Labor mobility, structural change and economic growth*

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Abstract

We develop a two sector growth model where the process of structural change in the sectoral composition of both employment and GDP is jointly determined by non-homothetic preferences and a labor mobility cost. This cost is paid by workers when they move to another sector and, therefore, it limits structural change. The two sectors are identified as the agriculture and non-agriculture sectors. We show that this model can explain the following patterns of development of the US economy in the period 1880-2000: (i) balanced growth of the aggregate variables in the second half of the last century; (ii) the process of structural change in the sectoral composition of employment; (iii) the process of structural change in the sectoral composition of GDP; (iv) convergence of the wages in the two sectors. We outline that the last two patterns can only be explained if we introduce the labor mobility cost. We also show that this cost generates a misallocation of production factors, implying a loss of GDP. We calibrate the model and we quantify that this loss amounts more than 30% of the GDP in the initial periods. During the transition, the loss of GDP decreases and, eventually, vanishes. Therefore, the elimination of the misallocation explains part of the increase in the GDP. We finally highlight that the misallocation introduces a mechanism through which cross-country differences in sectoral composition contribute to explain cross-country income differences.

JEL classification codes: O41, O47.

Keywords: structural change, non-homothetic preferences, labor mobility.

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1. Introduction

Recent multisector growth literature has built models aimed to explain both the balanced growth of aggregate variables and the process of structural change that we observe in most developed economies (see Acemoglu and Guerrieri, 2008; Boppart, 2014; Dennis and Iscan, 2008; Melck, 2002; Foellmi and Zweimuller, 2008; Kongsamut, Rebelo and Xie, 2001; Ngai and Pissariadis, 2007). On the one hand, the balanced growth of aggregate variables consists of an almost constant ratio of capital to GDP and an almost constant interest rate. On the other hand, the process of structural change consists of a large shift of both employment and production from agriculture to other sectors. This process, that is a common characteristic of most economies, is illustrated in the first two columns of Table 1 for the US economy during the period 1880 to 2000.

The aforementioned literature explains both balanced growth of aggregate variables and the process of structural change in the sectoral composition of employment. This literature can be divided in two lines. One line outlines that demand factors are the driving force of structural change (Kongsamut, Rebelo and Xie, 2001). These demand factors consist of income effects generated by non-homothetic preferences that drive structural change as the economy develops. The other line argues that supply factors are the driving force of structural change (Acemoglu and Guerrieri, 2008; Ngai and Pissariadis, 2007). These factors consist of changes in relative prices that through a substitution effect cause structural change. More recently, the literature combines both demand and supply factors in order to explain structural change (Boppart, 2014; Dennis and Iscan, 2008). However, none of these papers explains the magnitudes of the two patterns of structural change, the shift in employment and production from agriculture to other sectors. Buera and Kabosky (2009) argue that this literature does not explain these two features because it does not introduce sector specific factor distortions.

In this paper, we show that the two features of structural change can be explained when factor distortions cause sectoral wages differentials. In order to motive this conclusion, we use the definition of the labor income share (LIS) at the sectoral level and we decompose the ratio between the LIS in the agriculture sector and the LIS in the non-agriculture sector as the product of the following three ratios: the ratio between wages in the agriculture and non-agriculture sectors, the ratio between the employment shares in agriculture and in the non-agriculture sector and the ratio between the GDP shares in the non-agriculture and agriculture sectors.[1] If we assume that wages across sectors are equal, the first ratio equals one. In this case, we can use the US data for the sectoral composition of employment and GDP shown in Table 1 to compute the value of the ratio between the two sectoral LIS that is compatible with the process of structural change in both employment and GDP. Table 1 shows that the value of this ratio should be equal to 2.15 in the year 1880 and decreases until 1.05 in the year 2000. These values are problematic for two reasons. First, they show a declining long run trend in the ratio of

[1] The LIS in sector i is defined as $LIS_i = w_i L_i / P_i Y_i$ where w_i is the wage in sector i, L_i is the number of employed workers in this sector, P_i is the relative price and Y_i is the production in this sector. Using this definition, it is straightforward to obtain that the ratio between the LIS in sectors a and n is $LIS_a/LIS_n = (w_a/w_n)(u_a/u_n)\kappa_n/\kappa a$, where u_i is the employment share in sector $i = a, n$ and κ_i is the share of GDP produced in sector $i = a, n$.

LIS, which can only be explained if we consider large departures from the Cobb-Douglas production function. These departures are not supported by empirical estimates of the long run sectoral production functions.[2] Second, the value of the ratio between the two sectoral LIS that is consistent with the process of structural change is completely different from actual estimates of this ratio, which set its value approximately equal to 0.68.[3] This suggests that the two features of structural change cannot be explained if we assume that wages are equal across sectors. Moreover, empirical evidence clearly shows that wages are different across sectors, specially when we consider the agriculture and non-agriculture sectors (see Helwege, 1992; Caselli and Colleman, 2002; and Herrendorf and Schoellmany, 2014). Table 1 shows the relative wage between agriculture and non-agriculture sectors. As follows from the table, wages are lower in the agriculture sector and during the last century there has been a clear convergence of wages. However, still wage differentials across sectors are large. Using this data on relative wages, we compute the ratio between the sectoral LIS that is consistent with the two features of the process of structural change when wages are not equal across sectors. The last column of Table 1 shows this ratio. Note that after 1920 the value of this ratio is close to its empirical estimates and there are not trends.[4] This numerical analysis suggests that the introduction of sectoral wage differences is necessary to explain the two features of structural change.

The purpose of this paper is to show that a simple multisector growth model can explain the two features of structural change when wages do not equalize across sectors. To this end, we develop an exogenous two sector growth model with two features. First, preferences are non-homothetic due to the introduction of minimum consumption requirements, as in Kongsamut, et al. (2001) or Alonso-Carrera and Raurich (2014). Second, wages across sectors do not equalize. Differences of wages across sectors have been explained as the result of (i) differences in human capital across sectors (Caselli and Colleman, 2002; Herrendorf and Schoellmany, 2014); (ii) barriers to mobility (Hayashi and Prescott, 2008) or (iii) labor mobility cost (Lee and Wolpin, 2006; Raurich, et. al, 2014).[5] In this paper, we introduce a labor mobility cost.[6]

The labor mobility cost amounts for any cost that workers moving to another sector must pay. This amounts for reallocations expenses (transport and housing cost), formal training cost necessary to acquire the skills used in another sector or an opportunity

[2] Herrendorf, Herrington and Valentinyi (2014) estimate the elasticity of substitution between capital and employment and show that it is 1.58 for the agriculture sector, 0.8 for the manufacturing sector and 0.75 for the service sector. They conclude that Cobb-Douglas sectoral production functions capture the main technological forces in the US postwar structural change.

[3] This value is obtained from Valentinyi and Herrendorf (2008) that use data for the US in the period 1990-2000.

[4] Before 1920 data on relative wages is controversial as has been explained by Caselli and Colleman (2001). Therefore, meausurement errors in the value of relative wages may explain the low values of the ratio of LIS before 1920.

[5] Lee and Wolpin (2006) estimate that in the US mobility costs are substantial when mobility is across sectors (between 50% and 75% of average annual earnings). Artuc, et. al (2015) in a more recent paper estimate a substantially larger labor mobility cost when developing economies are considered. They show that this cost is on average 370% the annual wage in developing countries.

[6] Gollin, Lagakos, and Waugh (2014) show that the agriculture sector has a lower labor productivity once we control for human capital and number of hours employed. This suggests that the labor mobility cost explains part of the wage differences.

cost (working time lost looking for a job in a different sector). As moving out of the agriculture sector generally implies moving from a rural to an urban area, we assume that the relevant labor mobility cost is associated to reallocation expenses. As a consequence, we will assume that the unitary labor mobility cost is a fix cost and that it is only taken into account in the resource constraint of the non-agriculture sector. Artuc et al. (2015) estimate labor mobility cost for both developed and developing economies and they show that this cost as a fraction of annual wage is larger in developing economies. This implies that the labor mobility cost as a fraction of GDP declines along the development process. Note that this pattern is consistent with the assumption of a fix unitary labor mobility cost.

The introduction of the labor mobility cost divides the labor market in two sector specific labor markets. The labor supply in each market is determined by the existing number of workers in each sector and, thus, it is determined by the sectoral employment share. The labor demand in each market depends on the demand of consumption goods in every sector that in a model with non-homothetic preferences depends on economic development. In every period, market clearing determines the wages paid in each sector. Therefore, sectoral wage differences exist because the labor mobility cost prevents workers from moving instantaneously to the higher wage sector. However, as the economy develops, the labor mobility cost as a fraction of the GDP declines, which causes sectoral wage convergence.

The process of structural change is driven by both demand and supply factors. On the one hand, due to the non-homotheticity of preferences, the sectoral composition of consumption expenditures changes as the economy develops. Obviously, this is the classical demand factor explained in Kongsamut, et al. (2001). Economic development reduces the effect of the minimum consumption requirement on the sectoral composition and, eventually, this effect vanishes. As a consequence, preferences are homothetic in the long run, which implies that the equilibrium converges to a balanced growth path (BGP) in the long run. On the other hand, the supply factor is based on wage convergence, instead of the standard mechanism in the literature, based on relative price changes. Wage convergence implies that wages in the agriculture sector grow faster than wages in the non-agriculture sector. As a consequence, firms in the agriculture sector substitute labor for capital, making the technology more capital intensive and pushing workers out of the agriculture sector. This is the supply mechanism in this paper.

We calibrate this model to explain the process of structural change in the US, during the period 1880-2000. From numerical simulations, we show that the model explains (i) the convergence to a BGP, while there is structural change;[7] (ii) the process of structural change in the sectoral composition of employment; (iii) the process of structural change in the sectoral composition of GDP; and (iv) the convergence of wages across sectors. We outline that in the absence of the labor mobility cost the model does not explain sectoral wage convergence, nor the process of structural change in the sectoral composition of GDP.

The differences in sectoral wages introduce a misallocation of production factors: the sector with larger wages has a larger capital intensity. This misallocation causes a

[7]We follow Acemoglu and Guerrerie (2008) and Alonso-Carrera and Raurich (2014) and we claim that an equilibrium follows a BGP with structural change when the growth rate of capital to GDP is almost null, whereas the growth rate of the employment share is clearly different from zero.

loss of GDP. This GDP loss is not due to an inefficiency as in Restuccia, et. al. (2008), where barriers cause a GDP loss. Instead, it must be interpreted as the reduction in GDP with respect to the level that would be attained in the absence of the labor mobility cost. Intuitively, moving a worker from a low to a high wage sector increases the GDP. Therefore, the GDP loss will depend on the wage gap between the two sectors and on the size of the low wage sector (the agriculture sector). Both the wage gap and the size of the low wage sector were large in the US in the XIX century, implying a large GDP loss. We use the numerical simulation to account for the GDP loss and it turns out that it was about 30% of GDP in the last twenty years of the XIX century, it declines during the transition and eventually vanishes. As a consequence, part of the increase in the GDP during the transition, specially in the initial periods, is explained by the elimination of the misallocation.

The GDP loss introduces a mechanism through which cross-country differences in the sectoral composition of employment cause cross-country differences in income per capita. This mechanism implies that those countries specialized in the low wage sector, the agriculture sector, will suffer from a lower GDP. This conclusion is also obtained in Gollin, Parente and Rogerson (2004, 2007). In these papers, the specialization in the low wage sector is explained by home production or minimum consumption requirements. In contrast, in this paper, this specialization is explained as the result of either a larger intensity of the minimum consumption requirement or a larger labor mobility cost. The later can be the result of labor market regulations or larger reallocation expenses. The former can be explained by a lower level of development.

We measure the contribution of the misallocation in explaining cross-country income differences. We show that this contribution is not constant through time and it crucially depends on the initial source of differences in income per capita. We also show that the contribution of the misallocation is sizeable. It explains up to 18% of GDP differences when economies are initially differentiated by levels of technology and it explains up to 50% of GDP differences when economies are initially differentiated by the stock of capital.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 numerical solves the model and obtains the main results. Finally, concluding remarks are in Section 5.

2. Model

We build an exogenous two-sector growth model. We distinguish between the agriculture and the non-agriculture sector. We assume that the later is the numeraire of the economy and produces both a consumption good and an investment good. The agriculture sector only produces a consumption good.

2.1. Household

The economy is populated by an infinitely lived representative household, formed by a continuum of L members. Every member inelastically supplies one unit of time and the number of members is assumed to be constant. Therefore, the labor supply is inelastic, constant and equal to L. The household obtains income from capital and labor. This income is devoted to either consumption, investment or paying the cost of moving to another sector. Therefore, the budget constraint of the household is

$$rK + [w_a (1-u) + uw_n] L = (pc_a + c_n) L + \dot{K} + \pi \dot{u} L, \tag{2.1}$$

where r is the rental price of capital, K is the stock of capital, w_a is the wage obtained in the agriculture sector, w_n is the wage obtained in the non-agriculture sector, u is the fraction of workers employed in the non-agriculture sector, p is the relative price of agriculture goods in units of non-agriculture goods, c_a are the units consumed of goods produced in the agriculture sector, c_n are the units consumed of goods produced in the non-agriculture sector, π is the constant unitary labor mobility cost that every worker moving to another sector pays, and $\dot{u}$ is the fraction of workers that move every period.[8] We assume that the investment good is produced in the non-agriculture sector and, therefore, its relative price is normalized to one.

The representative households's utility function is

$$U = \int_0^{\infty} e^{-\rho t} L \left[\theta \ln (c_a - \widetilde{c}_a) + (1-\theta) \ln c_n\right] dt, \tag{2.2}$$

where $\widetilde{c}_a > 0$ is the minimum consumption requirement of agriculture goods; $\rho > 0$ is the subjective discount rate; and $\theta \in (0,1)$ measures the weight of the agriculture goods in the utility function. Note that this utility function is non-homothetic when $\widetilde{c}_a \neq 0$.

The representative household chooses the sectoral composition of consumption expenditures, the amount of consumption expenditures and the number of workers that every period move to the non-agriculture sector in order to maximize the utility function (2.2) subject to the budget constraint (2.1). By standard procedure, in Appendix A we find the first order conditions and rearrange them to summarize the necessary conditions for optimality in the following three conditions:

$$v = \theta + \frac{\widetilde{E}}{E} (1-\theta), \tag{2.3}$$

$$\frac{\dot{E}}{E} = \left(\frac{E - \widetilde{E}}{E}\right)(r - \rho) + \left(\frac{\widetilde{E}}{E}\right)\frac{\dot{p}}{p}, \tag{2.4}$$

and

$$w_n - w_a = r\pi, \tag{2.5}$$

where $E = pc_a + c_n$ is the value of consumption expenditures, $v = pc_a/E$ is the expenditure share in agriculture goods and $\widetilde{E} = p\widetilde{c}_a$ is the value at market prices of the minimum consumption requirement. Equation (2.3) determines the expenditure share in agriculture goods. Note that this share would be constant and equal to θ if $\widetilde{c}_a = 0$. In contrast, if $\widetilde{c}_a > 0$, preferences are non-homothetic and the sectoral composition of consumption expenditures decreases as the economy develops and consumption expenditure increases. This mechanism is the classical demand factor driving structural

[8] We write the budget constraint assuming that workers move from the agriculture to the non-agriculture sectors. This pattern of structural change is obtained in equilibrium.

change. Equation (2.4) is the Euler condition governing the intertemporal decision between consumption and savings. Finally, equation (2.5) is a non-arbitrage condition between two investment decisions: investment in capital goods and investment in moving out of the agriculture sector. The left hand side is the return from investing π units in moving a worker to another sector. The right hand side is the return from investing in capital π units. This non-arbitrage condition implicitly determines the number of workers moving out of the agriculture sector in every period and, thus, it determines the relative labor supplies in both sectors.

2.2. Firms

We assume that both sectors produce with the following constant returns to scale Cobb-Douglas technologies:

$$Y_a = [(1-s)K]^{\alpha_a} [A_a (1-u) L]^{1-\alpha_a} = A_a (1-u) L z_a^{\alpha_a}, \tag{2.6}$$

and

$$Y_n = (sK)^{\alpha_n} (A_n u L)^{1-\alpha_n} = A_n u L z_n^{\alpha_n}, \tag{2.7}$$

where $\alpha_a \in (0,1)$ and $\alpha_n \in (0,1)$ are, respectively, the capital output elasticities in the agriculture and non-agriculture sector, A_a and A_n are efficiency units of labor, s is the fraction of capital devoted to the non-agriculture sector, and $z_a = (1-s) K/A_a (1-u) L$ and $z_n = sK/A_n uL$ measure, respectively, capital intensity in the agriculture and non-agriculture sectors. We assume that efficiency units of labor grow in both sectors at the exogenous growth rate γ, implying that technological progress is unbiased and that the long run growth rate of GDP is γ. Finally, perfect competition implies that each production factor is paid according to its marginal product

$$w_i = A_i p_i (1-\alpha_i) z_i^{\alpha_i}, \tag{2.8}$$

and

$$r = p_i \alpha_i z_i^{\alpha_i - 1} - \delta, \tag{2.9}$$

where $\delta \in [0,1]$ is the depreciation rate and $i = a, n$.

As capital can move across sectors without cost, the marginal product of capital is identical across sectors. In contrast, the introduction of the labor mobility cost implies that wages can be different across sectors. We define the relative wage between the two sectors by $\lambda = w_a/w_n$. From using (2.8) and (2.9), we obtain that

$$z_a = \left(\frac{\lambda \psi A_n}{A_a}\right) z_n, \tag{2.10}$$

and

$$p = \left(\frac{\alpha_n}{\alpha_a}\right) \left(\frac{\lambda \psi A_n}{A_a}\right)^{1-\alpha_a} z_n^{\alpha_n - \alpha_a}, \tag{2.11}$$

where

$$\psi = \left(\frac{\alpha_a}{\alpha_n}\right) \left(\frac{1-\alpha_n}{1-\alpha_a}\right).$$

Equation (2.10) shows that the relationship between the sectoral capital intensities depends on the relative wage. As the economy develops, the relative wage increases, which causes an increase in the capital intensity of the agriculture sector relative to the capital intensity of the other sector. The intuition is as follows. An increase in the relative wage implies that wages in the agriculture sector increase relative to wages in the non-agriculture sector. As a consequence, firms in the agriculture sector choose a more capital intensive technology, by substituting labor for capital. This mechanism describes the supply factor driving structural change. This supply factor is different from the supply mechanism proposed by the literature and based on relative price changes caused by either biased technological change (Ngai and Pissariadis, 2007) or capital deepening and differences in capital output elasticities (Acemoglu and Guerrieri, 2008). This different supply mechanism has two relevant implications.

First, equation (2.11) shows that the relative price depends on (i) the relative wage, (ii) the ratio between the efficiency units of labor in the non-agriculture sector and the efficiency units of labor in the agriculture sector, and (iii) capital deepening. The new supply mechanism introduced in this paper implies an increase in the relative price of agriculture. In contrast, both the biased technological mechanism and capital deepening implies a reduction in this price. On the one hand, empirical evidence on TFP growth shows that TFP growth is larger in the agriculture sector and, thus, biased technological change reduces the relative price.[9] On the other hand, capital deepening implies a reduction in the relative price because estimates of the sectoral capital output elasticities suggest that this magnitude is larger in the agriculture sector (See Herrendorf and Valentinyi, 2008). As a consequence, this sector benefits the most from capital deepening, which causes the reduction in the relative price. As in the model we combine two different supply mechanisms, capital deepening with wage convergence, the relative price can either increase or decrease along the development process. Interestingly, this is consistent with the observed differences in the patterns of relative prices along the development process.[10]

Second, wage convergence implies that as the economy develops, the agriculture sector becomes a more capital intensive sector. This contributes to explain cross-country differences in sectoral capital intensities that clearly show that the agriculture sector is more relatively capital intensive in developed economies (see Alvarez-Cuadrado, et al., 2013). It should be noted that a classical supply mechanism would not explain this evidence. From using (2.10) and the definitions of z_n and z_a, it follows that neither different capital output elasticities, nor biased technological change can explain cross-country differences in relative capital intensities when the production function is Cobb-Douglas. To the best of our knowledge, cross-country differences in sectoral capital intensity has only been explained by Alvarez-Cuadrado et al (2013). Using CES production functions, they explain these differences as a result of different sectoral elasticities of substitution between capital and employment. We therefore offer a complementary explanation based on wage convergence. Note that wage convergence

[9] Dennis and Iscan (2009) show that TFP growth in the agriculture sector is larger than TFP growth in the non-agriculture sector in the US economy after 1930.

[10] Dennis and Iscan (2009) provide evidence that in the US relative prices of agriculture increase during the XIX century and decrease after 1920. **(Jaime, podrias mencionar como los precios han prsentado comportamientos dispares en diferentes economías).**

contributes to explain differences in sectoral capital intensities even if the production function is Cobb-Douglas.

3. Equilibrium

The non-agriculture sector produces a commodity that can be used either as a consumption good, as an investment good or to move to a different sector and, therefore, the resource constraint in this sector is

$$Y_n = Lc_n + \dot{K} + \delta K + \dot{u} L \pi.$$

The agriculture sector only produces a consumption good and thus the market clearing condition in this sector is $Lc_a = Y_a$, which can be rewritten as

$$1 - u = \frac{c_a}{A_a z_a^{\alpha_a}}. \tag{3.1}$$

Let $z = K/A_n L$ be the stock of aggregate capital per efficiency unit of labor in the economy. Thus, z measures the capital intensity of the economy. Using the definition of z, we obtain that

$$z_n = sz/u \tag{3.2}$$

and $z_a = (1-s) A_n z / (1-u) A_a$. From the last equation and (2.10), we obtain that

$$z_n \lambda \psi (1-u) = (1-s) z. \tag{3.3}$$

From using the equilibrium condition in the capital market and equations (3.2) and (3.3), we obtain

$$\frac{z}{z_n} = \lambda \psi (1-u) + u = \phi, \tag{3.4}$$

where ϕ measures the capital intensity of the economy relative to the capital intensity of the non-agriculture sector.

Note that without a mobility cost, wages will equalize implying that $\lambda = 1$ and $\phi = \psi(1-u) + u \equiv \phi^*$. However, the labor mobility cost implies that during the transition $\lambda < 1$ and then $\phi < \phi^*$. Therefore, the introduction of the labor mobility cost, by increasing the wages of the non-agriculture sector, increases capital intensity of this sector relative to the capital intensity of the economy. Thus, the labor mobility cost introduces a factors' misallocation, which is measured by the gap between ϕ and ϕ^* and it is

$$\phi^* - \phi = (1-u)\psi(1-\lambda).$$

The misallocation will cause a GDP loss. In order to see this, we define GDP as $Q = pY_a + Y_n$. Using (2.10), (2.11) and (3.4), GDP can be rewritten as

$$Q = \Omega \phi^{-\alpha_n} K^{\alpha_n} (A_n L)^{1-\alpha_n} = \Omega \phi^{-\alpha_n} z^{\alpha_n} A_n L, \tag{3.5}$$

where

$$\Omega = \left(\frac{\alpha_n}{\alpha_a}\right) \lambda \psi (1-u) + u = \left(\frac{\alpha_n}{\alpha_a}\right) \phi + u \left(\frac{\alpha_a - \alpha_n}{\alpha_a}\right), \tag{3.6}$$

and $\Omega\phi^{-\alpha_n}$ measures the sectoral composition component of the total factor productivity (TFP), which is given by $\Omega\phi^{-\alpha_n}A_n^{1-\alpha_n}$.[11]. By defining by Q^* the GDP level attained when $\lambda = 1$, we measure the GDP loss as a percentage of GDP by

$$\frac{Q^* - Q}{Q} = \left(\frac{\Omega^*}{\Omega}\right)\left(\frac{\phi^*}{\phi}\right)^{-\alpha_n} - 1,$$

where Ω^* is the value of Ω when $\lambda = 1$. Note that the loss of GDP depends on λ and on the the employment shares in agriculture $1 - u$. In the numerical simulations of Section 4, we show that the GDP loss has declined in the US during the last century as a result of both wage convergence and the decline of the employment share in the agriculture sector.

An important remark that follows from the expression of the TFP is that differences in the sectoral composition of employment cause differences in the TFP when either there are differences in capital output elasticities or there are differences in wages across sectors. If we had assumed that $\alpha_a = \alpha_n$ and that $\lambda = 1$, then $\Omega\phi^{-\alpha_n} = 1$ and differences in the sectoral composition would not imply differences in TFP levels. In other words, TFP increases when economies specialize in sectors with larger capital output elasticities or in sectors with larger wages. In the numerical analysis, we compare economies with different sectoral composition and we decompose the fraction of income differences explained by differences in sectoral wages and the fraction explained by differences in capital output elasticities. From this numerical analysis, we show that the main mechanism explaining income differences is based on differences in sectoral wages.

3.1. Sectoral Composition

In this subsection, we obtain the sectoral composition of consumption expenditures and of the employment shares and the relative wage, λ, as a function of the expenditure to GDP ratio, $e = E/Q$, the capital intensity, $z = K/A_nL$, the intensity of the minimum consumption requirement, measured by the ratio $\widetilde{e} = \widetilde{E}/Q$, and the intensity of the labor mobility cost, measured by $m = \pi/A_n$. Note that as the economy develops, both the intensity of the minimum consumption requirement and of the labor mobility cost declines and, eventually, converge to zero.

First, we use (2.3) and the definitions of e and $\widetilde{e}$ to obtain the expenditure shares

$$v = \theta + \frac{\widetilde{e}}{e}(1 - \theta). \tag{3.7}$$

The employment shares are obtained from combining (3.1), (2.10), (2.11) and (3.7) as follows

$$1 - u = v\left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{E}{\lambda\psi A_n L z_n^{\alpha_n}}\right).$$

Using (3.4) and (3.5), the previous equation simplifies as follows

$$1 - u = ve\left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{\Omega}{\lambda\psi}\right).$$

[11] Note that the aggregate production function is not Cobb-Douglas during the transition when there is structural change. In contrast, in the long run, when there is no structural change, it converges to a Cobb-Douglas production function.

From using the previous equations and the definition of Ω, we obtain the following expression of the employment share in the agriculture sector:

$$1 - u = \left(\frac{\alpha_a}{\alpha_n}\right)\left(\frac{veu}{\lambda\psi\,(1 - ve)}\right), \tag{3.8}$$

and

$$\Omega = \frac{u}{1 - ve}. \tag{3.9}$$

According to equation (3.8), sectoral change is driven by demand factors, measured by ve, and supply factors, measured by λ. Note that this equation provides a negative relationship between the employment share in the agriculture sector and λ. This relationship is explained by the reduction in the demand of workers of the agriculture sector due to the increase in the relative wage.

From using (2.7), (3.4), and (3.5), we obtain

$$\frac{Y_n}{Q} = \frac{u}{\Omega}. \tag{3.10}$$

The variable Ω determines the relationship between the agriculture share of GDP and the employment share in the agriculture sector. From using (3.6), the variables Ω can be rewritten as

$$\Omega = 1 + (1 - u)\left(\frac{1 - \alpha_n}{1 - \alpha_a}\right)(\lambda - 1) + (1 - u)\left(\frac{\alpha_a - \alpha_n}{1 - \alpha_a}\right).$$

Note that if there is no misallocation and there are no technological differences, then $\Omega = 1$. In this case, the relation between the employment share and the GDP share will be constant and indeed these two shares will be equal. However, as follows from Table 1, this is not consistent with actual data for the US economy. According to this data, the agriculture share of GDP is larger than the employment share, implying that the value of Ω should be smaller than one. As follows from the previous expression of Ω, the misallocation reduces the value of Ω and thus makes the model consistent with actual data. On the contrary, technological differences increase the value of Ω, as the agriculture sector is more capital intensive than the non-agriculture sector.[12] This analysis suggests that the misallocation must be introduced in order to explain the two dimensions of structural change.

The relative wage is obtained from the market clearing in the labor market. To obtain it, we rewrite the labor supply, (2.5), as follows

$$w_n - \lambda w_n = \pi r.$$

[12] Following Herrendorf and Valentinyi (2008), $\alpha_n = 0.33$ and $\alpha_a = 0.54$. Then, $\Omega = 1.2$ in the US during the period 1880-1900 when we assume that $\lambda = 1$. In this period, the average value of $u = 0.58$ and of $Y_n/Q = 0.75$. According to these values, Ω should be 0.75. Thus, in the absence of missallocation the sectoral composition of both GDP and employment cannot be explained. In fact, a value of Ω that is consistent with the sectoral composition of employment and GDP in the period 1880-1900 is attained when $\lambda = 0.31$.

Rearranging terms and using the labor demand, (2.8), and equations (2.9) and (3.4), we obtain

$$\lambda = 1 - m \left(\frac{\alpha_n \left(\frac{z}{\phi}\right)^{\alpha_n - 1} - \delta}{(1-\alpha_n)\left(\frac{z}{\phi}\right)^{\alpha_n}} \right). \tag{3.11}$$

In Appendix B we use (3.11) to obtain the relative wage in equilibrium and prove the following result.

Proposition 3.1. *The relative wage and the employment share satisfy*

$$\lambda = \widehat{\lambda}(e, z, m), \tag{3.12}$$

and

$$u = \frac{\lambda\psi\left(\frac{\alpha_n}{\alpha_a}\right)(1-ve)}{ve + \lambda\psi\left(\frac{\alpha_n}{\alpha_a}\right)(1-ve)}. \tag{3.13}$$

Furthermore, $\partial\lambda/\partial m < 0$ *and* $\partial u/\partial\lambda > 0$.

As follows from Proposition 3.1, λ is a decreasing function of the intensity of the labor mobility cost and u is an increasing function of λ. Therefore, a large mobility cost implies that the relative wage will be smaller and the agriculture share of employment will be larger. Both effects imply that the GDP loss increases with the labor mobility cost.

3.2. Equilibrium Dynamics

In a supplementary appendix we obtain a full system of differential equations characterizing the time path of the transformed variables: z, e, m and $\widetilde{e}$. Given initial conditions $\widetilde{e}_0$, m_0 and z_0, an equilibrium is a path of $\{e, \widetilde{e}, z, m, \lambda, v, u, \phi\}$ that solves this system of differential equations and satisfies equations (3.7), (3.12), (3.4), and (3.13), and the transversality condition $\lim_{t\to\infty} \frac{K}{c_n} e^{-\rho t} = 0$. Moreover, we define a balanced growth path (BGP) as an equilibrium along which both the ratio of capital to GDP and the interest rate remain constant.

Proposition 3.2. *There is an unique BGP along which the variables* $\{e, \widetilde{e}, z, m, \lambda, v, u, \phi\}$ *remain constant and its long run value is* $\widetilde{e}^* = 0, m^* = 0, \lambda^* = 1, v^* = \theta$,

$$\begin{aligned} e^* &= \frac{1-\alpha_n\Delta}{1+\Delta(\alpha_a-\alpha_n)\theta}, \\ u^* &= \frac{\psi\alpha_n(1-\theta e^*)}{\alpha_a\theta e^* + \psi\alpha_n(1-\theta e^*)}, \\ \phi^* &= \psi(1-u^*) + u^*, \end{aligned}$$

and

$$z^* = \left(\frac{\gamma+\delta+\rho}{\alpha_n}\right)^{\frac{1}{\alpha_n - 1}} \phi^*,$$

where $\Delta = (\delta+\gamma)/(\delta+\rho+\gamma)$. *Furthermore, this BGP is saddle path stable.* [13]

[13] The proof is available in a supplementary appendix.

Note that this BGP is attained asymptotically, as $\widetilde{e}^*$ and m^* converge to zero. Wages converge and, therefore, both the misallocation and the GDP loss disappear in the BGP. Moreover, along the BGP there is no structural change. Thus, asymptotically the economy converges to an equilibrium along which the interest rate and the ratio of capital to GDP remain constant and there is no structural change. As this only happens asymptotically, the analysis of transitional dynamics is particularly relevant. In the following section, we numerically analyze the transition and we show that aggregate variables exhibit a period of unbalanced growth followed by a long period in which they exhibit an almost constant time path of the interest rate and of the ratio of capital to GDP. We show that there is structural change during this period. We will then conclude that in this economy we observe (almost) balanced growth of aggregate variables and structural change.

The equilibrium is characterized by three state variables: capital intensity, z, intensity of the minimum consumption requirements, $\widetilde{e}$, and the intensity of the labor mobility cost, m. Saddle path stability implies that given initial conditions on these three state variables there is an unique equilibrium path converging to the steady state. In the following section, the uniqueness of the equilibrium path is used to calibrate and simulate the economy.

4. Transitional dynamics analysis: structural change

In this section we numerically simulate the economy in order to show the effects of the mobility cost on the process of structural change. To this end, we calibrate the parameters of the economy as follows. The parameter $\gamma = 2\%$ in order to have a long run GDP growth rate equal to 2%; we set $\alpha_a = 0.54$ and $\alpha_n = 0.33$ as estimated by Herrendorf and Valentinyi (2008); $\theta = 0.01$ is set to fit the long run expenditure share in agriculture obtained in Herrendorf, et al. (2013); $\rho = 0.32$ so that the long run interest rate equals 5.2%; $\delta = 5.6\%$ in order to obtain a long run ratio of investment to capital equal to 7.6%. We compare two different numerical simulations. In both simulations we assume that $z_0 = 0.75z^*$. The initial value of this state variable mainly determines the length of the transition of aggregate variables. We choose an initial value which is consistent with an almost constant time path of the interest rate and of the ratio of capital to GDP in the last 50 years of the simulation. The distinction between the two simulations is in the labor mobility cost. In a first numerical simulation, we assume that there is no mobility cost ($m_0 = 0$) and we set the initial condition on the other state variable, $\widetilde{e}_0 = 0.59$, to match the employment share in the US in the initial year 1880. In a second simulation, we assume that there is labor mobility cost and we set the initial conditions on the two state variables, $\widetilde{e}_0 = 0.28$ and $m_0 = 14.3$, to match both the value of the employment share in agriculture and the share of GDP produced in the agriculture sector in the US in the year 1880. The parameters and initial conditions of the two simulations are summarized in Table 2.

[Insert Table 2]

Figure 1 shows the first numerical simulation in which we assume that there is no mobility cost. In this case, wages equalize across sectors implying that the relative wage is equal to one and, thus, the simulation does not explain wage convergence. This implies that there is no misallocation and thus there is no GDP loss. Panel i shows that this simulation explains almost all the decline in the employment share in the agriculture sector. However, the model does not provide a reasonable explanation of the process of structural change in the sectoral composition of GDP as it overestimates the share agriculture in GDP during all the transition. In order to see this, we can rewrite (3.10) as $\Omega = u\left(Q/Y_n\right)$. From Table 1, we observe that $u < Y_n/Q$ implying that Ω should be substantially lower than one in order to explain the two dimensions of structural change. Note that Ω can be rewritten as

$$\Omega = 1 + (1-u)\left(\frac{\alpha_a - \alpha_n}{1-\alpha_a}\right) + (1-u)\left(\frac{1-\alpha_n}{1-\alpha_a}\right)(\lambda - 1).$$

The second addend amounts for the effect of the different capital output elasticities on Ω, whereas the third one amounts for the effect of the relative wage. In our calibration, $\alpha_a > \alpha_n$, implying that the second addend is positive. As a consequence, when the relative wage equals one, Ω is larger than one and the model fails to explain the two dimensions of structural change. This explains that the first numerical simulation overestimates the value of the production share and the performance of the simulation in explaining the process of structural change in the sectoral composition of GDP is poor.

[Insert Figure 1]

Figure 2 displays the second simulation, where a mobility cost is introduced. This cost, as a ratio of GDP, declines from 8% of GDP in the initial year to zero in the long run. This ratio declines both because GDP increases and because the process of sectoral change declines in the long run. The simulation explains the declining path of the employment share in the agriculture sector, the declining path of the share of GDP produced in the agriculture sector and the process of wage convergence. Moreover, the simulation explains almost all the decline in both the employment share in the agriculture sector and the agriculture share of GDP. Regarding wage convergence, the simulation explains the convergence of the relative wage, but it is not able to explain the level of this relative wage. We interpret this as partial evidence that there are other relevant explanations of the sectoral wage differences different from the mobility cost.[14] Finally, the performance of this simulation in explaining the process of structural change in the sectoral composition of GDP is clearly better than the previous simulation.

[Insert Figure 2 and Table 3]

Table 3 provides three measures of performance in order to compare the two simulations. As follows from these measures, the accuracy of both simulations in

[14] Candidates to explain the slow convergence in wages are metapreferences associated to working in one sector or different skills across sectors, among others. Other authors argue that wage differences can be explained by differences in the cost of living between urban and rural areas (see Esteban-Pretel and Sawada, 2014). Assuming that workers in urban areas are employed in the non-agriculture sector, whereas workers in rural areas can be employed in the agriculture sector, permanent sectoral wage differences are aimed to compensate for the differences in the cost of living.

explaining the process of structural change in the sectoral composition of employment is similar. For example, the coefficient of determination is 75% in Simulation 1 and 78% in Simulation 2. Thus, the performance is very similar and only slightly better in Simulation 2. A similar conclusion is attained if we compute the fraction of the reduction in the employment share of the agriculture sector in the period 1880-2000 explained by both simulations. Simulation 1 explains 96% of the reduction, whereas Simulation 2 explains 97%. The conclusion is completely different when we consider the performance of both simulations in explaining the process of structural change in the sectoral composition of GDP. Simulation 1, based on the absence of the labor mobility cost, has a very poor performance. In this simulation, the coefficient of determination is negative and the fraction of the reduction in the GDP share in the period 1880-2000 is 197%, implying that the simulated reduction in the GDP share almost doubles the reduction in actual data. In contrast, Simulation 2, based on the introduction of the labor mobility cost, has a very good performance. The coefficient of determination is 96% and the fraction of the reduction explained by this simulation is 91%. We can then safely conclude that the model with mobility cost provides a substantially better explanation of the process of structural change.

As explained in the previous section, the mobility cost introduces a misallocation of production factors that causes a loss of GDP. Panel (iv) in Figure 2 provides a measure of this loss as a percentage of GDP. This loss amounts initially 35% of GDP and declines and converges to zero as both the sectoral wage differences vanish and the labor share in the agriculture sector declines. Therefore, the elimination of the misallocation explains part of the increase in GDP during the transition.

The labor mobility cost also modifies the time path of the growth rate of GDP. As follows from Panel (vi) in Figure 2 the time path of the growth rate is hump-shaped. This finding is interesting as it is consistent with the observed development patterns.[15] Christiano (1989) and, more recently, Steger (2000, 2001) explain this hump-shaped patterns in models with minimum consumption requirements. In these models, a sufficiently intensive minimum consumption requirement deters initially investment, which explains the initial low growth. As the economy develops, the intensity of the minimum consumption requirement declines and both investment and growth initially increase. Eventually, the interest rate declines due to diminishing returns to capital and thus capital accumulation and the growth rate decline until they converge to its long run value. We contribute to this literature by showing that the hump-shaped growth pattern can be explained by the interaction of both capital accumulation and labor mobility. In this model, a large intensity of the labor mobility cost explains the initial low labor mobility and also a low initial capital accumulation. As this intensity declines, capital accumulation increases and the GDP loss declines due to the increase in the number of workers leaving the agriculture sector. These two changes imply an increase in the growth rate of GDP. Finally, diminishing returns to capital and labor imply that capital accumulation and labor mobility decline and finally converge, which explains the reduction in the growth rate of GDP. Note that both mechanisms, minimum consumption requirements and labor mobility cost, introduce complementary explanations of the hump-shaped time path of the GDP growth rate. Interestingly,

[15] Papageorgiou and Perez-Sebastian (2005) show that some fast growing economies exhibit a hump-shaped transition of the GDP growth rate.

the calibrated economy of Simulation 1, where there is no labor mobility cost, does not explain the hump-shaped time path of the GDP growth rate. This outlines the relevance of the complementarity between the two mechanisms in explaining the time path of the GDP growth rate.

[Insert Table 4]

An important stylized fact of the patterns of development in the US economy since the second half of the last century is the balanced growth of the aggregate variables. In this period, the interest rate and the ratio of capital to GDP remain almost constant, while the sectoral composition of employment and GDP change. In order to show that our simulations are consistent with this pattern of development, we follow Acemoglu and Guerrieri (2008) and we compute the average annual growth rate of the ratio of capital to GDP, the interest rate, the employment share in the agriculture sector and the agriculture share of GDP during the second half of the twenty century. Results are displayed in Table 4. As follows from this table, in both simulations the annual growth rate of both the interest rate and of the ratio of capital to GDP is almost null, which is consistent with the balanced growth of the aggregate variables. The annual growth rate of the employment share and of the GDP share are larger than 1% and consistent with actual data. Thus, the calibrated model is consistent with balanced growth and structural change.

[Insert Figure 3]

Figure 3 shows the simulated time path of the employment share when both demand and supply factors drive structural change (dashed line) and when only demand factors drive structural change (continuous line). The former employment share is obtained in Simulation 2 and the later is obtained by assuming that the relative wage does not increase. From the comparison between the two cases, it follows that demand factors explain most of the reduction in the employment share in the period 1880-2000. In fact, the supply factor based on wage convergence only explains 12% of the reduction in the employment share in the whole period. In contrast, wage convergence explains a much larger part of the reduction during the transition. As an example, wage convergence explains 40% of the fall in the share during the period 1880-1920.

4.1. Sensitivity Analysis

The purpose of this subsection is to increase our understanding of the effects of the minimum consumption requirement and of the labor mobility cost. To this end, we consider three exercises where we modify the value of these parameters in the benchmark of the calibrated economy of Simulation 2.

The first exercise is displayed in Figure 4. This figure shows the effects of changing the initial intensity of the minimum consumption requirement by comparing three economies that are differentiated only by the initial value of $\widetilde{e}_0$. The continuous line shows the calibrated economy of Simulation 2. In this benchmark economy, $\widetilde{e}_0 = 0.28$. The dashed line is an economy with a lower value of the initial intensity of the minimum consumption requirement, $\widetilde{e}_0 = 0.15$, and the doted line is an economy with almost zero initial intensity, $\widetilde{e}_0 = 0.01$. As follows from Panels i and iii of Figure 4, a larger minimum consumption requirement implies that both the employment share and the agriculture share of GDP are larger. This larger demand of labor in the agriculture sector implies that the relative wage is initially larger (see Panel ii). However, in this economy, wage convergence is slower. This happens because a larger initial intensity of the minimum consumption requirement reduces the willingness of agents to substitute intertemporally and, thus, in these economies agents are less willing to reduce current consumption to invest in either capital or in moving to a different sector. As a consequence, the reduction in the employment share of the agriculture sector is at a lower rate. Obviously, this explains the slower wage convergence.

[Insert Figures 4, 5 and 6]

Figure 5 shows the effects of changing the labor mobility cost by comparing three economies that have a different unitary mobility cost, π. The continuous line displays the benchmark economy of Simulation 2. The dashed lines displays an economy with a labor mobility cost that is 25% smaller than in the benchmark economy and the dotted line displays an economy with a mobility cost that is 75% smaller than in the benchmark economy. As follows from the comparison between these economies, a lower mobility cost implies a lower amount of workers in the agriculture sector (see Panel i), a larger relative wage (see Panel ii), a smaller GDP loss due to the misallocation (see Panel iv) and a lower mobility cost as a percentage of GDP (see Panel v). These differences in the GDP loss will affect the GDP growth rate, as shown in Panel vi. Note that the time path of growth rate in the low mobility cost economy is not hump-shaped. This happens because the reduction of the GDP loss is almost null in this economy and, as mentioned before, this reduction is necessary to explain the hump-shaped time path of the GDP growth rate. Note also that in the economy with a low mobility cost the GDP growth rate converges from below, as the growth rate is below its long run value almost every period. In the following section, we show that this low growth rate is explained by the negative impact that the sectoral composition has on the growth rate in those economies where there is a small reduction of the GDP loss.

Figure 6 compares three economies that are different in both the initial intensity of the labor mobility cost and of the minimum consumption requirement. The continuous line displays the benchmark economy of Simulation 2. The dotted lines displays an economy without labor mobility cost and the dashed line displays an intermediate situation with a positive but small labor mobility cost. In these economies, the initial intensity of the minimum consumption requirement has been calibrated to have the same initial employment share and, in fact, they exhibit a similar time path of the employment share (see panel i). However, there are relevant differences in the transitional dynamics of the other variables due to the fact that these economies have a different labor mobility cost. A larger mobility cost implies a smaller relative wage and, therefore, it implies a larger GDP loss (see panels ii and iv). Note that economies exhibiting a similar process of structural change in employment will exhibit different levels of GDP due to the differences in the GDP loss generated by the misallocation. The misallocation can be observed from the comparison between the employment share and the share of GDP produced in the agriculture sector. Those economies with a larger

GDP loss are economies with a lower agriculture share of GDP. In these economies, workers employed in the agriculture sector are extremely unproductive as follows from the comparison between the employment share and the GDP share (see Panels i and iii). This explains the lower level of GDP. We conclude from this analysis that in order to understand the effects of sectoral composition on GDP, multisector growth models must explain the sectoral composition of both employment and also GDP. Clearly, multisector growth models that only explain the time path of the employment share are not adequate to analyze the effects of structural change on GDP, as they do not consider the differences in productivity across sectors.

4.2. Development Patterns

The purpose of this subsection is to compare economies with different levels of development in order to understand how structural change contributes to explain differences between GDP levels. Therefore, in this section we work out two development exercises in which we explain the differences in GDP levels between two economies as the result of differences in technology, capital per capita and in the sectoral composition of employment. As mentioned in this paper, the sectoral composition affects the GDP through two different mechanisms: the misallocation and differences in capital output elasticities. According to the misallocation, a larger employment share in the agriculture sector reduces GDP per capita, as this sector has a lower wage. In contrast, according to the second mechanism, a larger employment share in agriculture increases GDP per capita, as capital output elasticity is larger in the agriculture sector. In Appendix C we obtain the contribution of each mechanism in explaining cross-country differences in GDP levels.

[Insert Figures 7 and 8]

Differences in development across economies has been typically explained by either differences in the level of technology or by differences in the capital stock.[16] In this section, we consider both. First, in Figures 7 and 8 we compare two economies, rich and poor, that are different only in the level of technology. The poor economy is the benchmark economy of Simulation 2 and the rich economy is build by assuming a technology level that is twice the level of the benchmark economy. Figure 7 compares these two economies by displaying the time path of several variables. As follows from Panel i, the poor economy devotes a larger fraction of employment to the agriculture sector. Due to this larger labor demand in the agriculture sector, the relative wage is initially larger in the poor economy (see Panel ii). In the more advance technological economy, labor mobility is larger as it is a richer economy.[17] This implies that the labor mobility cost is initially larger in the rich economy (see Panel iv) and the reduction of the employment share in the agriculture sector is faster. As a consequence, the relative wage converges faster in the rich economy, which implies that eventually the relative wage becomes larger in the rich economy. In Panel iv, the GDP loss is initially

[16] As explained in the previous subsection, this model can also explain different levels of development as the result of different minimum consumption requirements or different labor mobility costs.

[17] We can interpret this larger mobilty as the consequence of the introduction of new technologies (tractors) that accelarete structural change.

similar in both economies. This occurs because the effect on the GDP loss of a larger employment share in the poor economy is compensated by the initially larger relative wage. However, the differences in the GDP loss increase during the transition, as the rich economy experiments a faster reduction in the GDP loss. This is driven by the faster reduction in employment share and the faster wage convergence. Finally, the differences in the time path of the GDP loss explain the differences in GDP growth rates (see Panel vi). This again outlines the importance of the reduction in the misallocation to explain the patterns of GDP growth.

Figure 8 shows the differences in terms of GDP levels. Panel i displays the ratio of GDP between the rich and the poor economy. The initial GDP differences are explained only by technological differences (as shown in Panel ii, the contribution of technology to explain GDP differences is initially 100%). During the transition, there is a period of divergence followed by a period of convergence in the levels of GDP. This transition is explained by the impact that technological differences have on both capital accumulation and on the sectoral composition. On the one hand, the larger technological level implies a faster capital accumulation in the rich economy, which drives a permanent divergence in the GDP levels. As shown in Panel iii, the contribution of capital permanently increases. On the other hand, the larger technological level also implies a faster structural change in the rich economy. This faster structural change drives an initial period of divergence which is followed by a period of convergence. This is illustrated in Panel iv where the time path of the contribution of sectoral composition is hump-shaped. This hump-shaped contribution is obviously explained by the fact that eventually both economies convergence to the same sectoral composition and, thus, technological differences only have temporary effects on the sectoral composition (see Panel i in Figure 7). Note that the contribution of sectoral composition is sizeable, explaining up to 18% of the GDP differences between the two economies. Moreover, this contribution explains the period of convergence between the two economies. In fact, in the absence of the effect of sectoral composition, it would be a permanent divergence in the levels of GDP. As mentioned before, the contribution of sectoral composition on GDP differences is government by two different mechanisms: the misallocation and differences in capital output elasticities. In Panel v we show that the misallocation channel explains slightly more than 100% of the contribution of sectoral composition, implying that the other mechanism slightly reduces the contribution of sectoral composition. The reason is that capital output elasticity is larger in the agriculture sector and this reduces the GDP gap between the two economies, as the poor economy specializes in the agriculture sector.

[Insert Figures 9 and 10]

Figures 9 and 10 compare two economies that are different only in the initial level of capital. The rich economy has an initial capital stock that is 50% larger than the stock of capital in the poor economy. The poor economy is the benchmark economy of Simulation 2. As follows in the first two panels of Figure 9, the rich economy initially has a smaller employment share in the agriculture sector and a larger relative wage. This explains the initially smaller GDP loss in this economy. However, during the transition the accumulation of capital is stronger in the poor economy, as the stock of capital is initially smaller. This larger accumulation of capital requires a larger non-agriculture sector. As a consequence, the employment share in the non-agriculture sector eventually becomes slightly larger in the poor economy. It follows that the GDP loss becomes slightly smaller in the poor economy. These time paths of sectoral composition explain the results in Figure 10 that illustrate the contribution of capital and sectoral composition in explaining the differences in GDP levels between the two economies. Panel i shows that differences in GDP levels are transitory, as the differences in capital stocks vanish during the transition due to the faster accumulation of capital in the poor economy. The initial differences in GDP are explained by the contribution of capital (almost 50%) and the contribution of sectoral composition (slightly more than 50%). However, as differences in the sectoral composition of the two economies diminish, the contribution of sectoral composition decreases and it becomes negative when the employment share in the agriculture sector becomes larger in the rich economy. Obviously, the contribution of capital mirrors that of sectoral composition, implying that it is above 100% when the contribution of sectoral composition is negative. Finally, panels v and vi show that the mechanism through which sectoral composition affects the level of GDP is almost entirely the misallocation.

5. Concluding Remarks

We develop a two sector growth model where structural change is driven by both demand and supply factors. The demand factor is an income effect generated by non-homothetic preferences. The supply factor is a substitution effect generated by the change in the relative wage between the two sectors. In order to calibrate the economy, we identify the two sectors as the agriculture and non-agriculture sectors.

We show that this model can explain the following patterns of development: (i) balanced growth of the aggregate variables; (ii) structural change in the sectoral composition of employment; (iii) structural change in the sectoral composition of GDP; and (iv) convergence of the relative wage. We also show that in the absence of the labor mobility cost the last two patterns are not explained. We then conclude that a model of structural change should also include a theory of wage differentials that is consistent with the observed patterns of relative wages.

As wages are not equal across sectors, production factors are misallocated: the agriculture sector has smaller wages and lower capital intensity, whereas the non-agriculture sector has larger wages and a larger capital intensity. Obviously, this misallocation causes a loss of GDP. We measure this loss and we obtain that initially amounts 30% of the GDP. During the transition, the loss declines and, finally, vanishes. Therefore, the elimination of the misallocation explains part of the GDP growth, specially during the initial years of the transition, and affects the patterns of GDP growth.

The GDP loss introduces a relevant insight on the cross-country income differences: these differences can be explained by differences in the sectoral composition of employment when wages are different across sectors. In this paper, wage differences are explained by an exogenous and constant labor mobility cost. Future research should contribute to the understanding of the determinants of this labor mobility cost, which may include labor market regulations, fiscal policy, or geographical characteristics, among other determinants.

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Appendix

A. Solution of the representative household problem

The representative consumer maximizes the utility function (2.2) subject to the budget constraint (2.1). The Hamiltonian function is

$$\begin{aligned} H &= \theta \ln (c_a - \widetilde{c}_a) + (1-\theta) \ln c_n + \\ & \quad \eta \left\{ rK + \left[w_a (1-u) + u w_n \right] L - p c_a - c_n - \varphi L \pi \right\} + \mu \varphi \end{aligned}$$

where $\varphi = \dot{u}$. The first order conditions with respect to c_a, c_n, φ, K and u are, respectively,

$$\frac{\theta}{c_a - \widetilde{c}_a} = \eta p, \tag{A.1}$$

$$\frac{1-\theta}{c_n} = \eta, \tag{A.2}$$

$$\mu = \eta L \pi, \tag{A.3}$$

$$r - \rho = -\frac{\dot{\eta}}{\eta}, \tag{A.4}$$

$$\frac{\eta L}{\mu} (w_a - w_n) = \frac{\dot{\mu}}{\mu} - \rho. \tag{A.5}$$

From combining (A.1) and (A.2), we obtain (2.3) in the main text. We log-differentiate (A.1) to obtain

$$\frac{\dot{c}_a}{c_a - \widetilde{c}_a} + \frac{\dot{p}}{p} = -\frac{\dot{\eta}}{\eta} = \frac{\dot{c}_n}{c_n}$$

and from (A.3) we obtain

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{\eta}}{\eta}.$$

Using these two equations, we rewrite (A.4) as

$$r - \rho = \frac{\dot{c}_a}{c_a - \widetilde{c}_a} + \frac{\dot{p}}{p}, \tag{A.6}$$

and (A.5) as

$$\frac{w_a - w_n}{\pi} = \frac{\dot{\eta}}{\eta} - \rho. \tag{A.7}$$

Using the definition of E, (A.6) can be rewritten as (2.4) in the main text. Finally, (A.7) can be rewritten as (2.5) in the main text.

B. Proof of Proposition 3.1

From combining (3.8), (3.4) and the definition of Ω, we obtain (3.13). We substitute (3.4) and (3.13) into (3.11) and we use the definition of m to obtain

$$\left(\frac{1-\lambda}{\lambda \psi} \right) z \left(\frac{1-\alpha_n}{m} \right) \left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a} \lambda \psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}} \right) = \alpha_n - \delta \left(\frac{z}{\lambda \psi} \right)^{1-\alpha_n} \left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a} \lambda \psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}} \right)^{1-\alpha_n}.$$

This equation implicitly defines $\lambda = \widehat{\lambda}(e, z, m)$. From using the implicit function theorem, we obtain

$$\frac{\partial\lambda}{\partial m} = -\frac{\left(\frac{1-\lambda}{m^2}\right)\left(\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi\right)}{\left(\frac{1}{m}\right)\left[\left(\frac{1}{\lambda}\right)\frac{ve}{1-ve} + \lambda\frac{\alpha_n}{\alpha_a}\psi\right] + (1-\alpha_n)\,\delta\left(\frac{z}{\lambda\psi}\right)^{-\alpha_n}\left(\frac{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}\lambda\psi}{\frac{ve}{1-ve} + \frac{\alpha_n}{\alpha_a}}\right)^{-\alpha_n}\left(\frac{1}{\lambda}\right)\frac{ve}{1-ve}} < 0.$$

C. Development Accounting

The purpose of this appendix is to obtain the expression of the measures used in the development accounting exercises of Figures 8 and 10. To this end, we use (3.5) to decompose per capita GDP, $q = Q/L$, as $q = A_n^{1-\alpha_n}\Phi k^{\alpha_n}$,where $k = K/L$ is capital per capita, $A_n^{1-\alpha_n}\Phi$ amounts for the TFP and $\Phi = \Omega\phi^{-\alpha_n}$ measures the contribution of sectoral composition to the TFP. This contribution goes through two different mechanisms: sectoral differences in technologies $(\alpha_a \neq \alpha_n)$ and misallocation due to different sectoral wages $(\lambda \neq 1)$.

In what follows we explain the differences in GDP per capita between two economies (Rich and Poor) as the result of differences in technology, capital per capita, and sectoral composition. In a second step, we measure the relevance of the two mechanisms in the contribution of sectoral composition. In order to obtain to decompose the differences in GDP levels, we compute the rewrite the ratio of per capita GDPs as follows[18]

$$\log\left(\frac{q^R}{q^P}\right) = (1-\alpha_n)\log\left(\frac{A_n^R}{A_n^P}\right) + \log\left(\frac{\Phi^R}{\Phi^P}\right) + \alpha_n\log\left(\frac{k^R}{k^P}\right).$$

From this expression, we obtain the contribution to GDP of technology, capital per capita, and sectoral composition, that are, respectively,

$$C_A = \frac{(1-\alpha_n)\log\left(\frac{A_n^R}{A_n^P}\right)}{\log\left(\frac{q^R}{q^P}\right)} * 100,$$

$$C_\Phi = \frac{\log\left(\frac{\Phi^R}{\Phi^P}\right)}{\log\left(\frac{q^R}{q^P}\right)} * 100,$$

and

$$C_k = \frac{\alpha_n\log\left(\frac{k^R}{k^P}\right)}{\log\left(\frac{q^R}{q^P}\right)} * 100.$$

These magnitudes are displayed in Panels ii, iii, and iv of Figures 8 and 10.

We next measure the relevance of the two mechanisms determining the contribution of the sectoral composition. However, this decomposition cannot be done directly as these two mechanism generate complementaries. For our purpose, we follow the following steps:

[18] The superindex R amounts for the rich economy and the superindex P amounts for the poor economy

1. First, note that if $\alpha_a = \alpha_n$ and $\lambda = 1$ then $\Phi = 1$. This implies that we can decompose Φ as $\Phi = 1 + \Phi_\alpha + \Phi_\lambda$ where Φ_α measures the contribution of sectoral composition to GDP through sectoral different technologies and Φ_λ measures the contribution of sectoral composition to GDP through misallocation.

2. We obtain Φ_α from measuring the value of Φ when $\lambda = 1$ and u is the sectoral composition obtained when wages are different across sectors $(\lambda < 1)$. We, therefore, obtain Φ_α as follows:

$$\Phi_\alpha = \left(\Omega \phi^{-\alpha_n}\right)\big|_{\lambda=1} - 1 = \left[\left(\frac{\alpha_n}{\alpha_a}\right)\psi\left(1-u\right) + u\right]\left[\psi\left(1-u\right) + u\right]^{-\alpha_n}.$$

3. We compute the contribution of sectoral composition to GDP through misallocation (Φ_λ) by using Φ_α and Φ as follows

$$\Phi_\lambda = \Phi - \Phi_\alpha - 1.$$

4. We next compute the weight of the misallocation mechanism as the following ratio:

$$\varepsilon = \frac{\left(\frac{1+\Phi_\lambda^R}{1+\Phi_\lambda^P}\right)}{\left(\frac{\Phi^R}{\Phi^P}\right)}$$

   The numerator of this ratio measures the relative contribution of sectoral composition between the two countries due to the misallocation. Therefore, the ratio ε measures the fraction of the differences between the two countries in the contribution of the sectoral composition explained by the misallocation. We name this measure the weight of the misallocation mechanism and we display it in Panel v of Figures 8 and 10.

5. Finally, we compute the contribution of the misallocation to GDP as $C_{\Phi_\lambda} = \varepsilon C_\Phi$. This magnitude is displayed in Panel vi of Figures 8 and 10.

D. Figures and Tables

Table 1. Data on the US economy

	Ag. sh. of GDP	**Employ. share**	**Rel. wage**	**$\mathrm{LIS}_a/\mathrm{LIS}_n^*$**	**$\mathrm{LIS}_a/\mathrm{LIS}_n^*$**
1880-1900	0.251	0.412	0.203	2.151	0.438
1900-1920	0.174	0.304	0.257	2.082	0.535
1920-1940	0.117	0.222	0.333	2.169	0.723
1940-1960	0.071	0.135	0.413	2.021	0.834
1960-1980	0.041	0.049	0.602	1.202	0.723
1980-2000	0.021	0.022	0.697	1.054	0.735

Notes: Source. Historical statistics of the U.S; Caselli and Coleman (2002); Bureau of labor Statistic.
*This column shows the ratio of LIS obtained when wages are equal across sectors.
**This column shows the ratio of LIS obtained when wages are not equal across sectors.

Table 2. Parameters and initial conditions

Parameters	**Targets**
$\gamma = 0.02$	Long run growth rate of GDP is 2%
$\rho = 0.032$	Long run interest rate is 5.2%
$\theta = 0.01$	Long run expenditure share in agriculture*
$\delta = 0.056$	Long run ratio of capital to GDP is 7.6%
$\alpha_a = 0.54$	Labor income share in agriculture**
$\alpha_n = 0.54$	Labor income share in non-agriculture**
$L = 1;\ A_i(0) = 1$	Normalization
$z_0 = 0.75z^*$	Transition consistent with almost BGP.

	Initial conditions		**Targets. Year 1880**	
	$\widetilde{e}_0$	m_0	$u = 0.52$	$Y_n/Q = 0.73$
Sim. 1	0.59	0	$\surd$	$\times$
Sim. 2	0.28	14.3	$\surd$	$\surd$

Notes: *. Herrendorf, et al. (2013); **. Herrendorf and Valentinyi (2008).

Table 3. Performance of the simulations

	Employment Share			**Agriculture Share of GDP**		
	SSR	U-Theil	R^2	SSR	U-Theil	R^2
Sim 1	0.59	0.0636	0.758	1.097	0.2593	-0.0038
Sim 2	0.54	0.0607	0.78	0.031	0.0579	0.96

Table 4. Average annual growth rate in the last 50 years

	r	K/Q	$1-u$	pY_a/Q
Model 1	-0.08%	-0.04%	-1.81%	-1.77%
Model 2	-0.009%	-0.02%	-1.86%	-1.63%

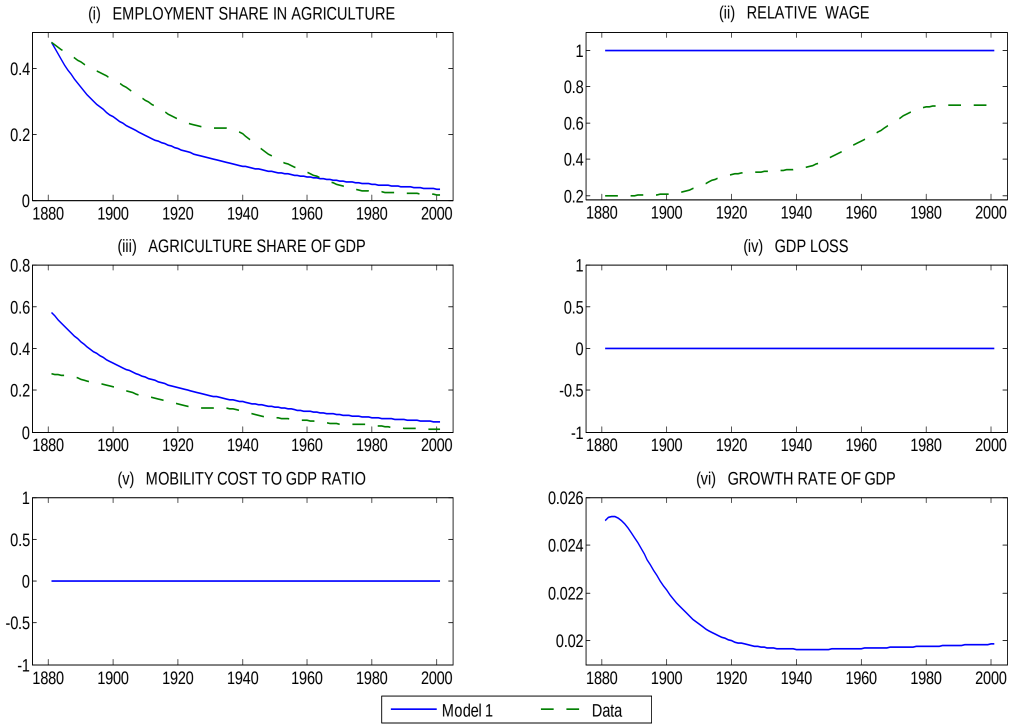

Figure 1. Numerical simulation without labor mobility cost

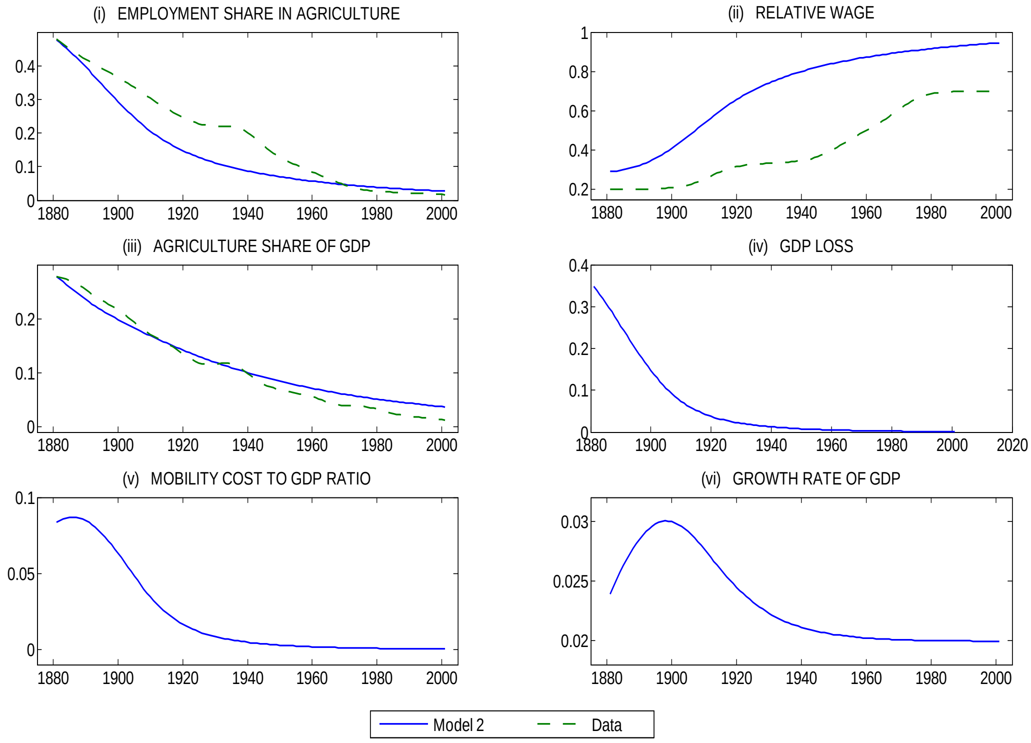

Figure 2. Numerical simulation with labor mobility cost

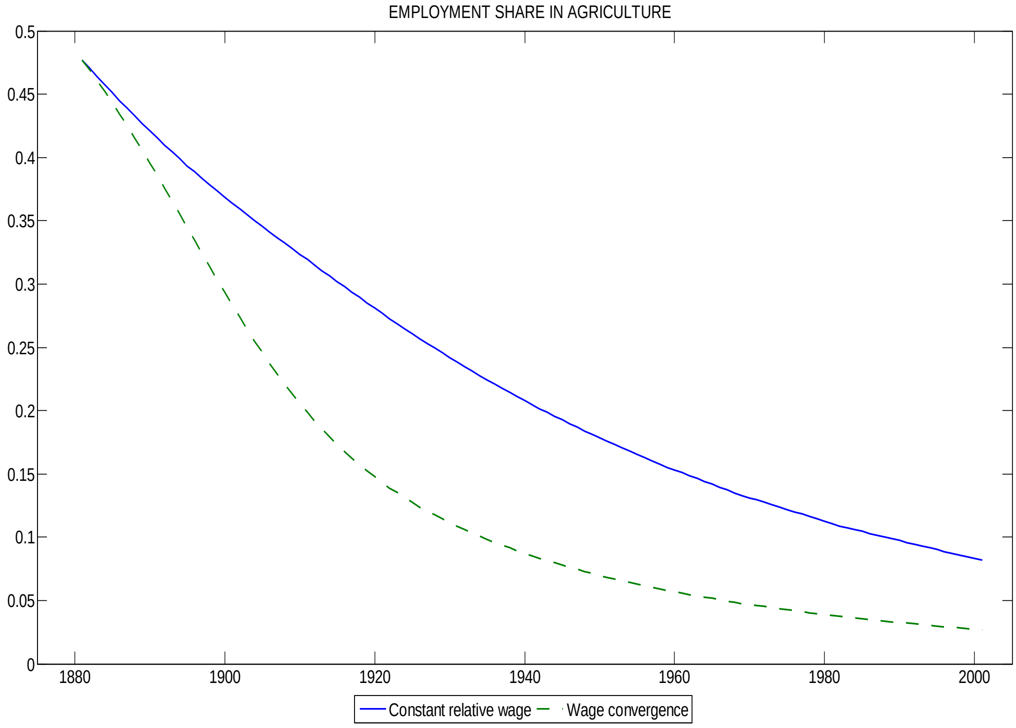

Figure 3. Demand and supply factors governing structural change

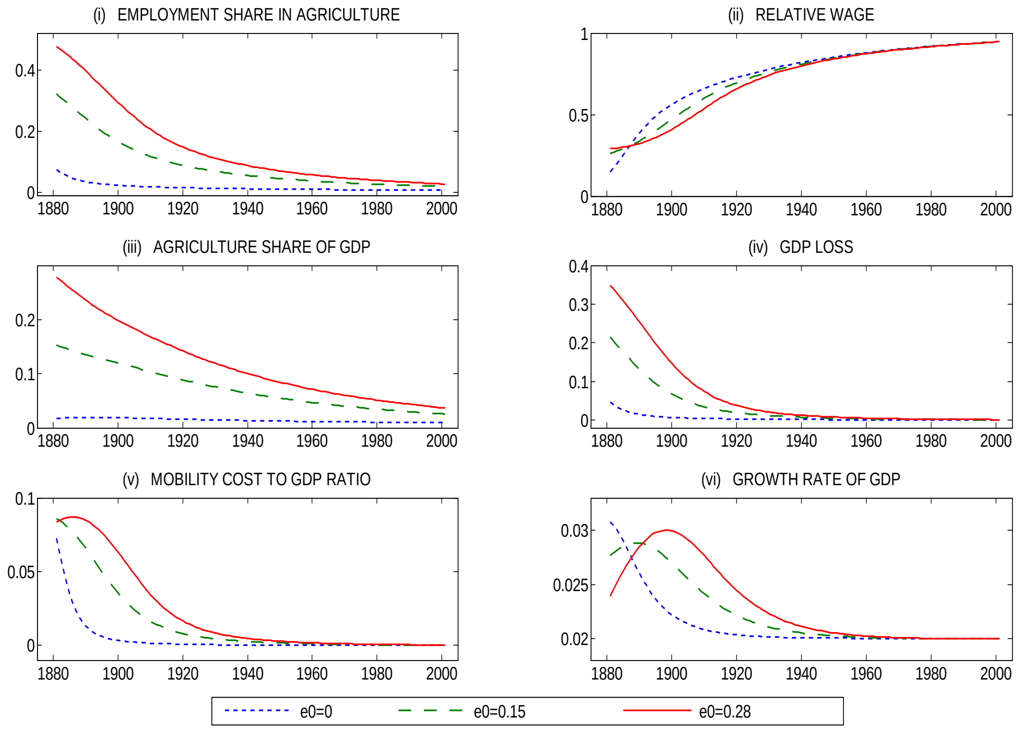

Figure 4. Economies with different initial minimum consumption intensity

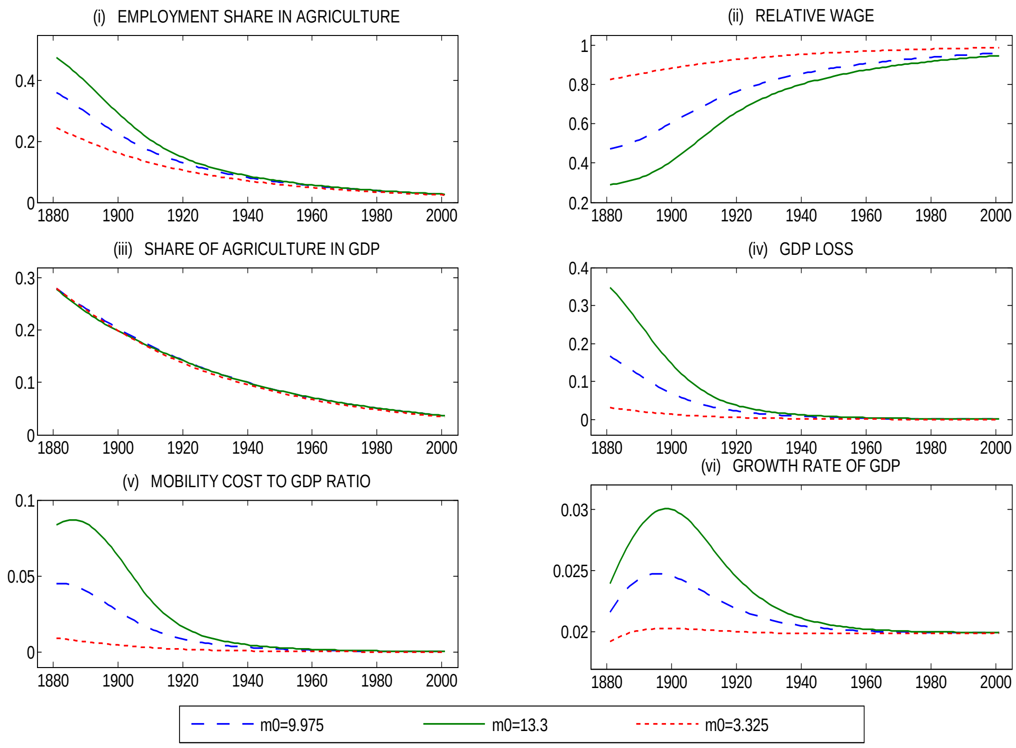

Figure 5. Economies with different labor mobility cost

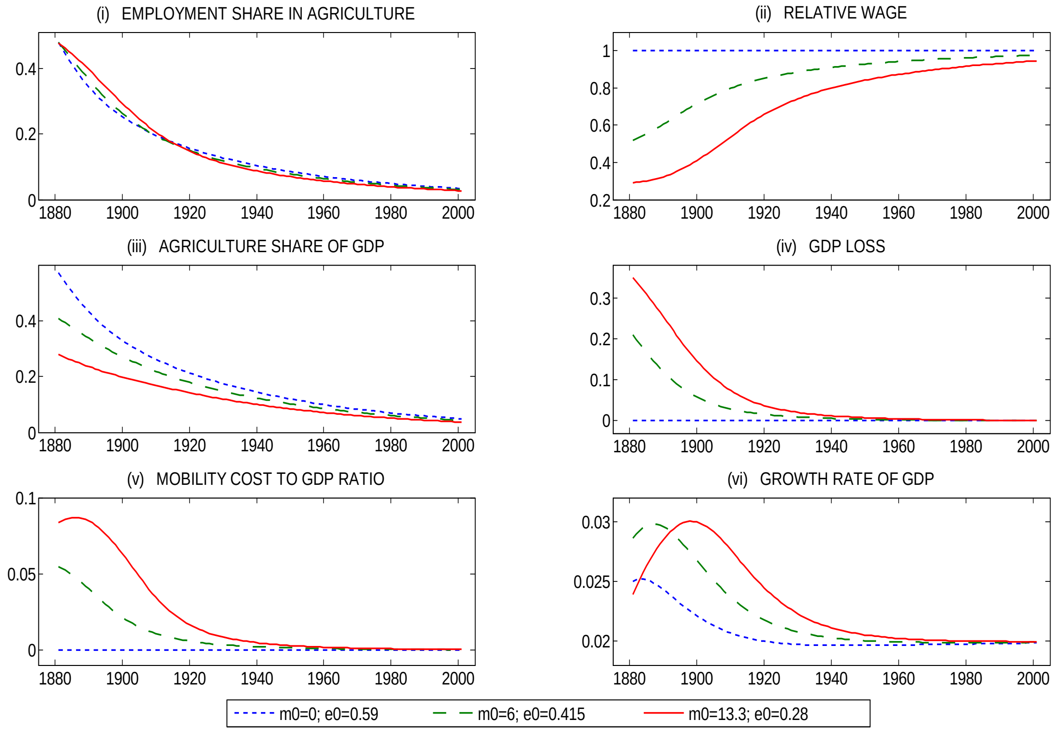

Figure 6. Economies with different labor mobility cost and minimum consumption requirements.

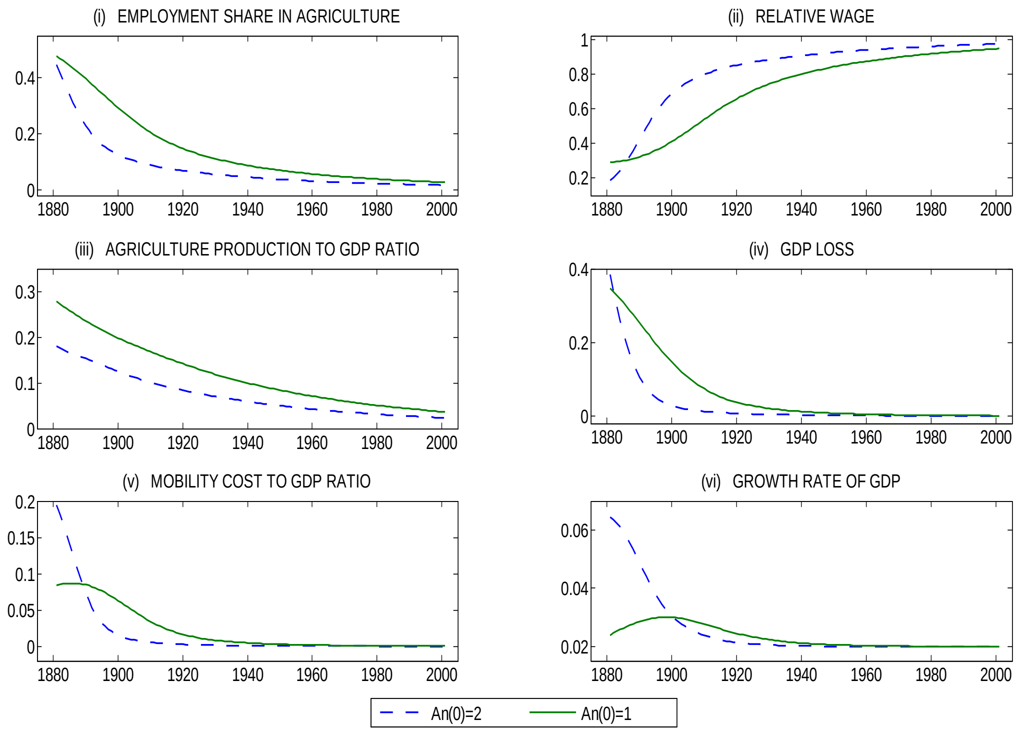

Figure 7 Economies with different initial technological levels.

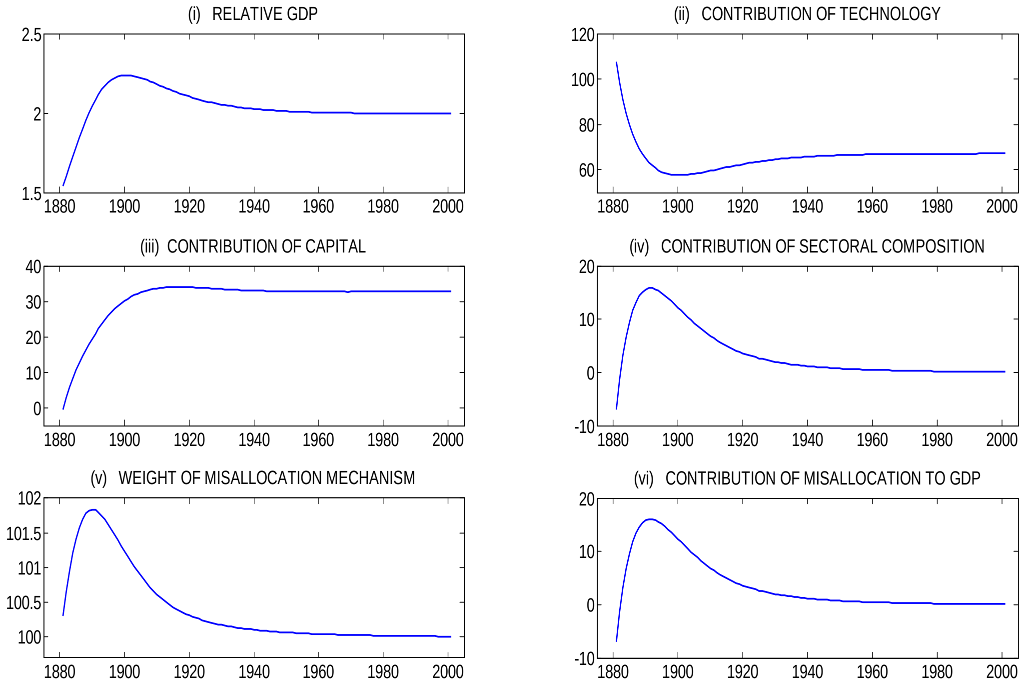

Figure 8. Development accounting between two economies with different technology.

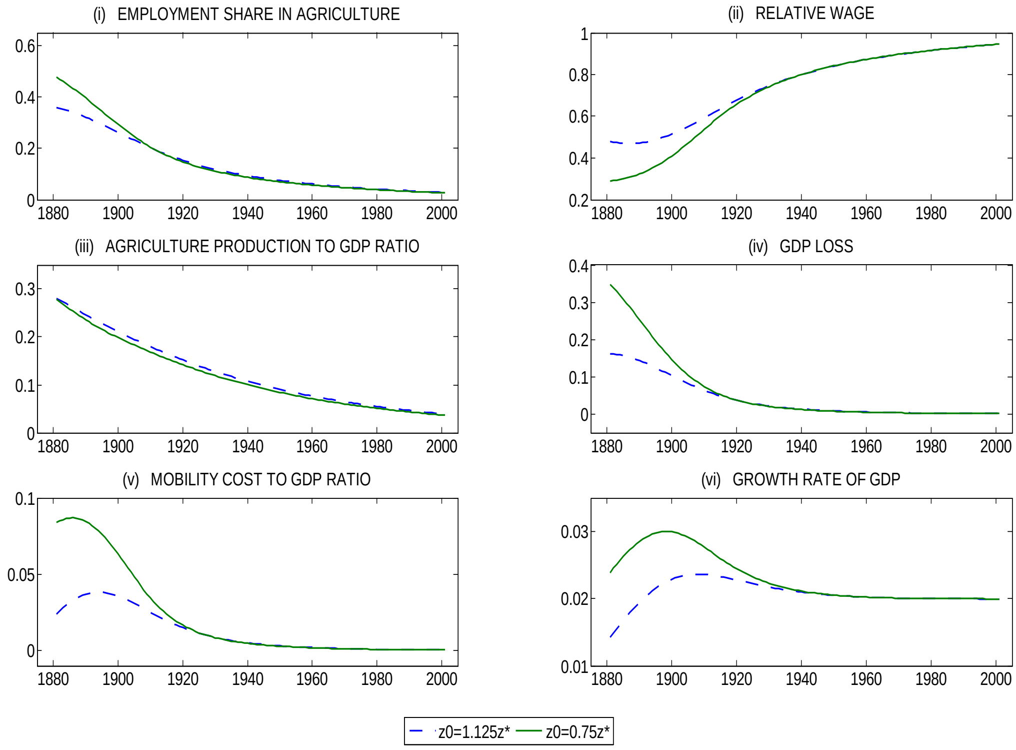

Figure 9 Economies with different initial capital stocks.

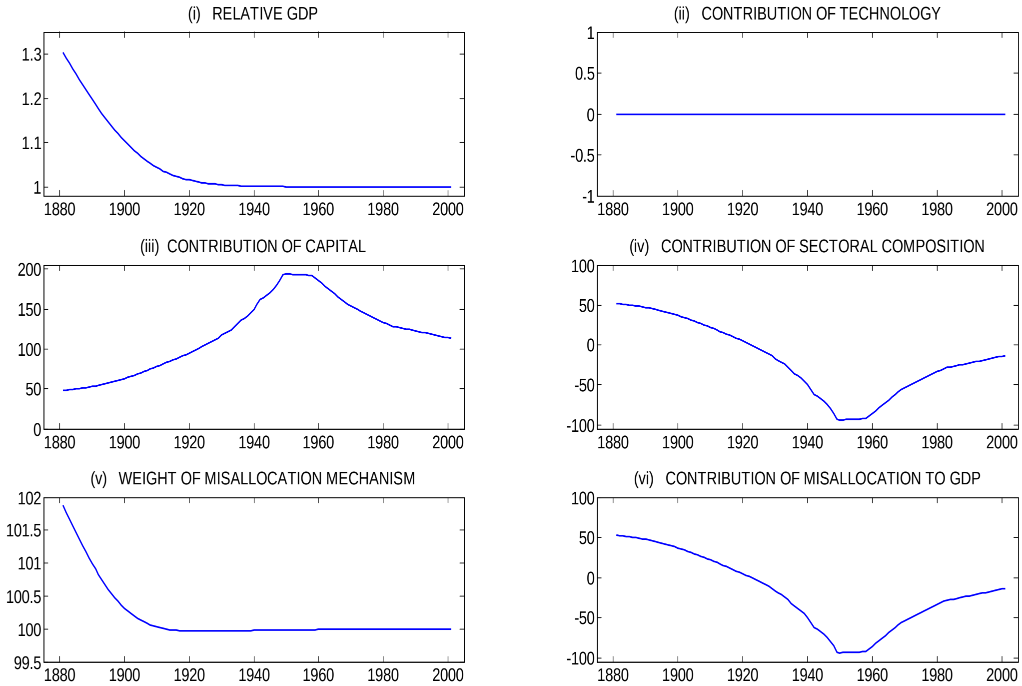

Figure 10. Development accounting between two economies with different capital stock.