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Working Paper 13/06

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Nonlinearities in the Relationship between Debt and Growth: Evidence from Co-Summability Testing*

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October 10, 2013

Abstract: We employ novel time series methods to investigate the presence of nonlinearities in the long-run relationship between public debt and growth. Analysing over two centuries of data for the United States, Great Britain, Sweden and Japan we find very limited evidence for nonlinear long-run relationships in these countries and further cannot support the notion that their equilibrium debt-growth relationship is identical. Both results weaken the case for a common 90% or indeed any common debt-to-GDP threshold recently popularised by the work of Reinhart and Rogoff (2010*b*) and others.

Keywords: economic growth; public debt; nonlinearity; summability, balance and co-summability

JEL classification: E62, C22, F34

*We are indebted to Vanessa Berenguer-Rico, Matei Demetrescu and Andrea Presbitero for helpful comments and suggestions. Vanessa also kindly provided the Gauss code which forms the basis for the analysis carried out. The usual disclaimers apply. Correspondence: Markus Eberhardt, School of Economics, University of Nottingham, Sir Clive Granger Building, University Park, Nottingham NG2 2RD, United Kingdom; email: markus.eberhardt@nottingham.ac.uk; fax: +44 115 95 14159; phone: +44 115 846 8416.

1 Introduction

"The latest research [by Reinhart and Rogoff] *suggests that once debt reaches more than about 90% of GDP the risks of a large negative impact on long term growth become highly significant."* George Osborne, Mais Lecture, February 24, 2010

"The study [Reinhart and Rogoff (2010*b*)] *found conclusive empirical evidence that total debt exceeding 90 percent of the economy has a significant negative effect on economic growth."* The Path to Prosperity – Restoring America's Promise, House Committee on the Budget, April 5, 2011

Despite the fiscal austerity measures and/or the rhetoric adopted by a number of governments and opposition parties over recent years, determining a causal link from public debt to growth as well as the potential nonlinearity of this relationship are widely regarded as unresolved or at best highly contentious empirical issues (International Monetary Fund, 2012; Égert, 2013; Panizza and Presbitero, 2013). As above quotes indicate the most influential research on the debt-growth nexus is unarguably the work by Reinhart and Rogoff (2010*b*) which has been adopted as justification for austerity measures by politicians on both sides of the Atlantic. Although recent revelations found fault with the descriptive analysis carried out in their paper, Reinhart and Rogoff maintain that "the weight of the evidence to date – including this latest comment [by Herndon, Ash and Pollin (2013)] – seems entirely consistent with our original interpretation of the data" (*Wall Street Journal* 'Real Time Economics' blog, April 16, 2013), namely that "high debt/GDP levels (90 percent and above) are associated with notably lower growth outcomes" (Reinhart and Rogoff, 2010*b*, p.577). Perhaps aware of the tension between the causal interpretation typically read into this type of statement and the descriptive nature of their analysis, Reinhart, Reinhart and Rogoff (2012) point to a set of empirical studies which adopt panel econometric methods to address both concerns regarding causality and the nature of the nonlinearity in the long-run debt-growth nexus (inter alia Kumar and Woo, 2010; Cecchetti, Mohanty and Zampolli, 2011; Checherita-Westphal and

Rother, 2012).[1] to support their findings.

This paper investigates the debt-growth nexus from a new angle and with a somewhat more modest aim, focusing on the persistence of the time series data used in the original Reinhart and Rogoff (2010*b*) study. We use annual data for over two centuries to investigate whether linear or various nonlinear specifications of the debt-growth nexus constitute 'long-run equilibrium relations' in four present-day OECD countries: the United States, Great Britain, Japan and Sweden. The former two economies are presently at the centre of a policy debate relating sustainable growth to fiscal austerity (e.g. US Senate Budget Committee, June 4, 2013), while the third is at times taken as an example for sustained growth at comparatively high levels of debt. Sweden (alongside the US and Britain) represents the country with the longest time series in our matched dataset. We follow the practice in the literature and adopt polynomial functions or piecewise linear (threshold) specifications to model the hypothesised nonlinearity. Difficulties for conventional time series analysis of a nonlinear relation between debt and growth arise given that the order of integration of the square (or cube) of an integrated variable or of an interaction between an integrated variable and a threshold dummy is not defined within the (co-)integration framework.[2]

Our analysis addresses this problem by adopting novel methods for summability, balance and co-summability testing (Berenguer-Rico and Gonzalo, 2013*a*,*b*). These concepts provide a framework encompassing integration and cointegration which however extends to non-linear relationships. Testing procedures are straightforward, building on least squares regressions involving re-scaled sums of the variables in the empirical model, with inference established via confidence intervals constructed from subsample estimates. We investigate the evidence for long-run equilibrium relationships between debt burden and per capita GDP levels in annual time series — since we are interested in the long-run relationship we adopt the levels variable for income, rather than its growth rate.[3]

[1]A further empirical study by Baum, Checherita-Westphal and Rother (2013) is also cited in Reinhart, Reinhart and Rogoff (2012) but argues to focus on the short-run relationship. A time series study for Italy by Balassone, Francese and Pace (2011) is also cited in support.

[2]Existing work on nonlinear cointegration (Park and Phillips, 2001; Wang and Phillips, 2009) is premised on the untested assumption of the existence of a long-run relationship in the data.

[3]For completeness we also analyse summability and balance adopting the per capita GDP

We provide two robustness checks for our main results: first, in order to determine whether results for the four economies investigated are out of line with those in a wider set of countries we carry out summability, balance and co-summability testing for a further 23 countries – mostly OECD economies but also a small number of developing countries (India, Peru, Sri Lanka) – for which long time series data are available. Second, we investigate balance and co-summability using a sixty-year moving data window to allow for structural breaks in the long-run debt-growth relationship and to avoid undue impact of global shocks on our full sample testing procedure. The window length of sixty years may appear somewhat arbitrary,[4] however this choice allows us not only to analyse the long-relationship evolving over two centuries, but also to focus on the post-WWII period which is at the centre of attention in most existing empirical studies, styuding results for subsamples ending in 2005 to 2010. This subsample approach also offers the opportunity to investigate post-WWII results when omitting the recent global financial crisis from the sample.

Our analysis in this study is thus narrowly focused on the question of nonlinearities in the long-run debt-growth relationship, bypassing any concerns over the direction of causation *which does not impact the statistical validity of our results*. If we cannot find any evidence for nonlinear long-run relations, then standard empirical specifications in the literature adopting thresholds or polynomial functions are fundamentally misspecified and the causal interpretation assigned in these studies is invalid. Results have important policy implications given that the most vocal academic supporters of fiscal austerity have pointed to these panel studies as providing empirically sound evidence for nonlinearities in the debt-growth relationship.

Our full sample analysis finds no evidence for any long-run relationship between debt and growth in the linear or nonlinear specifications for the four countries investigated. Results from the larger set of economies strongly support this conclusion. Subsample analysis does not fundamentally challenge this finding although it provides an indication that there may have been long-run relationships between debt and growth at different points in time, although not in

growth rate, for which results (see Tables 1 and 3) provide no evidence of balance in any country or specification. See also the discussion on 'growth' regressions in Section 5.

[4]Robustness checks suggest qualitatively similar results for alternative window-lengths.

the post-WWII period typically studied in the existing literature. The general patterns revealed by the subsample analysis supports the notion that the debt-growth relationship differs across countries and “with economic circumstances” (Larry Summers, Witness Statement to the US Senate Budget Committee, June 4, 2013). These findings challenge the apparent consensus in the empirical literature on the existence of a debt-growth threshold and caution policymakers against pursuing further austerity measures to reduce the debt burden in fear of ‘dangerous’ debt levels.

The remainder of the paper is structured as follows: the next section discusses the existing literature on debt and growth, with Section 3 providing the theoretical background for our econometric approach. Section 4 introduces the data and provides some discussion of the debt-growth nexus in each of the four countries. Results are provided in Section 5, Section 6 concludes.

2 Existing Literature

The existing empirical literature on the debt-growth nexus builds on somewhat tenuous theoretical foundations (see Panizza and Presbitero, 2013, for a recent survey). Some theoretical models argue that higher stocks of public debt may create increased uncertainty or even fear of future financial repression among investors and thus lead to a negative long-run relationship (Elmendorf and Mankiw, 1999; Cochrane, 2011) between debt and growth. Other work maintains that this negative relationship disappears once sticky wages and unemployment are taken into account in the modelling process (Greiner, 2011). The nonlinearity or debt threshold hypothesised and investigated in most empirical work can be motivated for developing countries by pointing to the issue of debt overhang (Krugman, 1988; Sachs, 1989), although it may be difficult to extend this argument to advanced economies such as those investigated in this paper. Nonlinearities may further arise if there is a tipping point of fiscal sustainability as is developed in Ghosh et al. (2013), however we are not aware of any theoretical models incorporating such debt tipping points into a framework for economic growth over the long-run.

As was suggested above, the work by Reinhart and Rogoff (Reinhart and Rogoff, 2009, 2010*a*,*b*, 2011) is largely descriptive in nature, although this should not distract from the significant contribution these authors have made to the literature in the construction of long data series for empirical analysis. Regression analysis of the debt-growth nexus conducted using panel data typically shares the unease about misspecification and endogeneity with the wider cross-country growth literature (see Durlauf, Johnson and Temple, 2005; Eberhardt and Teal, 2011, for a discussion of the latter). Empirical specifications in this literature are across the board partial adjustment models in the mould of Barro (1991) and Mankiw, Romer and Weil (1992) — regressing growth on a lagged level of per capita GDP and a measure for debt stock as well as typically a large number of control variables — in a pooled model specification, thus assuming away the possibility of parameter heterogeneity across countries.[5] The standard practice in the cross-country literature to average data over three- or five-year intervals in the panel is also adopted in all but the most recent papers (Checherita-Westphal and Rother, 2012; Panizza and Presbitero, 2012; Baum, Checherita-Westphal and Rother, 2013).[6] Samples differ significantly across existing studies, with the work by Kumar and Woo (2010), Cecchetti, Mohanty and Zampolli (2011), Checherita-Westphal and Rother (2012), Padoan, Silva and den Noord (2012), Panizza and Presbitero (2012) and Baum, Checherita-Westphal and Rother (2013) primarily focused on OECD and other high-income economies and thus most relevant to this study. Among these OECD country studies the only one to adopt a polynomial specification is the paper by Checherita-Westphal and Rother (2012), although this practice is quite popular in the study of developing economies (e.g. Cordella, Ricci and Ruiz-Arranz, 2010; Pattillo, Poirson and Ricci, 2011; Calderón and Fuentes, 2012; Presbitero, 2012). With the exception of Cecchetti, Mohanty and Zampolli (2011), who apply the within (fixed effects) estimator and thus cannot address concerns over

[5]Notable exceptions include studies by Caner, Grennes and Koehler-Geib (2010), Henderson and Parmeter (2013) and Kourtellos, Stengos and Tan (2012) which emphasise the heterogeneity of the debt-growth nexus across countries and adopt nonparametric methods to identify a threshold in the cross-section dimension.

[6]Baum, Checherita-Westphal and Rother (2013) suggest that annual data by definition capture the *short*-run behaviour of an equilibrium relationship. One might instead point to the vast time series econometric literature which maintains that distinction of long- and short-run is primarily a matter of model specification – e.g. by use of an explicitly dynamic autoregressive distributed lag or error correction model (Hendry, 1995).

reverse causality, all of the above implement their panel analysis adopting the Blundell and Bond (1998) System GMM estimator originally developed for firm-level panel data analysis.[7]

Despite different sample periods, country coverage, control variables, modelling of the nonlinearity and choice of moment conditions for identification, these studies come to remarkably similar conclusions, namely that beyond a threshold at around 90% debt-to-GDP the relationship between debt and growth is negative significant. However, as demonstrated by Panizza and Presbitero (2013), these findings are either not robust to small changes in the sample, suggesting the results are driven by outliers, or fail to formally test the coefficients on the pairwise linear terms, which on closer inspection typically cannot support the notion of a statistically significant change in the coefficient beyond the threshold.

All of the above studies are focused on pooled panel data modelling, implicitly assuming that the long-run equilibrium relationship between debt and growth is common to all countries in the sample. Existing research has found very different results when moving away from full sample analysis in homogeneous parameter regression models and investigating sub-samples along geographic, institutional or income lines (Kourtellos, Stengos and Tan, 2012; International Monetary Fund, 2012). There are a number of reasons to assume the equilibrium relationship between debt and growth could differ across countries. Vulnerability to public debt depends not only on debt levels, but also on debt composition (Inter-American Development Bank, 2006). Unfortunately, existing data for the analysis of debt and development often represent a mix of information relating to general and central government debt, debt in different denominations and with different terms attached (be they explicit or implicit). All of this implies that comparability of the debt data across countries may be compromised (Panizza and Presbitero, 2013). In addition, even assuming that debt stocks are comparable across countries and over time, the possible effect of public debt on GDP may depend on the reason why debt has been accumulated and on whether

[7]See Eberhardt and Helmers (2010) for a survey of this type of micro panel estimator. A thorough critique of this implementation in the macro panel context is beyond the scope of this paper; Bun and Sarafidis (2013) provide an analysis of the impact of nonstationary initial conditions on this set of estimators while Pesaran and Smith (1995) discuss the bias arising from heterogeneity misspecification.

it has been consumed or invested (and in the latter case in which economic activities). Furthermore, different stocks of debt may impinge differently on economic growth: debt can clearly hinder GDP growth when it becomes unsustainable, affecting interest rates and triggering a financial crisis, thus affecting the level of GDP. However, the capacity to tolerate high debts depends on a number of country-specific characteristics, related to past crises and the macro and institutional framework (Reinhart, Rogoff and Savastano, 2003; Kraay and Nehru, 2006; Manasse and Roubini, 2009). For these reasons we focus our analysis in this paper on country-by-country investigation of the long-run relationship between debt and growth.

A recent study which empirically investigates the debt-growth nexus with a time series econometric approach is the paper by Balassone, Francese and Pace (2011) on Italy (1861-2009). Adopting unit root and cointegration testing prior to estimation they establish a long-run relationship between per capita GDP, per capita capital stock and debt-to-GDP ratio (all in logs). They then go on to estimate (among other models) a piecewise linear specification for the debt-to-GDP ratio where values beyond a threshold of 100% are found to create a significantly stronger negative effects on growth – it is precisely this form of interaction between a threshold dummy and the debt-to-GDP ratio which is not defined under (linear) cointegration[8] and which necessitates the present analysis.[9]

3 Nonlinear Relations between Integrated Processes

In this section we highlight the difficulties arising for conventional time series analysis when assuming a non-linear model in the presence of integrated variables and discuss a novel approach to tackle these issues.

Suppose a time series relationship $y_t = f(x_t, \theta) + u_t$ for a nonstationary regressor $x_t \sim I(1)$, stationary u_t and some non-linear function $f(\cdot)$. Assuming for

[8]It should also be noted that cointegration does not imply causation from debt to growth.

[9]Adopting the threshold specification we find that the data series for ITA pass the balance test (CI low -2.738, $\hat{\delta}_y - \hat{\delta}_z = -1.319$, CI up 0.100), but none of the various thresholds adopted pass the co-summability test (100% threshold CI low 0.313, $\hat{\delta}_{\hat{e}_t}$=1.134, CI up 1.954; 90% CI low 0.486, $\hat{\delta}_{\hat{e}_t} = 1.052$, CI up 1.619; 70% CI low 0.910, $\hat{\delta}_{\hat{e}_t} = 1.695$, CI up 2.480; 50% CI low 0.811, $\hat{\delta}_{\hat{e}_t} = 1.471$, CI up 2.130) – see results section for notation.

illustration $f(x_t) = \theta x_t^2$, let $x_t = x_{t-1} + x_0 + \varepsilon_t$ and $\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$, then we know that

$$\mathbb{V}[x_t - x_{t-1}] \;\; = \;\; \sigma_\varepsilon^2 \quad \Rightarrow x_t \sim I(1) \tag{1}$$

In words, we can show that the Engle and Granger (1987, henceforth EG) 'characterisation' (read: definition) of a stationary process holds for Δx_t – finite variance is one of five EG characteristics. Now investigate the same property for Δx_t^2:

$$\mathbb{V}[x_t^2 - x_{t-1}^2] \;\; = \;\; \mathbb{E}[\varepsilon_t^4] + 4(t-1)\sigma_\varepsilon^4 - \sigma_\varepsilon^4 \quad \Rightarrow x_t^2 \sim I(?)$$

We can see that the finite variance characteristic is violated, given that the variance is a function of time. Since this problem cannot be solved by further differencing we are unable to determine the order of integration of x_t^2, which creates fundamental problems if the empirical analysis of $y_t = \theta x_t^2 + u_t$ is to be based on arguments of cointegration. The difficulty arises from the requirement of the EG characterisation to investigate the *differences* of a process, with the intrinsic linearity of the difference operator creating obvious problems for nonlinear processes.

Berenguer-Rico and Gonzalo (2013*b*) develop an alternative approach to integration,[10] based on the 'order of summability,' "a summary measure of the stochastic properties – such as persistence – of the time series without relying on linear structures" (p.3). This approach essentially investigates the rate of convergence of a rescaled sum of the variable of interest, say y_t. Using least squares we can estimate for $k = 1, \ldots, T$

$$Y_k^* = \beta^* \log k + U_k^* \tag{2}$$

where $Y_k^* = Y_k - Y_1$, $U_k^* = U_k - U_1$ and $Y_k = \log\left(\sum_{t=1}^{k}(y_t - m_t)\right)^2$. The deterministic component m_t can be accounted for by the partial mean of y_t, namely $m_t = (1/t)\sum_{j=1}^{t} y_j$ in case of a constant. Given the trending behaviour of our data we focus on the case of constant and linear trend terms, where $m_t =$

[10] Summability was first suggested by Gonzalo and Pitarakis (2006). The estimation method applied here goes back to McElroy and Politis (2007), with the subsampling methodology due to Politis, Romano and Wolf (1999).

$(1/t)\sum_{j=1}^{t} y_j - (2/t)\sum_{j=1}^{t}\left(y_j - (1/j)\sum_{i=1}^{j} y_i\right)$, i.e. partial demeaning of y_t is carried out twice. This implies

$$\hat{\beta}^* = \frac{\sum_{k=1}^{T} Y_k^* \log k}{\sum_{k=1}^{T} \log^2 k} \tag{3}$$

from which we then obtain our estimate of the order of summability $\hat{\delta}^* = (\hat{\beta}^* - 1)/2$. Summability is a more general concept than integration, but they are closely related: if a series x_t is integrated of order d, $d \geq 0$, then it is also summable of order d; however, not all $S(d)$ processes are also $I(d)$. Inference can be established using confidence intervals constructed from subsample estimation, whereby the above procedure is applied to $T - b + 1$ subsamples of length $b = int(\sqrt{T}) + 1$.

In a second step, following Berenguer-Rico and Gonzalo (2013*a*), the 'balance' of the empirical relationship is tested, namely the condition that the two sides of the empirical equation have the same order of summability: $S(\delta_y) = S(\delta_z)$ for $z = f(x_t, \theta) = \theta f(x_t)$. This balance test is equivalent to testing the null of $\beta_n \equiv (\beta_y - \beta_z) = 0$ in the least squares regression

$$Y_{yk}^* - Y_{zk}^* = \underbrace{(\beta_y - \beta_z)}_{\beta_n} \log k + (U_{yk} - U_{zk}) \tag{4}$$

where Y_{yk}^* is constructed from y and defined as in the summability analysis, and Y_{zk}^* is the partially demeaned sum of all regressors $Y_{zk} = \log\left(\sum_{t=1}^{k}(z_t - m_t)\right)^2$. In practice, all elements of z are summed, partially demeaned, their estimated β_z subtracted from that of y and the result divided by 2. Confidence intervals are constructed using subsample results in analogy with the summability analysis. Under the null of balance the resulting confidence interval includes zero; balancedness is a necessary but not sufficient condition for a long-run equilibrium relationship in the data.

Finally, let $\hat{e}_t$ be the least squares residuals from a balanced regression $y_t = \hat{\theta} g(x_t) + \hat{e}_t$, then 'strong co-summability' will imply the order of summability of $\hat{e}_t$ is statistically close to zero (Berenguer-Rico and Gonzalo, 2013*a*). We estimate the order of summability for $\hat{e}_t$ to determine whether our balanced model is co-summable.[11] Inference follows the subsampling approach as in the previous testing procedures and under the null of co-summability the confidence interval includes zero.

We adopt two specifications for nonlinearity in the debt-to-GDP ratio in line with standard approaches in the literature: firstly, in addition to the linear model (Model 1) we use polynomial specifications including linear and squared (Model 2) or linear, squared and cubed (Model 3) debt-to-GDP terms (in logarithms) — examples for this specification include Calderón and Fuentes (2012) and Checherita-Westphal and Rother (2012). Secondly, we adopt piecewise linear specifications where the debt-to-GDP ratio (in levels, not logs) is divided into two variables made up of values below and above a specified threshold, which is treated as exogenous (examples for this specification include Kumar and Woo, 2010; Panizza and Presbitero, 2012; Baum, Checherita-Westphal and Rother, 2013).[12] For Great Britain we adopt three threshold values: 90, 70 and 50 percent. For the United States and Japan we can only adopt the 50 percent threshold since over the 1870 to 2010 time horizon too few observations are above the other two thresholds: 12 (Japan: 17) for 70 percent and only 6 (Japan: 12) for 90 percent.[13]

Our balance and co-summability analysis thus investigates a number of specifications for the debt-growth relationship, inspired by the simple Reinhart and Rogoff (2010*b*) setup. The polynomial specifications are:

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \varepsilon_t \tag{5}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \varepsilon_t \tag{6}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \phi_3 x_t^3 + \varepsilon_t \tag{7}$$

where y is per capita GDP and x is the debt-to-GDP ratio (both in logarithms),

[11] The residual series $\hat{e}_t$ will sum to zero by default of the least squares principle if our specification includes an intercept; we therefore in practice do not subtract the estimate for the intercept term when constructing $\hat{e}_t$.

[12] We acknowledge that parts of the literature, including Baum, Checherita-Westphal and Rother (2013), employ threshold regression algorithms where the threshold value is determined endogenously. They find a statistically significant negative relationship beyond a threshold of 96% debt-to-GDP in a sample of 12 Eurozone countries over the 1990-2010 time horizon.

[13] In Sweden the debt-to-GDP ratio only surpasses the 50 percent threshold in 15 sample years (7% of observations). In Japan the 70 percent threshold applies to 17 years (14%) and the 90% threshold to 12 years (10%).

α_0 is an intercept, t a linear trend term with parameter φ and ε_t is white noise.

The threshold model specifications are based on

$$\begin{aligned} y_t &= \alpha_0 + \varphi t + \theta_1 X_t \times \mathbb{1}(X_t < \text{threshold}) \\ &\quad + \theta_2 X_t \times \mathbb{1}(X_t \geq \text{threshold}) + \varepsilon_t \end{aligned} \tag{8}$$

where $\mathbb{1}(X_t < \text{threshold})$ is an indicator function which is 1 for the debt-to-GDP ratio X_t below the threshold and 0 otherwise — similarly for $\mathbb{1}(X_t \geq \text{threshold})$ at and above the threshold.

In addition to the analysis for the full time horizon we investigate balance and co-summability using a window of sixty years, which we move along the time horizon from the 1800s to 2010. The purpose of this exercise is to provide both an indication of possible changes in the long-run debt-growth relationship over time as well as to safeguard the analysis from undue impact of severe shocks such as the two world wars. Due to the nature of the data this approach is only feasible for the polynomial specifications: as highlighted by Chinn (2012) in his review of Reinhart and Rogoff (2011) there are comparatively few episodes in developed economies where the debt-GDP ratio exceeds 90% and we can therefore not implement the moving window for the piecewise linear specification. Since our rolling window analysis represents a form of data mining we adjust the confidence intervals (CI) for our estimates following a standard Bonferroni correction, whereby CI* $= (1-\alpha/m)$ for the conventional confidence level $1-\alpha$ (we adopt $\alpha = .05$) and the number of sub-samples tested m (varies from 66 for Japan to 152 for the US, Great Britain and Sweden). In practice this makes the confidence intervals much wider, thus representing a much more conservative approach to rejecting the null hypothesis of balance or co-summability.

4 Data

We use annual per capita GDP series (in 1990 Geary-Khamis $) from an updated version (Bolt and van Zanden, 2013) of the data created by Maddison (2010) and the matched gross government debt-to-GDP ratio (in percent) from Reinhart and Rogoff (2009). The debt figures refer to total gross central government debt, comprising domestic and external debt.[14] Data coverage differs across countries: for the US, Britain and Sweden the series start in 1800, for Japan in 1872 (with a gap during 1940-1953) – all series end in 2010. Descriptive statistics are presented in the Data Appendix, where we also plot the levels and first differences of these data series. Data from a further 23 countries using the same sources are employed to carry out summability, balance and co-summability tests, providing a robustness check on our previous results. Here countries were included provided their per capita GDP and debt-ratio time series extended back to 1900 or earlier.

Figure 1 charts the evolution of the debt-to-GDP ratio for the four economies from the early 19th century to 2010, where in the spirit of Reinhart and Rogoff (2010*b*) we highlight periods with debt burden in excess of 90% of GDP. While the four time series all display idiosyncracies, it is nevertheless notable how similar in particular the patterns for British and American debt-to-GDP ratios are over much of the 20th century, albeit with substantially higher debt in the former. Britain is also the only economy studied which experienced sustained periods of debt-to-GDP above 90%. In Figure 2 we plot the debt-income relationship in each of the four countries, taking variables in deviation from the country-specific time-series mean. In all four economies the most significant turning points for the debt-growth nexus were marked by the Great War, the Great Recession of the late 1920s and World War II.

5 Results

Table 1 provides estimates of the order of summability for all model variables, including polynomial as well as threshold terms for debt. None of the confidence intervals for tests on per capita GDP *levels* or any of the debt variables include zero, thus rejecting the null of summability of order zero. The estimated order of summability for the per capita GDP *growth rates* in contrast is always very close to zero. These patterns are confirmed when we consult the results for the larger set of countries in Table TA-1 of the Technical Appendix: in 23 out

[14]For Great Britain the series are net rather than gross central (external and domestic) government debt. See Data Appendix for a number of exceptions for the extended analysis of 27 countries.

of 27 countries we identify a pattern whereby we cannot reject the null that the per capita GDP *growth rate* is S(0) but reject this null in the equivalent *levels* series.[15] In 25 out of 27 countries all three debt variables reject summability of order zero.[16] These findings highlight the significant persistence of the data and provide a strong motivation for the concerns over time series properties we argue are of primary importance when analysing the long-run debt-growth nexus. In analogy to integrated data, we run the risk of spurious results in any regressions containing these variables unless we can confirm our empirical models as balanced and co-summable.

Table 2 provides results from balance and co-summability testing using per capita GDP *levels* as dependent variable. We also provide the results for specifications using the per capita GDP *growth rates* as dependent variable in Table 3: the latter specifications do not constitute balanced empirical equations since none of the confidence intervals includes zero, regardless of whether we adopt polynomial or threshold specifications. The popularity of the 'growth' specification in the cross-country empirical literature is justified by the presence of a lagged level of per capita GDP as additional regressor. This quasi-error correction specification which provides estimates for a long-run *levels* relationship is however frequently misinterpreted as a growth equation (see Eberhardt and Teal, 2011, for a discussion). Our results support the notion that growth rates and debt levels do not share the same order of summability, which is in line with expectations of nonstationary log levels and stationary first differences (growth rates). For the specifications with per capita GDP *in levels* two of the nonlinear specifications for the United States (the threshold model and the polynomial specification with linear, squared and cubed terms) constitute unbalanced equations. In all other models studied we cannot reject balanced specifications. Co-summability is however rejected in *all* countries and specifications where we found balanced equations – note that the rejection is by no means marginal, with all confidence intervals some distance away from zero.

Investigation of the results for the larger set of countries in Table TA-2 of the Technical Appendix again confirms that these patterns of results are qualitatively

[15]For URY both series cannot reject S(0), for BRA, COL and PRT both series reject the S(0) null — see Table A-2 for country codes.

[16]For BRA all three polynomials cannot reject S(0), while (marginally) the same holds for the linear debt-to-GDP series for CAN.

identical across the additional 23 OECD and non-OECD countries investigated – we find a single case (polynomial specification with linear, squared and cubed debt terms for URY) which satisfies balancedness and co-summability. These results provide strong evidence against any nonlinear – or, for that matter, linear – specification in all countries investigated. This would imply that from a long-run perspective the debt-to-GDP ratio and per capita income do not move together, *precluding any causal relationship between these variables*.

There are two potential caveats with this analysis which we attempt to tackle in the following: firstly, it may be the case that our full-sample analysis insufficiently captures the serious shocks experienced by these economies over the past two centuries, e.g. the two world wars or the recent global financial crisis, and that the empirical testing conducted may be unduly affected by these global shocks. Secondly, our analysis has focused on time series covering over two centuries of data, implicitly assuming that the long-run equilibrium relationship between debt and growth is stable over this time horizon.

Instead of using the full time series data we therefore investigate a rolling window of sixty years to compute the balance and co-summability results. We report three sets of results from this exercise: (i) country-specific time-varying balance and co-summability statistics for the entire 152 subsamples (66 for Japan) of sixty years, presented in graphical form; (ii) comparison of the balance and co-summability subsample results for the United States, Great Britain and Sweden in graphical form – this is intended to uncover patterns of commonality and difference in the long-run equilibrium relationship across countries; (iii) balance and co-summability statistics for the final six sixty-year windows for the United States, Great Britain and Sweden, which allow us to focus on the post-WWII period while also analysing results omitting the most recent years covering the global financial crisis (2008-2010).[17]

Graphical results for our sub-sample analysis of balance and co-summability, including Bonferroni-adjusted confidence intervals, are presented in Figures 3 and 4. In each plot of the former the (broken) line and dots signify that the specification is *balanced*, whereas in the latter these identify *balanced and co-summable* specifications. In both cases we mark the end-year on the x-axis of

[17] We do not include Japan in this analysis since the gap in the debt series from 1941-53 prevents direct comparison with the sub-samples for the other three economies.

each plot – note that due to different data availability the time dimension of the plots differ between the US, Great Britain and Sweden on the one hand and Japan on the other. In line with the full-sample results we observe in Figure 3 that the vast majority of linear models (Model 1) across the four economies constitute balanced empirical models. While this is broadly also the case for the polynomial function with linear and squared debt terms (Model 2), the specifications further including a cubed debt term (Model 3) show a significant number of unbalanced subsamples for Great Britain and Japan, and to a lesser extent for Sweden.[18]

Once we move to the results for balance and co-summability in Figure 4, we can identify only a *minority* of sub-samples in any country or specification to be balanced and co-summable, thus broadly echoing the full sample results above.[19] During certain periods over the past two centuries consecutive subsamples were found to indicate a long-run equilibrium relationship, within each of the four economies none of the three model specifications reveals itself to clearly dominate the others in terms of balance and co-summability.

In Table 4 we compare the subsample periods for which the sixty-year data series constituted balanced and co-summable specifications in the data for the US, Great Britain and Sweden: Panel A refers to the linear model (Model 1), Panels B and C to the polynomial specifications with (in addition) squared and cubed debt terms, respectively (Models 2 and 3). For each country a shaded cell indicates the sixty-year subsample ending in the year specified constituted a balanced and co-summable specification, while the intensity of the shading indicates whether this property occurred in one (lightest), two (intermediate) or all three (darkest) countries. In the following we primarily focus on those 'episodes' of co-movement when the tests for *all three countries* agree on balance and co-summability: for both Models 1 and 2 we can identify clusters of such episodes in the 1860s (thus for the series starting in the 1800s), 1890s-1900s (1830s–1840s), 1950s (1890s) – in Model 2 there is evidence for pro-

[18]Interestingly, balancedness graph for the United States (Models 1 and 2), where the full time series test rejected balance, provides comparatively stronger evidence for balance in a majority of subsamples.

[19]The overall share of samples which satisfy balance and co-summability is as follows: USA 44%, GBR 49%, SWE 49%, JPN 24% (Model 1); USA 45%, GBR 43%, SWE 46%, JPN 24% (Model 2); USA 36%, GBR 13%, SWE 36%, JPN 12% (Model 3).

longed episodes in the 1960s and 1970s (1900s–1910s) – and thereafter isolated episodes in the 1970s and 1980s. The most recent episodes occurred in the early 2000s, although this cluster excludes the years of the global financial crisis. Taken together these episodes account for just 16% of all subsamples. Inbetween these episodes there are stretches where two countries have balanced and co-summable specifications (around 30% of subsamples in either model), although these are often clustered around the episodes just described. The remainder of subsamples is made up of single country episodes (37% in Model 1 and 28% in Model 2) and subsamples with no balance and co-summability in any country (18% in Model 1 and 27% in Model 2). [20] Model 3 has concurrent episodes in all three countries in just 4% of all subsamples, mainly in the 1960s, while a further 17% of subsamples have episodes in two countries. This leaves around 40% for the remaining two categories, respectively. Across all three models the proportion of concurrent episodes for one or two countries is roughly twice that respectively of episodes for all three countries,[21] providing an indication of the heterogeneity in the long-run debt-growth relationship across these three economies.

Much of the recent work on the debt-growth nexus has analysed data from the post-WWII period and we can similarly focus on those 60-year subsamples covering the end of our sample period. Table 5 provides balance and co-summability results for the last six subsamples (1946–2005, ..., 1951–2010) in the data for the US, Great Britain and Sweden. For Model 1 none of the US subsamples and only one each of the subsamples for Sweden and Great Britain 'pass' this test. For Model 2 none for Great Britain but two subsamples each for the US and Sweden pass the test, whereas for Model 3 no subsample across all three countries does so. Evidence for co-movement when restricting the sample

[20]If we refer back to Figure 2 we can remark that the first of these clusters, covering subsamples ending in the 1860s, occurred when all three countries substantially reduced their country-specific debt-burden (movement to the left in the figure) albeit with comparatively modest increase in growth in the US and Sweden (relatively flat line plots). None such pattern is revealed for the second cluster for subsamples ending in the 1890s and 1900s, while the third cluster with end years in the 1950s and 60s occurred when all three countries shifted from a relative debt build-up in years prior to and during WWII to significant debt reduction thereafter, whereby the latter period also represented a return to steady economic growth. The final cluster in the late 1990s and early 2000s again does not reveal any systematic patterns in the evolution of debt burden and growth across these three economies.

[21]For Model 1 one (two) episode(s) is 2.24 (1.72) times more common, for Model 2 the figures are 1.88 and 1.75, and for Model 3 4.33 and 9.83.

to the post-WWII period is thus quite limited. From these results it is further not obvious that inclusion or exclusion of the most recent crisis years (2007 onwards) yields systematically different results for our analysis.

Our robustness checks thus provide a number of important insights: firstly, there is no evidence that the full sample results for our four OECD economies differ from those for other countries considered in detail in a Technical Appendix. Secondly, there is no overwhelming evidence that these full sample results are severely distorted by global shocks or structural breaks in the long-run debt-growth relationship, given that only a minority of subsamples were found to be balanced and co-summable across all countries and specifications. Thirdly, having said that our results point to a distinct possibility that certain countries experienced linear or nonlinear co-movement between debt and income during certain periods of time over the past two centuries, although seemingly not during the post-WWII period.

6 Concluding Remarks

This study took an alternative approach to investigating the presence of nonlinearities in the long-run equilibrium relation between public debt and growth. Our empirical results for four OECD economies using data from the 1800s to 2010 and the various robustness checks carried out provide very limited evidence for nonlinear, or indeed linear, long-run relationships in these economies. There are certain subperiods over this long time horizon for which we can confirm co-movement between debt and income. We further conclude that the timing of these subperiods largely appears to differ across countries, thus questioning the standard approach in the empirical literature to analyse countries in a *pooled* panel data model, thus imposing parameter homogeneity.

Our results undermine some of the popular conclusions for this politically-charged issue which represent fiscal adjustment as a necessity for long-run economic stability and sustainability. We do not claim that a high debt burden is a matter of no concern for policymakers or that in the short-run debt may not be detrimental to growth. Instead, we highlight the absence of evidence for nonlinearities such as the popular 90% debt-to-GDP threshold in the long-run relationship with growth and development, which implies that aggressive austerity programs and government spending cuts may not result in the expected growth boost postulated.

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Tables and Figures

Table 1: Estimated Order of Summability

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	-	211	ln(GDP pc)	0.652	1.490	2.329
					Δln(GDP pc)	-0.524	0.066	0.657
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
				168	Debt/GDP $<$ 50%	0.313	0.825	1.337
				43	Debt/GDP $\geq$ 50%	0.691	1.409	2.127
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					Δln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
				100	Debt/GDP $<$ 90%	0.511	1.062	1.613
				111	Debt/GDP $\geq$ 90%	0.405	0.936	1.467
				86	Debt/GDP $<$ 70%	0.428	1.200	1.972
				125	Debt/GDP $\geq$ 70%	0.465	0.923	1.381
				64	Debt/GDP $<$ 50%	0.447	1.068	1.689
				147	Debt/GDP $\geq$ 50%	0.459	0.898	1.336
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					Δln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
JPN	1872	2010	2	125	ln(GDP pc)	0.824	2.070	3.315
					Δln(GDP pc)	-0.687	-0.001	0.685
					ln(Debt/GDP)	0.420	1.099	1.778
					ln(Debt/GDP) squared	0.371	1.108	1.845
					ln(Debt/GDP) cubed	0.406	1.115	1.823
				77	Debt/GDP $<$ 50%	0.195	1.019	1.843
				48	Debt/GDP $\geq$ 50%	0.503	1.325	2.147

Notes: For the US and GBR we also provide summability estimates for data below and above various debt/GDP thresholds: for the former this is only feasible for a 50 percent debt/GDP threshold, whereas for the latter we can test 90, 70 and 50 percent. Obs reports the number of observations. CI low and up indicate the 95% confidence interval for the summability estimate $S(\delta)$ constructed from subsampling – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted we allow for deterministic terms (constant and trend).

Table 2: Balance and Co-Summability — ln(GDP pc) specifications

						Balance				Co-Summability			
	Start	End	Gaps	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	-	211	-	-1.265	-0.507	0.252	$S(\delta_y) = S(\delta_z)$	0.467	1.050	1.633	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.611	-0.726	0.160	$S(\delta_y) = S(\delta_z)$	0.277	0.943	1.610	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.158	-1.145	-0.132	$S(\delta_y) \neq S(\delta_z)$				
					Threshold	-1.865	-0.942	-0.019	$S(\delta_y) \neq S(\delta_z)$				
GBR	1800	2010	-	211	-	-0.913	-0.178	0.558	$S(\delta_y) = S(\delta_z)$	0.664	1.202	1.739	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.705	-0.694	0.317	$S(\delta_y) = S(\delta_z)$	0.703	1.204	1.704	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.383	-1.137	0.109	$S(\delta_y) = S(\delta_z)$	0.706	1.203	1.700	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 90%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.726	1.163	1.601	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 70%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.720	1.175	1.629	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 50%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.526	1.131	1.736	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	-	211	-	-1.054	-0.350	0.354	$S(\delta_y) = S(\delta_z)$	0.793	1.577	2.362	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.660	-0.767	0.126	$S(\delta_y) = S(\delta_z)$	0.678	1.642	2.605	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.245	-1.108	0.028	$S(\delta_y) = S(\delta_z)$	0.716	1.636	2.556	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	14	125	-	-1.408	-0.539	0.330	$S(\delta_y) = S(\delta_z)$	0.478	1.097	1.716	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.187	-1.009	0.169	$S(\delta_y) = S(\delta_z)$	0.262	0.864	1.465	$S(\delta_{\hat{e}_t}) \neq 0$
				M =114	ln(Debt/GDP)^3	-2.778	-1.383	0.011	$S(\delta_y) = S(\delta_z)$	0.228	0.856	1.483	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold	-2.624	-1.264	0.095	$S(\delta_y) = S(\delta_z)$	1.186	2.261	3.336	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models we take the per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. Gaps indicates the number of missing observations. In all tests conducted we allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of the subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP). ‡ Results for balance do not differ across different threshold values since $X_t\mathbb{1}(X_t < \text{threshold}) + X_t\mathbb{1}(X_t \geq \text{threshold}) = X_t$.

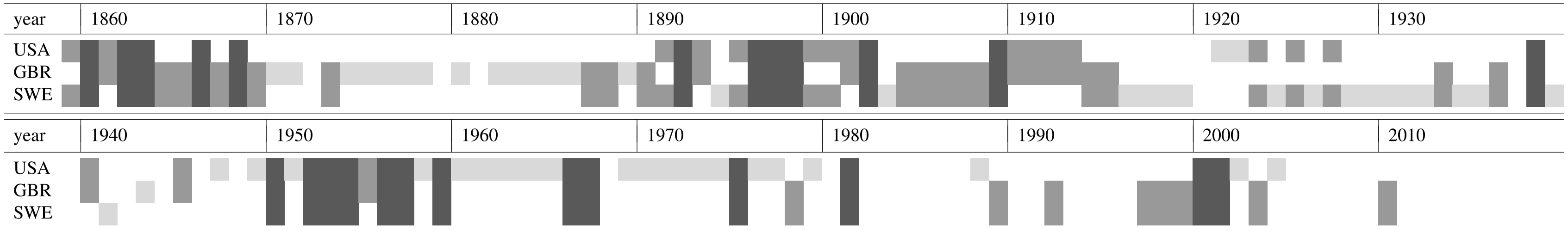
Table 3: Balance and Co-Summability — Δln(GDP pc) specifications

	Start	End	Gaps	Obs	Nonlinearity	Balance			
						CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict
USA	1801	2010	-	210	-	-2.199	-1.403	-0.606	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.642	-1.622	-0.603	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.237	-2.037	-0.836	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.024	-1.822	-0.620	$S(\delta_y) \neq S(\delta_z)$
GBR	1801	2010	-	210	-	-1.781	-0.944	-0.106	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.573	-1.458	-0.343	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.252	-1.903	-0.554	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.416	-2.007	-0.597	$S(\delta_y) \neq S(\delta_z)$
SWE	1801	2010	-	210	-	-2.201	-1.457	-0.713	$S(\delta_y) \neq S(\delta_z)$
				b =15	ln(Debt/GDP)^2	-2.806	-1.884	-0.963	$S(\delta_y) \neq S(\delta_z)$
				M =196	ln(Debt/GDP)^3	-3.301	-2.218	-1.135	$S(\delta_y) \neq S(\delta_z)$
JPN	1873	2010	14	122	-	-1.917	-1.192	-0.467	$S(\delta_y) \neq S(\delta_z)$
				b =12	ln(Debt/GDP)^2	-2.682	-1.659	-0.636	$S(\delta_y) \neq S(\delta_z)$
				M =111	ln(Debt/GDP)^3	-3.297	-2.043	-0.789	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.196	-1.960	-0.723	$S(\delta_y) \neq S(\delta_z)$

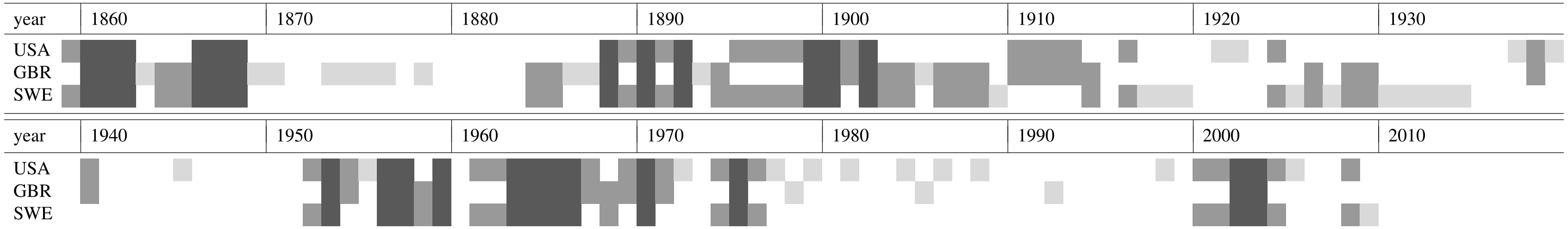
Notes: In all models we take the per capita GDP growth rate, Δln(GDP pc), as the dependent variable. See Table 2 for all other details. Since no model satisfies the balance test we do not carry out co-summability testing.

Table 4: Balance and Co-Summability — Cross-Country Comparison

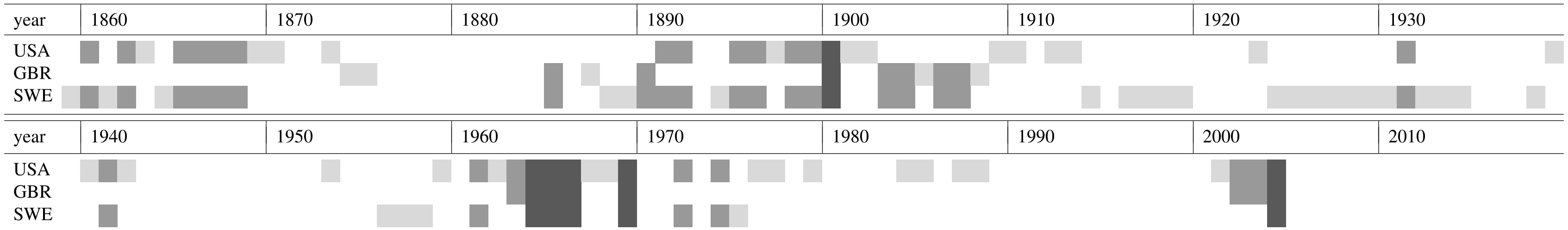
Panel A: Linear Specification (Model 1)



Panel B: Specification with squared debt (Model 2)



Panel C: Specification with squared and cubed debt (Model 3)



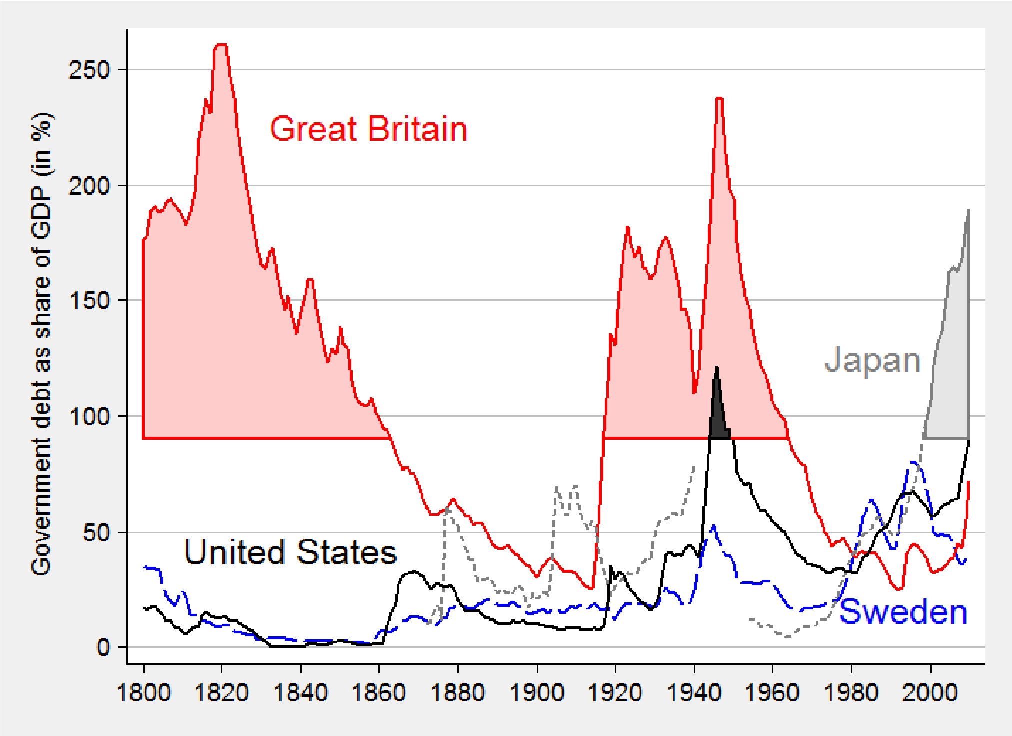
Notes: Increasing shading indicates the number (0-3) of countries for which the sixty-year subsample ending in the year indicated has a balanced and co-summable specification.

Table 5: Balance and Co-Summability — ln(GDP pc) specifications (sub-sample results for post-WWII period)

				Balance				Co-Summability			
	M	Start	End	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1	1946	2005	-1.954	-0.722	0.510	$S(\delta_y) = S(\delta_z)$	0.051	0.699	1.347	$S(\delta_{\hat{e}_t}) \neq 0$
		1947	2006	-2.360	-1.039	0.282	$S(\delta_y) = S(\delta_z)$	0.965	1.697	2.428	$S(\delta_{\hat{e}_t}) \neq 0$
		1948	2007	-2.207	-0.901	0.404	$S(\delta_y) = S(\delta_z)$	0.886	1.435	1.984	$S(\delta_{\hat{e}_t}) \neq 0$
		1949	2008	-2.188	-0.905	0.377	$S(\delta_y) = S(\delta_z)$	0.698	0.987	1.277	$S(\delta_{\hat{e}_t}) \neq 0$
		1950	2009	-2.066	-0.811	0.445	$S(\delta_y) = S(\delta_z)$	0.704	0.964	1.225	$S(\delta_{\hat{e}_t}) \neq 0$
		1951	2010	-1.757	-0.481	0.796	$S(\delta_y) = S(\delta_z)$	0.836	1.263	1.690	$S(\delta_{\hat{e}_t}) \neq 0$
	2	1946	2005	-2.975	-1.419	0.138	$S(\delta_y) = S(\delta_z)$	-0.250	0.347	0.944	$S(\delta_{\hat{e}_t}) = 0$
		1947	2006	-3.376	-1.733	-0.090	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-3.222	-1.597	0.027	$S(\delta_y) = S(\delta_z)$	0.504	1.573	2.641	$S(\delta_{\hat{e}_t}) \neq 0$
		1949	2008	-3.196	-1.591	0.014	$S(\delta_y) = S(\delta_z)$	-0.741	0.903	2.548	$S(\delta_{\hat{e}_t}) = 0$
		1950	2009	-2.931	-1.497	-0.063	$S(\delta_y) \neq S(\delta_z)$				
		1951	2010	-2.737	-1.110	0.517	$S(\delta_y) = S(\delta_z)$	0.777	1.169	1.561	$S(\delta_{\hat{e}_t}) \neq 0$
	3	1946	2005	-3.838	-2.011	-0.183	$S(\delta_y) \neq S(\delta_z)$				
		1947	2006	-4.226	-2.308	-0.390	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-4.075	-2.184	-0.294	$S(\delta_y) \neq S(\delta_z)$				
		1949	2008	-4.044	-2.170	-0.296	$S(\delta_y) \neq S(\delta_z)$				
		1950	2009	-3.776	-2.075	-0.374	$S(\delta_y) \neq S(\delta_z)$				
		1951	2010	-3.555	-1.629	0.297	$S(\delta_y) = S(\delta_z)$	0.790	1.217	1.644	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1	1946	2005	-1.421	-0.312	0.797	$S(\delta_y) = S(\delta_z)$	0.080	0.936	1.792	$S(\delta_{\hat{e}_t}) \neq 0$
		1947	2006	-1.725	-0.939	-0.153	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-1.645	-0.786	0.072	$S(\delta_y) = S(\delta_z)$	0.072	1.024	1.977	$S(\delta_{\hat{e}_t}) \neq 0$
		1949	2008	-1.631	-0.712	0.208	$S(\delta_y) = S(\delta_z)$	0.360	1.051	1.742	$S(\delta_{\hat{e}_t}) \neq 0$
		1950	2009	-1.798	-0.823	0.151	$S(\delta_y) = S(\delta_z)$	0.220	0.738	1.256	$S(\delta_{\hat{e}_t}) \neq 0$
		1951	2010	-1.685	-0.671	0.343	$S(\delta_y) = S(\delta_z)$	-0.145	0.551	1.247	$S(\delta_{\hat{e}_t}) = 0$
	2	1946	2005	-2.616	-1.132	0.352	$S(\delta_y) = S(\delta_z)$	0.158	0.780	1.402	$S(\delta_{\hat{e}_t}) \neq 0$
		1947	2006	-2.836	-1.735	-0.633	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-2.761	-1.600	-0.438	$S(\delta_y) \neq S(\delta_z)$				
		1949	2008	-2.815	-1.529	-0.243	$S(\delta_y) \neq S(\delta_z)$				
		1950	2009	-2.887	-1.617	-0.347	$S(\delta_y) \neq S(\delta_z)$				
		1951	2010	-2.790	-1.503	-0.216	$S(\delta_y) \neq S(\delta_z)$				
	3	1946	2005	-3.646	-1.805	0.036	$S(\delta_y) = S(\delta_z)$	0.057	0.648	1.238	$S(\delta_{\hat{e}_t}) \neq 0$
		1947	2006	-3.786	-2.405	-1.023	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-3.712	-2.275	-0.837	$S(\delta_y) \neq S(\delta_z)$				
		1949	2008	-3.808	-2.208	-0.607	$S(\delta_y) \neq S(\delta_z)$				
		1950	2009	-3.838	-2.277	-0.716	$S(\delta_y) \neq S(\delta_z)$				
		1951	2010	-3.766	-2.181	-0.595	$S(\delta_y) \neq S(\delta_z)$				
SWE	1	1946	2005	-1.555	-0.677	0.202	$S(\delta_y) = S(\delta_z)$	0.558	0.974	1.391	$S(\delta_{\hat{e}_t}) \neq 0$
		1947	2006	-1.612	-0.699	0.214	$S(\delta_y) = S(\delta_z)$	0.625	1.223	1.821	$S(\delta_{\hat{e}_t}) \neq 0$
		1948	2007	-1.552	-0.615	0.322	$S(\delta_y) = S(\delta_z)$	0.575	1.265	1.955	$S(\delta_{\hat{e}_t}) \neq 0$
		1949	2008	-1.629	-0.647	0.335	$S(\delta_y) = S(\delta_z)$	0.311	0.984	1.657	$S(\delta_{\hat{e}_t}) \neq 0$
		1950	2009	-1.550	-0.542	0.465	$S(\delta_y) = S(\delta_z)$	0.075	0.899	1.722	$S(\delta_{\hat{e}_t}) \neq 0$
		1951	2010	-1.231	-0.475	0.280	$S(\delta_y) = S(\delta_z)$	-0.174	1.127	2.428	$S(\delta_{\hat{e}_t}) = 0$
	2	1946	2005	-2.502	-1.311	-0.119	$S(\delta_y) \neq S(\delta_z)$				
		1947	2006	-2.552	-1.337	-0.122	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-2.486	-1.251	-0.017	$S(\delta_y) \neq S(\delta_z)$				
		1949	2008	-2.556	-1.276	0.004	$S(\delta_y) = S(\delta_z)$	-0.074	0.933	1.941	$S(\delta_{\hat{e}_t}) = 0$
		1950	2009	-2.462	-1.152	0.158	$S(\delta_y) = S(\delta_z)$	-0.260	0.881	2.023	$S(\delta_{\hat{e}_t}) = 0$
		1951	2010	-2.131	-1.077	-0.022	$S(\delta_y) \neq S(\delta_z)$				
	3	1946	2005	-3.286	-1.833	-0.381	$S(\delta_y) \neq S(\delta_z)$				
		1947	2006	-3.327	-1.863	-0.398	$S(\delta_y) \neq S(\delta_z)$				
		1948	2007	-3.256	-1.775	-0.295	$S(\delta_y) \neq S(\delta_z)$				
		1949	2008	-3.320	-1.796	-0.272	$S(\delta_y) \neq S(\delta_z)$				
		1950	2009	-3.235	-1.703	-0.171	$S(\delta_y) \neq S(\delta_z)$				
		1951	2010	-2.867	-1.570	-0.274	$S(\delta_y) \neq S(\delta_z)$				

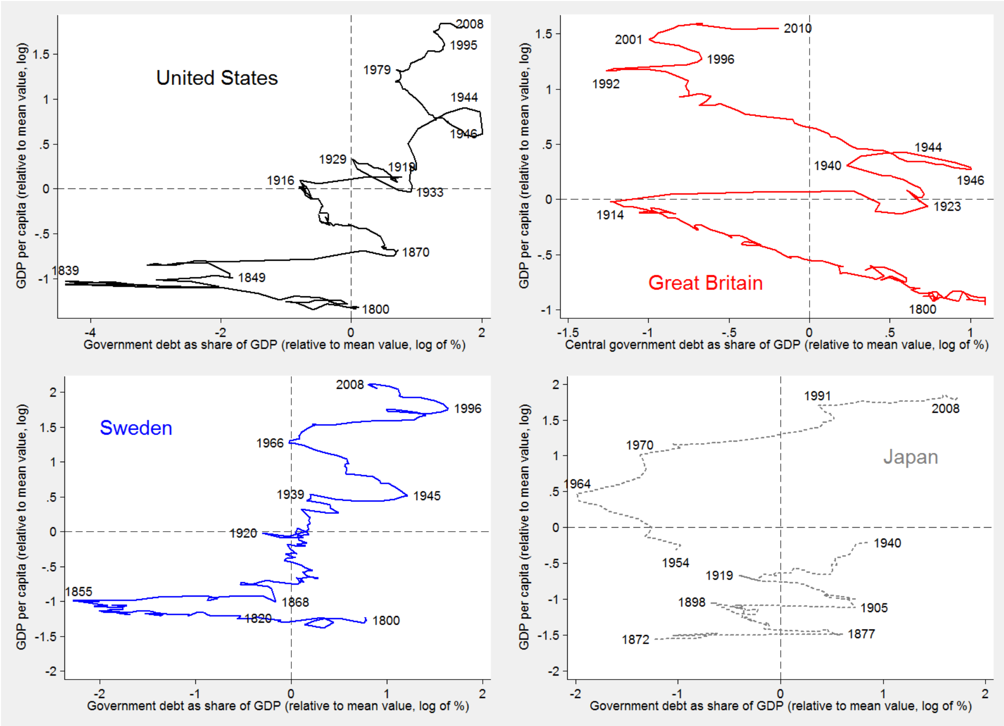
Notes: We present the results for sub-sample balance and co-summability testing for the United States, Britain and Sweden. Model (M) 1-3 refers to the specification: 1 – linear, 2 – linear and squared, 3 – linear, squared and cubed debt/GDP terms (in logs). In each case we report statistics from the sixty-year samples, with start and end years as indicated in the table, focusing on the period after 1945. See Table 2 for all other details.

Figure 1: Evolution of Debt/GDP ratios



Notes: The shaded areas represent the periods where debt/GDP exceeded 90%.

Figure 2: Debt Ratio and Income Per Capita



Notes: Debt ratios and per capita GDP series (both in logarithms) are presented in deviation from their country-specific time-series means (within transformation).

Figure 3: Balance Testing (Sub-Samples)

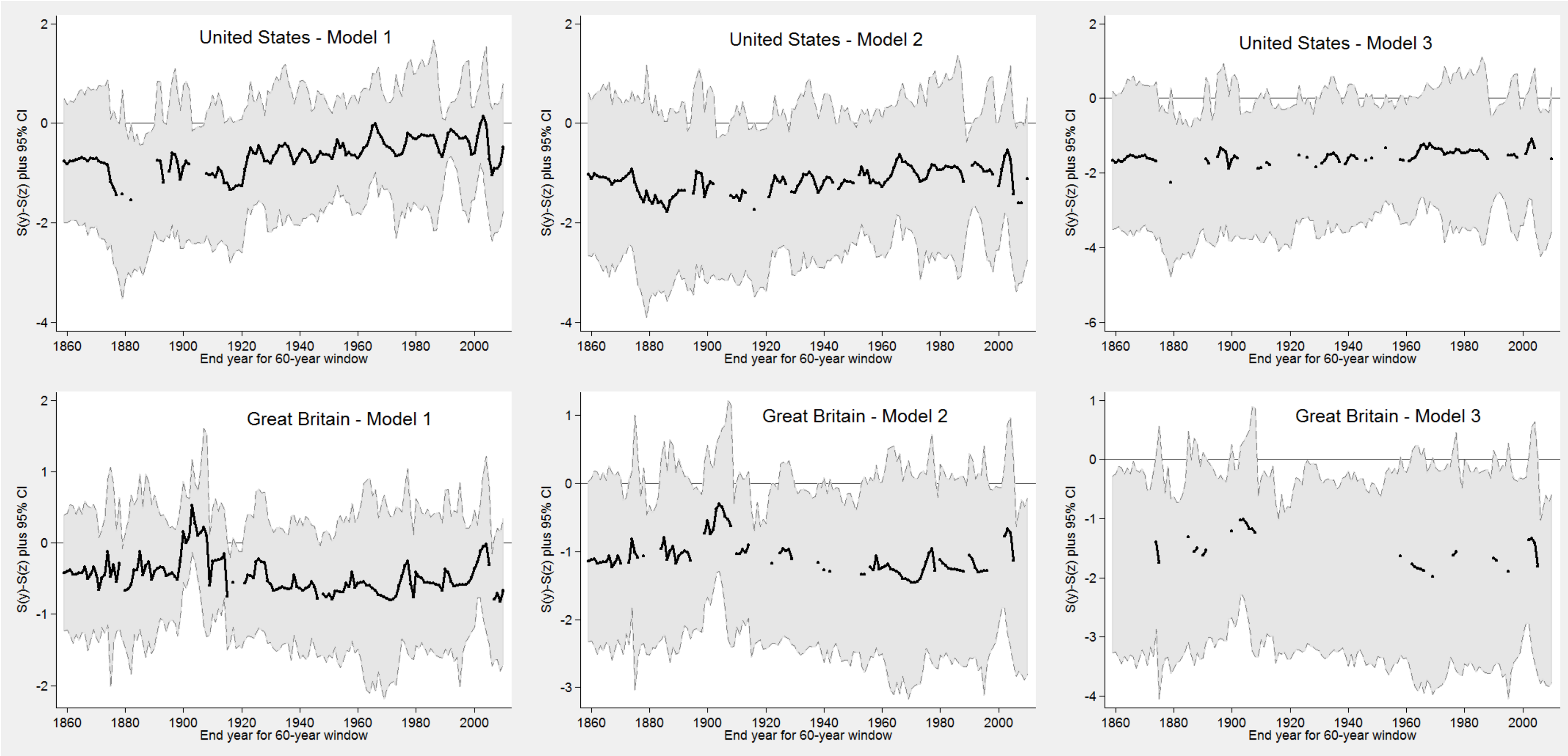


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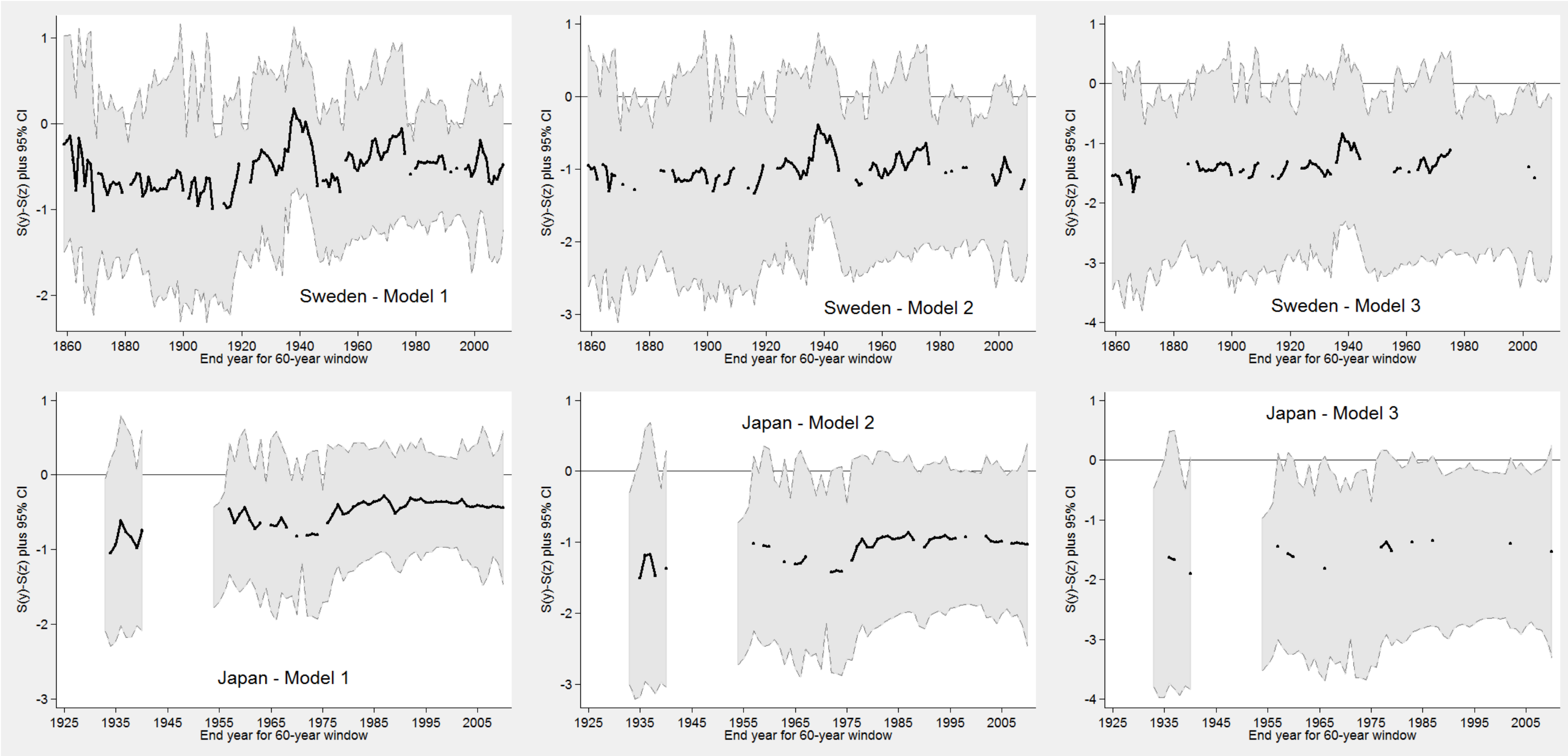
Figure 3: Balance Testing (Sub-Samples) continued



Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Balance statistic computed in a moving window of 60-year time periods; the solid line represents the balance estimate for consecutive windows: we only plot this when balance cannot be rejected. The coverage of the data differs across countries: for the US, Great Britain and Sweden we have data from 1800-2010 (152 subsamples), for Japan from 1872-2010 (with gaps; 66 subsamples). Model 1 refers to a specification with linear debt terms only, Model 2 to a specification with linear and quadratic debt terms, Model 3 further includes a cubed debt term. The graphs capture both subsequent end years in which subsamples were balanced as well as ‘isolated’ years.

Figure 4: Co-Summability Testing (Sub-Samples)

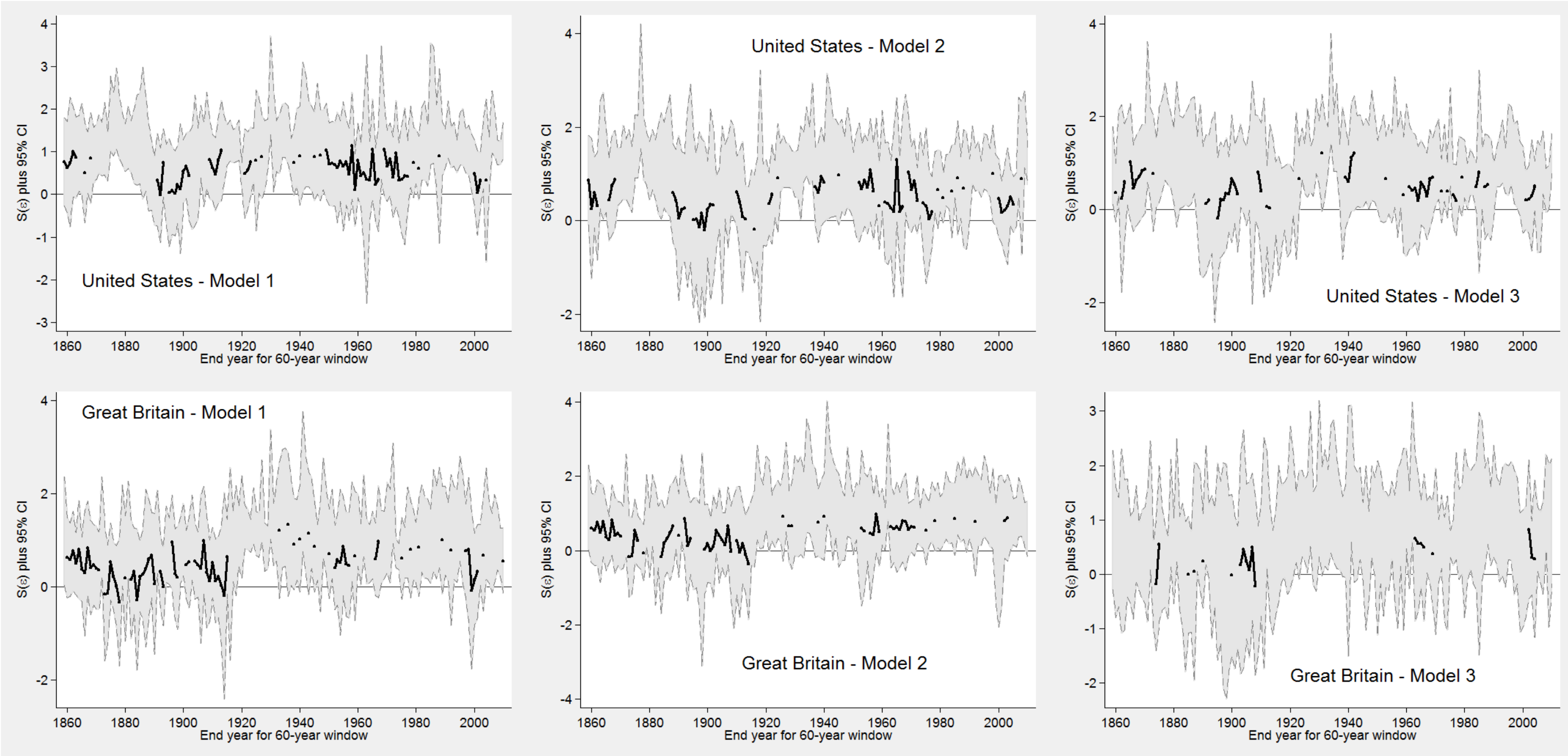


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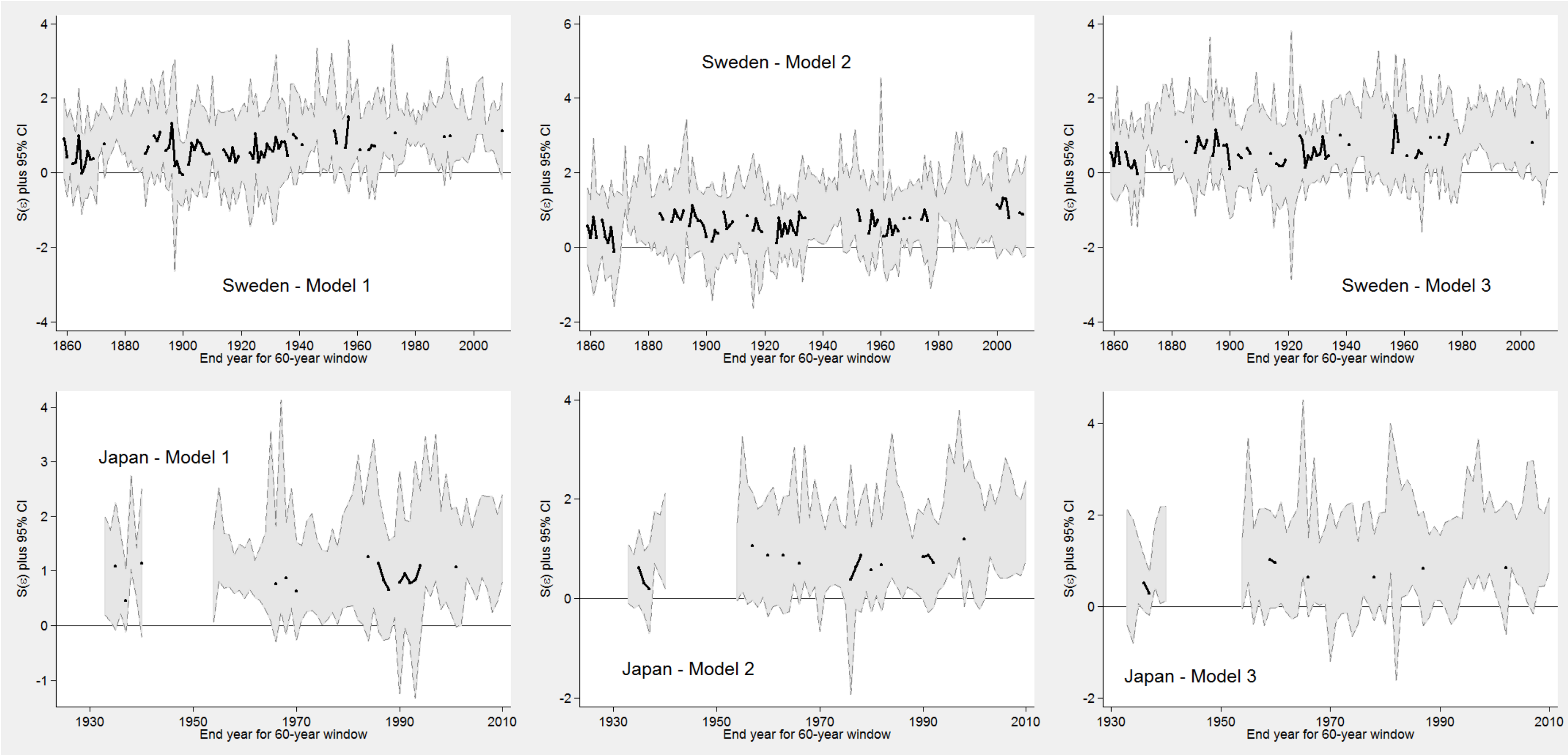
Figure 4: Co-Summability Testing (Sub-Samples) continued



Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Co-Summability statistic computed in a moving window of 60-year time periods. The solid black line represents the computed Co-Summability statistic — this line is only shown if the prerequisite balance test could not reject this feature for the specific subsample. We allow for an intercept in the co-summability analysis. For further details see Figure 3. The graphs capture both subsequent end years in which subsamples were balanced (line) as well as ‘isolated’ years (dots).

Data Appendix

The raw data for the analysis carried out is taken from the Excel spreadsheets available on `www.reinhartandrogoff.com` – the Reinhart and Rogoff (2009) companion website – and on `www.ggdc.net/maddison` – the Maddison Project website at the University of Groningen. In the following we describe a small number of changes we made to these data series.

The 'New' Maddison data provides two values for Great Britain's per capita GDP in 1851 – 2,330 and 2,718 – since this is where two data series come together: up to 1851 the estimates are taken from Van Zanden (2001), from 1851 onwards from the original Maddison (2010) estimates. We simple pick the arithmetic mean of the two values.

We interpolated the debt-GDP ratio in two cases for the additional results presented in a Technical Appendix where only a single observation was missing or (in case of India) where the recorded value was not credible (zero debt): Argentina 1866, India 1947.

Some more comments on various debt series: for Brazil we chose debt values starting from 1889 since prior to this date the series covered only external debts. For Great Britain the series are net rather than gross central (external and domestic) government debt. For Argentina, Italy, the Netherlands and New Zealand the debt series represent general rather than central government debt.

Table A-1: Descriptive Statistics

		Start	End	Gaps	Obs	Mean	Median	St.Dev.	Min	Max
USA	ln(GDP pc)	1800	2010	-	211	8.510	8.425	0.996	7.159	10.363
	Δln(GDP pc)				210	0.015	0.017	0.047	-0.241	0.171
	ln(Debt/GDP)				211	2.667	2.958	1.629	-5.878	4.798
	$\ln(Debt/GDP)^2$				211	9.754	9.090	6.397	0.010	34.552
	$\ln(Debt/GDP)^3$				211	31.026	25.870	36.127	-203.104	110.445
GBR	ln(GDP pc)	1800	2010	-	211	8.527	8.414	0.727	7.574	10.127
	Δln(GDP pc)				210	0.012	0.016	0.031	-0.114	0.091
	ln(Debt/GDP)				211	4.468	4.598	0.685	3.201	5.563
	$\ln(Debt/GDP)^2$				211	20.432	21.139	6.057	10.247	30.946
	$\ln(Debt/GDP)^3$				211	95.416	97.193	40.799	32.802	172.153
SWE	ln(GDP pc)	1800	2010	-	211	8.025	7.722	1.115	6.641	10.142
	Δln(GDP pc)				210	0.016	0.020	0.034	-0.094	0.120
	ln(Debt/GDP)				211	2.748	2.881	0.915	0.485	4.387
	$\ln(Debt/GDP)^2$				211	8.385	8.297	4.605	0.235	19.246
	$\ln(Debt/GDP)^3$				211	27.133	23.901	20.076	0.114	84.431
JPN	ln(GDP pc)	1872	2010	14	125	8.171	7.660	1.201	6.615	10.017
	Δln(GDP pc)				122	0.029	0.023	0.047	-0.091	0.162
	ln(Debt/GDP)				125	3.511	3.628	0.854	1.519	5.242
	$\ln(Debt/GDP)^2$				125	13.051	13.164	5.864	2.307	27.484
	$\ln(Debt/GDP)^3$				125	50.727	47.761	32.500	3.504	144.083

Notes: We provide the descriptive statistics for the levels variables included in our analysis (all in logarithms as indicated). Gap reports the number of missing observations.

Table A-2: Country Coverage (Extended Analysis)

Country	Argentina	Australia	Austria	Belgium	Brazil	Canada
Isocode	ARG	AUS	AUT	BEL	BRA	CAN
Start	1875	1861	1880	1846	1889	1870
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	20	12	0	0
Obs	136	150	111	153	122	141

Country	Chile	Colombia	Denmark	France	Germany	Greece
Isocode	CHL	COL	DNK	FRA	DEU	GRC
Start	1870	1899	1880	1880	1880	1848
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	23	37	15
Obs	141	112	131	108	94	163

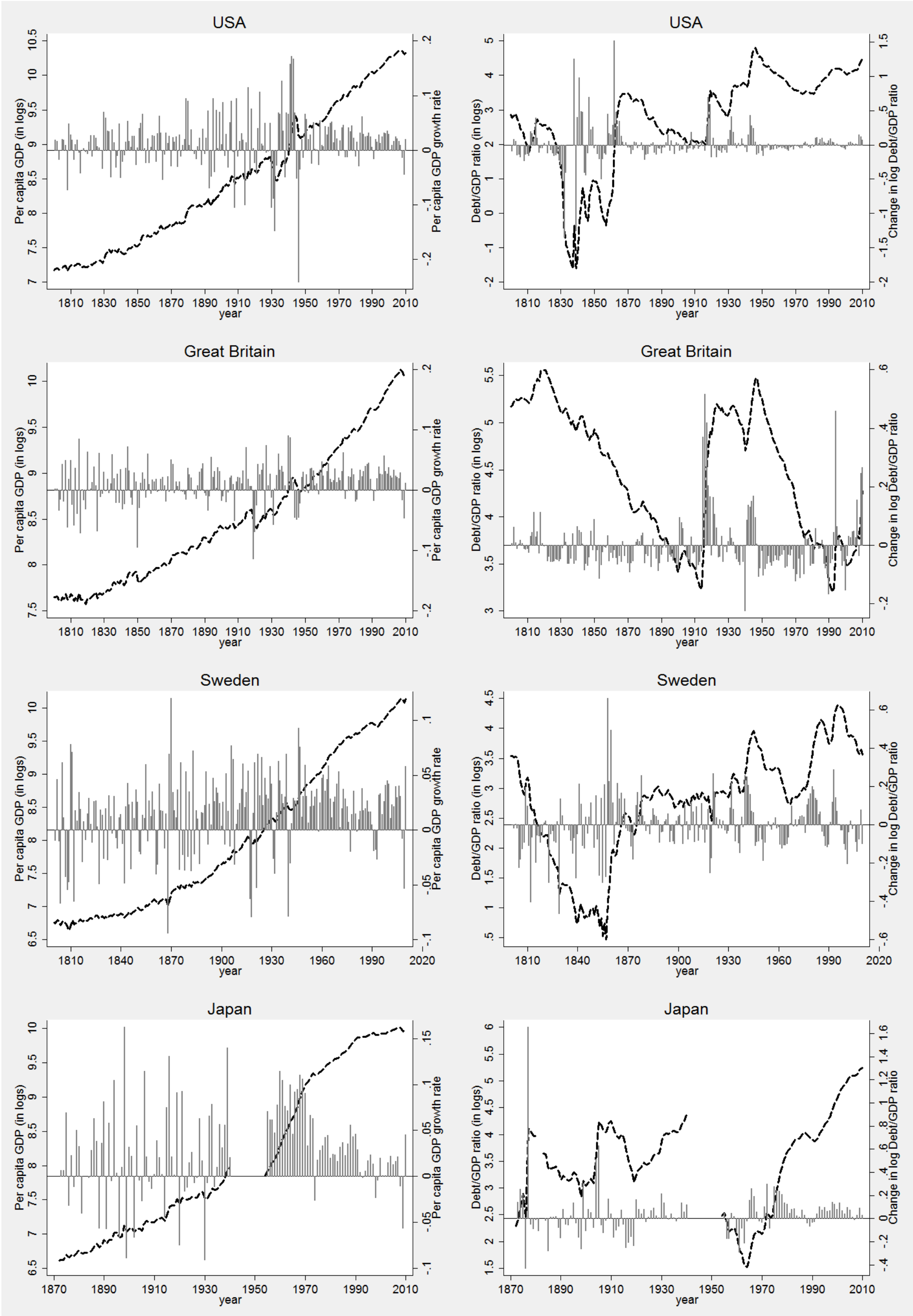
Country	India	Italy	Japan	Netherlands	New Zealand	Norway
Isocode	IND	ITA	JPN	NLD	NZL	NOR
Start	1884	1861	1872	1815	1870	1880
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	14	6	0	6
Obs	127	150	125	190	151	131

Country	Peru	Portugal	Spain	Sri Lanka	Sweden	Switzerland
Isocode	PER	PRT	ESP	LKA	SWE	CHE
Start	1883	1865	1850	1870	1800	1880
End	2010	2010	2010	2009	2010	2010
Gaps	14	0	4	35	0	16
Obs	114	146	157	105	211	115

Country	Great Britain	USA	Uruguay
Isocode	GBR	USA	URY
Start	1800	1800	1871
End	2010	2010	2009
Gaps	0	0	23
Obs	211	211	116

Notes: We present start and end years of per capita GDP and debt-ratio time series for the set of countries for which we report the summability, balance and co-summability results in a Technical Appendix.

Figure A-1: Income and debt evolution



Notes: These plots chart the evolution of per capita GDP (in logs, left column) and the debt/GDP ratio (in logs, right column) for our four OECD countries. In each plot the levels variable (left axis, dashed line) is graphed alongside the variable in first differences (right axis, gray bars).

Technical Appendix — not intended for publication

Table TA-1: Estimated Order of Summability — 27 countries

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
ARG	1875	2010	-	136	ln(GDP pc)	0.279	0.851	1.423
					Δln(GDP pc)	-0.855	-0.249	0.357
					ln(Debt/GDP)	0.117	0.661	1.205
					ln(Debt/GDP) squared	0.075	0.691	1.308
					ln(Debt/GDP) cubed	0.117	0.727	1.337
AUS	1861	2010	-	150	ln(GDP pc)	0.262	0.847	1.432
					Δln(GDP pc)	-0.222	0.464	1.151
					ln(Debt/GDP)	0.402	0.981	1.560
					ln(Debt/GDP) squared	0.300	0.978	1.656
					ln(Debt/GDP) cubed	0.308	1.011	1.714
AUT	1880	2010	20	111	ln(GDP pc)	0.056	0.775	1.495
					Δln(GDP pc)	-0.552	0.104	0.760
					ln(Debt/GDP)	0.546	1.225	1.904
					ln(Debt/GDP) squared	0.473	1.190	1.907
					ln(Debt/GDP) cubed	0.476	1.158	1.839
BEL	1846	2010	12	153	ln(GDP pc)	0.347	0.730	1.113
					Δln(GDP pc)	-0.484	0.163	0.810
					ln(Debt/GDP)	0.147	0.680	1.213
					ln(Debt/GDP) squared	0.131	0.675	1.220
					ln(Debt/GDP) cubed	0.106	0.673	1.240
BRA	1889	2010	-	122	ln(GDP pc)	0.650	1.157	1.664
					Δln(GDP pc)	0.094	1.064	2.035
					ln(Debt/GDP)	-0.385	0.376	1.137
					ln(Debt/GDP) squared	-0.510	0.321	1.152
					ln(Debt/GDP) cubed	-0.596	0.244	1.084
CAN	1870	2010	-	141	ln(GDP pc)	0.101	0.552	1.003
					Δln(GDP pc)	-1.235	-0.354	0.527
					ln(Debt/GDP)	-0.005	0.627	1.258
					ln(Debt/GDP) squared	0.024	0.657	1.291
					ln(Debt/GDP) cubed	0.136	0.690	1.244
CHL	1870	2010	-	141	ln(GDP pc)	0.145	0.717	1.289
					Δln(GDP pc)	-0.584	0.022	0.628
					ln(Debt/GDP)	0.135	0.818	1.500
					ln(Debt/GDP) squared	0.176	0.828	1.480
					ln(Debt/GDP) cubed	0.216	0.859	1.501
COL	1899	2010	-	112	ln(GDP pc)	0.817	1.537	2.257
					Δln(GDP pc)	0.374	1.157	1.940
					ln(Debt/GDP)	0.673	1.312	1.950
					ln(Debt/GDP) squared	0.745	1.243	1.740
					ln(Debt/GDP) cubed	0.766	1.179	1.593
DNK	1880	2010	-	131	ln(GDP pc)	0.141	0.677	1.213
					Δln(GDP pc)	-0.573	0.155	0.882
					ln(Debt/GDP)	0.389	1.182	1.976
					ln(Debt/GDP) squared	0.484	1.228	1.972
					ln(Debt/GDP) cubed	0.514	1.240	1.967

Table continued on the following page

Table TA-1: Estimated Order of Summability — 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
FRA	1880	2010	23	108	ln(GDP pc)	0.463	1.307	2.150
					Δln(GDP pc)	-0.490	0.226	0.942
					ln(Debt/GDP)	0.773	1.564	2.356
					ln(Debt/GDP) squared	1.153	2.336	3.519
					ln(Debt/GDP) cubed	1.557	3.156	4.755
DEU	1880	2010	37	94	ln(GDP pc)	0.549	1.239	1.929
					Δln(GDP pc)	-0.567	0.121	0.808
					ln(Debt/GDP)	0.514	0.940	1.367
					ln(Debt/GDP) squared	0.509	0.935	1.360
					ln(Debt/GDP) cubed	0.531	0.937	1.343
GRC	1848	2010	15	148	ln(GDP pc)	0.431	1.169	1.907
					Δln(GDP pc)	-0.534	0.138	0.809
					ln(Debt/GDP)	0.324	0.856	1.388
					ln(Debt/GDP) squared	0.406	0.960	1.515
					ln(Debt/GDP) cubed	0.367	0.990	1.613
IND	1884	2010	-	127	ln(GDP pc)	0.031	0.507	0.982
					Δln(GDP pc)	-0.925	-0.130	0.666
					ln(Debt/GDP)	0.279	0.991	1.702
					ln(Debt/GDP) squared	0.393	1.045	1.697
					ln(Debt/GDP) cubed	0.325	1.098	1.872
ITA	1861	2010	-	150	ln(GDP pc)	0.495	1.162	1.829
					Δln(GDP pc)	-0.210	0.364	0.939
					ln(Debt/GDP)	0.380	0.921	1.462
					ln(Debt/GDP) squared	0.384	0.937	1.491
					ln(Debt/GDP) cubed	0.390	0.953	1.517
JPN	1872	2010	14	125	ln(GDP pc)	0.953	2.503	4.054
					Δln(GDP pc)	-0.564	0.025	0.614
					ln(Debt/GDP)	0.420	1.099	1.778
					ln(Debt/GDP) squared	0.371	1.108	1.845
					ln(Debt/GDP) cubed	0.406	1.115	1.823
NLD	1815	2010	6	190	ln(GDP pc)	0.055	0.569	1.083
					Δln(GDP pc)	-0.353	0.304	0.961
					ln(Debt/GDP)	0.462	1.084	1.705
					ln(Debt/GDP) squared	0.477	1.089	1.702
					ln(Debt/GDP) cubed	0.528	1.097	1.666
NZL	1870	2010	-	141	ln(GDP pc)	0.098	0.503	0.909
					Δln(GDP pc)	-0.323	0.299	0.920
					ln(Debt/GDP)	0.479	0.960	1.441
					ln(Debt/GDP) squared	0.419	0.986	1.553
					ln(Debt/GDP) cubed	0.486	1.009	1.533
NOR	1880	2010	6	125	ln(GDP pc)	0.656	1.349	2.042
					Δln(GDP pc)	-0.179	0.579	1.337
					ln(Debt/GDP)	0.398	1.073	1.749
					ln(Debt/GDP) squared	0.394	1.086	1.778
					ln(Debt/GDP) cubed	0.394	1.101	1.808
PER	1883	2010	14	114	ln(GDP pc)	0.284	0.820	1.357
					Δln(GDP pc)	-0.073	0.665	1.404
					ln(Debt/GDP)	0.677	1.122	1.566
					ln(Debt/GDP) squared	0.683	1.063	1.444
					ln(Debt/GDP) cubed	0.676	1.009	1.342

Table continued on the following page

Table TA-1: Estimated Order of Summability — 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
PRT	1865	2010	-	146	ln(GDP pc)	0.464	1.087	1.709
					Δln(GDP pc)	0.010	0.802	1.594
					ln(Debt/GDP)	0.397	0.933	1.470
					ln(Debt/GDP) squared	0.347	0.940	1.533
					ln(Debt/GDP) cubed	0.381	0.945	1.510
ESP	1850	2010	4	157	ln(GDP pc)	0.212	0.767	1.322
					Δln(GDP pc)	-0.499	0.067	0.633
					ln(Debt/GDP)	0.394	0.994	1.595
					ln(Debt/GDP) squared	0.350	0.979	1.609
					ln(Debt/GDP) cubed	0.380	0.966	1.551
LKA	1870	2009	35	105	ln(GDP pc)	0.411	0.816	1.220
					Δln(GDP pc)	-0.319	0.379	1.078
					ln(Debt/GDP)	0.210	0.771	1.332
					ln(Debt/GDP) squared	0.240	0.797	1.354
					ln(Debt/GDP) cubed	0.224	0.822	1.420
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					Δln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
CHE	1880	2010	16	115	ln(GDP pc)	0.159	0.669	1.179
					Δln(GDP pc)	-0.690	-0.097	0.497
					ln(Debt/GDP)	0.506	1.265	2.023
					ln(Debt/GDP) squared	0.508	1.254	2.001
					ln(Debt/GDP) cubed	0.477	1.255	2.033
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					Δln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
USA	1800	2010	-	211	ln(GDP pc)	0.686	1.561	2.436
					Δln(GDP pc)	-0.522	0.052	0.627
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
URY	1871	2009	23	116	ln(GDP pc)	-0.128	0.539	1.206
					Δln(GDP pc)	-0.195	0.545	1.285
					ln(Debt/GDP)	0.419	1.061	1.704
					ln(Debt/GDP) squared	0.401	1.089	1.777
					ln(Debt/GDP) cubed	0.440	1.127	1.815

Notes: We report full sample order of summability estimates, CI low and up indicate the 95% confidence interval for the summability estimate $S(\hat{\delta})$ – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted we allow for deterministic terms (constant and trend). Country codes are detailed in the Data Appendix.

Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
ARG	1875	2010	-	136	-	-0.766	-0.109	0.549	$\hat{\delta}_y = \hat{\delta}_z$	1.483	2.654	3.826	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.506	-0.624	0.258	$\hat{\delta}_y = \hat{\delta}_z$	0.947	1.605	2.264	$S(\delta_{\hat{e}_t}) \neq 0$
				M =124	ln(Debt/GDP)^3	-2.151	-1.055	0.042	$\hat{\delta}_y = \hat{\delta}_z$	0.859	1.536	2.212	$S(\delta_{\hat{e}_t}) \neq 0$
AUS	1861	2010	-	150	-	-1.081	-0.421	0.239	$\hat{\delta}_y = \hat{\delta}_z$	0.894	1.378	1.861	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.504	-0.801	-0.098	$\hat{\delta}_y \neq \hat{\delta}_z$				
				138	ln(Debt/GDP)^3	-2.074	-1.145	-0.217	$\hat{\delta}_y \neq \hat{\delta}_z$				
AUT	1880	2010	20	111	-	-1.247	-0.398	0.452	$\hat{\delta}_y = \hat{\delta}_z$	0.451	1.038	1.626	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.083	-0.936	0.210	$\hat{\delta}_y = \hat{\delta}_z$	0.153	0.772	1.392	$S(\delta_{\hat{e}_t}) \neq 0$
				M =100	ln(Debt/GDP)^3	-2.818	-1.388	0.043	$\hat{\delta}_y = \hat{\delta}_z$	0.074	0.588	1.102	$S(\delta_{\hat{e}_t}) \neq 0$
BEL	1846	2010	12	153	-	-1.066	-0.397	0.273	$\hat{\delta}_y = \hat{\delta}_z$	0.103	0.531	0.960	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.693	-0.876	-0.060	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =141	ln(Debt/GDP)^3	-2.352	-1.273	-0.194	$\hat{\delta}_y \neq \hat{\delta}_z$				
BRA	1889	2010	-	122	-	-0.706	0.020	0.747	$\hat{\delta}_y = \hat{\delta}_z$	1.106	2.038	2.970	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.440	-0.486	0.469	$\hat{\delta}_y = \hat{\delta}_z$	0.658	1.274	1.891	$S(\delta_{\hat{e}_t}) \neq 0$
				M =111	ln(Debt/GDP)^3	-2.111	-0.854	0.403	$\hat{\delta}_y = \hat{\delta}_z$	0.724	1.144	1.563	$S(\delta_{\hat{e}_t}) \neq 0$

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Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
CAN	1870	2010	-	141	-	-0.774	-0.246	0.282	$\hat{\delta}_y = \hat{\delta}_z$	0.198	0.770	1.342	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.541	-0.763	0.016	$\hat{\delta}_y = \hat{\delta}_z$	0.172	0.745	1.317	$S(\delta_{\hat{e}_t}) \neq 0$
				M =129	ln(Debt/GDP)^3	-2.203	-1.197	-0.190	$\hat{\delta}_y \neq \hat{\delta}_z$				
CHL	1870	2010	-	141	-	-1.193	-0.095	1.004	$\hat{\delta}_y = \hat{\delta}_z$	0.626	1.120	1.614	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.919	-0.570	0.779	$\hat{\delta}_y = \hat{\delta}_z$	0.484	1.115	1.746	$S(\delta_{\hat{e}_t}) \neq 0$
				M =129	ln(Debt/GDP)^3	-2.529	-0.977	0.575	$\hat{\delta}_y = \hat{\delta}_z$	0.550	1.149	1.748	$S(\delta_{\hat{e}_t}) \neq 0$
COL	1899	2010	-	112	-	-1.513	-0.814	-0.116	$\hat{\delta}_y \neq \hat{\delta}_z$				
				b =12	ln(Debt/GDP)^2	-2.262	-1.329	-0.396	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =101	ln(Debt/GDP)^3	-2.896	-1.757	-0.617	$\hat{\delta}_y \neq \hat{\delta}_z$				
DNK	1880	2010	-	131	-	-1.076	-0.372	0.332	$\hat{\delta}_y = \hat{\delta}_z$	1.041	1.793	2.545	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.841	-0.899	0.043	$\hat{\delta}_y = \hat{\delta}_z$	0.600	1.648	2.696	$S(\delta_{\hat{e}_t}) \neq 0$
				M =120	ln(Debt/GDP)^3	-2.443	-1.309	-0.175	$\hat{\delta}_y \neq \hat{\delta}_z$				
FRA	1880	2010	23	108	-	-2.224	-0.770	0.684	$\hat{\delta}_y = \hat{\delta}_z$	0.818	1.175	1.531	$S(\delta_{\hat{e}_t}) \neq 0$
				b =11	ln(Debt/GDP)^2	-4.627	-2.110	0.406	$\hat{\delta}_y = \hat{\delta}_z$	0.442	1.081	1.719	$S(\delta_{\hat{e}_t}) \neq 0$
				M =98	ln(Debt/GDP)^3	-7.041	-3.419	0.203	$\hat{\delta}_y = \hat{\delta}_z$	0.393	1.117	1.840	$S(\delta_{\hat{e}_t}) \neq 0$

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Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
DEU	1880	2010	37	94	-	-1.173	-0.304	0.566	$\hat{\delta}_y = \hat{\delta}_z$	0.927	1.375	1.824	$S(\delta_{\hat{e}_t}) \neq 0$
				11	ln(Debt/GDP)^2	-2.015	-0.849	0.316	$\hat{\delta}_y = \hat{\delta}_z$	0.547	1.136	1.726	$S(\delta_{\hat{e}_t}) \neq 0$
				84	ln(Debt/GDP)^3	-2.738	-1.319	0.100	$\hat{\delta}_y = \hat{\delta}_z$	0.395	0.795	1.195	$S(\delta_{\hat{e}_t}) \neq 0$
GRC	1848	2010	15	148	-	-0.766	-0.084	0.598	$\hat{\delta}_y = \hat{\delta}_z$	0.270	0.930	1.589	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.680	-0.786	0.108	$\hat{\delta}_y = \hat{\delta}_z$	0.391	1.065	1.739	$S(\delta_{\hat{e}_t}) \neq 0$
				M =136	ln(Debt/GDP)^3	-2.463	-1.343	-0.223	$\hat{\delta}_y \neq \hat{\delta}_z$				
IND	1884	2010	-	127	-	-0.899	-0.254	0.392	$\hat{\delta}_y = \hat{\delta}_z$	0.163	0.709	1.256	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.605	-0.725	0.155	$\hat{\delta}_y = \hat{\delta}_z$	0.212	0.710	1.209	$S(\delta_{\hat{e}_t}) \neq 0$
				M =116	ln(Debt/GDP)^3	-2.177	-1.120	-0.064	$\hat{\delta}_y \neq \hat{\delta}_z$				
ITA	1861	2010	-	150	-	-1.121	-0.435	0.251	$\hat{\delta}_y = \hat{\delta}_z$	0.713	1.203	1.693	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.778	-0.956	-0.134	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =138	ln(Debt/GDP)^3	-2.446	-1.393	-0.341	$\hat{\delta}_y \neq \hat{\delta}_z$				
JPN	1872	2010	14	125	-	-1.408	-0.539	0.330	$\hat{\delta}_y = \hat{\delta}_z$	0.478	1.097	1.716	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.187	-1.009	0.169	$\hat{\delta}_y = \hat{\delta}_z$	0.262	0.864	1.465	$S(\delta_{\hat{e}_t}) \neq 0$
				M =114	ln(Debt/GDP)^3	-2.778	-1.383	0.011	$\hat{\delta}_y = \hat{\delta}_z$	0.228	0.856	1.483	$S(\delta_{\hat{e}_t}) \neq 0$

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Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

	Start	End	Gaps	obs	Nonlinearity	Balance				Co-Summability			
						CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
NLD	1815	2010	6	190	-	-1.127	-0.411	0.305	$\hat{\delta}_y = \hat{\delta}_z$	0.254	0.254	0.839	$S(\delta_{\hat{e}_t}) \neq 0$
				b =15	ln(Debt/GDP)^2	-1.952	-0.948	0.057	$\hat{\delta}_y = \hat{\delta}_z$	0.146	0.560	0.974	$S(\delta_{\hat{e}_t}) \neq 0$
				M =176	ln(Debt/GDP)^3	-2.668	-1.409	-0.151	$\hat{\delta}_y \neq \hat{\delta}_z$				
NZL	1870	2010	-	141	-	-1.188	-0.527	0.134	$\hat{\delta}_y = \hat{\delta}_z$	0.575	0.869	1.163	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.817	-1.067	-0.317	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =129	ln(Debt/GDP)^3	-2.529	-1.521	-0.513	$\hat{\delta}_y \neq \hat{\delta}_z$				
NOR	1880	2010	6	125	-	-0.670	0.001	0.672	$\hat{\delta}_y = \hat{\delta}_z$	0.723	1.318	1.913	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-1.408	-0.467	0.475	$\hat{\delta}_y = \hat{\delta}_z$	0.529	1.161	1.792	$S(\delta_{\hat{e}_t}) \neq 0$
				M =114	ln(Debt/GDP)^3	-2.004	-0.842	0.319	$\hat{\delta}_y = \hat{\delta}_z$	0.504	1.209	1.914	$S(\delta_{\hat{e}_t}) \neq 0$
PER	1883	2010	14	114	-	-1.477	-0.730	0.018	$\hat{\delta}_y = \hat{\delta}_z$	0.635	1.186	1.736	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.314	-1.301	-0.287	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =103	ln(Debt/GDP)^3	-3.037	-1.788	-0.539	$\hat{\delta}_y \neq \hat{\delta}_z$				
PRT	1865	2010	-	146	-	-1.117	-0.486	0.144	$\hat{\delta}_y = \hat{\delta}_z$	0.402	0.822	1.242	$S(\delta_{\hat{e}_t}) \neq 0$
				b =13	ln(Debt/GDP)^2	-1.781	-0.981	-0.181	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =134	ln(Debt/GDP)^3	-2.421	-1.390	-0.360	$\hat{\delta}_y \neq \hat{\delta}_z$				

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Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

						Balance				Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
ESP	1850	2010	4	157	-	-1.055	-0.384	0.287	$\hat{\delta}_y = \hat{\delta}_z$	0.253	0.911	1.570	$S(\delta_{\hat{e}_t}) \neq 0$
				b =14	ln(Debt/GDP)^2	-1.855	-0.907	0.040	$\hat{\delta}_y = \hat{\delta}_z$	0.232	0.840	1.449	$S(\delta_{\hat{e}_t}) \neq 0$
				M =144	ln(Debt/GDP)^3	-2.562	-1.351	-0.141	$\hat{\delta}_y \neq \hat{\delta}_z$				
LKA	1870	2010	35	106	-	-1.047	-0.452	0.142	$\hat{\delta}_y = \hat{\delta}_z$	0.253	0.696	1.139	$S(\delta_{\hat{e}_t}) \neq 0$
				b =11	ln(Debt/GDP)^2	-1.900	-1.028	-0.155	$\hat{\delta}_y \neq \hat{\delta}_z$				
				M =96	ln(Debt/GDP)^3	-2.619	-1.516	-0.414	$\hat{\delta}_y \neq \hat{\delta}_z$				
SWE	1800	2010	-	211	-	-1.054	-0.350	0.354	$\hat{\delta}_y = \hat{\delta}_z$	0.777	1.546	2.314	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.660	-0.767	0.126	$\hat{\delta}_y = \hat{\delta}_z$	0.658	1.602	2.546	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.245	-1.108	0.028	$\hat{\delta}_y = \hat{\delta}_z$	0.697	1.598	2.499	$S(\delta_{\hat{e}_t}) \neq 0$
CHE	1880	2010	18	115	-	-1.254	-0.315	0.624	$\hat{\delta}_y = \hat{\delta}_z$	0.588	1.033	1.479	$S(\delta_{\hat{e}_t}) \neq 0$
				b =12	ln(Debt/GDP)^2	-2.062	-0.814	0.434	$\hat{\delta}_y = \hat{\delta}_z$	0.639	1.036	1.434	$S(\delta_{\hat{e}_t}) \neq 0$
				M =104	ln(Debt/GDP)^3	-2.720	-1.229	0.263	$\hat{\delta}_y = \hat{\delta}_z$	0.445	0.974	1.503	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1800	2010	-	211	-	-0.913	-0.178	0.558	$\hat{\delta}_y = \hat{\delta}_z$	0.660	1.194	1.728	$S(\delta_{\hat{e}_t}) \neq 0$
				b =16	ln(Debt/GDP)^2	-1.705	-0.694	0.317	$\hat{\delta}_y = \hat{\delta}_z$	0.699	1.196	1.693	$S(\delta_{\hat{e}_t}) \neq 0$
				M =196	ln(Debt/GDP)^3	-2.383	-1.137	0.109	$\hat{\delta}_y = \hat{\delta}_z$	0.702	1.196	1.689	$S(\delta_{\hat{e}_t}) \neq 0$

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Table TA-2: Balance and Co-Summability — ln(GDP pc) specifications, 27 countries (continued)

	Start	End	Gaps	obs	Nonlinearity	Balance				Co-Summability			
						CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	-	211	-	-1.265	-0.507	0.252	$\hat{\delta}_y = \hat{\delta}_z$	0.467	1.049	1.631	$S(\delta_{\hat{e}_t}) \neq 0$
				16	ln(Debt/GDP)^2	-1.611	-0.726	0.160	$\hat{\delta}_y = \hat{\delta}_z$	0.277	0.943	1.609	$S(\delta_{\hat{e}_t}) \neq 0$
				196	ln(Debt/GDP)^3	-2.158	-1.145	-0.132	$\hat{\delta}_y \neq \hat{\delta}_z$				
URY	1871	2010	23	117	-	-1.233	-0.354	0.525	$\hat{\delta}_y = \hat{\delta}_z$	0.224	0.602	0.981	$S(\delta_{\hat{e}_t}) \neq 0$
				12	ln(Debt/GDP)^2	-2.091	-0.896	0.298	$\hat{\delta}_y = \hat{\delta}_z$	0.118	0.602	1.087	$S(\delta_{\hat{e}_t}) \neq 0$
				106	ln(Debt/GDP)^3	-2.814	-1.359	0.097	$\hat{\delta}_y = \hat{\delta}_z$	-0.156	0.477	1.111	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models we take the per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted we allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. $b = int\sqrt{T} + 1$ refers to the time series length of the subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).