UNIVERSITY OF NOTTINGHAM

Discussion Papers in Economics

Discussion Paper
No. 06/01

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January 2006

DP 06/01

ISSN 1360-2438

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Good and bad equilibria with the informal sector*

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Abstract

We examine whether an economy can have a bad (small or no formal sector, high taxes) as well as a good (small or no informal sector, low taxes) equilibrium. When the government maximizes instantaneous formal sector welfare, this can occur if the elasticity of average to marginal cost for the public good is less than one. More regard for the informal sector leads to a worse equilibrium, and a higher prevalence of multiple equilibria.

JEL Classification: H26, J24

Key words: informal sector, dynamics, multiple equilibria, tax evasion

*I thank Friedrich Schneider, Matthias Wrede and seminar participants in Linz and Groningen for helpful comments.

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1 Introduction

The informal sector, also known as black, underground, or unoffical economy, is large and growing around the world (see Table 1). There are two reasons why economists should worry about this. The first is that production in the informal sector is relatively inefficient. The informal sector producers try to avoid being caught. They cannot make use of public goods like the police force and courts of law to enforce contracts. This makes it difficult to sell products, to employ labour and above all, to borrow capital (Straub [34]). In addition, there is imperfect information about trading and employment opportunities in the informal sector (Anderberg et al. [4]).

The second reason to worry about the informal sector is that it doesn't pay taxes for the public goods that it does make use of. This means that the tax burden on the formal sector is larger than it would otherwise be.[1] As a result, the formal sector becomes less attractive as an employer and supplier of goods and services. More business moves to the informal sector, which means that taxes for the formal sector have to rise. There is the danger of a vicious circle, which may end in most or all of the economic activity being carried out "underground".

If this bad equilibrium were the only possible outcome, a fundamental change in policy or in the exogenous parameters would be needed to achieve a better outcome. But the outlook may not be quite as bleak, because by the same reasoning used above, a virtuous circle may also be possible. In a virtuous circle, the formal sector expands. There are more people to shoulder the tax burden, so tax rates decrease. This makes the formal sector more attractive, and business moves from the informal to the formal sector, etcetera. In the end, most or all of the economic activity takes place "above ground". In this case, all an economy in a vicious circle needs is a temporary policy change, rather than a fundamental change. The government needs to implement a one-off reduction in the size of the informal sector, enough for the formal sector to reach critical mass and to set the economy on the path to the good equilibrium.

[1] Schneider and Enste [30] point out that there are also positive fiscal effects of the shadow economy. The informal sector may supply intermediate products to the formal sector, and money earned in the informal sector is spent in the formal sector. These considerations do not apply to our model.

Table 1: Size of the informal sector as a percentage of official GDP

category	1990/1			1999/2000		
	lowest	highest	average	lowest	highest	average
Africa	22.1	47.3	33.9	28.1	59.4	41.2
Asia	8.2	43.2	20.9	11.3	52.6	26.3
Latin America	13.6	55.4	34.2	19.8	67.1	41.5
Transition countries	14.3	57.8	31.5	18.9	67.3	37.9
OECD	6.7[a]	22.8[a]	13.2[a]	8.6	28.7	16.8

Source: compiled from Schneider (2005)
a. 1989/1990

What we want to investigate is whether there can be multiple equilibria in an economy: a bad equilibrium with most or all economic activity in the informal sector, and a good one with most or all business in the formal sector. The existence of multiple equilibria would make it easier for policy makers to escape from a bad equilibrium. It could also explain why the size of the informal sector differs so much between otherwise similar countries, as we see in Table 1. One country may be caught in a bad equilibrium, while the other country, through good fortune or appropriate policy, is in the good equilibrium. While the idea of vicious and virtuous cycles is intuitively appealing and has been discussed in the literature,[2] it has hardly been modelled yet. The only formal model so far is by Johnson et al. [19], which as we shall see has some shortcomings.

The rest of the paper is organized as follows. In Section 2 we review the literature, paying special attention to Johnson et al. [19]. In Section 3, we introduce our model. In Section 4, we investigate the existence of multiple equilibria. We conclude with Section 5.

2 Review of the literature

2.1 General overview

In this section we shall review the literature on the informal sector, but first we discuss the related subjects of corruption and tax evasion.

In our model we ignore corruption, which is relevant for the informal sector in two

[2]E.g. Schneider and Enste [30], pp. 77-8, Mueller [25], p. 540.

ways. First, formal sector workers may have to bribe government officials in order to receive the required licences. They may have to pay to speed up the licencing process, or to receive them in the first place (Sarte [28]; Choi and Thum [10]). Secondly, informal sector workers may have the opportunity to bribe government officials in order to escape punishment.

Another link between corruption and informal sector activity is that both are illegal activities that can give rise to multiple equilibria. Multiple equilibria with corruption can arise because the probability of being caught (Lui [22], Mauro [24]), the punishment for corruption (Andvig and Moene [6]) or a bureaucrat's worries about his reputation (Tirole [35]) are decreasing in the number of corrupt bureaucrats.

The literature on tax evasion, started by Allingham and Sandmo [3] and reviewed by Andreoni et al. [5] and Slemrod and Yitzhaki [33] is typically concerned with the choice of a risk-averse agent how much of his income to declare to the tax agency. In our model, the choice between the formal and informal sector is all or nothing, and agents are risk-neutral. In addition, unlike in the tax evasion literature, tax evasion comes at the additional cost of not being able to use all public goods. Sandmo [27] discusses optimal taxation in an economy where the government only redistributes income (it does not provide public goods) and has a fixed revenue requirement. Taxpayers are divided into non-evaders and evaders. Only the latter can divide their labour between the formal and the informal sector. Sandmo [27] also analyzes a worker's labour supply, which we assume fixed. Cowell and Gordon [12] analyze how the tax rate affects tax evasion, taking the effect on public good provision into account. Falkinger [14] examines how tax evasion affects optimal public good provision. In both cases, the effect is ambiguous in general. In our comparatively simple model, an increase in the tax rate always increases the informal sector. The effect of informal sector size on public good provision is less relevant than the effect on tax rates, because the latter directly affects a worker's choice between sectors. An increase in the informal sector generally leads a welfare-maximizing government to raise the tax rate, but a formal sector welfare-maximizing government might respond either way.

Benjamini and Maital [7] were the first to point out that social stigmatization can

lead to multiple equilibria. When there are many (a few) honest taxpayers, a tax evader will experience high (low) social stigma costs, and thus tax evasion will be small (large). In the same vein, Schlicht [29] observed that multiple equilibria can occur when the psychic cost of disobedience decreases with evasion. This result has been refined by Gordon [18], Myles and Naylor [26] and Kim [20], and applied to sales tax evasion by Chang and Lai [9]. Cowell ([11], p. 110) identifies additional reasons why tax evasion becomes easier when it is more widespread: Tax evaders can learn from each other, and it becomes easier to find a "bent" accountant. Myles and Naylor [26] mention that corner equilibria become more prevalent when the probability of detection is decreasing in the number of tax evaders, without explicitly modelling how this might occur. In Myles and Naylor's [26] and Kim's [20] models, government policy is exogenous and tax revenue "disappears" from the economy.

The literature on the informal sector has been reviewed by Schneider and Enste [30, 31] and Gërxhani [17]. Loayza [21] analyzes the informal sector in an endogenous growth model with a congestible public good. As in our model, Loayza [21] assumes that the informal sector can only make use of a part of the public good. Unlike in our model, the congestible nature of the public good (e.g. roads) offers an additional channel through which the informal sector has a negative impact on the formal sector: The informal sector's use of the public good reduces its quality for the formal sector. Workers maximize intertemporal utility and can move instantaneously from one sector to the other. The government maximizes intertemporal social welfare. In our model, workers and the government are myopic, and workers do not move instantaneously to the sector with the highest payoff. Loayza [21] restricts the parameters such that there is a unique interior equilibrium. The purpose of our paper, however, is to see if there can be multiple equilibria.

Carillo and Pugno [8] assume a firm can employ workers formally and informally, but it can only give on-the-job training to formally employed workers, because it has a more long-run relation with them. Carillo and Pugno [8] assume Marshallian externalities from learning and show there may be two stable equilibria. In the "good" equilibrium the number and the size of firms, the proportion of official employment, overall output

and efficiency are greater than in the "bad" equilibrium.

2.2 Johnson, Kaufmann and Shleifer (1997)

In Johnson, Kaufmann and Shleifer's [19] (hereafter JKS) model, the government taxes and provides a public good for the formal sector, and the mafia does the same for the informal sector. The tax rates are constant, although JKS do discuss how the government and the mafia would set their tax rates (p. 170-1). Alexeev et al. [1, 2] model the government's and mafia's decision.

JKS find that there are two stable equilibria, one with everyone in the formal sector and one with everyone in the informal sector. When the formal sector is very large, the government has high tax revenues and provides a lot of its public good, so that the formal sector is an attractive place to work. The mafia, on the other hand, has low tax revenues because the informal sector is small. Public good provision in the informal sector is low, making it an unattractive place to work. Thus, all workers move to the formal sector and this is a stable equilibrium. By the same reasoning, there is a stable equilibrium with everyone in the informal sector. With two stable corner equilibria, the interior equilibrium (if there is just one) must be unstable.

While JKS take the government's and mafia's tax rates as given, one would expect that the two corner outcomes are also stable equilbria with endogenous tax rates. After all, when the informal sector is very small, the mafia (government) has low (high) potential revenues, and the government will always offer the more attractive tax-public good package.

Let us now look at JKS's model in more detail. L_F (L_I) is labour employed in the (in)formal sector. Official sector production Y_F is given by:

$$Y_F = QL_F$$

with Q the government's public good provision. With the wage rate normalized to one, after-tax profits Π_F are:

$$\Pi_F = p(1-t)QL_F - L_F$$

with t the government's tax rate. We have introduced product price p here to make sure that at least in equilibrium, profits are zero.

Analogously, profits in the informal Π_I sector are:

$$\Pi_I = p(1-s)RL_I - L_I$$

with r the mafia's tax rate and R its public good provision.

While JKS do not discuss this subject, it seems most natural and straightforward to assume identical firms. A firm's profit π_F in the formal sector would then be

$$\pi_F = p(1-t)Ql_F - l_F$$

with l_F the firm's labour input, and analogously in the informal sector.

The problem with this setup is that profits are linear in the labour input. This makes it difficult to specify a firm's profits and labour demand in out-of-equilibrium situations, which is necessary to study the dynamics. In the formal sector, for instance, firms would either not operate at all because they can only make a loss (if $p(1-t)Q < 1$), or they would make zero profit at any size (if $p(1-t)Q = 1$), or they would like to hire an infinite amount of labour as profits keep increasing (if $p(1-t)Q < 1$). Whereas product price p might adjust to ensure zero profits in the formal sector, there would be out of equilibrium profits or losses in the informal sector.

The easiest remedy for this is to eliminate the difference between firms and workers. Each worker produces a homogeneous good for his own consumption. His output is Q in the formal sector or R in the informal sector. Total production is QL_F in the formal and RL_I in the informal sector.

Formally, the government's tax revenue is given by $T = tQL_F$. The supply of public goods is increasing and concave in tax revenue: $Q = Q(T),\ Q' > 0,\ Q'' < 0$. Eliminating Q from the right-hand side of $Q = Q(tQL_F)$ yields $Q = q(tL_F)$, $q' > 0$, and analogously for the informal sector $R = r(sL_I)$.

In an interior equilibrium, both L_F and L_I are positive, and workers are indifferent between the two sectors. JKS's Figure 1, reproduced here as Figure 1a, purports to show that the interior equilibrium is unstable. On the horizontal axis is the relative size of the unofficial economy, which we shall simply set equal to L_I. JKS only give a verbal description of the two curves shown in Figure 1a. The thick line (the public

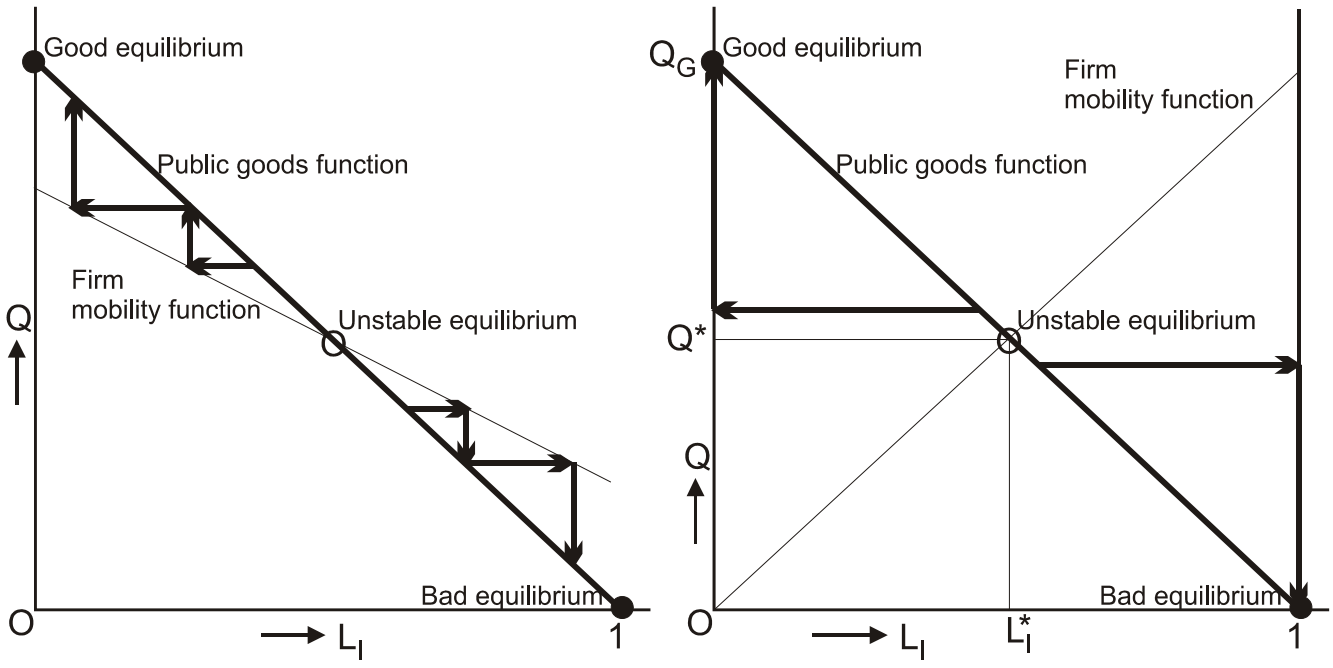

Figure 1: Multiple equilibria in the JKS model. a (left): JKS Figure 1. b (right): amended version

goods function) represents the public good level that the government can provide for a certain informal sector size according to $Q = q\left[t(1 - L_I)\right]$. The public goods curve is decreasing ($dQ/dL_I = -tq' < 0$), because the larger the informal sector, the lower tax revenues and public good provision by the government.

JKS claim that the thin line (the firm mobility function) is downward sloping and generally cuts the thick line from below. It is downward sloping because "the higher is the supply of public goods in the official economy, the fewer firms choose to operate unofficially." (p. 166) However, while a higher Q does make the official sector more attractive, this does not imply that the firm mobility function is downward sloping. On the curve, firms are indifferent between the official and the unofficial economy:

$$(1 - t)Q = (1 - s)r(sL_I)$$

We see that

$$\frac{dQ}{dL_I} = \frac{(1 - s)L_I r'(sL_I)}{1 - t} > 0$$

The firm mobility function thus has positive slope: The larger the unofficial economy, the larger the size of the public good offered by the mafia. In order to compete

with this, the government should increase its own public good provision.

The corrected picture, with upward sloping firm mobility function, is shown in Figure 1b. Figure 1a reproduces JKS's dynamics, where L_I assumes a non-equilibrium value, the government offers a Q on the tax collection curve, and the firms and the government respond to each other in turns. Carrying this over to Figure 1b reveals that the interior equilibrium (L_I^*, Q^*) is still unstable. When public good provision on the public good curve is below the firm mobility curve, all firms leave the official sector and the government's public good provision drops down to zero. This is the bad equilibrium. When public good provision is above the firm mobility curve, all firms move to the official sector and the government's public good provision increases to Q_G. This is the good equilibrium.

The dynamics in the present paper differ from JKS, for two reasons. First, while alternate moves may appeal in a two-player setup, it seems less adequate for a game between a government and a great number of workers. Secondly, while in JKS the government waits for the workers to adjust, it keeps offering the same amount of public good and charging the same tax rates. When workers are leaving (entering) the formal sector, this results in a budget deficit (surplus) for the government. This complication, which JKS overlook, does not arise in the present paper where the government balances its budget at each point in time.

3 The model

Our model builds on Johnson et al.'s [19] model, discussed in the previous section. There is a continuum of workers in the economy, of mass one. They either work in the formal sector F or in the informal sector I.[3] Let L_F (L_I) be the share of workers in the (in)formal sector, $L_I = 1 - L_F$. In either sector, a worker puts in the same and constant amount of work, and produces a homogeneous good for own consumption.

[3]This assumption, found in most of the theoretical literature (Alexeev et al. [2], Anderberg et al. [4], Choi and Thum [10], Johnson et al. [19], Loayza [21], Marcouiller and Young [23], Straub [34] and Carillo and Pugno [8], but not Alexeev et al. [1]), is justified when output in either sector is linear in labour. Alternatively, Cowell ([11], p. 95) suggests a person may prefer not to work in both sectors, because the tax authorities have a record of the activities of formal sector workers. This may make it easier to detect illicit activities of formal sector workers.

Per-capita production in the formal sector is $y_F = Q$.[4] In the informal sector it is $y_I = sQ,\ 0 < s < 1$. Q is the level of public good provision by the government. The formal sector enjoys the full benefit of public goods. The informal sector only enjoys a share s. We call s the degree of spillover of public goods from the formal to the informal sector. There are some "public" goods from which the informal sector is excluded, for instance the legal system to enforce contracts, and medical and social insurance. The informal sector does, however, benefit from other public goods like roads and public transport.[5]

The government levies a tax at rate t on the production of the formal sector. In the model, there are no (full-time) public sector workers.[6] We can say that all formal sector workers devote a fraction $1 - t$ of their time to producing the public good. A public good that fits this description very well is environmental protection. Everyone benefits from formal sector workers' efforts to abate the pollution resulting from their production.

A worker's consumption π_F in the formal sector is:

$$\pi_F = (1 - t)Q \tag{1}$$

In the informal sector, define $\sigma \equiv 1 - s$ as the implicit tax rate, so that consumption π_I is:

$$\pi_I = sQ = (1 - \sigma)Q \tag{2}$$

The government's tax revenues are $T = tL_F y_F = tL_F Q$. The government uses the tax revenues to produce the public good, according to $Q = Q(T),\ Q(0) = 0,\ Q' > 0,\ Q'' < 0$. We can also say that $T = C(Q)$ is the cost of producing Q, with $C(0) = C'(0) = 0,\ MC \equiv C' > 0$ for $Q > 0$ and $C'' > 0$. We thus have $C(Q) = tL_F Q$, which can be rewritten as:

$$AC(Q) = tL_F \tag{3}$$

[4]There might be an amount $y_0 > 0$ that workers in either sector can produce without any public good. We normalize this y_0 to zero. Since the government taxes Q, this implies that y_0 is exempt from taxation.

[5]Marcouiller and Young [23] allow informal sector workers to produce their own "order".

[6]Frey and Weck [16] model the case where the bureaucracy levies taxes on the formal sector in order to maximize the bureaucrat's wage, the size of the bureaucracy, or a combination of both.

with average cost $AC(Q) \equiv C(Q)/Q$. Totally differentiating with respect to Q yields:

$$\frac{dt}{dQ} = \frac{C'Q - C}{L_F Q^2} > 0 \tag{4}$$

Workers adjust sluggishly to a difference in remuneration between the sectors. They do not compare payoffs in both sectors on a day-to-day basis, let alone move instantaneously to the higher-paying sector. Workers have made sector-specific investments in real and human capital and in social networks and personal relationships (Schneider [32]). Formal-sector workers may have moral inhibitions about going underground, while informal-sector workers may fear awkward questions about their past as they become visible as tax payers (Cowell [11], p. 97). However, we will not formally model these barriers to adjustment.

When $t < \sigma$ (and $L_F < 1$), there will be a movement from the informal to the formal sector. When $t > \sigma$ (and $L_F > 0$), there will be a movement from the formal to the informal sector. When

- $t = \sigma$, or
- $t < \sigma$ and $L_F = 1$, or
- $L_F = 0$, so that $Q = 0$,

there will be no movement from one sector to another.

In equilibrium, no worker will want to move to another sector. Therefore, one of the three conditions above must hold. An equilibrium is stable if the economy returns to it after a small shock. It is easily seen that:[7]

Lemma 1 *When $L_F = 1$ is an equilibrium, it is stable. $L_F = 0$ is a stable equilibrium if and only if $\sigma < t$. An interior equilibrium L_F^* is stable if and only if $dt(L_F^*)/dL_F > 0$.*

The government's objective is to maximize the weighted welfare of formal and informal sector workers. It maximizes instantaneous welfare at each moment in time, taking the allocation of workers between formal and informal sector as given. While

[7] Figures 2 to 4 illustrate this Lemma.

for simplicity we assume an extreme degree of myopia on the part of the government, it seems likely that the government does not look too far into the future. The government may not be able to predict what happens in the more distant future, or it may not be interested, because it concentrates on winning the next elections. Moreover, a far-sighted government seems difficult to reconcile with the notion of multiple equilibria. A sufficiently far-sighted government would make sure not to get caught in a vicious circle, but rather set the economy on a path to the good equilibrium.

It seems natural to assume that the government's objective function would give formal sector workers a higher (at least not a lower) weight than informal sector workers, for the following three reasons. First, the feeling may be that the interests of the taxpayers should matter most when determining the tax rate (Cowell [11], Ch. 7). The workers who have decided not to pay taxes have forfeited the right to have their interests considered. Secondly, the formal sector is likely to be more visible, better organized and politically more powerful, for instance through trade unions (Frey [15]). Finally, as we shall see, the higher the weight the government places on formal sector welfare, the better the ultimate equilibrium is for everyone. Thus, a far-sighted government that has the interests of all its citizens at heart may choose to maximize instantaneous formal sector welfare.

4 Multiple equilibria?

4.1 General analysis

Government maximizes weighted welfare, putting a weight of one on the formal sector and a weight α, $0 \leq \alpha \leq 1$, on the informal sector. From (1), (2) and (3):

$$W \equiv L_F \pi_F + \alpha L_I \pi_I = [L_F + (1 - L_F)\alpha s]\, Q - C(Q) \tag{5}$$

Maximizing with respect to Q yields:

$$MC(Q) = L_F + (1 - L_F)\alpha s \tag{6}$$

Totally differentiating with respect to L_F, we find:

$$\frac{dQ}{dL_F} = \frac{1 - \alpha s}{C''} > 0 \tag{7}$$

The optimal tax rate follows from (3) and (6):

$$t = \frac{AC(1-\alpha s)}{MC - \alpha s} \tag{8}$$

Totally differentiating with respect to L_F yields:

$$\frac{dt}{dL_F} = (1-\alpha s)\left[\frac{C'Q(C'-\alpha s) - (C'-\alpha s + QC'')\,C}{Q^2(C'-\alpha s)}\right]\frac{dQ}{dL_F}$$

Substituting (7):

$$\frac{dt}{dL_F} = \frac{AC(1-\alpha s)^2}{(MC-\alpha s)^2}\left[\left(\frac{MC-\alpha s}{MC}\right)\varepsilon - 1\right] \tag{9}$$

where

$$\varepsilon \equiv \frac{dAC/AC}{dMC/MC} = \frac{C'\left[C'Q - C\right]}{QCC''} \tag{10}$$

is the elasticity of average to marginal cost for public good production.

For simplicity, we assume:

Condition 1 $\varepsilon - 1$ *does not change sign for* $MC \in (0,1]$, *where* ε *is defined by (10).*

For the Figures in this paper, we will use the following cost function that satisfies Condition 1:[8]

$$C(Q) = \theta Q^{\gamma} + \phi Q^{\beta}, \qquad \beta, \gamma \geq 1,\ \theta > 0 \tag{11}$$

for which the first and second derivatives, respectively, are:[9]

$$\begin{aligned} C'(Q) &= \theta\gamma Q^{\gamma-1} + \phi\beta Q^{\beta-1} > 0 \\ C''(Q) &= \theta\gamma(\gamma-1)Q^{\gamma-2} + \phi\beta(\beta-1)Q^{\beta-2} > 0 \end{aligned}$$

For this cost function, we find that

$$\varepsilon - 1 = \frac{-\theta\phi(\gamma-\beta)^2 Q^{\gamma+\beta-2}}{C''C}$$

Thus, when ϕ is negative/zero/positive, ε is larger than/equal to/smaller than one.[10]

[8] Calculations for the Figures are available from the author upon request.

[9] For $\phi < 0$, $C'(Q)$ is increasing for $C'(Q) \leq 1$ if and only if:

$$\left(\theta\gamma^{1-\beta}(-\phi\beta)^{\gamma-1}\left[\left(\frac{\beta-1}{\gamma-1}\right)^{\gamma-1} + \left(\frac{\beta-1}{\gamma-1}\right)^{\beta-1}\right]\right)^{\frac{1}{\gamma-\beta}} > \frac{1}{2}$$

[10] For $\phi \neq 0$, ε is not a constant, but it is always either below or above one.

Table 2: Stable equilibrium L_F with a formal sector welfare maximizing government

	$\varepsilon < 1$	$\varepsilon = 1$	$\varepsilon > 1$
$\sigma < t^F_{\min}$	0	0	0
$t^F_{\min} < \sigma < t^F_{\max}$	0 and 1	—	$0 < L_F < 1$
$\sigma > t^F_{\max}$	1	1	1

4.2 The government only cares about the formal sector

The formal sector welfare maximizing government F sets the weight α in (5) to zero. From (8) we see that $t^F = AC/MC < 1$.[11] From (9) it follows that:

Proposition 1 *The optimal tax rate for the formal sector welfare maximizing government F is increasing/constant/decreasing in L_F when the elasticity ε of average to marginal cost for public good production is larger than/equal to/smaller than one.*

Figures 2 to 4 illustrate the possible outcomes for the F government (the t^W and t^α curves will be discussed later). In Figure 2, $\varepsilon = 1$ so that the tax rate does not vary with L_F. In the Figure, the formal sector tax rate t is below the implicit informal sector tax rate σ, so that workers keep moving to the formal sector until they are all there. However, it could also occur that $t^F > \sigma$ so that everyone would move to the informal sector. In Figure 3, $\varepsilon > 1$ so that the tax rate is inceasing in L_F. In the Figure, the formal sector tax rate equals the implicit informal sector tax rate for $L_F = 0.9$. This is the unique stable equilibrium. However, it could also occur that t^F is everywhere below (above) σ, so that everyone would move to the (in)formal sector. Finally, in Figure 4, $\varepsilon < 1$ so that the tax rate is decreasing in L_F. In the Figure, there are two stable equilibria, one with everyone in the formal sector and one with everyone in the informal sector. The interior equilibrium is unstable. This is then the only case with a vicious as well as a virtuous cycle under the F government. But here again, t^F could be everywhere below (above) σ, so that everyone would move to the (in)formal sector.

To summarize, let $t^F_{\max}$ be the highest optimal tax rate. For $\varepsilon < (>)1$, this occurs at $L_F = 0(1)$. Let $t^F_{\min}$ be the lowest tax rate, which occurs at $L_F = 1(0)$ for $\varepsilon < (>)1$. The values of L_F in a stable equilibrium are then as shown in Table 2.

[11] t^F is not defined for $L_F = 0$, but this poses no problem, because there is no one to tax.

4.3 The government also weights the informal sector

Now let us examine what happens if the government weights the informal sector with a factor α, $0 < \alpha \leq 1$. Differentiating (6) totally with respect to α, we find:

$$\frac{dQ}{d\alpha} = \frac{(1 - L_F)s}{C''} \geq 0 \tag{12}$$

with strict inequality for $L_F < 1$. The more importance the government attaches to the informal sector, the more of the public good it will provide. This is because the government attaches more importance to the benefits of the public good to the informal sector, which doesn't pay for the public good itself. Combining (12) with (4), we see that the tax rate is increasing in the importance of the informal sector:

$$\frac{dt}{d\alpha} = \frac{dt}{dQ}\frac{dQ}{d\alpha} = \frac{(1 - L_F)s(C'Q - C)}{C''Q^2} \geq 0$$

A higher tax rate for the formal sector implies that the formal sector becomes less attractive. This will lead to a movement toward the informal sector, and to an equilibrium with a smaller formal sector[12] and the same tax rate.[13] Then by (3), the equilibrium Q is decreasing in α and it is clear from (1) and (2) that both formal and informal sector payoff decreases:

Proposition 2 *For any starting value of L_F, the higher the weight the government attaches to the informal sector, the lower the formal sector size and the lower the payoff in both sectors in the stable equilibrium to which the economy evolves.*

We have the somewhat paradoxical result that the more the government cares about the informal sector, the worse will be the outcome for everyone. The explanation for this paradox is that the government is extremely myopic. It maximizes weighted welfare, given the distribution of workers over formal and informal sectors. Long-run welfare would be best served by a government that maximizes the instantaneous welfare of the formal sector only. This government sets the lowest tax rate, thus attracting workers

[12] L_F will not change if initially $L_F = 0$, because it cannot decrease further, or $L_F = 1$, because marginal benefits on the RHS of (6) remain at 1.

[13] The tax rate remains at σ when the equilibrium is internal. The tax rate remains at $t(1)$ when the equilibrium remains at $L_F = 1$. When $L_F = 0$, the tax rate is irrelevant.

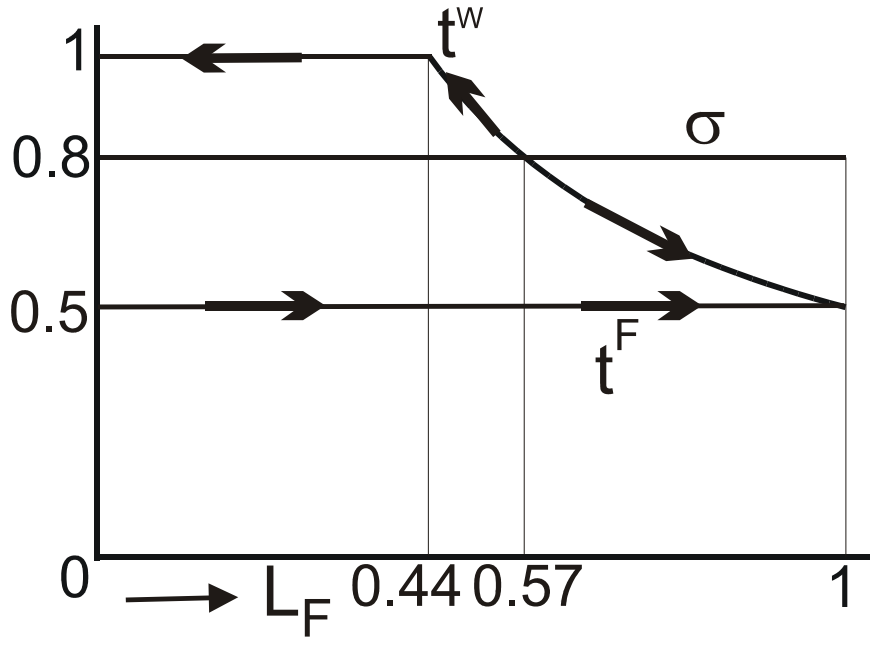

Figure 2: Dynamics with F and W governments, $\varepsilon = 1$ $[C(Q) = Q^2]$.

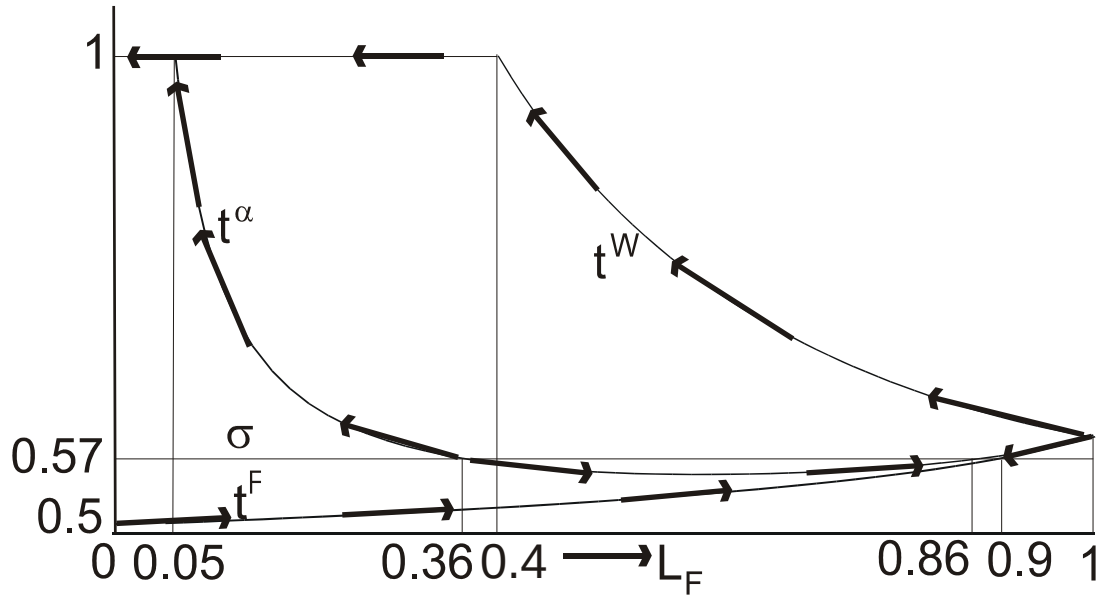

Figure 3: Dynamics with F, W and $\alpha = 0.13$ governments, $\varepsilon > 1$ $[C(Q) = 1.8Q^2 - Q^3]$.

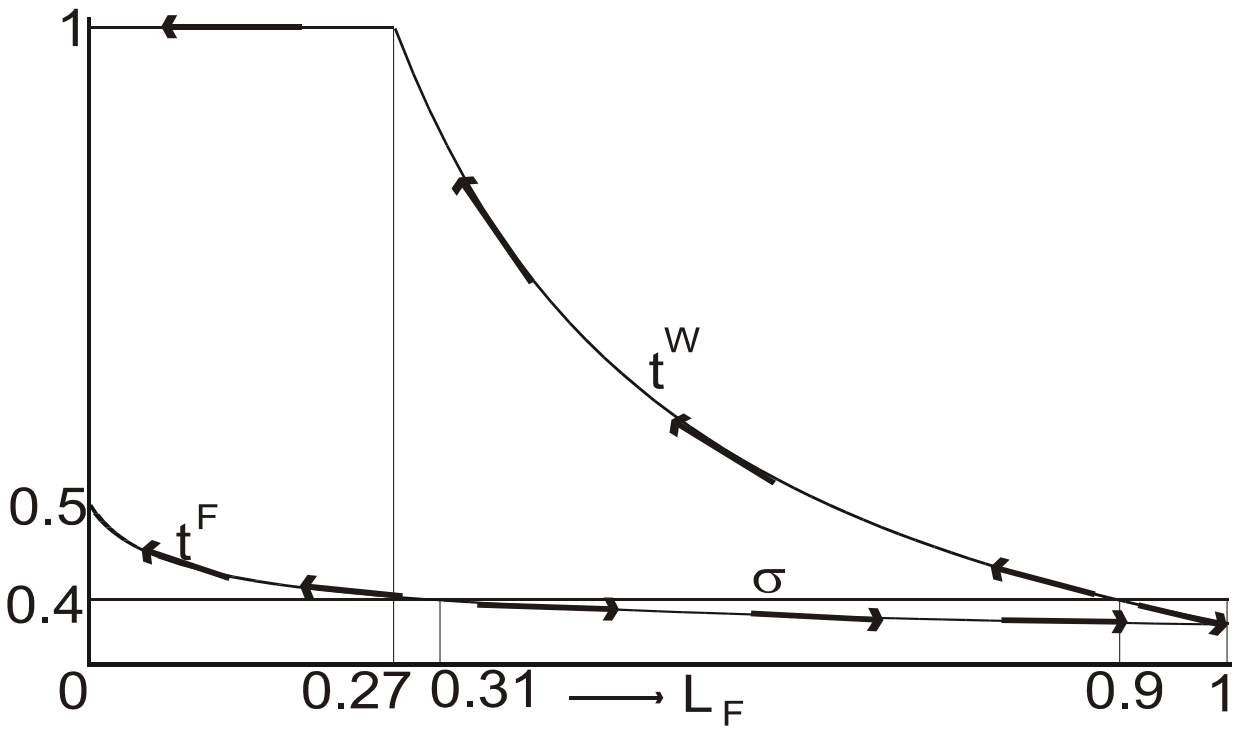

Figure 4: Dynamics with F and W governments, $\varepsilon < 1$ $[C(Q) = Q^3 + \frac{1}{4}Q^2]$.

to the formal sector and maximizing its size. The larger the formal sector, the better off everyone is in equilibrium.

When L_F is close to zero, we see from (6) that the government would like to set $MC(Q) = \alpha s$. However, because the government can only tax the small number of workers in the formal sector, it cannot collect enough tax revenues to produce the optimal amount of Q (call it Q^1). Therefore the government sets the tax at the maximum rate of one. Thus, for all $L_F \leq L_F^1$, the government sets $t = 1$, where L_F^1 is the L_F where $t = 1$ maximizes unconstrained welfare. Note that $L_F^1 < 1$, since by (3) $AC(Q^1) = L_F^1$ and $AC(Q^1) < MC(Q^1) = \alpha s$.

When L_F is small, the tax rate of one will drive formal sector workers to the informal sector. Thus, $L_F = 0$ is always a stable equilibrium for $\alpha > 0$.

Looking at dt/dL_F in equation (9), we see that the ratio before square brackets on the RHS is positive for $L_F > L_F^1$. When $\varepsilon \leq 1$, the expression in square brackets is negative and the tax rate is declining in L_F throughout. When $\varepsilon > 1$, the expression is negative for L_F close to L_F^1 (since $MC = \alpha s$ at L_F^1), but may become positive for

Table 3: Stable equilibrium L_F with a government that also weights the informal sector

	$\varepsilon \leq 1$	$\varepsilon > 1$
$\sigma < t^W_{\min}$	0	0
$t^W_{\min} < \sigma < t(1)$	—	0 and $0 < L_F < 1$
$\sigma > t(1)$	0 and 1	0 and 1

higher L_F. For simplicity, we assume that once dt/dL_F is positive, it does not turn negative for higher L_F. This is the case if and only if $d^2t/dL_F > 0$ at every point where $dt/dL_F = 0$. From (9), this condition can be rewritten as:

$$Q^2C'\left[C''\right]^2 - \left[C'' + QC'''\right]\left[C'Q - C\right] > 0$$

Cost function (11) satisfies this condition.

In Figures 2 to 4, t^W gives the tax rate for the W government that maximizes total welfare, setting $\alpha = 1$ in (5). Similar graphs can be drawn for any government with $\alpha > 0$. When $\varepsilon \leq 1$, t^W is always decreasing in L_F. In Figure 2 with $\varepsilon = 1$ and Figure 4 with $\varepsilon < 1$, there are two stable equilibria: the good and the bad corner. However, t^W could also be everywhere above σ, in which case the bad corner would be the only equilibrium. When $\varepsilon > 1$, t^W might also be monotonically decreasing in L_F. This is the case in Figure 3, where the bad corner is the only equilibrium.

For $\varepsilon > 1$, it is also possible that the optimal tax rate as a function of L_F has a U-shaped form. As a result, the good equilibrium could be an interior one. This possibility is illustrated in Figure 3. The t^α curve shows the optimal tax rate for a government with $\alpha = 0.13$ (call this the α government). The optimal tax rate is one for $L_F < 0.05$, but after that the tax rate moves closer and closer to the t^F curve for the F government. For the α government, the tax rate is increasing in L_F for high values of L_F, as it is everywhere for the F government. With the α government, there are two stable equilibria and one unstable equilibrium. For starting values of L_F below 0.36, the economy moves to the bad equilibrium with everyone in the informal sector. When L_F is above 0.36, the economy moves to the good equilibrium with $L_F = 0.86$.

The stable equilibria for a government that also cares about the informal sector ($\alpha > 0$ in (5)) are given in Table 3. The lowest tax rate $t^W_{\min}$ equals $t(1)$ for $\varepsilon \leq 1$, but

may be lower than $t(1)$ for $\varepsilon > 1$.

4.4 Comparison

Let us now compare the number of equilibria for the cases $\alpha = 0$ and $\alpha > 0$ (Tables 2 and 3). For this, we have to look at the tax rates at $L_F = 0$ and $L_F = 1$. At $L_F = 0$, the government sets the tax rate at one for $\alpha > 0$ and at $t^F(0)$ for $\alpha = 0$. At $L_F = 1$, there is no informal sector, so any government sets the tax rate at $t(1)$. There are three cases to consider:

1. $\varepsilon < 1$. There are two stable equilibria for $\alpha = 0$ if $t(1) < \sigma < t^F(0)$. There are two stable equilibria for $\alpha > 0$ if $\sigma > t(1)$. Thus, the conditions for multiple equilibria are less strict for $\alpha > 0$.

2. $\varepsilon = 1$. There is always one stable equilibrium for $\alpha = 0$. There are two stable equilibria for $\alpha > 0$ if $\sigma > t(1)$, the same condition as for $\varepsilon < 1$.

3. $\varepsilon > 1$. There is always one stable equilibrium for $\alpha = 0$. There are two stable equilibria for $\alpha > 0$ if the lowest tax rate $t^W_{\min}$ is below σ.

Thus, we find:

Proposition 3 *There are $C(Q)$ functions and values of s for which there is a single stable equilibrium with $\alpha = 0$, but there are two stable equilibria with $\alpha > 0$. The opposite cannot occur.*

Intuitively, when the government takes the interest of the informal sector into account, its policy depends on the size of the informal sector. When the informal sector is small, the interest of the formal sector prevails. The tax rate is low, which makes the formal sector relatively attractive. When the informal sector is large, its interest prevails in government policy. The tax rate is high, which makes the informal sector relatively attractive.

Proposition 3 is illustrated in Figures 2 to 4. In Figure 2, there are multiple equilibria (the good and bad corner) with the W government, whereas the F government

always leads to the good equilibrium. Figure 3 features the bad corner and a good interior equilibrium for a government with $\alpha = 0.13$ and a better interior equilibrium for the F government. In this case, the W government always leads to the bad corner equilibrium. In Figure 4, both the good and the bad corner are equilibria with both governments, however there are more starting points that lead to the bad equilibrium under the W government.

In these three Figures, the $\alpha > 0$ government sometimes leads to a worse outcome than the F government, and sometimes to the same outcome. As we know from Proposition 2, the $\alpha > 0$ government cannot lead to a better outcome than the F government.

5 Conclusion

The informal sector is large and growing around the world. There lures the danger of a vicious circle, with a growing informal sector and higher tax rates for the formal sector feeding on each other. However, an economy might also have the potential for a virtuous circle, with a growing formal sector and lower tax rates reinforcing each other. In this paper, we have investigated under which circumstances there can be vicious as well as virtuous cycles, leading to bad and good equilibria, respectively.

We have let the government maximize instantaneous welfare. While this could be relaxed in future work, a certain degree of myopia on the part of the government is required to explain multiple equilibria. If the government were sufficiently far-sighted, it would never get caught in a bad equilibrium.

A key variable in our analysis is the elasticity of average to marginal costs for public good production. When the government maximizes formal sector welfare, there can only be multiple equilibria if this elasticity is less than one. However, the elasticity could also be equal to or larger than one. And even if the elasticity is less than one, there might just be a single equilibrium (either the bad or the good corner outcome).

The higher the weight the government attaches to informal sector welfare, the worse the equilibrium that the economy ends up in. This result may seem puzzling at first, but it is due to the assumption that the government maximizes instantaneous welfare.

The more the government cares about the informal sector's benefits from the public good, the higher the tax rate it sets. This higher tax rate makes the formal sector less attractive, so that its size is lower in equilibrium.

When the government takes the informal sector into account, multiple equilibria are more likely to result. When the (in)formal sector is large, the government serves them with a low (high) tax rate, which makes the (in)formal sector relatively attractive.

In this paper we have abstracted from enforcement, where the government can punish tax evaders. Enforcement may also lead to multiple equilibria, if the probability of being caught is decreasing in the number of tax evaders (Myles and Naylor [26]). Dijkstra [13] introduces enforcement into the model of the present paper.

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