Universität St.Gallen

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Christiane Ernst and Christian Thöni

August 2009 Discussion Paper no. 2009-25

Department of Economics University of St. Gallen

Editor:	Martina Flockerzi University of St. Gallen Department of Economics Varnbüelstrasse 19 CH-9000 St. Gallen Phone +41 71 224 23 25 Fax +41 71 224 31 35 Email vwaabtass@unisg.ch
Publisher:	Department of Economics University of St. Gallen Varnbüelstrasse 19 CH-9000 St. Gallen Phone +41 71 224 23 25 Fax +41 71 224 31 35
Electronic Publication:	http://www.vwa.unisg.ch

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Christiane Ernst and Christian Thöni

Author's address: Dr. Christian Thöni
FEW-HSG
Varnbüelstrasse 14
9000 St. Gallen
Phone +41 71 2548
Fax +41 71 2302
Email christian.thoeni@unisg.ch
Website www.sew.unisg.ch/thoeni

Christiane Ernst
Email c.a.ernst@lse.ac.uk

[1] Acknowledgements: We thank the Brown-Bag Seminar Participants of the University of St. Gallen, Eva Deuchert, Therese Fässler, Martin Huber, Heinz Müller for helpful comments and suggestions.

Abstract

We report results from experimental first-price, sealed-bid, all-pay auctions for a good with a common and known value. We observe bidding strategies in groups of two and three bidders and under two extreme information conditions. As predicted by the Nash equilibrium, subjects use mixed strategies. In contrast to the prediction under standard assumptions bids are drawn from a bimodal distribution: very high and very low bids are much more frequent than intermediate bids. Standard risk preferences cannot account for our results. However, bidding behavior is consistent with the predictions of a model with reference dependent preferences as proposed by the prospect theory.

Keywords

All-pay Auction; Prospect Theory, Experiment

JEL Classification

D44, D72, D80, C91.

1. Introduction

Bidding behavior in all-pay auctions has so far only received limited attention in empirical auction research. This might be due to the fact that most of the applications of auction theory do not involve the all-pay rule. However, occurrences of this auction format, where every bidder pays her bid are numerous: lobbying battles, political campaigns, promotion tournaments in firms and applications for science grants (Klemperer, 2004).

In this article we report experimental data on bidding behavior from the simplest possible all-pay auction format. We conducted first-price, sealed-bid, common value auctions with two or three bidders and no uncertainty with regard to the value of the auctioned commodity. Every subject bids in an auction for a prize of 100 monetary units. Subjects choose their bids simultaneously, the highest bidder receives the prize, and all bidders pay their bid. We analyze bidding behavior in ten subsequent auctions and under two different information conditions. In the *NoRecall* treatment subjects do not receive any information about other subjects' bids during the ten rounds. In the *Recall* treatment subjects have full information about the bidding history in their group.

Previous evidence on bidding behavior in experimental all-pay auctions comes from Gneezy and Smorodinsky (2006), who conducted similar all-pay auctions with group sizes of four to 12. They report persistent overbidding, i.e., average bids were considerably higher than predicted by the Nash equilibrium for all group sizes.[1] While the existing literature focuses on aggregate outcomes, our aim is to investigate *individual* bidding strategies. Thus we study bidding behavior in the simplest case of auctions and in *small* groups.

We find that subjects indeed use mixed strategies, however, the observed distribution of bids shows interesting deviations from the predictions under standard assumptions. The mixed strategy Nash equilibrium under standard assumptions predicts uniformly distributed bids for groups of two players and a decreasing density function for larger group sizes. We find that subjects' bidding strategies differ sharply from these predictions: on average, subjects apply bimodal bidding strategies which give most weight to both very low and very high bids, resulting in a bimodal bidding function. Bimodal bidding occurs for both group sizes and information conditions. We show that bimodal bidding is consistent with prospect theory. If players are risk seeking in the domain of losses and risk averse in the domain of gains then equilibrium bidding strategies are bimodal.

[1] A related strand of literature focuses on the rent-seeking game introduced by Tullock (1967), where higher bids increase the bidder's share of the pie (or rent). See Millner and Pratt (1991), Shogren and Baik (1991), and Davis and Reilly (1998), Öncüler and Croson (2005) or Herrmann and Orzen (2008).

2. Theory and Experimental Design

The game is played in groups of n=2 or 3 bidders. All players simultaneously choose their bid $b_i \in \mathbb{R}_+$. The auctioned commodity has a value of unity for all bidders. A player's expected utility $u_i : \mathbb{R}_+^N \to \mathbb{R}$ is defined as:

$$u_i(b_i, b_{-i}) = \begin{cases} \frac{1}{M(b^{\max})} - b_i & \text{if } b_i = b^{\max} \geq b_j \text{ for all } j \neq i \\ -b_i & \text{otherwise} \end{cases}. \tag{1}$$

$M(b^{\max})$ counts the number of maximal bidders. Following, we intuitively derive the Nash equilibria of this game. A thorough theoretical treatment of this game is provided by Baye et al. (1996). Clearly the game cannot have an equilibrium in pure strategies since the best reply to every bid in [0,1) is to overbid by the smallest amount possible. In every mixed strategy equilibrium it must hold that bidders are indifferent between the mixed strategy and any pure strategy included in their mixed strategy. As long as the support of the mixed strategy includes zero, the expected utility in the mixed strategy must be zero. For n=2 this game has a unique equilibrium in mixed strategies where both players draw their bid from a uniform distribution over the support [0,1]. The expected utility then equals zero for both players.

Could the two players improve their situation by restraining the support of their mixed strategy from above, i.e., both drawing their bid from $[0,\overline{b}]$ with $\overline{b} < 1$? No, because this would offer the opportunity of earning a strictly positive utility by outbidding the other player with a pure strategy of bidding slightly more than $\overline{b}$. Could they improve by choosing their bid from $[\underline{b},1]$ with $\underline{b} > 0$? This is also not possible as bidding $\underline{b}$ would then result in a certain loss and the player would prefer to bid 0. To conclude, both players choose their bid from a uniform distribution with support [0,1] and earn an expected payoff of zero. Expected bids are 0.5 and expected standard deviation is $\sqrt{1/12} = 0.289$. Expected gross returns of the auctioneer are 1, which equals the value of the auctioned commodity.

For n=3 the theoretical solution becomes more complicated. There exists a unique *symmetric* equilibrium where all players draw their bid from $f(b_i) = 0.5 b_i^{-0.5}$, $b_i \in [0,1]$. In addition, there is a continuum of equilibria of the following kind: two players randomize on [0,1] while the third player randomizes continuously on an interval $[\underline{b},1]$ and concentrates the remaining mass at zero, with $0 \leq \underline{b} \leq 1$. The equilibria reach from $\underline{b} = 0$, which is the symmetric case to $\underline{b} = 1$, in which player three does not take part in the auction and the other two players choose their bids according to the equilibrium strategy in the two player case. Expected bids for the two players who randomize with full support range from one third to one half, expected bids from the third bidder range from zero to one third. All equilibria share the following features: the expected bids are one third, expected utility of all bidders is zero and the revenue for the auctioneer is unity. Standard deviations of the bids depend on the equilibrium and range from 0.298 for $\underline{b}=0$ to 0.333 for $\underline{b}=1$.

All equilibria also have in common that they are not lucrative for the bidders. It is straightforward to show that collusion can substantially increase the expected utility of the bidders. For example, if n=2 and both bidders bid zero, their expected utility is 0.5.[2] However, incentives to deviate from this collusive strategy are obviously very strong.

Our experimental subjects were first year students from the University of St. Gallen. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007). At the beginning of the experiment subjects were given a show-up fee of CHF 20 (about $18). Losses in the experiment were deducted from the show-up fee. The auctioned item was 100 ECU (Experimental Currency Unit). We conducted two treatments, the *Recall* and the *NoRecall* treatment.

In the *Recall* treatment subjects were allocated to groups of two or three subjects and played ten consecutive but independent all-pay auctions in unchanged groups. Full information about bids of group members in all previous rounds was provided.

In the *NoRecall* treatment subjects also played ten consecutive all-pay auctions for a price of 100 ECU. In each round they were randomly allocated to groups of either two or three subjects. They were informed about their group size but received no information at all about the outcome of the auction and the other subjects' bids. Prior to the experiment subjects were given detailed instructions (see Appendix). Bids were restricted in the interval [0,125] and a resolution up to three decimal places.[3]

We report results from 52 subjects in two experimental sessions. We apply a within subject design where all subjects played both treatments, changing the order of the treatments between sessions. The experiment lasted about an hour and the subjects earned on average CHF 19.4 (about $17.5), which means that, on average, subjects made a small loss of CHF .6 in the 20 auctions they played.

3. Results

We start by analyzing the data from the *Recall* treatment. In this treatment subjects had access to the whole history of bids within their group and played the game in stable groups. These are arguably the conditions most favorable for the establishment of a mixed strategy equilibrium.[4]

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[2] In our experiment the tie rule was that all maximum bidders pay their bid and one randomly chosen bidder receives the price.

[3] The upper bound of 125 was introduced to prevent subjects from making large losses due to erroneous entries. However, this upper bound was not communicated to the subjects in the instructions to prevent anchoring.

[4] Once a bidder learns that other bidders use a pure bidding strategy she can simply outbid the others by the smallest amount and earn a secure payoff, while the others make a loss.

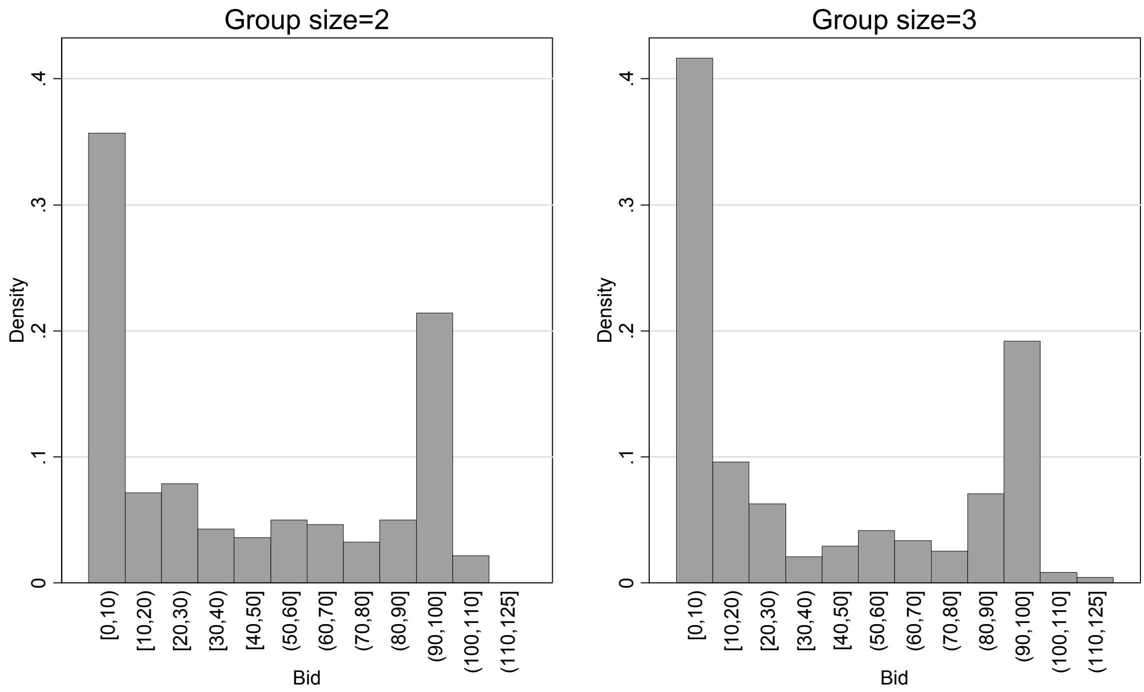
Average bids over the ten rounds were 42.0 in groups of two and 36.9 in groups of three. Compared to the Nash prediction of 50 and 33.3 respectively, we observe underbidding in groups of two and overbidding in groups of three. The differences are, however, not significant.[5] The underbidding in groups of two might partially be due to attempts of collusion. If both players bid zero and let chance decide about the winner, they have an expected profit of 50. The results by Huck et al. (2004) suggest that collusive strategies are most likely to be observed in small groups. Average bids are, however, not too informative with regard to the *bidding strategies* subjects played. If we calculate the standard deviation of the bids, we observe 40.0 in groups of two which is higher than predicted by the Nash equilibrium (28.9). In groups of three the observed standard deviation was 39.8 compared to the prediction which lies between 29.8 and 33.3.[6]

However, the most striking differences to the Nash prediction emerge if we take a closer look at the *distribution of bids*. Figure 1 shows histograms of the bids separated by group size. The distribution of bids is clearly *bimodal*. Very low and very high bids (up to 100) are much more frequent than intermediate bids.

[5] A conservative test based on the independent group averages does not allow to reject the null hypothesis that average bids are equal to the Nash prediction (p=.140 for groups of two and p=.262 for groups of three, two-sided Wilcoxon matched-pairs signed-ranks test).

[6] In groups of two the difference is significant at p=.036, in groups of three insignificant with p=.263 (two-sided Wilcoxon matched-pairs signed-ranks test).

Figure 1 **Histogram of Bids in Groups of Two and Three Subjects in the *Recall* Treatment**

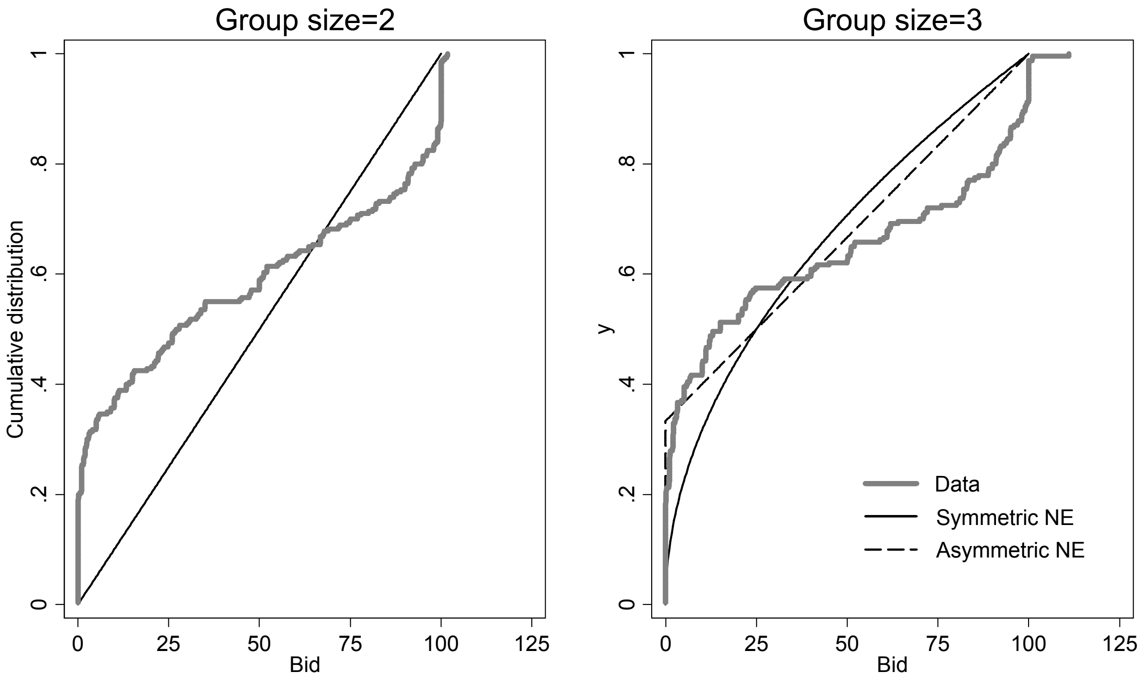


In the groups of three subjects very low bids are more frequent than very high bids. However, the overall adjustment of the bids to the larger group size is much smaller than predicted. In order to compare the observed bid to predicted bids we plot the cumulative distribution of bids. Figure 2 shows the cumulative distribution of the observed bids (bold lines) and the cumulative densities predicted by the mixed strategy Nash equilibrium (thin lines). In the right panel we account for the fact that multiple equilibria exist and depict the cumulative densities of two extreme cases: The kinked curve corresponds to the equilibrium where one player abstains from the auction (hence the intercept at one third) and the other two draw their bid from a uniform distribution; the smooth curve corresponds to the symmetric mixed strategy equilibrium.

For the smaller group size, prediction and data are obviously very different. In the auctions with three bidders the large mass at very low bids is compatible with an asymmetric mixed strategy equilibrium. Still, the mass of bids close to 100 is clearly not compatible with the prediction. If we apply Kolmogorov-Smirnov tests for the null hypothesis that the bids stem from the predicted densities we can reject the null hypotheses for both group sizes and all equilibria at $p < .001$.[7]

[7] Simple Kolmogorov-Smirnov tests yield p values of virtually zero. However, we have to take into account that observations within a group are not independent. We do this by using each group's Kolmogorov Smirnov test statistic as an observation. We then run a simulation ($n = 1000$) to calculate the test statistic for hypothetical bids drawn from the densities predicted by the symmetric Nash equilibria. In case of the asymmetric Nash equilibria we test the distribution of the non-zero bids against the predicted distribution most favorable to mass at very

Figure 2 **Nash Equilibrium (Thin Lines) and Observed Cumulative Distribution Functions of Bids by Group Size in the *Recall* Treatment (Bold Lines)**



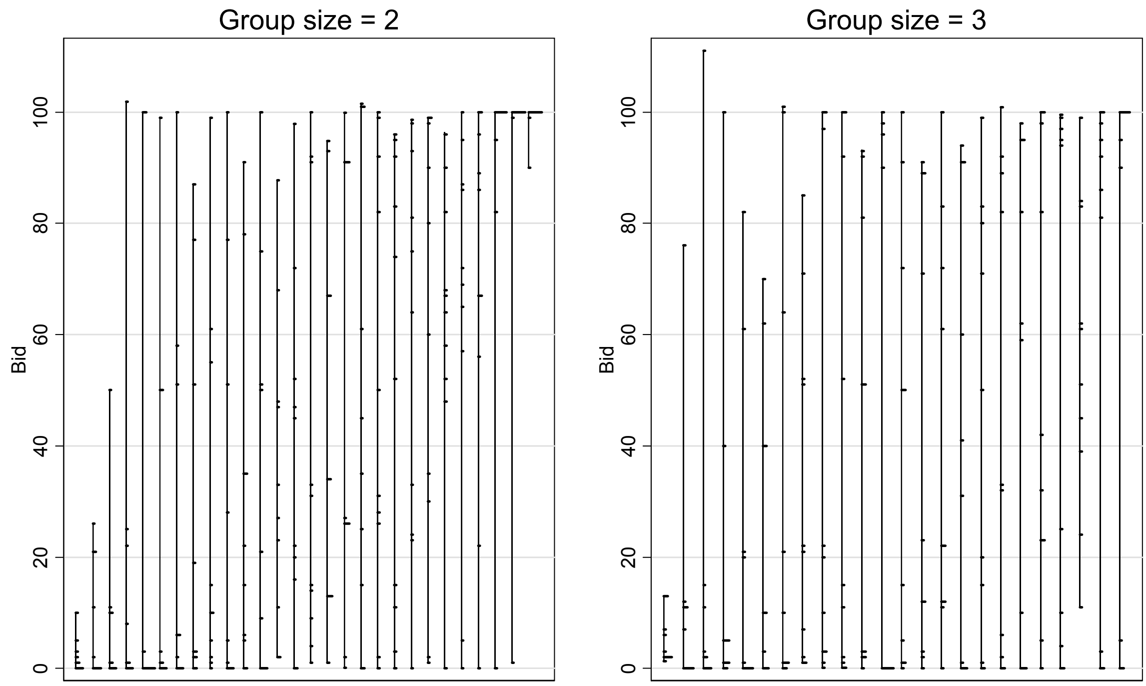
3.1. Individual Bidding Behavior

The data presented thus far did not contain information as to what extent the subjects actually played mixed strategies. In a next step we will look at individual patterns. Figure 3 depicts small histograms for individual bidding behavior over the ten rounds of the *Recall* treatment, for both group sizes. Each vertical line corresponds to one subject in the experiment and shows the spread of the bids. Subjects are sorted according to average bid. The length of the small horizontal spikes corresponds to the frequency of the corresponding bid (bids are rounded to integers). The overwhelming majority of the subjects bid in the whole range from zero to (almost) 100. Three quarter of the subjects have a spread of 90 or more in their ten bids. The majority of the subjects changed their bid frequently during the ten rounds. If we calculate the number of *different* bids a subject chose, we obtain an average of 7.96 different bids. More than a quarter of the subjects chose different bids in all ten rounds. Additionally we can look at the number of changes in a subject's bid from one round to the next. In 90.4 percent of the cases subjects changed their bid from t to t+1.[8]

high bids, which is the asymmetric equilibrium with one player abstaining from the auction. The test statistics for our data are always higher than for the simulated data. A Wilcoxon rank-sum test gives $p < .001$ in all cases.

[8] These numbers refer to all subjects in the *Recall* treatments, irrespective of group size.

Figure 3 Individual Histograms of the Bids in the Ten Rounds of the *Recall* Treatment



Note. Vertical lines show the spread of the bids of each individual, small horizontal lines depict the frequency of the corresponding bid (bids are rounded to integers).

In order to test for time effects we ran OLS and Tobit estimates for the subjects' bids. To take into account statistical dependence of observations within groups we estimate standard errors using the group as cluster. Table 1 reports the results of the OLS estimates (Tobit results are almost identical). Explanatory variables in Model (1) are the *Round* number, a dummy for the group size, a dummy for the order of the two treatments, *Recall* and *NoRecall*.[9] None of these variables explains the bidding behavior and the model as a whole is insignificant. This is surprising since the theoretically predicted expected bids differ considerably with group size (33.3 vs. 50). As shown above, subjects in groups of three overbid and subjects in groups of two underbid. Thus, in our data the influence of the group size on bids – in absolute terms – is much smaller than predicted. This is in line with Gneezy and Smorodinsky (2006), who find that subjects' bidding behavior does not sufficiently react to the number of competitors at least in early rounds. They also report that bids decrease over time, reducing the amount of overbidding.[10]

[9] In our data we also have information about socio-economic characteristics of the participants, such as gender, age, wealth of the family of origin and urban background. In all estimates these controls proved to be insignificant (alone and jointly). We did not include them in the estimates reported in Table 1.

[10] Gneezy and Smorodinsky (2006) discuss the logit equilibrium proposed by Anderson et al. (1998) concept as an explanation for the insufficient adjustment of bids to group size. In such an equilibrium players do not play

In order to check whether there is pressure towards the equilibrium in our experiment we have to keep in mind that the direction is not clear. In groups of two, bids should increase, in groups of three they should decrease. We allow for different time trends in Model (2) with the interaction term. However, the model does not benefit from this at all. Finally, in Model (3) we add lagged variables. This gives us a model with some explanatory power. A subject's bid in the previous round is positively related to her bid in the actual round. The maximal bid in the previous round has no effect. The most interesting result is the (weakly) significant coefficient for the dummy indicating whether the subject won the previous auction. Winners of the previous auction tend to reduce their bid in the actual round (controlled for the bid in the previous round). Choosing a high but losing bid seems to increase bids, while a win induces subjects to bid more conservatively in the next round.

Table 1 OLS Estimates for Bids in the *Recall* and *NoRecall* Treatment

	Dependent variable: Bid in *t*				
	Recall			*NoRecall*	
	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)
Group size=3 (D)	-5.362	-7.258	2.552	-16.837***	-19.113**
	(6.728)	(9.548)	(9.089)	(5.083)	(9.248)
Round	0.048	-0.111	0.975	-0.706	0.027
	(0.506)	(0.745)	(0.990)	(0.545)	(0.711)
Round x Group size=3		0.345	-1.139		0.405
		(0.998)	(1.065)		(1.154)
Recall→NoRecall (D)	4.570	4.570	1.911	-20.252**	-11.919**
	(7.366)	(7.373)	(5.111)	(7.857)	(5.000)
Bid in *t*-1			0.265**		0.438***
			(0.094)		(0.083)
Maximum bid in *t*-1			0.030		
			(0.071)		
Won auction in *t*-1 (D)			-11.332*		
			(6.007)		
Constant	39.758***	40.633***	27.032**	72.034***	40.635***
	(5.537)	(6.629)	(9.804)	(5.260)	(7.207)
F-test	0.3	0.2	2.6	6.9	24.0
Prob > F	0.860	0.935	0.045	0.001	0.000
R^2	0.007	0.007	0.064	0.119	0.313
N	520	520	468	520	468

Notes. OLS estimates for bid in the *Recall* and in the *NoRecall* treatment. In parentheses we report robust standard errors using the group as cluster. * p<0.1, ** p<0.05, *** p<0.01.

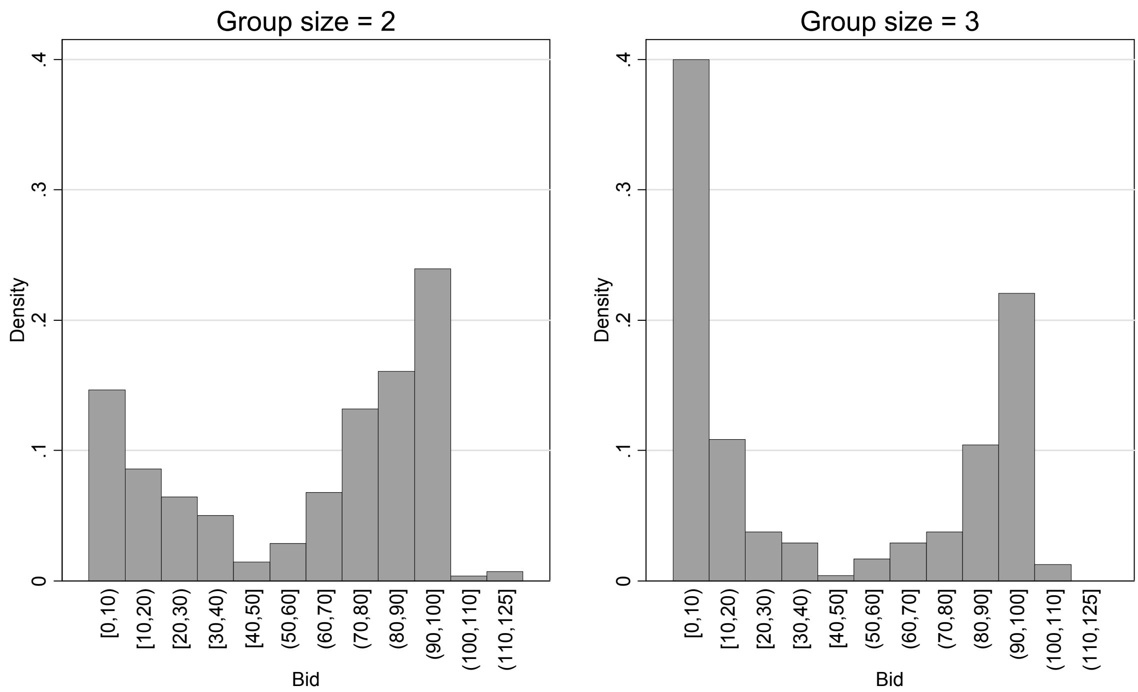
strict best response but play strategies with higher expected value with higher probability. In the extreme case (very large error term) strategies are drawn from the uniform distribution, thus trivially generating a positive relation between the sum of bids and the number of bidders. Bimodal distributions of bids are, however, incompatible with this solution concept. Our results suggest that deviations from Nash bidding behavior under standard assumptions is not driven by errors as formalized in the logit equilibrium.

3.2. Reducing Information

Thus far we have only analyzed data from the *Recall* treatments, where subjects learn other subjects' bids and can adapt their bids accordingly in the next round. In the *NoRecall* treatments subjects receive no feedback at all from the game and cannot find out the success of their strategies. In such a game it is even more difficult to infer a subject's bidding strategy from the observed bids. Even if subjects play a mixed strategy in the sense that they use a random draw from a probability distribution to determine their bid, it is unclear whether they draw every round or only once in the experiment.

It was in fact the case that subjects changed their bids less frequently in the *NoRecall* treatment compared to the *Recall* treatment, despite the fact that in the *NoRecall* treatment group size changed over time. During the ten rounds, subjects chose on average 6.60 different bids (as opposed to 7.96 in the *Recall* treatment). Still, the overall distribution of bids was clearly bimodal for both groups of two and three subjects. Figure 4 shows histograms for the bids in the *NoRecall* treatments.

Figure 4 Histogram of Bids: Groups of Two and Three Subjects in the *NoRecall* Treatment



Models (4) and (5) in Table 1 apply the estimation of Models (1) and (2) to the *NoRecall* data. Interestingly we observe effects of the group size and the order of the treatments. Bids are lower by 16.8 units (standard error: 5.08) in groups of three compared to those of groups of two. The lack of adjustment of bids to group size observed in the *Recall* treatment (and also reported by Gneezy and Smorodinsky, 2006) disappears if we observe within subject variation of the group size. In fact, the size of the effect is very close to the predicted effect of 16.7 (=50–33.3). The dummy for the order of the two treatments is significant as well. When subjects have experienced the *Recall* treatment before playing the *NoRecall* treatment, bids are lower by 20.3 units (standard error: 7.86) compared to when they start with the *NoRecall* treatment. On the other hand, time effects remain insignificant. Model (5) presents the OLS estimate including the bid from the previous round as explanatory variable, which again has a significant positive influence. We do not add the other lagged variables to the equation because subjects in the *NoRecall* treatment had no information about the outcome of the auction during the ten rounds.

Comparing the data from the *NoRecall* treatment to the *Recall* treatment also provides us with information about the subjects' attempts to collude. Collusive strategies can only be signaled by the subjects in the *Recall* treatment, because only there, the other bidder learns about the bids. In the groups with two subjects a bid of exactly zero was chosen in 5.4 percent of the cases in the *Recall* treatment but only in one percent of the cases in the *NoRecall* treatment. This difference is highly significant. For groups of three we find that the percentage of zero bids drops from 9.2 to 6.7 percent when we compare *Recall* and *NoRecall*, and the difference is not significant at all. We conclude from these observations that some of the bids in the two bidder auctions are motivated by collusive attempts, however already in groups of three collusion is sufficiently difficult to achieve that it does not influence bidding behavior.[11]

4. Why Bimodal Bidding?

Why should one adopt a bimodal bidding strategy? Given that the subjects in our experiment did not play according to Nash strategies, a profit maximizing player would not be indifferent between all possible bids when playing against this population. We can use the observed distribution of bids to calculate the bid that maximizes expected profits, given the bimodal bidding behavior of the other subjects. The optimal bid lies somewhat above the lower mode of the bimodal bidding strategy in the region between 5 and 15. The worst thing to do is to bid slightly below the upper mode of the bidding strategy used by our subjects.

There is already a great amount of empirical evidence which demonstrates that many people do not behave like expected profit maximizers in risky situations (see e.g. Dohmen et al., 2005). It is thus natural to explore the game theoretic predictions under different assumptions with regard to the utility function. The prime concern is risk preferences. The predictions derived in Section 2 hold only for players whose utility is linear in the monetary payoff of the game. Let us consider an arbitrary utility

[11] We use probit estimates for b_i=0 with the treatment dummy, round effects and the treatment order dummy as explanatory variables. The treatment dummy is highly significant for groups of two bidders ($p = .003$) and insignificant for groups of three bidders ($p = .340$).

function $u(x_i)$ where x_i is the monetary payoff and $u'>0$. For every mixed strategy equilibrium with *n* homogeneous players it must hold that:

$$EU(b_i) = F(b_i)^{n-1}\ u(1-b_i) + \left(1-F(b_i)^{n-1}\right) u(-b_i) = u(0)\ . \tag{2}$$

The expected utility of every bid b_i used in the strategy equals the probability of winning the auction times the utility in case of a win, plus the probability of losing the auction times the respective utility. Both terms depend on *F*( ), which is the cumulative density of the bidding strategy of the other bidders. For a mixed strategy with full support to be a best reply this expression must be constant and equal to the utility of bidding zero. Following we assume $u(0)=0$ without loss of generality. Thus the utility of winning the auction will be positive and losing the auction will yield zero or negative utility. From this expression we can easily derive the cumulative density of the equilibrium bidding strategy with arbitrary (but symmetric) utility functions as:

$$F(b_i) = \left[\frac{u(-b_i)}{u(-b_i)-u(1-b_i)}\right]^{\frac{1}{n-1}} \qquad \text{with } 0 \le b_i \le 1\ . \tag{3}$$

What densities $f(b_i)$ can we generate with standard risk preferences? If we introduce different risk preferences into the utility function using a function with hyperbolic absolute risk aversion, we can only predict unimodal bidding strategies in groups of two players. Relative to the risk neutral case, risk aversion shifts mass towards low bids while risk seeking preferences shift mass towards high bids.[12] However, in light of the arguments by Rabin (2000) it is not very plausible that behavior in small stake games is explainable by the standard expected utility maximizing approach. Explaining behavior which deviates from expected payoff maximization in small stake situations calls for a different explanation – an explanation that does not rely on a single concave or convex utility function in overall wealth.

It turns out that the bidding strategies observed in our experiment can be explained by the curvature of the utility function if we allow for concave and convex regions. This leads us to a very well known theoretical alternative to expected utility theory, namely Kahneman and Tversky's (1979) prospect theory. A core element of prospect theory is that players evaluate their outcome relative to a reference point. If they earn more than their reference point, they are in the domain of gains, otherwise in the domain of losses. Kahneman and Tversky propose a 'value function' that is concave in the domain of gains and convex in the domain of losses. In addition to that, Kahneman and Tversky introduce a 'loss aversion' parameter, which incorporates the notion that most people suffer more

[12] For example, if we assume a utility function with constant relative risk aversion (CRRA), such as $u(x)=x^{(1-\gamma)}/(1-\gamma)$ we cannot produce a bimodal bidding function for groups of two bidders. For groups of three it is possible to generate bimodal bidding functions but it requires strong risk loving preferences.

from the loss of a certain amount of money than they enjoy the win of the same amount. For illustration purposes we use the parametric specification proposed in Tversky and Kahneman (1992):

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0 \\ -\lambda(-x)^{\alpha} & \text{else} \end{cases} . \tag{4}$$

We denote the amount of money a player earns in an auction by x; α is a parameter for the curvature of the value function. Risk aversion in gains and risk seeking in losses requires $0 < \alpha < 1$. λ is a shifting parameter in the domain of losses, which is larger than one for loss aversion. We assume that in every auction the reference point is the actual wealth when entering the auction. Thus, winning the auction with a bid below 100 puts a player in the domain of gains while losing the auction with a positive bid puts a player in the domain of losses.[13] A second integral part of prospect theory is the probability weighting function which maps objective probabilities into subjective probabilities. For simplicity we do not consider the probability weighting function in our context because, unlike in the typical application of prospect theory, subjects in our game do not know the probability of winning and losing.[14]

Our experimental data offers an unusual approach to estimating preference parameters of prospect theory. Levy and Levy (2002) argue that empirical evidence on prospect theory focused largely on lotteries involving either positive or negative payoffs, whereas real decisions under risk usually involve positive and negative outcomes. They face their subjects with a set of mixed prospects and conclude form their data that behaviour is better explained by an reversed S-shaped value function rather than the S-shaped value function proposed by Kahneman and Tversky. Thus, unlike in prospect theory, Levy and Levy suggest a value function with $\alpha > 1$.[15]

We use a least squares method to fit a cumulative density for the bids to our data for both group sizes. We pool all data from the *Recall* and *NoRecall* treatments. The estimates for the preference

[13] The reference point is usually defined as the status quo. Köszegi and Rabin (2006) discuss the role of reference point determination and present a model where the reference point can differ from the status quo. In our context we think that an outcome of zero is a natural reference point.

[14] In our context the probability weighting function as usually assumed in cumulative prospect theory does not offer additional predictive power. If subjective probabilities deviate from objective probabilities, then equation (3) provides a condition for the subjective densities in equilibrium. In prospect theory it is usually assumed that changes in probabilities close to zero and one have more weight than changes in intermediate probabilities. Thus, in the extreme case it could be that two prospect theory agents draw their bids from the uniform distribution and perceive the winning probabilities as depicted by the bold line in the right panel of Figure 5, i.e., we could have an equilibrium where probability weighting offsets the effects of the prospect theory specific shape of the utility function.

[15] Baucells and Heukamp (2004) however show that the experimental results reported by Levy and Levy (2002) are in line with an s-shaped value function when probability weighting is taken into account.

parameters are $\alpha = .48$, and $\lambda = .86$.[16] The left panel in Figure 5 depicts the value function. The right panel depicts the fitted cumulative densities and the observed bids. Thus, unlike Levy and Levy our data is well organized by an S-shaped value function. A reverse S-shaped value function would predict a mode at intermediate bids, i.e., exactly the opposite of what we observe.

Figure 5 **Left Panel: Prospect Theory Value Function with Estimated Parameters. Right Panel: Observed (Bold Lines) and Predicted (Thin Lines) Cumulative Densities for Both Group Sizes**

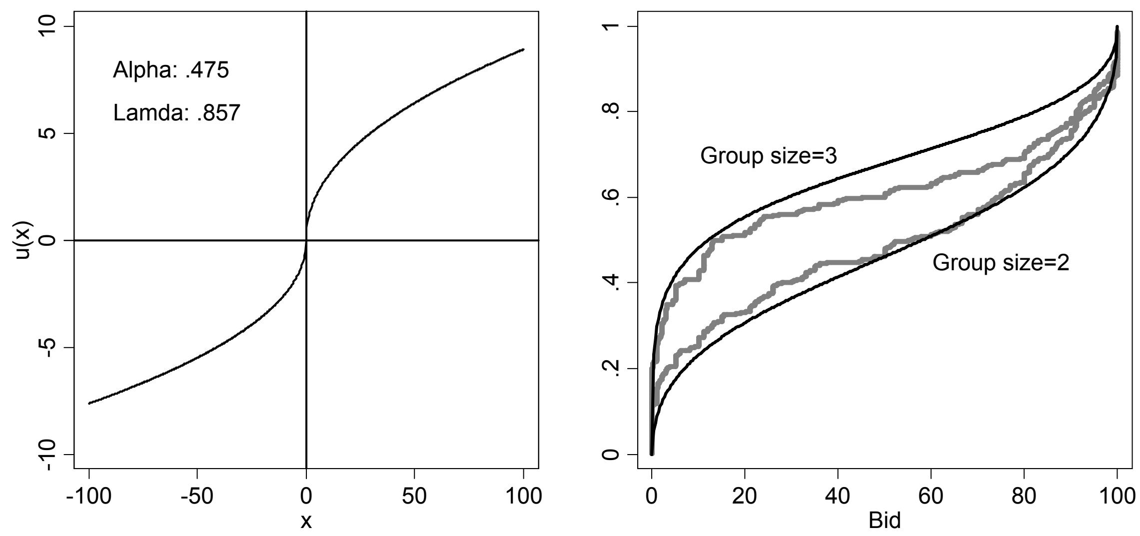


Figure 5 shows that the combination of risk aversion in the domain of gains and risk seeking behavior in the domain of losses as proposed by Kahneman and Tversky (1979) can account for bimodal bidding behavior. Intuitively the subjects use a make-or-break strategy, i.e., they either submit a very low bid and hope for the lucky punch or they submit a very high bid in order to increase the winning probability. The potentially high loss connected with this strategy is acceptable due to the risk seeking preferences in the domain of losses.

Surprisingly, the estimate for the loss aversion parameter λ is smaller than one, which means the opposite of loss aversion, i.e., some sort of loss tolerance. This is at odds with many observations from experiments on loss aversion in risky choice situations (see e.g. Gächter et al., 2007 or Abdellaoui et al. 2007). We think that the main difference between this literature is that we observe risky choices in a strategic situation with competitive characteristics. We speculate that there is a

[16] The literature provides a relatively wide range of parameter estimates. Tversky and Kahneman (1992) report $\alpha = .88$ and $\lambda = 2.25$; Camerer and Ho (1994) find $\alpha = .32$; Wu and Gonzalez (1996) find $\alpha = .50$, the latter two do not include the loss aversion parameter. Booij et al. (2007) estimate the parameters in a representative sample allowing for different powers in gains and losses. They report no significant differences for the power in gains and losses and find $\alpha = .86$ and $\lambda = 1.58$.

preference for competing, which makes winning in an auction more attractive than earning the same amount of money in a simple lottery. Similar effects are also found in market entry games with a competitive structure (Fischbacher and Thöni, 2008).

Prospect theory can account for bimodal bidding and for the fact that mass is shifted from the higher mode to the lower mode of the distribution when the number of contestants in the auction is increased. This corresponds to the observations in our experiment and also to the data reported by Gneezy and Smorodinsky (2006). Furthermore, by allowing a λ lower than one we can account for the fact that in larger groups bids are typically higher than predicted by the Nash equilibrium under standard assumptions.

5. Conclusion

We investigated bidding strategies in very simple common value all-pay auctions with no pure strategy equilibria. Bidders in our experiment use mixed strategies that are remarkably different from the mixed strategies predicted by the Nash equilibrium under standard assumptions. Bidders in our experiment drew their bids not from uniform or unimodal densities but from bimodal densities. They seemed to apply an all-or-nothing strategy, where they either chose a very low bid as a low risk strategy or a very high bid with high winning probabilities but large potential losses. Bimodal bidding strategies are observed under two very distinct information conditions: They occur when bidders are in stable groups with full information about the bidding history and also when bidders do not receive any information about other bidders' strategies.

The bimodality in the distribution of bids cannot be explained by standard risk preferences but fits very well to the S-shaped value function proposed by Kahneman and Tversky (1979) in their prospect theory. We use our data to estimate preference parameters. For the curvature of the value function we find values that are comparable with the values reported in the literature. For the second ingredient of prospect theory's value function – loss aversion – we find strikingly different results. The observed bidding strategies are best explained when assuming the contrary of loss aversion. The reason for this is presumably not because our subjects like losses, but because the competitive structure of the game offers them additional utility when they win the auction.

This hypothesized additional utility for winning an auction can additionally explain why bids tend to become excessive in larger groups, leading to systematic losses for the bidders in such all-pay auctions. Anderson et al. (1998) propose the logit equilibrium as a solution concept to account for excessive bids. In this framework players are boundedly rational in the sense that they make random errors when choosing their bid. The probability of choosing a strategy that is not best reply is negatively related to the expected payoff of that strategy. The distribution of bids we observe in our data makes this explanation highly unlikely, because a bimodal distribution of bids is not compatible with this kind of erroneous bids. Errors simply shift the densities predicted by the Nash equilibrium under standard assumptions towards the uniform distribution, but cannot produce a second mode at high bids.

What are the consequences of our bidders deviating from strategies derived under standard assumptions? On the one hand bimodal bidding densities increase the variance of the bids. On the other hand, the fact that bidders appear to gain utility from winning the auction that goes beyond the financial benefits reduces their expected monetary payoff.

Many competitive situations in the real world involve aspects of all-pay auctions, like lobbying battles or competing for research money. While our experiment certainly represents a very stylized situation we still think it is informative for real life situations. People are aware of the fact that mixed strategies are optimal but they seem to be especially attracted to pure strategies at the boundaries of the strategy space and seem to be considerably more tolerant to the risk of losing money than in non-competitive situations.

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6. Appendix: Experimental Instructions

General explanation for the participants

You are now taking part in an economic experiment that is financed by different research-promoting facilities. If you are reading the following explanation carefully, you can – depending on your decisions – make a considerable amount of money. Hence, it is important that you read this explanation carefully.
The instructions you will receive are for your private information only.

During the experiment absolute silence is required. Communication is prohibited.

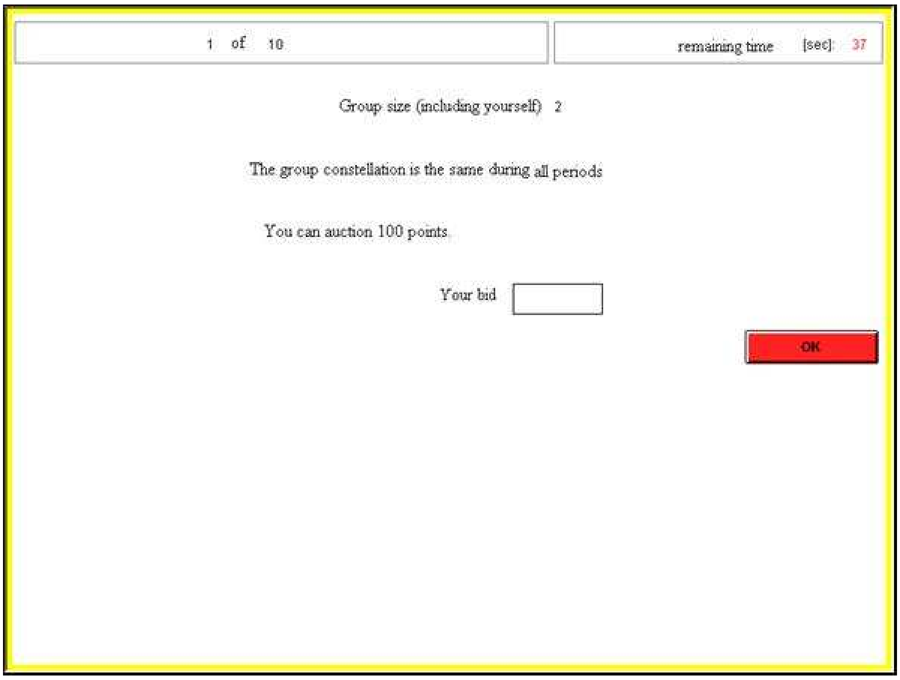
If you have any questions please direct them towards us. Non-observance of these rules will lead to exclusion from the experiment and any payments.
During the experiment we do not talk about Swiss Francs. Your income will be calculated in points. At the end of the experiment, the attained points will be transferred into Swiss Frances, where

1 point = 1 centime.

At the end of the experiment you will receive your earned points (in CHF) plus CHF 20 for showing up, in cash. If you make a loss, it will be deducted from the CHF 20. You cannot make an overall loss.
The next pages describe the detailed procedure of the experiment.

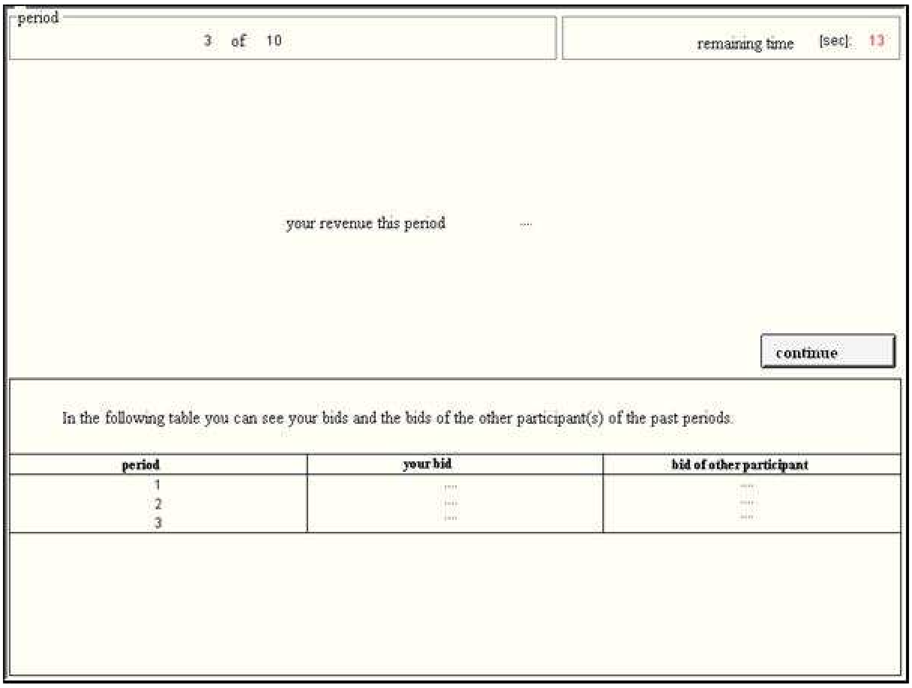
Experiment Instructions: Recall Treatment

You are taking part in an auction. In total there are 10 rounds and in each of these rounds a prize of 100 points is auctioned. You will be put in a group of either two or three participants. Hence there will be one other or two other bidders in your group besides yourself. **The group composition will be constant during the 10 rounds**, i.e. in each round you are in the same group with either one or two other participants. You will not know who else in this room is in your group; your identity will be kept secret. When the auction begins you will have to place your bid. All participants do so at the same time. You can place a bid up to three decimal places. In each group the participant placing the highest bid wins. If more than one participant bid the same highest value, the computer randomly assigns the winner. Different to usual auctions you might know, not only the winner but **all bidders** have to pay their bid. As soon as the experiment starts you will see the following screen:



On the screen, you can see whether you are placed in a group with 2 or 3 participants. In the right hand upper corner you can note the remaining time you have to place your bid. Type your bid into the field. After submitting the bid [pressing OK] you will not be able to change it again. You only bid once in each round. You can place bids from and with zero and up to three decimal places.

As soon as all the participants have submitted their bids, the computer will calculate the highest bid and determine the earnings in points. It will then appear a screen showing the outcome of the auction:



You will be notified if you won the auction and of your revenue in this round. Additionally you will see the bids of the other participant(s) as well as your own. Subsequently press the 'continue' button to proceed to the next round.

An example to clarify the rules:
Assuming that in a group of three participants the following bids are submitted:
Anton: 10 points
Berta: 50 points
Claus: 80 points

Claus wins the auction and has a revenue of 20 points (=100-80) in this round. Anton and Berta do not win, but have to pay their bid nevertheless. The revenue of Anton is therefore -10 and that of Berta is -50 in this round.

Do you have any questions?

Experiment Instructions: NoRecall Treatment

<u>Explanation to the second experiment</u>

The second experiment also consists of 10 rounds, in each of which 100 points are to be auctioned. There are **two important modifications**:

- **You will not receive any information whether or not you won the auction in this round**. You will also not receive any information about the bids of the other participants.
- **The constellation of the group changes in each round.** The group size varies between 2 and 3. The computer will randomly assign you in a group of 2 or of 3. You will be able to see the size on the screen.

Do you have any questions?