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Abstract

In this paper we consider a regulated monopoly that can pad its costs to increase its cost reimbursement. Even while padding is inefficient the optimal incentive scheme tolerates some padding of costs to reduce the information rents paid to low cost types. It is shown that high cost firms pad costs more than low cost firms. We also show that cost padding moves pricing away from Ramsay optimal pricing toward more monopolistic pricing rules. We show that when auditing of total costs is costly, low cost firms face a fixed price contract and engage in no cost padding. High cost firms do less well but do engage in padding to increase the verified cost. If padded costs can be audited at some cost, low cost types engage in cost padding but high cost types do not. We also endogenize the distribution of cost types by allowing firms to engage in a pre-contractual, non-observable or verifiable cost-reducing investment. The firm adopts a mixed strategy and this determines the distribution of cost types at the contracting stage. An example is given to show how the equilibrium distribution is computed.

KEYWORDS: Regulation; cost padding; costly state falsification; costly state verification; endogenous screening.

JEL Codes: D8; L2; L5.

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1 Introduction

It is a serious concern among regulatory agencies that firms engage in accounting contrivances and cost padding to increase their remuneration from the regulator. Firms usually have many ways of diverting funds to raise or pad costs: increasing salaries and expense claims, "gold-plating" of expenditures, charging other equipment to project costs, advertising for corporate image, charging for depreciated assets, not reporting of cost reducing improvements and so on.

The extent of such cost padding is difficult to quantify as by definition it is hidden. However it can be substantial. Some evidence derives from legal action. In one case of the US government versus the defense contractor Sundstrand in 1989, a sum of $200 million was recovered as the court found that Sundstrand had co-mingled commercial and government costs. In another case from 1985, a U.S. federal grand jury indicted the General Electric Company on charges that it had falsified claims for work on a nuclear warhead system. It was alleged that the government had been defrauded of at least $800,000 between January 1980 and April 1983 because the company had entered exaggerated charges on employee time cards. Another piece of evidence comes from a 1984 audit by the U.S. Department of Defense inspector-general, which found that contractors

1For examples of such reports related to public utilities, the reader is referred to McAfee and McMillan (1988, government contracting - North America), Quiggin (1998, electricity - Australia); Kerr (1998, water - New Zealand); Department of Transportation and Regional Services (2000, transport - Australia); Ontario Federation of Agriculture (1999, energy - Canada); Watson (2000, public utilities - Australia), and the OECD study by Gonenc, Maher, and Nicoletti (2000) that compares the incentives that price-cap regulation provides for cost-padding in electricity and telecommunications. For examples in procurement contracts, the reader is referred to Manoj (2000, Shipping - India) and Higgs (1998, military - U.S.A).

2Another case from 1969 concerned the supply of rocket motor assemblies for the GENIE weapons system by McDonnell-Douglas to the US military. The armed services board of contract appeals found that McDonnell-Douglas had failed to disclose actual experienced manufacturing hours and failed to disclose information about inventories and the latest available prices and quotations on purchased parts. The board found that the government was entitled to a $54,235 price reduction.
were inflating charges for spare parts and tools in over one third of all cases.\textsuperscript{3,4}

Whilst an important concern of regulators cost padding is relatively neglected in the main theoretical studies of regulation.\textsuperscript{5} In Baron and Myerson (1982) for example it is assumed that the regulator is totally unable to observe the firm’s costs and therefore the firm has no incentive to pad costs. Equally in Laffont and Tirole (1993)[Chapter 2] it is assumed that the regulator observes the firm’s costs perfectly but is unable to observe the firm’s effort in cost reduction. Again the firm has no incentive to pad costs as the true cost is perfectly and costlessly monitored. This paper considers an intermediate case where the regulator can perfectly observe total cost, that is true cost plus padded costs but cannot disentangle the two components.\textsuperscript{6} What we show is that the optimal regulatory environment will tolerate some cost padding. The optimum regulatory contract will try to penalise those with high costs and reward those with low costs to reduce cost padding activities. However, since true costs are unknown to the regulator and since reimbursement is socially costly, the regulator will still need to offer higher reimbursement to those with

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\textsuperscript{3}This overpricing amounted in total to only 6 per cent of the value of the equipment in question. Reports of contractors’ overcharging the Pentagon however, appear regularly: thus it has been claimed that hammers selling for $7 in hardware stores were charged to the Pentagon at $436; that a small plastic cap worth 75 cents was charged at $1,118 per unit; that a 25-cent plastic washer was charged at $400 per unit. In 1984, the Air Force paid $7,600 for a coffee maker for use in a Lockheed transport aircraft. Although often cited these inflated prices are in part consequence of the Pentagon’s accounting rules: a large proportion of the price of the $436 hammer consists of overhead and extra labour costs charge in accordance with Pentagon regulations (see McAfee and McMillan (1988)).

\textsuperscript{4}Some indirect evidence of the potential scale of cost padding may be garnered from the study of hierarchical organizations by McAfee and McMillan (1995) who cite the studies by Berliner (1957), Schiff and Lewin (1968) and Schiff and Lewin (1970). Berliner interviewed former managers of Soviet firms and found as one manager reported "an enormous amount of falsification in all branches of production and in their accounting systems..." (p.161). Schiff and Lewin studied the efficiency of divisions within three large U.S. corporations and produced estimates of the size of cost-padding within divisions of the same company to be between 20 and 25%.

\textsuperscript{5}For a survey see e.g. Laffont (1994).

\textsuperscript{6}As we explain below this model is very different from the model of Laffont and Tirole (1993)[Chapter 12] that does address issues of cost padding.
higher costs and won’t be able to eliminate cost padding completely. This obviously has
important legal and policy implications. If evidence of cost padding is found, then this
should not be taken as a *prima facia* case of contract violation if the contract was initially
designed to make allowance for some padding of costs.

Another serious concern of regulators and the main concern addressed by the work
of Laffont and Tirole (1993) is the extent to which firms have an incentive to engage
in cost reducing activity. We address this issue in a different way by allowing firms to
influence costs by undertaking a cost reducing activity or investment at a *pre-contractual*
stage. As in Laffont and Tirole (1993) this cost reducing activity or investment is not
observable by the regulator and hence not contractible. With cost reducing activity
undertaken before the contract is signed, it is shown that a firm will engage in a mixed
strategy. The firm wishes to keep the regulator uninformed about its cost reduction in
order to extract information rents. If it were to adopt a pure strategy for its cost reduction
activity decision, the regulator would anticipate this and offer an appropriate fixed price,
high power incentive contract. The distribution of cost types used by the regulator in
designing the optimum contract is then determined endogenously by the firm’s mixed
strategy and depends on the underlying technology parameters. We show how this leads
to underinvestment in cost reducing activity. It is also shown that the monotone hazard
property holds, so that given assumptions on preferences and technology, there is no
bunching in the optimum contract. An example is computed to show how the equilibrium
distribution of cost types is determined.

In the second part of the paper we consider auditing of costs by the regulator. We
restrict attention to perfect and deterministic but costly auditing and consider two cases.
In the first case we move in the direction of Baron and Myerson (1982) and assume that
total costs are only observable if the regulator pays an auditing cost. This is similar to
the model of Baron and Besanko (1984). In the second case we move in the direction
of Laffont and Tirole (1993) and assume that true cost, that is cost net of padded costs,
can be observed by the regulator if costly auditing is undertaken. In both case auditing
of high cost firms is undertaken. However, when total costs are audited it is low cost firms that are given a high powered incentive contract and high cost types engage in cost padding whereas when padded costs are audited it is the low cost types that engage in cost padding activities.

There is a small literature that directly considers cost-padding in regulated firms. In these models the firm operates subject to some regulatory constraint. In Albon and Kirby (1983) the regulatory constraint is a profit target which the firm is not allowed to exceed. They assume that padded costs add directly to the utility of the firm and show that it will simply pad costs so that the profit target is met at the monopoly outcome. In Daughety (1984) the regulator imposes a profit target that profit over revenue cannot exceed some positive fraction less than one. In his model padded costs add nothing to utility but he shows how costs may be padded so that the regulatory constraint is met at the revenue maximizing output where marginal revenue equals zero.7 In these models however, the regulatory constraint is exogenously given and it is important to determine if there might still be cost padding when the regulator optimally determines the regulatory environment. In Section 4 we show how cost padding induces a move away from Ramsay pricing toward greater restriction of output. This coincides with the results of Daughety (1984). This conclusion is reinforced when the distribution of types is endogenous as firms will invest less for any given output level.

This paper contributes to and draws together three different literatures to address the problem of regulation. Firstly our treatment of cost padding as a hidden action problem follows from the literature on costly-state falsification. Secondly our treatment of auditing follows the literature on costly-state verification. Thirdly, the treatment of pre-contractual cost reducing activity is related to the literature on endogenous screening.

Cost padding in our model is a hidden action of the firm. The firm acts to pad costs to increase the cost reimbursement. In this we follow the literature on costly-state falsification initially proposed and analysed by Lacker and Weinberg (1989). They

7See Waterson (1988) for a summary of this type of model.
showed that optimal contracts may tolerate some falsification in equilibrium and were able to demonstrate conditions under which no falsification would be optimal. The form of the contract crucially depends on whether or not non-contingent transfers are allowed. The intuition is simple. Non-contingent contracts imply fixed transfers that eliminate any incentives for falsification. However, these type of contracts either defeat the purpose of having a contract (e.g. insurance) or violate a participation constraint of the agent or are costly to the principal as in the current paper. In general, it is optimal to tolerate some falsification of costs.\footnote{These costly state falsification model has been extended by Maggi and Rodríguez-Clare (1995) who consider a general agency model with risk neutral principal and agent and by Crocker and Morgan (1998) who allow for risk aversion and consider falsification and fraud in insurance contracts.}

There is an obvious connection between costly-state falsification and costly-state verification of the type considered by Townsend (1979), Gale and Hellwig (1985) and Williamson (1986). In costly state falsification, the agent can falsify the state at some cost. In costly-state verification, the principal can verify the state at some cost. In this paper we marry both problems.\footnote{We consider only deterministic monitoring with commitment. A number of authors consider stochastic auditing, e.g. Boyd and Smith (1994), Mookherjee and Png (1989), Chander and Wilde (1998) and auditing without commitment, e.g. Khalil (1997) and Jost (1996).} The firm as an agent of the regulator can pad or falsify its costs. The regulator may be able to verify or audit either total costs or actual (non-padded costs) at some cost to itself. In Laffont and Tirole (1993, chapter 12) in contrast audits allow regulators to receive a signal that is correlated with the degree of cost-padding. The quality of the signal is exogenous and auditing is costless. Beyond its applicability for regulatory practice, our approach might be of more general interest. The combined costly-state verification and costly-state falsification framework might have interesting applications for finance, insurance and taxation.

Finally, the paper follows the literature on endogenous screening, González (2000) and Gul (2001), in deriving the distribution of cost types as an equilibrium outcome from an initial investment choice of the firm. Since the distribution of cost types is
derived endogenously, the properties of the cost-reimbursement contract depend only on the fundamental technology and preference parameters of the model and do not depend on arbitrary assumptions about the distribution of types which are difficult to specify econometrically.

The paper proceeds by examining the issues of cost padding, endogeneity of the cost type distribution and the auditing of costs in turn and is organized as follows: The next section outlines the basic model with procurement and solves for incentive compatible falsification contracts for a given type distribution. Section 3 endogenizes the type distribution by considering the firm’s strategy for pre-contractual cost reducing activity. Section 4 extends the model to consider monopoly regulation and addresses the issue of deviations from Ramsay pricing implied by cost padding activities of firms. In Section 5 and 6 we return to the procurement model with an exogenous type distribution and consider costly auditing of total and padded costs in turn. Section 7 concludes and suggests ideas for future research.

2 The Procurement Model

In this section we consider a procurement model where the regulator commissions a firm to undertake a public project which is worth \( V \) to consumers. The total costs incurred by the firm are \( c \in \mathbb{R}_+ \). The regulator reimburses the firm its costs and pays in addition a transfer \( t \). Thus the transfer is net of all measured costs and is a tax if negative. We follow Laffont and Tirole (1993) and introduce a shadow cost of public funds. Letting \( 1 + \lambda > 1 \) denote the shadow cost of public funds, the social cost of the transfer is \((1 + \lambda)(t + c)\).\(^{10}\) The regulator is assumed to maximize the sum of consumer surplus plus producer surplus (profits) net of the social transfer cost. Thus the social welfare for a given cost and transfer

\(^{10}\)We shall show that \( t + c > 0 \), so the sum \( t + c \) is the funds that are transferred to the firm even if \( t < 0 \) in some cases.
is

\[ V + r - (1 + \lambda)(t + c) \]

where \( r \) is the producer surplus.

The firm has a cost of realizing the public project of \( g(k) \), where \( k \in \mathcal{K} \subset \mathbb{R}_+ \) is the stock of investment capital. We assume that \( \mathcal{K} \) is an interval of the form \([k, \bar{k}]\) and that costs are a declining and convex function of the capital input: \( g'(k) < 0 \) and \( g''(k) > 0 \). Thus a higher \( k \) reduces production cost but there are diminishing returns to cost reduction.\(^{11}\) We assume that each unit of capital costs one unit.\(^{12}\) The long-run efficient level of cost reducing investment which maximizes \( V - g(k) - k \) is \( k^* \), where \( g'(k^*) = -1 \). We assume that \( g'(0) < -1 \) so that \( k^* > 0 \). We also assume that the project is worthwhile so \( V - g(k^*) - k^* > 0 \). We denote the range of \( k \) where the project is exante profitable as \([k_{\min}, k_{\max}]\), where \( k_{\min} = 0 \) if \( V - g(0) \geq 0 \).

The total cost \( c \) of a firm with cost reducing investment of \( k \) comprises of two components \( c = g(k) + x \), where \( g(k) \) is the underlying or true cost when the project is undertaken. The variable \( x \) can represent either the extent to which the firm can falsely report its costs or any additional or irrelevant costs undertaken by the firm once the true underlying cost is known. That is the amount \( x \) can be either a real but essentially unnecessary expenditure undertaken by the firm or an accounting contrivance that raises the perceived costs as seen by the regulator. We will refer to \( x \) as cost padding whether this comes from false reporting or actual but wasteful expenditure.\(^{13}\) The regulator only observes the total cost \( c \), whereas the firm observes the true cost \( g(k) \) before deciding on the level of cost padding, \( x \).

\(^{11}\)In this section \( k \) is fixed and firms are distinguished by their capital stock. In the next section we shall allow firms to choose the extent of their investment in cost reduction.

\(^{12}\)This is for simplicity and we could allow the cost of capital to be some constant \( \rho \) - the competitive price - or allow for some increasing cost function.

\(^{13}\)Since the cost padding arises because of the firm’s monopoly position in supplying the regulator it may also be interpreted as a measure of the extent of \( x \)-inefficiency and suggest that the analysis may be applied more generally to any monopolistic firm.
As discussed in the introduction there are many ways in which firms can pad costs: advertising and sponsorship, transfer of funds across divisions, unnecessary remuneration increases, larger than normal allowances for depreciation, not reporting on cost-saving improvements, and various other perks as well as other costly accounting contrivances. The regulator observes only total cost $c$ and does not observe cost reducing investment $k$ or padded costs $x$, so is unable to disentangle these two components. The investment in cost reduction is hidden information of the firm and padded costs are a hidden action of the firm as far as the regulator is concerned. The regulator makes some probability assessment of $k$, on $\mathcal{K} = [\underline{k}, \bar{k}]$ with a hazard rate $h(k)$ and a distribution function

$$F(k) = 1 - e^{\int_{\underline{k}}^{k} h(\kappa) d\kappa}.$$  

So that if $F(k)$ is differentiable, the density function satisfies $f(k) = h(k)(1 - F(k))$.

We will assume a monotone hazard property that $h'(k) > 0$ applies in this section and give a proof that this monotonicity property does hold when the level of cost reduction is determined endogenously.

We assume that the extra padded cost $x$ generates some utility benefit for the firm $\psi(x)$. The benefit function $\psi(x)$ is assumed to satisfy $\psi(0) = 0$, $0 \leq \psi'(x) \leq 1$, $\psi''(x) < 0$ and $\psi'''(x) > 0$.\textsuperscript{14} That is an increase in padded costs by one unit generates a positive gain in utility but not as much as the costs incurred and marginal benefit is declining with costs. It seems natural to assume that the extra expenditures do generate some utility benefit to the firm and are not simply pure waste. Also the assumption that the marginal benefit is less than one is consistent with the positive shadow cost of public funds. The analogy here is that diverting costs to the contracted project from another area of business has a positive shadow cost for the firm itself. The assumption on the third derivative is made to avoid the possibility of the optimum contract being stochastic. 

As has already been mentioned, there are two interpretations of cost padding. Either it may be a real expenditure that has a direct utility benefit of it may be a costly accounting contrivance that raises the perceived costs as seen by the regulator. To see

\textsuperscript{14}These are fairly standard assumptions (see Crocker and Morgan (1998)).
this suppose that the benefit function $\psi(x)$ is written as $\psi(x) = x - d(x)$, where the assumptions on $\psi(x)$ imply that $d(0) = 0$, $0 \leq d'(x) \leq 1$, $d''(x) > 0$ and $d'''(x) < 0$. The function $d(x)$ can then be interpreted as the cost of falsifying the reported costs. With this interpretation $c$ is the cost observed by the regulator and $g(k)$ is the cost incurred by the firm as before. However, the difference $x = c - g(k)$ is not real expenditure, but an accounting contrivance that raises the costs seen by the regulator. The function $d(c - g(k))$ measures the costs of falsifying the accounts and depends upon the extent of the falsification $c - g(k)$. The profits of the firm are given by the transfer $t$ plus the reimbursed costs observed by the regulator $c$, less the true costs of production $g(k)$, less the cost of falsifying the accounts $d(c - g(k))$ less the capital cost $k$. Thus the profits of the firm are

$$\pi = t + c - g(k) - d(c - g(k)) - k = t + x - d(x) - k = t + \psi(x) - k$$

showing the equivalence of both interpretations.\textsuperscript{15} It is to be emphasized though that these two alternatives represent very different situations as in one case there is a real expenditure that generates utility benefits whereas in the later case it is an accounting contrivance which has real costs.

The objective of the regulator is to design the transfer to maximize social welfare by making the transfer $t$ depend upon the observed costs $c$. This is the hidden action problem, with the action $x$ being taken once the level of cost reducing investment $k$ and hence cost $g(k)$ is determined and known by the firm, but unobserved by the regulator. For a given transfer $t$ and choice of cost padding $x$ the firm’s net utility benefit is

$$\pi = t + \psi(c - g(k)) - k.$$

The firm will want to choose $x = c - g(k)$ to maximize profits and this choice will depend on how $t$ responds to different costs observed by the regulator. It is important to remember that cost padding tightens even further the informational constraints faced

\textsuperscript{15}The second interpretation in terms of the cost of falsifying the accounting cost is equivalent to the standard model of costly-state falsification (see e.g. Crocker and Morgan (1998)).
by regulators imposed by the superior knowledge that firms possess about their costs. Indeed the desire to pad costs only arises in circumstances where the regulator needs to extract information rents from the firm so as to avoid socially costly transfers to the firm. Absent these considerations there would be no cost padding.

Using the standard approach in hidden action problems we apply the revelation principle and restrict attention to direct mechanisms where the firm reports its investment in cost reduction $\hat{k}$ and impose incentive compatibility constraints that the firm will have no incentive to misreport its investment level. The taxation principle then applies and a contract that specifies total cost $c(\hat{k})$ and transfer $t(\hat{k})$ is equivalent to one specifying a transfer function $t(c)$ that depends directly on observed costs. We will show that $t$ and $c$ are negatively related.

Given the contract $t(\hat{k})$ and $c(\hat{k})$, the firm has the option not to participate, so a firm with investment $k$ will only participate if

$$t(\hat{k}) + \psi(c(\hat{k}) - g(k)) \geq 0.$$ 

Define $\pi(k, \hat{k}) = t(\hat{k}) + \psi(c(\hat{k}) - g(k)) - k$ to be the profit the firm earns with investment $k$ when it announces $\hat{k}$. The participation constraint is $\pi(k, \hat{k}) \geq -k$ as the investment in cost reduction $\hat{k}$ is a sunk cost. The incentive compatibility constraints are

$$\pi(k, k) \geq \pi(k, \hat{k}) \quad \forall \ k, \hat{k} \in [k, \bar{k}].$$

The firm will choose to announce the level of cost reducing investment $\hat{k}$ that maximizes profits given the contract $(t(\hat{k}), c(\hat{k}))$ it faces. We will suppose that these functions are continuous and piecewise differentiable so that a first-order approach can be used almost everywhere. Then the derivative of the profit function is $\pi_2(k, \hat{k}) = \dot{t}(\hat{k}) + \psi'(c(\hat{k}) - g(k))\dot{c}(\hat{k})$ where subscripts denote partial derivatives and dots denote derivatives with respect to $k$. The incentive compatibility constraint means that the first-order condition for this maximization problem is satisfied when $k = \hat{k}$ so that

$$\pi_2(\hat{k}, \hat{k}) = \dot{t}(\hat{k}) + \psi'(c(\hat{k}) - g(\hat{k}))\dot{c}(\hat{k}) = 0.$$
Hence substituting in this first-order condition, the derivative of the profit function becomes

$$\pi_2(k, \hat{k}) = (\psi'(c(\hat{k}) - g(k)) - \psi'(c(\hat{k}) - g(\hat{k})))\hat{c}(\hat{k}).$$

We will assume for the moment that $\hat{c}(\hat{k}) < 0$.\(^{16}\) Then since $\psi''(x) < 0$ and $g'(k) < 0$, we have $\pi_2(k, \hat{k}) > 0$ for $\hat{k} < k$ and $\pi_2(k, \hat{k}) < 0$ for $\hat{k} > k$. Thus we have a global maximum at $\hat{k} = k$ when $\pi_2(k, k) = 0$. Letting $r(k) = \pi(k, k) + k = t(k) + \psi(c(k) - g(k))$ denote the producer surplus for the given $k$, the first-order condition can be written more simply as

$$\hat{r}(k) = -\psi'(c(k) - g(k))g'(k) \quad (1)$$

and the participation constraint can be written as

$$r(k) \geq 0 \quad \forall \ k \in [k, \bar{k}] \quad (2)$$

The problem for the regulator is to maximize social welfare. For a given cost and transfer social welfare is

$$V + r - (1 + \lambda)(t + c) = V - \lambda r - (1 + \lambda)(c - \psi(c - g(k))) \quad (3)$$

and the profit of the firm is $\pi(k) = t(k) + \psi(c(k) - g(k)) - k$. The regulator’s problem is to choose $r(k)$ and $c(k)$, to maximize expected social welfare

$$\int_{k}^{\bar{k}} (V - \lambda r(k) - (1 + \lambda)(c(k) - \psi(c(k) - g(k)))) dF(k)$$

subject to the incentive compatibility constraint (1) and subject to the participation constraint (2). Given the incentive constraint (1), $r(k)$ is an increasing function (given $c(k) - g(k) > 0$) so the participation constraint can be simplified to $r(k) \geq 0$. We can see that rent enters negatively into the objective function (3), given $\lambda > 0$, so this constraint will hold as an equality,

$$r(k) = 0. \quad (4)$$

\(^{16}\)It will be shown later that the monotone hazard rate assumption $h'(k) > 0$, together with the assumption that $\psi'''(x) \geq 0$ is enough to ensure that the constraint is satisfied.
We therefore have a standard optimal control problem with one endpoint fixed (the lower endpoint) and the other free. Letting $\phi(k)$ be the costate variable. The Hamiltonian function is:

$$H(r, c, \phi, k) = (V - \lambda r - (1 + \lambda)(c - \psi(c - g(k))))dF(k) + \phi \psi'(c - g(k))g'(k). \quad (5)$$

The first-order conditions for the solution are obtained from maximizing (5) subject to (1) and (4) are:

$$\dot{\phi}(k) = -\lambda dF(k) \quad (6)$$

$$\phi(\bar{k}) = 0 \quad (7)$$

$$(1 + \lambda)(1 - \psi'(c(k) - g(k)))dF(k) = \phi(k)\psi''(c(k) - g(k))g'(k) \quad (8)$$

Using the conditions (6) and (7) we have $\phi(k) = \lambda(1 - F(k)) \geq 0$, so that substituting into (8), $c(k)$ is determined by the following equation

$$\psi'(c(k) - g(k)) = 1 - \frac{\lambda \psi''(c(k) - g(k))g'(k)}{h(k)} \quad (9)$$

where $h(k) = \frac{f(k)}{(1 - F(k))}$ is the hazard rate and $f(k)$ is the density function. The second term on the right hand side of this equation is positive, so $\psi'(c(k) - g(k)) < 1$ with $\psi'(c(k) - g(k)) \to 1$ as $k \to \bar{k}$ provided $h(k) \to \infty$ as $k \to \bar{k}$. Thus costs are always padded except possibly at the highest capital level. This is the classical efficiency at the top result.

Given that $c(k)$ is decreasing in $k$ the total reimbursement $t(k) + c(k) = r(k) - \psi(c(k) - g(k)) + c(k)$ is also decreasing in $k$. The derivative of the right-hand side with respect to $k$, is $\dot{r}(k) - \psi'(c(k) - g(k))(\dot{c}(k) - g'(k)) + \ddot{c}(k)$. Using equation (1) this is equal to $\dot{c}(k)(1 - \psi'((c(k) - g(k))))$. Since $\dot{c}(k) < 0$ and $\psi'(x) < 1$, it follows that $t(k) + c(k)$ is decreasing in $k$. Since $r(k) = 0$ it follows that $t(k) < 0$. Thus at low values of $k$ (high costs) there is cost padding which is tolerated but the regulator imposes a tax on the firm in order to compensate for the known additional costs. In fact $t'(k) > 0$ as from the incentive compatibility condition (1), $t$ and $c$ are negative related, i.e. $\dot{t}(k) = -\ddot{c}(k)(\psi'(c(k) - g(k))) > 0$. 

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We now want to check the conditions for \( \dot{c}(k) < 0 \) as assumed. Differentiating equation (9) gives
\[
\dot{c}(k) - g'(k) = -\frac{\lambda}{1+\lambda} \left( \psi''(c(k) - g(k))(g''(k)h(k) - g'(k)h'(k)) \right)
\]
It is easy to check that if \( \psi'''(x) \geq 0 \) and \( h'(k) > 0 \), then the right hand side is negative and therefore \( \dot{c}(k) < 0 \).

The following example specifies the technology \( g(k) \), the hazzard function \( h(k) \), the shadow cost of public funds \( \lambda \) and the preferences \( \psi(x) \) and shows how the level of cost padding \( x(k) \) and the optimum contract \( c(k) \) and \( r(k) \) can be calculated. The expected level of cost padding can also be computed. In the example it is \( \frac{1}{3}(8 - 5\sqrt{2}) \).

**Example 1** Let \( g(k) = 5 - 2\sqrt{2}\sqrt{1+k} \), which implies \( k_{\min} = V - 1 - 2\sqrt{2}\sqrt{V - 2} \) and \( k_{\max} = V - 1 + 2\sqrt{2}\sqrt{V - 2} \); \( \psi(x) = x - \frac{1}{2}x^2 \), \( \lambda = 1 \) and \( h(k) = \frac{1}{(1-k)} \) so that the distribution is uniform on \([0,1]\) provided \( V \geq 5 - 2\sqrt{2} \). From equation (9) the amount of cost padding is \( x(k) = \frac{1}{2}\frac{\sqrt{2}(1-k)}{\sqrt{1+k}} \), so the costs reimbursed are \( c(k) = 5 - 2\sqrt{2}\sqrt{1+k} + \frac{1}{2}\frac{(1-k)\sqrt{2}}{\sqrt{1+k}} \). From equation (1) producer surplus is \( r(k) = k + 2\sqrt{2}(\sqrt{1+k} - 1) - 2 \log_{e}(1+k) \) and the expected level of cost padding is \( \frac{1}{3}(8 - 5\sqrt{2}) \).

### 3 Choice of Investment

In this section we endogenize the determination of the investment in cost reduction. Suppose that the firm can choose its investment in cost reduction at a pre-contractual stage.\(^{17}\) The firm’s strategy is a distribution on \( K \). The regulator’s strategy is the contract offered.

\(^{17}\)Again this is different from Laffont and Tirole (1993, chapter 1). They have effort in cost reduction after the contract is signed. They also allow for a capital investment, which may be contractible or non-contractible but is also undertaken after the contract is signed. Allowing for investment in cost reduction to be pre-contractual both simplifies the analysis and may in many circumstances be more realistic.
There cannot be an equilibrium in which the firm chooses a pure strategy. If the firm were to choose a pure strategy, then the regulator’s best response would be to offer a contract that simply reimbursed the known costs of the firm’s investment choice, i.e. to offer a fixed price contract.¹⁸ In such a situation no information rent is paid to the firm, no costs are padded. The firm will however, be able to extract rents from the regulator by adopting a mixed strategy for the choice of $k$. The regulator has beliefs about the firm’s choice of investment which are represented by the hazard rate $h(k)$. Given these beliefs the regulator will offer the contract outlined in Section 2. If the firm is to adopt a mixed strategy, each possible choice of investment level $k$ must give rise to the same level of profit. Thus $t(k) + \psi(c(k) - g(k)) - k = t(k') + \psi(c(k') - g(k')) - k'$ for any $k$ and $k'$ in the support of the distribution. We shall show that this determines the hazard rate uniquely and will assume that the firm randomizes precisely according to this distribution.¹⁹

Next we argue that if $k$ can be chosen from $\mathcal{K} \subset \mathbb{R}_+$ then the support of the distribution must be the subinterval $[0, k^*] \subset \mathbb{R}_+$. First suppose that $k$ can be chosen from $\mathcal{K} \subset \mathbb{R}_+$, then it is shown in the Appendix that if the support of the distribution is discrete with investments $\{k_0, k_1, \ldots, k_S\} \subset \mathcal{K}$, then the firm can choose some $k \in \mathcal{K}$ between some $k_i$ and $k_{i+1}$, report $k_i$ and can raise profits. The intuition is that the contract imposes downward incentive compatibility constraints so that the firm is indifferent between choosing $k_{i+1}$ and reporting $k_i < k_{i+1}$, that is of reporting a higher cost. Since incentive compatibility is not imposed for all investment choices, the firm can raise profits by reducing investment somewhat and reporting higher than actual costs. Thus given that the support of the distribution is an interval and given that the mixed strategy assures the same profit from each $k$ chosen with positive probability and also given the assumed differentiability of the functions, the choice of $k$ must satisfy

$$-\psi'(c(k) - g(k))g'(k) - 1 = 0.$$  

¹⁸A fixed price contract is incentive compatible but the firm’s best response to any feasible fixed price contract is to choose the efficient level of investment $k^*$. Thus the only pure strategy equilibrium is for the firm to choose $k^*$ and the regulator to reimburse $g(k^*)$.

¹⁹As with any mixed strategy equilibrium, the equilibrium is not strict.
This equation determines the cost function $c(k)$ directly. It can be seen directly from equation (11) that $\dot{c}(k) - g'(k)$ is negative since $\psi''(x) < 0$ and $g'' > 0$, and hence that the second-order condition that $c(k)$ is decreasing is satisfied. Thus there is no bunching in equilibrium. Note too that equation (11) shows directly that $\dot{U}c(k) - g'(k)$ is negative since $\psi''(x) < 0$ and $g'' > 0$, and hence that the second-order condition that $c(k)$ is decreasing is satisfied. Thus there is no bunching in equilibrium. 

In equilibrium this hazard rate is the hazard rate for the mixed strategy choice of the firm. In addition it can be seen from equation (10) that the hazard rate is restricted such that $h'(k)/h(k) > g''(k)/g'(k)$. Thus the hazard rate need not be monotonically increasing and an example is given below where it is not.

Next consider the determination of the endpoints of the distribution $k$ and $\bar{k}$ in $\mathcal{K}$. At the pre-contractual stage the firm has the option not to undertake any investment in cost reducing activity. With $k$ known the regulator will leave the firm with no profits. The firm can do no worse than this, so there is an ex-ante constraint that

$$t(k) + \psi(c(k) - g(k)) - k \geq 0$$

or $r(k) \geq k$. Since $r(k) = 0$, this implies $\bar{k} = 0$. This is really the hold-up problem. Once the firm has invested in cost reducing activity, the regulator can extract the entire rent from the highest cost firm and thus in order to have non-negative profits ex ante, not investing must be an option. Equally at the top endpoint $c(\bar{k}) = g(\bar{k})$ from equation (9) and this implies $\psi'(0) = 1$ so from equation (11) $g'(\bar{k}) = -1$ so that $\bar{k} = k^*$. For $k < k^*$,

\[\text{20The expected costs of the investment should also be subtracted but this does not affect the maximization problem.}\]
$g'(k) < -1$, so that from equation (11), $c(k) > g(k)$ and costs are padded. The following example illustrates the optimum contract and shows how the hazard rate is computed.

**Example 2** Let $g(k) = 2(1 - \sqrt{k})$ so that $k_{\text{min}} = V - 2\sqrt{V - 1}$ and $k_{\text{max}} = V + 2\sqrt{V - 1}$; $\psi(x) = x - \frac{1}{2}x^2$, $\lambda = 1$. Then $k^* = 1$ and $k_{\text{min}} = 0$ provided $V \geq 2$. From (11) $x(k) = 1 - \sqrt{k}$ and $c(k) = 3(1 - \sqrt{k})$. From (1) $t(k) = \frac{1}{2}(3k - 1)$ and from (9), $h(k) = \frac{1}{2(\sqrt{k} - k)}$ so that the equilibrium distribution function is $F(k) = \sqrt{k}$ on $[0, 1]$. The transfer as a function of observed costs is $t(c) = 1 - c + \frac{1}{6}c^2$. The expected level of cost padding is $\frac{1}{2}$.

The implication of the hold-up problem is that the regulator will always face some high cost firms. This would potentially be a problem if the regulator had the right to exclude high cost firms from the contract. If the regulator has this right, it will choose some $k_\ast \geq k$ such that those firms with $k < k_\ast$ are excluded. It is clear that the amount of cost padding undertaken by a given firm is unaffected by this exclusion as the amount of cost padding is determined by local incentive constraints. Equally the regulator will offer the high cost type $k_\ast$ a zero rent from the contract. Clearly for consistency in this case we require that the regulator optimally chooses $k_\ast = 0$. The condition for this to be true is

$$\lambda + (V - (1 + \lambda)(c(0) - \psi(c(0) - g(0))))h(0) \geq 0$$

where $c(0)$ satisfies equation (11) and given that $f(0) = h(0)$. This equation says that the project should cover costs and falsification costs at zero investment adjusted by the social cost of funds and is satisfied if $V$ is sufficiently large and $g'(0)$ is finite.

### 4 Variable size project

In this section, we allow for a variable project size so that, in addition to the optimal cost-reimbursement rule, we can also analyze optimal output and pricing decisions. Let $q$ denote the output and $V(q)$ the value of the project to the consumers, where $V(0) = 0, V'(q) > 0, V''(q) < 0$. The firm has a marginal cost of realizing the public project of
The firm’s true total variable cost is equal to $g(k, q)$ and the short-run marginal cost is $g_q(k, q)$. In addition to production costs, total costs $C$ also include the level of cost padding $x$, i.e. $C = g(k, q) + x$. As before, $\psi(x)$ is the firm’s benefit from cost padding.

The first-best optimum is to set the marginal social value of an extra unit of output equal to the short run marginal cost of public funds.

$$ V'(q) = (1 + \lambda)g_q(q, k) $$

Denote the first-best level of output as $q^{FB}(k)$. In the case of of a private good, $V'(q) = p + \lambda p + \lambda qP(q)$ where $p$ is price and $P(q)$ is the inverse demand curve. The Lerner index is given by $L = \frac{p - g_q(k, q)}{p}$ and the Ramsay index is given by $R = \frac{\lambda - 1}{1 + \lambda \eta}$. Thus the at the social optimum the Ramsay pricing rule $L = R$ holds and the Lerner index is proportional to the inverse of the elasticity of demand $\eta = -\frac{dg}{dp}/p$ with the factor of proportionality $\frac{\lambda}{1+\lambda}$.

Now consider the case where the regulator can only observe $C$, but where $k$ is exogenous. In this case the analysis proceeds as in Section 2 except that the regulator also determines an output level $q(k)$ in addition to cost reimbursement and transfer rules $C(k)$ and $t(k)$. We shall show that when the regulator cannot observe the extent of cost padding, it will adapt the Ramsay pricing rule so $L > R$ and $q(k) < q^{FB}(k)$. Proceeding as before the incentive compatibility constraint is

$$ r(k) = -\psi'(C(k) - g(k, q(k)))g_k(k, q(k)) $$

This is virtually identical to equation (1) except that $g_k(k, q(k))$ replaces $g'(k)$. The second-order conditions require that $\dot{C}(k) < 0$ and $\dot{q}(k) > 0$. The participation constraint is the same as equation (2) except that $r(k) = t(k) + \psi(C(k) - g(k, q(k)))$. Given $C(k) - g(k, q(k)) > 0$ the incentive constraint (13) shows that $r(k)$ is an increasing function and as before the participation constraint can be simplified to $r(k) \geq 0$.

\footnote{We shall assume that the derivatives satisfy $g_q(k, q) > 0$, $g_k(k, q) < 0$, $g_{qq}(k, q) > 0$, $g_{kq}(k, q) < 0$ and $g_{kk}(k, q) > 0$. If we have just two inputs $k$ and $l$, and a production function $f_l(k, l)$, then the sign of $g_{kq}(k, q)$ is the same as the sign of $(f_{ll}f_k - f_{lk}f_l)$. This latter term is not signed but is negative for most typical production functions.}

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Social welfare is for a given cost and transfer

\[ V(q) + r - (1 + \lambda)(t + C) = V(q) - \lambda r - (1 + \lambda)(C - \psi(C - g(k, q))). \quad (14) \]

Since the rent enters negatively into the objective function (14), and given \( \lambda > 0 \), the participation constraint will hold as an equality, and is given by equation (4). The regulator’s problem is to choose \( r(k) \), \( C(k) \) and \( q(k) \), to maximize expected social welfare

\[
\int_{\underline{k}}^{k} (V(q) - \lambda r(k) - (1 + \lambda)(C(k) - \psi(C(k) - g(k, q(k))))dF(k)
\]

subject to the incentive compatibility constraint (13) and subject to the participation constraint (4). The first-order conditions for the solution are given by equation (6), equation (7) and

\[
(1 + \lambda)(1 - \psi'(C(k) - g(k, q(k))))dF(k) = \phi(k)\psi''(C(k) - g(k, q(k)))g_k(k, q(k)) \tag{15}
\]

\[
(V'(q) - (1 + \lambda)\psi'(C(k) - g(k, q(k))))dF(k) = \\
\phi(k)\psi''(C(k) - g(k, q(k)))g_q(k, q(k)) + \psi'(C(k) - g(k, q(k)))g_{qk}(k, q(k)) \tag{16}
\]

As before \( \phi(k) = \lambda(1 - F(k)) \geq 0 \), so there is an efficiency at the top result with \( C(\bar{k}) = g(\bar{k}, q(\bar{k})) \) and \( q(\bar{k}) = q^{FB}(\bar{k}) \). Then substituting into equation (15), gives

\[
\psi'(C(k) - g(k, q(k)))) = 1 - \frac{\lambda}{1 + \lambda} \frac{\psi''(C(k) - g(k, q(k)))g_k(k, q(k))}{h(k)} \tag{17}
\]

which has exactly the same form as in equation (9) for the procurement case. Substituting equation (17) into equation (16) gives

\[
V'(q) - (1 + \lambda)g_q(k, q(k)) = -\frac{\lambda}{h(k)}\psi'(C(k) - g(k, q(k)))g_{qk}(k, q(k)). \tag{18}
\]

Since we have assumed that \( g_{qk}(k, q(k)) < 0 \) and \( \psi'(x) > 0 \) it is seen that the effect of cost padding is to restrict output so \( q(k) < q^{FB}(k) \) for each \( k \) except the highest level \( \bar{k} \).

For a purely private good the Lerner index \( L = \frac{p - g_{q}(k, q(k))}{p} \) satisfies

\[
L = \frac{\lambda}{(1 + \lambda)} \left( \frac{1}{\eta} - \frac{\psi'(C(k) - g(k, q(k)))g_{qk}(k, q(k))}{ph(k)} \right). \tag{19}
\]
so the effect is to raise the Lerner index $L$ above the Ramsay index $R = \frac{\lambda}{(1+\lambda) \eta}$. Thus cost padding under optimal regulation will tend to restrict output and raise prices. This is a similar conclusion to that found in Daughety (1984) with an arbitrary regulatory constraint.

**Example 3** Assume that there is a Cobb-Douglas production function $q = k^{1/2}l^{1/2}$, then with the price of labour equal to unity $g(k, q) = \frac{q^2}{4k}$. Assume too that demand is unit elastic $q(p) = \frac{1}{p}$; $\psi(x) = x - \frac{1}{2}x^2$, $\lambda = 1$ and $h(k) = \frac{1}{(1-k)^2}$ so that the distribution is uniform on $[0, 1]$. Then $V'(q) = \frac{1}{q}$, $q^{FB}(k) = \frac{k}{2}$ and the solution is $C(k) = \frac{(1+k)}{2(1+\sqrt{k})^2}$; $q(k) = \frac{k}{1+\sqrt{k}}$; $x(k) = \frac{(1-k)}{2(1+\sqrt{k})^2}$; $^\ddot{r} = \frac{1+3\sqrt{k}}{2(1+\sqrt{k})^3}$ and $r(k) = \frac{5(1+\sqrt{k})^{-1}}{2} + 3 \log_e (1 + \sqrt{k}) - 4$ and the expected level of cost padding is $\frac{3}{2} - 2 \log_e 2$.

Finally consider the case where the firm chooses the cost reducing investment $k$. As in Section 3 the firm will adopt a mixed strategy for the choice of $k$ and the profits of the firm must be the same for each possible level of cost reduction, hence

$$
\psi'(C(k) - g(k, q(k)))g_k(k, q(k)) + 1 = 0. \quad (20)
$$

The first-best level of investment is $k^{FB}(q)$ where $g_k(k^{FB}(q), q) + 1 = 0$. Since it has already been shown that $q(\bar{k}) = q^{FB}(\bar{k})$, it follows that $\bar{k} = k*$ where $k*$ and $q* = q^{FB}(k*)$ solve both $q_k(k*, q*) + 1 = 0$ and $V'(q*) = (1+\lambda)g_q(k*, q*)$. For a given level of output $q$, it follows from equation (20) that $k < k^{FB}(q)$, since $\psi'(C(k) - g(k, q(k)))$ and $g_k(k, q, q) > 0$. Thus investment is lower and costs higher for any given output level before any additional padded costs are added. The optimal solution for $C(k)$, $q(k)$ and the hazard rate $h(k)$ is determined by equation (20) together with equations (17) and (18), with the rent function satisfying $r(k) = \text{const.} + k$.

## 5 Auditing of Total Costs

In this section we return to the procurement model but move in the direction of Baron and Myerson (1982) and assume that the firm’s cost are unobservable except if an audit
is undertaken. We shall assume that the distribution of cost types is exogenous and assume that the audit is costly and deterministic and perfectly reveals the firm’s total cost \( c = g(k) + x \). We follow Townsend (1979) and assume that there is a fixed monitoring cost \( \mu \). If the monitoring cost is paid, the regulator can verify the total cost or equivalently the reported state \( k \). The previous sections have assumed implicitly that \( \mu = 0 \) so total cost (true plus padded) is always verified.

As in the previous sections a contract specifies a transfer \( t(k) \) and the cost reimbursement \( c(k) \). If the monitoring cost is not paid there is no verification and the only feasible contract must offer a total remuneration that is independent of \( k \). Let \( K \) be the set of states where the contract calls for verification and let \( K^c \) denote its complement in the support of the distribution where no verification is undertaken. On \( K^c \), the total remuneration \( t(k) + c(k) \) is independent of \( k \). Let \( T \) denote this total remuneration. As falsification is costly, there will be no falsification on \( K^c \) and the rent of the firm is \( r(k) = T - g(k) \). There is "no falsification without verification".

Given that the total remuneration is a constant \( T \) on \( K^c \), the incentive compatibility conditions with costly audits are:

\[
T - g(k) \geq t(\hat{k}) + \psi(c(\hat{k}) - g(k)) \quad \forall \ k \in K^c \quad \& \quad \forall \ \hat{k} \in K 
\]

(21-a)

\[
t(k) + \psi(c(k) - g(k)) \geq T - g(k) \quad \forall \ k \in K \quad \& \quad \forall \ \hat{k} \in K^c
\]

(21-b)

\[
t(k) + \psi(c(k) - g(k)) \geq t(\hat{k}) + \psi(c(\hat{k}) - g(k)) \quad \forall \ k \in K \quad \& \quad \forall \ \hat{k} \in K
\]

(21-c)

Following the methodology of Townsend (1979), we shall show that for a fixed monitoring cost \( \mu \), the verification and non-verification regions are intervals. In particular there is some \( \gamma \) such that \( K = [\underline{k}, \gamma) \) and \( K^c = [\gamma, \bar{k}] \). That is monitoring occurs for low \( k \) (high costs) and no verification takes place when \( k \) is high (low costs). In Section 2 it has been assumed the \( \mu = 0 \) so that \( \gamma = \bar{k} \) and \( K = [\underline{k}, \bar{k}] \).

To show this suppose that \( K \) and \( K^c \) are not intervals of the type conjectured. Then there must exist some intervals \( K' = [k_1, k_2] \subseteq K^c \) and \( K'' = (k_2, k_3] \subseteq K \). By
construction the contact is feasible on $K'$, so $T - g(k') \geq 0$ for each $k' \in K'$. Since $g(k)$ is decreasing it follows that $T - g(k'') > 0$ for each $k'' \in K''$. The social welfare for some level of $k \in K$ is similar to that given by equation (3) with an appropriate subtraction of the auditing cost evaluated using the shadow cost of public funds:

$$V + t(k) + \psi(c(k) - g(k)) - (1 + \lambda)(t(k) + c(k) + \mu).$$

The social welfare for each $k$ on the non-verification region $K^c$ is

$$V + (T - g(k)) - (1 + \lambda)T = V - g(k) - \lambda T.$$

Suppose that the region $K''$ is changed to a no verification region and the total transfer $T$ is paid. This is feasible since $T - g(k'') > 0$ and reduces expenditure on auditing. The change in social welfare is:

$$\lambda(t(k) + c(k) - T) + ((c(k) - g(k)) - \psi(c(k) - g(k))) + (1 + \lambda)\mu. \quad (22)$$

It follows directly from the incentive compatibility condition that $t(k) + c(k) \geq T + (c(k) - g(k)) - \psi(c(k) - g(k))$ and since $x - \psi(x) > 0$, we have $t(k) + c(k) \geq T$ for all $k \in K$. As all three terms in equation (22) are positive, the change increases social welfare, thus establishing a contradiction and proving that the two regions are intervals of the type conjectured.\textsuperscript{22}

Given that $K = [k, \gamma)$ and $K^c = [\gamma, \bar{k}]$ the incentive compatibility constraints can be reduced simply to

$$\hat{r}(k) = -\psi'(c(k) - g(k))g'(k) \quad \forall \ k \in [k, \gamma)$$

$$T = r(\gamma) + g(\gamma)$$

The former has already been derived in Section 2. To see the latter note that $t(k) + \psi(c(k) - g(k)) + g(k)$ is decreasing in $k$ as $(1 - \psi'(c(k) - g(k))) \geq 0$. The first incentive compatibility condition, equation (21-a) requires that $T \geq t(\hat{k}) + \psi(\hat{k}(k) - g(k)) + g(k)$. Changing $\hat{k}$ has no effect on the right-hand-side of this inequality, but the right hand side

\textsuperscript{22}Except possibly at a set of measure 0.
on $K^c$ is maximized at $k = \gamma$. Thus we have $T \geq r(\gamma) + g(\gamma)$. Similarly the second incentive constraint, equation (21-b) shows that on $K$, $r(k) + g(k)$ is minimized at $k = \gamma$ and hence $T \leq r(\gamma) + g(\gamma)$.

Now write

$$B(\gamma, r(\gamma)) = \int_{\gamma}^{k} (V - g(k) - \lambda T) dF(k) = (V - \lambda(r(\gamma) + g(\gamma)))(1 - F(\gamma)) - \int_{\gamma}^{k} g(k) dF(k)$$

where $r(\gamma) + g(\gamma)$ has been substituted for $T$. The objective of the regulator is to choose $r(k), c(k)$ and $\gamma$ to maximize expected social welfare

$$\int_{\gamma}^{k} (V - \lambda r(k) - (1 + \lambda)(c(k) - \psi(c(k) - g(k)) + \mu) dF(k) + B(\gamma, r(\gamma)).$$

The first-order conditions (6) and (8) still apply on $[k, \gamma]$ together with the transversality condition $\phi(\gamma) = \lambda(1 - F(\gamma))$. In addition there is a first-order condition for the choice of $\gamma$. For an interior solution for $\gamma$ this condition is

$$\mu = -(x(\gamma) - \psi(x(\gamma))) - \frac{\lambda}{(1 + \lambda)} \frac{g'(\gamma)}{h(\gamma)}$$

where $x(\gamma) = c(\gamma) - g(\gamma)$. Given that $h(k) \to \infty$ as $k \to \bar{k}$, $x(k) \to 0$ there will always be some region where no verification takes place if $\mu > 0$. On the other hand if

$$\mu \geq -(x(\bar{k}) - \psi(x(\bar{k}))) - \frac{\lambda}{(1 + \lambda)} \frac{g'(\bar{k})}{h(\bar{k})}$$

then no verification will take place at all. In the case of Example 1, there will be no verification if $\mu \geq \frac{1}{2\sqrt{2}}$. The following example shows how Example 1 is modified for $\mu \in (0, \frac{2\sqrt{2} - 1}{4})$.

**Example 4** Let $g(k) = 5 - 2\sqrt{2}\sqrt{(1 + k)}$, $\psi(x) = x - \frac{1}{2}x^2$, $\lambda = 1$, $\mu = \frac{4\sqrt{3} - 1}{24}$ and $h(k) = \frac{1}{(1-k)}$ so that the distribution is uniform on $[0, 1]$. Then $\gamma = \frac{1}{2}$ and as in Example 1 the amount of cost padding is $x(k) = \frac{1}{(1-k)}\sqrt{(1 + k)}$, on $[0, \frac{1}{2}]$; costs reimbursed are $c(k) = 5 - 2\sqrt{2}\sqrt{(1 + k)} + \frac{1}{2}(1-k)\sqrt{2}$ on $[0, \frac{1}{2}]$; the producer surplus is $r(k) = k + 2\sqrt{2}\sqrt{(1 + k)} - 1 - 2\log_e(1 + k)$ on $[0, \frac{1}{2}]$ and $T = \frac{11}{2} - 2\sqrt{2} - 2\log_e(\frac{3}{2})$. 

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To summarize the results of this section we find that for large \( \mu \), none of the firm types are audited. For lower but positive \( \mu \) low cost firms \( (k > \gamma) \) face a fix price contract; High cost firms \( (k < \gamma) \) are audited and their total transfer \( t + c \) falls with \( k \), i.e. rises with cost. This reflects the fact that high cost firms not only have high cost but pad costs by more than low cost firms.

6 Auditing of Padded Costs

In this section of the paper we reconsider the procurement model and move in the direction of Laffont and Tirole (1993) and assume that the true cost \( g(k) \) is observable but only at some monitoring cost \( \zeta > 0 \), while \( c(k) \) can be costlessly observed. Again we assume for this section that the distribution of types is exogenous. In Section 2 it has been implicitly assumed that \( \zeta \) is large enough that no verification of true costs was undertaken.

As in the previous section we find that where verification is optimal it occurs only for high cost (low \( k \)) types. In contrast to the previous section, however, cost padding takes place only in the non-verification region, that is low cost types engage in cost padding and high cost types do not.

As before let \( K \) be the set of states where the contract calls for verification and let \( K^c \) denote its complement. In the verification region \( K \), true costs are revealed and we shall assume that sufficient penalties can be imposed to discourage any misreporting of types in \( K \). Thus the regulator will only reimburse the true cost \( g(k) \) in this region and pay no transfer, \( t(k) = 0 \). In order for the contract to be incentive compatible therefore, the following constraints must be met

\[
0 \geq t(\hat{k}) + \psi(c(\hat{k}) - g(k)) \quad \forall \ k \in K \quad \& \quad \hat{k} \in K^c \quad (23-a)
\]
\[
t(k) + \psi(c(k) - g(k)) \geq 0 \quad \forall \ k \in K^c \quad \& \quad \hat{k} \in K \quad (23-b)
\]
\[
t(k) + \psi(c(k) - g(k)) \geq t(\hat{k}) + \psi(c(\hat{k}) - g(k)) \quad \forall \ k \in K^c \quad \& \quad \hat{k} \in K^c \quad (23-c)
\]
Notice that the first constraint applies only for \(\hat{k} \in K^c\) such that \(c(\hat{k}) - g(\hat{k}) > 0\) since we assume that hiding of costs is infeasible.

First, we show that the verification and non-verification regions are intervals. Precisely as in the previous section there is a \(\gamma\) such that \(K = [\hat{k}, \gamma)\) and \(K^c = [\gamma, \bar{k}]\). As before assume to the contrary that there exist intervals \(K' = [k_1, k_2] \subseteq K^c\) and \(K'' = (k_2, \bar{k}] \subseteq K\). Then choose some \(k' \in K'\) such that \(r(k') = t(k') + \psi(c(k') - g(k')) > 0\). A type \(k'' \in K''\) can then pad their costs by an amount \((c(k') - g(k')) > 0\) and claim to be a type \(k'\). The resulting surplus would be \(t(k') + \psi(c(k') - g(k')) > 0\) which is greater than the zero surplus if type \(k''\) reported honestly.

Having established that \(K = [\hat{k}, \gamma)\) and \(K^c = [\gamma, \bar{k}]\), it is easy to show that equations (23-a) and (23-b) imply that \(r(\gamma) = 0\). It is also easy to check that inequality (23-a) is satisfied. Consider a type \(k \in K\) misreporting a type \(\hat{k} \in K^c\). This is only feasible if \(c(\hat{k}) > g(k)\). We also know that misreporting of type \(\hat{k}\) by type \(\gamma \in K^c\) is unattractive as \(r(\gamma) = t(\gamma) + \psi(c(\gamma) - g(\gamma)) \geq t(\hat{k}) + \psi(c(\hat{k}) - g(\gamma))\). But \(g(k) > g(\gamma)\) and \(r(\gamma) = 0\), so \(\psi(c(\hat{k}) - g(\gamma)) > \psi(c(\hat{k}) - g(k))\) and \(0 > t(\hat{k}) + \psi(c(\hat{k}) - g(k))\), so misreporting of \(\hat{k}\) by type \(k\) is suboptimal.

The regulator’s objective is to choose the functions \(r(k)\) and \(c(k)\) and the cut-off point \(\gamma\) to maximize expected social welfare. Let

\[
A(\gamma) = \int_{\hat{k}}^{\gamma} (V - (1 + \lambda)(g(k) + \zeta))dF(k)
\]

then expected social welfare is

\[
A(\gamma) + \int_{\gamma}^{\bar{k}} (V - \lambda r(k) - (1 + \lambda)(c(k) - \psi(c(k) - g(k))))dF(k).
\]

The first-order conditions (6) and (8) still apply on \([\gamma, \bar{k}]\) together with the transversality condition \(\phi(\bar{k}) = 0\). The first-order condition for an interior solution for \(\gamma\) is

\[
\zeta = (x(\gamma) - \psi(x(\gamma))) - \frac{\lambda}{(1 + \lambda)} \frac{\psi'(x(\gamma))g'(\gamma)}{h(\gamma)}
\]

where \(x(\gamma) = c(\gamma) - g(\gamma)\).
Again given that $h(k) \to \infty$ as $k \to \bar{k}$, $x(k) \to 0$ there will always be some verification of true cost if $\zeta > 0$. On the other hand if

$$\zeta \geq (x(k) - \psi(x(k))) - \frac{\lambda}{(1 + \lambda)} \frac{\psi'(x(k))g'(k)}{h(k)}$$

then no verification of true cost will take place and this is the situation considered in Section 2. In the case of Example 1, there will be no verification if $\zeta \geq \frac{2\sqrt{2}-1}{4}$. The following example shows how Example 1 is modified for $\zeta \in (0, \frac{2\sqrt{2}-1}{4})$.

**Example 5** Let $g(k) = 5 - 2\sqrt{2}\sqrt{(1 + k)}$, $\psi(x) = x - \frac{1}{2}x^2$, $\lambda = 1$, $\zeta = \frac{4\sqrt{3}-1}{24}$ and $h(k) = \frac{1}{(1-k)}$ so that the distribution is uniform on $[0, 1]$. Then $\gamma = \frac{1}{2}$ and as in Example 1 the amount of cost padding is $x(k) = \frac{1}{2}\sqrt{2(1-k)}$, on $\left(\frac{1}{2}, 1\right]$; costs reimbursed are $c(k) = 5 - 2\sqrt{2}\sqrt{(1 + k)} + \frac{1}{2}\frac{(1-k)\sqrt{2}}{\sqrt{(1+k)}}$ on $\left(\frac{1}{2}, 1\right]$ and the true cost $5 - 2\sqrt{2}\sqrt{(1 + k)}$ on $[0, \frac{1}{2}]$; the producer surplus is $r(k) = k + 2\sqrt{2}(\sqrt{(1 + k)} - 1) - 2\log_e(1 + k)$ on $\left(\frac{1}{2}, 1\right]$ and 0 elsewhere.

To summarize the results of this section we find that for large $\zeta$, all firm types are unaudited. For lower but positive $\zeta$ low cost firms ($k > \gamma$) face a low powered incentive contract; high cost firms ($k < \gamma$) are audited and they have their true cost remunerated.

## 7 Conclusion

This paper has shown how some cost padding will be tolerated in optimal regulatory contracts. It also induces a move away from Ramsay pricing and implies weaker price regulation than without cost padding. The distribution of types has been endogenized by allowing firms to undertake a pre-contractual investment in cost reduction.

The paper falls between Baron and Myerson (1982) who assume that the regulator cannot observe firms costs and Laffont and Tirole (1993) who assume that the regulator observes true costs. Therefore we have considered the effect of a deterministic audit scheme with commitment that allows in one case for the auditing of total cost and in the other case
for the auditing of padded cost. In both cases auditing of high cost firms is undertaken. However, with auditing of total costs, it is low cost firms that are given a high powered incentive contract and high cost types engage in cost padding whereas for auditing of padded costs it is the low cost types that engage in cost padding activities.

It should be noted that we have restricted attention to deterministic audit schemes with commitment. However, it is well known that audit costs can be significantly reduced if stochastic mechanisms are allowed. When Townsend (1979) introduced the costly-state verification framework he anticipated this possibility.\textsuperscript{23} Equally the regulator may not have an ex post incentive to monitor even in the states where the contract requires it. Both these difficulties raise complex problems\textsuperscript{24} so are beyond the scope of the current paper.

The paper may also have wider applicability. Firstly, the marriage of the costly-state falsification and costly-state verification frameworks might be of interest beyond regulation and procurement as the conflict between agents attempting to falsify and principals striving to verify arise in many areas of economic interest. For example, in finance lenders take costly actions to verify the returns of projects while borrowers, have an incentive to underreport the true returns. Insurance companies spend a huge amount of money in an attempt to curtail insurance fraud. Lastly, tax authorities allocate huge budgets for auditing income return files while clever accountants make huge profits by manipulating accounts so that their clients’ tax liabilities are reduced. Secondly, endogenizing the type distribution may produce use extra insights in a variety of incentive problems.

\textsuperscript{23}Other examples of either stochastic verification or stochastic auditing include: Border and Sobel (1987), Mookherjee and Png (1989), and Chander and Wilde (1998).

\textsuperscript{24}Stochastic auditing results are only known in the discrete state case and addressing commitment issues requires a detailed specification of the auditing game.
Appendix

This appendix outlines the finite state case and shows that if the feasible set of investment in cost reduction is an interval then incentive compatible contracts will be defined on a sub-interval of this set.

Consider first the case where the level of investment in cost reduction can be chosen only from a discrete set of points. For simplicity consider the case with two choices \( k_0 \) and \( k_1 \) where \( k_1 > k_0 \). The true cost in each case is \( g_0 = g(k_0) \) and \( g_1 = g(k_1) \). The contract specifies the cost to be reimbursed \( c_0 \) and \( c_1 \) and the transfers to be made \( t_0 \) and \( t_1 \). Let \( v \) denote the regulator’s probability assessment of the investment \( k_0 \) and \( (1 - v) \) be the probability assessment of the investment \( k_1 \). The objective function for the regulator is

\[
v(V - \lambda r_0 - (1 + \lambda)(c_0 - \psi(c_0 - g_0))) + (1 - v)(V - \lambda r_1 - (1 + \lambda)(c_1 - \psi(c_1 - g_1))).\]

The incentive compatibility constraints are

\[
r_0 = t_0 + \psi(c_0 - g_0) \geq t_1 + \psi(c_1 - g_0) \\
r_1 = t_1 + \psi(c_1 - g_1) \geq t_0 + \psi(c_0 - g_1).
\]

and the participation constraints are

\[
r_0 = t_0 + \psi(c_0 - g_0) \geq 0 \\
r_1 = t_1 + \psi(c_1 - g_1) \geq 0.
\]

As is standard it can be shown that \( r_0 = 0, c_1 = g_1 \), that is no cost padding by the low cost type and \( r_1 = r_0 + \Psi(c_0 - g_0) \) where

\[
\Psi(c_0 - g_0) = \psi(c_0 - g_0 + (g_0 - g_1)) - \psi(c_0 - g_0).
\]

Substituting these values into the objective function and differentiating with respect to \( c_0 \) gives

\[
\psi'(c_0 - g_0) = 1 + \frac{\lambda}{(1 + \lambda)} \frac{(1 - v)}{v} \Psi'(c_0 - g_0),
\]

Note that \( \Psi' < 0 \) so that \( c_0 > g_0 \) and there is cost padding by the high cost type.
**Example 6** Consider an example where \( g_0 = 2, g_1 = 0, \lambda = 1 \) and \( \psi(x) = x - \frac{1}{8}x^2 \). Then \( \Psi(x) = \frac{1}{2}(3 - x), c_0 = \frac{(1+\upsilon)}{\upsilon} \) and \( c_1 = 0 \).

When the firm can choose \( k \) before the contract is signed, the profits from both the choice of \( k_0 \) and \( k_1 \) must be the same as discussed in Section 3. This means \( r_0 - k_0 = r_1 - k_1 \). Given the solution to the incentive problem, this can be rewritten as

\[
\Psi(c_0 - g_0) = (k_1 - k_0).
\]

This determines the cost reimbursement \( c_0 \) and the first-order condition is then used to determine \( \upsilon \).

**Example 7** Consider the previous example and suppose \( k_0 = 0 \) and \( k_1 = 1 \). (This is consistent with the cost reduction function \( g(k) = 2(1 - \sqrt{k}) \).) Then \( c_0 - g_0 = 1 \) and \( c_0 = 3 \). From the previous example \( c_0 = \frac{(1+\upsilon)}{\upsilon} \), so this can be solved to give \( \upsilon = \frac{1}{2} \), \( t_0 = -\frac{7}{8} \) and \( t_1 = 1 \).

Now suppose that the firm can choose any \( k \in [k_0, k_1] \) and is not restricted to the set \( \{k_0, k_1\} \). Let \( \pi(k_0, k) \) denote the level of profits from choosing \( k \) but reporting \( k_0 \). By definition \( \pi(k_0, k) = t_0 + \psi(c_0 - g(k)) - k \). Equally the profit from the contract are

\[
\begin{align*}
\pi_0 &= t_0 + \psi(c_0 - g(k_0)) - k_0 \\
\pi_1 &= t_0 + \psi(c_0 - g(k_1)) - k_1
\end{align*}
\]

where the latter equation uses the binding incentive compatibility constraint for the low cost type. If the original contract is to remain incentive compatible, it must be the case that \( \pi(k_0, k) < \pi_0 = \pi_1 \). Using the previous equations

\[
\begin{align*}
\pi(k_0, k) &= \pi_0 + k_0 - \psi(c_0 - g(k_0)) + \psi(c_0 - g(k_)) - k \\
&= \pi_1 + k_1 - \psi(c_0 - g(k_1)) + \psi(c_0 - g(k)) - k.
\end{align*}
\]

Therefore the following incentive conditions must hold

\[
\begin{align*}
\psi(c_0 - g(k_1)) - \psi(c_0 - g(k)) &> k_1 - k \\
k - k_0 &> \psi(c_0 - g(k)) - \psi(c_0 - g(k_0)).
\end{align*}
\]
Taking $k = \frac{1}{2}(k_0 + k_1)$ this implies that
\[
\psi(c_0 - g(k_1)) - \psi(c_0 - g(k)) > \psi(c_0 - g(k)) - \psi(c_0 - g(k_0))
\]
which contradicts the condition that $\psi'' < 0$ as $c_0 - g(k_1) > c_0 - g(k) > c_0 - g(k_0)$.

Note that this argument generalizes directly to the finite state case as it only uses the condition that the downward incentive compatibility constraint is binding and does not use the special features of the top and bottom state. Thus if the firm can choose its investment in cost reduction from an interval, a contract restricted to a discrete subset of points is not incentive compatible. The only case which is properly incentive compatible is the continuum case where the distribution function is defined on some subinterval of $\mathcal{K} \subseteq \mathbb{R}_+$. This justifies the use of the continuum case described in the paper.

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