Influence of vibrational wave form on intruder clustering

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(Received 9 February 2006; accepted 16 May 2006; published online 29 June 2006)

It has recently been shown that, within a two-dimensional granular bed subjected to sinusoidal, vertical vibration, neutrally buoyant intruders attract each other over a distance of up to five intruder diameters. The interaction between these intruders is the net result of attractive and repulsive forces which occur at different parts of the vibration cycle. Here we show that these forces may be manipulated by altering the vibration wave form to vary the strength of the overall attraction, or even to produce a weak repulsion. This ability is important for controlling the mixing of larger grains held within a granular bed. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2218317]

The ability to separate or mix granular materials is of major importance in a variety of industrial processes, many of which involve mechanical vibration.[1,2] Numerous authors have considered the behavior of a single large intruder, a "Brazil nut," moving within a bed of smaller identical host particles subjected to vertical, sinusoidal vibration.[3] Whether the intruder rises or sinks depends principally upon the density of the intruder with respect to the density of the host grains.[4]

Recently, we reported simulations of a two-dimensional granular bed containing two or more larger intruders and subjected to sinusoidal, vertical vibration.[5] If the intruders are neutrally buoyant, it is found that two intruders are attracted to each other and several intruders tend to cluster. There is an effective attraction between neutrally buoyant intruders which has a range of about five intruder diameters. Subsequently this intruder-intruder attraction has been confirmed experimentally.[6]

Our computer simulations showed that the force between intruders varies over the cycle of sinusoidal oscillation.[5] The question then arises: If we change the oscillatory wave form from that of a pure sine wave, can the strength of the forces be manipulated so that the net effect is a reduced attraction or even a repulsion? Here we report experiments which use a wave form consisting of a fundamental plus a harmonic. It is shown that control of clustering may be achieved by adjusting the relative phase of the two components.

Our experiments used glass spheres with diameters in the range of 2–2.2 mm, and Dural disks of 6 mm diameter and 2.1 mm thickness as the intruders. Holes were drilled in the center of each disk to make them neutrally buoyant when the bed was fluidized by vertical vibration.[6] The spheres and disks were held within a rectangular glass cell of width of 175 mm and height of 77 mm, formed by two glass plates separated by 2.3 mm. Around 900 glass spheres were used, sufficient to form a bed 11 particles deep. The cell was vibrated on a transducer assembly designed to produce one-dimensional motion aligned to within 0.2° of the vertical, the motion being monitored using accelerometers. The phase and amplitude response of this system to pure sine waves was measured from 10 to 200 Hz with the cell in place, so that the applied voltage wave form required to produce an arbitrary oscillation of the cell could be deduced. Motion was recorded using a high speed camera; image recognition software was used to identify and track the behavior of the intruders.

We chose a wave form for the vertical displacement of the cell, $z(t)$, of the form

$$z(t) = A_1 \cos(2\pi f t) + A_2 \cos(2n\pi f t + 2\pi Q), \qquad (1)$$

where f is the fundamental frequency and Q is a phase parameter. Initially we chose a wave form consisting of a fundamental of 20 Hz and its second harmonic ($n=2$), setting $A_2/A_1=0.4$. For this wave form we observed that both the kinetic activity of the bed and the tendency of the intruders to cluster depended strongly upon Q. However, for values of A_1 which are sufficiently large for bed fluidization, the bed collides with the base on alternate cycles of vibration for some ranges of Q. The bed may then undergo "arching" in which the collisions occur on alternate cycles at the right and left hand sides of the bed.[7] There are substantial ranges of Q for which the motion changes erratically between a state where the whole bed is thrown in unison and an arching state. This made a detailed investigation of intruder behavior difficult, so an alternative wave form which did not present this problem was used for further studies.

A wave form based on a fundamental plus its fourth harmonic ($n=4$) was found to resolve the problem of arching. We used $f=10$ Hz and $A_2/A_1=0.40$, with A_1 and Q being used as the control parameters. Initially we set $A_1=2.8$ mm and studied the effect of varying Q. Seven intruder particles were placed in the central third of the cell, and their motion followed for a period of 100 s. This process was repeated a number of times so that the average behavior could be found.

For all values of Q the intruders remain neutrally buoyant. However, for some values of Q they cluster while for others they spread apart. In these shallow beds we are principally interested in the horizontal separation of the intruders. To quantify this we introduce a parameter

$$\Phi(t) = \frac{1}{N(N-1)} \sum_{i}^{N} \sum_{j \neq i}^{N} |x_i(t) - x_j(t)|, \qquad (2)$$

which provides a measure of how close the N intruders are in the horizontal direction. The quantity $x_i(t)$ is the horizontal position of intruder i from the center of the cell. $\Phi(t)$ is a measure of the average spacing of the intruders. For a heavily clustered group $\Phi(t)$ is small, whereas large values

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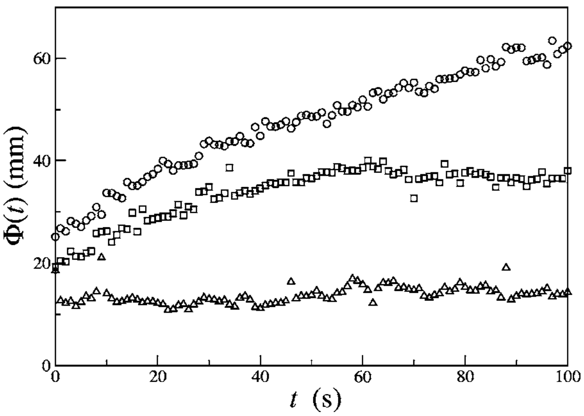

FIG. 1. The behavior of $\Phi(t)$ with time for $f=10$ Hz, $n=4$, and $A_1=2.8$ mm with $A_2/A_1=0.40$ and $Q=0.5$ (triangles), $Q=0.4$ (squares), and $Q=0.1$ (circles).

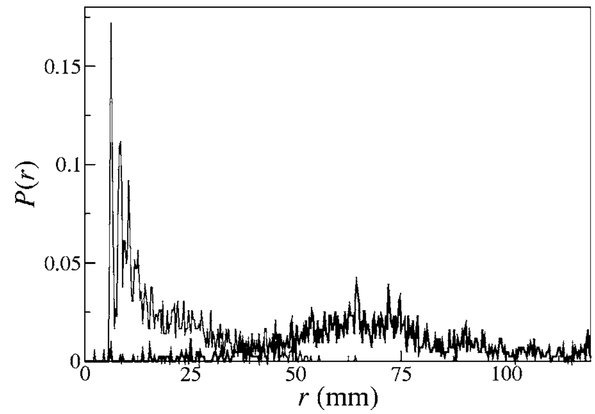

FIG. 3. The pair probability density $P(r)$ for intruders excited by a 10 Hz fundamental plus its fourth harmonic. The heavier line shows the data for $Q=0.1$, while the lighter line shows the data for $Q=0.5$. The data are taken with $A_1=2.8$ mm and $A_2/A_1=0.40$.

of $\Phi(t)$ indicate that the intruders are spread out. For $N=7$ intruders which are equally spaced in the x direction, the spacing between a pair of adjacent intruders is $3\Phi(t)/8$.

For each value of Q the average behavior of $\Phi(t)$ was obtained. The initial conditions were always the same: The seven intruders were grouped close together in the central third of the cell. During each run filming was begun 5 s after the application of vibration.

Figure 1 shows how $\Phi(t)$ evolves for three values of the phase parameter Q. For $Q=0.5$, the intruders bunch into a circular cluster and remain bunched over the period of measurement; $\Phi(t)$ remains roughly constant with a value of about 14 mm. For $Q=0.4$ the quantity $\Phi(t)$ increases but tends to a limit by 80 s. Clustering is weaker with the intruders strung out horizontally with a mean separation between adjacent intruders of about two intruder diameters. In contrast, for $Q=0.1$, the intruders are still moving apart at 100 s, by which time the mean spacing between adjacent intruders is about four diameters. In this case, the mean-square spacing increases roughly as t, the dependence expected for noninteracting particles undergoing a random walk. There is no longer any tendency to cluster.

Figure 2 shows the variation with Q of the mean value of $\Phi(t)$ between 80 and 100 s, $\bar{\Phi}$, for $A_1=2.8$ mm. The arrowed line indicates those values of Q for which $\Phi(t)$ has reached a plateau by 100 s. Throughout this region clustering occurs to some extent, with very tight bunching close to $Q=0.5$. In the region around $Q=0.1$, the region outside that indicated by the arrowed line, there is no evidence for an attractive interaction.

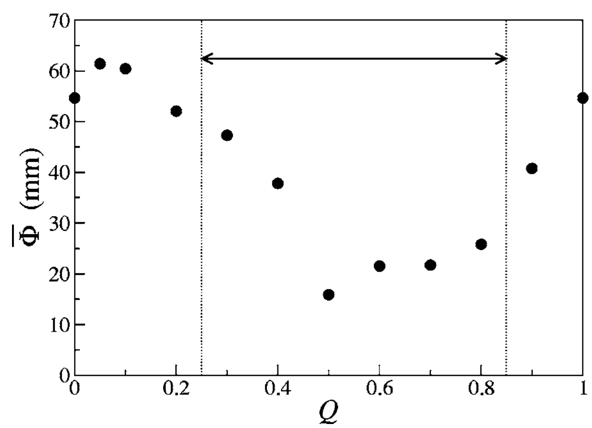

FIG. 2. The mean value of $\Phi(t)$ between 80 and 100 s, $\bar{\Phi}$, as a function of the phase parameter Q. Here $A_1=2.8$ mm and $A_2/A_1=0.40$. The arrowed line indicates those values of Q for which $\Phi(t)$ is constant at 100 s.

It is clear that by changing Q at fixed A_1 and A_2 we can cause the intruders to cluster strongly, to cluster weakly, or to move apart. For wave forms consisting of a fundamental plus its fourth harmonic, the kinetic activity of the bed does not appear from camera images to vary strongly with Q. However, the possibility exists that changing the wave form alters the average granular kinetic energy of the bed and that this increase in kinetic energy removes the tendency to cluster. To test this, further experiments were conducted to investigate the effect of increasing the kinetic activity through changes in A_1, keeping $A_2/A_1=0.40$ and $Q=0.1$. We have already seen in Fig. 1 that for $A_1=2.8$ mm the value of $\Phi(t)$ steadily increases with time; the intruders are not clustered. For $A_1=3.1$ mm, $\Phi(t)$ reaches the value of 55 mm where it remains, while for $A_1=3.5$ mm, $\Phi(t)$ quickly reaches a value of about 40 mm. The higher values of A_1, for which the kinetic activity is visually greater, correspond to stronger clustering than do lower values of A_1. Within this parameter range, shaking the system harder *increases* the clustering.

We have also examined the behavior of a pair of intruders. Two intruders were placed in the central region of the bed, horizontally 18 mm apart and half way up the bed. A number of values of Q were studied, all with A_1 set equal to 2.8 mm and with $A_2/A_1=0.40$. Many runs were carried out to obtain the average motion. A run was stopped if either of the intruders reached a sidewall of the cell. The positions of the intruders were measured from the digital footage, and the probability $P(r)dr$ of finding the intruders with separations between r and $r+dr$ was calculated. Figure 3 shows $P(r)$ for $Q=0.5$, a value for which strong clustering of multiple intruders has been observed. The figure shows that the intruders attract strongly, the form of $P(r)$ being very similar to that observed for sinusoidal excitation.[6] $P(r)$ is large at short distances, with a number of subsidiary peaks corresponding to integer numbers of host particles separating the two intruders. The data for $Q=0.1$, a value for which multiple intruders move apart, show a very different form of $P(r)$. There is an extremely small probability of finding the intruders at short separations. Rather, there is a very broad peak in the probability at around $r\simeq 70$ mm. This suggests that the intruders weakly repel one another and perhaps that they are weakly repelled by the end walls of the cell. The data for other values of Q show a gradual transition between these two forms.

What causes these clustering phenomena? From studies of purely sinusoidal excitation it has been observed that, just after the bed impacts with the base, a shock wave passes up through the bed.[8] When this wave reaches the intruders it tends to push them apart. However, during the subsequent bed flight the region just above the intruders develops a deficiency in host particles.[5] Consequently, as the bed lands, there are more collisions pushing the intruders together than pushing them apart. This attractive part of the process lasts until the next shock wave reaches the intruders. For sinusoidal vibration, the two effects lead to an overall net attraction.[5] We speculate that the strength of this attraction increases with flight time because there is more time for the deficiency in host particles to develop, favoring the attractive part of the process.

For nonsinusoidal vibration we have determined, from camera images, the vertical positions of the base of the cell and the bottom of the granular bed as a function of time. Figure 4 (top) shows the resulting trajectories for $Q=0.5$, a value for which maximum clustering occurs. During each cycle, the bed suffers three impacts, a, b, and c, after which a shock wave is observed to pass upward through the bed. We note that the flight time between a and b is long. Figure 4 (bottom) shows the corresponding diagram for $Q=0.1$, a value for which no clustering is observed. Here, the bed suffers four impacts, a–d, and the flight times are all relatively short. Thus we see that for $Q=0.5$, for which clustering occurs, there are only three impacts associated with repulsion but a long flight time needed for attraction. On the other hand, for $Q=0.1$, there are more impacts per cycle and shorter flight times. We believe that it is these differences in the number of impacts and flight times which change the degree of clustering.

It has been shown by experiment that nonsinusoidal wave forms may be used to manipulate the interaction between neutrally buoyant intruders vibrated within a granular bed of finer particles. The parameter space of nonsinusoidal wave forms is boundless, and we have only investigated a small region. Nevertheless, our results have important implications for the manipulation of granular materials, including the possibility of efficiently creating homogeneous granular mixtures.

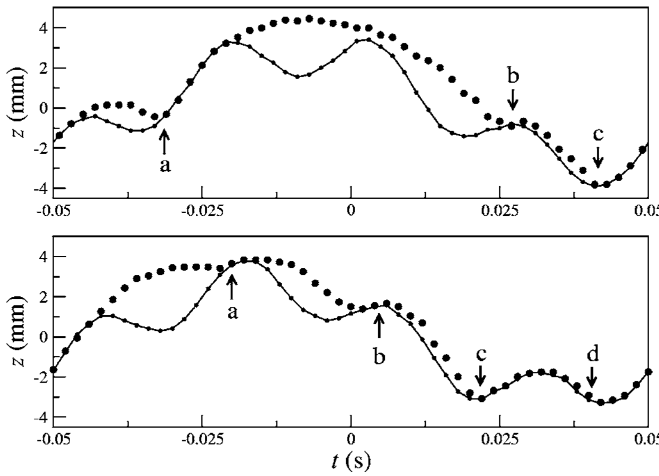

FIG. 4. The effect of throwing the granular bed using a wave form consisting of a fundamental and its fourth harmonic. Here $f=10$ Hz, $A_1=2.8$ mm, $A_1/A_2=0.40$, and $Q=0.5$ (top) and $Q=0.1$ (bottom). In each case the motion of the cell is shown as a continuous line, and the motion of the bed is shown as bold filled circles. The arrows indicate bed impacts.

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