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Abstract

We model a general choice environment via probabilistic choice correspondences, with (possibly) incomplete domain and infinite universal set of alternatives. We offer a consistency restriction regarding choice when the feasible set contracts. This condition, 'contraction consistency', subsumes earlier notions such as Chernoff's Condition, Sen's α and β, and regularity. We identify a restriction on the domain of the stochastic choice correspondence, under which contraction consistency is equivalent to the weak axiom of revealed preference in its most general form. When the universal set of alternatives is finite, this restriction is also necessary for such equivalence. Analogous domain restrictions are also identified for the special case where choice is deterministic but possibly multi-valued. Results due to Sen (Rev Econ Stud 38: 307-317, 1971) and Dasgupta and Pattanaik (Econ Theory 31: 35-50, 2007) fall out as corollaries. Thus, conditions are established, under which our notion of consistency, articulated only in reference to contractions of the feasible set, suffices as the axiomatic foundation for a general revealed preference theory of choice behaviour.

Keywords and Phrases: Stochastic choice correspondence, Contraction consistency, Regularity, Chernoff's condition, Weak axiom of revealed preference, Weak axiom of stochastic revealed preference, Complete domain, Incomplete domain.

JEL Classification Numbers: D11, D71

• This paper is, to a very great extent, the consequence of numerous conversations with Prasanta Pattanaik, though I alone am responsible for all remaining errors and weaknesses. I also acknowledge my intellectual debt to the late Dipak Banerjee, who introduced me to revealed preference theory.

1. Introduction

The revealed preference approach to the general theory of choice seeks to: (i) offer intuitively plausible *a priori* consistency postulates as axiomatic foundations for choice behaviour, and (ii) deduce the logical implications of such consistency postulates in specific analytical contexts. The natural starting point for developing a notion of consistent choice would appear to be the context of 'set contraction'. Suppose, starting from some initial choice situation, i.e., some collection of alternatives that are available to the decision-maker, the feasible set contracts, in that some alternatives become unavailable. What kind of choice behaviour in the new situation should be deemed 'reasonable', in the sense of being intuitively consistent with choice in the initial situation?

The basic answer offered appears to be: alternatives initially chosen should not be rejected because *other* alternatives have been eliminated. The exact form this intuitive answer takes however varies. Under the assumption that a single, unique, alternative is picked from each feasible set, so that choice behaviour is represented via deterministic choice functions (DCFs), Chernoff (1954) required the alternative initially chosen to continue to be chosen in the new situation, unless eliminated by the contraction of the feasible set. The condition of 'regularity' generalizes this formulation to the context of stochastic choice functions (SCFs); i.e., to contexts where a single alternative is picked from a given feasible set, but exactly which alternative is chosen is determined according to some probabilistic rule. Regularity requires the probability, of the chosen alternative lying in some collection $C \subseteq B \subseteq A$, not to fall when the feasible set contracts from A to B. Sen (1969) considered the parallel generalization of DCFs to deterministic choice correspondences (DCCs), which permit multiple alternatives to be chosen from a feasible set, but only in a non-probabilistic fashion.[1] His α and β conditions together require the following. Suppose some subset of alternatives, say C, is chosen from the initial feasible set A. Then, assuming not every member of C is eliminated by the contraction of the feasible set to B, the choice set from B should consist of all surviving members of C.[2] Nandeibam (2003) has offered a probabilistic version of this condition, which he terms regularity, in the context of a *finite* universal set of alternatives.

Probabilistic choice and choice of multiple alternatives have both featured extensively, but usually independently, in the literature.[3] In the context of individual decision-making, it is widely

[1] What we term a 'choice function' is often also referred to in the literature as an 'element-valued choice function', to demarcate it from our 'choice correspondence', which is termed a 'set-valued choice function'.

[2] Assuming $C \cap B$ is non-empty, α requires the choice set of B to be some *superset* of $C \cap B$, while β requires it to be some *subset* of $C \cap B$ unless its intersection with C is empty. Consequently, together, the two conditions require the choice set of B to be exactly $C \cap B$. While β is often termed 'expansion consistency', it is evident from the above formulation that it can be equivalently interpreted as a consistency restriction on choice from a contracted feasible set.

[3] Recent examples of the literature on probabilistic choice include Bandyopadhyay, Dasgupta and Pattanaik (2004, 2002, 1999), Barbera and Pattanaik (1986), Dasgupta and Pattanaik (2008, 2007), McCausland (2008), McFadden (2005) and Nandeibam (2007). Contributions in the DCC framework include Arrow (1959)

recognized that random preferences and preferences that generate multiple best alternatives appear independently plausible on intuitive, as well as empirical, grounds. Furthermore, in many theoretical contexts, it is helpful to represent an aggregation of individual, deterministic, choice correspondences by means of a probability distribution.[4] Thus, a unified framework that simultaneously permits the choice counterparts of both these properties, by means of stochastic choice correspondences (SCCs), is of considerable interest. Within this unified framework, how should one interpret the notion of choosing consistently when the feasible set contracts? Since most choice problems in economic contexts involve universal sets that are *infinite*, the condition of regularity in Nandeibam (2003) needs to be suitably expanded. The first purpose of this paper is to offer such an expansion.

Our second, more substantive, purpose is to advance this notion of 'contraction consistent' choice as the axiomatic foundation for a general revealed preference theory. This necessitates an additional step. Our interpretation should suffice to generate choice restrictions across two feasible sets, neither of which includes the other. Versions of the weak axiom of revealed preference (WARP), designed for DCFs, DCCs and SCFs, have been developed explicitly to cover such cases. Predictive implications of the standard theory in alternative analytical contexts are typically deduced from corresponding versions of the weak axiom, whether singly or in conjunction with other conditions. Our notion of contraction consistent choice in the general context of SCCs should therefore imply a correspondingly expanded version of WARP, which subsumes all earlier, restrictive, versions. Accordingly, we offer a version of the weak axiom, expanded to cover SCCs, which is shown to follow from our notion of contraction consistency, under reasonable domain restrictions.

Analogous exercises were performed by Dasgupta and Pattanaik (2007) in the context of SCFs, and by Sen (1971) in the context of DCCs. The analysis in the present paper, carried out in the completely general context of SCCs, thus subsumes, integrates and supersedes these earlier findings.

Section 2 formalizes the idea of representing choice behaviour via SCCs. We introduce our two consistency postulates for SCCs, viz. contraction consistency (NC) and the weak axiom of stochastic revealed preference (WASRP), in section 3. Our NC expands Nandeibam's (2003) notion of regularity to permit universal sets which are not-necessarily finite, whereas our WASRP expands the version in Dasgupta and Pattanaik (2007) to permit multi-valued choice. In section 3, we also clarify how our two consistency postulates subsume and unify various analogous notions available in the literature. The relationship between the two postulates is discussed in section 4. While WASRP necessarily implies NC, if the SCC is not defined for some subsets of the universal set of

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and Sen (1971) in the general choice context, and Richter (1966), Afriat (1967) and Varian (1982) in the specific context of consumers' demand.

[4] For example, Bandyopadhyay, Dasgupta and Pattanaik (2002) discuss how, in the context of consumers' choice, the analytical construct of a stochastic demand function may usefully be deployed to provide the aggregate representation of a collection of individual, deterministic, demand functions. The literature on collective choice has analyzed at length the implications of following various probabilistic social decision rules, especially random dictatorship (e.g. Nandeibam, 2008, 2003, 1996; Pattanaik and Peleg, 1986).

alternatives,[5] it can violate WASRP while satisfying NC. We identify a restriction on the domain of the SCC under which the two conditions are equivalent. This restriction permits the domain to be 'incomplete', i.e., not defined for some subsets of the universal set. Our domain restriction also happens to be necessary for NC to imply WASRP, when the universal set is finite[6]. We then provide a domain restriction which suffices for NC to imply WASRP, for the special case of DDCs, i.e., of degenerate SCCs. As before, this restriction also turns out to be necessary when the universal set is finite. Lastly, we clarify how Sen's (1971) result regarding the equivalence between the combination of his α and β conditions and WARP for DCCs, and the key findings of Dasgupta and Pattanaik (2007), all follow as special cases of our general analysis. Section 5 concludes. Proofs are relegated to the appendix.

2. Stochastic choice correspondence

Let X denote the (non-empty) universal set of alternatives. Given any set T, $r(T)$ will denote the class of all possible non-empty subsets of T and $R(T)$ will denote the power set of T (i.e., $R(T) \equiv r(T) \bigcup \{\phi\}$, where ϕ denotes the empty set). Thus, $Z \subseteq r(X)$ will denote a non-empty class of non-empty subsets of the universal set of alternatives. Given two sets, T and T', $[T \setminus T']$ will denote the set of all elements of T that do not belong to T'.

Definition 2.1. Let $\phi \neq Z \subseteq r(X)$. A *stochastic choice correspondence* (SCC) over Z is a rule S which, for every $A \in Z$, specifies exactly one finitely additive probability measure Q_A on $(r(A), R(r(A)))$ ($r(A)$ being the set of outcomes and $R(r(A))$ being the relevant algebra in $r(A)$).

Consider a given non-empty collection of non-empty subsets of the universal set of alternatives (X), say Z. Z represents the collection of all the different feasible sets of alternatives that the decision-maker may face. Notice that Z may possibly be *incomplete*, in that some subsets of X may not belong to Z. Suppose an SCC, say S, is defined over the domain Z. Then S is a complete specification of choice behaviour when faced with different permissible feasible sets, i.e., different members of the collection Z. Let $A \in Z$ denote some permissible feasible set. Faced with the feasible set A, any non-empty subset from A (i.e., any member of the class $r(A)$) may be picked as the choice set (i.e., the collection of chosen alternatives). Given $a \subseteq r(A)$, the SCC specifies the probability, $Q_A(a)$, that the choice set actually picked will belong to the collection a. $S(A), S(B)$ etc. will be denoted, respectively, by Q_A, Q_B etc. Thus, an SCC captures the idea that, given a

[5] This is the case in many theoretical contexts, including the standard theory of consumers' behaviour.

feasible set: (i) one may pick a subset with multiple elements, and (ii) one may choose among the alternative subsets available in a probabilistic fashion.

An SCC is the most flexible tool available for modelling choice behaviour. Intuitively, SCFs, DCCs and DCFs are all special classes of SCCs. Formally, we define the following.

Definition 2.2. Let $\phi \neq Z \subseteq r(X)$, and let S be some SCC over Z.

(i) S is *singular*, iff, for all $A \in Z$, and for every $a \subseteq r(A)$, $Q_A(a) = Q(\{a_i \in a \mid |a_i| = 1\})$.

(ii) S is *degenerate*, iff, for all $A \in Z$, there exists $a \in r(A)$ such that $Q(\{a\}) = 1$.

An SCC is singular when the probability of choosing a non-singleton set is zero. It is degenerate when choice is, in effect, deterministic. An SCC is singular *and* degenerate when exactly one alternative is picked, that too in a deterministic fashion. We shall identify a singular SCC with an SCF, a degenerate SCC with a DCC, and a singular *and* degenerate SCC with a DCF.

Remark 2.3. An SCC over Z may be constructed as an aggregate representation of n (possibly different) DCCs over Z. Given any $A \in Z$, and any $a \subseteq r(A)$, let $p_A(a)$ denote the proportion of such DCCs which specify a choice set belonging to the collection a. Then one can construct an SCC as an aggregate representation of these n DCCs by identifying $Q_A(a)$ with $p_A(a)$.

3. Contraction consistency and the weak axiom of stochastic revealed preference

We now introduce our two rationality, or consistency, postulates for choice behaviour.

Definition 3.1. Let $\phi \neq Z \subseteq r(X)$. An SCC over Z satisfies *contraction consistency* (NC) iff, for all $A, B \in Z$ such that $B \subseteq A$, and for all non-empty $C \subset r(B)$,

$$Q_B(C) \geq Q_A(\{s \subseteq A \mid (s \cap B) \in C\}). \qquad (3.1)$$

Consider some initial feasible set $A \in Z$, and some $B \subseteq A$, $B \in Z$. Let C be an arbitrary (non-empty) collection of subsets of B. Consider the probability of choosing a subset whose overlap with B happens to be a member of C. Intuitively, it seems reasonable to argue that this probability should not fall when the feasible set is reduced from A to B, since such a move only eliminates alternatives outside B. This is the requirement imposed by our contraction consistency.[7]

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[6] This is the case commonly considered, for example, in social choice contexts.

[7] The LHS in (3.1) is 1 if $C = r(B)$. Hence NC implies (3.1) must hold for all non-empty $C \subseteq r(B)$.

In the special case of a finite universal set of alternatives, our NC is equivalent to the version of regularity provided by Nandeibam (2003). We proceed to clarify how our NC relates to earlier, analogous, restrictions in the literature. Definitions 2.2 and 3.1 imply the following.

Observation 3.2. Let $\phi \neq Z \subseteq r(X)$.

(i) A *singular* SCC over Z satisfies NC iff, for all $A, B \in Z$ such that $B \subseteq A$, and for all non-empty $C \subseteq B$, $Q_B(\{\{x\} \mid x \in C\}) \geq Q_A(\{\{x\} \mid x \in C\})$.

(ii) A *degenerate* SCC over Z satisfies NC iff, for all $A, B \in Z$ such that $B \subseteq A$, and for all non-empty $C \subseteq B$, [$Q_B(\{C\}) = 1$ if $Q_A(\{s \subseteq A \mid (s \cap B) = C\}) = 1$].

(iii) A *singular and degenerate* SCC over Z satisfies NC iff, for all $A, B \in Z$ such that $B \subseteq A$, and for all $x \in B$, [$Q_B(\{\{x\}\}) = 1$ if $Q_A(\{\{x\}\}) = 1$].

When choice is confined to singleton subsets of alternatives, Observation 3.2(i) implies NC is equivalent to the condition of regularity specified in the literature with regard to SCFs (e.g. Nandeibam, 2008, 1996; Pattanaik and Peleg, 1986). This requires the probability, of the chosen alternative lying in a subset C of B, not to decrease when we contract the feasible set from A to B.[8] When choice is restricted to be deterministic (but possibly non-singleton), Observation 3.2(ii) implies the equivalence of NC with the combination of Sen's (1969) α and β conditions, specified in relation to DCCs. This requires the following. If some alternative $x \in B$ is chosen under A, then $y \in B$ will be rejected under B if, and only if, y is also rejected under A. If choice is constrained to be deterministic *and* singleton, Observation 3.2(iii) implies the equivalence of NC with Chernoff's (1954) restriction: $x \in B$ must continue to be chosen under B if x is chosen under $A \supseteq B$.

Our next step is to introduce a generalized version of WARP - one that is applicable to SCCs.

Definition 3.3. Let $\phi \neq Z \subseteq r(X)$**.** An SCC over Z satisfies the *weak axiom of stochastic revealed preference* (WASRP) iff, for all $A, B \in Z$, and for all non-empty $C \subset r(A \cap B)$,

$$Q_A(r(A \setminus B)) \geq Q_B(\{s' \subseteq B \mid (s' \cap A) \in C\}) - Q_A(\{s \subseteq A \mid (s \cap B) \in C\}). \tag{3.2}$$

Consider two feasible sets $A, B \in Z$. Let C be some arbitrary collection of subsets which are available under both A and B. Consider the probability of choosing a subset whose overlap with $A \cap B$ belongs to C. Suppose this probability rises when the feasible set changes from A to B (so that the RHS of the inequality in (3.2) is positive). It seems reasonable to argue that this rise occurs only because the move eliminates some alternatives. But, in that case, the magnitude of the increase should not exceed the initial probability of choosing a subset comprised exclusively of such alternatives, which is $Q_A(r(A \setminus B))$. This is the restriction imposed on SCCs by our WASRP. [9]

We now note that Definitions 2.2 and 3.3 together imply the following.

Observation 3.4. Let $\phi \neq Z \subseteq r(X)$.

(i) A *singular* SCC over Z satisfies WASRP iff, for all $A, B \in Z$, and for every non-empty $C \subseteq (A \cap B)$, $Q_B(\{\{x\} \mid x \in (B \setminus A)\}) \geq Q_A(\{\{x\} \mid x \in C\}) - Q_B(\{\{x\} \mid x \in C\})$.

(ii) A *degenerate* SCC over Z satisfies WASRP iff, for all $A, B \in Z$, and for every non-empty $C \subseteq (A \cap B)$, if $Q_A(\{s \subseteq A \mid (s \cap B) = C\}) = 1$, then: $Q_B(r(B \setminus A)) = 1$ when $Q_B(\{s' \subseteq B \mid (s' \cap A) = C\}) = 0$.

(iii) A *singular and degenerate* SCC over Z satisfies WASRP iff, for all $A, B \in Z$, and for every $x \in [A \cap B]$, if $Q_A(\{\{x\}\}) = 1$, then: $Q_B(\{\{x'\} \mid x' \in (B \setminus A)\}) = 1$ when $Q_B(\{\{x\}\}) = 0$.

Observation 3.4 clarifies how our WASRP integrates and subsumes earlier versions of the weak axiom. Our WASRP, when confined to singular SCCs, becomes equivalent to WASRP for SCFs, introduced by Bandyopadhyay *et al*. (1999) in the context of consumers' demand and reformulated by Dasgupta and Pattanaik (2007) for the general choice context. This requires the probability, under A, of the chosen alternative belonging to some subset of $A \cap B$, not to exceed the probability, under B, of the chosen alternative either lying in that subset or being unavailable under A. When confined to degenerate SCCs, our WASRP is equivalent to WARP for DCCs (Arrow, 1959; Sen, 1971). This requires: if an alternative is rejected in one situation, it cannot be chosen in another, so long as some alternative chosen in the first is also available in the second. Lastly, in the case of a degenerate and singular SCC, our WASRP is equivalent to WARP for DCFs, introduced by Samuelson (1938) in the context of consumers' choice, and reformulated by Houthakker (1950) for the general choice context. This requires, when the alternative chosen under A is also available under B, that the alternative chosen under B be either identical to that chosen under A, or else unavailable under A.

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[8] Dasgupta and Pattanaik (2007) specify the quantifier erroneously in their definition (Def. 2.6(i)) of regularity.

[9] When $C = r(A \cap B)$, (3.2) must hold trivially. Hence WASRP implies (3.2) for all non-empty $C \subseteq r(A \cap B)$. Dasgupta and Pattanaik (2008) have presented a version of WASRP in the specific context of consumers' choice, modelled via stochastic demand correspondences. The version introduced in this paper implies their version, when translated to our general choice context.

Remark 3.5. Consider a collection of n DCCs defined over some domain Z. As discussed earlier (Remark 2.3), one can construct an SCC over Z as an aggregate representation of these n DCCs. It can be checked that, if all such DCCs individually satisfy Sen's (1969) α and β conditions, then the SCC so constructed must satisfy NC. Analogously, if the constituent DCCs all satisfy WARP, then their aggregate SCC representation must satisfy our WASRP.

4. Results

We are now ready to characterize the relationship between our two consistency restrictions on SCCs, viz. contraction consistency and the weak axiom of stochastic revealed preference. We first introduce some notation, before proceeding to present and discuss our central results.

Notation 4.1. Let $\Im$ be the set of all non-empty $Z \subseteq r(X)$ such that, for all $A, B \in Z$, at least one of the following two conditions holds:

$$|A \cap B| \leq 1; \tag{4.1}$$

$$\text{there exist } \tilde{A}, \tilde{B} \in Z \text{ such that: } \tilde{A} \subseteq A,\ \tilde{B} \subseteq B,\ [\tilde{A} \cap \tilde{B}] = [A \cap B] \text{ and } (\tilde{A} \cup \tilde{B}) \in Z. \tag{4.2}$$

Proposition 4.2. Let $\phi \neq Z \subseteq r(X)$.

(i) *An SCC over Z satisfies NC if it satisfies WASRP.*

(ii) *An SCC over Z satisfying NC also satisfies WASRP when $Z \in \Im$.*

Proof: See the Appendix.

Corollary 4.3. *An SCC over $Z \in \Im$ satisfies NC if and only if it satisfies WASRP.*

By Proposition 4.2(i), WASRP implies NC, irrespective of the domain of the SCC. By Proposition 4.2(ii), NC implies WASRP when the domain of the SCC is restricted to the class $\Im$. Thus, by Proposition 4.2, NC and WASRP are equivalent when the domain of the SCC belongs to the class $\Im$. Since $r(X) \in \Im$, this in turn implies the equivalence of the two consistency restrictions for SCCs with *complete* domain, i.e., SCCs defined over all possible non-empty subsets of the universal set of alternatives. Since the class $\Im$ may also contain members other than $r(X)$, Corollary 4.3 implies that NC and WASRP may be equivalent even if the domain of the SCC is *incomplete*, i.e., even if the SCC is not defined for some possible non-empty subsets of the universal set of alternatives.

Recall now that, when the SCC is constrained to be singular, the restrictions imposed by our NC and WASRP turn out to be equivalent, respectively, to those imposed on an SCF by the condition of regularity and the version of WASRP advanced by Dasgupta and Pattanaik (2007). Dasgupta and

Pattanaik (2007) show that, for SCFs, their version of WASRP implies regularity, while the converse also holds when the domain of the SCF is restricted to the class $\Im$.[10] Thus, our Proposition 4.2 and Corollary 4.3 extend these findings beyond their SCF-based context, so that these central results in Dasgupta and Pattanaik (2007) fall out as a special case of our more general, SCC-based, analysis.

Proposition 4.2(ii) provides a *sufficient* domain restriction for NC to imply WASRP. Is this also *necessary*? With a particular infinite universal set of alternatives, Dasgupta and Pattanaik (2007) provide an example of an SCF with domain $Z \notin \Im$ which satisfies regularity, yet violates their WASRP. It follows that, when X is infinite and the domain of a (singular) SCC falls outside the class $\Im$, satisfaction of NC need not imply the satisfaction of our WASRP for SCCs. Dasgupta and Pattanaik (2007) also show that, when the universal set of alternatives is finite, given any arbitrary domain $Z \notin \Im$, one can *always* construct an SCF which satisfies regularity, yet violates WASRP for SCFs. Thus, given a finite universal set of alternatives, whenever the domain falls outside the class $\Im$, there necessarily exists a (singular) SCC that satisfies NC, yet violates our WASRP. It follows that, in a general setting which makes no prior assumption regarding the cardinality of the universal set of alternatives, NC implies WASRP *only if* the domain of the SCC belongs to the class $\Im$. In this sense, our domain restriction turns out to be not only sufficient, but also necessary, for NC to imply WASRP. We note these findings formally below for the sake of completeness.

Proposition 4.4 (Dasgupta and Pattanaik, 2007). Let $\phi \neq Z \subseteq r(X)$.

(i) *If X is infinite, there may exist an SCC over some $Z \notin \Im$ which satisfies NC but violates WASRP.*

(ii) *If X is finite, for every $Z \notin \Im$, there must exist an SCC over Z which satisfies NC but violates WASRP.*

Lastly, consider the special case where SCCs are constrained to be degenerate. This is the environment typically considered in traditional investigations. We now specify a restriction on the domain that turns out to be both necessary and sufficient for NC to imply WASRP in this case.

Notation 4.5. Let $\phi \neq Z \subseteq r(X)$. Let $\overline{\Im}$ be the set of all Z such that, for all $A, B \in Z$, we have (4.1) or (4.3) or (4.4) below:

$$A \bigcup B \in Z\,; \tag{4.3}$$

for all distinct $x, y \in A \bigcap B$, [for some $\tilde{A}, \tilde{B} \in Z$: $\tilde{A} \subseteq A$, $\tilde{B} \subseteq B$, $\{x, y\} \subseteq [\tilde{A} \bigcap \tilde{B}]$ and $(\tilde{A} \bigcup \tilde{B}) \subset (A \bigcup B)$]. (4.4)

[10] Notice that $[A \bigcap B] \in Z$ implies (4.2). The corresponding domain restriction in Dasgupta and

Remark 4.6. While $\Im \subseteq \overline{\Im}$, it is not true that $\overline{\Im} \subseteq \Im$: Dasgupta and Pattanaik (2007) provide a domain that belongs to $\overline{\Im}$, but falls outside $\Im$. Thus, $\overline{\Im}$ is, in general, a larger class than $\Im$.

Notation 4.7. Let $\phi \neq Z \subseteq r(X)$. Let $\hat{\Im}$ be the set of all Z such that, for all $A, B \in Z$, we have (4.1) or (4.3) or (4.5) below:

for all distinct $x, y \in A \cap B$, [for some $\tilde{A}, \tilde{B} \in Z$: $\tilde{A} \subseteq A$, $\tilde{B} \subseteq B$, $\{x, y\} \subseteq [\tilde{A} \cap \tilde{B}]$, $(\tilde{A} \cup \tilde{B}) \subset (A \cup B)$ and $(\tilde{A} \cup \tilde{B}) \in Z$]. (4.5)

Lemma 4.8. (i) $\hat{\Im} \subseteq \overline{\Im}$ *and* (ii) *if* X *is finite,* $\hat{\Im} = \overline{\Im}$.

Proof: See the Appendix.

Remark 4.9. Regardless of the cardinality of X, $\Im \subseteq \hat{\Im}$ (but it is not true that $\hat{\Im} \subseteq \Im$[11]). Thus, Lemma 4.8(i) implies $\Im \subseteq \hat{\Im} \subseteq \overline{\Im}$. When X is infinite, there exist domains that belong to $\overline{\Im}$ but not to $\hat{\Im}$. To see this, consider the following example. Let $X = [-2,-1] \cup \{0,0.5\} \cup [1,2]$, and let $Z = \{[1,a] \cup \{0,0.5\} \mid 1 < a \leq 2\} \cup \{[-b,-1] \cup \{0,0.5\} \mid 1 < b \leq 2\}$. Then, for every $A \in Z$, either (i) $A = [1,a] \cup \{0,0.5\}$ for some $a \in (1,2]$ or (ii) $A = [-b,-1] \cup \{0,0.5\}$ for some $b \in (1,2]$. Notice that, (i) $\{0,0.5\} \notin Z$, and (ii) for all $a, b \in (1,2]$, $([1,a] \cup \{0,0.5\} \cup [-b,-1]) \notin Z$. It can then be checked that $Z \in \overline{\Im}$ but $Z \notin \hat{\Im}$.

Proposition 4.10. *Let* $\phi \neq Z \subseteq r(X)$. *Then*:

(i) *for every* $Z \in \hat{\Im}$, *if a degenerate SCC* S *over* Z *satisfies NC, then* S *must also satisfy WASRP;*

(ii) *if* X *is finite, for every* $Z \notin \hat{\Im}$, *there must exist a degenerate SCC over* Z *which satisfies NC but violates WASRP.*

Proof: See the Appendix.

Dasgupta and Pattanaik (2007) show that, given a finite X, the class $\overline{\Im}$ provides a domain restriction that is both necessary and sufficient for Chernoff's Condition to imply WARP for DCFs. In light of Lemma 4.8(ii), Proposition 4.10, in effect, generalizes this result beyond their context of

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Pattanaik (2007) (their Notation 4.1) thus contains a redundancy, which is eliminated in Notation 4.1 here.

[11] In light of Lemma 4.8(ii), the example with finite X in Dasgupta and Pattanaik (2007) (their Example 5.1), which shows that Z may belong to $\overline{\Im}$, yet fall outside $\Im$, also suffices to establish this claim.

DCFs to that of DCCs. In light of Lemma 4.8(i) and Remark 4.9, part (i) of Proposition 4.10 also goes beyond the analysis in Dasgupta and Pattanaik (2007) by providing a sufficient domain restriction for Chernoff's condition to imply WARP, regardless of the cardinality of the domain.

Furthermore, Proposition 4.10(i) offers a sufficient domain restriction under which the combination of Sen's α and β conditions turns out to be equivalent to WARP for DCCs. Recall that, when the SCC is degenerate, the restrictions imposed by NC and WASRP are equivalent, respectively, to those imposed on a DCC by the combination of Sen's α and β conditions and WARP. Sen (1971) establishes the equivalence, between WARP and the combination of his α and β conditions, for DCCs whose domain Z includes every *two-element* subset of the universal set of alternatives (i.e., when, for all distinct $x, y \in X$, $\{x, y\} \in Z$). Evidently, any such domain must belong to our class $\hat{\Im}$; additionally, there may exist domains which belong to $\hat{\Im}$ but fall outside the class identified by Sen.[12] Thus, our Proposition 4.9(i) subsumes Sen's result, by showing that the class of domains over which his α and β conditions imply WARP for DCCs is, in general, larger than the one he identifies. Furthermore, Sen does not address the issue of necessity, which is covered by Proposition 4.10(ii) (for the general case where the universal set is not constrained to be infinite).

By Proposition 4.2(ii), an SCC over $Z \in \Im$ must satisfy WASRP if it satisfies NC, regardless of whether it is degenerate or non-degenerate. Recall however that there exist domains which belong to $\hat{\Im}$ but not $\Im$ (Remark 4.9). Thus, Propositions 4.4(ii) and 4.10(i) together imply that there are domains over which: (i) every degenerate SCC satisfying NC also satisfies WASRP, but (ii) there exist non-degenerate SCCs satisfying NC which violate WASRP.

By Proposition 4.10(ii), given any arbitrary finite X and any arbitrary domain Z outside the class $\hat{\Im}$, there exists at least one degenerate SCC over Z which satisfies NC but not WASRP. A weaker version of this necessity claim (with regard to the domain restriction $\hat{\Im}$) can be extended to the case of infinite X. Even when X is infinite, there may exist SCCs with domain $Z \notin \hat{\Im}$ which satisfy NC but violate WASRP. Consider for example the domain specified in Remark 4.9 above. Define a singular and degenerate SCC over Z as follows: (i) for every $A \in Z$ such that $A = [1, a] \cup \{0, 0.5\}$ for some $a \in (1,2]$, $Q_A\{\{0\}\} = 1$; and (ii) for every $A \in Z$ such that $A = [-b, -1] \cup \{0, 0.5\}$ for some $b \in (1,2]$, $Q_A\{\{0.5\}\} = 1$. This SCC satisfies NC, but violates WASRP. Thus, NC does not imply WASRP if $Z \notin \hat{\Im}$ - even when the universal set of alternatives is infinite. However, we do not know whether, given *any arbitrary* infinite X, there necessarily exists a degenerate SCC satisfying NC but violating WASRP, for *every* possible Z outside the class $\hat{\Im}$.

We suspect this is so, but, since we have been unable so far to construct a general example for the infinite case, this remains an open question. The issue is analogously unresolved for infinite X with regard to non-degenerate SDCs defined over domains outside the class $\Im$ (recall Proposition 4.4).

5. Conclusion

In this paper, we have considered a general choice context, where decision-makers may choose probabilistically among (possibly multi-element) subsets of a given feasible set of alternatives. We have modelled such choice behaviour in terms of an SCC with possibly incomplete domain, i.e., one which need not be defined over all possible non-empty subsets of the universal set of alternatives. We have introduced a minimal consistency postulate, viz. contraction consistency, which restricts choice behaviour when the feasible set is contracted, as well as a generalized version of WARP. The first condition generalizes the condition of regularity in Nandeibam (2003), while the second subsumes the version introduced in Dasgupta and Pattanaik (2007). Our substantive results identify the relationship between the two conditions. While the latter necessarily implies the former regardless of the domain of the SCC, the reverse relationship does not hold. We have identified a restriction on the domain of the SCC, under which the two consistency postulates turn out to be equivalent. This restriction includes a complete domain, while also permitting the domain to be incomplete. When the universal set of alternatives is finite, we have shown that this domain restriction constitutes a necessary, as well as sufficient, condition for contraction consistency to imply our generalized version of the weak axiom. We have also identified another domain restriction as both necessary and sufficient for the two conditions to be equivalent when one constrains SCCs to be degenerate, in addition to assuming the universal set to be finite. This condition suffices even when the universal set is infinite. Our results subsume the SCF-based analysis in Dasgupta and Pattanaik (2007) within the more general environment of probabilistic multi-valued choice. Key results for the deterministic non-singleton choice environment, due to Sen (1971), also turn out to be implied as special cases of our analysis.

The major thrust of our analysis lies in advancing our notion of contraction consistency as the foundational axiom for a general revealed preference approach to the theory of choice. Considerations of plausibility, transparency and weak requirements would all appear to support its claim. This condition permits the immediate generalization of all earlier, restrictive analyses based on Chernoff's Condition, the combination of Sen's α and β conditions, or regularity, to an expanded environment of probabilistic multi-element choice from possibly infinite feasible sets. Our analysis shows that one may utilize NC to achieve such a generalization even when some version of the weak axiom of revealed preference is necessary to generate significant empirical or predictive consequences. This is when the context of the theory makes it reasonable to assume that the SCC is

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[12] Suppose $X = \{a, b, c, d, e\}, A = \{a, b, c, d\}, B = \{b, c, d, e\}, Z = \{A, B, X\}$. Then $Z \in \hat{\Im}$, even though Z violates Sen's restriction, since the domain does not contain any two-element set of alternatives.

defined over a domain sufficient for NC to imply WASRP. Our results characterize the general conditions under which this can be achieved; conditions which may be applied to specific theoretical contexts in future investigations. [13] Future work may also seek to identify domain restrictions under which NC suffices to imply rationalizability of the SCC in terms of probabilistic preference orderings.

Appendix

Proof of part (i) of Proposition 4.2. Let $\phi \neq Z \subseteq r(X)$, and let S be some SCC over Z satisfying WASRP. Consider any $A, B \in Z$ such that $B \subseteq A$, and any non-empty $C \subset r(B)$. To show that S satisfies NC, we need to show that (3.1) holds. Let $D \equiv [r(B) \setminus C]$. By WASRP, noting $B \subseteq A$, and $\phi \neq D \subset r(B)$, we get:

$$Q_A(r(A \setminus B)) + Q_A(\{s \subseteq A \mid (s \cap B) \in D\}) \geq Q_B(D). \quad \text{(N1)}$$

Notice now that:

$$Q_A(r(A \setminus B)) + Q_A(\{s \subseteq A \mid (s \cap B) \in D\}) + Q_A(\{s \subseteq A \mid (s \cap B) \in C\}) = 1 = Q_B(D) + Q_B(C) \quad \text{(N2)}$$

Together, (N1) and (N2) imply (3.1). ◊

We shall establish part (ii) of Proposition 4.2 via the following three Lemmas.

Lemma X1. *Let $\phi \neq Z \subseteq r(X)$, and let S be some SCC with domain Z. For all $A, B \in Z$, if $|A \cap B| \leq 1$, then the SSC, S, must satisfy (3.2) for all non-empty $C \subset r(A \cap B)$.*

Proof of Lemma X1. If $|A \cap B| \leq 1$, then either $[A \cap B] = \phi$ or $A \cap B$ is a singleton. Thus, there does not exist any non-empty $C \subset r(A \cap B)$ if $|A \cap B| \leq 1$. Hence, (3.2) must hold trivially. ◊

Lemma X2. *Let $\phi \neq Z \subseteq r(X)$, and let S be some SCC with domain Z that satisfies NC. Let $A, B \in Z$ be such that $(A \cup B) \in Z$ and $|A \cap B| \geq 2$. Then, for all non-empty $C \subset r(A \cap B)$, the SSC, S, must satisfy (3.2).*

[13] For example, consumers routinely face governments who tax/subsidize and ration goods. Since the intersection of two 'budget triangles' in a two-good world can be generated as the consumer's feasible set under an appropriate tax-subsidy regime (where purchases above and below some threshold are taxed/subsidized at differential rates), it appears natural to include the intersection of two budget triangles in the domain of the SCC. NC can then replace WASRP as the foundational rationality axiom for demand analysis in the two-good case. Analogously, in many collective choice contexts, given any two feasible sets in the domain, it may be intuitively reasonable to include their union in the domain as well. Dasgupta (2005) has introduced a consistency restriction on competitive firm behaviour which implies neither profit maximization nor cost minimization, but nevertheless suffices to generate the standard predictions regarding supply behaviour. It would be useful to investigate whether his consistency condition in turn can be derived from restrictions akin to NC.

Proof of Lemma X2. Consider any $A, B \in Z$ such that $E \equiv (A \cup B) \in Z$, $\overline{C} \equiv [A \cap B] \neq \phi$, $|\overline{C}| \geq 2$; and any non-empty $C \subset r(\overline{C})$. Let $D \equiv (r(\overline{C}) \setminus C)$. Note that $\phi \neq D \subset r(\overline{C})$. Suppose (3.2) is violated, so that:

$$Q_B(\{s' \subseteq B \mid (s' \cap A) \in C\}) > Q_A(r(A \setminus B)) + Q_A(\{s \subseteq A \mid (s \cap B) \in C\}). \tag{N3}$$

We shall show that (N3) yields a contradiction, given NC. First notice that, by NC,

$$Q_B(r(B \setminus A)) \geq Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap B) \in r(B \setminus A)\}) \geq Q_E(r(B \setminus A)). \tag{N4}$$

Now let $\theta \equiv \{s' \subseteq B \mid (s' \cap \overline{C}) \in D\}$, $\gamma \equiv \{s \subseteq A \mid (s \cap \overline{C}) \in D\}$ and $\varpi \equiv \{s \subseteq A \mid (s \cap \overline{C}) \in C\}$. Consider any $\tilde{s} \subseteq E$ such that $(\tilde{s} \cap A) \in \gamma$. Then $(\tilde{s} \cap \overline{C}) \in D$, so that $(\tilde{s} \cap B) \in \theta$. Hence, $\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \gamma\} \subseteq \{\tilde{s} \subseteq E \mid (\tilde{s} \cap B) \in \theta\}$. Thus, by NC,

$$Q_B(\theta) \geq Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap B) \in \theta\}) \geq Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \gamma\}), \tag{N5}$$

$$Q_A(r(A \setminus B)) + Q_A(\varpi) \geq Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in r(A \setminus B)\}) + Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \varpi\}). \tag{N6}$$

By (N3) and (N6),

$$Q_B(\{s' \subseteq B \mid (s' \cap A) \in C\}) > Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in r(A \setminus B)\}) + Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \varpi\}) \tag{N7}$$

From (N4), (N5) and (N7),

$$\begin{aligned} &Q_B(r(B \setminus A)) + Q_B(\{s' \subseteq B \mid (s' \cap A) \in D\}) + Q_B(\{s' \subseteq B \mid (s' \cap A) \in C\}) > \\ &Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in r(A \setminus B)\}) + Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \varpi\}) + Q_E(\{\tilde{s} \subseteq E \mid (\tilde{s} \cap A) \in \gamma\}) \\ &\quad + Q_E(r(B \setminus A)). \end{aligned} \tag{N8}$$

Since C and D partition $r(\overline{C})$, clearly, $r(A \setminus B), \varpi$ and γ partition $r(A)$. Hence, (N8) implies:

$$1 = Q_B(r(B)) > Q_E(r(E)) = 1.$$

This contradiction implies (3.2) must hold for all non-empty $C \subset A \cap B$ when $[A \cup B] \in Z$. ◊

Lemma X3. *Let $\phi \neq Z \subseteq r(X)$, and let S be some SCC with domain Z that satisfies NC. Let $A, B, \tilde{A}, \tilde{B} \in Z$ be such that $\tilde{A} \subseteq A$, $\tilde{B} \subseteq B$, and $[A \cap B] = [\tilde{A} \cap \tilde{B}]$. Then, for all non-empty $C \subset [A \cap B]$, the SSC, S, must satisfy:*

$$Q_A(r(A \setminus B)) + Q_A(\{s \subseteq A \mid (s \cap B) \in C\}) \geq Q_{\tilde{A}}(r(\tilde{A} \setminus B)) + Q_{\tilde{A}}(\{\tilde{s} \subseteq \tilde{A} \mid (\tilde{s} \cap B) \in C\}), \tag{N9}$$

$$Q_B(\{s' \subseteq B \mid (s \cap A) \in C\}) \leq Q_{\tilde{B}}(\{\tilde{s}' \subseteq \tilde{B} \mid (\tilde{s}' \cap A) \in C\}). \tag{N10}$$

Proof of Lemma X3. Consider any non-empty $C \subset [A \cap B]$, let $D \equiv [r(A \cap B) \setminus C]$ and let $\tilde{\gamma} \equiv \{\tilde{s} \subseteq \tilde{A} \mid (s \cap B) \in D\}$, $\tilde{\varpi} \equiv \{\tilde{s} \subseteq \tilde{A} \mid (s \cap B) \in C\}$. Since $\tilde{A} \subseteq A$, by NC,

$$Q_{\tilde{A}}(\tilde{\gamma}) \geq Q_A\left(\left\{s \subseteq A \mid \left(s \cap \tilde{A}\right) \in \tilde{\gamma}\right\}\right). \qquad \text{(N11)}$$

From (N11), noting that $r\left(\tilde{A} \setminus B\right), \tilde{\gamma}$ and $\tilde{\varpi}$ partition $r\left(\tilde{A}\right)$, we get:

$$Q_{\tilde{A}}\left(r\left(\tilde{A} \setminus B\right)\right) + Q_{\tilde{A}}(\tilde{\varpi})$$
$$\leq Q_A\left(r\left(A \setminus \tilde{A}\right)\right) + Q_A\left(\left\{s \subseteq A \mid \left(s \cap \tilde{A}\right) \in r\left(\tilde{A} \setminus B\right)\right\}\right) + Q_A\left(\left\{s \subseteq A \mid \left(s \cap \tilde{A}\right) \in \tilde{\varpi}\right\}\right). \qquad \text{(N12)}$$

Notice now that:

$$Q_A\left(\left\{s \subseteq A \mid \left(s \cap \tilde{A}\right) \in \tilde{\varpi}\right\}\right) = Q_A(\{s \subseteq A \mid (s \cap B) \in C\}); \qquad \text{(N13)}$$

$$Q_A\left(r\left(A \setminus \tilde{A}\right)\right) + Q_A\left(\left\{s \subseteq A \mid \left(s \cap \tilde{A}\right) \in r\left(\tilde{A} \setminus B\right)\right\}\right) = Q_A(r(A \setminus B)). \qquad \text{(N14)}$$

Together, (N12)-(N14) yield (N9). Noting $\tilde{B} \subseteq B$, (N10) follows directly from NC. ◊

Proof of part (ii) of Proposition 4.2. Let $Z \in \mathfrak{I}$, and let S be some SCC with domain Z which satisfies NC. Consider any $A, B \in Z$, and any non-empty $C \subset (A \cap B)$. To establish that the SSC, S, satisfies WASRP, we need to show that S satisfies (3.2). If $|A \cap B| \leq 1$, then (3.2) holds by Lemma X1. Suppose $|A \cap B| \geq 2$. Then, since $Z \in \mathfrak{I}$, there exist $\tilde{A}, \tilde{B} \in Z$ such that: $\tilde{A} \subseteq A$, $\tilde{B} \subseteq B$, $[\tilde{A} \cap \tilde{B}] = [A \cap B]$ and $\left(\tilde{A} \cup \tilde{B}\right) \in Z$. Since $\left(\tilde{A} \cup \tilde{B}\right) \in Z$, by Lemma X2,

$$Q_{\tilde{A}}\left(r\left(\tilde{A} \setminus B\right)\right) + Q_{\tilde{A}}\left(\left\{\tilde{s} \subseteq \tilde{A} \mid (s \cap B) \in C\right\}\right) \geq Q_{\tilde{B}}\left(\left\{\tilde{s}' \subseteq \tilde{B} \mid (\tilde{s}' \cap A) \in C\right\}\right).$$

In light of Lemma X3, we immediately get (3.2). ◊

Proof of Lemma 4.8. Let $\phi \neq Z \subseteq r(X)$. Part (i) follows directly from Notations 4.5 and 4.7.

(ii) Let X be finite, and let $Z \in \overline{\mathfrak{I}}$. Suppose $Z \notin \hat{\mathfrak{I}}$. Then there exist $A, B \in Z$ which satisfy (4.4) but violate (4.5). Thus, there must exist distinct $x, y \in A \cap B$ and $\tilde{A}_0, \tilde{B}_o \in Z$, such that: [$\{x, y\} \subseteq \tilde{A}_0 \subseteq A$, $\{x, y\} \subseteq \tilde{B}_0 \subseteq B$, $\left(\tilde{A}_0 \cup \tilde{B}_0\right) \subset (A \cup B)$ and $\left(\tilde{A}_0 \cup \tilde{B}_0\right) \notin Z$]. But then, since $\tilde{A}_0, \tilde{B}_o \in Z \in \overline{\mathfrak{I}}$, there must exist $\tilde{A}_1, \tilde{B}_1 \in Z$, such that: [$\{x, y\} \subseteq \tilde{A}_1 \subseteq A_0$, $\{x, y\} \subseteq \tilde{B}_1 \subseteq B_0$, $\left(\tilde{A}_1 \cup \tilde{B}_1\right) \subset (A_0 \cup B_0)$ and $\left(\tilde{A}_1 \cup \tilde{B}_1\right) \notin Z$]. Proceeding in this fashion, we have an infinite sequence of ordered pairs, $\left(\tilde{A}_1, \tilde{B}_1\right), \left(\tilde{A}_2, \tilde{B}_2\right)$ … , such that, for every positive integer $i \in \{1, 2, \ldots\}$, [$\{x, y\} \subseteq \tilde{A}_{i+1} \subseteq \tilde{A}_i$, $\{x, y\} \subseteq \tilde{B}_{i+1} \subseteq \tilde{B}_i$, $\tilde{A}_i, \tilde{B}_i \in Z$ and $\left(\tilde{A}_{i+1} \cup \tilde{B}_{i+1}\right) \subset \left(\tilde{A}_i \cup \tilde{B}_i\right)$]. This, however, contradicts the assumption that X is finite. Hence, $Z \in \overline{\mathfrak{I}}$ implies $Z \in \hat{\mathfrak{I}}$, so that $\overline{\mathfrak{I}} \subseteq \hat{\mathfrak{I}}$. Since $\hat{\mathfrak{I}} \subseteq \overline{\mathfrak{I}}$ by part (i) of Lemma 4.8, part (ii) of Lemma 4.8 follows. ◊

Proof of Proposition 4.10.

(i) Let $Z \in \hat{\Im}$. Let S be a degenerate SCC with domain Z, which satisfies NC. Suppose S violates WASRP. Then:

there exist $A, B \in Z$ and distinct $x, y \in A \cap B$, such that: $Q_A(\{s \subseteq A \mid x \in s\}) = 1$ and $Q_B(\{s' \subseteq B \mid x \notin s', y \in s'\}) = 1$. (N15)

Noting Lemma X2, (N15) implies $(A \cup B) \notin Z$. Then, since $Z \in \hat{\Im}$,

there exist $\tilde{A}, \tilde{B} \in Z$, such that: [$\{x, y\} \subseteq \tilde{A} \subseteq A$, $\{x, y\} \subseteq \tilde{B} \subseteq B$, $(\tilde{A} \cup \tilde{B}) \subset (A \cup B)$ and $(\tilde{A} \cup \tilde{B}) \in Z$]. (N16)

Now, since the SCC satisfies NC, (N15)-(N16) imply:

$Q_{\tilde{A}}(\{\tilde{s} \subseteq \tilde{A} \mid x \in \tilde{s}\}) = 1$ and $Q_{\tilde{B}}(\{\tilde{s}' \subseteq \tilde{B} \mid x \notin \tilde{s}', y \in \tilde{s}'\}) = 1$. (N17)

(N17) and Lemma X2 together imply $(\tilde{A} \cup \tilde{B}) \notin Z$, which contradicts (N16).

(ii) In light of Lemma 4.8(ii), part (ii) of Proposition 4.10 follows directly from part (ii) of Proposition 5.4 in Dasgupta and Pattanaik (2007). ◊

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