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Maria Montero

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Karina Terry
Centre for Decision Research and Experimental Economics
School of Economics
University of Nottingham
University Park
Nottingham
NG7 2RD
Tel: +44 (0) 115 95 15620
Fax: +44 (0) 115 95 14159
karina.terry@nottingham.ac.uk

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Bargaining in Legislatures: A New Donation Paradox

Maria Montero*

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Abstract

It is well known that proposers have an advantage in the canonical model of bargaining in legislatures: proposers are sure of being part of the coalition that forms, and, conditional on being in a coalition, a player receives more as a proposer than as a coalition partner. In this paper I show that, if parties differ in voting weight, it is possible for a party to donate part of its proposing probability to another party and be better-off as a result. This can happen even if the recipient never includes the donor in its proposals. Even though actually being the proposer is valuable, having a higher *probability* of being proposer may be harmful.

Keywords: legislative bargaining, weighted majority games, voting paradoxes.

J.E.L. classification: C78, D72.

*School of Economics, University of Nottingham, University Park, NG7 2RD Nottingham (United Kingdom); e-mail: maria.montero@nottingham.ac.uk.

1 Introduction

A donation paradox occurs when a player transfers an apparently valuable prerogative to another player but is better-off as a result (Kadane et al., 1999). The donation paradox in power indices was identified by Felsenthal and Machover (1995). A power index exhibits the donation paradox when it is possible for a player to increase its power (as measured by the index) by donating part of its weight to another player. Felsenthal and Machover (1998, p. 258-259) see the donation paradox as something that should not occur for measures of what they call P-power (the voter's expected share in a fixed purse that is divided by voting).

In this paper, I identify a donation-type paradox that arises as an equilibrium phenomenon in the context of legislative bargaining. The most influential model of bargaining in legislatures is due to Baron and Ferejohn (1989). In this model, n parties must divide a budget by majority rule. The parties have opposed preferences in the sense that each party would like to have the whole budget for itself. One of the parties is randomly selected to make a proposal, and the remaining parties accept or reject. Being the proposer is valuable in this model: the proposer is guaranteed to be in the coalition that forms and, conditional on being part of the coalition, a player gets more as a proposer than as a responder. Baron and Ferejohn analyze simple majority rules with symmetric voters, but the proposer advantage occurs for any distribution of votes and any qualified majority as long as no player has a monopoly on making proposals and there are no veto players (see Harrington 1990, proposition 1; Okada 1996, theorem 1; Montero 2006, corollary 3). When each player has one vote, having a higher probability of being proposer can never hurt a player (Eraslan (2002)). The present paper shows that, if players have different weights, it may be possible for a player to gain from donating some of its *proposing probability* to a recipient and be better-off. This can happen even though the recipient never includes the donor in its proposals (either before or after the donation). The effect is triggered by the fact that the donor is disproportionately likely to receive proposals by third parties after the donation.

2 The model

2.1 Weighted majority games

Let $N = \{1, ..., n\}$ be the set of players. $S \subseteq N$ ($S \neq \varnothing$) represents a generic coalition of players, and $v : 2^n \to \mathbb{R}$ with $v(\varnothing) = 0$ denotes the characteristic function. We have a weighted majority game iff there exist n nonnegative numbers (weights) $w_1, ..., w_n$ and a nonnegative number q such that $v(S) = 1$ if $\sum_{i \in S} w_i \geq q$ and 0 otherwise. A coalition S is called *winning* iff $v(S) = 1$ and *losing* iff $v(S) = 0$. It is called *minimal winning* iff $v(S) = 1$ and $v(T) = 0$ for all T such that $T \subset S$. The set of all winning coalitions is denoted by W. A player who belongs to all winning coalitions is called a *veto player*.

A weighted majority game admits a *homogeneous representation* if there exists a vector of nonnegative numbers $w_1^h, ..., w_n^h$ and a nonnegative number q^h such that $v(S) = 1$ if and only if $\sum_{i \in S} w_i^h = q^h$.

2.2 The bargaining procedure

Let (N, v) be weighted majority game. We interpret this game as a transferable payoff game where n players decide by majority rule on the division of a (perfectly divisible) budget.

Bargaining proceeds as follows: At every round $t = 1, 2, ...$ Nature selects a player randomly to be the proposer according to some probability distribution $\theta = (\theta_i)_{i \in N}$, where $\theta_i \geq 0$ for all i and $\sum_{i \in N} \theta_i = 1$. The selected player proposes a payoff vector $(x_i)_{i \in N}$. This payoff vector must be feasible ($\sum_{i \in N} w_i \leq 1$) and no player can receive a negative payoff ($x_i \geq 0$ for all i in N). Given a proposal, all players vote "yes" or "no " sequentially (the order does not affect the results). If the total number of votes in favor is at least q, the proposal is implemented and the game ends. Otherwise the game proceeds to the next period in which Nature selects a new proposer (always with the same probability distribution). Players are risk-neutral and share a discount factor $\delta \leq 1$.

The probability θ_i is player i's *recognition probability*. Two common assumptions are *equal recognition probabilities* ($\theta_i = \frac{1}{n}$ for all i in N), and (for weighted majority games) *proportional recognition probabilities*, $\theta_i = \frac{w_i}{w(N)}$ for all i in N.

A pure strategy for player i is a sequence $\sigma_i = (\sigma_i^t)_{t=1}^{\infty}$, where σ_i^t, the t-th round strategy of player i, prescribes

1. A *proposal* $(x_i)_{i \in N}$.

2. A *response function* assigning "yes" or "no" to all possible proposals of the other players.

Players are free to condition their actions on the history of the game up to time t; however we will study equilibria in which they choose not to do so. The solution concept is *stationary subgame perfect equilibrium* (SSPE). Stationarity requires that players follow the same (possibly mixed) strategy at every round t: the probability that the proposer makes a given proposal is the same for all t regardless of history, and the response function depends only on the current proposal and not on what happened in previous rounds.

Given an SSPE σ^* we will denote the associated *expected payoff* for player i (computed at the beginning of the game, before Nature chooses the proposer) by $y_i(\sigma^*)$ -we will drop σ^* to simplify notation-. The expected payoff given that a proposal is rejected is called the *continuation value.* Continuation values play a very important role in any SSPE: because incredible threats are ruled out by subgame perfection, a responder must accept any payoff strictly higher than their continuation value. Moreover, when the equilibrium is stationary the continuation value is the same at all subgames for given σ^*: after a proposal is rejected a period elapses and the players do the same they would do at time 1 all over again, thus player i's continuation value is simply δy_i.

2.3 The proposer advantage

The proposer advantage was originally established by Baron and Ferejohn (1989) and Harrington (1990) for symmetric games. Because of symmetry, each player expects $\frac{1}{n}$ if the game goes to the next period. Since the proposer needs only to convince $q-1$ players to vote for the proposal, it can offer $\frac{1}{n}$ to $q-1$ players and pocket the remaining $1 - \frac{q-1}{n} = \frac{1}{n} + \frac{n-q}{n}$. Thus there is a proposer advantage as long as $q < n$. Introducing discounting leads to an even greater proposer advantage. Okada (1996) shows that there is a proposer advantage in general games assuming that each player is recognized with probability $\frac{1}{n}$ and $\delta < 1$. This result can be easily generalized to any recognition probabilities and to $\delta = 1$ provided that no player has veto power or a recognition probability equal to 1 (if a player has veto power, there is still a proposer advantage if $\delta < 1$).

Lemma 1 *Let $[q; w]$ be a weighted majority game. If there are no veto players and no player with $\theta_i = 1$, then there is a proposer advantage in the sense that a player earns strictly more as a proposer than as a coalition partner in an SSPE. The requirement of no veto players can be replaced by $\delta < 1$.*

Proof. Let y_i be the expected equilibrium payoff for player i. In a stationary equilibrium a player has the same y_i in each period and, since there is 1 unit to divide, $\sum_{j \in N} y_j \leq 1$.

Any player with $\theta_i > 0$ must have $y_i > 0$. As a proposer, player i can always exclude some k with $y_k > 0$ and offer everybody else slightly more than their continuation value; the proposal will pass as k is not a veto player.[1] Since $\sum_{j \in N \backslash \{i,k\}} y_j < 1$, it follows that i has a positive payoff as a proposer. Moreover, given that no player can be allocated a negative payoff as a responder, any player with a positive recognition probability must have a positive expected payoff overall.

Player i receives δy_i as a coalition partner in an SSPE. As a proposer, it receives $1 - \sum_{j \in S \backslash \{i\}} \delta y_j$, where

$$S \in \arg \min_{S \ni i, S \in W} \sum_{j \in S} y_j.$$

This is because a player must accept any offers above δy_j. If offers were above δy_j, the proposer could undercut the offer slightly and it would still be accepted. Thus j must receive exactly δy_j as a coalition partner, and the proposer receives $1 - \sum_{j \in S \backslash \{i\}} \delta y_j$ for some S. If S were not the

[1] If there was no k with $y_k > 0$ it would be even easier for i to have a positive payoff as a proposer by offering all players slightly more than 0.

solution to the minimization problem, there would be another coalition that could be proposed with coalition partners getting slightly more than their continuation value; they would have to accept and the proposer would be better-off.

The difference between proposer and coalition partner payoff is $1 - \delta \sum_{j \in S} y_j$. This is always positive because, since no player has a monopoly on making proposals there is at least one player $k \neq i$ who can make proposals (and will therefore have $y_k > 0$) and since there are no veto players that player can be excluded.

The requirement of there being no veto players can be replaced by $\delta < 1$. Then there must be a proposer advantage because the sum of the continuation values of all players is strictly less than 1.

If a player has a monopoly on making proposals the advantage of being proposer is not defined. ■

3 A new donation paradox

Suppose there are four parties in the legislature, controlling 3, 2, 2 and 1 votes respectively, and 5 votes are needed to pass a proposal. We consider two possible scenarios: each party is recognized with a probability proportional to its number of votes ($\theta = \left(\frac{3}{8}, \frac{2}{8}, \frac{2}{8}, \frac{1}{8}\right)$), or alternatively each party is recognized with equal probability (thus $\theta = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$). Both scenarios are plausible: in the first case, we can think of a voter with three votes as a party composed of three members, each of them with one vote, who always follow party discipline, and each member is selected with equal probability; in the second case, we can think of parties as being treated equally in terms of voice even though they have different numbers of votes. Because the medium-size players have the same recognition probability in both scenarios, we can view the move from one scenario to the other as the large player "donating" some of its recognition probability to the small player.

Eraslan and McLennan (2006) show that all SSPE have the same expected payoffs, therefore if we are only interested in payoffs and not in strategies it is enough to find one equilibrium.

Note that players of the same type must have the same payoff in equilibrium if they have the same recognition probability (Montero, 2002, lemma 2). This result also follows from Eraslan and McLennan's uniqueness result (if equilibrium payoffs are unique they must be symmetric). Thus, we can set $y_2 = y_3$ and use y_2 to denote both player 2 and player 3's payoffs. We will also focus on equilibrium strategies that are symmetric in the sense that the two players of the same type play the same strategy, and are treated symmetrically by other players' strategies.

What coalitions do players propose in equilibrium? The answer is straightforward for the largest and the smallest player.

The large player always proposes $\{1, 2\}$ or $\{1, 3\}$ (each with probability 0.5 since we focus on symmetric strategies). The small player is of no use to the large player as a coalition partner: adding the small player to a coalition that already contains the large player never turns a losing coalition into a winning one.

Similarly, the large player is of little use to the small player as a coalition partner. The natural coalition for the small player to propose is the only minimal winning coalition to which it belongs, $\{2, 3, 4\}$. A coalition like $\{1, 2, 4\}$ could conceivably be proposed if $y_1 \leq y_2$, but this is never the case for the recognition probabilities we consider.

As for a medium-size player like player 2, it can propose coalition $\{1, 2\}$ or coalition $\{2, 3, 4\}$, depending on how y_1 compares with y_2+y_4. If $y_1 = y_2+y_4$ we have a competitive situation in the sense that players that can replace each other in a minimal winning coalition receive the same payoff.

Proposition 2 *Consider the weighted majority game* $[5; 3, 2, 2, 1]$*, and let* $\theta = \left(\frac{3}{8}, \frac{2}{8}, \frac{2}{8}, \frac{1}{8}\right)$*. The equilibrium payoff vector in any SSPE with* $\delta = 1$ *is* $y = \left(\frac{5}{14}, \frac{4}{14}, \frac{4}{14}, \frac{1}{14}\right)$.

Proof. Suppose the large player proposes to each medium-sized player with probability 0.5, the small player proposes to both medium-sized players, and each medium-sized player randomizes between proposing to the large player (with probability λ) and proposing coalition $\{2, 3, 4\}$ (with probability $1 - \lambda$). Suppose moreover that each coalition partner is offered its continuation value (so, for example, player 2 proposes $(y_1, 1-y_1, 0, 0)$), and players accept any offer that gives them at least their continuation value. Note that a mixed strategy can only be optimal for a medium-sized player if $y_1 = y_2 + y_4$. The following system of equations determines the expected payoffs derived from these strategies and the equilibrium value of λ.

$$\begin{aligned}
y_1 &= \frac{3}{8}(1-y_2) + \frac{4}{8}\lambda y_1 \\
y_2 &= \frac{2}{8}(1-y_1) + \frac{3}{8}\frac{1}{2}y_2 + \frac{2}{8}(1-\lambda)y_2 + \frac{1}{8}y_2 \\
y_4 &= \frac{1}{8}(1-2y_2) + \frac{4}{8}(1-\lambda)y_4 \\
y_1 &= y_2 + y_4
\end{aligned}$$

The solution to this system is $y_1 = \frac{5}{14}$, $y_2 = \frac{4}{14}$, $y_4 = \frac{1}{14}$ and $\lambda = \frac{1}{2}$.

The strategies described above constitute an equilibrium because responders are offered their continuation values, and proposers are proposing to the cheapest possible coalition partners. ■

Proposition 3 *Consider the weighted majority game* $[5; 3, 2, 2, 1]$, *and let* $\theta = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$. *The equilibrium payoff vector in any SSPE with* $\delta = 1$ *is* $y = \left(\frac{3}{8}, \frac{2}{8}, \frac{2}{8}, \frac{1}{8}\right)$.

Proof. Suppose the large player proposes to each medium-sized player with probability 0.5, the medium-size players propose to the large player, and the small player proposes to both medium-sized players. Suppose moreover that each coalition partner is offered its continuation value (so, for example, player 2 proposes $(y_1, 1-y_1, 0, 0)$) and players accept any offer that gives them at least their continuation value. Then continuation values are found from the following system of equations

$$\begin{aligned}
y_1 &= \frac{1}{4}(1-y_2) + \frac{2}{4}y_1 \\
y_2 &= \frac{1}{4}(1-y_1) + \frac{1}{4}\frac{1}{2}y_2 + \frac{1}{4}y_2 \\
y_4 &= \frac{1}{4}(1-2y_2)
\end{aligned}$$

The solution to this system of equations is $y_1 = \frac{3}{8}$, $y_2 = \frac{2}{8}$ and $y_4 = \frac{1}{8}$.

The strategies described above constitute an equilibrium because responders are offered their continuation values, and proposers are proposing to the cheapest possible coalition partners. In particular, a medium-sized player would compare proposing to the large player (and paying $\frac{3}{8}$) with proposing to the other two players (and paying $\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$). Because the alternative coalition is no better than the one that is being proposed, there is no profitable deviation. ■

What is the effect of the donation from player 1 to player 4? The direct effect is negative: player 4 always proposes coalition $\{2, 3, 4\}$, so if players did not change their strategies it would be the case that y_2 would go up (as the medium-sized players receive more proposals), y_4 would go up (as the small player is more likely to be recognized) and y_1 would go down (as the large player is less likely to be recognized). But then it would no longer be optimal for players 2 and 3 to play a mixed strategy, as $y_1 < y_2 + y_4$. In the new equilibrium, the medium-sized players are more likely to propose to the large player than before. This indirect effect (the large player is more likely to receive proposals from the medium-sized players) brings the equilibrium back to a competitive situation in which $y_1 = y_2 + y_4$. Nevertheless, the individual values of y_1, y_2 and y_4 are not the same as before, and player 1 is better-off in this new competitive equilibrium.

More generally, there is a range of probabilities such that player 1 can move from a competitive allocation to another competitive allocation that is more favorable by donating some probability to player 4. Fix the probability of being proposer for a medium-size player at $\frac{1}{4}$, and let θ be the probability that the large player is selected to be proposer; then the small player is selected with probability $\frac{1}{2} - \theta$. If we only consider recognition probabilities such that a larger player cannot be selected less often than a smaller player, the relevant range of values for θ is $\frac{1}{4} \leq \theta < \frac{1}{2}$. It turns out that the equilibrium is always such that a medium-size player is indifferent between proposing to the large player and proposing to the other medium-size player and the small player, or equivalently $y_1 = y_2 + y_4$. Let λ be the probability that a medium-size player proposes to the large player. Then expected payoffs are found from the following equations

$$\begin{aligned}
y_1 &= \theta(1-y_2)+\frac{1}{2}\lambda y_1 \\
y_2 &= \frac{1}{4}(1-y_1)+\frac{\theta}{2}y_2+\frac{1}{4}(1-\lambda)y_2+\left(\frac{1}{2}-\theta\right)y_2 \\
y_4 &= \left(\frac{1}{2}-\theta\right)(1-2y_2)+\frac{1}{2}(1-\lambda)y_4 \\
y_1 &= y_2+y_4
\end{aligned}$$

The solution for λ is $2(1-2\theta)$. It starts at 1 for $\theta=\frac{1}{4}$, and is approaches 0 when θ approaches $\frac{1}{2}$. This is intuitive: if a player is less likely to be proposer with strategies being unchanged, it becomes cheaper and will receive more proposals. What is surprising is the overcompensation, so that the player is better-off when it is less likely to be proposer. It turns out that $y_1=\frac{2(1-\theta)}{5-4\theta}$, which is decreasing in θ. Payoffs for the other two types are $y_4=\frac{1-2\theta}{5-4\theta}$ (which is decreasing in θ as one would intuitively expect; the direct effect of the donation is stronger than the indirect effect), and $y_2=\frac{1}{5-4\theta}$ (which must be increasing in θ since the other payoffs are decreasing in θ).

4 Discussion

It is known that the indirect effect can offset the direct effect. Montero (2002) shows that, for apex games and symmetric protocols, all values $0<\theta_1\leq 0.5$ lead to the same expected payoffs. If the apex player becomes the proposer more often, it receives proposals less often so that the competitive solution $y_1=(n-2)y_2$ is maintained.

An important difference between the game $[5;3,2,2,1]$ and apex games is that the competitive payoff vector is unique for apex games because apex games have a unique homogeneous representation (up to rescaling). The indifference condition $y_1=(n-2)y_2$ together with $y_1+(n-1)y_2=1$ determines expected payoffs uniquely.

The game $[5;3,2,2,1]$ has many competitive payoff vectors because it has many homogeneous representations, so that assuming that the outcome is competitive does not lead to a unique payoff vector. For example, $[7;4,3,3,1]$ is a homogeneous representation of the same game. If we normalize the weights so that they add up to 1, it is easy to compute all homogeneous representations. Clearly, players 2 and 3 must have the same weight in any homogeneous representation. Denote the weights by w_1, w_2 and w_4 respectively. Normalization implies that

$$w_1 + 2w_2 + w_4 = 1 \tag{1}$$

Homogeneity implies that $w_1 + w_2 = 2w_2 + w_4$, or

$$w_1 = w_2 + w_4 \tag{2}$$

Note that the homogeneity condition is the same as the indifference condition that we obtained previously for a medium-sized player, but with weights instead of payoffs. Solving this system we obtain

$$w_2 = 1 - 2w_1 \tag{3}$$

$$w_4 = 3w_1 - 1 \tag{4}$$

It turns out that w_2 is negatively related to w_1, whereas w_4 is positively related to w_1.

Since $\{1, 4\}$ and $\{2, 3\}$ are losing coalitions, there are two additional constraints: $w_2 > w_4$ guarantees that $\{1, 4\}$ is losing, and $w_4 > 0$ guarantees that $\{2, 3\}$ is losing. Taking these constraints into account we find that any value w_1 such that $\frac{1}{3} < w_1 < \frac{2}{5}$ leads to a homogeneous representation (the corresponding intervals for the other two players are $\frac{1}{3} > w_2 > \frac{1}{5}$ and $0 < w_3 < \frac{1}{5}$).

If we assume a "competitive" equilibrium in which $y_1 = y_2 + y_4$ (equivalent votes receive the same payoff), expected payoffs must be proportional to *some* homogeneous representation, and the payoffs of 1 and 4 must vary together. This goes some way towards explaining the phenomenon (if a donation from 1 to 4 affects y_1 and y_4 it must have a paradoxical effect) though it does not explain why payoffs change when 1 donates probability to 4 instead of remaining constant.

5 Concluding remarks

Being recognized as a proposer is always a good thing *ex post*. However, having a higher recognition probability can hurt a player. The reason is that the indirect effect of this donation may outweigh the direct effect: the recipient is now less likely to receive proposals, and that effect more than compensates for the increase in the recognition probability.

In the example the paradox seems to be connected to the fact that the set of minimal winning coalitions is not rich enough, so that the homogeneous representation of the game is not unique. Identifying a class of games for which the paradox does not occur (besides apex games) would be an interesting topic for future research.[2]

[2] To the best of my knowledge, there are no general results on the comparative statics of changing recognition probabilities. Kalandrakis (2006) shows that *any* expected payoffs can be obtained for *some* recognition probabilities, but contains no claims on what probabilities lead to what payoffs.

References

[1] Baron, David P., and Ferejohn, John A. (1989) "Bargaining in Legislatures". *American Political Science Review* **83**, 1181-1206.

[2] Eraslan, H. (2002) "Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model". *Journal of Economic Theory* **103**, 11-30.

[3] Eraslan, H. and McLennan, A. (2006) "Uniqueness of Stationary Equilibrium Payoffs in Coalitional Bargaining". Typescript.

[4] Felsenthal, D.S. and Machover, M. (1995). "Postulates and Paradoxes of Relative Voting Power: A Critical Re-Appraisal". *Theory and Decision* **38**, 195-229.

[5] Felsenthal, D.S. and Machover, M. (1998). *The Measurement of Voting Power: Theory and Practice, Problems and Paradoxes*, Edward Elgar Publishing.

[6] Harrington, J. E. (1990). "The Power of the Proposal Maker in a Model of Endogenous Agenda Formation". *Public Choice* **64**, 1-20.

[7] Kadane, J. B., Stone, C. A. and Wallstrom, G. (1999). "The Donation Paradox for Peremptory Challenges". *Theory and Decision* **47**, 139-151.

[8] Kalandrakis, T. (2006). "Proposal Rights and Political Power". *American Journal of Political Science* **50**, 441-448.

[9] Montero, M. (2002) "Noncooperative Bargaining in Apex Games and the Kernel". *Games and Economic Behavior* **41**, 309-321.

[10] Okada, A. (1996) "A Noncooperative Coalitional Bargaining Game with Random Proposers". *Games and Economic Behavior* **16**, 97-108.