An example of such separation is shown in figure 4

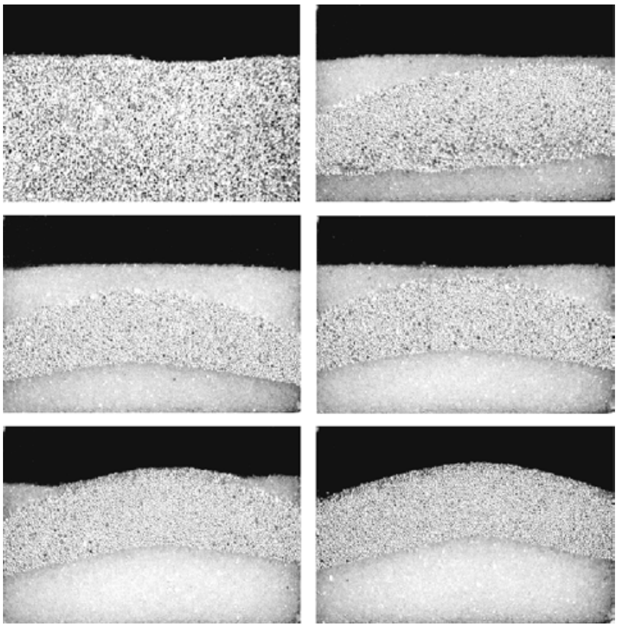

Fig. 5 Stages in the separation to the "symmetric bronze on top" configuration via the symmetric sandwich configuration. The images show the behaviour of a 50% : 50% glass-bronze mixture of $300 - 355\mu m$ diameter grains vibrated at 180Hz at $\Gamma = 10$, after times of 0, 2, 7, 19, 32 and 40 minutes from top left to bottom right

for 80Hz and $\Gamma = 5$. A glass region forms at the bottom of the granular bed. The upper surface of the developing bronze region undergoes Faraday tilting, and a second glass region, having a nearly horizontal upper surface develops at the bottom of the slope. The volume of this glass region diminishes with time, aided by the convection currents within the bronze region. Eventually, an almost pure tilted bronze layer is found above a pure glass region.

The line "γ" separates the region B from a region C in which the behaviour of figure 1c is found. An example of such a behaviour is shown in figure 5 for 180Hz and $\Gamma = 10$. Here a lower and an upper glass region forms, separated by a bronze layer. This layer is bowed in shape under the influence of the convection currents in both the bronze and glass layers. The upper glass layer thins as glass passes through the bronze layer, until eventually a single bronze-rich layer covers a single lower glass layer. At these high frequencies, however, the granular system is not tilted. Rather it is bowed, influenced by the convection currents within the layers. Within the shaded region D the separation is less complete, with visible quantities of bronze within the glass regions. Here, the boundaries between the bronze and glass regions are broader than the extremely sharp boundaries found outside the region D.

Figure 2b shows the corresponding data for a 1000–1180 μm 50% : 50% bronze- glass mixture, included to illustrate how the various features change with particle size. All of the same phenomena are found. However the boundary between the B and C behaviours, "γ", occurs at lower values of frequency and Γ for the larger grained mixture. At larger grain sizes, the region of poor separation, shown as D, extends to lower frequencies. Within this region, the separation is worse than for the smaller particle size, with more diffuse separation boundaries and partial mixing through global convection currents. By grain sizes of $1500\mu m$ good separation is only found at very low frequencies.

Table 1 The separation onsets for 50% : 50% glass-bronze mixtures of various equal mean sizes based on a time scale of two minutes. Values of Γ for the onset of separation are given for three frequencies. Estimated values of Γ for which the gap between an upper bronze and a lower glass bed is equal to $d/10$ are given in brackets

Mean size (μm)	Γ at 50Hz	Γ at 100Hz	Γ at 150Hz
90–125	10.0 (4.3)	10.5 (7.3)	11.0 (10.1)
150–200	4.3 (3.5)	5.9 (5.0)	6.9 (7.0)
300–355	2.3 (2.5)	3.1 (3.6)	3.6 (4.9)
600–710	1.8 (2.0)	2.5 (3.3)	3.2 (5.3)
1000–1180	1.8 (2.2)	2.1 (4.2)	2.5 (–)

For smaller grain sizes, a higher Γ is required to produce granular motion; the dynamics leading to separation is slower. Table 1 shows the values of Γ corresponding to the "α" line, for three frequencies and for a range of particle sizes. It may be seen that the values of Γ rise extremely rapidly as the particle size falls to $100\mu m$. It is the slowing of the dynamics for small particle sizes, and the need for extremely large values of Γ which limit the size of particles for which effective separation may be obtained.

We observed that all well separated configurations are close to two-dimensional in form, with little variation normal to the principal box faces. For boxes with a cross-sectional aspect ratio considerably greater than unity, the mean granular convection is also approximately two dimensional at low frequencies, except close to box corners. This could be confirmed by inspection of the motion at the upper granular surface and by following tracker beads. For our aspect ratio of 4:1 this approximate 2D convective behaviour was found throughout the low-frequency "bronze-on-top" region (e.g. A of figures 2), most movement being parallel to the plane of the principal box faces. Within the "tilted sandwich" regions of the $f - \Gamma$ plane we observed components of mean granular movement normal to the principal box faces, increasingly so as the frequency was raised (e.g. the regions B of figure 2). Within the higher frequency regions where "sandwich" or "symmetric bronze on top" behaviour is found (e.g. the regions C of figure 2) the convective behaviour was no longer close to 2D, with granular up-welling on the central axis of the box. Three-dimensional behaviour was particularly evident within the regions D of figure 2 where the separation is observed to be poor.

3.2 Varying the composition

In this section we consider the effect of varying the composition for mixtures in which the diameters of both glass and bronze grains lies in the range 300–355 μm, an intermediate size within the range which we have studied.

Firstly we compare the behaviour of a 75% bronze, 25% glass mixture with that of the 50% : 50% mixture reported above. The system dynamics are on a very similar time scale in the two cases. At low frequencies the 75% : 25% mixture separates into the "tilted bronze on top" configuration. At higher frequencies the system forms a tilted sandwich which slowly transforms into the "tilted bronze on top" configuration. However, the meta-stable "sandwich" configuration, found at high values of frequency and Γ for the 50% : 50% mixture ("C" in figure 2b), is not found for the 75% : 25% mixture for any values of frequency < 200Hz and $\Gamma < 20$, perhaps because the greater proportion of bronze lowers the stability of the "sandwich" configuration.

Secondly we compare the behaviour of a 25% bronze, 75% glass mixture with that of the 50% : 50% mixture. Here the time scale for the approach to equilibrium separation is slowed by the greater amount of the more heavily fluid-damped component. The quality of final separation is less complete than for the other compositions studied, with of order 0.1% of bronze within the glass, and 1% of glass within the bronze. At low frequencies direct separation into the "bronze on top" configuration occurs as for the other compositions. At intermediate frequencies, the "tilted bronze on top" form was still the final configuration but, due to the greater proportion of glass, it was approached via a full "sandwich" configuration, the process being very slow since extra glass must pass down through the central bronze layer. The greater proportion of glass also stabilises the sandwich configuration corresponding to C of figure 2b and this region extends to lower values of frequency and Γ for the 25% bronze:75% glass sample.

3.3 Separation of glass and bronze grains of different mean diameters

In air-immersed systems $S = \rho_b d_b^2/\rho_g d_g^2$ is the principal parameter determining whether fluid-driven separation occurs and, if so, which component forms uppermost [19]. Here the suffices b and g refer, for example, to bronze and glass components. The parameter S represents the ratio of the influences of the air on the components of the binary granular mixture. For $S = 1$, the effect of the air is equal on the two components and no separation is expected. For S appreciably greater than unity the b component is expected to separate above the g component at low frequencies, while for S appreciably less than unity the g component will have a tendency to separate uppermost. For liquid-immersed systems, the appreciable fluid density also affects the fluid-grain interactions and the situation is more complex. Nevertheless, S remains a useful control parameter in experiments in which the ratio of the fluid interactions with the two species is varied through changes in the relative particle sizes. For the mixtures which we have just described, the bronze and glass grains have equal sizes and $S \sim 3.6$. For this value of S, bronze is expected to be thrown higher during vibration and to separate above the glass, as we observe at low frequencies.

We have investigated three 50% : 50% glass-bronze mixtures in which the two species are of different diameters in order to investigate the behaviour for systems in which S is close to or less than unity. The first of these uses 425–500 μm diameter bronze and 1000–1180 μm diameter glass; $S \approx 0.9$. At low frequencies, the system tilts, forming a very badly separated bronze on top configuration. The bronze contains many percent of glass, the glass contains many percent of bronze, and the boundary between the upper and lower layers is ill-defined extending over many grain diameters. At higher frequencies, a glass layer forms above a mixture of bronze and glass. In its upper parts, the glass is almost pure. However, closer to the bronze-rich region the glass is very impure, the boundary with the lower bronze-rich region being very ill-defined.

The second mixture uses 200–300 μm bronze and 500–600 μm glass for which $S \approx 0.7$. At low frequencies, this system too forms a very badly separated "bronze on top" configuration with an ill-defined boundary between the upper and lower layers. At high frequencies, a tilted partial sandwich forms with poor separation of bronze and glass and ill-defined boundaries.

We have also investigated a mixture of 300–355 μm bronze and 1000–1180 μm glass for which $S \approx 0.3$. At low frequencies, we find a slight tendency for the bronze to collect at the top of the bed under vibration. The separation is very poor, the proportion of bronze varying smoothly between the lower and upper regions. However, above 100Hz, the system first forms a glass-bronze-glass sandwich with a sharp upper boundary and diffuse lower boundary. On a time scale of tens of minutes this changes to an "glass on top" configuration with well separated horizontal layers of glass and bronze and very sharp separation boundaries.

4 Granular convection in the separated state

We have shown that excellent separation may occur when S has a value appreciably greater than unity. Under these conditions the separated bronze and glass regions may each contain a very low proportion of the other component and the boundaries between the regions are defined to within one grain diameter. In addition, separate granular convection cells exist within each separated region but there are no global convection patterns which would lead to mixing.

The nature of the granular convection flows are shown schematically for $S > 1$ in figure 6 for three principal situations which may occur during separation. Figure 6a shows the convective flows after separation into the "tilted bronze on top" configuration, the usual form found at low frequencies. The flow velocities are considerably greater in the less damped upper bronze region than in the lower glass region, which is more damped by the fluid. Note that the circulations are in the same sense. At the interface, therefore, the flows are in opposite directions; there appears to be a discontinuity in the velocity field at the sharp boundary between the two regions. Figure 6b shows the convective flows in the "tilted sandwich" configurations, the usual metastable state found at intermediate frequencies during separation to an eventual "tilted bronze on top" state. The quicker principal circulation in the bronze layer and the slower circulation in the lower glass layer are in the same sense. Again, there appears to be a discontinuity in the velocity field at the sharp boundary between them. The circulation in the upper glass region is observed to be in the opposite sense to the principal circulation in the bronze, and only the differences in the two circulation rates introduces an apparent velocity discontinuity at the boundary between them. Figure 6c shows the convective flows for the case of a "symmetric bronze on top" configuration, where there are double Muchowski rolls in both the glass and bronze regions. Here too, it should be noted that there is an apparent velocity discontinuity at the sharp interface boundary.

We must now consider two important questions. How can excellent separation exist in the presence of what appears to be high granular shear at the interface boundaries? Why is the convection entirely within each separated region rather than acting global? To address these questions we have examined the granular motion using high speed digital photography,

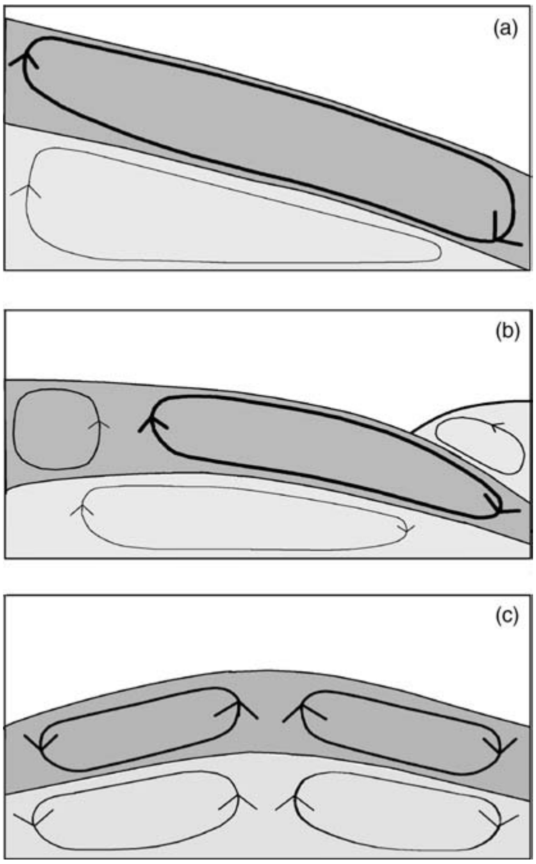

Fig. 6 Schematic diagram showing the convection patterns found in a separated "bronze on top" configuration, a, in the "tilted sandwich configuration, b, and in the "symmetric bronze on top" configuration, c. The thicker arrowed lines indicate faster flow than the thinner arrowed lines

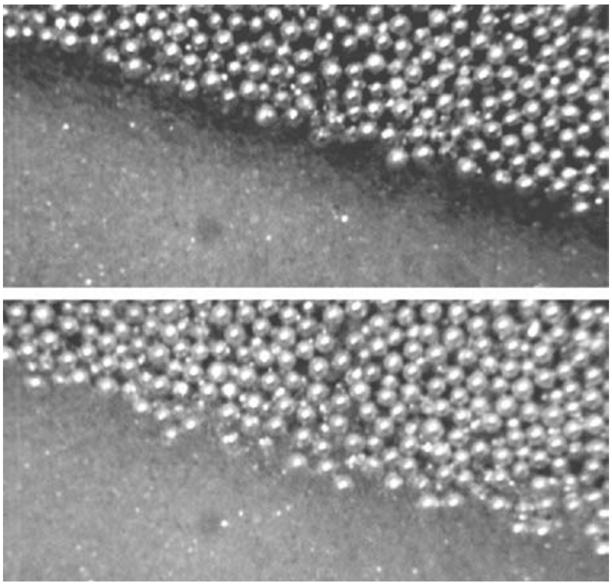

Fig. 7 High speed camera images of the separated "bronze on top" configuration observed for a 50% : 50% mixture of $600 - 710\mu m$ bronze and $300 - 355\mu m$ glass during vibration at 15Hz and $\Gamma = 5$. The bottom image was taken at a vibration phase when the gap is closed, while the top image was taken at the phase of maximum gap

studying the motion of the granular beds during flight. In water-immersed systems the vertical granular motion with respect to the box is much reduced from the values which would be expected in air or in vacuo, and we have been constrained to restrict our studies to the low frequency regime where the "bronze on top" configuration is found. In this region we clearly observe that gaps open up between the bottom of the box and the lower glass region over part of the vibration cycle and that a gap opens up between the lower glass region and the bronze region over a somewhat more extended part of the vibration cycle. Such a gap is shown in figure 7. Importantly, we observe that *the granular convection occurs only while the gap is open*. Thus convection occurs when the separated regions are not actually in contact with each other. This is the reason why separated beds are stable against what appears to be considerable velocity shear at the interface between them, and why distinct convection flows appear in the separated regions. The gaps reduce in size as the frequency is increased and it is difficult to confirm this mechanism using photography at frequencies above 50Hz. However, it seems likely that at high frequencies too, convective movement occurs principally during those phases in which there are adequate gaps between the separated regions.

5 The dynamics of multiple granular beds

The separation phenomena which we have described here have many similarities to the separation phenomena reported for fine binary mixtures in air [18, 19]. The mechanisms behind air-driven separation have been identified following a series of experimental studies and recent DEM simulation

[19,20]. A binary granular bed, initially well mixed, is thrown by vertical vibration of sufficient amplitude and air is forced first downwards and then upwards through it during each cycle. The component having the smaller value of the product ρd^2 will be more greatly influenced by the fluid flow and will tend to move with respect to the other component in response to the flow. However, clusters of grains of the same type will experience much the same forces and will tend to stay together. Over many cycles these processes may lead to almost complete separation with, at least at low frequencies, the less damped component, that component having the greater value of ρd^2, uppermost.

This explanation for separation, based on differential interaction with a fluid forced through a granular bed by vibration, is expected to apply quite generally. From our high speed camera observations of the present water-immersed systems and from the many similarities between the present observations and the separation effects observed for air we are confident that we are observing the results of a similar mechanism here.

We may gain some insight into the well separated state through an approximate approach based on Kroll's model for a vertically thrown bed. Kroll considered a bed of fixed porosity moving in one dimension as a single rigid object within an incompressible fluid [27]. From Newton's laws, conservation of fluid volume and Darcy's law for flow through the bed, Kroll derived an equation for the motion of the bed during flight. This treatment is valid for those ranges of frequency and Γ for which the bed takes off, undergoes flight, lands and settles within each cycle. Here we extend the work of Kroll to include multiple beds and the non-zero density of the fluid within which the beds are thrown.

Consider two horizontal layers of distinct composition, moving in the vertical direction within an incompressible fluid of density ρ_f (figure 8). The beds have been thrown from a platform moving as $z_p = a\sin(\omega t)$. The lower bed, of material labelled by the suffix 1 is of depth h_1, while the upper bed, of properties labelled 2 is of depth h_2. During flight, the pressures below these beds differ from ambient by ΔP_1 and ΔP_2, respectively. Application of Newton's laws leads to the following two flight equations for the beds.

$$h_1(1-\phi_1)(\rho_1+\lambda\rho_f)\ddot{z}_1 = -h_1(1-\phi_1)\rho_1 g + h_1(1-\phi_1)\rho_f(g+[1+\lambda]\ddot{z}_p) + \Delta P_1 - \Delta P_2 \quad (1)$$

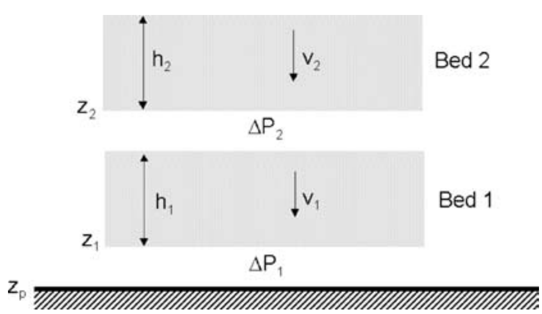

Fig. 8 Schematic diagram showing two granular beds, 1 and 2, being thrown vertically from a platform moving as $z_p(t)$

$$h_2(1-\phi_2)(\rho_2+\lambda\rho_f)\ddot{z}_2 = -h_2(1-\phi_2)\rho_2 g + h_2(1-\phi_2)\rho_f(g+[1+\lambda]\ddot{z}_p) + \Delta P_2 \quad (2)$$

Here ϕ_i ($i = 1,2$) is the bed porosity and g is the acceleration due to gravity. The term in $\lambda\rho_f$ on the left hand side of each equation represents the virtual mass correction due to the fluid, while the term in λ on the right hand side of each equation represents the corresponding correction for buoyancy in the accelerating frame. The virtual mass and buoyancy corrections are related by the requirement for correct behaviour if the whole system falls freely under gravity. For an isolated spherical particle, $\lambda = 0.5$ [28]. However, for a granular bed λ is not known [29]. We set λ to zero, noting that small values of λ do not greatly affect the gaps and landing times deduced from these equations. In addition, we have ignored "history corrections", also largely unknown for granular beds [28–31].

We now suppose that the pressure gradients across the two beds result in fluid flows through the beds of superficial velocities v_1 and v_2 respectively. In relating these fluid flows to the pressure gradients, we note that the Reynolds' number may appreciably exceed unity at low frequencies and high values of Γ. We therefore use the expressions

$$\Delta P_1 - \Delta P_2 = -h_1(L_1 v_1 + T_1|v_1|v_1) \quad (3)$$

and

$$\Delta P_2 = -h_2(L_2 v_2 + T_2|v_2|v_2) \quad (4)$$

Here L_1 and L_2 represent the laminar permeabilities of the two beds (Darcy's law), while the terms in T_1 and T_2 represent the non-linear corrections [33]. Finally, conservation of fluid volume offers

$$v_1 = \dot{z}_1 - \dot{z}_p \quad \text{and} \quad v_2 - v_1 = \dot{z}_2 - \dot{z}_1 \quad (5)$$

Note from the equations (5) that $v_2 = \dot{z}_2 - \dot{z}_p$.

The equations (3) and (4) may be used to eliminate ΔP_1 and ΔP_2 from equations (1) and (2). Using the equations (5), we then may write equations (1) and (2) in terms of the rates of change of the positions of the beds with respect to the platform, v_1 and v_2.

$$\dot{v}_1 + \gamma_1 v_1 + \delta_1|v_1|v_1 + \alpha_1 g = -\alpha_1\ddot{z}_p \quad (6)$$

$$\dot{v}_2 + \gamma_2 v_2 + \delta_2|v_2|v_2 + \alpha_2 g = -\alpha_2\ddot{z}_p \quad (7)$$

Here, the laminar fluid drag parameters, γ_i, are equal to $L_i/(\rho_i(1-\phi_i))$, while the non-linear terms δ_i are equal to $T_i/(\rho_i(1-\phi_i))$. The buoyancy parameter α_i is given by $(\rho_i - \rho_f)/\rho_i$. It should be noted that the equations (6) and (7) for the two beds are uncoupled; each bed is not influenced by the presence of the other while in separated flight. This is to be expected for motion within an incompressible fluid. Further, these flight equations do not contain either h_1 or h_2, the bed depths. A similar treatment of three or more layers also results in uncoupled flight equations of the same form. Again, the equations do not contain the bed thicknesses.

In applying this one-dimensional model to real situations, we have ignored horizontal motion of the grains and the fluid. The justification for this is that, over the range of f and

Γ that we have investigated, the horizontal motion is small compared to the vertical motion. Although these horizontal components lead to the slow granular convection such as that shown in Fig. 6, this motion is small within each vibrational cycle. Also, since each flight equation does not contain the bed thickness, all parts of a tilted bed or a bed of variable thickness will undergo the same vertical flight unless the horizontal components of the fluid flow are appreciable in comparison with the vertical components. In the present situations, the angles of the bed surfaces to the horizontal are usually less than 20^o and the horizontal fluid flows are not then expected to substantially modify the vertical flight equations.

A numerical solution of the equations (6) and (7), together with the motion of the platform, readily offers the time dependence of the bed positions, the gaps and the landing times for any specific situation. The porosity of a loose random packing of spheres is close to 0.42. The bed porosities ϕ_i (i = 1,2) are expected to be only slightly greater than this during flight [24, 25], especially so since the effective coefficient of restitution of liquid immersed grains is close to zero at low Stokes' numbers [32]. Although we observe that the kinetic activity in a bed of greater ρd^2 is, in flight, greater that the activity of a bed of lower ρd^2, we take the porosity of both to be 0.42. The γ_i and δ_i may be obtained from the bed equation of Ergun [33]

$$\gamma_i = \frac{150(1-\phi_i)\eta}{\phi_i^3 \rho_i d_i^2} \quad \text{and} \quad \delta_i = \frac{1.75\rho_f}{\phi_i^3 \rho_i d_i}. \tag{8}$$

Before we use this model, the range of applicability must be considered. For any particular particle size, there will be a range of low values of frequency and Γ for which a bed lands sufficiently early in the vibratory cycle for settling to occur before it takes off again in the next cycle. The dynamics of one cycle do not continue into the next. The model may be applied if this is true for each bed. However, at higher values of frequency and Γ, one or more beds may not have time to settle before they are next thrown. The dynamics spill over from one cycle to the next. Experimentally, complex behaviour may then be observed, the bed motions repeating with a period of more than one cycle. The assumptions of the model are not then valid.

The model may be used to estimate the limits of its applicability. We have indicated in figures 2a and 2b, using broken lines, the values of frequency and Γ for which a bronze bed is estimated to land 0.5 ms before its next take-off. For both particle sizes the broken lines correspond quite well to the boundaries lines "γ". It appears that the boundaries "γ" separate the conditions for repeated single cycle behaviour from those for continuous dynamics; it is within the continuous dynamics regime that the "sandwich" configuration appears to be metastable.

Good separation requires a strong separation mechanism that sorts the grains; in our case this is provided by the differential effect of the fluid. In addition, excellent and stable separation requires the presence of appreciable gaps, or at least regions between the beds sufficiently enhanced in porosity to allow relative motion during flight. Further, the beds should not collide during flight, when their porosity is higher than at rest. The model allows us to examine when these conditions should occur.

In considering the flight of a pair of beds we note that, in the absence of buoyancy corrections ($\alpha_i = 1$) and non-linearity, a gap will exist throughout flight if $\gamma_1 \neq \gamma_2$. This is equivalent to $S \neq 1$. This may be the case for a low density fluid such as air and for shallow beds where compressibility effects do not play an important role [32]. For water-immersed systems, however, differential buoyancy plays a significant role and non-linear drag may be significant. It is not then possible to exactly match the flights of separated layers of different particle size and density. Numerical solutions of equations 6 and 7 show that, for bronze and glass, collisions will occur in flight for a range of S. This range has an upper limit at about $S \approx 1.1$ for all particle sizes, depending only weakly on frequency and Γ. The lower limit of S is approximately 0.7 for smaller particles sizes and 0.4 for the larger particle sizes used here, depending somewhat upon frequency and Γ. Good separation is unlikely to occur within these ranges, whatever Γ is used.

For the situation where, following separation, a bed of **greater** ρd^2 (bed 2) lies above a bed of **lower** ρd^2 (bed 1), for example in a "bronze on top" configuration, a gap is possible throughout flight once S exceeds ≈ 1.1. Vibration of sufficient amplitude is also required, since the magnitude of the gap increases with Γ and a gap sufficient for relative motion between beds is likely to be on the scale of the particle diameter. We note that for mid-sized particles the "α" onset lines correspond to maximum gaps of about 0.1 particle diameters, suggesting that a gap of at least this order is a necessary condition for the effective separation of spherical grains. The values of Γ needed to achieve this maximum gap have been calculated for each particle size and for three frequencies and are included in Table 1 (figures in brackets). For very small particles and for low frequencies, very high values of Γ are needed to initiate grain-grain motion in heavily damped systems such as this, a phenomenon sometime known as "sludging" [1]. Granular motion leading to separation is only achieved for values of Γ corresponding to a gap already considerably greater than d/10. The "α" onset lines has been defined in terms of the observation of large-scale structure within a defined time scale. However, at high frequencies and for larger particle sizes, separation just above the "α" line is poor as figures 2a and 2b show. Very good separation only occurs for values of Γ well above the "α" line, again in agreement with the calculations presented in Table 1.

We may also consider the situation where, following separation, a bed of **lower** ρd^2 (bed 2) lies above a bed of **higher** ρd^2 (bed 1). In the present study, this situation is found, for example, in the central bronze and upper glass beds of the "tilted sandwich configuration" (figure 1b). Both the central and upper beds will take flight together at times given by $\sin(\omega t) = 1/\Gamma$. Early in flight, the gap opening below this pair of beds drives fluid down through them. Since the central bed is less affected by the fluid flow it would, if free, fly higher than the upper bed. It being constrained, the two beds stay together. Their common motion can be treated by a single flight equation. However, as the gap below this composite bed closes, fluid is force up through them. The glass and bronze layers will now separate and a gap will open between them. However, this gap will be considerably smaller than in the earlier case which we considered, that in which the bronze was above the glass. Separate convection cells may still exist in the presence of strong separation. However, in the absence of an adequate gap, grains may be lost from the more compact upper glass layer into the somewhat less compact bronze layer. In the long term, the configuration may not be as stable, reverting to the "bronze on top" configuration, as is observed experimentally.

We have seen that good separation is not expected for a range of $1.1 \geq S \geq 0.4, 0.7$ because of collisions in flight. However, the criterion that, for excellent separation, a gap of a size comparable to the particle diameter must open up between beds, extends both the upper and lower limits of the range of S offering poor separation by amounts which depend upon the particle sizes. It should be noted that, for bronze and glass grains, the great disparity in particle sizes may also influence separation for S substantially less than 1.0.

The model may also be used to compare the flights of air and water-immersed systems, taking the dynamic viscosity, η as 50 times higher for water than for air (2.10^{-5}Pa s for air, 1.10^{-3}Pa s for water). For binary systems of different density the bed flights cannot be matched precisely and we note that there is no exact scaling between air and water immersed systems involving a single scaling factor. However, over the frequency range 10–40Hz and for a thrown bronze bed, an approximate match occurs with $\rho_b \mathrm{d}_b^2$ about 7–7.5 times larger for water than for air. For a glass bed, even the best match is very poor and occurs for $\rho_g \mathrm{d}_g^2$ about 5–7 times higher. Experimentally, the size ranges for comparable separation occur for particle sizes about 5–6 times higher in water than in air, a little lower than the mean of the figures given above.

6 Summary

We have studied the behaviour of water-immersed bronze glass systems under vertical vibration and have found a range of conditions for almost complete separation. The presence of a fluid, in this case water, offers a strong separation mechanism in addition to the weak tendencies to separate through differences in size or density, tendencies which are often thwarted by convection. We have examined the conditions for excellent separation and have shown that they require the existence of an appreciable gap between the separated beds as they are thrown. Our results suggest that a maximum "gap" of at least $d/10$ is required to ensure a region of enhanced porosity sufficient to allow relative movement of the beds. It is the existence of these regions which enable separation to be maintained through local convection within each separated bed without global mixing.

The predictions of a simple model are used to examine the values of the parameter S for which poor separation is expected. The experimental observations are broadly in agreement with these predictions. The model suggests that the "tilted sandwich" configuration is unlikely to be stable, but will revert to the "tilted bronze on top" configuration, as we indeed observe. We also examine the ratio of particle sizes for similar separation phenomena in air and water immersed systems and again find reasonable agreement with experiment.

Our analysis also suggest that the "sandwich" behaviour found at higher values of frequency and Γ corresponds to dynamics which spill over from one vibration cycle to the next. It is likely that a more detailed model such as that offered by DEM simulation techniques will be needed to fully understand the bed dynamics and modes of separation in this region.

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