Measuring Macro-Financial Conditions Using a Factor-Augmented Smooth-Transition Vector Autoregression*

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Abstract

In this paper, financial stress is measured by explicitly linking current financial conditions with the macroeconomy. We define financial stress as the regime in which exogenous negative financial shocks have stronger negative effects on growth and inflation. The financial stress periods are identified using a large unbalanced panel of financial variables with an embedded method for variable selection. We use a novel factor-augmented vector autoregressive model with smooth regime changes (FAST-VAR). The unobserved factor is jointly estimated with the parameters of a smooth transition function that describe the weights given to the financial stress regime over time. As by-product of our approach, we provide a measure of financial conditions extracted from a nonlinear factor modelling approach, which has the advantage of being more strongly related to business cycle phases.

Keywords: factor-augmented VAR, Gibbs variable selection, financial crisis

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1 Introduction

Financial Conditions indexes (FCI) aim to measure exogenous systematic shifts in financial variables that could be related to future economic activity. Policymakers and market analysts use FCIs to anticipate episodes of financial stress with negative effects in economic activity. Because it is hard to measure U.S. financial conditions using one or two financial variables, FCIs have been constructed by extracting common components from large panels of financial time series as surveyed by Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010). Unfortunately, there is no consensus measure of financial conditions. Ideally, an FCI should provide an early warning system for financial distresses that effect macroeconomic activity. The indexes are normally constructed by finding a measure of financial conditions and, then, examining the impact on future economic activity. However, Kliensen, Owyang and Vermann (2012) show that several popular measures of financial conditions are highly– but not perfectly–correlated with each other, suggesting the definition of financial stress varies across researchers.

Instead of measuring financial stress as exogenous changes in the financial markets that could have a potential impact on the macroeconomy, we measure financial stress by explicitly linking current financial conditions with the macroeconomy. We define financial stress as the periods in which exogenous negative financial shocks have stronger negative effects on growth and inflation if compared with the no stress periods. We provide a measure of the likelihood of being in the financial stress regime in the current period. We identify financial stress periods using a large unbalanced panel of financial variables with an embedded method for covariate selection, so irrelevant financial variables are excluded from the financial conditions factor. As by-product of our modelling approach, we provide a measure of financial conditions similar to the ones surveyed by Hatzius et al. (2010), but extracted from a nonlinear factor modelling approach. The advantage of allowing for nonlinearity in the factor dynamics is that the extracted factor is more strongly related with business cycle phases.

We identify periods of financial stress using a novel factor-augmented vector autoregressive model with smooth regime changes (FASTVAR). The model has two regimes, allowing for dynamics changes depending on the financial condition factor. The proposed model augments the smooth transition vector autoregressive model (surveyed by Van Dijk, Terasvirta and Franses (2002)) with an unobserved factor as in Bernanke, Boivin and Eliasz (2005). Thus, the strength of the relation between financial conditions and economic activity depends explicitly on the unobserved financial conditions factor linked to a set of observed financial variables. The unobserved factor is jointly estimated with the parameters of a smooth transition function that describe the weights given to each regime over time. We also include a step in the estimation that allows for covariate selection to determine the composition of the data vector included in the financial conditions factor.

Our approach differs from the literature as follows. Hatzius et al. (2010) filters the time series of financial variables to exclude the effect of macroeconomic conditions before building their financial condition index (Brave and Butters (2012) also follow similar approach). Our approach relates financial stress to periods in which the effect of financial shocks are stronger in future economic activity and inflation. This means that we do not clean our financial variables of macroeconomic effects before the estimation because the financial shocks within a VAR approach will be by definition exogenous. Our approach also differs from the FAVAR approach of Koop and Korobilis (2013) who remove the effect of macroeconomic variables from financial variables using an adequate measurement equation. The restriction we impose to disentangle part of the factor dynamics from macroeconomic variables is to assume that lagged macroeconomic variables do not enter the factor equation within the VAR.

The issue that the transmission of shocks may depend on the level of financial stress has also being addressed by Davig and Hakkio (2010) and Hubrich and Tetlow (2011) in a Markov-switching approach, and by Dahlhaus (2012) in a smooth transition approach. However, they all use an observed financial conditions index, computed outside the modelling approach, to help to identify regime changes. Our approach jointly estimates the financial conditions index and the regimes of financial stress. The difficulty of the joint estimation is that a method that deals with nonlinearity in the state-space representation of the model is required.

Koop and Korobilis (2013) allow for time variation in the parameters of the VAR—that is, changes are exogenous and are not related with financial conditions. A method to choose variables to enter factors was also performed by Kaufmann and Schumacher (2012) using sparse priors in the context of dynamic factor models, and Koop and Korobilis (2013) using model averaging in FAVAR models.

Following Hatzius et al. (2010) and Brave and Butters (2012), we consider a unbalanced panel of 23 financial indicators. Estimation of the model is conducted in a Bayesian environment using Metropolis-Hastings steps to draw the transition function parameters and a vector of indicator variables determining the financial series entering the factor. Because of the nonlinearity in the autoregressive parameters, the factor must be estimated using a nonlinear filter. We use the extended Kalman filter, implying that we use a first-order approximation of the state equation.

The balance of the paper proceeds as follows: Section 2 describes the general FASTVAR model with model indicators used for model selection. Section 3 outlines the Gibbs sampler used to estimate the model parameters, the factor, and the posterior distributions for the model inclusion indicators. In this section, we also describe the data used. Section 4 presents the results for the model estimated with one factor, representing our macro-financial stress index. Section 5 summarizes and offers some conclusions.

2 The Empirical Model

In this section, we propose a method to identify financial stress periods: periods in which financial shocks have stronger effects on macroeconomic variables. We begin by describing a vector autoregressive model that links an exogenously-defined financial condition index to economic activity. Then, we propose a Factor-Augmented Smooth Transition vector autoregressive model (FASTVAR) that allows for the joint estimation of a financial condition factor and the time-varying weights for the financial stress regime.

2.1 The Smooth Transition VAR Model

Let f_t represent the period$-t$ value of a financial conditions index. For now, assume that f_t is scalar, observed, and exogenously determined. Define $\mathbf{z}_t$ as an $(N_z \times 1)$ vector of macroeconomic variables of interest—e.g., GDP growth, employment, inflation, etc. Suppose that the effect of a shock to financial conditions on macroeconomic variables is linear, but that financial conditions are also affected by macroeconomic variables—in particular, current economic activity. In this case, the dynamic response can be evaluated in a standard VAR framework. Define the $((N_z + 1) \times 1)$ vector $\mathbf{y}_t = [\mathbf{z}_t', f_t]'$, where the ordering of f_t last is intentional and provides the identifying restriction used to construct impulse responses. The VAR in question is then:

$$\mathbf{y}_t = A(L)\,\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $A(L)$ is a matrix polynomial in the lag operator, $\boldsymbol{\varepsilon}_t \sim N(0, \Omega)$, and we have suppressed any constants and trends. The matrices $A(L)$ drive the transmission of financial shocks—shocks to f_t—to macroeconomic variables $\mathbf{z}_t$. However the transmission in this specification cannot change over time or with the level of financial stress. Suppose that the transmission mechanism changes over time and depends on the size and sign of the financial conditions index; then, we can write:

$$\mathbf{y}_t = [1 - \pi_t(f_{t-1}; \gamma, c)]\, A_1(L)\,\mathbf{y}_{t-1} + \pi_t(f_{t-1}; \gamma, c)\, A_2(L)\,\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t; \tag{2}$$

where $A_1(L)$ and $A_2(L)$ are matrices of lag polynomials, $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}_{N_z+1}, \Omega_t)$, and Ω_t the variance-covariance matrix. If f_t is observed, the model described in (2) is a standard smooth transition vector autoregression (STVAR) as in Van Dijk et al. (2002). In the parlance of the

STAR models, f_{t-1} is the transition variable and $\pi_t(f_{t-1})$ is the transition function, where $0 \leq \pi_t(f_{t-1}) \leq 1$. The transition function $\pi_t(f_{t-1})$ determine the time-varying weights of each set of autoregressive parameters $A_1(L)$ and $A_2(L)$ on the path of $\mathbf{y}_t$.

The transition function can take a number of forms. One example is a first-order logistic transition function of the form:

$$\pi_t(f_{t-1}; \gamma, c) = [1 + \exp(-\gamma(f_{t-1} - c))]^{-1}, \tag{3}$$

where $\gamma \geq 0$ is the speed of transition and c is a fixed threshold. In (3), the regime process is determined by the sign and magnitude of the deviation of lagged financial conditions, f_{t-1}, from the threshold c. If f_{t-1} is less than c, the transition function, $\pi_t(f_{t-1})$ gives more weight to the autoregressive parameters of the first regime, $A_1(L)$. The coefficient γ determines the speed of adjustment: as $|\gamma| \to \infty$, the transition becomes sharper and the regime switches resemble a pure threshold model. At $\gamma = 0$, the model collapses to a linear model. Smooth transition and threshold vector autoregressions have been employed to measure asymmetries in the dynamic effects of monetary shocks (Weise, 1999; Ravn and Sola, 2004), and in the effect of credit conditions on economic activity (Balke, 2000).

In the model (2) and (3), a shocks propagates differently depending on the (lagged) state of financial conditions. Shocks to macro variables have regime-dependent effects that can be determined conditional on ambient financial conditions. Shocks to financial conditions, on the other hand, have two effects. Conditional on the regime, the response to a financial conditions shock can be computed as standard (state-dependent) impulse response. In addition, shocks to financial conditions can cause a change in future macroeconomic dynamics by driving the economy away from one regime towards the other.

2.2 The Factor-Augmented STVAR

The STVAR model in the preceding subsection relies on the fact that f_t is observed. This could be true if one used an observed proxy for financial stress or if one used a constant weight measure, as the financial condition indexes surveyed by Hatzius et al. (2010). But how can we be sure we are properly modeling financial conditions such that we correctly identify financial stress periods? As a consequence, we estimate the financial conditions index as a factor within a factor-augmented STVAR based on a vector of financial variables, $\mathbf{x}_t$.

Let f_t be the factor that summarizes the comovements across N_x demeaned financial series, $\mathbf{x}_t$:

$$\mathbf{x}_t = \boldsymbol{\beta} f_t + \mathbf{u}_t, \tag{4}$$

where $\boldsymbol{\beta}$ is the matrix of factor loadings and u_{it} are iid $N\left(0, \sigma_i^2\right)$. The model (2), (3), and (4) comprise the FASTVAR model. The factor is jointly determined by the cross-series movements in the financial variables and the behavior of the macroeconomic variables.

One of the central issues in the literature measuring financial stress is how to determine which financial series should comprise $\mathbf{x}_t$. For example, Kliensen et al. (2012) surveyed 11 different indexes that were constructed with 4 up to 100 indicators. While more series may provide a more complete view, increasing the cross-sectional dimension of $\mathbf{x}_t$ may result in estimated factors that do not truly represent financial stress. We are interested in determining the set of financial variables that alters the underlying dynamics of the macroeconomy—that is, which financial variables switch the macroeconomic dynamics from $A_1(L)$ to $A_2(L)$ and vice versa.

To accomplish this, we augment (4) with a set of model inclusion dummies, $\Lambda = [\lambda_1, ..., \lambda_{N_x}]'$, $\lambda_i \in \{0, 1\}$. The inclusion dummies indicate whether a particular financial series should be included in the set of variables that make up the factor—that is, if $\lambda_i = 1$, x_i is included in the set of variables that determine the factor. If $\lambda_i = 0$, x_i is excluded of the estimation of the factor; the effect of $\lambda_i = 0$ is to set the factor loading associated with the ith element of $\mathbf{x}_t$ to zero. We can then rewrite (4) as

$$\mathbf{x}_t = (\boldsymbol{\Lambda} \odot \boldsymbol{\beta}) f_t + \mathbf{u}_t. \tag{5}$$

The vector of inclusion indicators, Λ, can be estimated along with the other parameters in the model.

2.2.1 The State-Space representation

The state-space form of the model consisting of (2), (3), and (4) summarizes the assumptions behind the FASTVAR model that we have made thus far. For exposition, we assume that $p = 1$ and $N_z = 2$. The measurement equation is:

$$\begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \end{bmatrix} = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & (\boldsymbol{\Lambda} \odot \boldsymbol{\beta}) \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ f_t \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{u}_t \end{bmatrix}; u_t \sim iidN(0, \sigma_i^2). \tag{6}$$

This differs from the FAVAR specification of Bernanke et al. (2005) by excluding the macroeconomic variables $\mathbf{z}_t$ as observable factors in the measurement equation of the financial variables $\mathbf{x}_t$.

The state equation is:

$$\begin{aligned} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ f_t \end{bmatrix} &= \begin{bmatrix} c_1 \\ c_1 \\ 0 \end{bmatrix} + \begin{bmatrix} a_{1,11} & a_{1,12} & a_{1,13} \\ a_{1,21} & a_{1,22} & a_{1,23} \\ 0 & 0 & a_{1,33} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \\ f_{t-1} \end{bmatrix} \\ &+\pi_t(f_{t-1}; \gamma, c) \left\{ \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} z_{1,t-1} \\ z_{2,t-1} \\ f_{t-1} \end{bmatrix} \right\} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{ft} \end{bmatrix}, \end{aligned} \tag{7}$$

where $\varepsilon_t \sim N(0, \Omega)$, $\pi_t(f_{t-1}; \gamma, c) = [1 + \exp(-\gamma(f_{t-1} - c))]^{-1}$, and $d_{ij} = a_{2,ij} - a_{1,ij}$ measures the change in the autoregressive coefficients across regimes. In equation (7), we include additional restrictions in the factor dynamics, excluding the possibility of direct dynamic effects of macroeconomic variables on the financial factor. This is justified by the fact that the factor is estimated/filtered using both the measurement (6) and the state (7) equations, and we would like to relate the factor more strongly to financial variables in $\mathbf{x}_t$ than economic variables in $\mathbf{z}_t$.

2.2.2 Impulse Response Functions and Financial Conditions

The FASTVAR allows for asymmetric transmission of financial shocks, which affect directly the f_t equation, on macroeconomic variables. However, asymmetries will only prevail if estimates of d_{ij} do not collapse to zero or, alternatively, if the transmission of shocks differs even though the shocks' size and sign are invariant. We split the data on macroeconomic variables and the estimated factor into two subsets to verify whether the dynamic transmission changes with regimes. The first subset refers to the histories during the lower regime, $\pi_t(f_{t-1}; \gamma, c) \leq 0.5$, and the other subset refers to the upper regime, $\pi_t(f_{t-1}; \gamma, c) > 0.5$. Based on these two sets of histories, we compute generalized impulse responses conditional on the regime as suggested by Koop, Pesaran and Potter (1996), and applied by Galvao and Marcellino (2013). The responses measure the effect of a one-unit shock to financial conditions on the endogenous variables, assuming (i) a specific set of histories at the impact (either lower or upper regime), and (ii) that the regimes may change over horizon. We simulate data to compute the conditional expectations of $\mathbf{y}_{t+h}$ with and without the shock to compute responses:

$$IRF_{h,v,s} = \frac{1}{T_s} \sum_{t=1}^{T_s} \left\{ E[\mathbf{y}_{t+h} | \mathcal{F}_t^{(s)}, v_t = v] - E[\mathbf{y}_{t+h} | \mathcal{F}_t^{(s)}] \right\},$$

where T_s is number of histories in regime s, $\digamma_t^{(s)}$ is a history from regime s (typically includes $\mathbf{z}_t, ..., \mathbf{z}_{t-p+1}$ and $f_t, ..., f_{t-p+1}$), and $v_t = v$ is the shock vector. In the empirical application, we use 200 draws from the disturbances distribution to compute each conditional expectation using a given set of FASTVAR parameters. The $IRF_{h,v,s}$ measures the reponses of both macroeconomic variables and the factor at horizon h from shock v that hit the model in regime s (either the lower or the upper regime defined using the transition function as above). This approach for computing impulse responses takes the nonlinear dynamics of the FASTVAR fully in consideration

We use the estimated impulse responses to identify a given regime as the “financial stress” regime. For example, if negative shocks to the financial conditions factor f_t have significantly more persistent effects (larger cumulative effects) in the lower regime in contrast with the upper regime, then this implies that the lower regime is the financial stress regime, based on the definition of financial stress firstly described in the Introduction. The estimated transition function over time $\pi_t(f_{t-1}; \gamma, c)$ measures the weights given to the low (no) stress regime over time. If the current transition function is such that $\pi_t(f_{t-1}; \gamma, c) \leq 0.5$, then we should expect that unexpected changes in the financial factor f_t would have stronger effects on future macroeconomic variables.

3 Estimation

We estimate the model using the Gibbs sampler with a Metropolis-in-Gibbs step. Let Θ collect all of the model parameters. We can partition the set of model parameters into blocks: (1) $\mathbf{\Psi} = [A_1(L), A_2(L)]$, the VAR coefficients; (2) $\mathbf{\Omega}$, the VAR variance-covariance matrix; (3) γ and c, the transition speed and the threshold; (4) β, Λ, and $\mathbf{f}_T = \{f_t\}_{t=1}^{T}$, the factor loadings, the inclusion indicators, and the factor; and (5) $\{\sigma_{it}^2\}_{i=1}^{N_x}$, the variances of financial variables. The algorithm samples from each block, conditional on the other blocks. After a suitable number of draws are discarded to achieve convergence, the set of conditional draws form the joint distribution of the whole model.

3.1 Priors

Table A: Priors for Estimation			
Parameter	Prior Distribution	Hyperparameters	
$vec(\mathbf{\Psi})$	$N(\mathbf{m}_0, \mathbf{M}_0)$	$\mathbf{m}_0 = \mathbf{0}_N$; $\mathbf{M}_0$=10$\mathbf{I}_N$	$N = 2N_z(N_z+1)P + 2N_z + 2P$
Ω^{-1}	$W\left(\frac{\nu_0}{2}, \frac{D_0}{2}\right)$	$\nu_0 = 1000$; $D_0 = \mathbf{I}_N$	
γ	$\Gamma(\mathbf{g}_0, \mathbf{G}_0)$	$\mathbf{g}_0 = 6$; $\mathbf{G}_0 = 3$	$\Delta_\gamma = 0.2$
c	$Unif(\mathbf{c}_L, \mathbf{c}_H)$	$\mathbf{c}_L = f_{0.05}$; $\mathbf{c}_H = f_{0.95}$	
σ_n^{-2}	$\Gamma(\omega_0, W_0)$	$\omega_0 = 1$; $W_0 = 1$	$\forall n$
β_n	$N(\mathbf{b}_0, \mathbf{B}_0)$	$\mathbf{b}_0 = -\mathbf{100}$; $\mathbf{B}_0$=0.01	$\forall n$
λ_n	ρ_0	$\rho_0 = 0.1$	$\forall n$

We assume a proper normal–inverse-Wishart prior for the VAR(P): Each regime-dependent coefficient matrix has a multivariate normal prior; the covariance matrix is inverse Wishart. The threshold in the transition function has a uniform prior bounded by the 5th and 95th quantiles of the distribution of the factors; the transition speed has a gamma prior. We also adopt a normal–inverse-gamma prior for the factor equation: Each of the factor loadings has a normal prior and each variance is inverse gamma. The prior for the inclusion indicator is set such that more weight is assigned to excluding variables. This makes the factor estimated over, ex ante, as parsimonious a vector of financial indicators as possible. Table A presents the prior hyperparameters.

3.2 Drawing $\mathbf{\Psi}$ conditional on $\Theta_{-\mathbf{\Psi}}, \mathbf{f}_T, \mathbf{z}_{d,T}$ and $\mathbf{x}_T$

Conditional on $\pi_t(f_{t-1})$, a draw from the posterior distributions for the VAR parameters is a straightforward application of Chib (1993) and Chib and Greenberg (1996). Rewrite the VAR of $\mathbf{y}_t = [\mathbf{z}_t', f_t]'$ as:

$$\mathbf{y}_t = \boldsymbol{\theta}_t \widetilde{\boldsymbol{\Psi}} + \varepsilon_t, \tag{8}$$

where $\widetilde{\boldsymbol{\Psi}}$ is the $(2(N_z+1)N_zP + 2N_z + 2P \times 1)$ stacked vector of parameters,

$$\boldsymbol{\theta}_t = \begin{bmatrix} I_{N_z} \otimes \widehat{\mathbf{y}}_{t-1} & \mathbf{0}_{2P} \\ 0 & \widehat{\mathbf{f}}_{t-1} \end{bmatrix},$$

$$\widehat{\mathbf{y}}_{t-1} = \left[\pi_t(f_{t-1})\,\mathbf{y}_{t-1}^p, (1-\pi_t(f_{t-1}))\,\mathbf{y}_{t-1}^p\right],$$

$\mathbf{y}_{t-1}^p = \left[1, \mathbf{y}_{t-1}', ..., \mathbf{y}_{t-p}'\right]$,

$$\widehat{\mathbf{f}}_{t-1} = \left[\pi_t(f_{t-1})\,\mathbf{f}_{t-1}^p, (1-\pi_t(f_{t-1}))\,\mathbf{f}_{t-1}^p\right],$$

and $\mathbf{f}_{t-1}^p = \left[f_{t-1}', ..., f_{t-p}'\right]'$. Then, given the prior $N(\mathbf{m}_0, \mathbf{M}_0)$, the (stacked) joint parameter vector can be drawn from

$$\boldsymbol{\Psi} \sim N(\mathbf{m}, \mathbf{M}),$$

where

$$\mathbf{M} = \left(\mathbf{M}_0^{-1} + \sum_{t=1}^{T} \boldsymbol{\theta}_t' \Omega_t^{-1} \boldsymbol{\theta}_t\right)^{-1}$$

and

$$\mathbf{m} = \mathbf{M}\left(\mathbf{M}_0^{-1}\mathbf{m}_0 + \sum_{t=1}^{T} \boldsymbol{\theta}_t' \Omega_t^{-1} \mathbf{y}_t\right).$$

3.3 Drawing $\widetilde{c}, \widetilde{\gamma}$ conditional on $\Theta_{-[\widetilde{c},\widetilde{\gamma}]}, \mathbf{f}_T, \mathbf{z}_{d,T}$ and $\mathbf{x}_T$

The prior on the hyperparameters of the transition equation is jointly normal-gamma. Given the prior, the posterior is not a standard form; γ, however, can be drawn using a Metropolis-in-Gibbs step (Lopes and Salazar, 2005). To do this, we first draw the candidates, γ^* and c^*, separately from random walk gamma and normal proposal densities, respectively:

$$\gamma^* \sim G\left(\frac{\left(\gamma^{[i-1]}\right)^2}{\Delta_\gamma}, \frac{\gamma^{[i-1]}}{\Delta_\gamma}\right)$$

and

$$c^* \sim Unif\left(\mathbf{c}_L, \mathbf{c}_H\right),$$

where the superscript $[i-1]$ represents the values retained from the past Gibbs iteration and Δ_γ is a tuning parameter, and the bounds of the uniform distribution are chosen such that there proposed threshold always lies on the interior of the distribution of the factors for the current factor draw. The joint candidate vector is accepted with probability $a = \min\{A, 1\}$, where

$$\begin{aligned} A &= \frac{\prod_t \phi\left(\mathbf{z}_t | \pi_t\left(f_{t-1}|\gamma^*, c^*\right), \mathbf{\Psi}, f_t\right)}{\prod_t \phi\left(\mathbf{z}_t | \pi_t\left(f_{t-1}|\gamma^{[i-1]}, c^{[i-1]}\right), \mathbf{\Psi}, f_t\right)} \\ &\quad \times \frac{dUnif\left(c^* | \mathbf{c}_L, \mathbf{c}_H\right)}{dUnif\left(c^{[i-1]} | \mathbf{c}_L, \mathbf{c}_H\right)} \frac{dG\left(\gamma^* | \left(\gamma^{[i-1]}\right)^2 / \Delta_\gamma, \gamma^{[i-1]}/\Delta_\gamma\right)}{dG\left(\gamma^{[i-1]} | \left(\gamma^{[i-1]}\right)^2 / \Delta_\gamma, \gamma^{[i-1]}/\Delta_\gamma\right)}, \end{aligned}$$

$\gamma^{[i]}$ represents the last accepted value of γ, $dUnif\,(.)$ is the uniform pdf, and $dG\,(.)$ is the gamma pdf.

3.4 Drawing $\mathbf{\Omega}$ conditional on $\Theta_{-\mathbf{\Omega}}, \mathbf{f}_T, \mathbf{z}_{d,T}$ and $\mathbf{x}_T$

Here, we describe the draw of the homoskedastic VAR variance-covariance matrix; extension to the regime-dependent structure described above is straightforward. Rewrite (2) in terms of the residual:

$$\boldsymbol{\varepsilon}_t = \mathbf{y}_t - A_1(L)\mathbf{y}_{t-1} - \pi_t\left(f_{t-1}\right) D(L)\mathbf{y}_{t-1}.$$

Then, given the prior $W\left(\nu_0, D_0\right)$ for $\boldsymbol{\Omega}^{-1}$, the posterior is

$$\boldsymbol{\Omega}^{-1} \sim W\left(\nu, D\right),$$

where

$$\nu = \frac{\nu_0 + T}{2},$$

$$D = \frac{D_0}{2} + \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t',$$

and $W\left(.,.\right)$ is the Wishart distribution.

3.5 Drawing β, and Λ conditional on $\Theta_{-\beta,\Lambda}, \mathbf{z}_{d,T}$, $\mathbf{f}_t$ and $\mathbf{x}_T$

In a standard factor-augmented VAR, the factors can be drawn by a number of methods including the Kalman filter and the factor loadings are conjugate normal. In our case, we have two issues that can complicate estimation. First, because the composition of the vector of data determining the factor is unknown, we must sample the inclusion indicators, loadings, and factors jointly. This joint draw requires a Metropolis step. Second, because the factors also affect the regimes through the transition equation, the state-space representation is nonlinear and a standard Kalman filter cannot be used.

The joint draw proceeds as follows. Our plan is to draw Λ via a reversible-jump Metropolis step; however, a new candidate Λ^* invalidates the β from the previous draw. Thus, it is more efficient to draw β and Λ jointly. Define the joint proposal density, $q\left(\beta^*, \Lambda^*\right)$, as

$$q\left(\beta^*, \Lambda^*\right) = q\left(\beta^* | \Lambda^*\right) q\left(\Lambda^*\right).$$

First, we draw a set of inclusion candidates, Λ^*, from $q\left(\Lambda^*\right)$. Then, conditional on these candidates, we draw a candidate factor loading, β^*, from $q(\beta^*|\Lambda^*)$. This allows us to simplify the acceptance probability of the joint candidate.

3.5.1 Drawing the Inclusion Indicator Candidate

The financial stress index may be sensitive to small shocks in the financial variables because of the nonlinearities in the transition function, making variable selection important. Let $\Lambda^{[i-1]} = \left[\lambda_1^{[i-1]}, ..., \lambda_{N_x}^{[i-1]}\right]$ represent the last iteration's draw of the matrix of inclusion indicator with $\lambda^{[i-1]} \in \{0, 1\}$. We draw an index candidate, n^*, from a discrete uniform with support 1 to N_x. The candidate Λ^* is then

$$\Lambda^* = \left[\lambda_1^{[i-1]}, ..., \lambda_{n-1}^{[i-1]}, 1 - \lambda_n^{[i-1]}, \lambda_{n+1}^{[i-1]}, ..., \lambda_{N_x}^{[i-1]}\right],$$

which essentially turns the n^* switch on from off or vice versa.

3.5.2 Drawing the Loading Candidate

Conditional on the factors and variances, the factor loadings can be drawn from a normal posterior given the normal prior, $N(b_0, B_0)$. Moreover, because the $x's$ are assumed to orthogonal conditional on the factors, we can draw the candidate loadings one at a time: $\beta_n^* \sim N(\mathbf{b}_n, \mathbf{B}_n)$, where

$$\mathbf{b}_n = \mathbf{B}_n^{-1}\left(B_0^{-1}b_0 + \sigma_n^{-2} f_T' \mathbf{x}_{nT}\right)$$

and

$$\mathbf{B}_n^{-1} = B_0^{-1} + \sigma_n^{-2} f_T' f_T.$$

3.5.3 Accepting the Draw

Once we have a set of proposals, we accept them with probability

$$A_{n,\gamma} = \min\left\{1, \frac{|\mathbf{B}^*|^{1/2}}{|\mathbf{B}|^{1/2}} \frac{\exp\left(\frac{1}{2}\mathbf{b}^*\mathbf{B}^{*-1}\mathbf{b}^*\right)}{\exp\left(\frac{1}{2}\mathbf{b}\mathbf{B}^{-1}\mathbf{b}\right)} \frac{\pi\left(\Lambda^*\right)}{\pi\left(\Lambda^{[i-1]}\right)} \frac{q\left(\Lambda^*\right)}{q\left(\Lambda^{[i-1]}\right)}\right\}, \tag{9}$$

where $\mathbf{b}^*$ and $\mathbf{B}^*$ are defined and $\mathbf{b}_n$ and $\mathbf{B}_n$ are defined for $\Lambda^{[i-1]}$.

3.6 Drawing the Factor

To implement the extended Kalman filter, we rewrite the model in its state space representation. The state variable is $\boldsymbol{\xi}_t = \mathbf{y}_t^p$ as defined above; let $\mathbf{Y}_t = [\mathbf{z}_t', \mathbf{x}_t']'$. Then,

$$\begin{aligned} \mathbf{Y}_t &= H\boldsymbol{\xi}_t + \mathbf{e}_t, \\ \boldsymbol{\xi}_t &= G\left(\boldsymbol{\xi}_{t-1}\right) + \mathbf{v}_t, \end{aligned}$$

where

$$H = \begin{bmatrix} \mathbf{I}_{N_z+1} & \mathbf{0}_{N_z\times 1} & \mathbf{0}_{N_z\times N_c} \\ \mathbf{0}_{N_x\times N_z+1} & \boldsymbol{\Lambda}\odot\boldsymbol{\beta} & \mathbf{0}_{N_x\times N_c} \end{bmatrix},$$

$\mathbf{e}_t = \left[\mathbf{0}_{N_z\times 1}', \mathbf{u}_t'\right]'$, $\mathbf{v}_t = \left[\boldsymbol{\varepsilon}_t', \mathbf{0}_{(N_c+1)\times 1}'\right]'$, $N_c = (N_x+1)(P-1)$, $E_t\mathbf{e}_t'\mathbf{e}_t = \mathbf{R}$, and $E_t\mathbf{v}_t'\mathbf{v}_t = \mathbf{Q}$. Note that, in general, both $\mathbf{Q}$ and $\mathbf{R}$ will be singular. The function $G(.)$ is

$$G\left(\boldsymbol{\xi}_{t-1}\right) = [1 - \pi_t\left(f_{t-1}\right)]A_1(L) + \left(\pi_t\left(f_{t-1}\right)\right)A_2(L)]\,\mathbf{y}_{t-1},$$

which is nonlinear in the state variable.

We can then draw $\boldsymbol{\xi}_T \sim p\left(\boldsymbol{\xi}_{T|T}, \mathbf{P}_{T|T}\right)$ which is obtained from the extended Kalman filter (EKF). The EKF utilizes a (first order) approximation of the nonlinear model.The EKF, then, uses the familiar Kalman prediction and update steps to generate the posterior distributions for the state variable, $\boldsymbol{\xi}_t \sim p\left(\boldsymbol{\xi}_{t|t}, \mathbf{P}_{t|t}\right)$. The distribution $\boldsymbol{\xi}_{T-1} \sim p\left(\boldsymbol{\xi}_{T-1|T}, \mathbf{P}_{T-1|T}\right)$ is obtained via smoothing and preceding periods are drawn recursively.

3.7 Drawing $\boldsymbol{\sigma}^2$ conditional on $\boldsymbol{\Psi}_{-\sigma^2}, \mathbf{Z}_T$ and $\mathbf{X}_T$

Given the inverse gamma prior, the measurement variances can be drawn from an inverse gamma posterior, $\sigma_i^{-2} \sim \Gamma(\omega_i, W_i)$, where

$$\omega_i = \frac{1}{2}(\omega_0 + T),$$

$$W_i = \frac{1}{2}(W_0 + \mathbf{u}_{it}\mathbf{u}'_{it}),$$

and

$$\mathbf{u}_{it} = \mathbf{x}_{it} - \Lambda_i f_t.$$

4 Empirical Results

4.1 Data

To measure financial stress through its effects on the transition dynamics of macroeconomic variables, we require two sets of data. First, we need financial data with which we can search for common fluctuations. Second, we need a set of macroeconomic variables. For the former, we consider a unbalanced panel consisting of a vector of 23 financial series also used in Hatzius et al. (2010). These financial indicators include term spreads, risk spreads, Treasury rates, commercial paper rates, and survey data. The data ends in September, 2012. All variables are monthly, and described in Table 1. The selection of variables encompass all subgroups described in Hatzius et al. (2010), Brave and Butters (2012) and Kliensen et al. (2012).

Because the financial data are monthly, we use the monthly growth rate in industrial production as our main economic indicator. We also include a monthly inflation measure, the rate of change of headline CPI. Both series are seasonally adjusted.

4.2 Financial Conditions Factor and Transition Function.

The factor estimated within a FASTVAR model differs from the one obtained with a typical dynamic factor model or with a FAVAR by allowing for smooth transition nonlinearity and financial variables (covariate) selection. We start by exploiting the relative importance of each one of these features in the estimation of the factor. Table 2 presents the posterior mean of each element in Λ using the FASTVAR and also an equivalent linear FAVAR. Both models use a VAR of order one. The covariate selection is more clear-cut in the case of the linear specification with many values equal to either zero or one. The selection within the FASTVAR model differs from the one with FAVAR. This is an initial evidence that nonlinearity matters for estimating the financial conditions factor. An interesting result is that slope of the yield curve (10y3msp) is not chosen with both models while the predictability of the slope for U.S. recessions is frequently reported (Rudebusch and Williams, 2009). A possible explanation is that we are selecting variables for short horizon predictability (one month), while the slope is more important at longer horizons (one year).

Figure 1 includes the estimates of three financial condition factors from different modelling approaches. Negative values are generally associated with recession phases. The FASTVAR estimates are the posterior means presented with 68 percent intervals. We also present posterior mean estimates using the linear specification with covariate selection (as in Table 2), and the FASTVAR with no covariate selection (factor computed with all 23 financial variables). The FASTVAR estimates with no covariate selection are normally not far from the estimates with covariate selection, but they are outside the FASTVAR 68 percent interval. The estimates obtaining with the linear specification tell a very different story, and the link with business cycle phases is not as clear. For example, financial conditions deteriorate during the 2001 and the 2008 recessions, but not in earlier recessions. The fact that the factor dynamics may change over time has a clear impact on measuring financial conditions.

Figure 2 plots the mean of the posterior distribution of the transition function, and 68 percent intervals. The transition function for each period is computed at each Gibbs iteration using the current draws of the transition parameters and the (lag of the) factor. As opposed to the Markov-switching VAR model, in the FASTVAR model, the economy can reside in the transition state between the two extreme regimes. These values represent the weights given to the upper regime. Values near zero imply that the economy is in the lower regime. Figure 2 also presents the transition function estimates when there is no covariate selection. The time-variation is not very different from the full FASTVAR model, but the transition function values are generally higher, implying that impulse response functions of these two models may differ.

At this stage, one may be tempted to define the lower regime as a financial stress regime; however, we still need to evaluate whether the transmission of financial shocks changes across regimes.

4.3 Impulse Responses

Figure 3 presents the responses of one-unit decrease in the financial condition index if the shock hits either in the lower or in the upper regime. These are generalized responses, that is, they allow for regime switching over horizons and are computed conditional on lower and upper regime histories as described in section 2.2.2. Looking at the factor estimates in Figure 1, we can say sizeable decreases in the financial condition indexes are normally associated with recession periods. The plots present the mean response over 700 draws from the posterior distributions of all FASTVAR parameters, and include 68% intervals.

The responses of IP growth and inflation are clearly asymmetric across regimes. Financial shocks have significantly more persistent effects on growth and inflation in the lower regime in contrast with the upper regime. As a consequence, we can classify the lower regime as the financial stress regime. The negative cumulative effects after two years in the financial stress regime are 40% larger for IP growth and 400% larger for inflation. The effect of financial shocks on the factor are also more persistent in the financial stress regime, but the difference is only significant at short horizons. We also find similar results if the shock is negative, but small (0.1).

The FASTVAR model is also able to generate asymmetric responses depending on the sign of the shock. Figure 4 presents the response of inflation and IP growth computed at the posterior mean values of the parameters for positive and negative one-unit shocks. It is clearly, in particular for inflation responses during the financial stress regime, that negative shocks have more persistent effects than positive shocks. This implies that shocks that improve financial conditions will have slower beneficial effects on growth and on raising inflation than shocks that deteriorate financial conditions if the economy is in the financial stress regime.

4.4 The Measure of Financial Stress

Based on the results so far, we can associate financial stress periods with periods in which negative financial shocks have stronger negative effects on growth and inflation and that positive shocks have a reduced effect. In this part, we investigate how financial stress periods identified using our novel methodology are related to NBER recession periods. Figure 5 presents NBER and financial stress periods, and also includes our estimated financial conditions factor.

The proportion of recession months between 1981M11 and 2012M9 is 13 percent, while the proportion of financial stress months is 27 percent. This implies that we have periods of financial stress during expansion months. This is expected since even if negative financial shocks hit during financial stress months, they may be small such that the economy would be still in an expansion. More interesting is the fact that we identify financial stress periods during all the four recessions in the period. This suggests that our measure of financial stress may work as a real-time identification of periods in which the economy is more subject to negative financial shocks that could, with a higher likelihood than normal times, lead to a recession. Although the financial stress measure is good to pick up the beginning of recessions, it is less useful to signal the end of recessions since financial stress periods normally finish four to six months after the end of each NBER dated recession.

5 Conclusions

The financial crisis emphasized the importance of identifying periods of high financial stress as these periods can have important and detrimental effects on the macroeconomy. In this paper, we construct an index of macro-financial conditions which—by design—includes only variables which alter the economic dynamics between financial conditions and macroeconomic variables such as industrial production and inflation.

We find that financial conditions, in general, do affect macroeconomic dynamics by altering the underlying state of the economy. In periods of high financial stress, the financial shocks have more persistent effects on macroeconomic variables.

We also find that the financial variables that do affect macroeconomic conditions tend to be risk spreads and survey measures of financial conditions. Many yield spreads are excluded.

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Table 1 – Financial Variables

	Description	Sample
10y	annual growth rate of the 10 year treasury rate	1981M9-2012M9
FFR3msp	fed fund rates - 3month tbill rates	1981M9-2012M9
2y3msp	2-year treasury rates – 3-month tbill rates	1981M9-2012M9
10y3msp	10-year treasury rates – 3-month tbill rates	1981M9-2012M9
baa10ysp	Baa corporate rates – 10-year treasury rates	1981M9-2012M9
30mort10ysp	30-year mortgage rates – 10-year treasury rates	1981M9-2012M9
tedsp	TED spread	1981M9-2012M9
creditsp	citibank corporate credit spread	1981M9-2012M9
exchrate	annual growth rate of the exchange rate	1981M9-2012M9
wilrate	annual growth rate of the Wishire 5000	1981M9-2012M9
houseinf	annual growth rate of the national house index	1981M9-2012M9
creditrate	annual growth rate of bank credit of commercial banks	1981M9-2012M9
compaperrate	annual growth rate of commercial paper outstanding	1981M9-2012M9
moneyrate	annual growth rate of money stock (zero maturity)	1981M9-2012M9
nfibsurv	%credit was harder to get than last time	1981M9-2012M9
migoodsurv	%good-%bad conditions for buying large goods	1981M9-2012M9
mihousesurv	%good-%bad conditions for buying a house	1981M9-2012M9
miautosurv	%good-%bad conditions for buying a car	1981M9-2012M9
vix	VIX (monthly average)	1990M1-2012M9
jumbospread	Jumbo rates - 30-year conventional rates	1998M6-2012M9
OIS spread	3-month libor rates - overnight index swap rates	2001M12-2012M9
highyieldspre	High-yield corporate rates – Baa corporate rates	1997M1-2012M9
oil price	price of oil relative to a 2-year moving average	1981M9-2012M9

Table 2 – Covariate Selection: Posterior Mean.

	Nonlinear (FASTVAR)	Linear (FAVAR)
10y	0.46	1.00
FFR3msp	0.72	1.00
2y3msp	0.44	1.00
10y3msp	0.38	0.00
baa10ysp	0.83	1.00
30mort10ysp	0.88	0.00
tedsp	0.84	0.98
creditsp	0.74	0.84
exchrate	0.68	0.00
wilrate	0.85	0.69
houseinf	0.62	1.00
creditrate	0.44	0.97
compaperrate	0.52	1.00
moneyrate	0.44	0.27
nfibsurv	0.71	0.06
migoodsurv	0.97	0.07
mihousesurv	0.83	1.00
miautosurv	0.77	1.00
vix	0.71	1.00
jumbospread	0.63	1.00
OIS spread	0.83	0.01
highyieldspre	0.97	1.00
oil price	0.50	0.17

Note: Based on 7000 draws of the posterior distribution (9000 draws with 2000 discharged).

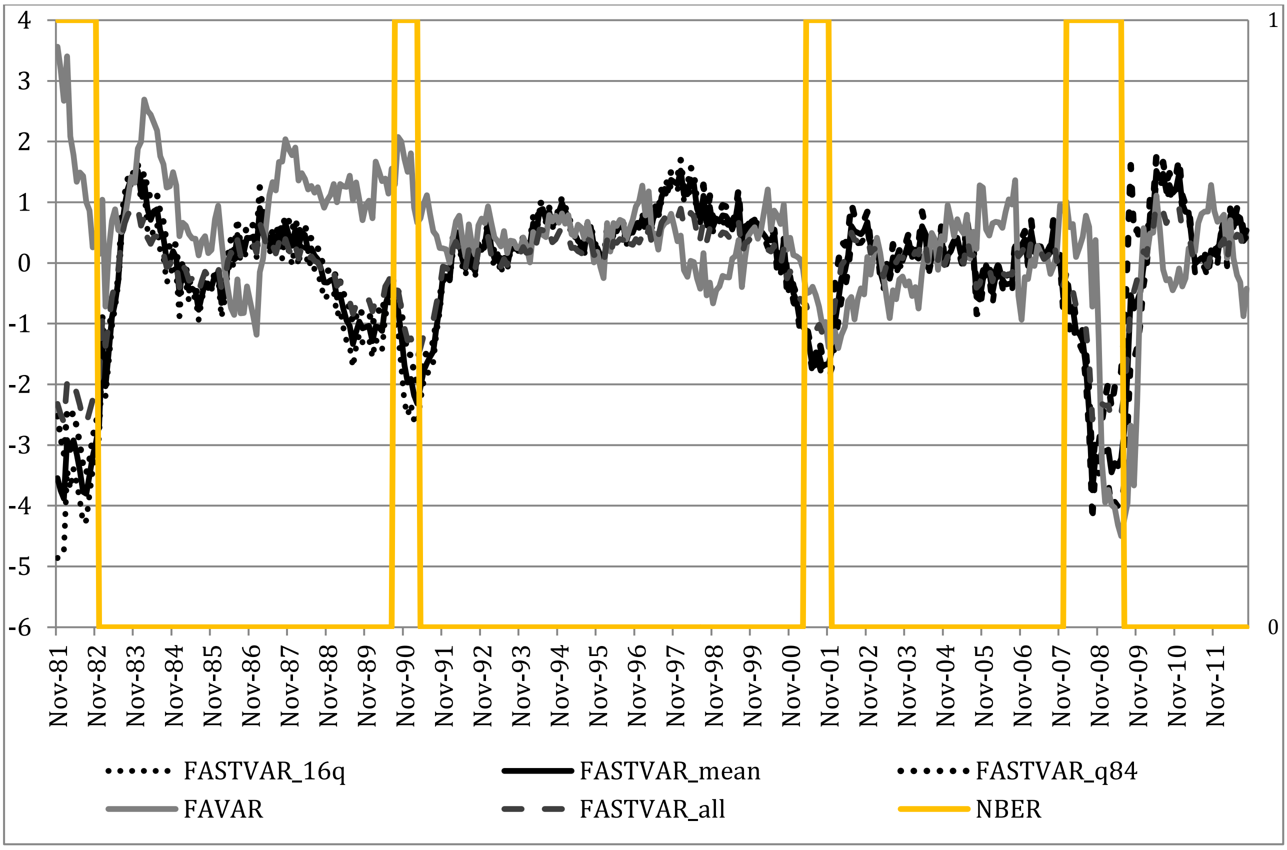

Figure 1: Comparing Financial Factor Measures: FASTVAR, FAVAR with covariate selection and FASTVAR with no covariate selection.

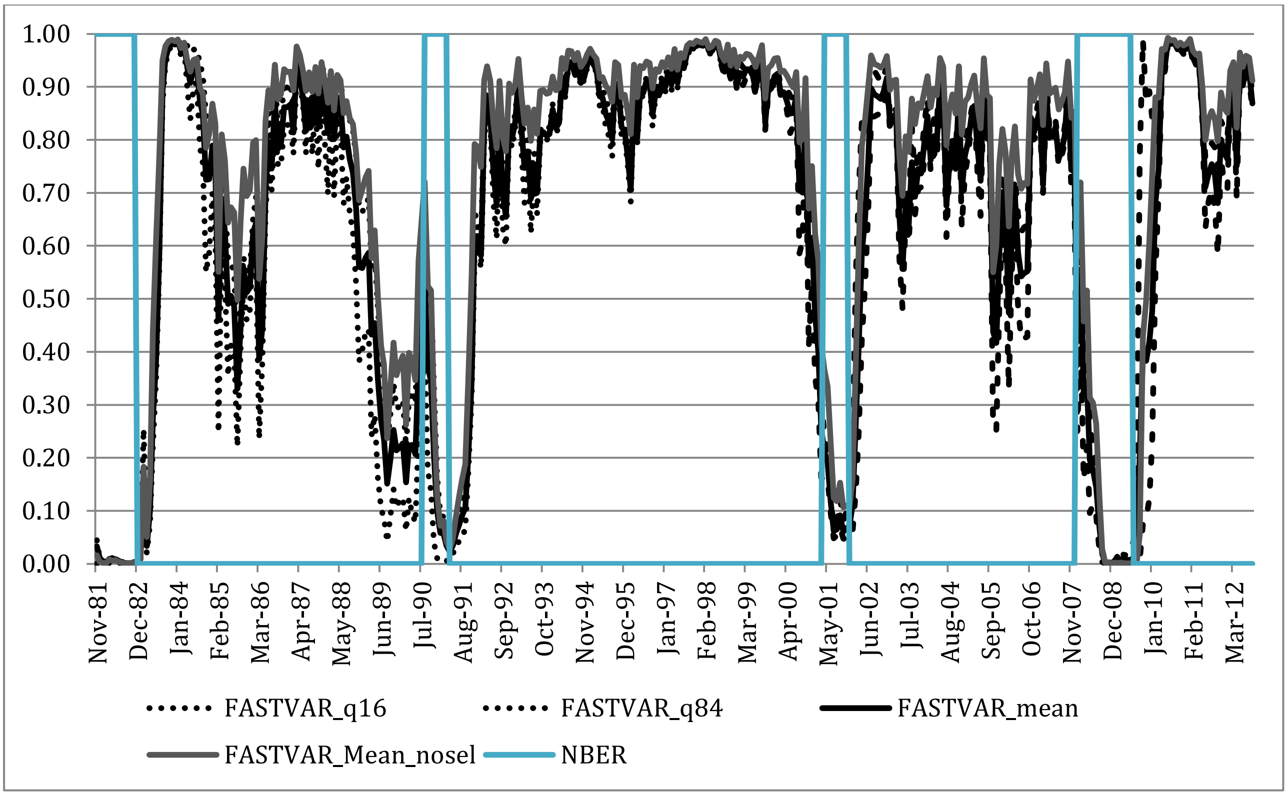

Figure 2: Transition Function values: FASTVAR and FASTVAR with no covariate selection.

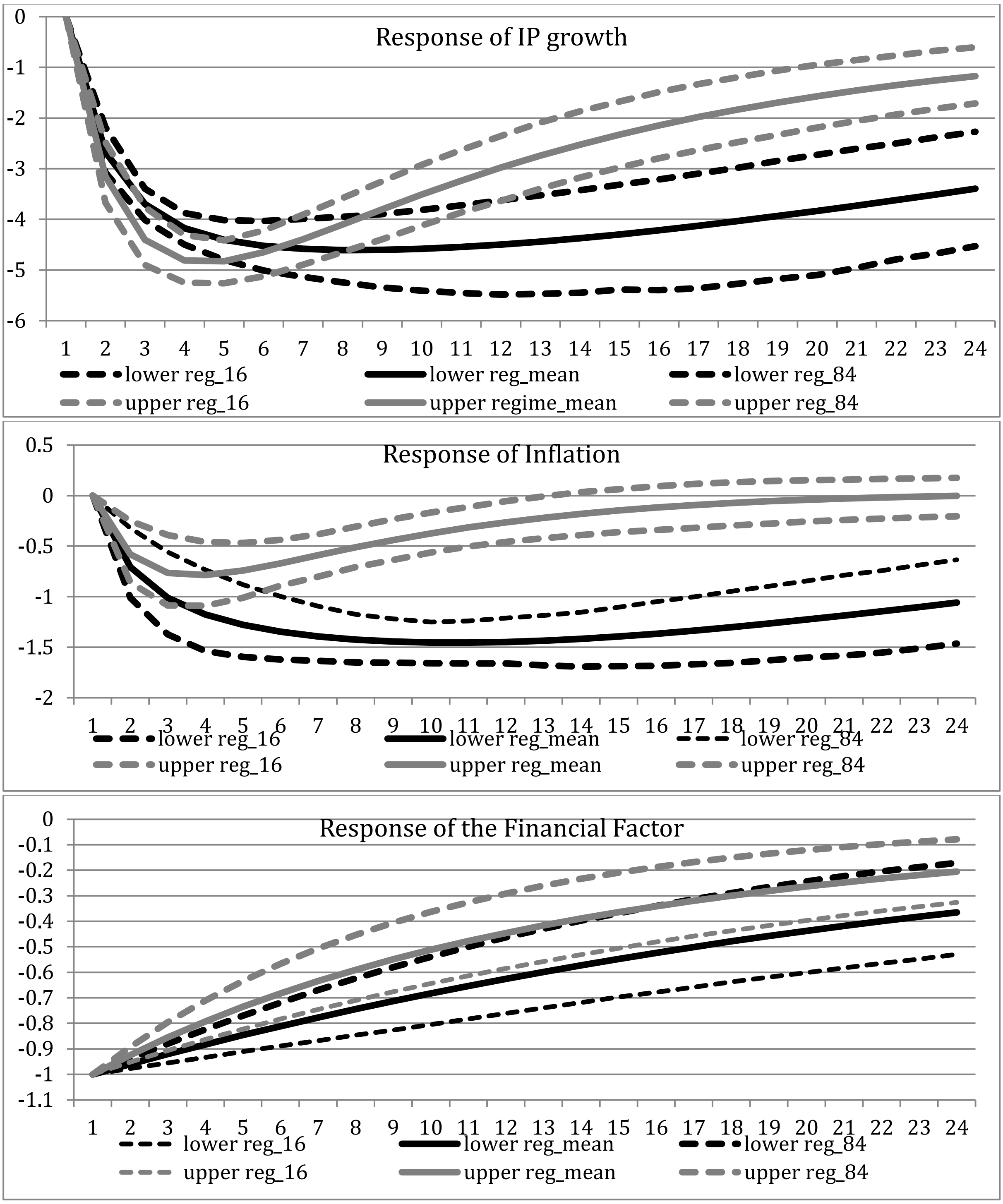

Figure 3: Responses to a -1p.p. shock to Financial Conditions in the lower and upper regimes (Computed with 700 equally-spaced draws of the posterior distribution of FASTVAR parameters.)

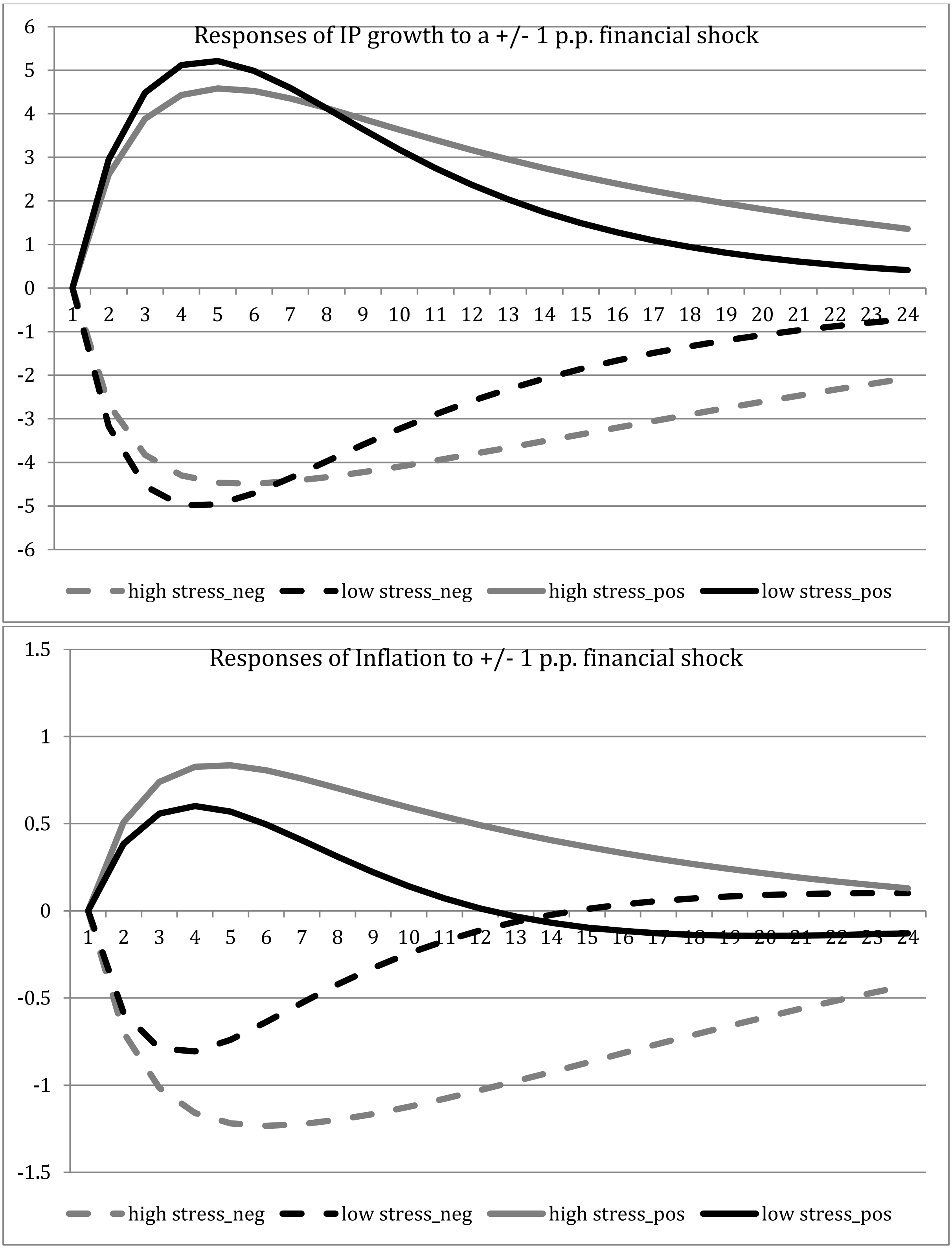

Figure 4: Asymmetries from different shock signs computed at the posterior mean of the FASTVAR parameters.

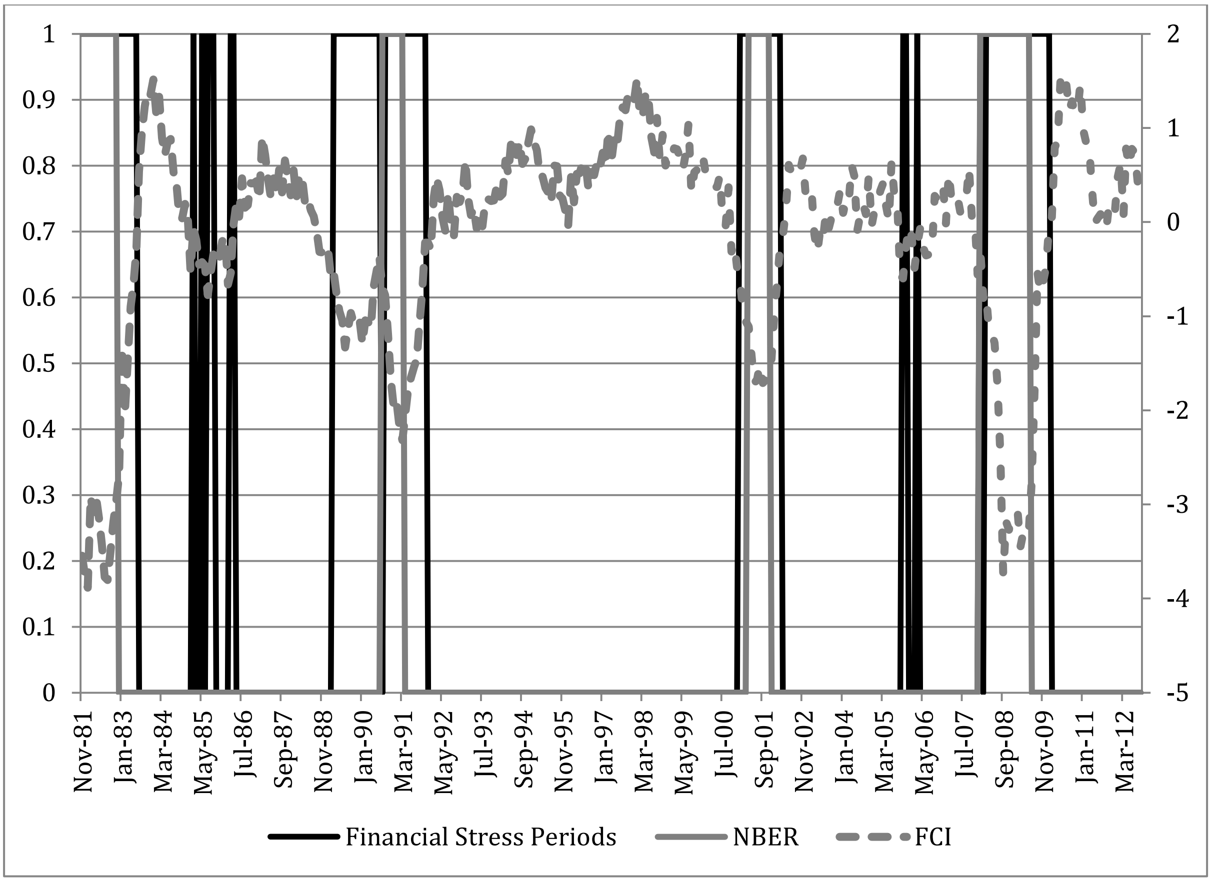

Figure 5: NBER recessions, Financial Stress Periods and the Financial Conditions Factor.