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## *Working Paper 15/02*

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with public goods

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# Pareto-improving social security reform with public goods

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4 March 2015

***Key words:***

Social security, Pareto-improving, consolidated budgets, public debt

***Abstract:***

A social security reform may be Pareto-improving by releasing finance to provide more public goods, either directly if the two budgets are consolidated or indirectly through increasing the demand for public debt.

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## 1. Introduction

The introduction of a pay-as-you-go social security scheme provides a free gift to a first generation which receives benefits without having made contributions, but its termination entails a welfare loss to a final generation, which has made contributions while losing a claim to prospective benefits. The latter means that a reform or a reduction in the level of the provision of social security cannot be Pareto-improving – whether or not the economy is dynamically efficient and whether or not the scheme is fiscally burdensome and sustainable.<sup>2</sup>

Some authors have suggested that in principle, at least, a reform might be Pareto-improving by generating side-effects, which may be channelled to compensate potential losers. In a seminal contribution, Homburg (1990) showed that this could arise through reducing the distortionary labour taxes raised to pay for benefits. Space does not permit a review of the literature – in Roberts (2013) – except to say that the possible gains generally depend on some form of production externality.

Two main criticisms have been levelled at this approach. First, although gains from reform might in theory be possible, they are unlikely to be of the magnitude required for providing sufficient compensation to losers. The second is a nicer, theoretical point: if the gains arise only from reducing distortionary taxes, a reform of social security *per se* cannot be Pareto-improving, and welfare concerns should focus instead on revising the tax system. However, taking a more pragmatic view about the infeasibility of eliminating all distortions, social security reform might be advocated on the basis of *facilitating* rather than *constituting* a possible Pareto-improvement. This change in semantics is more conducive to discussing whether policy changes might incur generation-specific costs.

The present paper presents a model where neither of these two criticisms need apply. Public goods are assumed to make a competing claim for public finances, and taxes are not distortionary. Benefit recipients might be willing to forgo some amount of state-financed private consumption in exchange for a greater provision of public goods. A scope for a trade-off exists, either if the two public budgets are consolidated or if cuts in social security increase the demand for public debt. It is then shown that single period cuts to social security may be Pareto-improving, which implies that within a steady state the same applies for

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<sup>2</sup> See Breyer (1989).

permanent reductions. *Section 2* considers the first of these two possibilities, *Section 3* the second of them, and *Section 4* concludes.

## 2. The model

### 2.1 Household utility

A version of the Diamond (1965) overlapping generations model is presented. Young households born at time  $t$  obtain utility over two periods from both private consumption,  $c_t^t$  and  $c_{t+1}^t$ , and from public goods,  $g_t$  and  $g_{t+1}$ ,

$$V_t^t = (1-\sigma)\left((1-\eta^Y)\ln c_t^t + \eta^Y \ln g_t\right) + \sigma\left((1-\eta^O)\ln c_{t+1}^t + \eta^O \ln g_{t+1}\right) \quad (1)$$

The term  $\sigma$ ,  $0 < \sigma < 1$ , represents relative time-preference, while  $\eta^j$ ,  $0 < \eta^j < 1$   $j = Y, O$ , is the relative preference for public goods, which may vary across generations. If the public good, say, healthcare, is particularly favoured by the old,  $\eta^O > \eta^Y$ .

The household budget constraints are

$$c_t^t = w_t - \tau_t^{P,Y} - \tau_t^{G,Y} - s_t, \quad c_{t+1}^t = p_{t+1} - \tau_{t+1}^{G,O} + R_{t+1}s_t. \quad (2)$$

The young work for a wage,  $w_t$ , from which they save,  $s_t$ , and pay taxes,  $\tau_t^{P,Y}$  and  $\tau_t^{G,Y}$ , to support, respectively, social security payments and the public good. Old households receive a public pension,  $p_{t+1}$ , in addition to the return,  $R_{t+1}s_t$ , on their previous period's saving, and are taxed,  $\tau_t^{G,O}$ , but only to support the public good. Their utility is

$$V_t^{t-1} = (1-\eta^O)\ln\left(p_t - \tau_t^{G,O} + R_t s_{t-1}\right) + \eta^O \ln g_t. \quad (3)$$

Initially, without public debt, saving leads only to the capital. The standard assumptions apply: saving leads to investment with a lag; there is full depreciation of the previous capital stock within a period; and the population growth factor,  $1+n$ , is also the support ratio. Dynamic efficiency, that is  $1+n < R$ , is also assumed. Small case variables denote per head of young population, in which terms macroeconomic equilibrium is

$$k_{t+1} = (1+n)^{-1} s_t \quad (4)$$

## 2.2 *Pareto-improving pension reform under a consolidated budget*

A consolidated public budget for social security,  $p_t$ , and the public good,  $g_t$ , is first considered,

$$p_t + g_t = (1+n)(\tau_t^{P,Y} + \tau_t^{G,Y}) + \tau_t^{G,O} \quad (5)$$

The fact of population ageing and so declining support ratios, leading to imbalance in social security budgets, furnishes some support for this assumption. Defining the net transfers of the old as  $p_t^N \equiv p_t - \tau_t^O$  ( $p_t^N \geq -\tau_t^O$ ) and the total taxes of the young as  $\tau_t^Y \equiv \tau_t^{P,Y} + \tau_t^{G,Y}$ , and treating the supply of public goods,  $g_t$ , as an endogenously determined residual means that the privately maximized utilities of the young and old in period  $t$  are

$$V_t^t = \left( (1-\sigma)(1-\eta^Y) + \sigma(1-\eta^O) \right) \ln \left( w_t - \tau_t^Y + \frac{p_{t+1}^N}{R_{t+1}} \right) + \\ (1-\sigma)\eta^Y \ln \left( (1+n)\tau_t^Y - p_t^N \right) + \sigma\eta^O \ln \left( (1+n)\tau_{t+1}^Y - p_{t+1}^N \right) + \sigma(1-\eta^O) \ln R_{t+1} + \dots \quad (6.1)$$

$$V_t^{t-1} = (1-\eta^O) \ln (s_{t-1} R_t + p_t^N) + \eta^O \ln \left( (1+n)\tau_t^Y - p_t^N \right). \quad (6.2)$$

The subsequent level of capital is

$$k_{t+1} = \frac{\sigma(1-\eta^O)(w_t - \tau_t^Y) - (1-\sigma)(1-\eta^Y) \left( p_{t+1}^N / R_{t+1} \right)}{(1+n) \left( \sigma(1-\eta^O) + (1-\sigma)(1-\eta^Y) \right)} \quad (7)$$

Wage income is the base for saving base, so that  $\partial k_{t+1} / \partial w_t > 0$ , and capital raises the marginal product of labour,  $\partial w_t / \partial k_t > 0$ , so that there will be monotonic adjustment,  $\partial k_{t+1} / \partial k_t > 0$ , to any steady state. Thus, with dynamic efficiency, a fall in the capital stock hurts all future generations. Thus, the analysis is restricted to cases excluding this possibility.

We first consider the reform of *current* net pension benefits,  $p_t^N$ . Equation (6.2) implies that the utility for the old is maximized where

$$p_t^{N,O,t-1*} = (1 - \eta^O)(1 + n)\tau_t^Y - \eta^O s_{t-1} R_t, \quad (8)$$

which may even be negative where either their preferences for public goods,  $\eta^O$ , or their asset incomes,  $s_{t-1} R_t$ , are sufficiently high. By contrast, the currently young and all future generations are better off with the lowest possible current values for  $p_t^N$ , as this leads to the highest disposable income for the first group [equation (6.1)] and, so, the greatest largest amount of capital to be inherited by their successors [equation (7)].

*Proposition One:* Under consolidated budgets, there is a Pareto-improving, current social security reform: a partial one, if  $0 \leq p_t^{O,t-1*} < p_t$ ; or a full one, if  $p_t^{O,t-1*} \leq 0 < p_t$

A cut in current social security benefits will be Pareto-improving, if the old prefer a rebalancing towards expenditure on public goods without the young having to be taxed more.

A parallel case is a commitment at time  $t$  to a *prospective* reform at time  $t+1$ , where the promise of  ${}_t p_{t+1}^N$  over-rides an earlier “commitment” to pay  ${}_t \tilde{p}_{t+1}^N$ . This prospect is irrelevant to the old, while the young would most prefer

$${}_t p_{t+1}^{N,Y,t*} \equiv \frac{(1 - (1 - \sigma)\eta^Y - \sigma\eta^O)(1 + n)\tau_{t+1}^Y - \eta^O R_{t+1}\sigma(w_t - \tau_t^Y)}{1 - (1 - \sigma)\eta^Y}, \quad (9)$$

according to their intertemporal utility in equation (6.1). Provided that this value is eneath a previous “commitment”,  $\tilde{p}_{t+1}^N$ , so that future taxes do not have to rise, this is another Pareto-improving change.<sup>3</sup>

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<sup>3</sup> It can be verified that a prospective reform is dynamically-consistent. The anticipation  $p_{t+1}^N = p_{t+1}^{N,Y,t*}$  induces a current level of saving by the young, such that their preference when they are old is  $p_{t+1}^{N,O,t} > 0$ , where  $p_{t+1}^{N,O,t} = {}_t p_{t+1}^{N,Y,t*}$ .

*Proposition Two: Under consolidated budgets, there is a Pareto-improving, prospective social security reform: a partial one, if  $0 <_t p_{t+1}^{N,Y,t*} < \tilde{p}_{t+1}^N$  ; or a full one, if  ${}_t p_{t+1}^{N,Y,t*} \leq 0 < \tilde{p}_{t+1}^N$ .*

### 3. Further considerations

#### 3.1 Separate budgets

Under separate budgets, equation (5) splits into two,

$$p_t = (1+n)\tau_t^{P,Y}, \quad g_t = (1+n)\tau_t^{G,Y} + \tau_t^{G,O}, \quad \text{where } \tau_t^{G,Y} = \tau_t^{G,O} = \tau_t. \quad (10)$$

First, it is straightforward that an independent social security reform cannot be Pareto-improving under separate budgets, since this is equivalent to the base case without the allowance for a compensating change in the supply of public goods. Second, as for most models, there is possible scope for a Pareto-improving but isolated change in the provision of public goods, except that, for the present one, the capital accumulation effects enter the calculation.

#### 3.2 Public debt

Public debt is now introduced as complementary store of value for households and as another means of financing public expenditure by the state. Separate budgets are assumed, where public debt is issued only to finance public goods, leaving the social security account in balance. Equations (10) become

$$p_t = (1+n)\tau_t^{P,Y}, \quad v_t x_t = g_t - (2+n)\tau_t^G + (d + \bar{v})x_{t-1}, \quad (11)$$

Public debt consists of single-period bonds which pay a coupon  $d$  and have a fixed redemption or book value of  $\bar{v}$ . In each period  $t$ , there are  $x_t$  issues sold at the market price,  $v_t$ , whether at a discount or a premium, and the previous debt is serviced and redeemed by the payment  $(d + \bar{v})x_{t-1}$ , as another component of public expenditure.

The household budget constraints are now

$$c_t^t = w_t - \frac{p_t}{1+n} - \tau_t^G - s_t^K - v_t x_t, \quad c_{t+1}^t = p_{t+1} - \tau_t^G - R_{t+1} s_t^K + (d + \bar{v}) x_t. \quad (12)$$

The young save by acquiring both public debt,  $v_t x_t$ , and capital assets,  $s_t^K$ , which alone are “productive” in augmenting the capital stock,

$$k_{t+1} = (1+n)^{-1} s_t^K \quad (13)$$

A demand for public debt is now determined by assuming risky returns to capital with risk-averse households. Instead of utility from future private consumption being certain at  $\ln(c_{t+1})$ , we postulate a function,  $(1/(1+\alpha))\ln E(c_{t+1}) + (\alpha/(1+\alpha))\ln c_{t+1,\min}$ , where  $E(c_{t+1})$  and  $c_{t+1,\min}$  are the mean and the minimal values of consumption. The term  $\alpha$  parameterizes risk-aversion: risk-neutrality is defined where  $\alpha = 0$ , and absolute risk-aversion, where  $\alpha \rightarrow \infty$ , gives rise to *maxmin* behaviour.<sup>4</sup> With consumption uncertainty confined to the second period, the two-period utility function in (1) becomes

$$V_t^t = (1-\sigma)\left((1-\eta^Y)\ln c_t^t + \eta^Y \ln g_t\right) + \sigma\left((1-\eta^O)\left((1/(1+\alpha))\ln E(c_{t+1}^t) + (\alpha/(1+\alpha))\ln(c_{t+1,\min}^t)\right) + \eta^O \ln g_{t+1}\right) \quad (14)$$

Risk is deemed to relate to the returns on capital, which may implicitly be due to productivity shocks, from which wages are assumed to be immune. It is also assumed that  $R_{t+1}^{K,\min} = 0$ : that in the worst possible state neither the interest nor the principal on investment assets is returned. This conveniently prevents quadratic outcomes, so that the *partial* solutions for the household’s asset demands become

$$s_t^K = \lambda \left( w_t - \tau_t^G - \frac{p_t}{1+n} - \frac{\alpha}{1+\alpha} \left( \frac{w_t - \tau_t^G - \frac{p_t}{1+n} + \frac{p_{t+1} - \tau_{t+1}^G}{E(R_{t+1}^K)}}{1 - \frac{d + \bar{v}}{E(R_{t+1}^K)v_t}} \right) \right), \quad (15)$$

where

<sup>4</sup> This specification favours *down-side* over *symmetric* risk, which may be relevant. See Ang, Chen and Xing (2006).



$$\lambda \equiv \frac{\sigma(1-\eta^O)}{\sigma(1-\eta^O) + (1-\sigma)(1-\eta^Y)}, \quad (16)$$

and

$$v_t x_t = \frac{d+\bar{v}}{R_{t+1}^K} x_t + \lambda \alpha \left( \frac{w_t - \tau_t^G - \frac{p_t}{1+n} + \frac{p_{t+1} - \tau_{t+1}^G}{E(R_{t+1}^K)}}{(1+\alpha)x_t + \left(1 + (1+\alpha) \frac{(1-\sigma)(1-\eta^Y)}{\sigma(1-\eta^O)}\right) \left(\frac{p_{t+1} - \tau_{t+1}^G}{d+\bar{v}}\right)} \right) x_t \quad (17)$$

The effect of a prospective social security reform on the current demand for public debt,  $\partial x_t v_t / \partial p_{t+1}$ , is of uncertain sign, because of countervailing wealth and substitution effects. The effect of a current reform, however, unambiguously raises the demand for public debt,  $\partial x_t v_t / \partial p_t < 0$ , through reducing taxes and thus raising the disposable income of the young. An increased demand for public debt then allows the government to provide more public goods [equation (11)], which may be Pareto-improving.

We provide a particular example by employing the simplifying restriction,  $p_{t+1} = \tau_{t+1}^G$ , thus reducing equations (15) and (17) to

$$s_t^K = \frac{\lambda}{1+\alpha} \left( w_t - \tau_t^G - \frac{p_t}{1+n} \right) - \frac{d+\bar{v}}{E(R_{t+1}^K)} x_t, \quad (18)$$

$$x_t v_t = \frac{d+\bar{v}}{E(R_{t+1}^K)} x_t + \frac{\lambda \alpha}{1+\alpha} \left( w_t - \tau_t^G - \frac{p_t}{1+n} \right) \quad (19)$$

We consider falls in  $p_t$  for a fixed number of bond issues,  $x_t$ . Equation (19) shows this raises capital,  $\partial s_t^K / \partial p_t < 0$ , thereby also making future generations better off. Thus, the possibility of Pareto-improvement is ascertained with respect to the utility of the old, evaluated by combining equations (6.2), (12) and (20) in

$$V_t^{t-1} = (1-\eta^O) \ln(R_t s_{t-1} + p_t - \tau_t^G) + \eta^O \ln \left( (2+n)\tau_t^G - (d+\bar{v})x_{t-1} + \frac{d+\bar{v}}{E(R_{t+1}^K)} x_t + \frac{\lambda \alpha}{1+\alpha} \left( w_t - \tau_t^G - \frac{p_t}{1+n} \right) \right)$$

This is at a maximum where

$$p_t^{O,t-1*} = (1-\eta^O)(1+n) \left( \frac{1+\alpha}{\lambda\alpha} \left( (2+n)\tau_t^G - (d+\bar{v})x_{t-1} + \frac{d+\bar{v}}{E(R_{t+1}^K)} x_t \right) + w_t - \tau_t^G \right) - \eta^O (R_t s_{t-1} - \tau_t^G) \quad (20)$$

*Proposition Three: If public goods are at least partly funded through public debt, there is a Pareto-improving, current social security reform: a partial one, if  $0 \leq p_t^{O,t-1*} < p_t$ ; or a full one, if  $p_t^{O,t-1*} \leq 0 < p_t$*

With regard to equation (21), there are some parameter values for which  $p_t^{O,t-1*} = \tau_t^G$ .

Thus, the above simplifying restriction,  $p_{t+1} = \tau_{t+1}^G$ , on which this equation is based, may also be regarded as a Pareto-efficient outcome within a steady state.

#### 4. Conclusion

If benefit-receiving households jointly value public goods with private consumption, a robust case cannot be made against social security reform on the basis of Pareto-optimality. This point is unrelated to any demographic pressures for reform. However, if these, factors, namely, falling support ratios, encourage consolidated budgeting and, if that of population ageing shifts preferences towards public goods, a possibility of a Pareto-improving reform may arise, where none had existed, although other reasons may remain for or against an actual reform.<sup>5</sup>

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<sup>5</sup> These might include the generational distribution of income. Mulligan and Sala-i-Martin (2004) find that the over 65s have higher relative incomes in all the Continental European countries in their sample, but not in the US and the UK.

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