



# **Growth and Welfare Maximization in Models of Public Finance and Endogenous Growth**

by

**Florian Misch, Norman Gemmell and Richard Kneller**

## **Abstract**

This paper evaluates the trade-off between growth and welfare maximization from two perspectives. Firstly, it synthesizes and extends endogenous growth models with public finance to compare the growth and welfare maximizing tax rates. Secondly, it examines the distinct model outcomes in terms of the growth rates and welfare levels. This comparison highlights the range of trade-offs: the growth maximizing tax rate can lie above, below, or on the welfare maximizing equivalent. We find however that even relatively large differences in growth and welfare maximizing tax rates translate into relatively small differences in growth rates, and, in some cases, welfare levels.

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## **Outline**

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# 1 Introduction

Comparisons between the growth maximizing and welfare maximizing fiscal policy over the long-run which have been a central issue in models of public finance and growth are important from a policy-making perspective: Although the maximization of welfare is typically characterized as the priority of benevolent governments, imperfect knowledge about the preferences of the household may prevent them from pursuing a first-best strategy to achieve this objective. An obvious option for a second-best strategy is growth maximization because in practice changes in welfare are more difficult to measure than income levels or growth. However, policy makers often perceive a distinction between the provision of social public services to meet objectives related to social welfare and the policies necessary to achieve higher growth rates.<sup>1</sup> These issues are also important in current policy debates, especially with respect to appropriate fiscal policies for developing countries.<sup>2</sup>

This paper evaluates the conclusions regarding the trade-off between growth and welfare maximization from two perspectives. The first compares the welfare maximizing and growth maximizing tax rates found in models of public finance and growth. In so doing we synthesize as well as extend the theoretical literature. The key outcome of this exercise is to highlight the range of conclusions possible regarding the trade-off between growth and welfare maximization that can be drawn from this class of theoretical models. The growth maximizing tax rate can be the same as, higher or lower than the welfare maximizing equivalent, as a result of small changes in model assumptions about the nature of the effects of fiscal policy. As is well known, in the Barro (1990) model with a flow of productive public *services*, the growth and welfare maximizing tax rates coincide, whereas in the Futagami et al. (1993) model with productive public *capital*, the growth maximizing tax rate exceeds the welfare maximizing tax rate. In contrast, in a model with one

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<sup>1</sup>For instance, the Tanzanian National Strategy for Growth and Reduction of Poverty contains a cluster for ‘Growth and the reduction of income poverty’ and another cluster on ‘Improved quality of life and social well-being’ that both list the provision of various public services and various types of public investment. The division into two clusters reflects the perception that there are trade-offs.

<sup>2</sup>See for example World Bank (2007).

utility-enhancing, and one productive, public service derived from the flow of public spending, the growth maximizing tax rate lies below the welfare maximizing rate.<sup>3</sup>

We make two further extensions to the Barro (1990) framework. In the first extension we allow for the possibility that public services or public capital entail mixed effects; the same public service/capital may simultaneously be productive as well as utility-enhancing. In developing countries where the government typically provides more rudimentary public services, it is likely that few public services entail purely productive or purely utility-enhancing effects. For example, public transportation infrastructure may not only be productive because it facilitates access to hospitals, and primary health facilities but may also be productive because they ensure that the labour force remains fit for work. Agénor and Neanidis (2006) provide a survey of empirical evidence on the impact of health on growth and the impact of infrastructure on health outcomes.

We also extend the model to allow for greater complementarity between productive public services and private capital than in the Cobb-Douglas case (the elasticity of substitution is assumed to be lower than one). Public services provided by the government fundamentally differ from private inputs, such that it may be very costly for firms to substitute for them. For example, poor quality road surfaces may require firms to purchase special, more expensive, vehicles for the transportation of goods.

We consider various combinations of these assumptions. In the final set of models public services are assumed to yield productive as well as utility-enhancing effects, and the elasticity of substitution is assumed to be lower than one. Since closed-form solutions cannot be obtained in such cases, it is shown numerically, that with public capital that entails mixed effects, the Futagami et al. (1993) results no longer holds, and that with a higher degree of complementarity, the same is true for the Barro (1990) result. Overall,

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<sup>3</sup>Throughout the paper, the term ‘Barro Model’ refers to the main model developed in Barro (1990), and the term ‘Futagami Model’ refers to the model developed in Futagami et al. (1993). The term ‘public services’ denotes public services derived from the flow of public spending, whereas the term ‘public capital’ is equivalent to public services derived from the stock of public capital.

these additions serve to further decrease the generality of the conclusion regarding the relationship between growth and welfare maximizing tax rates.

The second contribution of the paper is to provide an evaluation of the extent to which growth and welfare maximization yield distinct outcomes in terms of the growth rates and welfare levels along the balanced growth path. This is a question that is often ignored in the literature, even though differences in outcomes determine the trade-off between growth and welfare maximization. This analysis is provided through numerical simulations of policies and outcomes under growth and welfare maximization for a wide range of parameter sets, and in particular which nest different degrees of complementarity between public services/capital and private capital.<sup>4</sup>

The results from this exercise are striking and serve to modify the policy conclusions that might be drawn from the first part of the paper. Even when the differences between the tax rate necessary to achieve growth compared to welfare maximization are relatively large, we find that this translates into relatively small differences between growth rates. For models with public services, they also translate into relatively small differences in welfare levels. That is, even where there is uncertainty about how a particular form of public service or capital affects the production function or the utility function, in practice growth maximization yields growth outcomes (and in the case of public services, welfare outcomes) that are very close to those found under welfare maximization. We establish that this holds for a large array of possible parameter combinations and therefore would appear a robust result in the developments made since the original Barro (1990) model. This finding suggests that with public services, growth maximization may be a suitable second-best strategy for benevolent governments. It occurs in part because the growth rate is a central determinant of welfare, but also because policy is relatively ineffective between the welfare and the growth rate maximum. It should be remembered, in addition, that this result occurs in a class of models that ensure long run impacts of fiscal policy, and which typically form the reference point for any theoretical discussion of fiscal policy and long-run

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<sup>4</sup>We model this using a distribution function for each exogenous model parameter, allowing us to generate a large number of possible parameter sets.

growth.<sup>5</sup>

The remainder of the paper is organized as follows. The next section develops the models. Section 3 derives the equilibrium in the market economy. Section 4 analytically compares the growth and welfare maximizing tax rates. Section 5 provides some numerical comparisons of growth and welfare maximizing tax rates, while section 6 uses numerical examples to compare the growth rates, and welfare levels along the balanced growth path (under both growth and welfare maximization). Finally, section 7 summarizes the results and discusses some policy implications.

## 2 The Models

The public finance growth framework we adopt in the paper is based on Barro (1990). We assume that there is a large number of identical and infinitely lived households that is normalized to one, and that population growth is zero. The household produces a single composite good which can be used for consumption or physical capital accumulation. To incorporate the notion of complementarity between private and public services later in the paper the production function is a generalized version of that found in Barro (1990). Output is produced using private capital ( $k$ ) and a non-rival and non-excludable productive public service ( $g$ ):

$$y = (\theta k^v + \alpha g^v)^{\frac{1}{v}} \quad (1)$$

where  $\theta = 1 - \alpha$ . The parameter  $v$  determines the elasticity of substitution given by:

$$s = \frac{1}{1 - v} \quad (2)$$

The government levies a proportional tax on output at rate  $\tau$ , to provide public services. Hence:

$$g = \tau y \quad (3)$$

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<sup>5</sup>See Turnovsky (2004) for an excellent discussion of the long-run growth effects of fiscal policy.

The instantaneous utility function is

$$u(c, g) = \frac{(g^\beta c^{1-\beta})^{1-\sigma}}{1-\sigma} \quad (4)$$

implying that public services are both productive and utility-enhancing if  $\alpha > 0$  and  $\beta > 0$ .

The use of general specifications for the utility and the production function allows us to nest different models defined by particular parameter values. Most obviously, with  $\beta = 0$ , public services are solely productive, and with  $v = 0$ , output is produced using Cobb-Douglas technology such that the model is identical to the Barro model.<sup>6</sup> Other models considered in the paper include a version in which  $v = 0$  and  $\beta > 0$ , such that output is produced using Cobb-Douglas technology and public services entail mixed effects (referred to as Model 2). Model 4, which allows that the possibility that the elasticity of substitution is lower and that the degree of complementarity is larger than in the case of Cobb-Douglas technology, while public services are solely productive, refers to the case when  $v \leq 0$  and  $\beta = 0$ . Model 6 refers to the case when  $v \leq 0$  and  $\beta \geq 0$  and therefore includes the Barro model, Model 2 (public services have mixed effects but the production function exhibits an elasticity of substitution equal to one) as well as Model 4 (public services solely affect the production function but the elasticity of substitution is equal to or less than one) as special cases.

The Futagami et al. (1993) Model - in which output is a function of public capital ( $k_G$ ) - can be generalized to allow for mixed public capital and public-private complementarity, such that (1) is rewritten as:

$$y = (\theta k^v + \alpha k_G^v)^{\frac{1}{v}} \quad (5)$$

Similarly (3) and (4) become:

$$\dot{k}_G = \tau y \quad (6)$$

and

$$u(c, k_g) = \frac{(k_G^\beta c^{1-\beta})^{1-\sigma}}{1-\sigma} \quad (7)$$

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<sup>6</sup>Implicitly, total factor productivity is assumed to be 1.

respectively, implying that public capital is not only productive but also utility-enhancing if  $\alpha > 0$  and  $\beta > 0$ . This model is identical to the Futagami Model under particular parameter settings: If  $\beta = 0$ , public capital is solely productive, and with  $v = 0$ , output is produced using Cobb-Douglas technology. Model 3 refers to the case when  $v = 0$  and  $\beta > 0$  which means that output is produced using Cobb-Douglas technology and that public capital entails mixed effects. Model 5 refers to the case when  $v \leq 0$  and  $\beta = 0$ . Model 7 refers to the case when  $v \leq 0$  and  $\beta \geq 0$  and therefore includes the Futagami Model, Model 3 as well as Model 5 as special cases.

Instead of assuming that public services have mixed effects, it can be assumed that the government provides one purely productive public service and one purely utility-enhancing public service. In an extended version of the base model in Barro (1990) (which, for the purposes of this paper, is referred to as Model 1), the instantaneous utility function is

$$u(c, h) = \frac{(h^\beta c^{1-\beta})^{1-\sigma} - 1}{1 - \sigma} \quad (8)$$

where  $h$  denote utility enhancing public services which are derived according to

$$h = \tau_h y \quad (9)$$

Output is produced using Cobb-Douglas technology:

$$y = k^{1-\alpha} g^\alpha \quad (10)$$

where

$$g = \tau_g y \quad (11)$$

Of course, in practice governments would not typically levy two distinct income taxes, one for each public service,  $\tau_h$  and  $\tau_g$ . However, this specification simplifies (but does not change) the comparisons between the growth and welfare maximizing tax rates.

Table 1 summarizes the key features of the models described above.



Table 1: Model Summary

Model	Stock/Flow	$v$	Effect of public services/capital
Barro	public services	1	productive
Futagami	public capital	1	productive
Model 1	public services	1	1 prod. & 1 utility-enhancing
Model 2	public services	1	mixed
Model 3	public capital	1	mixed
Model 4	public services	$\leq 1$	productive
Model 5	public capital	$\leq 1$	productive
Model 6	public services	$\leq 1$	mixed
Model 7	public capital	$\leq 1$	mixed

### 3 The Equilibrium in the Market Economy

This section derives the equilibrium of the market economy in Models 6 and 7. The equilibrium in Model 6 (which incorporates the Barro Model, Model 2 and Model 4 as special cases) can be characterized as follows. The representative household chooses the consumption path to maximize lifetime utility  $U$  given by

$$U = \int_0^{\infty} u(c(t))e^{-\rho t} dt \quad (12)$$

subject to the respective production function of the model as well as the household's resource constraint

$$\dot{k} = (1 - \tau)y - c \quad (13)$$

taking  $\tau$ ,  $g$  and  $k_0$  as given.<sup>7</sup> There are no transitional dynamics, and the economy is always on the balanced growth path where  $c$ ,  $k$  and  $y$  all grow at the same rate  $\gamma$ .

The first order conditions that are derived from the present-value Hamiltonian can be written as

$$(1 - \beta) (g^{\beta(1-\sigma)} c^{\sigma\beta-\beta-\sigma}) e^{-\rho t} = \lambda \quad (14)$$

<sup>7</sup>The time subscript is omitted whenever possible. A dot over the variable denotes its derivative with respect to time.

and

$$(1 - \tau)y_k = -\frac{\dot{\lambda}}{\lambda} \quad (15)$$

where  $y_k$  is the derivative of output with respect to private capital and  $\lambda$  is the costate variable. In addition, the transversality condition has to be fulfilled:

$$\lim_{t \rightarrow \infty} [\lambda k] = 0 \quad (16)$$

Rearranging (14), taking logs and differentiating with respect to time yields

$$\frac{\dot{c}}{c} = \frac{1}{\sigma\beta - \beta - \sigma} \frac{\dot{\lambda}}{\lambda} + \frac{\rho}{\sigma\beta - \beta - \sigma} + \frac{-\beta(1 - \sigma)}{\sigma\beta - \beta - \sigma} \frac{\dot{g}}{g} \quad (17)$$

Since  $g = \tau y$  and given that the tax rate is constant,  $\gamma = \frac{\dot{g}}{g} = \frac{\dot{y}}{y}$  along the balanced growth path. Therefore, after setting  $\frac{\dot{g}}{g} = \frac{\dot{c}}{c}$ , and after substituting for  $\frac{\dot{\lambda}}{\lambda}$  using (15), (17) can be rewritten to yield

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (18)$$

Since  $\beta$  does not enter the latter expression, it can be noted that in the market economy, the presence of mixed effects of public services does not affect the growth rate. Disregarding the policy choice, it can be shown that the transversality condition (16) is always fulfilled if  $\sigma > 1$ . Therefore, for simplicity, it will be assumed throughout the paper that  $\sigma > 1$  so that the choice of  $\tau$  is unconstrained by the transversality condition.

For the case of the equilibrium in the market economy, Model 7 (which incorporates the Futagami Model, and Models 3 and 5 as special cases) can be characterized as follows. The representative household again maximizes lifetime utility given by (12) subject to the respective production function of the model and the household's resource constraint (13) taking  $\tau$ ,  $k_G > 0$  and  $k_0 > 0$  as given.

The first order conditions that are derived from the present-value Hamiltonian can be written as

$$(1 - \beta) (k_G^{\beta(1-\sigma)} c^{\sigma\beta - \beta - \sigma}) e^{-\rho t} = \lambda \quad (19)$$

and

$$(1 - \tau)y_k = -\frac{\dot{\lambda}}{\lambda} \quad (20)$$

where  $y_k$  is the derivative of output with respect to private capital and  $\lambda$  is the costate variable. In addition, the transversality condition has to be fulfilled.

Rearranging (19), taking logs and differentiating with respect to time, then yields:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma\beta - \beta - \sigma} \frac{\dot{\lambda}}{\lambda} + \frac{\rho}{\sigma\beta - \beta - \sigma} - \frac{\beta(1 - \sigma)}{\sigma\beta - \beta - \sigma} \frac{\dot{k}_G}{k_G} \quad (21)$$

The growth rate of public capital is

$$\frac{\dot{k}_G}{k_G} = \frac{\tau y}{k_G} \quad (22)$$

Substituting  $\frac{\dot{\lambda}}{\lambda}$  and  $\frac{\dot{k}_G}{k_G}$  in (21) using (20) and (22) yields the growth rate of consumption:

$$\frac{\dot{c}}{c} = \frac{((1 - \tau)y_k - \rho + \beta(1 - \sigma)\frac{\tau y}{k_G})}{\beta + \sigma - \sigma\beta} \quad (23)$$

Along the balanced growth path,  $\gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{k}_G}{k_G}$  so that (23) can be rewritten as

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (24)$$

The Appendix shows that the equilibrium of the models is saddlepoint stable within the relevant parameter ranges, and that the balanced growth path is unique.<sup>8</sup>

In Model 1, where productive and utility-enhancing public services are separate, utility maximization is subject to the households' resource constraint now given by:

$$\dot{k} = (1 - \tau_h - \tau_g)y - c \quad (25)$$

The households take  $\tau_g$ ,  $\tau_h$ ,  $g$ ,  $h$  and  $k_0$  as given. There are no transitional dynamics, and the economy is always on the balanced growth path. It can be shown that the growth rate of the economy can be expressed as

$$\gamma = \frac{\dot{c}}{c} = \frac{1}{\sigma} ((1 - \tau_h - \tau_g)y_k - \rho) \quad (26)$$

which is equivalent to (24) where  $\tau_h + \tau_g = \tau$ .

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<sup>8</sup>It is again assumed that  $\sigma > 1$  so that the transversality condition is fulfilled.

## 4 Growth and Welfare Maximizing Policies

### 4.1 The Base Models

This section derives the growth maximizing tax rate,  $\tau^*$ , and the welfare maximizing tax rate,  $\tau^{**}$ , in the Barro Model, in the Futagami Model, and in Model 1. Inserting (A.5) in (18), the growth rate in the Barro Model can be expressed as

$$\gamma = \frac{1}{\sigma} \left( (1 - \tau)(1 - \alpha)\tau^{\frac{\alpha}{1-\alpha}} - \rho \right) \quad (27)$$

Maximizing the latter expression for  $\tau$  yields the familiar growth maximizing tax rate for the Barro Model,  $\tau_B^*$ , as:

$$\tau_B^* = \alpha \quad (28)$$

Output net of taxation can be written as

$$y = (1 - \tau)\tau^{\frac{\alpha}{1-\alpha}}k$$

It can easily be seen that maximizing output net of taxation at every point of time yields the same tax rate as maximizing the growth rate. Therefore, in the Barro Model, there are no trade-offs between growth maximization and welfare maximization, and the growth and the welfare maximizing tax rate coincide:

$$\tau_B^* = \tau_B^{**} = \alpha \quad (29)$$

Inserting (A.16) in (24), the growth rate in the Futagami Model can be expressed as

$$\gamma = \frac{1}{\sigma} \left( (1 - \tau)(1 - \alpha) \left( \frac{\tau}{\gamma} \right)^{\frac{\alpha}{1-\alpha}} - \rho \right) \quad (30)$$

Using implicit differentiation, it can be shown that the growth maximizing tax rate is

$$\tau_F^* = \alpha \quad (31)$$

Under welfare maximization in the market economy, the government maximizes (12) subject to (13) and (6) while taking the first order conditions of

the households as given. Futagami et al. (1993) have shown that the growth maximizing tax rate exceeds the welfare maximizing one:

$$\tau_F^* = \alpha > \tau_F^{**} \quad (32)$$

The reason is that when public services are derived from the stock of public capital, consumption is foregone in the process of accumulating public capital (Turnovsky (1997)) so that maximizing output and maximizing the growth rates are no longer identical. This effect is termed the ‘capital accumulation effect’.

Turnovsky (1997) derives an expression for the welfare maximizing tax rate of a centrally planned economy and considers relative congestion. For the market economy, no closed-form solution for the welfare maximizing tax rate can be found. Along the lines of Ghosh and Roy (2004), it is shown in the Appendix that the welfare maximizing tax rate,  $\tau_F^{**}$ , has to satisfy equations (A.44) and (A.46) when  $\beta = 0$  (which are both restated here for convenience):

$$y_{k_G} = (1 - \tau)y_k \quad (33)$$

$$\tau \frac{y}{k_G} = \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (34)$$

Expressions for  $\frac{y}{k}$ ,  $\frac{y}{k_G}$ ,  $y_k$  and  $y_{k_G}$  are derived in the Appendix .

The growth rate in Model 1 is similar to the one in the Barro Model (27):

$$\gamma = \frac{1}{\sigma} \left( (1 - \tau_h - \tau_g)(1 - \alpha)\tau_g^{\frac{\alpha}{1-\alpha}} - \rho \right) \quad (35)$$

It is obvious that under growth maximization,  $\tau_h$  is zero<sup>9</sup> because  $\tau_h$  has an unambiguously negative effect on the growth rate. In contrast to this, under welfare maximization,  $\tau_h$  is positive if  $\beta > 0$ . This effect is termed the ‘utility-enhancement effect’.

Following an extension of his base model, by Barro (1990), there are several papers that assume a utility function of the form  $u = u(c, h)$  and

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<sup>9</sup> $\tau_h = 0$  only holds under growth maximization if one ignores the fact that with no spending on  $h$ , households do not derive any utility from economic activity. Therefore, a more realistic assumption would be that under growth maximization,  $\tau_h$  still has to be positive.

compare the growth and welfare maximizing tax rate, including Lau (1995), Park and Philippopoulos (2002), as well as Greiner and Hanusch (1998). Of such models, Model 1 probably captures the distinction between growth and welfare maximization with utility-enhancing public services that is typically perceived among policy makers. In this model, the welfare maximizing level of taxation clearly exceeds the growth maximizing level, and maximizing welfare unambiguously lowers the growth rate.

The next subsection extends the above models to allow for ‘mixed’ public services and complementarity between public and private capital.

## 4.2 The Extended Models

This sub-section first analytically derives the growth and welfare maximizing tax rates in models with *public services* that incorporate mixed effects, complementarity, or both (Models 2, 4 and 6), and then considers the equivalent results in models with *public capital* (Models 3, 5 and 7). To simplify the exposition, in Models 4 to 7, the elasticity of substitution is assumed to be  $\frac{1}{2}$  (implying  $v = -1$ ), which is halfway between the Cobb-Douglas and Leontief technologies. This yields a larger (smaller) degree of complementarity between the inputs to private production than Cobb-Douglas (Leontief) technology.

In Model 2, the growth maximizing tax rate,  $\tau_2^*$ , corresponds to the Barro Model since the production functions are identical. Hence:

$$\tau_2^* = \alpha \quad (36)$$

In Models 4 and 6, using (A.9) and (A.8) to substitute for  $y_k$  in (18), and with  $v = -1$ , the growth rate can be written as

$$\gamma = \frac{1}{\sigma} \left( (1 - \tau) \left( \theta + \frac{\alpha\theta}{(\tau - \alpha)} \right)^{-2} \theta - \rho \right) \quad (37)$$

Maximizing the latter expression yields the growth maximizing tax rates,  $\tau_4^*$  and  $\tau_6^*$ :

$$\tau_4^* = \tau_6^* = \frac{1}{2} (\sqrt{\alpha^2 + 8\alpha} - \alpha) \quad (38)$$

Since there are no transitional dynamics in Models 2, 4 and 6, the welfare maximizing tax rate can be derived as follows. With  $x = \frac{c}{k}$ , lifetime utility can be written as

$$U = \int_0^\infty \frac{\left(\left(\frac{g}{k}\right)^\beta x^{1-\beta} k(t)\right)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \quad (39)$$

Along the balanced growth path,  $x$  and  $\frac{g}{k}$  are constant, and  $\frac{\dot{k}}{k} = \gamma$ , such that:

$$k = k_0 e^{\gamma t} \quad (40)$$

Therefore, along the balanced growth path, (39) can be expressed as (ignoring the constants):

$$U = \frac{1}{1-\sigma} \left[ \frac{\left(\left(\frac{g}{k}\right)^\beta x^{1-\beta}\right)^{1-\sigma}}{\rho - (1-\sigma)\gamma} \right] \quad (41)$$

Maximizing the latter expression (after substituting for  $x = \frac{c}{k} - (1-\tau)\frac{y}{k}$  and for  $\frac{g}{k}$  using (A.8), and setting  $\beta = 0$  for Model 4) yields the welfare maximizing tax rate of Model 2 ( $\tau_2^{**}$ ), Model 4 ( $\tau_4^{**}$ ) and Model 6 ( $\tau_6^{**}$ ). Closed-form solutions cannot be obtained.

In Model 3, the growth maximizing tax rate corresponds to the Futagami Model since the production functions are identical, hence:

$$\tau_3^* = \alpha \quad (42)$$

In Model 5 and 7, the growth maximizing tax rate can be found by maximizing (24) using implicit differentiation in a similar way to the Futagami Model. However, no closed-form solution exists. Similarly, there are no closed-form solutions available for the welfare maximizing tax rate in Model 3, 5 and 7.<sup>10</sup> We therefore rely on numerical simulations, discussed in sections 5 and 6.

There are only few models in the existing literature that consider mixed public services and mixed public capital that are both utility-enhancing and

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<sup>10</sup>As shown in the Appendix, the welfare-maximizing tax rate has to satisfy equations (A.44), (A.46) and (A.47). Expressions for  $\frac{y}{k}$ ,  $\frac{y}{k_G}$ ,  $y_k$  and  $y_{k_G}$  which differ between the models are derived in the Appendix.

productive.<sup>11</sup> In part due to their complexity, the growth maximizing and the welfare maximizing income tax rates are generally not compared for the decentralized economy. The most straightforward version can be found in Baldicci (2005), who develops a Barro-style model in which the government provides mixed public services derived from the flow of public spending. He finds that within a centrally planned economy, the welfare maximizing tax rate exceeds the growth maximizing one.

A more complicated approach is adopted in a series of papers by Agénor. Agénor and Neanidis (2006) introduce a model in which final output is produced using private capital, public services derived from spending on infrastructure and effective labour which in turn depends on health services and on the share of educated workers in production. Educated labour is produced using, among other inputs, raw labour, health services and public spending on education. Health services in turn are produced using educated labour, public spending on health and spending on infrastructure, and in addition to being productive are also utility-enhancing.

Agénor and Neanidis (2006) compare the growth maximizing allocation of government revenue in a decentralized economy to the welfare maximizing allocation of government revenue in *a centrally planned economy* treating the tax rate and the share of one category of public spending as exogenously set. They examine trade-offs between the three spending shares: infrastructure, education and health. They find that the welfare maximizing shares of infrastructure spending, and of education spending, in total revenue when offset by spending on health are lower than the growth maximizing shares (in the first case, the share of spending on education and in the second case, the share of spending on infrastructure is exogenously set). This implies that the welfare maximizing share of health spending is higher than the growth maximizing equivalent.

Agénor (2005) presents a model with health and infrastructure public services. The former are both productive and utility-enhancing. In the first

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<sup>11</sup>Comparing welfare maximizing and growth maximizing fiscal policies within a model in which public capital is productive and utility-reducing due to negative welfare effects of growth (pollution) is proposed for further research by Greiner and Kuhn (2003).



version of the model, both types of public services are derived from the flow of public spending on health and infrastructure. He shows that in a *centrally planned* economy, the welfare maximizing tax rate and the welfare maximizing share of health spending in total government revenue exceed the corresponding growth maximizing equivalents in a decentralized economy. In the second version of the model, health services are derived from the stock of health capital that is accumulated using public spending on health and spending on infrastructure. In this case the stock of health capital entails mixed effects. However, welfare maximizing policies are not derived for this version of the model.

Likewise, there are a few papers that consider CES technology within endogenous growth models with public spending. Even here, growth and welfare maximizing policies are typically not compared for a decentralized economy. Devarajan et al. (1996) introduce a model in which output is produced using private capital and two productive public services with CES technology, but they do not study optimal policies. Baier and Glomm (2001) alternatively consider a case in which output is produced using labour, private capital and public capital and allow for varying degrees of substitutability between public and private capital. They show numerically that the growth maximizing size of the public sector increases as the elasticity of substitution decreases. Finally, Ott and Turnovsky (2006) introduce a model in which the government provides a non-excludable and an excludable public service that are both subject to congestion financed by income taxes and user fees. They find that for the centrally planned economy with CES technology, the growth and welfare maximizing expenditure shares in output (which essentially correspond to the tax rates) coincide. Ott and Turnovsky (2006) also derive welfare maximizing fiscal policies for the decentralized economy.

## **5 Growth and Welfare Maximizing Income Tax Rates: Numerical Comparisons**

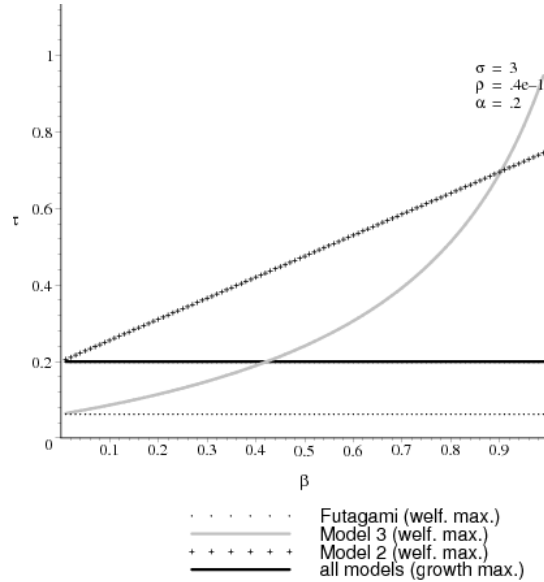
Due to the lack of closed-form solutions in several versions of the model, in this section we present numerical comparisons between growth and welfare

maximization. Specifically, we plot the growth and welfare maximizing tax rates as functions of  $\beta$  in the alternative models. This allows us to compare the growth and welfare maximizing tax rates across a wide range of parameter configurations, and provides an indication of the magnitude of potential differences. Again, for simplicity, the elasticity of substitution in models 4 to 7 is assumed to be  $\frac{1}{2}$  (implying  $\nu = -1$ ).

Figure 1 plots the welfare and growth maximizing tax rates in the Futagami Model, Model 2 and Model 3. The growth and welfare maximizing tax rates of the Barro Model are also implicitly considered since they correspond to the growth maximizing tax rate that coincides across these models. As shown in the previous section, due to the capital accumulation effect, the welfare maximizing tax rate in the Futagami Model is lower than the growth maximizing one. In contrast to this, the welfare maximizing tax rate in the model with mixed public services (Model 2) exceeds the growth maximizing tax rate. That is, due to the simultaneous utility-enhancing effect of public services, higher levels are desirable from a welfare perspective. As might be expected from this configuration of results when we consider the model with mixed public capital (Model 3), the impact of increasing  $\beta$  on the relative position of the welfare maximizing tax rate is ambiguous. The utility-enhancement effect and the capital accumulation effect oppose each other. For low values of  $\beta$  the welfare maximizing tax rate is below the growth maximizing rate, whereas for high values of  $\beta$  it lies above it, and

there exist a particular value of  $\beta$  when both tax rates are identical.

Figure 1: The tax rate as a function of  $\beta$



In Figure 2 we plot the welfare and growth maximizing tax rates for Models 4 and 6. In Model 4 - the model with public services but an elasticity of substitution less than one - the welfare maximizing tax rate is not affected by  $\beta$ . However, it no longer matches the growth maximizing tax rate, and the welfare maximizing tax rate is always lower. As Barro (1990) predicts, the elasticity of substitution affects the relationship between the welfare and growth maximizing tax rates. The reason is that maximizing output net of taxation is no longer identical to maximizing the growth rate. When we allow for mixed public services in the model with an elasticity of substitution less than one (Model 6), we find that the welfare maximizing tax rate increases with  $\beta$ , and its position with regard to the growth maximizing tax rate is ambiguous. A smaller elasticity of substitution lowers the welfare maximizing tax rate, whereas the utility-enhancement effect (which increases with  $\beta$ ) raises it. For low values of  $\beta$  the welfare maximizing tax rate is below the

growth maximizing rate, whereas the reverse is true for high values of  $\beta$ .

Figure 2: The tax rate as a function of  $\beta$

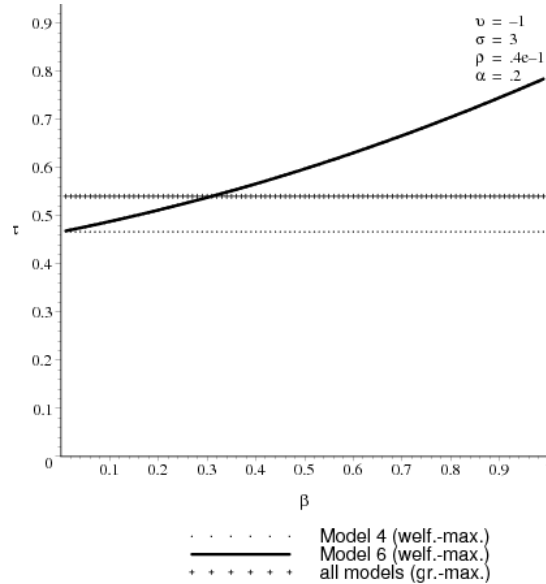
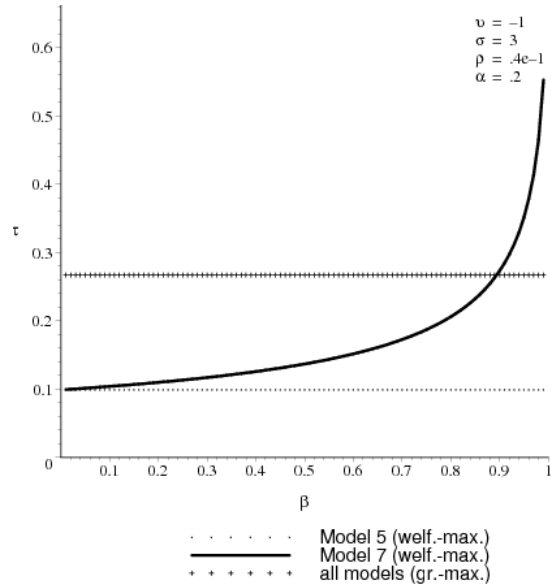


Figure 3 plots the results for Models 5 and 7, with public capital and an elasticity of substitution less than one. Model 5 refers to the case where public capital affects only the production function and Model 7 where it affects both production and utility (i.e. it is mixed). In Model 5, the welfare maximizing tax rate is not affected by  $\beta$ . Compared to the Futagami Model with public capital, the effect of the change in the assumption regarding the elasticity of substitution is to accentuate the difference between the growth and welfare maximizing tax rates. The welfare maximizing tax rate is lower because of the capital accumulation effect, but also because the elasticity of substitution is less than one. In Model 7, the fact that public capital also affects utility results in the welfare maximizing tax rate increasing with  $\beta$  as before, and its position with regard to the growth maximizing tax rate is again ambiguous. The small elasticity of substitution and the capital accumulation effect lower the welfare maximizing tax rate, whereas the utility-enhancement effect raises

it, such that it crosses the growth maximizing tax rate at some point.

Figure 3: The tax rate as a function of  $\beta$



To summarize this section: it is possible to show that small changes in the underlying assumptions of the models can lead to fundamentally different conclusions from comparisons between the growth and welfare maximizing tax rates. The reasons for divergence of the growth and welfare maximizing tax rate in these models include differences between output and growth maximization, utility-enhancing effects of public spending and accumulation of public capital by the government. Without knowledge of the way that public services or capital affect production or utility governments cannot be sure whether the welfare maximizing tax rate is expected to be above, below or to be the same as the growth maximizing tax rate, nor about the size of that difference. Several generalizations are possible however.

(1) The use of public capital, as in the Futagami Model, tends to yield outcomes in which the welfare maximizing tax rate is below the growth maximizing rate.

(2) The use of ‘mixed’ public services - that affect both production and utility - generates a welfare maximizing tax rate that lies above the growth maximizing rate.

Table 2: Overview of Results

Model	Relationship between $\tau^*$ and $\tau^{**}$
Barro	$\tau^* = \tau^{**}$
Futagami	$\tau^* > \tau^{**}$
Model 1	$\tau^* < \tau^{**}$
Model 2	$\tau^* < \tau^{**}$
Model 3	ambiguous
Model 4	$\tau^* < \tau^{**}$
Model 5	$\tau^* < \tau^{**}$
Model 6	ambiguous
Model 7	ambiguous

(3) In models in which the elasticity of substitution between public services and private capital is less than one, the welfare maximizing tax rate lies below the growth maximizing rate.

As a consequence it is possible to generate versions of the public policy growth models in which these differences in tax rates are magnified or become ambiguous. Table 2 summarizes the range of results from the alternative models.

## 6 Growth Rates and Welfare Levels under Growth and Welfare Maximization: Numerical Comparisons

In this section of the paper we turn to the comparison of the outcomes that result from the different versions of the public finance and growth models considered above. In particular we are interested in whether the ambiguous nature of the differences in tax rates with welfare and growth maximization translate into large or small differences in outcomes. We perform this exercise by quantifying differences between the growth rates and, for models with public services, welfare levels along the balanced growth path under growth and welfare maximization. The motivation is that while the extent of trade-offs between both government objectives is ultimately determined by

differences in outcomes, most papers solely focus on differences in policies.

An exception is Monteiro and Turnovsky (2007) who develop a two-sector endogenous growth model with physical and human capital. The government provides one public service that enhances the production of final output and one public service that enhances the production of human capital. Both are derived from the flow of public expenditure. They present steady state growth rates and steady state welfare levels for several different combinations of the tax rate and public spending composition (under two alternative settings of the remaining model parameters). Whereas utility is derived from consumption, which in turn is derived from final output, the welfare benefits of spending on the production of human capital are less direct. They therefore find a trade-off between growth and welfare maximization.

The previous section has shown that it is difficult to draw specific conclusions from comparisons between the growth and welfare maximizing tax rates. As a result trade-offs in terms of fiscal policies are very difficult to predict if the precise model specification, and the specific values of key parameters, are unknown. To deal with this model and parameter uncertainty we numerically evaluate the growth rates and welfare levels along the balanced growth path for a large number of different values of the exogenous model parameters. By doing so it is hoped that general conclusions about growth and welfare maximization can be derived even under model and parameter uncertainty.

The procedure used consists of two steps:<sup>12</sup> First, a large number of values for each exogenous model parameter were generated. No assumptions regarding the specific parameter values were made. Rather, each parameter is allowed to vary across some (plausible) range. The lower bound ( $l$ ) and the upper bound ( $u$ ) are chosen to reflect theoretical restrictions, econometric estimates and/or anecdotal evidence where available. Two alternative distributions are assumed between the lower and upper bound. First, a Uniform distribution, and second, a symmetric Normal distribution (with mean,  $\mu = \frac{(l+u)}{2}$ , and standard deviation,  $s = \frac{(u-l)}{1.96}$ ) are used. 7728 parameter sets were then generated based on 7728 independent draws for each distribution.

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<sup>12</sup>We use the same method as Salhofer et al. (2001) who apply it in a different context.

Table 3: Exogenous Parameter Ranges and Distribution

	$s$	$u$	Distribution 1	Distribution 2
$\sigma$	1.001	3	Uniform	Normal
$\rho$	0.02	0.06	Uniform	Normal
$\alpha$	0.1	0.45	Uniform	Normal
$\beta$	0	0.6	Uniform	Normal
$v$	-1	-0.001	Uniform	Normal

Table 4: Normal Parameter Distribution

	Mean	Std. Dev.	Min.	Max.
$\sigma$	2.002	0.443	1.001	2.999
$\rho$	0.04	0.009	0.02	0.06
$\alpha$	0.273	0.078	0.1	0.45
$\beta$	0.299	0.134	0	0.6
$v$	-0.499	0.224	-1	-0.001
N			7728	

Each parameter set includes values for all exogenous parameters in Models 6 and 7. Table 3 summarizes the parameter assumptions; Tables 4 and 5 summarize the simulated distributions resulting from the 7728 independent draws.

Secondly, the maximization procedures, and the resulting outcomes in both models, were solved numerically for the Uniformly and Normally distributed parameter values. The growth and welfare maximizing tax rates,  $\tau^*$  and  $\tau^{**}$  were calculated in the same way as in Models 6 and 7 (where  $v$  takes the values within the range defined above). To compare both tax rates, the

Table 5: Uniform Parameter Distribution

	Mean	Std. Dev.	Min.	Max.
$\sigma$	2.002	0.581	1.001	2.999
$\rho$	0.04	0.012	0.02	0.06
$\alpha$	0.273	0.101	0.1	0.45
$\beta$	0.296	0.173	0	0.6
$v$	-0.501	0.287	-1	-0.001
N			7728	



relative difference is calculated as:

$$\left| \frac{(\tau^{**} - \tau^*)}{\tau^*} \right| \times 100 \quad (43)$$

We then compare the growth rates and welfare levels that result from these different growth and welfare maximizing fiscal policies. For Model 6, growth rates and welfare levels along the balanced growth path under growth and welfare maximization,  $\gamma^*$  and  $\gamma^{**}$  as well  $W^*$  and  $W^{**}$ , are calculated. The level of welfare along the balanced growth path is calculated based on (41). To compare growth rates and welfare under both objectives, it is again useful to calculate relative differences, given by

$$\frac{(\gamma^* - \gamma^{**})}{\gamma^*} \times 100 \quad (44)$$

and

$$\frac{(W^{**} - W^*)}{W^{**}} \times 100 \quad (45)$$

respectively. In Model 7, due to transitional dynamics, (41) is not identical to lifetime utility. That is, while the welfare maximizing tax rate yields the highest possible lifetime utility, it does not necessarily represent the highest welfare levels *along the balanced growth path*. Since no expression for lifetime utility is available for models with public capital, the comparison between welfare under growth and welfare maximization in these models is not feasible.

Summary statistics for Model 6 are shown in Tables 6 and 7. The tables show that, for both distributions, the mean and standard deviation of the relative difference between the growth and welfare maximizing tax rates are much larger than for the relative difference between the growth rate and welfare under growth and welfare maximization. The mean difference in tax rates is calculated at 14%, while the mean difference in growth rates that result from these is less than 2.4% and the mean of the relative difference of welfare levels is less than 4.3%. For the Normal distribution, differences are smaller than for the Uniform distribution, reflecting the fact that under the Normal distribution the probability of extreme values is smaller. The standard deviations (of relative differences) are also large in absolute terms

Table 6: Model 6 with Normal Parameter Distribution

Variable	Mean	Std. Dev.	Min.	Max.
$\tau^*$	0.475	0.111	0.12	0.738
$\tau^{**}$	0.505	0.096	0.179	0.752
relative difference b/w tax rates	10.412	13.044	0	195.055
$\gamma^*$	0.104	0.054	0.01	0.423
$\gamma^{**}$	0.102	0.053	0.01	0.419
relative difference b/w growth rates	1.61	2.346	0	20.577
relative difference b/w welfare	2.544	4.982	0	77.562
N		7728		

Table 7: Model 6 with Uniform Parameter Distribution

Variable	Mean	Std. Dev.	Min.	Max.
$\tau^*$	0.466	0.143	0.105	0.749
$\tau^{**}$	0.499	0.126	0.125	0.772
relative difference b/w tax rates	14.181	20.984	0.001	261.516
$\gamma^*$	0.114	0.076	0.007	0.583
$\gamma^{**}$	0.112	0.075	0.007	0.581
relative difference b/w growth rates	2.384	3.478	0	28.568
relative difference b/w welfare	4.268	9.559	0	185.425
N		7728		

for taxes but small for growth. This is a key result: the trade-offs in terms of tax policies of the type found in previous sections exaggerate the trade-offs in terms of growth rates and welfare levels. For example, the largest relative difference in tax rates generated from the different parameterizations is 195%. This generates a difference in growth rates of just 16%. Figures 4 and 5 plot the relative difference in tax rates against the relative difference in growth (Figure 4) and welfare levels (Figure 5) for all of the generated parameterization sets (based on the Normal distribution). While there is a positive correlation between the relative differences in tax rates and relative differences in outcomes in both figures, large differences in tax rates are associated with generally larger differences in growth or welfare outcomes. Differences in the optimal tax rate are consistently associated with smaller differences in outcomes, especially in the case of growth rates (Figure 4).

Figure 4: Relative differences of tax rates and growth rates  
(public services ; Normal distribution)

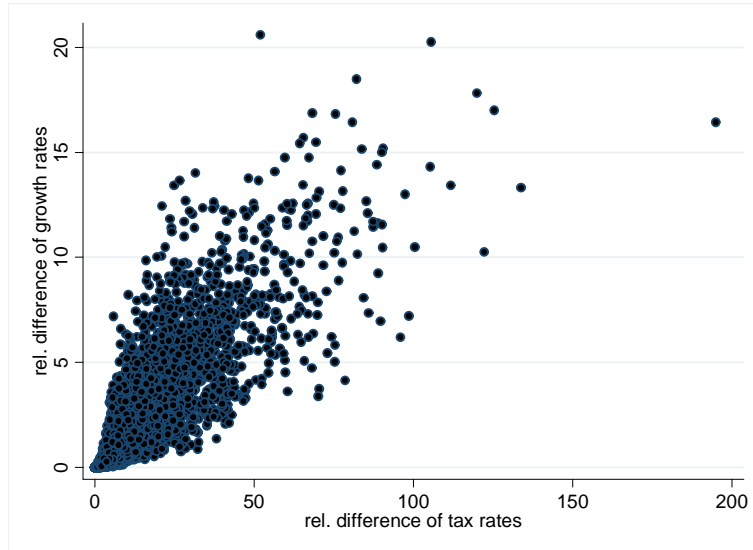
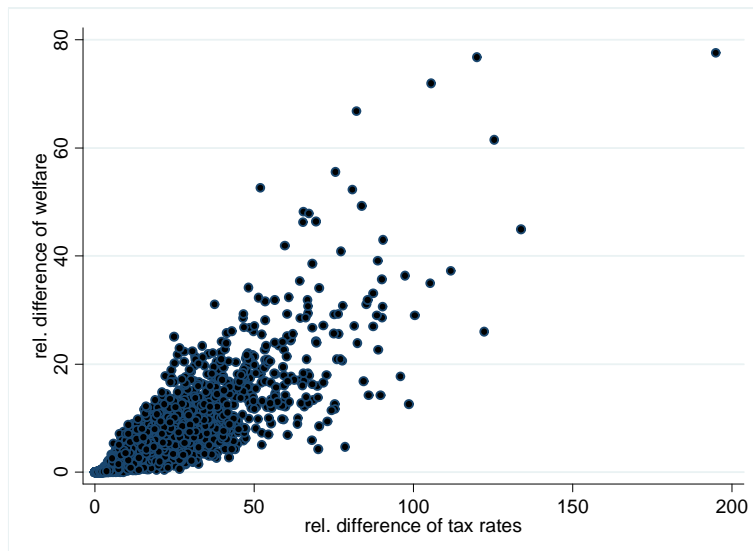


Figure 5: Relative differences of tax rates and welfare levels  
(public services ; Normal distribution)



Figures 6 and 7 shed more light on the distribution of the relative dif-

ferences for the Normal distribution. They show that for 75% of the parameter sets that we generate, the relative differences between growth rates and welfare levels are generally below 5%. This suggests that trade-offs between growth and welfare maximization tend to be very small in most cases, and that therefore, maximizing growth and maximizing welfare roughly yield identical outcomes. The key result in this section is not therefore a result of particular combinations of parameter values we might have chosen, but instead holds for a large number of alternative sets. The reason appears to be the relative ‘flatness’ of both the tax-growth curve and the tax-welfare curve between the growth and welfare maximizing tax rates. Hence, in this region fiscal policy is relatively ineffective. Since growth is essential for welfare, the tax rates typically do not differ to the extent that growth rates (and hence welfare) fundamentally differ under both objectives.

Figure 6: Relative differences of growth rates  
public services ; Normal distribution)

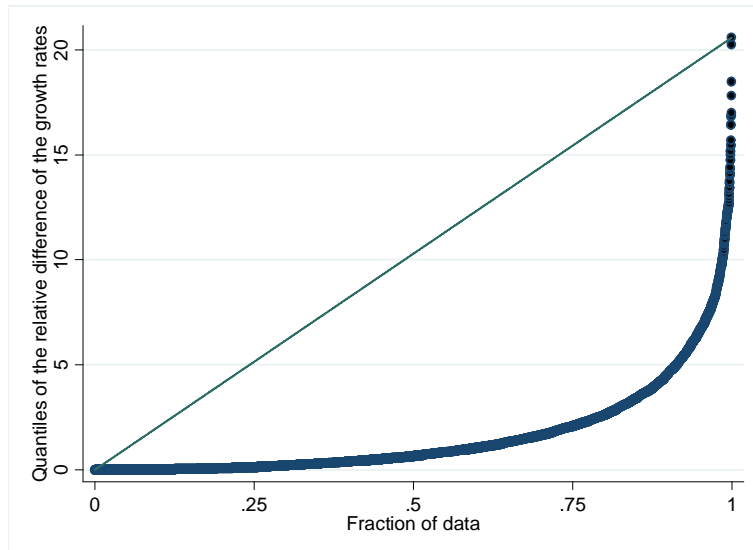
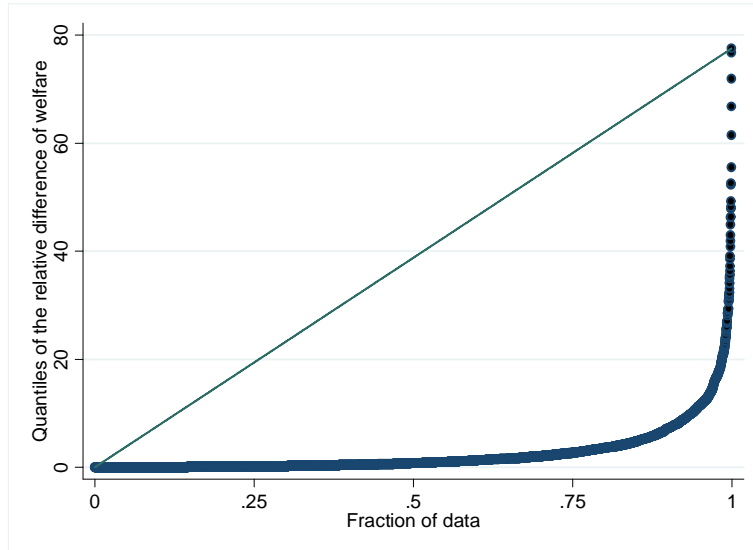


Figure 7: Relative differences of welfare  
(public services ; Normal distribution)



Tables 8 and 9 show the summary statistics for Model 7. First, the tables show that, as in the case of Model 6, for both distributions the mean and the standard deviation of the relative difference between the growth and welfare maximizing tax rates are much larger than the equivalent relative differences in growth rates under growth and welfare maximization. Thus, the assumed parameter distribution does not seem to matter. Figure 8, based on the Normal distribution, confirms that while there is a correlation between the relative differences in tax rates and relative differences in outcomes, the former tend to be much larger.

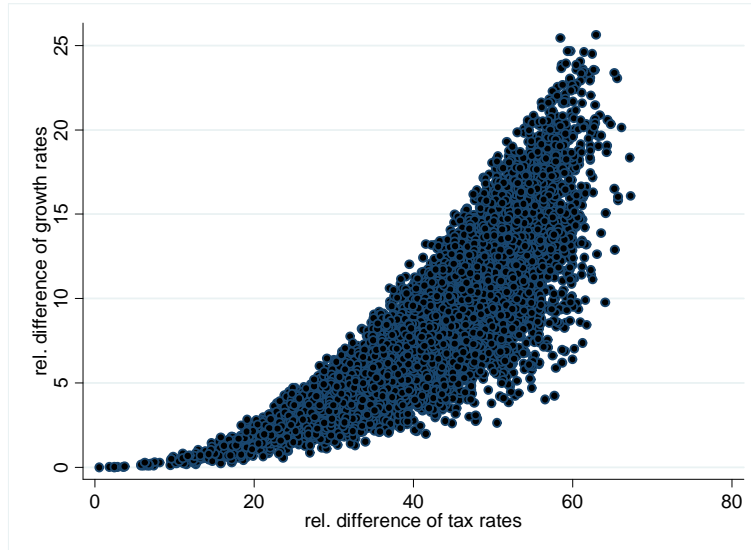
Table 8: Model 7 with Normal Parameter Distribution

Variable	Mean	Std. Dev.	Min.	Max.
$\tau^*$	0.321	0.06	0.112	0.471
$\tau^{**}$	0.185	0.054	0.054	0.434
relative difference b/w tax rates	42.776	10.306	0.593	67.345
$\gamma^*$	0.269	0.046	0.184	0.495
$\gamma^{**}$	0.246	0.052	0.148	0.494
relative difference b/w growth rates	8.840	4.542	0.001	25.614
number of observations when $W^* > W^{**}$	7727			
N	7728			

Table 9: Model 7 with Uniform Parameter Distribution

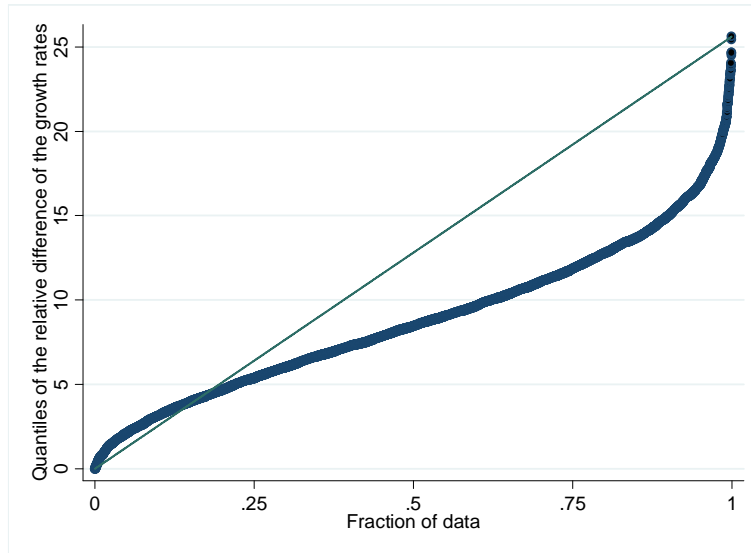
Variable	Mean	Std. Dev.	Min.	Max.
$\tau^*$	0.318	0.08	0.104	0.493
$\tau^{**}$	0.19	0.073	0.039	0.501
relative difference b/w tax rates	41.121	13.767	0.02	67.882
$\gamma^*$	0.278	0.064	0.185	0.618
$\gamma^{**}$	0.257	0.072	0.138	0.618
relative difference b/w growth rates	8.523	5.719	0	28.018
number of observations when $W^* > W^{**}$	7727			
N	7728			

Figure 8: Relative differences of tax rates and growth rates  
(public services ; Normal distribution)



Secondly, Tables 8 and 9 also show that the mean relative difference in growth rates between growth and welfare maximization is below 9%. Compared to the model with public services, this is noticeably larger. The reason is that with public capital there are transitional dynamics, with total welfare driven to a lesser extent by the growth rate along the balanced growth path. Therefore, growth and welfare maximizing tax rates, and hence growth rates, differ much more with public capital. Figure 9 sheds more light on the distribution of the relative differences for the Normal distribution case. They show that for 75% of the parameter sets, the relative difference between growth rates is less than 12.5% (e.g. 3% compared with 3.375%). This suggests that growth rate trade-offs between growth and welfare maximization tend to be moderate in most cases. Given that no expression for lifetime utility is available when transitional dynamics occur, trade-offs in terms of welfare cannot be analyzed. However, as shown in Tables 7 and 8, along the balanced growth path welfare is typically larger under growth maximization than under welfare maximization because the welfare maximizing policy reflects transitional dynamics. This implies that along the balanced growth path, the welfare maximizing policy in Model 7 may not be optimal.

Figure 9: Relative differences of tax rates and growth rates  
(public services ; Normal distribution)



## 7 Conclusions

This paper has considered the difference between growth and welfare maximization by comparing income tax rates under both maximizations in different versions of a model of endogenous growth with fiscal policy. It has also compared growth rates and welfare levels as outcomes of fiscal policy in these different models. Several conclusions can be drawn from this exercise.

Firstly, comparisons between the growth and welfare maximizing tax rates across several different models show that the central results of the existing literature are not robust to small changes in their underlying assumptions. The results depend crucially on the way that fiscal policy is assumed to be effective. The Barro (1990) result does not hold if public services entail mixed effects, or if it is assumed that the elasticity of substitution is less than one. The Futagami et al. (1993) result no longer holds if public capital entails mixed public services. Likewise, it was shown that even if public services enter the utility function, the relationship between the growth and welfare



maximizing tax rates is ambiguous, and the two may even coincide. These comparisons show that for this class of endogenous growth models, without exact knowledge of the model parameters, differences between the growth and welfare maximizing fiscal policies are hard to predict.

The second conclusion modifies the policy concerns raised by the first. The relative differences between growth and welfare maximizing tax rates tend to be much larger than relative differences between growth rates for models with public services and public capital, and welfare levels for models with public services. It was shown that relative welfare and growth trade-offs in models with public services are very small, while in models with public capital, the growth trade-off is larger but still seems moderate. Parameter uncertainty was handled by assuming a distribution function of all parameters instead of adopting specific values. Conditional on the general class of models, this would appear to imply that the choice between growth and welfare maximization is unlikely to have large impacts on growth and, in the case of models with public services, on welfare levels.

While this might motivate investigations of whether the same conclusions hold in a different class of growth models, it should be remembered that the Barro Model and its extensions form a key reference point in policy discussions of the long-run growth effects of fiscal policy. Possible extensions include adding additional factors that cause the welfare maximizing tax rate to diverge from the growth maximizing rate, such as adjustment costs and public good congestion previously identified in the literature. Alternatively the framework might be usefully extended to allow for several sources of public revenue or, in the spirit of Devarajan et al. (1996), several public services. Future research would therefore ideally compare growth and welfare maximization for several different instruments of fiscal policy in models that contain several sources for divergence between the growth and welfare maximizing tax rates.

The results of this paper, though derived from relatively abstract models, have important policy implications. The knowledge available to governments is inevitably imperfect: they typically face informational constraints with regard to household preferences and the magnitude of the utility-enhancing

effects of public services. The results of this paper suggest that the welfare trade-offs between growth and welfare maximization may be small; we find many cases where the growth maximizing fiscal policy yields roughly the same welfare outcome as the welfare maximizing policy. If growth rates are indeed susceptible to fiscal policy, and if the growth enhancing effect of public services are easier to measure, then benevolent governments might reasonably seek to maximize the growth rate instead as a second-best strategy. In addition, the results show that fiscal policy tends to be relatively ineffective in altering welfare levels and growth rates between the growth and welfare maximizing tax rates, at least in models with public services. Changes in fiscal policy within this interval can be expected to have only a small impact on the economy.

## A Appendix

### A.1 Expressions for $y$ , $y_k$ and $y_{kG}$

In the Barro Model and in Model 2, the production function is

$$y = g^\alpha k^{1-\alpha} \quad (\text{A.1})$$

Since

$$g = \tau y \quad (\text{A.2})$$

(A.1) can be rearranged so that

$$\frac{y}{k} = \tau^{\frac{\alpha}{1-\alpha}} \quad (\text{A.3})$$

Using (A.2) and (A.3) yields an expression for  $\frac{g}{k}$ :

$$\frac{g}{k} = \tau^{\frac{1}{1-\alpha}} \quad (\text{A.4})$$

Differentiating (A.1) for  $k$  and using (A.4) for substitute for  $\frac{g}{k}$  yields

$$y_k = (1 - \alpha)\tau^{\frac{\alpha}{1-\alpha}} \quad (\text{A.5})$$

In Models 4 and 6, the production function is

$$y = (\theta k^v + \alpha g^v)^{\frac{1}{v}} \quad (\text{A.6})$$

Using (A.2) to substitute for  $g$  in (A.6) and rearranging yields

$$\frac{y}{k} = \theta^{\frac{1}{v}} (1 - \alpha\tau^v)^{-\frac{1}{v}} \quad (\text{A.7})$$

Using (A.2) and (A.7) yields an expression for  $\frac{g}{k}$ :

$$\frac{g}{k} = \tau\theta^{\frac{1}{v}} (1 - \alpha\tau^v)^{-\frac{1}{v}} \quad (\text{A.8})$$

Differentiating (A.1) for  $k$  yields

$$y_k = \left(\theta + \alpha \left(\frac{g}{k}\right)^v\right)^{\frac{1}{v}-1} \theta \quad (\text{A.9})$$

with  $\frac{g}{k}$  defined according to (A.8).

In the Futagami Model and in Model 3, the production function is

$$y = k_G^\alpha k^{1-\alpha} \quad (\text{A.10})$$

Since along the balanced growth path, private capital, public capital, consumption and output all grow at the same rate given by (24),

$$y = \frac{\dot{y}}{\gamma} \quad (\text{A.11})$$

Using the (A.11) to substitute for  $y$  in  $\dot{k}_g = \tau y$  yields

$$\dot{k}_g = \tau \frac{\dot{y}}{\gamma} \quad (\text{A.12})$$

Integrating and rearranging yields

$$k_g = \tau \frac{y}{\gamma} \quad (\text{A.13})$$

Substituting for  $k_g$  in (A.10) and rearranging yields

$$\frac{y}{k} = \left( \frac{\tau}{\gamma} \right)^{\frac{\alpha}{1-\alpha}} \quad (\text{A.14})$$

Using (A.14) and (A.12),  $z = \frac{k_G}{k}$  can be written as

$$z = \left( \frac{\tau}{\gamma} \right)^{\frac{1}{1-\alpha}} \quad (\text{A.15})$$

Differentiating (A.10) for  $k$  and using (A.15) to substitute for  $z$  yields

$$y_k = (1 - \alpha) \frac{\tau^{\frac{\alpha}{1-\alpha}}}{\gamma} \quad (\text{A.16})$$

In Models 5 and 7, the production function is

$$y = (\theta k^v + \alpha k_G^v)^{\frac{1}{v}} \quad (\text{A.17})$$

Manipulations yield expressions for  $\frac{y}{k}$  and for  $\frac{y}{k_G}$

$$\frac{y}{k} = (\theta + \alpha z^v)^{\frac{1}{v}} \quad (\text{A.18})$$

$$\frac{y}{k_G} = (\theta z^{-v} + \alpha)^{\frac{1}{v}} \quad (\text{A.19})$$

Differentiating (A.17) for  $k$  yields

$$y_k = (\theta + \alpha z^v)^{\frac{1}{v}-1} \theta \quad (\text{A.20})$$

Differentiating (A.17) for  $k_G$  yields

$$y_{k_G} = (\theta z^{-v} + \alpha)^{\frac{1}{v}-1} \alpha \quad (\text{A.21})$$

An expression for  $z$  can be found in the same way as above.

## A.2 Uniqueness and Stability in Model 7

Let  $x = \frac{c}{k}$  and  $z = \frac{k_G}{k}$ . Together with the transversality condition,  $\lim_{t \rightarrow \infty} [\lambda k] = 0$ , and with the initial conditions,  $x_0 > 0$  and  $z_0 > 0$ , the dynamics of the market economy can be expressed as a system of two differential equations:

$$\frac{\dot{x}}{x} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} \quad (\text{A.22})$$

and

$$\frac{\dot{z}}{z} = \frac{\dot{k}_G}{k_G} - \frac{\dot{k}}{k} \quad (\text{A.23})$$

From (24), (13) and (22), respectively,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (\text{A.24})$$

$$\frac{\dot{k}}{k} = (1 - \tau) \frac{y}{k} - x \quad (\text{A.25})$$

$$\frac{\dot{k}_G}{k_G} = \tau \frac{y}{k_G} \quad (\text{A.26})$$

Setting  $\frac{\dot{x}}{x} = 0$  in (A.22) and solving for  $x$  yields its steady state value,  $\tilde{x}$ :

$$\tilde{x} = (1 - \tau) \frac{y}{k} - \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (\text{A.27})$$

Using (A.27) to substitute for  $x$  in (A.25), and using (A.25) and (A.26) to substitute for  $(\frac{\dot{k}}{k})$  and  $(\frac{\dot{k}_G}{k_G})$  in (A.23) yields

$$F = \tau \frac{y}{k_G} - \frac{1}{\sigma} (1 - \tau)y_k + \frac{\rho}{\sigma} \quad (\text{A.28})$$

Using (A.19) and (A.20) to substitute for  $\frac{y}{k_G}$  and  $y_k$ , respectively, in (A.28) yields

$$F = \tau(\alpha z^{-v} + \beta)^{\frac{1}{v}} + \frac{1}{\sigma}((1 - \tau)((\alpha + \beta z^v))^{\frac{1}{v}-1} \alpha v - \rho) \quad (\text{A.29})$$

Differentiating for  $z$  and rearranging yields

$$\frac{1}{\sigma}((1 - \tau)((\alpha + \beta z^v))^{\frac{1}{v}-2} v \beta z^v < \tau(\alpha z^{-v} + \beta)^{\frac{1}{v}-1} z^{-v} \quad (\text{A.30})$$

It can be seen that if  $v \leq 0$ ,  $F$  is a monotonically decreasing function of  $z$  so that there is a unique positive value,  $\tilde{z}$ , that satisfies  $F = 0$ . From (A.27), there is a unique positive value of  $x$  as well. Thus, the growth path is unique.

To investigate the dynamics in the vicinity of the unique steady state equilibrium, equations (A.22) and (A.23) can be linearized to yield

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x - \tilde{x} \\ z - \tilde{z} \end{bmatrix} \quad (\text{A.31})$$

where  $\tilde{x}$  and  $\tilde{z}$  denote the steady state values of  $x$  and  $z$ . From (A.22) and (A.23),  $\dot{x}$  and  $\dot{z}$  can be rewritten as follows:

$$\dot{x} = \left( \frac{1}{\sigma}(1 - \tau)(\alpha + \beta \tilde{z}^v)^{\frac{1}{v}-1} \alpha - \frac{\rho}{\sigma} - (1 - \tau)(\alpha + \beta \tilde{z}^v)^{\frac{1}{v}} + \tilde{x} \right) \tilde{x} \quad (\text{A.32})$$

and

$$\dot{z} = \left( \tau(\alpha \tilde{z}^{-v} + \beta)^{\frac{1}{v}} - (1 - \tau)(\alpha + \beta \tilde{z}^v)^{\frac{1}{v}} + \tilde{x} \right) \tilde{z} \quad (\text{A.33})$$

Saddlepoint stability requires that the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A.31) must be negative:

$$\det J = a_{11}a_{22} - a_{12}a_{21} \quad (\text{A.34})$$

Given the complexity of the matrix, this condition cannot be verified analytically. However, using numerical simulations, it can be shown that the condition holds for every parameter set that was generated under the Normal distribution using the method outlined in Section 6. This strongly supports the assumption that the equilibrium is saddlepath stable.

### A.3 The Welfare Maximizing Tax Rate with Public Capital

The present-value Hamiltonian that corresponds to the maximization problem of the government can be written as follows:

$$H = \left( \frac{(k_G^\beta c^{1-\beta})^{1-\sigma} - 1}{1-\sigma} \right) e^{-\rho t} + \lambda ((1-\tau)y - c) + \mu \tau y \quad (\text{A.35})$$

The first order conditions for the government with respect to  $\tau$  and  $k_G$  are hence

$$-\lambda y + \mu y = 0 \quad (\text{A.36})$$

$$\beta(k_G^\beta c^{1-\beta})^{-\sigma} k_G^{\beta-1} c^{1-\beta} e^{-\rho t} + \lambda(1-\tau)y_{k_G} + \mu y_{k_G} = -\dot{\mu} \quad (\text{A.37})$$

Rearranging (A.36) yields

$$\mu = \lambda \quad (\text{A.38})$$

which implies that

$$\dot{\mu} = \dot{\lambda} \quad (\text{A.39})$$

In addition, the transversality condition has to be fulfilled:

$$\lim_{t \rightarrow \infty} [\mu k_G] = 0 \quad (\text{A.40})$$

The government takes the first order conditions of the household, (19) and (20), as given which are restated here for convenience:

$$\lambda = (1-\beta)(k_G^\beta c^{1-\beta})^{-\sigma} k_G^{\beta} c^{-\beta} e^{-\rho t} \quad (\text{A.41})$$

and

$$\lambda(1-\tau)y_k = -\dot{\lambda} \quad (\text{A.42})$$

Dividing (A.37) by (A.42) leads to

$$\frac{\beta(k_G^\beta c^{1-\beta})^{-\sigma} k_G^{\beta-1} c^{1-\beta} e^{-\rho t} + \mu y_{k_G}}{\lambda(1-\tau)y_k} = \frac{-\dot{\mu}}{-\dot{\lambda}} \quad (\text{A.43})$$

Taking into account (A.38), substituting for  $\lambda$  using (A.41), and rearranging yields

$$1 + \frac{\beta}{1-\beta} \frac{x}{z} y_{k_G} = (1-\tau) \frac{y_k}{y_{k_G}} \quad (\text{A.44})$$

with  $x = c/k$  and  $z = k_g/k$ . When  $x$  and  $z$  are constant,

$$\frac{\dot{k}_G}{k_G} = \frac{\dot{c}}{c} \quad (\text{A.45})$$

which can be written as

$$\tau \frac{y}{k_G} = \frac{1}{\sigma} ((1 - \tau)y_k - \rho) \quad (\text{A.46})$$

From (A.27),

$$x = (1 - \tau) \frac{y}{k} - \frac{1}{\sigma} ((1 - \tau)y_k) + \frac{\rho}{\sigma} \quad (\text{A.47})$$

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