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Product innovation, prisoner's dilemma and welfare

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Abstract: It is usually believed that innovation increases profits of the firms and also social welfare. In a duopoly model with product innovation, we show that both these believe may go wrong. We show that if the cost of innovation is not very large, prisoner's dilemma occurs under product innovation, i.e., each firm earns lower profit when all the firms do innovation compared to the situation when neither firm does innovation. We also show that innovation is welfare reducing if the cost of innovation is not very small.

JEL Classifications: D21; D43

Key Words: Cross-licensing; Prisoner's dilemma; Product innovation; Welfare

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Abstract: It is usually believed that innovation increases profits of the firms and also social welfare. In a duopoly model with product innovation, we show that both these believe may go wrong. We show that if the cost of innovation is not very large, prisoner's dilemma occurs under product innovation, i.e., each firm earns lower profit when all the firms do innovation compared to the situation when neither firm does innovation. We also show that innovation is welfare reducing if the cost of innovation is not very small.

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1. Introduction

It is usually believed that innovation increases profits of the firms and also social welfare. In this paper, we show that both these common believe may go wrong in a strategic environment with product innovation.

In a duopoly model with product innovation, we show that if the cost of innovation is not very large, prisoner's dilemma occur under product innovation, i.e., each firm earns lower profit when all the firms do innovation compared to the situation where neither firm does innovation. We also show that though product innovation increases the number of products, it reduces social welfare if the cost of innovation is not very small.

Hence, given the positive relationship between the complexity of a technology and the cost of innovation, this paper suggests that though it is always privately beneficial to have fewer firms with a particular product, it is socially beneficial to have fewer (more) firms with a particular product if the technology of that product is sufficiently complex (very simple). So, our results may have important implications for government policies trying to encourage innovation.

Though the present paper can be related to the earlier work of Lambertini and Rossini (1998), who also show prisoner's dilemma under product innovation, our paper differs from them in several important ways. First, we differ from them in terms of product innovation. We consider that product innovation allows a firm to invent a new product, and therefore, gives the innovating firm access to more products, whereas product innovation in their paper increases the degree of horizontal product differentiation. So, while higher competition due to a new product is the reason for prisoner's dilemma in our analysis, positive externality due to higher product differentiation is the reason for their result.[1] Secondly, in contrast to them, where prisoner's dilemma reduces innovation, we show that prisoner's dilemma encourages innovation. Moreover, Bernhofen and Bernhofen (1999) explain that the argument of Lambertini and Rossini (1998) is misleading and show that the parameter configuration that most likely generates the equilibrium with no innovation in Lambertini and Rossini (1988) is least likely to generate prisoner's dilemma. In contrast, we show that prisoner's dilemma occurs *whenever* we have the unique equilibrium where both firms do innovation. Thirdly, we also consider the welfare effects of innovation, while they are silent on this matter.

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[1] There are several other recent papers, which, unlike the present paper, consider that product innovation either determine the degree of product differentiation or the quality of a product (Cabrales and Motta, 2001, Lambertini et al., 2002 and Lin and Saggi, 2002), or it increases value of the current product or creates a replacement technology (De Bijl and Goyal, 1995, Rosenkrans, 1995 and 1996, Bonanno and Haworth, 1998, Costa and Dierickx, 2001 and Wickelgren, 2004). Though Bae (2002) considers that product innovation creates a new product with a higher quality, that paper allows innovation by a single firm only.

Our result can also be related to Mills and Smith (1996), where asymmetric equilibrium occurs in a symmetric game under *process innovation* and thus, explain heterogeneity of firms as an equilibrium phenomenon. We show that this conclusion may not extend to *product innovation*. The symmetric game in this paper always has the symmetric equilibrium and therefore, in equilibrium, heterogeneity does not occur.

The rest of the paper is organized as follows. Section 2 describes the model, and shows the equilibrium under innovation and its implications on the profits. Section 3 looks at the welfare implication of innovation. Section 4 concludes. The proofs are relegated to the appendices.

2. The model

Assume that there are two firms, 1 and 2. Imagine the situation where firm 1 (firm 2) has the technology to produce a product x (y). We assume that the inverse market demand function for x and y are respectively

$$P_x = a - x - gy \tag{1}$$

$$P_y = a - y - gx \tag{2}$$

where $g \in [0,1]$ shows the degree of product differentiation between the products and the notations have usual meanings.[2] However, each firm can invent the technology of the other product by investing the amount K.[3]

[2] This inverse demand function is generated from the utility function $U = a(x+y) - \frac{1}{2}(x^2 + y^2 + 2gxy)$.

[3] We assume that either this economy imposes process patent, which allows the same product to be produced under different processes, or the patents of these products have already expired, which allows a firm to invent (or imitate) the product of its competitor. However, even if patent has expired, it may not be costless to invent (or imitate) the technology of the competitor. For example, the disutility from innovation or the opportunity cost of innovation generates the positive cost of innovation.

We consider the following two-stage game. At stage 1, firms 1 and 2 decide simultaneously whether to invent the technologies for y and x respectively. So, if a firm does innovation, it has access to both the technologies. At stage 2, the firms compete in the product market like Cournot duopolists with respective products. For simplicity, we normalize the costs of production to zero.

We assume that if both firms produce the same product, consumers consider this particular product of these firms as a perfect substitute. However, this product is an imperfect substitute of the other product. So, if both firms produce both products, if both firms produce x and only one firm produces y, and if both firms produce y and only one firm produces x, the inverse market demand functions for x are respectively:

$$P_x = a - x_1 - x_2 - gy_1 - gy_2 \tag{3}$$

$$P_x = a - x_1 - x_2 - gy \tag{4}$$

$$P_x = a - x - gy_1 - gy_2 \tag{5}$$

where the subscripts 1 and 2 stand for the outputs of firms 1 and 2 respectively. We get the similar inverse market demand functions for product y also.

If neither firm innovates, we find that the optimal profits of firms 1 and 2 are[4]

$$\pi_1(NR,NR) = \pi_2(NR,NR) = \frac{a^2}{(2+g)^2}. \tag{6}$$

If both firms innovate and produce both the products, the optimal profits of firms 1 and 2 are

$$\pi_1(R,R) = \pi_2(R,R) = \frac{2a^2}{9(1+g)} - K. \tag{7}$$

[4] The first (second) argument in the profit function stands for the innovation strategy (i.e., innovation (R) or no innovation (NR)) of firm 1 (firm 2).

If only firm 1 does innovation, the optimal profits of firms 1 and 2 are respectively

$$\pi_1(R,NR) = \frac{a^2(13-5g)}{36(1+g)} - K \tag{8}$$

$$\pi_2(R,NR) = \frac{a^2}{9}. \tag{9}$$

If only firm 2 does innovation, the optimal profits of firms 1 and 2 are respectively

$$\pi_1(NR,R) = \frac{a^2}{9}. \tag{10}$$

$$\pi_2(NR,R) = \frac{a^2(13-5g)}{36(1+g)} - K. \tag{11}$$

Since, $\frac{a^2}{(2+g)^2} > \frac{2a^2}{9(1+g)}$, the following proposition is immediate from (6) and (7).

Proposition 1: *If both firms do innovation, the net profit of each firm is lower compared to the situation where neither firm does innovation.*

The reason for the above proposition is as follows. If neither firm does innovation, each firm produces one product, which faces competition from an imperfect substitute. But, if both firms do innovation, each firm produces both the products and therefore, each product faces competition not only from an imperfect substitute but also from a perfect substitute. Hence, product innovation by both firms creates more competition in the product market and reduces profit of each firm.

Proposition 2: *Product innovation creates prisoner's dilemma when* $K < K_2$.

Proof: See Appendix A. Q.E.D.

The above result is in contrast to Lambertini and Rossini (1998) and shows that if the costs of innovation are not very large, prisoner's dilemma occurs for any degree of product differentiation and induces innovation.

So far, we have assumed that the firms decide only on innovation before production. However, since both firms do innovation for $K < K_2$ and the innovation requires K amount of investment, it immediately implies that the firms have the incentive to cross-license their technologies to save the cost of innovation. However, since $\frac{a^2}{(2+g)^2} > \frac{2a^2}{9(1+g)}$, cross-licensing cannot eliminate prisoner's dilemma even if it increases profits of the firms under innovation.

Hence:

Proposition 3: *Cross-licensing occurs when* $K < K_2$. *However, prisoner's dilemma remains even under licensing.*

In contrast to Eswaran (1994), which shows cross-licensing is unprofitable between the competing technologies in a static game, the above result shows cross-licensing can be profitable in a static game. In Eswaran (1994), the firms get the technology of its competitor only through licensing, whereas the present paper allows the firms to invent the technology of the other firm, which makes cross-licensing between the firms profitable in our analysis.

Now, we determine all possible equilibria of this game. We find from (6) – (11) that 'both firms do innovation' is the equilibrium if

$$K < \frac{a^2(1-g)}{9(1+g)} \equiv K_1 \qquad (12)$$

and 'neither firm does innovation' is the equilibrium if

$$K > \frac{a^2(13-5g)}{36(1+g)} - \frac{a^2}{(2+g)^2} \equiv K_2, \qquad (13)$$

where $K_2 < K_1$.

Proposition 4: *(i) If $K < K_2$, both firms do innovation.*

(ii) If $K \geq K_2$, neither firm does innovation.

Proof: See Appendix B. Q.E.D.

The above result is in contrast to Mills and Smith (1996), where asymmetric equilibrium occurs under process innovation and explains heterogeneity of firms endogenously. We show that their result may not hold under product innovation, since there is always symmetric equilibrium in our symmetric game.

3. Welfare implications

Now we analyze how the innovation affects social welfare, which is the summation of consumer surplus and the net industry profit.

If neither firm does innovation, social welfare is

$$W(NR, NR) = \frac{a^2(3+g)}{(2+g)^2}, \qquad (14)$$

whereas social welfare is

$$W(R, R) = \frac{8a^2}{9(1+g)} - 2K \qquad (15)$$

when both firms do innovation.

Proposition 5: *Assume* $K < K_2$. *Welfare is higher (lower) under 'both firms doing innovation' compared to 'neither firm doing innovation' if the cost of innovation is very small (not very small).*

Proof: See Appendix C. Q.E.D.

The above proposition considers welfare implication of innovation, which occurs for $K < K_2$. If $K \in (K_2, K_1)$, even if 'both firms doing innovation' is a possible equilibrium, we have seen that equilibrium selection induces the firms to avoid this equilibrium and 'neither firm doing innovation' is the solution of the game. Since, (14) is greater than (15) for $K \in (K_2, K_1)$, the equilibrium selection by the firms in case of multiple equilibria (i.e., for $K \in (K_2, K_1)$) helps to avoid socially desirable innovations.

4. Conclusion

In contrary to the common believe, we show that, product innovation may reduce both profits of the firms and social welfare. If the cost of innovation is not very large, product innovation creates prisoner's dilemma, where each firm earns lower profit when all the firms do innovation compared to the situation when neither firm does innovation. We also show that if the cost of innovation is not very small, product innovation reduces social welfare. We also find that our symmetric game always has the symmetric equilibrium and contradicts the previous literature explaining heterogeneity of firms through process innovation.

Hence, our results may have important implications for government policies targeting innovation, and suggest that a country may prefer to adopt policies that reduce the number of products.

Appendix

Appendix A: Proof of Proposition 2: It follows from (6) – (11) that doing innovation is the dominant strategy of each firm if $K < K_2$. Further, Proposition 1 suggests that if both firms innovate, the profit of each firm is lower compared to the situation where neither firm innovates. This proves the result.

Q.E.D.

Appendix B: Proof of Proposition 4: (i) Since, $K_2 < K_1$, it immediately follows from (12) and (13) that both firms innovate when $K < K_2$.

(ii) If $K > K_1$, it immediately follows from (12) and (13) that here neither firm does innovation. However, if $K \in (K_2, K_1)$, we have two pure strategy equilibria: (i) where both firms innovate and (ii) where neither firm innovates. Proposition 1 shows that each firm earns higher profit in the latter equilibrium than the former equilibrium, and appealing to the focal point[5] argument, we can say that the higher profit of each firm in the latter equilibrium may act as the focal point of this game and induces neither firm to innovate. So, neither firm does innovation is the solution if $K \geq K_2$. Q.E.D.

Appendix C: Proof of Proposition 5: 'Both firms doing innovation' is the equilibrium for $K < K_2$. We see that (15) is continuous and decreasing in K, and it is greater than (14) at $K = 0$ but less than (14) at $K = K_2$. Hence, there exists a value of K, say K^C, such that welfare is higher (lower) under 'both firms doing innovation' than 'neither firm doing innovation' if $K \in [0, K^C)$ ($K \in (K^C, K_2)$). Q.E.D.

References

Bae, H., 2002, 'Product innovation, sell-off, and entry deterrence', *International Journal of Industrial Organization*, **20**: 577 – 88.

Bernhofen, D. M. and L. T. Bernhofen, 1999, 'On the likelihood of a prisoners' dilemma in a differentiated duopoly', *Economics Letters*, **64**: 291 – 94.

Bonanno, G. and B. Haworth, 1998, 'Intensity of competition and the choice between product and process innovation', *International Journal of Industrial Organization*, **16**: 495 – 510.

Cabrales, A. and M. Motta, 2001, 'Country asymmetries, endogenous product choice and the timing of trade liberalization', *European Economic Review*, **45**: 87 – 107.

Costa, A. L. and I. Dierickx, 2002, 'Licensing and bundling', *International Journal of Industrial Organization*, **20**: 251 – 67.

De Bijl, P. W. J. and S. Goyal, 1995, 'Technological change in markets with network externalities', *International Journal of Industrial Organization*, **13**: 307 – 25.

Eswaran, M., 1994, 'Cross-Licensing of Competing Patents as a Facilitating Device', *Canadian Journal of Economics*, **27**: 689 –708.

Fudenberg, D. and J Tirole, 1991, *Game theory*, The MIT Press, Massachusetts.

Lambertini, L. and G. Rossini, 1998, 'Product homogeneity as a prisoner's dilemma in a duopoly with R&D', *Economics Letters*, **58**: 297 – 301.

Lambertini, L., S. Poddar and D. Sasaki, 2002, 'Research joint ventures, product differentiation, and price collusion', *International Journal of Industrial Organization*, **20**: 829 – 54.

[5] See, e.g., Fudenberg and Tirole (1991) for the focal point argument.

Lin, P. and K. Saggi, 2002, 'Product differentiation, process R&D, and the nature of market competition', *European Economic Review*, **46**: 201 – 11.

Mills, D. E. and W. Smith, 1996, 'It pays to be different: endogenous heterogeneity of firms in an oligopoly', *International Journal of Industrial Organization*, **14**: 317 – 29.

Rosenkranz, S., 1995, 'Innovation and cooperation under vertical product differentiation', *International Journal of Industrial Organization*, **13**: 1 – 22.

Rosenkranz, S., 1996, 'Quality improvements and the incentive to leapfrog', *International Journal of Industrial Organization*, **15**: 243 – 61.

Wickelgren, A. L., 2004, 'Innovation, market structure and the holdup problem: investment incentives and coordination', *International Journal of Industrial Organization*, **22**: 693 – 713.