The following PhD vacancies and research topics within the School of Mathematical Sciences were compiled in November 2013 and were correct at the time of publication.

A regularly updated list of available PhD projects can be viewed online at:

www.maths.nottingham.ac.uk/postgraduate/projects

For further guidance on pursuing a PhD in any of these areas, please consult the School of Mathematics website or contact the relevant members of academic staff as listed below.

Algebra and Analysis

LOT Groups

Supervisor: Martin Edjvet (martin.edjvet@nottingham.ac.uk)

A labelled oriented graph is a connected, finite, directed graph in which each edge is labelled by a vertex. A labelled oriented graph gives rise to a group presentation whose generating set is the vertex set and whose defining relations say that the initial vertex of an edge is conjugated to its terminal vertex by its label. A group G is a LOG group if it has such a presentation. A LOT group is a LOG group for which the graph is a tree. Every classical knot group is a LOT group. In fact LOT groups are characterised as the fundamental groups of ribbon n-discs in Dn+2. The most significant outstanding question on the topology of ribbon discs is: are they aspherical? The expected answer is yes. Howie has shown that it is sufficient to prove that LOT groups are locally indicable. An interesting research project would be to study the following two questions: is every LOT group locally indicable; is every LOT group an HNN extension of a finitely presented group?


Equations over groups

Supervisor: Martin Edjvet (martin.edjvet@nottingham.ac.uk)

Let G be a group. An expression of the form g1 t … gk t=1 where each gi is an element of G and the unknown t is distinct from G is called an equation over G. The equation is said to have a solution if G embeds in a group H containing an element h for which the equation holds. There are two unsettled conjectures here. The first states that if G is torsion-free then any equation over G has a solution. The second due to Kervaire and Laudenbach states that if the sum of the exponents of t is non-zero then the equation has a solution. There have been many papers published in this area. The methods are geometric making use of diagrams over groups and
curvature. This subject is related to questions of asphericity of groups which could also be studied.

**Quadratic forms and forms of higher degree, nonassociative algebras**

**Supervisor:**  **Susanne Pumpluen** ([susanne.pumpluen@nottingham.ac.uk](mailto:susanne.pumpluen@nottingham.ac.uk))

Dr. Pumplün currently studies forms of higher degree over fields, i.e. homogeneous polynomials of degree \(d\) greater than two (mostly over fields of characteristic zero or greater than \(d\)). The theory of these forms is much more complex than the theory of homogeneous polynomials of degree two (also called quadratic forms). Partly this can be explained by the fact that not every form of degree greater than two can be “diagonalized”, as it is the case for quadratic forms over fields of characteristic not two. (Every quadratic form over a field of characteristic not two can be represented by a matrix which only has non-zero entries on its diagonal, i.e. is diagonal.)

A modern uniform theory for these forms like it exists for quadratic and symmetric bilinear forms (cf. the standard reference books by Scharlau or Lam) seems to be missing, or only exists to some extent. Many questions which have been settled for quadratic forms quite some time ago are still open as as soon as one looks at forms of higher degree. It would be desirable to obtain a better understanding of the behaviour of these forms. First results have been obtained. Another related problem would be if one can describe forms of higher degree over algebraic varieties, for instance over curves of genus zero or one.

Dr. Pumplün is also studying nonassociative algebras over rings, fields, or algebraic varieties. Over rings, as modules these algebras are finitely generated over the base ring. Their algebra structure, i.e. the multiplication, is given by any bilinear map, such that the distributive laws are satisfied. In other words, the multiplication is not required to be associative any more, as it is usually the case when one talks about algebras. Her techniques for investigating certain classes of nonassociative algebras (e.g. octonion algebras) include elementary algebraic geometry. One of her next projects will be the investigation of octonion algebras and of exceptional simple Jordan algebras (also called Albert algebras) over curves of genus zero or one. Results on these algebras would also imply new insights on certain algebraic groups related to them.

Another interesting area is the study of quadratic or bilinear forms over algebraic varieties. There are only few varieties of dimension greater than one where the Witt ring is known. One well-known result is due to Arason (1980). It says that the Witt ring of projective space is always isomorphic to the Witt ring of the base field.

If you want to investigate algebras or forms over algebraic varieties, this will always involve the study of vector bundles of that variety. However, even for algebraically closed base fields it is usually very rare to have an explicit classification of the vector bundles. Hence, most known results on quadratic (or symmetric bilinear) forms are about the Witt ring of quadratic forms, e.g. the Witt ring of affine space, the projective space, of elliptic or hyperelliptic curves. An
explicit classification of symmetric bilinear spaces is in general impossible because it would involve an explicit classification of the corresponding vector bundles (which admit a form). There are still lots of interesting open problems in this area, both easier and very difficult ones.

Cohomology Theories for Algebraic Varieties

Supervisor: Alexander Vishik (alexander.vishik@nottingham.ac.uk)

After the groundbreaking works of V. Voevodsky, it became possible to work with algebraic varieties by completely topological methods. An important role in this context is played by the so-called Generalized Cohomology Theories. This includes classical algebraic K-theory, but also a rather modern (and more universal) Algebraic Cobordism theory. The study of such theories and cohomological operations on them is a fascinating subject. It has many applications to the classical questions from algebraic geometry, quadratic form theory, and other areas. One can mention, for example: the Rost degree formula, the problem of smoothing algebraic cycles, and u-invariants of fields. This is a new and rapidly developing area that offers many promising directions of research.

Quadratic Forms: Interaction of Algebra, Geometry and Topology

Supervisor: Alexander Vishik (alexander.vishik@nottingham.ac.uk)

From the beginning of the 20th century it was observed that quadratic forms over a given field carry a lot of information about that field. This led to the creation of rich and beautiful Algebraic Theory of Quadratic Forms that gave rise to many interesting problems. But it became apparent that quite a few of these problems can hardly be approached by means of the theory itself. In many cases, solutions were obtained by invoking arguments of a geometric nature. It was observed that one of the central questions on which quadratic form theory depends is the so-called "Milnor Conjecture". This conjecture, as we now understand it, relates quadratic forms over a field to the so-called motivic cohomology of this field. Once proven, this would provide a lot of information about quadratic forms and about motives (algebro-geometric analogues of topological objects) as well. The Milnor Conjecture was finally settled affirmatively by V. Voevodsky in 1996 by means of creating a completely new world, where one can work with algebraic varieties with the same flexibility as with topological spaces. Later, this was enhanced by F. Morel, and now we know that quadratic forms compute not just the cohomology of a point in the "algebro geometric homotopic world", but also the so-called stable homotopy groups of spheres as well. It is thus no wonder that these objects indeed have nice properties.

Therefore, by studying quadratic forms, one actually studies the stable homotopy groups of spheres, which should shed light on the classical problem of computing such groups (one of the central questions in mathematics as a whole). So it is fair to say that the modern theory of quadratic forms relies heavily on the application of motivic topological methods. On the other
hand, the Algebraic Theory of Quadratic Forms provides a possibility to view and approach the
devic world from a rather elementary point of view, and to test the new techniques developed
there. This makes quadratic form theory an invaluable and easy access point to the forefront of
modern mathematics.

Regularity conditions for Banach function algebras

Supervisor: Joel Feinstein (joel.feinstein@nottingham.ac.uk)

Banach function algebras are complete normed algebras of bounded, continuous, complex-valued functions defined on topological spaces. There are very many different examples with a huge variety of properties. Two contrasting examples are the algebra of all continuous complex-valued functions on the closed unit disc, and the subalgebra of this algebra consisting of those functions which are continuous on the closed disc and analytic on the interior of the disc. In the second of these algebras, any function which is zero throughout some non-empty open set must be constantly zero. This is very much not the case in the bigger algebra: indeed Urysohn’s lemma shows that for any two disjoint closed subsets of the closed disc, there is a continuous, complex-valued function defined on the disc which is constantly 0 on one closed set and constantly 1 on the other (algebras of this type are called regular algebras).

Most Banach function algebras have some features in common with one or the other of these two algebras. The aim of this project is to investigate a variety of conditions, especially regularity conditions, for Banach function algebras, and to relate these conditions to each other, and to other important conditions that Banach function algebras may satisfy.

Regularity conditions have important applications in several areas of functional analysis, including automatic continuity theory and the theory of Wedderburn decompositions. There is also a close connection between regularity and the theory of decomposable operators on Banach spaces.

Properties of Banach function algebras and their extensions

Supervisor: Joel Feinstein (joel.feinstein@nottingham.ac.uk)

Banach function algebras are complete normed algebras of bounded continuous, complex-valued functions defined on topological spaces. There are very many different examples with a huge variety of properties. Two contrasting examples are the algebra of all continuous complex-valued functions on the closed unit disc, and the subalgebra of this algebra consisting of those functions which are continuous on the closed disc and analytic on the interior of the disc. In the second of these algebras, any function which is zero throughout some non-empty open set must be constantly zero. This is very much not the case in the bigger algebra: indeed Urysohn’s lemma shows that for any two disjoint closed subsets of the closed disc, there is a continuous,
complex-valued function defined on the disc which is constantly 0 on one closed set and constantly 1 on the other (algebras of this type are called regular algebras).

Most Banach function algebras have some features in common with one or the other of these two algebras. The aim of this project is to investigate a variety of conditions (including regularity conditions) for Banach function algebras, to relate these conditions to each other, and to other important conditions that Banach function algebras may satisfy, and to investigate the preservation or introduction of these conditions when you form various types of extension of the algebras (especially ‘algebraic’ extensions such as Arens-Hoffman or Cole extensions).

**Meromorphic Function Theory**

Supervisor:  **James Langley** ([james.langley@nottingham.ac.uk](mailto:james.langley@nottingham.ac.uk))

A meromorphic function is basically one convergent power series divided by another: such functions arise in many branches of pure and applied mathematics. Professor Langley has supervised ten PhD students to date, and specific areas covered by his research include the following.

Zeros of derivatives. There is a long history of work on these questions which can be viewed as natural generalizations of the classical Picard theorem to the effect that an entire function which omits two values is a constant. Langley’s best known result is probably his 1993 proof of Hayman’s 1959 conjecture on the zeros of a meromorphic function and its second derivative. More recently, Langley, Bergweiler and Eremenko settled the last outstanding case of a conjecture of Wiman from 1911 concerning the derivatives of real entire functions, and Langley has extended much of this work to meromorphic functions.

Differential equations in the complex domain, in particular the growth and oscillation of solutions of linear differential equations. This has been an active area in recent years and there are potential applications.

Properties of compositions of entire functions. This area has connections to complex dynamics, in which the famous Julia and Mandelbrot sets arise, and which has been a very successful field of research in recent years.

Critical points of potentials associated with distributions of charged points and wires, and zeros of related meromorphic functions. This research was awarded an EPSRC Project Studentship in 2003-4.

**Compensated convex transforms and their applications**

Supervisor:  **Kewei Zhang** ([kewei.zhang@nottingham.ac.uk](mailto:kewei.zhang@nottingham.ac.uk))
This aim of the project is to further develop the theory and numerical methods for compensated convex transforms introduced by the proposer and to apply these tools to approximations, interpolations, reconstructions, image processing and singularity extraction problems arising from applied sciences and engineering.

Endomorphisms of Banach algebras

Supervisor:  **Joel Feinstein** ([joel.feinstein@nottingham.ac.uk](mailto:joel.feinstein@nottingham.ac.uk))

Compact endomorphisms of commutative, semisimple Banach algebras have been extensively studied since the seminal work of Kamowitz dating back to 1978. More recently the theory has expanded to include power compact, Riesz and quasicompact endomorphisms of commutative, semiprime Banach algebras.

This project concerns the classification of the various types of endomorphism for specific algebras, with the aid of the general theory. The algebras studied will include algebras of differentiable functions on compact plane sets, and related algebras such as Lipschitz algebras.

Analytic functions, quasiregular mappings, and iteration

Supervisor:  **Daniel Nicks** ([dan.nicks@nottingham.ac.uk](mailto:dan.nicks@nottingham.ac.uk))

Complex dynamics is the study of iteration of analytic functions on the complex plane. This has been a remarkably active area of research in recent years. A rich mathematical structure is seen to emerge amidst the chaotic behaviour. Its appeal is enhanced by the intricate nature of the Julia sets that arise, and fascinating images of these fractal sets are widely admired.

Complex dynamics exploits many beautiful results from classical complex analysis. Among these is Nevanlinna's theory of value distribution, which offers a powerful generalisation of Picard's theorem that an entire function omitting two values must be constant.

Quasiregular mappings of n-dimensional real space generalise analytic functions on the complex plane. One can therefore attempt to develop a theory of quasiregular iteration parallel to the results of complex dynamics. Such a theory is just beginning to emerge, lying between the well-studied analytic case (where many powerful tools are available) and general iteration in several real variables, which is much less well-understood.

Hydrodynamic limit of Ginzburg-Landau vortices

Supervisor:  **Matthias Kurzke** ([matthias.kurzke@nottingham.ac.uk](mailto:matthias.kurzke@nottingham.ac.uk))
Many quantum physical systems (for example superconductors, superfluids, Bose-Einstein condensates) exhibit vortex states that can be described by Ginzburg-Landau type functionals. For various equations of motion for the physical systems, the dynamical behaviour of finite numbers of vortices has been rigorously established. We are interested in studying systems with many vortices (this is the typical situation in a superconductor). In the hydrodynamic limit, one obtains an evolution equation for the vortex density. Typically, these equations are relatives of the Euler equations of incompressible fluids: for the Gross-Pitaevskii equation (a nonlinear Schrödinger equation), one obtains Euler, for the time-dependent Ginzburg-Landau equation (a nonlinear parabolic equation), one obtains a dissipative variant of the Euler equations.

There are at least two interesting directions to pursue here. One is to extend recent analytical progress for the Euler equation to the dissipative case. Another one is to obtain hydrodynamic limits for other motion laws (for example, mixed or wave-type motions).

This is a project mostly about analysis of PDEs, possibly with some numerical simulation involved.


**Dynamics of boundary singularities**

**Supervisor:** Matthias Kurzke ([matthias.kurzke@nottingham.ac.uk](mailto:matthias.kurzke@nottingham.ac.uk))

Some physical problems can be modelled by a function or vector field with a near discontinuity at a point. Specific examples include boundary vortices in thin magnetic films, and some types of dislocations in crystals. Typical static configurations can be found by minimizing certain energy functionals. As the core size of the singularity tends to zero, these energy functionals are usually well described by a limiting functional defined on point singularities.

This project investigates how to obtain dynamical laws for singularities (typically in the form of ordinary differential equations) from the partial differential equations that describe the evolution of the vector field. For some such problems, results for interior singularities are known, but their boundary counterparts are still lacking.

This project requires some background in the calculus of variations and the theory of partial differential equations.


**Vectorial Calculus of Variations, Material Microstructure, Forward-Backward Diffusion Equations and Coercivity Problems**
Supervisor: **Kewei Zhang** ([kewei.zhang@nottingham.ac.uk](mailto:kewei.zhang@nottingham.ac.uk))

This aim of the project is to solve problems in vectorial calculus of variations, forward-backward diffusion equations, partial differential inclusions and coercivity problems for elliptic systems. These problems are motivated from the variational models for material microstructure, image processing and elasticity theory. Methods involve quasiconvex functions, quasiconvex envelope, quasiconvex hull, Young measure, weak convergence in Sobolev spaces, elliptic and parabolic partial differential equations, and other analytic and geometric tools.

**Industrial and Applied Mathematics**

**Crystallisation in Polymers**

Supervisor: **Richard Graham** ([richard.graham@nottingham.ac.uk](mailto:richard.graham@nottingham.ac.uk))

Polymers are very long chain molecules and many of their unique properties depend upon their long chain nature. Like simple fluids many polymer fluids crystallise when cooled. However, the crystallisation process is complicated by the way the constituent chains are connected, leading to many curious and unexplained phenomena. Furthermore, if a polymer fluid is placed under flow, this strongly affects both the ease with which the polymer crystallises and the arrangement of the polymer chains within the resulting crystal. This project will develop and solve models for polymer dynamics and phase transitions using a range of analytical, numerical and stochastic techniques, with the ultimate aim of improving our understanding of polymer crystallisation. The project offers the opportunity to collaborate with a wide range of scientists working in the field, including several world-leading experimental groups.

**Dynamics of entangled polymers**

Supervisor: **Richard Graham** ([richard.graham@nottingham.ac.uk](mailto:richard.graham@nottingham.ac.uk))

Polymers are extraordinarily long molecules, made out of chains of simpler molecules. They occur everywhere in our everyday lives, including in the DNA chains that make up our genetics, in many high-tech consumer products and in the simple plastic bag. Often these applications depend crucially on the way that the polymer chains move. This is especially true in concentrated polymer liquids, where the chain dynamics are controlled by how the chains become entangled with each other. A powerful mathematical framework for describing these entangled systems has been under development for some time now, but the ideas have yet to be fully developed, tested and exploited in practical applications. Working on this PhD project will give the opportunity to train in a wide range of mathematical techniques including analytical work, numerical computations and stochastic simulation and to apply these to problems of real practical impact. This lively research field involves mathematicians, scientists and engineers and
a keenness to learn from and co-operate with researchers from a range of backgrounds would be a real asset in this project.

**Instabilities of fronts**

Supervisors: **Stephen Cox** ([stephen.cox@nottingham.ac.uk](mailto:stephen.cox@nottingham.ac.uk))  
**Paul Matthews** ([paul.matthews@nottingham.ac.uk](mailto:paul.matthews@nottingham.ac.uk))

Chemical reactions often start at a point and spread through a reactant, much as a fire spreads through combustible material. The advancing zone in which the reactions take place is called a reaction front. In the simplest cases, the reaction front is smooth (flat, cylindrical or spherical), but it may develop irregularities due to instability. Sometimes the instability is so strong that it destroys the front itself; in other cases, it just results in a slight modulation to the front shape. This project involves studying a partial differential equation, the Nikolaevskiy equation, that describes the nonlinear development of the instability of a front. Numerical simulations of the Nikolaevskiy equation show highly complicated, chaotic solutions. This project will involve a mix of numerical simulations and analytical work to understand the behaviour of the Nikolaevskiy equation and of front instabilities in general.

**Power converters**

Supervisors: **Stephen Cox** ([stephen.cox@nottingham.ac.uk](mailto:stephen.cox@nottingham.ac.uk))  
**Stephen Creagh** ([stephen.creagh@nottingham.ac.uk](mailto:stephen.creagh@nottingham.ac.uk))

In a wide range of applications, it is necessary to convert one electrical power supply to another, of different voltage or frequency. Power converters are devices which achieve this, but they often suffer highly undesirable instabilities, which significantly compromise their operation. The goal of this project is to develop mathematical models for existing power converter technologies and to use these to provide a detailed description of their operation and a thorough understanding of the instability. Through mathematical modelling, it may prove possible to improve existing power converter designs to reduce or eliminate the stability problems! This project will be theoretical in nature, relying largely on analytical and numerical techniques for differential equations, and will involve significant interaction with the Power Electronics Group in the Department of Electrical and Electronic Engineering.


**Class-D audio amplifiers**

**Supervisors: Stephen Cox** ([stephen.cox@nottingham.ac.uk](mailto:stephen.cox@nottingham.ac.uk))  
Stephen Creagh ([stephen.creagh@nottingham.ac.uk](mailto:stephen.creagh@nottingham.ac.uk))

The holy grail for an audiophile is distortion-free reproduction of sound by amplifier and loudspeaker. This project concerns the mathematical modelling and analysis of class-D audio amplifiers, which are highly efficient and capable of very low distortion. Designs for such amplifiers have been known for over 50 years, but only much more recently have electronic components been up to the job, making class-D amplifiers a reality. (Class-D amplifiers rely on very high frequency - around 1MHz - sampling of the input signal, and so test their components to the limit.) Unfortunately, while the standard class-D design offers zero distortion, it has poor noise characteristics; when the design is modified by adding negative feedback to reduce the noise, the amplifier distorts. By a further modification to the design it is possible to eliminate (most of) the distortion. This project involves modelling various class-D designs and determining their distortion characteristics, with the aim of reducing the distortion. The project will be largely analytical, applying asymptotic methods and computer algebra to solve the mathematical models. Simulations in matlab or maple will be used to test the predictions of the mathematical models.


**Mathematical modelling and analysis of composite materials and structures**

**Supervisor: Konstantinos Soldatos** ([kostas.soldatos@nottingham.ac.uk](mailto:kostas.soldatos@nottingham.ac.uk))

Nottingham has established and maintained, for more than half a century, world-wide research leadership in developing the Continuum Theory of fibre-reinforced materials and structures. Namely, a theoretical mechanics research subject with traditional interests to engineering and, more recently, to biological material applications. The subject covers extensive research areas...
of mathematical modelling and analysis which are of indissoluble adherence to basic understanding and prediction of the elastic, plastic, visco-elastic or even viscous (fluid-type) behaviour observed during either manufacturing or real life performance of anisotropic, composite materials and structural components.

Typical research projects available in this as well as in other relevant research subjects are related with the following interconnected areas:

Linearity and non-linear elasticity, static and dynamic analysis of homogeneous and inhomogeneous, anisotropic, solid structures and structural components;

Forming flow (process modelling) of fibre-reinforced viscous resins;

Plastic behaviour modelling as well as study of failure mechanisms/modes of fibrous-composites;

Mathematical modelling and analysis of the behaviour of thin walled, anisotropic structures in terms of high-order, linear or non-linear, ordinary and partial differential equations;

Development and/or use of various analytical, semi-analytical and/or numerical mathematical methods suitable for solving the sets of differential equations which emerge in the above research areas.

The large variety of topics and relevant problems emerging in these subjects of Theoretical Mechanics and Applied Mathematics allow considerable flexibility in the formation of PhD projects. A particular PhD project may accordingly be formed/designed around the strong subjects of knowledge of a potential post-graduate student. The candidate’s relevant cooperation is accordingly desirable and, as such, will be appreciated at the initial, but also at later stages of tentative research collaboration.

**Dynamics of coupled nonlinear oscillators**

**Supervisor:**  **Paul Matthews** ([paul.matthews@nottingham.ac.uk](mailto:paul.matthews@nottingham.ac.uk))

Coupled oscillators arise in many branches of science and technology and also have applications to biological systems. One spectacular example is swarms of fireflies that flash in synchrony. This research field is an expanding area in applied mathematics because of the many applications within physics and biology and because of the variety of behaviour which such systems can exhibit. Recent work on coupled oscillators has revealed some interesting novel results: nonlinear oscillators can synchronise to a common oscillation frequency even if they have different natural frequencies, provided the coupling is above some threshold; the breakdown of synchronisation as the coupling strength decreases involves periodic behaviour and chaos. The project involves extending and improving this work in two ways. First, the oscillator model used in earlier work was simple and idealised; the model will be refined to make it more realistic.
Second, most earlier work used a simple linear global coupling so that each oscillator is equally coupled to all of the others. In most practical examples this is not the case and a coupling law over a two- or three-dimensional lattice would be more appropriate, with stronger coupling between nearer pairs of oscillators. The research will be carried out using a combination of numerical and analytical techniques.

Dynamo action in convection

Supervisor: Paul Matthews (paul.matthews@nottingham.ac.uk)

The magnetic fields of the Earth and Sun are maintained by dynamo action. Fluid motions are generated by thermal convection. The kinetic energy of these fluid motions is then converted to magnetic energy, in a manner similar to that of a bicycle dynamo. Dynamo theory studies how this conversion takes place. It is known that in order for a dynamo to work efficiently, the fluid flow must exhibit chaos.

This project will investigate dynamo action in convection, using 3D numerical simulation of the equations for the fluid motion and the magnetic field. An existing computer program will be used to study the dynamo problem. A sequence of numerical simulations will be carried out to determine the conditions under which convection is able to generate a magnetic field, whether rotation is necessary or advantageous for dynamo action, the underlying topological stretching mechanism responsible for the magnetic field generation.

The project is also suitable for analytical work, either based on an asymptotic analysis of the equations, or in investigating or proving 'anti-dynamo' theorems.

Nonlinear penetrative convection

Supervisor: Paul Matthews (paul.matthews@nottingham.ac.uk)

The phenomenon of convection, in which heat is transferred by fluid motion, occurs very commonly in nature. Examples include in the Earth's atmosphere, the interior of the Sun, the Earth's liquid outer core, lakes and oceans. The most commonly used mathematical model for convection assumes that a layer of fluid is bounded above and below by boundaries that are maintained at a fixed temperature. This is not a good model for most of the environmental applications, where typically part of the fluid layer is thermally unstable and part is stable. Convection in the unstable layer overshoots and penetrates into the stable layer. This phenomenon, known as 'penetrative convection', has received relatively little investigation. The research project will study penetrative convection in the nonlinear regime. An existing computer program will be adapted to investigate penetrative convection numerically, and analytical work will be carried out using asymptotic methods and methods of bifurcation theory. In particular, the extent of penetration into the stable layer and the possibility of instability to a mean flow will be explored.
Coupling between optical components

Supervisor:   Stephen Creagh (stephen.creagh@nottingham.ac.uk)

Evanescent coupling between different optical components is a very important process in optical communications. In this effect, light travelling along an optical fibre effectively spills out a little bit into the region of space immediately surrounding the fibre itself and can then leak into and become captured by other, nearby optical components. Among other uses, this mechanism forms a basis for optical switches, which transfer light from one fibre to another, and for wavelength filters, which selectively transmit or redirect light in only certain frequency ranges.

This project will investigate the coupling between cylindrical and spherical optical components in two and three dimensions using the geometry of the underlying ray solutions. The aim will be to exploit and generalise approximations which have been developed in the context of quantum waves but which should be equally applicable to the optics problem.

Modelling of Advanced Gas Film Bearings

Supervisor:   Stephen Hibberd (stephen.hibberd@nottingham.ac.uk)

High speed gas film bearings are designed to work with no contact and very small gaps for a range of industrial applications. The application of gas-lubrication has the potential to provide improved bearing performance, particularly in very high differential speed operation, compared to existing classical bearings. Current applications of gas film bearings include hard disk drives, industrial generators and small engines. Such bearings have inherent advantages, particularly if the design of the bearing can provide hydrodynamic capabilities, i.e. making use of local dynamic effects, to maintain sufficient gap between the rotating parts. Additional advantages can be obtained from reducing the bearing gap however this could also result in catastrophic collision between the rotor and stator from induced vibrations, particularly at increasingly high rotation rates. Bearing development is currently focussing on designs that rotate at extremely small clearances and high speeds. It is these characteristics that can lead to significant instabilities as a result of the dynamic behaviour of the gas film and potential thermal and mechanical distortions.

Mathematical modelling of high speed gas-lubricated bearings is ongoing at Nottingham in collaboration with Rolls-Royce Aerospace plc (via a University Technology Centre (UTC)). The fundamental fluid dynamics is of compressible air films and classical extensions of an analysis from (Reynolds) lubrication theory for thin film flow. However, in practical applications the dynamics of the fluid-structure interaction, innovations in the bearing surface design to enhance lift (steps, grooves and foils), together with possible thermal considerations in the air gap are required. The relative displacements of components can be considerable and thus can only realistically be described by nonlinear analysis. Several recent and current PhD studies have
identified the importance of significant mathematical and numerical modelling aspects to inform fundamental development of ‘next-generation’ air-film technology.

A background in applied mathematics is required for the following PhD project and an interest in fluid mechanics and mathematical and numerical methods. Projects will be supervised by Dr S Hibberd, Prof KA Cliffe (Mathematical Sciences) and Prof H Power (Engineering) and advised by a member of the Advanced Seal Team, Rolls-Royce Aerospace plc.

**Reduced order air-film models**

For modelling general air-flow domains (e.g. designs with limited symmetry) combined with the addition of lift generating features with complex geometries the existing classical Reynolds equation models for the air-film need to be refined. Enhanced air-bearing studies have been developed at Nottingham in which an amount of complexity has been introduced into the modelling of air-films. Numerical modelling approaches have also been evaluated particularly for transient simulations with a coupling to a detailed structural model. Within an industrial context an important aspect for use in design is that the modelling of the processes are effective in incorporating the key physical aspects, providing sufficiently accurate results and within a manageable computation timeframe. Thus a challenge in the modelling is to incorporate all these aspects. A PhD project developing the mathematical and dynamic evaluation will have a broad remit including both the generation of the model and the numerical solutions. An initial problem will be to link the dynamics of a film model to that of a stator/rotor structural model.

**Robust modelling of advanced lift generating features**

An active area of research activity is air-film modelling of flow with lift generating features. An asymptotic approach has been used to link modelling discontinuous air-film thicknesses due to over simple radial and azimuthal steps. In practice, complex lift generating features that include spiral grooves, herring bone grooves, recessed pockets etc. are used. Such features are incorporated to enhance the hydrodynamic characteristics of the bearings and provide improved steady-state dynamic stability through producing additional lift. Modelling these systems is limited – typically using general numerical flow solvers. A PhD project is directed toward the analysis of these advanced lift generating features. A significant component of this project would be the creation of a modelling approach and numerical scheme capable of handling the complex geometry of the different lift generating features. Using this new modelling capability the flow behaviour generated by the different types of lift generating features will be analysed and compared. A capability to model the flow behaviour driven by a range of lift generating features would enable further investigation to optimise performance, for example to produce the most lift. This may offer an option to employ formal optimisation techniques to the problem. An aim is to establish sophisticated ways of mathematically ‘reducing’ the complexity of a model and demonstrating robustness and understanding what detailed information is lost due to model reduction.


Optical limits of left-handed media

Supervisor: Keith Hopcraft (keith.hopcraft@nottingham.ac.uk)

Recent investigations into the optics of 'left-handed' materials with refractive index equal to -1 have revealed the possibility of creating a super-lens that is not diffraction limited, these being capable of resolving arbitrarily finely. The solution to Maxwell’s equations in this limit is singular, and clearly presents problems for calculating the performance of such systems. We have recently obtained a renormalised full wave solution for a slab of arbitrary thickness of negative refractive index material for the case when the dielectric and magnetic constants differ arbitrarily from their ideal value of -1 but the refractive index remains at the critical value. This produces finite valued fields throughout all space and enables the robustness of the super-lensing effect to be quantified and solved analytically for a variety of systems. One such problem is to consider the effect of a defect on one surface of the lens. This defect excites 'plasmons' which travel along the surface, but whose fields become amplified by the negative refractive index material. These fields interact with the incident illuminating field, and the image of source plus defect can be calculated. The work will quantify whether the super-lensing properties also resolve the characteristics of the defect. Another problem that can be tackled is to consider the optical behaviour of anisotropic left-handed materials, which present the prospect of cloaking objects to the electromagnetic field and novel polarisation effects. The project will be principally analytical in nature and will involve the mathematics and physics of wave propagation and scattering.

Solitons in higher dimensions

Supervisor: Jonathan Wattis (jonathan.wattis@nottingham.ac.uk)

The localisation of energy and its transport is of great physical interest in many applications. The mechanisms by which this occurs have been widely studied in one-dimensional systems; however, in two- and three-dimensional systems a greater variety of waves and wave phenomena can be observed; for example, waves can be localised in one or both directions.

This project will start with an analysis of the nonlinear Schrodinger equation (NLS) in higher space dimensions, and with more general nonlinearities (that is, not just $\gamma=1$). Current interest in the Bose-Einstein Condensates which are being investigated in the School of Physics and Astronomy at Nottingham makes this topic particularly timely and relevant.

The NLS equation also arises in the study of astrophysical gas clouds, and in the reduction of other nonlinear wave equations using small amplitude asymptotic expansions. For example, the reduction of the equations of motion for atoms in a crystal lattice; this application is particularly
intriguing since the lattice structure defines special directions, which numerical simulations
show are favoured by travelling waves. Also the motion of a wave through a hexagonal
arrangement of atoms will differ from that through a square array of atoms. The project will
involve a combination of theoretical and numerical techniques to the study such systems.

Modelling the vibro-acoustic response of complex structures

Supervisor: Gregor Tanner (gregor.tanner@nottingham.ac.uk)

The vibro-acoustic response of mechanical structures (cars, airplanes, ...) can in general be well
approximated in terms of linear wave equations. Standard numerical solution methods comprise
the finite or boundary element method (FEM, BEM) in the low frequency regime and so-called
Statistical Energy Analysis (SEA) in the high-frequency limit. Major computational challenges
are posed by so-called mid-frequency problems - that is, composite structures where the local
wave length may vary by orders of magnitude across the components.

Recently, I proposed a set of new methods based on ideas from wave chaos (also known as
quantum chaos) theory. Starting from the phase space flow of the underlying - generally
chaotic - ray dynamics, the new method called Dynamical Energy Analysis (DEA) interpolates
between SEA and ray tracing containing both these methods as limiting cases. Within the new
theory SEA is identified as a low resolution ray tracing algorithm and typical SEA assumptions
can be quantified in terms of the properties of the ray dynamics. I have furthermore developed
a hybrid SEA/FEM method based on random wave model assumptions for the short-wavelength
components. This makes it possible to tackle mid-frequency problems under certain constraints
on the geometry of the structure.

The PhD project will deal with extending these techniques towards a DEA/FEM hybrid method as
well as considering FEM formulations of the method. The work will comprise a mix of analytic
and numerical skills and will be conducted in close collaboration with our industrial partners
inuTech GmbH, Nuremburg, Germany and Jaguar/landrover, Gaydon, UK.

Wave chaos in acoustics and elasticity, G. Tanner and N. Soendergaard, J. Phys. A 40, R443 -
R509 (2007).

Dynamical Energy Analysis - determining wave energy distributions in complex vibro-acoustical

Ruin, Disaster, Shame!

Supervisor: Keith Hopcraft (keith.hopcraft@nottingham.ac.uk)

Naturally occurring disasters, such as a freak wave that inundates a ship, a bear market that
plunges an economy into recession, or those caused by extremes in weather resulting from
'global warming', cannot be avoided. But they can be planned for so that their devastating effects can be ameliorated. This project will study the mathematical properties extremal events that are caused by a stochastic process exceeding a threshold. It forms part of a larger programme that will generate data from an optical analogue of extremal events – the generation of caustics, and from analyses of financial and climate data. The project will investigate the extrema produced by a non-gaussian stochastic process that is represented mathematically by the nonlinear filtering of a signal, and will determine such useful quantities as the fluctuations in number of extremal events, and the time of occurrence to the next event. The project will involve modelling of stochastic processes, asymptotic analysis, simulation and data processing. Direct involvement with the experimental programme will also be encouraged. A Case Award supplement may be available for a suitably qualified candidate.

Projects in the mechanics of crystals

Supervisor:  **Gareth Parry** ([gareth.parry@nottingham.ac.uk](mailto:gareth.parry@nottingham.ac.uk))

The aim is to understand different aspects of plastic behaviour in complex defective crystals. It is not surprising that methods of traditional continuum mechanics play a role in this area of materials science, but it is perhaps unexpected that classical ideas of differential geometry are central to an appreciation of the issues involved. A student of traditional background in either pure or applied mathematics will be guided, first of all, in reading and in other preparatory exercises, in such a way as to strengthen his or her knowledge appropriately. Possible research projects in this area are the following.

Locking mechanisms in defective crystals: The traditional view regarding the propagation of defects such as dislocations is that they move under the influence of stress until they encounter some imperfection or other inhomogeneity. The aim of the project is to quantify this idea in the context of a mathematical model of defective crystals.

Slip in complex crystals: Existing work details the types of slip that are allowed in a continuum theory of crystals for which the appropriate "state" is given by prescribing three linearly independent vector fields at each point of the region occupied by the crystal. Many crystals do not fit into this scheme, since more than three vector fields are required in the model. The project will extend the existing work to encompass these more complex cases.

Geometrical structure of defective crystals: In some cases, the geometrical structure of defective crystals derives from the structure of certain three-dimensional Lie groups. This connection has not been exploited at all in the past, and the project will begin the cross-fertilization of mathematics and mechanics in this context.

Constitutive functionals for defective crystals: Specifying relationships between stress and strain, subject to certain invariance requirements, is a classic and well-developed procedure in traditional continuum mechanics. In the presence of defects, the procedure is not so clear cut first of all one has to decide on appropriate measures of changes in geometry (like strain) and
then decide if ideas like stored energy and stress are realistic. Finally, symmetry requirements deriving from microscopic considerations have to be derived. The project thus provides an essential prerequisite for any study of the continuum mechanics of this type of crystal.

Variational problems: In the classical calculus of variations formulation of the theory of elasticity, the task is to find the infimum of the "stored energy" functional, sometimes accomplished by choosing a function (representing the elastic deformation) which actually provides a minimum of the functional. In defective crystals, the corresponding task is to find the infimum of an appropriate functional by choosing two functions representing (i) the elastic deformation (ii) the rearrangement (or slip) of the crystal. The project will consist of the study of mathematical problems of this type.

Thermodynamics of defective crystals: It is accepted in the materials science community that friction and energy dissipation are involved in the slip of one crystal lattice plane over another and that temperature effects are important in the mechanics of crystals allowed to deform by slip. The task is to incorporate modern thermodynamic ideas into a theory of the mechanics of such processes.

The frequency of catastrophes

Supervisor: Keith Hopcraft (keith.hopcraft@nottingham.ac.uk)

We have recently developed analytical stochastic models that are capable of describing the frequency of discrete events that have (essentially) an arbitrary distribution, including such extreme cases as when the mean does not exist. Such models can be used to investigate the frequency of rare or extremal events, and can be used to quantify the size of fluctuations that are generated by systems that are close to a critical point, where correlations have a dominating role. The current interest on global climate change provides an interesting and important area with which to apply these models. Climate records provide a detailed source of data from which one can deduce extremal events, such as the number of times the temperature or precipitation exceeds the mean during a period and the models then provide the capacity to estimate the future frequency of such occurrences. The work will involve time-series analysis of climate records, stochastic model building and solution of those models using analytical and numerical techniques.

Caustics: optical paradigms of complex systems

Supervisor: Keith Hopcraft (keith.hopcraft@nottingham.ac.uk)

A complex system is multi-component and heterogeneous in character, the interactions between its component parts leading to collective, correlated and self-organising behaviours. Manifestations of these behaviours are diverse and can range from descriptions of matter near a critical point, through turbulence, to the organising structures that emerge in societies. The
interactions which generate these behaviours are always nonlinear and often triggered by the system crossing a threshold, the frequency of crossing this barrier provides an important characteristic of the system under consideration. The pattern of caustics observed on the bottom of a swimming pool is one commonly experienced manifestation of such a threshold phenomenon, the caustics being caused by the stationary points of the water’s surface. This illustrates how a continuous fluctuation- i.e. the water's surface, leads to the occurrence of a discrete the number of events — the caustics. The project will investigate the how the number of caustics depends on the properties of the surface and propagation distance (i.e. the depth of the swimming pool). The work will be mainly analytical in nature, involving elements of stochastic model building and their solution, with some simulation. There is a possibility of comparing models with experimental data of light propagation through 'model swimming-pools' and entrained fluids.

**Monitoring classically entangled systems**

Supervisor:  **Keith Hopcraft** ([keith.hopcraft@nottingham.ac.uk](mailto:keith.hopcraft@nottingham.ac.uk))

Models for the dynamics of populations usually investigate the time-behaviour of the mean population size, but neglect the fluctuations which can substantially modify the behaviour of the system, particularly if the populations are of small size. With the aid of newly developed models, the opportunity now exists to investigate analytically the full stochastic behaviour of two or more interacting populations. This enables the effect or indeed the nature of their interactions to be probed through monitoring, or 'counting', the number comprising the component populations using different methods of detection. One model which is particularly interesting to consider is when two populations become mixed or entangled through their interaction, and to discover what measurements made on the system are sensitive to detecting the influence of the interaction. The work will be principally analytical in nature, involving stochastic model building and their solution, with some simulation.

**Inertial Effects in 3-D Rimming Flow**

Supervisors:  **Stephen Hibberd** ([stephen.hibberd@nottingham.ac.uk](mailto:stephen.hibberd@nottingham.ac.uk))  
**Andrew Cliffe** ([andrew.cliffe@nottingham.ac.uk](mailto:andrew.cliffe@nottingham.ac.uk))

Droplet-cooled oil films develop on the internal surfaces of an aero-engine bearing chamber are a primary mechanism in removing heat from the chamber as oil is continuously collected and externally cooled and recycled. Studying interacting oil flow and thermal processes within a simplified bearing chamber geometry provides useful information on the trends and characteristics which can arise under different applied flow conditions and insight to the effect of chamber design parameters may have on oil degradation and cooling of chamber walls.
In a generic bearing chamber the oil film is typically driven by the combined mechanisms of a strong shearing airflow, associated by high-speed rotating part, within the chamber and by the added momentum of droplets carried within this airflow as they enter the film. Within the film the classical balance of forces is described by the Navier-Stokes equations but usefully simplified for thin film flow. The heat transfer mechanism is predominantly the cooling effect of the droplets as they enter the (relatively) hot film heated by the surrounding walls and airflow.

In an ongoing PhD study (ED Kay), finishing in June 2013, a two-dimensional thin-film rimming flow within a fixed cylinder has been studied subject to a very high surface shear and oil droplet entering the flow. Unlike previous leading-order thin-film models inertial effects have been included relevant to increasingly high-speed applications. Using this approach leading non-linear inertial effects are retained to analyse the more general flow patterns to provide comparison to other existing thin-film asymptotic studies. Importantly the study investigates the effect of inertia on some typical rimming-flow solutions and presents a greater insight to existing and new film solutions which arise from including inertia effects.

A new PhD study is proposed to extend this current work to include non-axisymmetric geometries.

A background in applied mathematics is required for the following PhD project and an interest in fluid mechanics and mathematical and numerical methods. Projects will be supervised by Dr S Hibberd, Prof KA Cliffe (Mathematical Sciences) and Prof H Power (Engineering) and advisor from Rolls-Royce Aerospace plc.

Mathematical modelling of film flow and dynamics of bearings is ongoing at Nottingham in collaboration with Rolls-Royce Aerospace plc (via a University Technology Centre (UTC - http://www.nottingham.ac.uk/~eazsm/utc_website/ ). In ‘next generation’ aero-engines the technology will involve greater engine speeds and increasing engine core temperatures and require ever more appropriate analytical and numerical modelling to support / verify UTC and RR ‘in-house’ CFD and other computational capabilities.

Film flow characteristics of droplet cooling in a simplified bearing chamber,


Chemical reaction at threshold

Supervisor:  Stephen Creagh (stephen.creagh@nottingham.ac.uk)

Chemical reaction rates are best calculated from first principles by examining a restricted set of molecular configurations separating reactants from products. In terms of molecular dynamics,
this means using the geometry of lower-dimensional subsets to characterise bottlenecks in phase space controlling the flow from reactants to products. A particularly interesting case is where a bottleneck closes as a parameter such as the total energy of the system is varied. A naive simulation using molecular trajectories would then predict that the reaction simply stops. However, wave-mechanical or quantum effects can be important in molecular motion and can lead to reaction even when the energy of the reactants is too small to allow this in a simulation using trajectories. This project will involve applying methods which have been developed using wave asymptotics to understand how this reaction process works in some explicit models for molecular reaction. The work will involve a combination of dynamical theory of the molecular motion with quantum or wave-mechanical theory, which is approached using semiclassical or asymptotic methods.

**Classical and quantum Chaos in 3-body Coulomb problems**

**Supervisor:** Gregor Tanner (gregor.tanner@nottingham.ac.uk)

The realisation that the dynamics of 2 particles interacting via central forces is fundamentally different from the dynamics of three particles can be seen as the birth of modern dynamical system theory. The motion of two particles (for example the earth-moon problem neglecting the sun and other planets) is regular and thus easy to predict. This is not the case for three or more particles (especially if the forces between all these particles are of comparable size) and the resulting dynamics is in general chaotic, a fact first spelt out by Poincaré at the end of the 19th century. An important source for chaos in the three-body problem is the possibility of triple collisions, that is, events where all three particles collide simultaneously. Triple collisions form essential singularities in the equation of motions, that is, trajectories can not be smoothly continued through triple collision events. This is related to the fact, that the dynamics at the triple collision point itself takes place on a collision manifold of non-trivial topology.

During the project, the student will be introduced to scaling techniques which allow to study the dynamics at the triple collision point. We will in particular consider three-body Coulomb problems, such as two-electron atoms, and study the influence of the triple-collision on the total dynamics of the problem. As a long term goal, we will try to uncover the origin of approximate invariants of the dynamics whose existence is predicted by experimental and numerical quantum spectra of two-electron atoms such as the helium atom.


**Electromagnetic Compatibility - can chaos theory help you keep your mobile phone on in a plane?**
Modern technology is typically stuffed with electronic componenty. Devices ranging from a mobile phone to a pc to an Airbus A380 will have many internal electronic components operating at high frequencies and therefore radiating electromagnetic waves. If the waves radiated from one component are strong enough, they can interfere with the functioning of another component somewhere else in the unit. The field of Electromagnetic Compatibility (EMC) aims to mitigate these effects by better understanding the emitted radiation.

This project will develop a model to calculate the electromagnetic radiation within an enclosure containing electronic sources --- a desktop pc providing one explicit example. In tackling this problem we combine methods from chaos theory and the theory of high frequency asymptotics developed in the context of quantum mechanics. Starting from a ray model for the propagation of EM radiation, we will take advantage of tools from the theory of dynamical systems to characterise ray propagation in complex enclosures such as a pc including all the electronic components. In addition we need to model aspects of the underlying linear wave problems in order to draw conclusions about wave effects such as diffraction or interference.

Wave propagation and complexity - a transfer operator approach

Supervisors:  **Gregor Tanner** ([gregor.tanner@nottingham.ac.uk](mailto:gregor.tanner@nottingham.ac.uk))  
**Stephen Creagh** ([stephen.creagh@nottingham.ac.uk](mailto:stephen.creagh@nottingham.ac.uk))

Modelling wave propagating inside complex enclosures is important for a variety of physical and engineering applications. In the electromagnetic world, wireless communication within buildings, or characterising electromagnetic interference within devices, is a formidable challenge. Likewise, modelling noise and vibration in a car, airplane or train is a hard problem. It is often said that engineers can model every aspect of car on the computer these days, except how it will sound!

These problems are most efficiently solved by splitting the whole structure into substructures, determining the solutions for each sub-structure and then coupling these local solutions. The sub-system solutions can be determined on the boundary (such as the building walls) rather than on the higher-dimensional interior.

This project aims to investigate the use of boundary-based calculations to provide an analytical tool to develop approaches such as ray-based propagation. The main idea will be to provide an explicit relationship between the wave arriving at a boundary and the wave leaving it in terms of a so-called transfer operator. This transfer operator characterises the effect of passage through the interior of the cavity to give the wave arriving at the boundary at one part in terms of the wave leaving it elsewhere. It is related to the numerical boundary-based approaches such as the Boundary Element Method but better suited to developing analytical approximations.
Network performance subject to agent-based dynamical processes

Supervisors:  Keith Hopcraft  (keith.hopcraft@nottingham.ac.uk)  
Simon Preston  (simon.preston@nottingham.ac.uk)

Networks – systems of interconnected elements – form structures through which information or matter is conveyed from one part of an entity to another, and between autonomous units. The form, function and evolution of such systems are affected by interactions between their constituent parts, and perturbations from an external environment. The challenge in all application areas is to model effectively these interactions which occur on different spatial- and time-scales, and to discover how:

i)   the micro-dynamics of the components influence the evolutionary structure of the network, and

ii)  the network is affected by the external environment(s) in which it is embedded.

Activity in non-evolving networks is well characterized as having diffusive properties if the network is isolated from the outside world, or ballistic qualities if influenced by the external environment. However, the robustness of these characteristics in evolving networks is not as well understood. The projects will investigate the circumstances in which memory can affect the structural evolution of a network and its consequent ability to function.

Agents in a network will be assigned an adaptive profile of goal- and cost-related criteria that govern their response to ambitions and stimuli. An agent then has a memory of its past behaviour and can thereby form a strategy for future actions and reactions. This presents an ability to generate ‘lumpiness’ or granularity in a network’s spatial structure and ‘burstiness’ in its time evolution, and these will affect its ability to react effectively to external shocks to the system. The ability of externally introduced activists to change a network’s structure and function - or agonists to test its resilience to attack - will be investigated using the models. The project will use data of real agent’s behaviour.

Fluctuation Driven Network Evolution

Supervisors:  Keith Hopcraft  (keith.hopcraft@nottingham.ac.uk)  
Simon Preston  (simon.preston@nottingham.ac.uk)

A network’s growth and reorganisation affects its functioning and is contingent upon the relative time-scales of the dynamics that occur on it. Dynamical time-scales that are short compared with those characterizing the network’s evolution enable collectives to form since each element remains connected with others in spite of external or internally generated ‘shocks’ or fluctuations. This can lead to manifestations such as synchronicity or epidemics. When the network topology and dynamics evolve on similar time-scales, a ‘plastic’ state can emerge.
where form and function become entwined. The interplay between fluctuation, form and function will be investigated with an aim to disentangle the effects of structural change from other dynamics and identify robust characteristics.

Mathematical Medicine and Biology

Stochastic Threshold Models: from single nodes to networks

Supervisors: Ruediger Thul (ruediger.thul@nottingham.ac.uk)  Stephen Coombes (stephen.coombes@nottingham.ac.uk)

The behaviour of excitable systems can often be captured with a simpler threshold description. The Integrate-and-Fire model of a neuron is a great example as is the Fire-Diffuse-Fire model of calcium wave propagation in cardiac cells. The project will use tools from dynamical systems theory (bifurcation theory, nonsmooth vector fields, scientific computation) and stochastic processes (Markov chains, Wiener processes) to analyse models with threshold noise. Beginning with studies of single units the project will build up to understand the collective behaviour of interacting threshold units, with applications in neuroscience (rhythm generation, neural computation) and cardiac dynamics (wave propagation, coherent oscillations and arrhythmias).

The role of space in sub-cellular cardiac alternans

Supervisors: Ruediger Thul (ruediger.thul@nottingham.ac.uk)  Stephen Coombes (stephen.coombes@nottingham.ac.uk)

Cardiac arrhythmia are the leading cause of the death in the UK, killing more people each year than breast cancer, lung cancer and AIDS combined. Among the different kinds of cardiac arrhythmia, atrial fibrillation is the most common one. Here, the smaller chambers of the heart beat so fast that they cannot pump blood anymore, shifting all the blood propelling work of the heart to the larger chambers. This situation is especially problematic under conditions when blood needs to be pumped more quickly (e.g. during exercise) and among the elderly when the heart generally becomes weaker.

One of the precursors of atrial fibrillation are cardiac alternans. Here the heart still exhibits a regular rhythm, but with alternating strength. Only every second heart beat is strong enough to sufficiently contract the heart. A particular form of this arrhythmia are sub-cellular alternans where different parts of a single cell oscillate out of phase. Such a cell does not contract at all, and a group of such cells significantly impairs cardiac contractility. To better understand the emergence of atrial fibrillation and to design treatments, it is vital to gain a comprehensive picture of cardiac alternans.
In this project, we will use a recently developed three dimensional model of an atrial myocyte [2] to investigate the emergence of sub-cellular cardiac alternans. In contrast to earlier work, our approach does not require an ad-hoc compartmentalisation of the cell, but we can work with a realistic representation of the cellular morphology. In turn, this will allow us to better characterise the interaction between different sub-cellular processes that shape cardiac alternans. The challenge is to develop the analysis of alternans for a spatially extended cell model that complements numerical simulations and allows us to predict the onset of alternans more efficiently. Keeping in mind that any drug treatment acts first at the single cell level, our approach will help to identify potential targets for pharmaceutical intervention in cardiac therapy.

**Spatio-temporal patterns with piecewise-linear regulatory networks**

Supervisor:  
**Etienne Farcot** (etienne.farcot@nottingham.ac.uk)

A number of fascinating and important biological processes involve various kinds of spatial patterns: spatial patterns on animal skins, or the very regular organ arrangements found in plants (called phyllotaxis) for instance. These patterns often originate at very small scales, and their onset can only be seen using very recent microscope and image analysis techniques. Among several families of models for biological patterning, one of the simplest is based on the idea that mobile substances (called morphogens) are acting upstream of their targets, which respond locally to a globally defined gradient pattern. In this project one will consider models where targets are themselves mobile morphogens, potentially regulating their own input. One will study the effect of such spatial feedback on patterning. To do so, one will rely on a class of models which are biologically relevant, tractable analytically, and not much studied yet in a context with spatial interactions. A class of models which meet all this criteria is provided by piecewise-linear differential equations.

**Spine morphogenesis and plasticity**

Supervisors:  
**Stephen Coombes** (stephen.coombes@nottingham.ac.uk)  
**Ruediger Thul** (ruediger.thul@nottingham.ac.uk)

Mathematical Neuroscience is increasingly being recognised as a powerful tool to complement neurobiology to understand aspects of the human central nervous system. The research activity in our group is concerned with developing a sound mathematical description of sub-cellular processes in synapses and dendritic trees. In particular we are interested in models of dendritic spines [1], which are typically the synaptic contact point for excitatory synapses. Previous work in our group has focused on voltage dynamics of spine-heads [2]. We are now keen to broaden the scope of this work to include developmental models for spine growth and
maintenance, as well as models for synaptic plasticity [3]. Aberrations in spine morphology and density are well known to underly certain brain disorders, including Fragile X syndrome (which can lead to attention deficit and developmental delay) and depression [4]. Computational modelling is an ideal method to do in-silico studies of drug treatments for brain disorders, by modelling their action on spine development and plasticity. This is an important complementary tool for drug discovery in an area which is struggling to make headway with classical experimental pharmaceutical tools.

The mathematical tools relevant for this project will be drawn from dynamical systems theory, biophysical modelling, statistical physics, and scientific computation.

Stochastic Neural Network Modelling

Supervisors: Ruediger Thul (ruediger.thul@nottingham.ac.uk)
Stephen Coombes (stephen.coombes@nottingham.ac.uk)

Large scale studies of spiking neural networks are a key part of modern approaches to understanding the dynamics of biological neural tissue. One approach in computational neuroscience has been to consider the detailed electrophysiological properties of neurons and build vast computational compartmental models. An alternative has been to develop minimal models of spiking neurons with a reduction in the dimensionality of both parameter and variable space that facilitates more effective simulation studies. In this latter case the single neuron model of choice is often a variant of the classic integrate-and-fire model, which is described by a non-smooth dynamical system with a threshold. It has recently been shown that one way to model the variability of neuronal firing is to introduce noise at the threshold level. This project will develop the analysis of networks of synaptically coupled noisy neurons. Importantly it will go beyond standard phase oscillator approaches to treat strong coupling and non-Gaussian noise. One of the main mathematical challenges will be to extend the Master-Stability framework for networks of deterministic limit cycle oscillators to the noisy non-smooth case that is relevant to neural modelling. This work will determine the effect of network dynamics and topology on synchronisation, with potential application to psychiatric and neurological disorders. These are increasingly being understood as disruptions of optimal integration of mental processes sub-served by distributed brain networks.

S Coombes, R Thul and K C A Wedgwood 2012 Nonsmooth dynamics in spiking neuron models, Physica D, DOI: 10.1016/j.physd.2011.05.012


J Hlinka and S Coombes 2012 Using computational models to relate structural and functional brain connectivity, European Journal of Neuroscience, to appear
Cell signalling

Supervisor: **John King** ([john.king@nottingham.ac.uk](mailto:john.king@nottingham.ac.uk))

Cell signalling effects have crucial roles to play in a vast range of biological processes, such as in controlling the virulence of bacterial infections or in determining the efficacy of treatments of many diseases. Moreover, they operate over a wide range of scales, from subcellular (e.g. in determining how a particular drug affects a specific type of cell) to organ or population (such as through the quorum sensing systems by which many bacteria determine whether or not to become virulent). There is therefore an urgent need to gain greater quantitative understanding of these highly complex systems, which are well-suited to mathematical study. Experience with the study of nonlinear dynamical systems would provide helpful background for such a project.

Modelling DNA Chain Dynamics

Supervisor: **Jonathan Wattis** ([jonathan.wattis@nottingham.ac.uk](mailto:jonathan.wattis@nottingham.ac.uk))

Whilst the dynamics of the DNA double helix are extremely complicated, a number of well-defined modes of vibration, such as twisting and bending, have been identified. At present the only accurate models of DNA dynamics involve large-scale simulations of molecular dynamics. Such approaches suffer two major drawbacks: they are only able to simulate short strands of DNA and only for extremely short periods (nanoseconds). The aim of this project is to develop simpler models that describe vibrations of the DNA double helix. The resulting systems of equations will be used to simulate the dynamics of longer chains of DNA over long timescales and, hence, allow larger-scale dynamics, such as the unzipping of the double helix, to be studied.

Multiscale modelling of vascularised tissue

Supervisor: **Markus Owen** ([markus.owen@nottingham.ac.uk](mailto:markus.owen@nottingham.ac.uk))

Most human tissues are perfused by an evolving network of blood vessels which supply nutrients to (and remove waste products from) the cells. The growth of this network (via vasculogenesis and angiogenesis) is crucial for normal embryonic and postnatal development, and its maintenance is essential throughout our lives (e.g. wound healing requires the repair of damaged vessels). However, abnormal remodelling of the vasculature is associated with several pathological conditions including diabetic retinopathy, rheumatoid arthritis and tumour growth.

The phenomena underlying tissue vascularisation operate over a wide range of time and length scales. These features include blood flow in the existing vascular network, transport within the tissue of blood-borne nutrients, cell division and death, and the expression by cells of growth
factors such as VEGF, a potent angiogenic factor. We have developed a multiscale model framework for studying such systems, based on a hybrid cellular automaton which couples cellular and subcellular dynamics with tissue-level features such as blood flow and the transport of growth factors. This project will extend and specialise our existing model to focus on particular applications in one of the following areas: wound healing, retinal angiogenesis, placental development, and corpus luteum growth. This work would require a significant element of modelling, numerical simulation and computer programming.

Self-similarity in a nanoscale island-growth

Supervisor: Jonathan Wattis (jonathan.wattis@nottingham.ac.uk)

Molecular Beam Epitaxy is a process by which single atoms are slowly deposited on a surface. These atoms diffuse around the surface until they collide with a cluster or another atom and become part of a cluster. Clusters remain stationary. The distribution of cluster sizes can be measured, and is observed to exhibit self-similarity. Various systems of equations have been proposed to explain the scaling behaviour observed. The purpose of this project is to analyse the systems of differential equations to verify the scalings laws observed and predict the shape of the size-distribution. The relationship of equations with other models of deposition, such as reactions on catalytic surfaces and polymer adsorption onto DNA, will also be explored.

Sequential adsorption processes

Supervisor: Jonathan Wattis (jonathan.wattis@nottingham.ac.uk)

The random deposition of particles onto a surface is a process which arises in many subject areas, and determining its efficiency in terms of the coverage attained is a difficult problem.

In one-dimension the problem can be viewed as how many cars can be parked along a road of a certain length; this problem is similar to a problem in administering gene therapy in which polymers need to be designed to package and deliver DNA into cells.

Here one wishes to know the coverage obtained when one uses a variety of polymer lengths to bind to strands of DNA.

The project will involve the solution of recurrence relations, and differential equations, by a mixture of asymptotic techniques and stochastic simulations.

Robustness of biochemical network dynamics with respect to mathematical representation

Supervisor: Etienne Farcot (etienne.farcot@nottingham.ac.uk)
In the recent years, a lot of multi-disciplinary efforts have been devoted to improving our understanding of the dynamics of interactions between the many types of molecules present in biological cells. This has led to a widespread viewpoint where networks of genes, proteins and other biochemical species are considered at once, as complex dynamical systems from which the global state of cells emerge. Several mathematical formalisms are used to represent these systems, from discrete or boolean models to differential equations. One striking fact, especially regarding models of developmental processes, is that a number of relevant properties of these networks can be captured similarly by all these formalisms, like for instance the property of bistability. One possible interpretation of this independence with respect to formalism is that biological regulatory systems are most often extremely robust. The project will start by developing parallel models – using different formalisms – of actual biological networks whose behaviour is known. Elaborating on these examples the theoretical and practical implications of this notion of robustness will be explored.

**Nuclear signalling in eukaryotes: modelling spatio-temporal patterns of intracellular calcium**

**Supervisors:** Ruediger Thul ([ruediger.thul@nottingham.ac.uk](mailto:ruediger.thul@nottingham.ac.uk))
Stephen Coombes ([stephen.coombes@nottingham.ac.uk](mailto:stephen.coombes@nottingham.ac.uk))

Calcium is critically important for a large number of cellular functions, such as muscle contraction, cardiac electrophysiology, secretion, synaptic plasticity, and adaptation in photoreceptors [1]. Mechanisms by which a cell controls its Calcium concentration are of central interest in cell physiology. The recent use of Calcium specific fluorescent reporter dyes and digital videomicroscopy has begun to reveal the complexity of Calcium dynamics in spatially extended cellular systems. Calcium signalling in a wide diversity of cell types frequently occurs as Calcium oscillations. These do not generally occur uniformly throughout the cell but are initiated at a specific site and spread in the form of waves. The fluorescent imaging of localised elementary Calcium release events has now made it clear that Calcium release is a stochastic process that occurs at spatially discrete sites that are clusters of receptors in the endoplasmic or sarcoplasmic reticulum. Mathematical modelling is an ideal tool for capturing the details of how intracellular Calcium waves spread throughout a cell and subserve physiological and pathological signals, especially in light of current resolution limitations of imaging technologies. In particular the stochastic Fire-Diffuse-Fire (FDF) model [2] is an ideal starting point for the development of a computationally economical framework to allow fast simulations of realistic cell geometries with large numbers of release sites. This project will focus on developing cell models to track waves that allow targeted release of calcium in the nuclear region of eukaryotes. Nuclear oscillations in calcium are especially important as they can drive downstream responses for gene expression. The model will be extended to include calcium decoders (such as a nuclear-localised calcium and calmodulin-dependent protein kinase) and developed to model the symbiotic signalling pathway of legumes during root nodulation [3].
This project will mainly draw from the toolbox of computational cell biology (including GPU programming in Python) to address important open problems in cellular calcium signalling in plants.


**Spirals and auto-soliton scattering: interface analysis in a neural field model**

Supervisors:  **Stephen Coombes** ([stephen.coombes@nottingham.ac.uk](mailto:stephen.coombes@nottingham.ac.uk))  
**Daniele Avitabile** ([daniele.avitabile@nottingham.ac.uk](mailto:daniele.avitabile@nottingham.ac.uk))

Neural field models describe the coarse grained activity of populations of interacting neurons. Because of the laminar structure of real cortical tissue they are often studied in 2D, where they are well known to generate rich patterns of spatio-temporal activity. Typical patterns include localised solutions in the form of travelling spots as well as spiral waves [1]. These patterns are naturally defined by the interface between low and high states of neural activity. This project will derive the dimensionally reduced equations of motion for such interfaces from the full nonlinear integro-differential equation defining the neural field. Numerical codes for the evolution of the interface will be developed, and embedded in a continuation framework for performing a systematic bifurcation analysis. Weakly nonlinear theory will be developed to understand the scattering of multiple spots that behave as auto-solitons, whilst strong scattering solutions will be investigated using the scatter theory that has previously been developed for multi-component reaction diffusion systems [2].


**STIM-ORAI dependent Calcium oscillations**

Supervisors:  **Ruediger Thul** ([ruediger.thul@nottingham.ac.uk](mailto:ruediger.thul@nottingham.ac.uk))  
**Stephen Coombes** ([stephen.coombes@nottingham.ac.uk](mailto:stephen.coombes@nottingham.ac.uk))

Calcium oscillations have long been recognised as a main pathway with which cells translate external stimuli into intracellular responses such as enzyme secretion, neurotransmitter
production or cell contraction [1]. Over the last years, it has emerged that calcium oscillations often do not occur uniformly across a cell, but that either different parts of a cell oscillate out of phase with respect to each other or that cellular oscillations actually correspond to traveling calcium waves. The importance of space in shaping intracellular calcium oscillations has recently been highlighted by the discovery of the STIM-ORAI machinery [2]. Here, translocation of intracellular molecules (STIM) to designated areas close to the cell membrane (ORAI) are responsible for initiating and maintaining calcium oscillations.

A large body of experimental data convincingly suggests that most of the information of intracellular calcium oscillations is encoded in their frequency and sometimes in their amplitude. The STIM-ORAI system now shows that only if the cellular calcium oscillations occur through STIM and ORAI certain genes are activated. Intracellular calcium oscillations that look identical but involves different molecular partners fail to initiate a genetic response [3].

In this project, we will develop a spatially extended model for STIM-ORAI induced calcium oscillations that will explain the still unknown mechanisms behind the long periods of calcium oscillations. Introducing the pathway that is responsible for gene activation, we will study the signalling cascade that links calcium oscillations to gene expression with a special emphasis on the emergence of calcium microdomains and exchange mechanisms between the cell cytoplasm and nucleus. Given the importance of STIM-ORAI dependent oscillations in cells of the immune system, our work has direct implications to strengthen human health.

The project will involve model design based on published experimental data and mathematical techniques for partial differential equations, delayed differential equations and stochastic processes.

### Modelling signal processing and sexual recognition in mosquitoes: neural computations in insect hearing systems

**Supervisors:** Daniele Avitabile ([daniele.avitabile@nottingham.ac.uk](mailto:daniele.avitabile@nottingham.ac.uk))  
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Insects have evolved diverse and delicate morphological structures in order to capture the inherently low energy of a propagating sound wave. In mosquitoes, the capture of acoustic energy, and its transduction into neuronal signals, is assisted by the active mechanical participation of actuators called scolopidia. When a sound wave reaches the head of a mosquito, the antenna oscillates under the action of the external pressure field (passive component) and of the force provided by the mechanical actuators (active component). The latter is particularly relevant for sexual recognition: when a male mosquito hear the flyby of a female, his antennal oscillation are greatly amplified by the scolopidia. In other words, the antenna of a male is tuned very sharply around the frequency and intensity of a female flyby.
Recent studies have shown that mosquitoes of either sex use both their antenna and their wing beat to select a partner: understanding how their hearing system works could help us controlling the population of species that carry viral diseases.

Even though some models of mosquitoes hearing systems have been proposed in the past, a number of key questions remain unanswered. Where do the mechanical actuators get their energy? How do they twitch? How is the mechanical motion of the antenna transformed into an electric signal? Do neurones control the mechanical motion? How does the brain of a mosquito process the neural information and distinguish various sources of sound? Is the sexual recognition entirely based on sound perception, or is it also influenced by olfactory signals? Is the antenna sensitive to sounds from different directions?

AVITABILE, D, HOMER, M, CHAMPNEYS, AR, JACKSON, JC and ROBERT, D, 2010. Mathematical Modelling Of The Active Hearing Process In Mosquitoes Journal Of The Royal Society Interface. 7(42), 105-122


Mechanistic models of airway smooth muscle cells - application to asthma

Supervisor:  Bindi Brook (bindi.brook@nottingham.ac.uk)

Lung inflammation and airway hyperresponsiveness (AHR) are hallmarks of asthma, but their interrelationship is unclear. Excessive shortening of airway smooth muscle (ASM) in response to bronchoconstrictors is likely an important determinant of AHR. Hypercontractility of ASM could stem from a change in the intrinsic properties of the muscle, or it could be due to extrinsic factors such as chronic exposure of the muscle to inflammatory mediators in the airways with the latter being a possible link between lung inflammation and AHR. The aim of this project will be to investigate the influence of chronic exposure to a contractile agonist on the force-generating capacity of ASM via a cell-level model of an ASM cell. Previous experimental studies have suggested that the muscle adapts to basal tone in response to application of agonist and is able to regain its contractile ability in response to a second stimulus over time. This is thought to be due to a transformation in the cytoskeletal components of the cell enabling it to bear force, thus freeing up subcellular contractile machinery to generate more force. Force adaptation in ASM as a consequence of prolonged exposure to the many spasmogens found in asthmatic airways could be a mechanism contributing to AHR seen in asthma. We will develop and use a cell model in an attempt to either confirm this hypothesis or determine other mechanisms that may give rise to the observed phenomenon of force adaptation. blah.

The role of contractile unit reorganization in force generation in airway smooth muscle, B S Brook and O E Jensen, Mathematical Medicine and Biology, 2013. doi:10.1093/imammb/dqs031

Computational Cell Biology

Supervisors: Ruediger Thul (ruediger.thul@nottingham.ac.uk)
Stephen Coombes (stephen.coombes@nottingham.ac.uk)

Computational Cell Biology (CSB) uses techniques from nonlinear dynamical systems, partial differential equations and stochastic processes to gain a deeper insight into the reliability and robustness of cellular signalling cascades. By combining analytical and numerical approaches, CSB plays a major role in the discovery and quantitative descriptions of key biological processes. Mathematical models that are tailored to specific biological questions can yield answers that are still out of reach for cutting-edge experimental approaches.

In the current project we will explore how the spatial arrangement of the molecular machinery affects cellular signal transduction. A key feature of cells is to translate external stimuli into cellular responses. Cells in the pancreas produce insulin when extracellular markers indicate high blood sugar levels, neurons in the brain release chemical messengers to their neighbours upon electrical stimulation, and heart cells contract more effectively when they experience a rush of adrenaline. Cells use advanced molecular machinery to trigger the appropriate reaction for a given stimulus. In recent years, convincing evidence has emerged that cells employ spatio-temporal patterns to achieve this task. For instance, complex oscillations and travelling waves of intracellular calcium have been observed, where the frequency of the oscillations and the spatial spread of the waves tightly correlate with the external stimulation.

C.P. Fall, E.S. Marland, J.N. Wagner, J.J. Tyson, Computational Cell Biology, Springer, 2002

Synchronisation and propagation in human cortical networks

Supervisor: Reuben O'Dea (reuben.o'dea@nottingham.ac.uk)

Around 25% of the 50million epilepsy sufferers worldwide are not responsive to antiepileptic medication; improved understanding of this disorder has the potential to improve diagnosis, treatment and patient outcomes. The idea of modelling the brain as a complex network is now well established. However, the emergence of pathological brain states via the interaction of large interconnected neuronal populations remains poorly understood. Current theoretical study of epileptic seizures is flawed by dynamical simulation on inadequate network models, and by the absence of customised network measures that capture pathological connectivity patterns.

This project aims to address these deficiencies via improved computational models with which to investigate thoroughly the influence of the geometry and connectivity of the human brain on epileptic seizure progression and initiation, and the development of novel network measures.
with which to characterise epileptic brains. Such investigations will be informed by exhaustive patient datasets (such as recordings of neural activity in epilepsy patients and age-matched controls), and will be used to study (i) improved diagnostic strategies, (ii) the influence of treatment strategies on seizure progression and initiation, and (iii) the identification of key sites of epilepsy initiation.

**Multiscale analysis of growing deformable tissues**

**Supervisor:** Reuben O'Dea ([reuben.o'dea@nottingham.ac.uk](mailto:reuben.o'dea@nottingham.ac.uk))

Biological tissue is distinguished from materials described historically by continuum mechanical theory by its ability to grow and remodel adaptively, regulated by complex processes occurring within autonomous discrete cells. Current continuum approaches are wholly unable to represent comprehensively this detail and, in general, employ ad hoc modelling strategies, applied at the macroscale, with little or no rigorous consideration of the underlying dynamics; effective descriptions obtained via multiscale analyses have not addressed adequately the combination of (i) discrete cell behaviour (including population expansion), and (ii) micromechanics (the latter consideration incorporating the influence of relevant tissue microstructure and mechanical properties on macroscale tissue dynamics).

This project will seek to address this deficiency, by developing improved mathematical representations of a wide class of deformable, adaptively remodelling materials, accommodating the influence both of microscale (cell-scale) and mesoscale (tissue structure and mechanics) effects on corresponding macroscale (tissue-scale) formulations, exploiting a combination of multiscale asymptotic homogenisation techniques, mathematical modelling and detailed numerical simulation. Collaboration with experimental experts at the University of Nottingham and Keele University will inform and validate the models developed.

**Patterns of synchrony in discrete models of gene networks**

**Supervisor:** Etienne Farcot ([etienne.farcot@nottingham.ac.uk](mailto:etienne.farcot@nottingham.ac.uk))

One of the greatest challenges of biology is to decipher the relation between genotype and phenotype. One core difficulty in this task is that this relation is not a map; the proteins which are produced thanks to the information contained in the genome are themselves used to control which parts of the genome are being used in a given situation. To understand the effect of this feedback between genes and their product it is crucial to consider the dynamics of this process.

The term 'gene network' refers to a set of genes which regulate each other; understanding the dynamics of gene networks is thus crucial to decipher the genotype-phenotype relation. Mathematical models of gene networks have been proposed since the 1960's, among which the class of Boolean models has proved very successful. Because of the discrete nature of these models, the effect of time is often described using representations inspired by manufactured
computing device, where the genes are updated in parallel, or in series. However, the updating scheme of genes could in principle be much more general. In this project, one will investigate the effect of such a generalization. One will consider arbitrary update schemes, both deterministic and stochastic, notably in relation to the dynamics of continuous models of gene networks.

Cell cycle desynchronization in growing tissues

Supervisor:  **Etienne Farcot** (etienne.farcot@nottingham.ac.uk)

A very general phenomenon is the fact that coupled oscillators tend to naturally synchronize [1]. This simple fact takes many forms observable in real life: synchronization of applause after a concert, of neural cells, of flashing fireflies, and many other. A complete understanding of this phenomenon, depending on the particular dynamics of individual oscillators or the nature of their coupling, is still an on-going topic of mathematical research. However general, the synchronization of coupled oscillators is not a universal rule. In this project, one will study a situation where it indeed does not seem to occur: the division cycles of cells in a growing tissue does not seem to be synchronized, as observed in recent data form plant tissues. A plausible explanation is that the divisions of cell induce a change in the coupling between cells, which is mostly due to physical or chemical exchanges between neighbouring cells. Relying on simplified representations, one will consider the effect of growth, whereby the coupling structure of a system changes in time, on synchronization in populations of oscillators.


Bottom-up development of multi-scale models of airway remodelling in asthma: from cell to tissue.

Supervisor:  **Bindi Brook** (bindi.brook@nottingham.ac.uk)

Airway remodelling in asthma has until recently been associated almost exclusively with inflammation over long time-scales. Current experimental evidence suggests that broncho-constriction (as a result of airway smooth muscle contraction) itself triggers activation of pro-remodelling growth factors that causes airway smooth muscle growth over much shorter time-scales. This project will involve the coupling of sub-cellular mechano-transduction signalling pathways to biomechanical models of airway smooth muscle cells and extra-cellular matrix proteins with the aim of developing a tissue-level biomechanical description of the resultant growth in airway smooth muscle.

The mechano-transduction pathways and biomechanics of airway smooth muscle contraction are extremely complex. The cytoskeleton and contractile machinery within the cell and ECM proteins surrounding it are thought to rearrange dynamically (order of seconds). The cell is
thought to adapt its length (over 10s of seconds). To account for all these processes from the bottom-up and generate a tissue level description of biological growth will require the combination of agent-based models to biomechanical models governed by PDEs. The challenge will be to come up with suitably reduced models with elegant mathematical descriptions that are still able to reproduce observed experimental data on cell and tissue scales, as well as the different time-scales present.

While this study will be aimed specifically at airway remodelling, the methodology developed will have application in multi-scale models of vascular remodelling and tissue growth in artificially engineered tissues.

**Bacterial swimming and invasion**

The Bdellovibrio bacterium is a motile parasite that preys on larger bacteria such as E. coli. A single Bdellovibrio attaches to and then invades its host, in which it grows and divides before destroying the host cell wall, releasing a new generation of parasites into the environment. This project will use mathematical modelling to address some of the biomechanical features of this cycle, including cell swimming (driven by a single rotary flagellum), adhesion and invasion. The project is collaborative with Prof RE Sockets in the School of Biology.

**Blood flow in the placenta: understanding transport in a disordered porous medium**

The placenta is the life-support system for a growing fetus. Within the placenta, maternal and fetal blood supplies are brought into close proximity to allow efficient exchange of nutrients and waste products. Fetal blood vessels are arranged in tree-like structures while maternal blood flows between the branches of the trees. In modelling flow and transport in the placenta, both in health and disease, it is important to understand how the spatial arrangement of the branches (and, in particular, randomness) affects the overall transport efficiency of the organ. This project will investigate this problem using a variety of computational and analytical techniques. The project offers the opportunity to engage with experiments and to address some fundamental problems of relevance to a range of porous medium flows.


**Multiscale mechanics in plant growth**

Plant cells typically contain a pressurized vacuole surrounded by a stiff cell wall. The cell wall is a fibre-reinforced material that can be softened by enzymes, allowing the cells to expand. Growth of a plant tissue arises through the coordinated expansion of its constituent cells. The
anisotropic arrangement of cellulose fibres in a plant cell wall allows tissues to grow in specific directions and to form particular shapes, often with the generation of internal (residual) stresses. This project will use theoretical models to examine how the microstructure of the cell wall influences large-scale deformations of multicellular plant tissues. Biological background for this study will be provided through collaborations with the Centre for Plant Integrative Biology.


**On the dynamics of the Lighthouse model for spiking neural networks**

Supervisor: Stephen Coombes (stephen.coombes@nottingham.ac.uk)

One of the holy grails of the theoretical neuroscience community is to develop a tractable model of neural tissue. This must necessarily involve a single cell model, capable of generating spikes of activity (so-called action-potentials), that when connected into a synaptic network can generate the rich repertoire of behaviour seen in a real nervous system. For all of the popular conductance-based single neurons models, and also the simpler integrate-and-fire variety, the understanding of network dynamics has proved elusive. In essence this is because we have not yet developed an appropriate mathematical framework to understand the neurodynamics of spiking networks. To date progress in this area has been restricted to firing rate neural models, which cannot adequately capture known spike-train correlations. Interestingly, the recently proposed Lighthouse model of Hermann Haken is a candidate single neuron model that may allow a bridge to be built between spiking neuron models and firing rate descriptions. Indeed in the limit of slow synaptic interactions it may be shown to reduce to the oft-studied Amari firing rate model. Importantly the Lighthouse model is sufficiently simple that it may also be analysed at the network level, even for fast synaptic responses. Hence, a comprehensive study of a network of synaptically coupled Lighthouse neurons may pave the way for the development of a specific exactly soluble neurodynamics. This may also shed light on how best to develop a more general approach valid for more detailed models of coupled spiking neurons. This project will pursue the study of the Lighthouse network using techniques from dynamical systems theory and statistical physics, building upon emerging techniques and principles from the physics of complex systems. As it will closely focus on the generation of realistic spike-train correlations from a mathematical model it will benefit enormously from locally available multi-electrode array data collected from both in-vitro and in-vivo neuronal ensembles.


**Shear induced chaos in neuronal networks**

Supervisor: Stephen Coombes (stephen.coombes@nottingham.ac.uk)
Shear induced chaos has recently been shown to be an important mechanism for determining the response of conductance based models of single neurons to time-dependent (typically periodic) input [i]. This PhD project will develop a natural phase-amplitude coordinate system [ii] for describing reduced networks of synaptically interacting neurons. Network states, including phase-locking, synchrony, heteroclinic cycles, and routes to chaos, will be analysed using techniques from dynamical systems theory (both analytical and numerical) to understand fundamental aspects of information processing within the central nervous system including network reliability in the presence of shear.

K K Lin, K C A Wedgwood, S Coombes and L-S Young 2013 Limitations of perturbative techniques in the analysis of rhythms and oscillations, Journal of Mathematical Biology, Vol 66, 139-161


Pain matrices and their analysis: a combined neuroimaging, statistical and modelling analysis

Supervisors: Stephen Coombes (stephen.coombes@nottingham.ac.uk)  
Theodore Kypraios (theodore.kypraios@nottingham.ac.uk)

Scientific background

There is increasing evidence to suggest that chronic pain is a disease that can alter brain function. In particular neuroimaging studies have demonstrated structural remapping and functional reorganisation of brain circuits under various pain conditions. In parallel, preclinical models have demonstrated that chronic pain causes long-term neuroplasticity. For a recent review see [1].

In theory, physiological changes at the single-unit, multi-unit, and circuitry levels can be used as predictors of pain, and ultimately to guide targeted neuromodulation of specific brain regions for therapeutic purposes. The Pain Imaging group at Nottingham is developing circuit level imaging biomarkers (using MRI techniques) to track such physiological changes. The complementary statistical techniques for prediction (and identification of brain states associated with pain) and computational modelling that would allow in-silico design of pain therapies are skill sets that exist within the School of Mathematical Sciences. Thus Nottingham is well positioned to develop multidisciplinary research into the mechanisms of pain-related phenomena in the brain that can offer insights into novel approaches for the diagnosis, monitoring, and management of persistent pain.

Aims and objectives

In light of recent breakthroughs in the statistical analysis of brain network signals [2] and computational models of interacting neuronal populations [3], as well as locally available data
sets from the Pain Imaging group, our aim is to equip a PhD student with multi-disciplinary skills for understanding how humans experience pain. The objective is for them to develop a novel systems perspective of pain as a complex multidimensional experience that can be understood with the modern tools of applied mathematics and statistics.

Although activation patterns may vary, the regions most consistently reported to have increased blood-oxygen-level-dependent signals associated with experimentally induced pain include the thalamus, somatosensory cortex, anterior cingulate cortex, prefrontal cortex, insula, and the cerebellum, forming a so-called pain matrix. We will develop network models of this system of interacting neural populations building on recent work in [3]. This will allow us to explore the mechanisms for the emergence of functional connectivity associated with normal activation of the ‘pain matrix’, and dysfunctional connectivity associated with the experience of chronic pain. The transition between the two states will be studied, with a particular focus on the dependence of the functional connectivity patterns on the dynamics of a sub-population, the dynamics of synaptic currents, and plasticity of interconnections (and of course disturbances in each, mimicking various forms of sensitisation, channelopathies, sub-circuit over-activation, etc.). The development of an in silicomodel will also allow the design of restorative stimulation protocols, such as via deep brain stimulation or patient-controlled real-time feedback, to alleviate pain. The mathematical challenge will be to understand how a dysfunctional ‘pain matrix’ state induced within the model environment can be coaxed back to a normal activation pattern.

Statistical methods will be developed to decode neuroimaging signals and predict a sensory pain experience on the basis of spatially correlated fMRI voxels. Exponential random graph models (ERGMs) will allow us to gain deeper insights into the complex neurobiological interactions and changes that occur in many disorders. Although ERGMs have been extensively utilised in social science to analyse highly complex networks, it is only until recently that they have been successfully used to study brain networks using resting fMRI data showing some very promising results [2].

**Training**

The student will do a laboratory rotation in the Pain Imaging group, to appreciate the data sets that are available to work with. Training on Neuroimaging data acquisition and analysis will be provided by participation at the MSc Translational Neuroimaging (Course director: D Auer)

The student will learn about advanced techniques in Computational Neuroscience by attending the course G14TNS Theoretical Neuroscience (School of Mathematical Sciences). The student will also learn about advanced statistical computational techniques such as Markov Chain Monte Carlo (MCMC) by attending the course G14CST and courses from the Academy for PhD Training in Statistics (APTS).

Nonlinear dynamics in the heart - the role of atrial myocytes

Supervisor: Ruediger Thul (ruediger.thul@nottingham.ac.uk)

A human heart beats more than a billion times during the average lifespan, and is required to do so with great fidelity. The ventricular (larger) chambers of the heart are responsible for generating the force that propels blood to the lungs and body [1]. Under sedentary conditions, the atrial (smaller) chambers make only a minor contribution to blood pumping. However, during periods of increased hemodynamic demand, such as exercise, atrial contraction increases to enhance the amount of blood within the ventricles before they contract. This `atrial kick' is believed to account for up to 30% extra blood pumping capacity. Deterioration of atrial myocytes, i.e. muscle cells, with ageing causes the loss of this blood pumping reserve, thereby increasing frailty in the elderly. Atrial kick is also lost during atrial fibrillation (AF), the most common form of cardiac arrhythmia. The stagnation of blood within the atrial chambers during AF can cause thrombus formation, leading to thromboembolism. Approximately 15% of all strokes occur in people with AF. As shown in numerous reports, the genesis and maintenance of AF is causally linked to the dysregulation of calcium signalling, which is bidirectionally coupled to the membrane potential of the cell [2-4].

In this project, we will investigate how changes in the membrane potential lead to changes in the intracellular calcium concentration, which in turn feeds back to the temporal evolution of the membrane potential. We will employ a recently developed three-dimensional model of an atrial myocyte with a biologically realistic distribution of calcium release sites. Through detailed numerical simulations we will achieve a better understanding of how the specific morphology of atrial myocytes impacts on the membrane driven generation of calcium transients, and how clinically relevant pathologies like early after depolarisation and delayed after depolarisation are shaped through the interaction of localised calcium transients near the plasma membrane and the membrane potential itself.


Mathematical Physics

Critical random matrix ensembles
Supervisor:  Alexander Ossipov (alexander.ossipov@nottingham.ac.uk)

In Random Matrix Theory (RMT) one deals with matrices whose entries are given by random variables. RMT has a great number of applications in physics, mathematics, engineering, finance etc. In this project, a particular class of random matrix ensembles --- critical random matrix models will be studied. These models describe statistical properties of disordered systems at a point of the quantum phase transition. Using RMT one can compute various critical exponents, correlation functions and other physically important quantities.

In the language of RMT, the criticality implies very special properties of eigenvalues and eigenvectors of random matrices. One can show, for example, that eigenvectors in critical models are fractal. For certain models fractality of eigenvectors can be investigated with the help of rather simple random matrices, such as two by two matrices in the simplest case. In other cases, much more sophisticated methods, such as supersymmetry, should be employed. Some steps in this direction have been taken recently, but very few general results are available at the moment. The aim of the project is to close this gap.


Models of Quantum Geometry
Supervisor:  John Barrett (john.barrett@nottingham.ac.uk)

There are a number of projects investigating aspects of quantum geometry models and related mathematics. This study uses techniques from topology, algebra, category theory and geometry.
The motivation is to study models that include gravity, working towards solving the problem of quantum gravity.

Quantum gravity in three dimensions is defined using state sum models. A recent development is the definition of observables in these models and the possibility of coupling Feynman diagrams of matter particles to quantum gravity. Possible projects are:

To calculate examples of observables in state sum models and develop a geometric understanding of their properties.

To explore the relation of quantum gravity coupled to matter to the usual Feynman diagrams.

To investigate fermionic state sum models

**Diffeomorphism Invariant Gauge Theories**

**Supervisor:** Kirill Krasnov ([kirill.krasnov@nottingham.ac.uk](mailto:kirill.krasnov@nottingham.ac.uk))

Yang-Mills gauge theory is a dynamical theory of connection (gauge field) that requires the background spacetime metric for its construction. Diffeomorphism invariant gauge theories are similarly dynamical theories of connection, but can be written down without any background structure like the metric. It can be shown that simplest non-trivial such theories (for the gauge group SU(2)) are theories of gravity. It can also be shown that for more complicated gauge groups these theories describe both gravity and the usual Yang-Mills gauge theory, as well as some typically massive matter. The study of these theories has just begun, and this project can involve looking into either classical (e.g. understanding details of the gravity/other interactions unification in this language) or quantum aspects (e.g. doing perturbative quantum computations with gravity in this framework).


**Acceleration, black holes and thermality in quantum field theory**

**Supervisor:** Jorma Louko ([jorma.louko@nottingham.ac.uk](mailto:jorma.louko@nottingham.ac.uk))

Hawking's 1974 prediction of black hole radiation continues to inspire the search for novel quantum phenomena associated with global properties of spacetime and with motion of observers in spacetime, as well as the search for laboratory systems that exhibit similar phenomena. At a fundamental level, a study of these phenomena provides guidance for developing theories of the quantum mechanical structure of spacetime, including the puzzle of the microphysical origin of black hole entropy. At a more practical level, a theoretical control of the phenomena may have applications in quantum information processing in situations where gravity and relative motion are significant, such as quantum communication via satellites.
Specific areas for a PhD project could include:

Model particle detectors as a tool for probing nonstationary quantum phenomena in spacetime, such as the onset of Hawking radiation during gravitational collapse. See arXiv:1206.2055 and references therein.

Black hole structure behind the horizons as revealed by quantum field observations outside the horizons. See arXiv:1001.0124 and references therein.


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**Scattering in disordered systems with absorption: beyond the universality**

Supervisor:  **Alexander Ossipov** ([alexander.ossipov@nottingham.ac.uk](mailto:alexander.ossipov@nottingham.ac.uk))

The study of wave scattering in quantum systems with disorder or underlying classical chaotic dynamics is essential for an understanding of many different physical systems. These include, for example, light propagation in random media, transport of electrons in quantum dots, transmission of microwaves in waveguides and cavities, and many others.

An important feature of any real experiment on scattering is the presence of absorption. As the result, not all the incoming flux is either reflected or transmitted through system, but part of it is irreversibly lost in the environment.

In recent years, considerable progress has been made in the study of scattering in disordered or chaotic quantum systems in the presence of absorption, see e.g Fyodorov, Savin & Sommers, (2005). However almost all results known so far are restricted by the so called "universal limit" described by the conventional Random Matrix Theory. The idea of the suggested project is to go beyond the "universal limit" and to investigate properties of the scattering matrix in lossy systems for the case of a quasi-one-dimensional disordered waveguide. This model describes for example electron dynamics in a thick disordered wire or propagation of light or microwave radiation in a slab geometry. There are two recent advances making an analytical treatment of this problem feasible. The first one is a discovery of a kind of fluctuation dissipation relation between the properties of an open system in the presence of absorption and a certain correlation function of its closed counterpart. This can be exploited, for example, to relate statistics of scattering characteristics to eigenfunction fluctuations in closed systems (Ossipov & Fyodorov, 2005). The second one is a new analytical insight into properties of quasi-one-dimensional disordered conductors, see Skvortsov & Ostrovsky, (2006).

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**Non-Parametric methods in quantum state estimation**

Supervisor:  **Madalin Guta** ([madalin.guta@nottingham.ac.uk](mailto:madalin.guta@nottingham.ac.uk))

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The recent advances in Quantum Information and Quantum Computation [1] have brought a paradigm shift in the way we think about encoding and manipulating information. Atoms and photons are carriers of a new type of information and thanks to the modern technology we have reached the point where we can measure individual quantum systems. A fundamental implication of these developments is that statistical inference based on data obtained by measuring a limited number of individual systems, will play a much greater role in quantum theory.

Quantum Homodyne Tomography is an estimation technique in quantum optics which allows the statistical reconstruction of the state of a mode of light from the random results of homodyne measurements on identically prepared pulses of light [2,3]. Experimenters use it to establish that they have succeeded in creating non-classical states such as squeezed light and Schrodinger cats. The estimators currently used in the physics literature involve arbitrary choices of smoothing, binning or truncation parameters that tend to destroy the interesting 'quantum' patterns that the experimenter is looking for.

The objective of this project is to provide physicists with powerful and reliable estimators for quantum homodyne tomography and other measurement schemes [4], by using methods based on state-of-the-art statistical techniques. More accurate estimation methods for quantum states will benefit Quantum Engineering by reducing the number of required measurements, and thus the costs.

In the last four years I brought some contributions [5,6] in this direction, and I established collaborations with top Mathematical Statistics groups: Richard Gill (Leiden), Cristina Butucea (Paris).


**Statistical problems in quantum filtering and control**

Supervisor: **Madalin Guta** (madalin.guta@nottingham.ac.uk)
The ability to manipulate, control and measure quantum systems is a central issue in Quantum Computation. The Theory of Quantum Filtering [1] offers a clear conceptual understanding of continuous-time measurements and has been used extensively at interpreting experimental data in quantum optics [2]. Mathematically, we deal with an extension of the classical theory used in engineering at estimating an unobservable signal of interest from some available noisy data. In Quantum Filtering we are interested in determining the time evolution of a quantum system by continuously monitoring the environment with whom the system interacts.

The result is the so called filtering equation, or stochastic Schrodinger equation [3] for the state of the system conditioned on the path of measurement outcomes. The next step is to use the knowledge of the conditional state to steer the system towards a certain goal by constantly acting upon it in a feedback loop. Different feedback control strategies are now available [4,5], and the principles of Quantum Filtering & Feedback Control have been successfully implemented in squeezing of atomic ensembles [6].

The theoretical framework of quantum filtering relies on the assumption that all parameters of the model are known in advance, including the initial state of the system, the interaction strength and the detection losses. In practical situations this is not always the case and one may actually be interested in estimating such parameters in the first place. This project aims at developing new tools for statistical problems in the framework of continuous-time measurements. A variety of problems can be considered:

Estimation of state and dynamical parameters. The adaptive measurement [7] of the phase of a coherent state is a beautiful illustration of the usefulness of feedback in improving the quality of estimators in quantum statistics. A general optimality principle for estimation in continuous-time measurements has not yet been formulated.

Asymptotic statistical properties of multi-particle systems. Recent state-of-the-art experiments [6,8] use large ensembles of spins in interaction with the electromagnetic field. By making a Gaussian approximation for the joint state of the qubits, one can describe the spins-field dynamics with a drastically reduced number of parameters. This investigation can be pursued in other cases such as atomic squeezing, and more generally in time-asymptotics of continuous time measurements.

Sufficient statistic for the measurement output process, dimensional reduction. In a continuous-time measurement the output is a stochastic process which may be difficult to handle. For statistical inference problems one can restrict to estimators based on a sufficient statistic of this process. For example in homodyne detection of light leaking from a cavity, a single integral of the detector signal is sufficient statistic for the family of models indexed by the initial state of the mode of light. When the dimension of the observed system is large one would like to further reduce the dimension of the sufficient statistic without compromising the statistical quality of the data.

The student will benefit from the expertise of Prof. Viacheslav Belavkin who is a pioneer in Quantum Filtering & Control, and from my collaboration links with: Luc Bouten (Caltech), Hans Maassen (Nijmegen), Jonas Kahn (Paris).
V.P. Belavkin, Quantum stochastic calculus and quantum nonlinear filtering, Journal of Multivariate Analysis, 42, (1992), 171--201


Projects on quantized detector networks

Supervisor: George Jaroszkiewicz (george.jaroszkiewicz@nottingham.ac.uk)

QDN is an approach to quantum physics developed by the Supervisor over the period 2001 onwards. Its aim is to follow the original quantum principles laid down in 1925 by Heisenberg, one of the pioneers of quantum mechanics. In this approach, the objective is to rewrite the laws of physics in terms of observer-apparatus information exchange. This contrasts with the standard approach to quantum mechanics, which supposes that observers extract information from systems under observation (SUOs).

This is not a small difference. It alters in a fundamental way the conception of what quantum mechanics is really about and requires a novel mathematical formulation reminiscent of that used to describe quantum computation. In QDN, SUOs are not regarded as strange wave-particle objects governed by quantum wave equations and collapsing into particles when they are observed. Rather, QDN views quantum mechanics in its proper perspective as the physically correct way of discussing observer-apparatus information exchange. Historically, the recognition that this was what really mattered (rather than the classical world view that things "exist" on their own independently of observation) became known as empiricism/instrumentalism. Famous names associated with this are the philosophers Locke and Hume in the days before quantum mechanics was discovered. In the quantum era, Heisenberg and Feynman are two outstanding individuals who attempted to create concrete mathematical theories based on this idea. Heisenberg's success led to "matrix mechanics", which was the first real quantum theory, whilst Feynman attempted to describe electrodynamical interactions between charges without the use of electromagnetic waves or the photon concept. This was the subject of his PhD thesis.

It would be a mistake to regard instrumentalism as metaphysical. In fact, the emphasis in QDN is to strip away one traditional layer of metaphysics, the SUO concept, and deal only with what can be known for sure, which is the existence of the observer and their apparatus. QDN can deal with time-dependent apparatus, including changes in Hilbert space dimensions, multiple
observers, and gives a proper insight into relativistic quantum mechanics (traditionally a source of problems).

A major review of QDN will be published by World Scientic in the journal International Journal of Modern Physics B on 30 December 2007:


Potential PhD students should be interested in the fundamentals of physics and have a knowledge of classical mechanics, quantum mechanics and relativity. The following specific areas suitable for a PhD dissertation are offered within the QDN programme. Variants of these programmes are possible.

1. Relativity and QDN In this project, the aim is to develop further the relationship between quantum mechanics and relativity (in both special and general forms) from the QDN perspective. There is reason to believe that fundamental problems in "quantum gravity" would be handled differently when observer-apparatus issues are looked at properly.

2. Quantum Optics and QDN The review quoted above describes some QDN applications to quantum optics. Quantum optics is an excellent scenario for QDN, as the sort of experiments actually done are generally amenable to a modularized approach. The aim in this project will be to develop more examples in quantum optics and explore the possibility of writing a computer-algebra package to allow for the description of quantum optics experiments of arbitrary complexity.

3. Mixed States and QDN In this project, the aim will be to develop further the application of the fundamental "signal algebra" which is embedded in QDN to partial observations: these are situations where the observer extracts only some information from their apparatus, resulting in what are equivalent to mixed states in standard QM. This area should have many potential avenues of investigation, including teleportation, cryptography and tomography. The relationship between QDN and the POVM (positive operator valued measure) approach in quantum physics will be central to this project.

4. Continuous degrees of freedom and QDN In this project, the aim will be to investigate how QDN can (or perhaps cannot) deal with SUOs with an infinite number of degrees of freedom in the conventional description of them. This means for example not only looking at atomic theory, etc, but also quantum field theory and more exotic questions such as to the physical meaning of "space". QDN takes a specific view about the latter, regarding it as contextual.

5. Contextuality, probability and QDN In this project, the aim will be to develop further understanding of the relationship between context, probability, information, and quantum mechanics. It will require investigation in to various areas such as complexity, Bayesian probability, the measurement problem in quantum mechanics, and if required, issues related to the renormalization programme in quantum field theory and the apparent failure of the conventional "quantum gravity" and super string programmes.
Recent PhDs supervised by the Supervisor in this area are Jon Eakins Classical and Quantum Causality in Quantum Field Theory, University of Nottingham PhD (awarded 2004) Jason Ridgway-Taylor Elements of Classical and Quantum Theories from Classical and Quantum Bits, University of Nottingham PhD (awarded 2007)


Quantum correlations in many-body systems

Supervisor: Gerardo Adesso (gerardo.adesso@nottingham.ac.uk)

The behaviour of physical systems at the microscopic scale obeys the laws of quantum mechanics. Quantum systems can share a form of quantum correlations known as entanglement, which is nowadays acknowledged as a resource for enhanced information processing. However, there are more general types of quantum correlations, beyond entanglement, that can be present in separable quantum states.

This project deals with the characterisation of the nonclassicality of correlations in multipartite quantum systems. Interesting aspects of this project are the elucidation of the relationship between these more general forms of quantum correlations, as quantified e.g. by the "quantum discord", and entanglement in mixed multipartite quantum states. Another theme will be the identification of experimentally friendly schemes to engineer quantum correlations, and detect them in practical demonstrations, as well as rigorously assessing the usefulness of quantum correlations beyond entanglement as resources for next-generation quantum information protocols.


D. Girolami, M. Paternostro, G. Adesso; Non-classicality indicators and extremal quantum correlations in two-qubit states; arXiv:1008.4136
Quantum aspects of frustration in spin lattices

Supervisor: Gerardo Adesso (gerardo.adesso@nottingham.ac.uk)

Recently, a number of tools developed in the framework of quantum information theory have proven useful to tackle founding open questions in condensed matter physics, such as the characterization of quantum phase transitions and the scaling of correlations at critical points. Our contribution to the field dealt with a method, based on quantum informational concepts, to identify analytically factorized (unentangled) ground states in many-body spin models, which constitute an exact solution to generally non-exactly solvable models for specific values of the Hamiltonian parameters. In presence of frustration, ground state factorization is suppressed. Therefore the factorizability provides a qualitative handle on the degree of quantum frustration.

This project will build on these premises and will seek for genuine signatures of quantum versus classical frustration in spin systems, a topic of great relevance for condensed matter. Frustrated quantum models may play a key role for high-temperature superconductivity and for certain biological processes. The relationship between frustration, disorder and entanglement is yet largely unexplored.


Quantum information with non-Gaussian states

Supervisor: Gerardo Adesso (gerardo.adesso@nottingham.ac.uk)

Quantum information with continuous variable systems is a burgeoning area of research which has recorded astonishing theoretical and experimental successes, mainly thanks to the manipulation and exploitation of Gaussian states of light and matter. However, quite recently a number of tasks have been individuated which can not be perfectly implemented by using Gaussian states and operations only, and another set of processes is being explored where some non-Gaussianity has been recognised as an advantageous ingredient to sharply improve performances of quantum communication.

In this project the student will investigate the limitations of the Gaussian scenario in different contexts such as quantum communication, computation and estimation and, more generally, quantum technology. This is paralleled by recent progresses in the experimental generation of
non-Gaussian states, which further motivate their application in quantum information science. Special emphasis will be put on devising efficient methods to quantify the entanglement in selected classes of non-Gaussian states, using techniques whose complexity is not exceedingly large compared to the usual tools (quadrature measurements, homodyne detection) which are effective for Gaussian states.


**Developing new relativistic quantum technologies**

Supervisor: **Ivette Fuentes** ([ivette.fuentes@nottingham.ac.uk](mailto:ivette.fuentes@nottingham.ac.uk))

Relativistic quantum information is an emerging field which studies how to process information using quantum systems taking into account the relativistic nature of spacetime. The main aim of this PhD project is to find ways to exploit relativity to improve quantum information tasks such as teleportation and to develop new relativistic quantum technologies.

Moving cavities and Unruh-Dewitt type detectors promise to be suitable systems for quantum information processing [1,2]. Interestingly, motion and gravity have observable effects on the quantum properties of these systems [2,3]. In this project we will find ways to implement quantum information protocols using localized systems such as cavities and detectors. We will focus on understanding how the protocols are affected by taking into account the non-trivial structure of spacetime. We will look for new protocols which exploit not only quantum but also relativistic resources for example, the non-local quantum correlations present in relativistic quantum fields.


D. E. Bruschi, I. Fuentes, & J. Louko Voyage to Alpha Centauri: Entanglement degradation of cavity modes due to motion accepted in Physical Review D Rapid Communications

N. Friis, D. E. Bruschi, J. Louko & I. Fuentes Motion generates entanglement Submitted to Physical Review D

**Many-body localization in quantum spin chains and Anderson localization**
Properties of wave functions in many-body systems is very active topic of research in modern condensed matter theory. Quantum spin chains are very useful models for studying quantum many-body physics. They are known to exhibit complex physical behaviour such as quantum phase transitions. Recently, they have been studied intensively in the context of many-body localization.

The key idea of this project is to explore the similarity between Hamiltonians of the spin chain models and the Anderson models on d-dimensional hypercube. In such models, single particle wave functions can be localized in space due to the famous phenomenon of the Anderson localization.

Understanding of the relation between many-body localization and Anderson localization, quantum phase transitions in spin chains and the Anderson metal-insulator transition will be the main topic of this project.

**The density of states of non-Hermitian Hamiltonians**

Supervisor: **Alexander Ossipov** (alexander.ossipov@nottingham.ac.uk)

Observables in quantum mechanics are usually described by Hermitian operators. However, in some cases non-Hermitian Hamiltonians may play an important role in quantum mechanics. For example, they appear as effective Hamiltonians describing dynamics of open quantum systems.

Statistical properties of complex quantum systems can be studied with the help of the Random Matrix Theory (RMT), which can be used for Hermitian and for non-Hermitian Hamiltonians.

The aim of this project is to investigate statistical properties of the eigenvalues of a certain class of non-Hermitian Hamiltonians applying analytical techniques developed in RMT.

**Entanglement of non-interacting fermions at criticality**

Supervisor: **Alexander Ossipov** (alexander.ossipov@nottingham.ac.uk)

Entanglement of the ground state of many-particle systems has recently attracted a lot of attention. For non-interacting fermions, the ground state entanglement can be calculated from the eigenvalues of the correlation matrix of the single particle wavefunctions. For this reason, the nature of the single particle wavefunctions is crucially important for understanding of the entanglement properties of a many-body system.

The ground state entanglement is well understood now for free fermions, whose wavefuncions are simple plane waves. However, there are almost no analytical results available in the case where wavefuncions are non-trivial.
This project will explore the ground state entanglement at the quantum critical point of the metal-insulator transition, where single particle wavefunctions are known to have self-similar fluctuations, characterised by non-trivial fractal dimensions.

**Topological Resonances on Graphs**

**Supervisor:** Sven Gnutzmann ([sven.gnutzmann@nottingham.ac.uk](mailto:sven.gnutzmann@nottingham.ac.uk))

If a light wave in a resonator between two almost perfect mirrors shows resonance if the wavelength is commensurate with the distance between the two mirrors. If this condition is satisfied it will decay much slower than at other wavelengths which are not commensurate. This is one of the simplest mechanisms for a resonance in a wave system. There are other well known mechanisms that rely on complexity and disorder. It has recently been observed that a network of wire may have a further mechanism that leads to resonances. This mechanism relies on cycles in the network and leads to various signatures which cannot be explained using other well-known mechanisms for resonances. In this project these signatures will be analysed in detail.


**Quantum Chaos in Combinatorial Graphs**

**Supervisor:** Sven Gnutzmann ([sven.gnutzmann@nottingham.ac.uk](mailto:sven.gnutzmann@nottingham.ac.uk))

Graphs consist of V vertices connected by B bonds (or edges). They are used in many branches of science as simple models for complex structures. In mathematics and physics one is strongly interested in the eigenvalues of the V x V connectivity matrix C of a graph. The matrix element C_{ij} of the latter is defined to be the number of bonds that connect the i'th vertex to the j'th vertex.

In this PhD project the statistical properties of the connectivity spectra in (generally large) graph structures will be analysed using methods known from quantum chaos. These methods have only recently been extended to combinatorial graphs (Smilansky, 2007) and allow to represent the density of states and similar spectral functions of a graph as a sum over periodic orbits. The same methods have been applied successfully to metric graphs and quantum systems in the semiclassical regime for more than two decades.


Supersymmetric field theories on quantum graphs and their application to quantum chaos

Supervisor: Sven Gnutzmann (sven.gnutzmann@nottingham.ac.uk)

Quantum graphs are a paradigm model for quantum chaos. They consist of a system of wires along which waves can propagate. Many properties of the excitation spectrum and the spatial distribution of standing waves can be mapped exactly onto a supersymmetric field theory on the network. In a mean-field approximation one may derive various universal properties for large quantum graphs. In this project we will focus on deviations from universal behaviour for finite quantum graphs with the field-theoretic approach.


GNUTZMANN, S., KEATING, J.P. and PIOTET, F., 2008. Quantum Ergodicity on Graphs PHYSICAL REVIEW LETTERS. VOL 101(NUMB 26), 264102


Pseudo-orbit expansions in quantum graphs and their application

Supervisor: Sven Gnutzmann (sven.gnutzmann@nottingham.ac.uk)

Quantum graphs are a paradigm model to understand and analyse the effect of complexity on wave propagation and excitations in a network of wires. They have also been used as a paradigm model to understand topics in quantum and wave chaos where the complexity has a different origin while the mathematical framework is to a large extent analogous.

Many properties of the waves that propagate through such a network can be described in terms of trajectories of a point particle that propagates through the network. The ideas is to write a property of interest as a sum over amplitudes (complex numbers) connected to all possible trajectories of the point particle. These sums remain challenging objects for explicit evaluations. Recently a numer of advanced methods for their summation have been introduced. The latter are built on so-called pseudo-orbits. In this project these methods will be developeled further and applied to questions related to quantum chaos and random-matrix theory.
Daniel Waltner, Sven Gnutzmann, Gregor Tanner, Klaus Richter, A sub-determinant approach for pseudo-orbit expansions of spectral determinants, arXiv:1209.3131 [nlin.CD]


The ten-fold way of symmetries in quantum mechanics. An approach using coupled spin operators.

Supervisor: Sven Gnutzmann (sven.gnutzmann@nottingham.ac.uk)

About 50 years ago Wigner and Dyson proposed a three-fold symmetry classification for quantum mechanical systems -- these symmetry classes consisted of time-reversal invariant systems with integer spin which can be described by real symmetric matrices, time-reversal invariant systems with half-integer spin which can be described by real quaternion matrices, and systems without any time-reversal symmetry which are described by complex hermitian matrices. These three symmetry classes had their immediate application in the three classical Gaussian ensembles of random-matrix theory: the Gaussian orthogonal ensemble GOE, the Gaussian symplectic ensemble GSE, and the Gaussian unitary ensemble GUE. In the 1990's this classification was extended by adding charge conjugation symmetries -- symmetries which relate the positive and negative part of a spectrum and which are described by anti-commutators.

The classification was completed by Altland and Zirnbauer who have shown that there are essentially only seven further symmetry classes on top of the Wigner-Dyson classes leading to what is now known as the 'ten-fold way'. All symmetry classes have applications in physics. The new symmetry classes are realised by various cases of the Dirac equation and the Bogoliubov-de Gennes equation. For a long time people have thought of these symmetries only in the context of many-body physics or quantum field theory. However there are simple quantum mechanical realisations of all ten symmetry classes which in terms of two coupled spins where the classification follows from properties of the coupling parameters and of the irreducible SU(2) representations on which the spin operators act. This project will explore these simple representations in the quantum mechanical and semiclassical context. One goal will be to understand the implications of the quantum mechanical symmetries for the corresponding classical dynamics which appears in the semiclassical limit of large spins.


Nonlinear waves in waveguide networks

Supervisor: Sven Gnutzmann (sven.gnutzmann@nottingham.ac.uk)

Many wave guides (such as optical fibres) show a Kerr-type effect that leads to nonlinear wave propagation. If the wave guides are coupled at junctions then there is an additional element of complexity due to the non-trivial connectivity of wave guides. In this project the impact of the structure and topology of the network on wave propagation will be studied starting from simple geometries such as a Y-junctions (three waveguides coupled at one junction), a star (many waveguides at one junction), or a lasso (a waveguide that forms a loop and is connected at one point to a second waveguide).


The statistics of nodal sets in wavefunctions

Supervisor: Sven Gnutzmann (sven.gnutzmann@nottingham.ac.uk)

If a membrane vibrates at one of its resonance frequencies there are certain parts of the membrane that remain still. These are called nodal points and the collection of nodal points forms the nodal set. Building on earlier work this project will look at the statistical properties of the nodal set -- e.g. for 3-dimensional waves the nodal set consists of a collection of surfaces and one may ask questions about how the area of the nodal set is distributed for an ensemble of membranes or for an ensemble of different resonances of the same membrane. This project will involve a strong numerical component as wavefunctions of irregular membranes need to be found and analysed on the computer. Effective algorithms to find the area of the nodal set, or the number of domain in which the sign does not change (nodal domains) will need to be developed and implemented.


Number Theory

Number theory in a broad context

Supervisor: Ivan Fesenko (ivan.fesenko@nottingham.ac.uk)

Ivan Fesenko studies zeta functions in number theory using zeta integrals. These integrals are better to operate with than the zeta functions, they translate various properties of zeta functions into properties of adelic objects. This is a very powerful tool to understand and prove fundamental properties of zeta functions in number theory. In the case of elliptic curves over global fields, associated zeta functions are those of regular models of the curve, i.e. the zeta function of a two dimensional object. Most of the classical work has studied arithmetic of elliptic curves over number fields treating them as one dimensional objects and working with with generally noncommutative Galois groups over the number field, such as the one generated by all torsion points of the curve. The zeta integral gadget works with adelic objects associated to the two dimensional field of functions of the curve over a global field and using commutative Galois groups. The latter has already been investigated in two dimensional abelian class field theory and it is this theory which supplies adelic objects on which the zeta integral lives. For example, Fourier duality on adelic spaces associated to the model of the curve explains the functional equation of the zeta function (and of the L-function of the curve).

The theory uses many parts of mathematics: class field theory, higher local fields and several different adelic structures, translation invariant measure and integration on higher local fields (arithmetic loop spaces), functional analysis and harmonic analysis on such large spaces, groups endowed with sequential topologies, parts of algebraic K-theory, algebraic geometry. This results in a beautiful conceptual theory.

There are many associated research problems and directions at various levels of difficulty and opportunities to discover new objects, structures and laws.

Computational methods for elliptic curves and modular forms

Supervisor: Christian Wuthrich (christian.wuthrich@nottingham.ac.uk)

Computational Number Theory is a fairly recent part of pure mathematics even if computations in number theory are a very old subject. But over the last few decades this has changed dramatically with the modern, powerful and cheap computers. In the area of explicit computations on elliptic curves, there are two subjects that underwent a great development
recently: elliptic curves over finite fields (which are used for cryptography) and ‘descent’ methods on elliptic curves over global fields, such as the field of rational numbers.

It is a difficult question for a given elliptic curve over a number field to decide if there are infinitely many solutions over this field, and if so, to determine the rank of the Mordell-Weil group. Currently, there are only two algorithms implemented for finding this rank, one is the descent method that goes back to Mordell, Selmer, Cassels,... and the other is based on the work of Gross, Zagier, Kolyvagin... using the link of elliptic curves to modular forms. While the first approach works very well over number fields of small degree, it becomes almost impossible to determine the rank of elliptic curves over number fields of larger degree. The second method unfortunately is not always applicable, especially the field must be either the field of rational numbers or a quadratic extension thereof.

There is another way of exploiting the relation between elliptic curves and modular forms by using the p-adic theory of modular forms and the so-called Iwasawa theory for elliptic curves. Results by Kato, Urban, Skinner give us a completely new algorithm for computing the rank and other invariants of the elliptic curve, but not much of this has actually been implemented. Possible PhD projects could concern the further development of these new methods and their implementation.

**Statistics and Probability**

**Index policies for stochastic optimal control**

Supervisor:  **David Hodge** ([david.hodge@nottingham.ac.uk](mailto:david.hodge@nottingham.ac.uk))

Since the discovery of Gittins indices in the 1970s for solving multi-armed bandit processes the pursuit of optimal policies for this very wide class of stochastic decision processes has been seen in a new light. Particular interest exists in the study of multi-armed bandits as problems of optimal allocation of resources (e.g. trucks, manpower, money) to be shared between competing projects. Another area of interest would be the theoretical analysis of computational methods (for example, approximative dynamic programming) which are coming to the fore with ever advancing computer power.

Potential project topics could include optimal decision making in the areas of queueing theory, inventory management, machine maintenance and communication networks.

**Uncertainty quantification in palaeo-climate reconstruction**

Supervisor:  **Richard Wilkinson** ([r.d.wilkinson@nottingham.ac.uk](mailto:r.d.wilkinson@nottingham.ac.uk))
The climate evolves slowly. Even if we stopped emitting green-houses gases today, we wouldn't see the full effect of the damage already done for at least another 100 years. The instrumental record of past climate and weather goes back at most 300 years, and before then we have to rely on indirect (and inaccurate) data sources. Because of the slow evolution of the climate, this is like only having a very small number of accurate observations, and so consequently we have very little information that can be used to assess the accuracy of climate simulators, which are the key tool used for predicting how the climate will behave in the future.

An important source of information on what the climate was like in the past comes from proxy data sources such as pollen taken from lake deposits, or measurements of the air-content (specifically the ratio of oxygen-18 to oxygen-16) stored in glaciers hundreds of thousands of years ago. Reconstructing past climate from these data sources is a difficult task as the measurements are noisy, correlated, and don't have accurate dates attached to them, yet the task is important if we are to understand how the climate evolves and hence be able to predict the future.

In this project, we will look at statistical methods for accurate palaeo-climate reconstruction, and aim to provide believable uncertainty quantifications that accurately represent our degree of confidence/ignorance about what the climate was like in the past. The complex nature of the problem means that it is likely that state-of-the-art Monte Carlo methods will be needed, as well as potentially developing new methods in order to do the inference.

**Semi-Parametric Time Series Modelling Using Latent Branching Trees**

**Supervisor:** Theodore Kypraios ([theodore.kypraios@nottingham.ac.uk](mailto:theodore.kypraios@nottingham.ac.uk))

A class of semi-parametric discrete time series models of infinite order where we are be able to specify the marginal distribution of the observations in advance and then build their dependence structure around them can be constructed via an artificial process, termed as Latent Branching Tree (LBT). Such a class of models can be very useful in cases where data are collected over long period and it might be relatively easy to indicate their marginal distribution but much harder to infer about their correlation structure. The project is concerned with the development of such models in continuous-time as well as developing efficient methods for making Bayesian inference for the latent structure as well as the model parameters. Moreover, the application of such models to real data would be also of great interest.

**Bayesian methods for analysing computer experiments**

**Supervisor:** Richard Wilkinson ([r.d.wilkinson@nottingham.ac.uk](mailto:r.d.wilkinson@nottingham.ac.uk))

Computer experiments (ie simulators) are used in nearly all areas of science and engineering. The statistical analysis of computer experiments is an exciting and rapidly growing area of
statistics which looks at the question of how best to learn from computer models. Examples of the types of challenges faced, and possible areas for a Ph.D, are given below.

(i) Computer models are often process models where the likelihood function is intractable (as is common in genetics, ecology, epidemiology etc) and so to do inference we have to use likelihood-free techniques. Approximate Bayesian computation (ABC) methods are a new class of Monte Carlo methods that are becoming increasingly popular with practitioners, but which are largely unstudied by statisticians, and there remains many open questions about their performance. Application areas which use ABC methods are mostly biological (genetics and ecology in particular), but their use is growing across a wide range of fields.

(ii) Expensive simulators which take a considerable amount of time to run (eg, climate models), present the challenge of how to learn about the model (its parameters, validity, or its predictions etc.) when we only have a limited number of model evaluations available for use. A statistical tool developed in the last decade is the idea of building statistical emulators of the simulator. Emulators are cheap statistical models of the simulator (models of the model) which can be used in place of the simulator to make inferences, and are now regularly used in complex modelling situations such as in climate science. However, there are still many questions to be answered about how best to build and then use emulators. Possible application areas for these methods include climate science, and engineering (such as ground-water flow problems for radio-active waste), as well as many others.

(iii) "All models are wrong, but some are useful" - In order to move from making statements about a model to making statements about the system the model was designed to represent, we must carefully quantify the model error - interest lies in what will actually happen, rather than in what your model says will happen! Failure to account for model errors can mean that different models of the same system can give different predictions (see for example the controversy regarding the differing predictions of the large climate models - none of which account for model error!). Assessing and incorporating model error is a new and rapidly growing idea in statistics, and is done by a combination of subjective judgement and statistical learning from data. The range of potential application areas is very wide, but in particular meteorology and mechanical engineering are areas where these methods are needed.

Wilkinson, Approximate Bayesian computation (ABC) gives exact results under the assumption of model error, in submission. Available as arXiv:0811.3355.


Ion channel modelling

Supervisor: Frank Ball (frank.ball@nottingham.ac.uk)

The 1991 Nobel Prize for Medicine was awarded to Sakmann and Neher for developing a method of recording the current flowing across a single ion channel. Ion channels are protein molecules that span cell membranes. In certain conformations they form pores allowing current to pass across the membrane. They are a fundamental part of the nervous system. Mathematically, a single channel is usually modelled by a continuous time Markov chain. The complete process is unobservable but rather the state space is partitioned into two classes, corresponding to the receptor channel being open or closed, and it is only possible to observe which class of state the process is in. The aim of single channel analysis is to draw inferences about the underlying process from the observed aggregated process. Further complications include (a) the failure to detect brief events and (b) the presence of (possibly interacting) multiple channels. Possible projects include the development and implementation of Markov chain Monte Carlo methods for inferences for ion channel data, Laplace transform based inference for ion channel data and the development and analysis of models for interacting multiple channels.

Optimal control in yield management

Supervisor: David Hodge (david.hodge@nottingham.ac.uk)

Serious mathematics studying the maximization of revenue from the control of price and availability of products has been a lucrative area in the airline industry since the 1960s. It is particularly visible nowadays in the seemingly incomprehensible price fluctuations of airline tickets. Many multinational companies selling perishable assets to mass markets now have large Operations Research departments in-house for this very purpose. This project would be working studying possible innovations and existing practices in areas such as: customer acceptance control, dynamic pricing control and choice-based revenue management. Applications to social welfare maximization, away from pure monetary objectives, and the resulting game theoretic problems are also topical in home energy consumption and mass online interactions.

Stochastic Processes on Manifolds

Supervisor: Huiling Le (huiling.le@nottingham.ac.uk)

As well as having a wide range of direct applications to physics, economics, etc, diffusion theory is a valuable tool for the study of the existence and characterisation of solutions of partial differential equations and for some major theoretical results in differential geometry, such as the 'Index Theorem', previously proved by totally different means. The problems which arise in all these subjects require the study of processes not only on flat spaces but also on curved spaces or manifolds. This project will investigate the interaction between the geometric
structure of manifolds and the behaviour of stochastic processes, such as diffusions and martingales, upon them.

**Analytic methods in probability theory**

This research focuses on interactions between probability theory, combinatorics and topology. Topics of particular interest:

- dependence, limit theorems for dependent variables with applications to dynamical systems, examples and counterexamples to limit theorems;
- modern analytic methods in probability theory such as stochastic orderings, Stein's type operator, contraction, Poincare - Hardy - Sobolev type inequalities and their applications;
- stochastic analysis of rare events such as Poisson approximation and large deviations;
- Random groups, Hausdorff dimension and related topics

**Statistical Theory of Shape**

Supervisor:  **Huiling Le** ([huiling.le@nottingham.ac.uk](mailto:huiling.le@nottingham.ac.uk))

Devising a natural measure between any two fossil specimens of a particular genus, assessing the significance of observed 'collinearities' of standing stones and matching the observed systems of cosmic 'voids' with the cells of given tessellations of 3-spaces are all questions about shape.

It is not appropriate however to think of 'shapes' as points on a line or even in a euclidean space. They lie in their own particular spaces, most of which have not arisen before in any context. PhD projects in this area will study these spaces and related probabilistic issues and develop for them a revised version of multidimensional statistics which takes into account their peculiar properties. This is a multi-disciplinary area of research which has only become very active recently. Nottingham is one of only a handful of departments at which it is active.

**Automated tracking and behaviour analysis**

Supervisor:  **Christopher Brignell** ([chris.brignell@nottingham.ac.uk](mailto:chris.brignell@nottingham.ac.uk))

In collaboration with the Schools of Computer and Veterinary Science we are developing an automated visual surveillance system capable of identifying, tracking and recording the exact movements of multiple animals or people. The resulting data can be analysed and used as an early warning system in order to detect illness or abnormal behaviour. The three-dimensional
targets are, however, viewed in a two dimensional image and statistical shape analysis techniques need to be adapted to improve the identification of an individual's location and orientation and to develop automatic tests for detecting specific events or individuals not following normal behaviour patterns.

**Asymptotic techniques in Statistics**

Supervisor:  **Andrew Wood (andrew.wood@nottingham.ac.uk)**

Asymptotic approximations are very widely used in statistical practice. For example, the large-sample likelihood ratio test is an asymptotic approximation based on the central limit theorem. In general, asymptotic techniques play two main roles in statistics: (i) to improve understanding of the practical performance of statistics procedures, and to provide insight into why some procedures perform better than others; and (ii) to motive new and improved approximations. Some possible topics for a Ph.D. are

- Saddlepoint and related approximations
- Relative error analysis
- Approximate conditional inference
- Asymptotic methods in parametric and nonparametric Bayesian Inference

**Statistical Inference for Ordinary Differential Equations**

Supervisors:  **Theodore Kypraios (theodore.kypraios@nottingham.ac.uk)**  
**Simon Preston (simon.preston@nottingham.ac.uk)**  
**Andrew Wood (andrew.wood@nottingham.ac.uk)**

Ordinary differential equations (ODE) models are widely used in a variety of scientific fields, such as physics, chemistry and biology. For ODE models, an important question is how best to estimate the model parameters given experimental data. The common (non-linear least squares) approach is to search parameter space for parameter values that minimise the sum of squared differences between the model solution and the experimental data. However, this requires repeated numerical solution of the ODEs and thus is computationally expensive; furthermore, the optimisation's objective function is often highly multi-modal making it difficult to find the global optimum. In this project we will develop computationally less demanding likelihood-based methods, specifically by using spline regression techniques that will reduce (or eliminate entirely) the need to solve numerically the ODEs.

**Bayesian approaches in palaeontology**
PhD Vacancies 2014
School of Mathematical Sciences

www.nottingham.ac.uk/mathematics/research

Supervisor: Richard Wilkinson (r.d.wilkinson@nottingham.ac.uk)

Palaeontology provides a challenging source of problems for statisticians, as fossil data are usually sparse and noisy. Methods from statistics can be used to help answer scientific questions such as when did species originate or become extinct, and how diverse was a particular taxonomic group. Some of these questions are of great scientific interest - for example - did primates coexist with dinosaurs in the Cretaceous? There is no hard evidence either way, but statistical methods can be used to assess the probability that they did coexist.

This project will involve building a stochastic forwards model of an evolutionary scenario, and then fitting this model to fossil data. Quantifying different sources of uncertainty is likely to play a key part in the analysis.


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Supervisor: Christopher Fallaize (chris.fallaize@nottingham.ac.uk)

In statistical shape analysis, objects are often represented by a configuration of landmarks, and in order to compare the shapes of objects, their configurations must first be aligned as closely as possible. When the landmarks are unlabelled (that is, the correspondence between landmarks on different objects is unknown) the problem becomes much more challenging, since both the correspondence and alignment parameters need to be inferred simultaneously.

An example of the unlabelled problem comes from the area of structural bioinformatics, when we wish to compare the 3-d shapes of protein molecules. This is important, since the shape of a protein is vital to its biological function. The landmarks could be, for example, the locations of particular atoms, and the correspondence between atoms on different proteins is unknown. This project will explore methods for unlabelled shape alignment, motivated by the problem of protein structure alignment. Possible topics include development of:

- i) efficient MCMC methods to explore complicated, high-dimensional distributions, which may be highly multimodal when considering large proteins;
- ii) fast methods for pairwise alignment, needed when a large database of structures is to be searched for matches to a query structure;
- iii) methods for the alignment of multiple structures simultaneously, which greatly exacerbates the difficult problems faced in pairwise alignment.


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High-dimensional molecular shape analysis

Supervisor: Ian Dryden (ian.dryden@nottingham.ac.uk)

In many application areas it is of interest to compare objects and to describe the variability in shape as an object evolves over time. For example in molecular shape analysis it is common to have several thousand atoms and a million time points. It is of great interest to reduce the dimension to a relatively small number of dimensions, and to describe the variability in shape and coverage properties over time. Techniques from manifold learning will be explored, to investigate if the variability can be effectively described by a low dimensional manifold. A recent method for shapes and planar shapes called principal nested spheres will be adapted for 3D shape and surfaces. Also, other non-linear dimension reduction techniques such as multidimensional scaling will be explored, which approximate the geometry of the higher dimensional manifold. The project will involve collaboration with Dr Charlie Laughton of the School of Pharmacy.


Uncertainty quantification for models with bifurcations

Supervisors: Ian Dryden (ian.dryden@nottingham.ac.uk) Andrew Cliffe (andrew.cliffe@nottingham.ac.uk)

The project will consider Uncertainty Quantification (UQ) when there are bifurcations or discontinuities in the models. Gaussian Process Emulators (GPE) and Generalised Polynomial Chaos (gPC) techniques will be used to construct fast approximations to high-cost deterministic models. Also, an important component of Bayesian UQ is the difficult task of elicitation of the prior distributions of the parameters of interest, which will be investigated. We will exploit the flexibility in the choice of GPE covariance function to deal with cases where the dependence on the inputs is not smooth. Lack of smoothness can be handled by dividing the parameter space into elements and using of gPC or GPE on each element, but this is difficult to do automatically. We propose to compute the hypersurfaces at which discontinuities occur, using techniques from numerical bifurcation theory, as preparation for discretising with gPC or GPE methods. Bifurcations arise in carbon sequestration applications, and radioactive waste disposal is another area where elicitation and Bayesian emulation are useful.

Statistical analysis of neuroimaging data

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The activity of neurons within the brain can be detected by function magnetic resonance imaging (fMRI) and magnetoencephalography (MEG). The techniques record observations up to 1000 times a second on a 3D grid of points separated by 1-10 millimetres. The data is therefore high-dimensional and highly correlated in space and time. The challenge is to infer the location, direction and strength of significant underlying brain activity amongst confounding effects from movement and background noise levels. Further, we need to identify neural activity that are statistically significant across individuals which is problematic because the number of subjects tested in neuroimaging studies is typically quite small and the inter-subject variability in anatomical and functional brain structures is quite large.

**Identifying fibrosis in lung images**

Supervisor: Christopher Brignell (chris.brignell@nottingham.ac.uk)

Many forms of lung disease are characterised by excess fibrous tissue developing in the lungs. Fibrosis is currently diagnosed by human inspection of CT scans of the affected lung regions. This project will develop statistical techniques for objectively assessing the presence and extent of lung fibrosis, with the aim of identifying key factors which determine long-term prognosis. The project will involve developing statistical models of lung shape, to perform object recognition, and lung texture, to classify healthy and abnormal tissue. Clinical support and data for this project will be provided by the School of Community Health Sciences.

**Modelling hospital superbugs**

Supervisor: Philip O’Neill (philip.oneil@nottingham.ac.uk)

The spread of so-called superbugs such as MRSA within healthcare settings provides one of the major challenges to patient welfare within the UK. However, many basic questions regarding the transmission and control of such pathogens remain unanswered. This project involves stochastic modelling and data analysis using highly detailed data sets from studies carried out in hospital, addressing issues such as the effectiveness of patient isolation, the impact of different antibiotics, and the way in which different strains interact with each other.

**Modelling of Emerging Diseases**

Supervisor: Frank Ball (frank.ball@nottingham.ac.uk)

When new infections emerge in populations (e.g. SARS; new strains of influenza), no vaccine is available and other control measures must be adopted. This project is concerned with addressing questions of interest in this context, e.g. What are the most effective control
measures? How can they be assessed? The project involves the development and analysis of new classes of stochastic models, including intervention models, appropriate for the early stages of an emerging disease.

**Structured-Population Epidemic Models**

 Supervisor:  **Frank Ball** ([frank.ball@nottingham.ac.uk](mailto:frank.ball@nottingham.ac.uk))

The structure of the underlying population usually has a considerable impact on the spread of the disease in question. In recent years the Nottingham group has given particular attention to this issue by developing, analysing and using various models appropriate for certain kinds of diseases. For example, considerable progress has been made in the understanding of epidemics that are propagated among populations made up of households, in which individuals are typically more likely to pass on a disease to those in their household than those elsewhere. Other examples of structured populations include those with spatial features (e.g. farm animals placed in pens; school children in classrooms; trees planted in certain configurations), and those with random social structure (e.g. using random graphs to describe an individual's contacts). Projects in this area are concerned with novel advances in the area, including developing and analysing appropriate new models, and methods for statistical inference (e.g. using pseudo-likelihood and Markov chain Monte Carlo methods).

**Bayesian model choice assessment for epidemic models**

 Supervisor:  **Philip O'Neill** ([philip.oneill@nottingham.ac.uk](mailto:philip.oneill@nottingham.ac.uk))

During the last decade there has been a significant progress in the area of parameter estimation for stochastic epidemic models. However, far less attention has been given to the issue of model adequacy and assessment, i.e. the question of how well a model fits the data. This project is concerned with the development of methods to assess the goodness-of-fit of epidemic models to data.

**Epidemics on random networks**

 Supervisor:  **Frank Ball** ([frank.ball@nottingham.ac.uk](mailto:frank.ball@nottingham.ac.uk))

There has been considerable interest recently in models for epidemics on networks describing social contacts. In these models one first constructs an undirected random graph, which gives the network of possible contacts, and then spreads a stochastic epidemic on that network. Topics of interest include: modelling clustering and degree correlation in the network and analysing their effect on disease dynamics; development and analysis of vaccination strategies, including contact tracing; and the effect of also allowing for casual contacts, i.e. between
individuals unconnected in the network. Projects in this area will address some or all of these issues.

