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Why Praise Inequality? Public Good Provision, Income Distribution and Social Welfare

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Abstract:

We consider a two-person Cournot game of voluntary contributions to a public good with identical individual preferences, and examine equilibrium aggregate welfare under a separable, symmetric and concave social welfare function. Assuming the public good is pure, Itaya, de Meza and Myles (*Econ. Letters*, 57: 289-296; 1997) have shown that maximization of social welfare precludes income equality in this setting. We show that their case breaks down when the public good is impure: there exist individual preferences under which maximization of social welfare necessitates exact income equalization. Even if the public good is pure, any given, positive level of income inequality can be shown to be socially excessive by suitably specifying individual preferences. Thus, sans knowledge of individual preferences, one cannot reject the claim that a marginal redistribution from the rich to the poor will improve social welfare, regardless of how small inequality is in the status quo.

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1. Introduction

Rich individuals often voluntarily contribute towards the provision of public goods that are intrinsically important for the well-being of the poor, but have negligible impact on their incomes. Examples of such public goods that routinely acquire rich patrons include places of worship, ethnic festivals, literary and cultural activities, sports clubs, civic/neighborhood amenities (including parks, museums, theatres, community halls, libraries), scientific research, etc. A large literature exists on voluntary provision of public goods with negligible income consequences. This literature typically examines how redistribution of income might influence the equilibrium level of voluntary provision.[1] A related issue, which has received much less attention, is how voluntary provision affects the normative case for income equalization. Given identical individual preferences, represented by some indirect utility function that is strictly concave in income, maximization of any social welfare function that is symmetric and concave in utilities requires income equalization when all consumption is private. Would voluntary public goods provision by the rich suffice to negate this conclusion?

In a well-known contribution, Itaya *et al*. (1997) argued that this is indeed the case. Examining the Nash equilibrium of a two-person Cournot game of voluntary contributions to a pure public good, they showed that maximization of social welfare *necessarily* implies inequality in incomes.[2] In light of this finding, however, two additional questions immediately suggest themselves.

First, while Itaya *et al.* (1997) focused on 'pure' public goods, where individual contributions are perfect substitutes, public goods in reality are often better conceptualized as 'impure'. Individuals may derive greater utility from an additional unit of the public good if they themselves provide it, because of the 'warm glow' from the act of giving *per se*, or due to other private benefits.[3] Does maximization of social welfare preclude equalization of income when public goods are impure?

Second, even in the empirically restrictive case of pure public goods, does social optimality impose a *positive lower bound* on inequality, regardless of individual preferences? If not, any arbitrary level of inequality, however small, could be deemed socially excessive under some configuration of individual preferences. Thus, it would not be possible to reject greater equalization independently of individual preferences, even if inequality is arbitrarily small in the status quo. While Itaya *et al*. (1997) reject *exact* equalization independently of individual preferences, they remain silent on this issue of 'virtual', or 'almost exact', equalization. Yet, reducing inequality to arbitrarily small levels appears to be of more substantive interest than the limiting construct of exact equalization.

The purpose of this paper is to answer these two questions. As in Itaya *et al.* (1997), we consider a two-person Cournot game of voluntary contributions to a public good with identical individual

[1] See Dasgupta and Kanbur (2007, 2005), Cornes and Sandler (2000, 1996), Bergstrom *et al.* (1986), *etc*.

[2] A related contribution is Cornes and Sandler (2000), who show that redistribution from the poor to the rich can be Pareto-improving in certain contexts. However, unlike Itaya *et al*. (1997), theirs is not a *general* defence of inequality. For example, no inequality expanding redistribution is Pareto-improving in a two-person society.

[3] See, for example, Dasgupta and Kanbur (2007), Cornes and Sandler (1994) and Andreoni (1990). The pure public good model implies any income redistribution that leaves the set of contributors unchanged will have no impact on equilibrium consumption bundles: a hypothesis that is typically rejected in empirical investigations.

preferences, and examine equilibrium social welfare under a separable, symmetric and concave social welfare function. We show that, when the public good is impure, there exist individual preferences under which maximization of social welfare entails exact equalization of income. Thus, the case that Itaya *et al.* (1997) make against equality turns out to be vulnerable in the presence of private benefits from giving. Furthermore, when the public good is pure, any given level of income inequality can be deemed socially excessive by suitably specifying individual preferences. Our results imply that voluntary provision of public goods, by itself, does not necessitate income inequality as a precondition for the maximization of social welfare. Indeed, there exist preference configurations which make equality socially essential under such voluntary provision. Furthermore, sans knowledge of individual preferences, one cannot dismiss the claim that a marginal redistribution from the rich to the poor will improve social welfare, regardless of how small inequality is in the status quo.

Section 2 sets up the model; Section 3 presents our results. Proofs are provided in the Appendix.

2. The model

Consider a two person society,[4] where preferences are given by:

$$u_i = u(x_i, y_i + \eta y_{-i});$$

x_i, y_i, y_{-i} denoting, respectively, private consumption by individual $i \in \{1,2\}$, the amount of the public good provided by i, and the amount provided by the other person; $\eta \in (0,1]$. Thus, individuals have identical preferences. The public good is *pure* in the special case where $\eta = 1$; it is *impure* in the remaining cases $\eta \in (0,1)$. Both prices are set equal to unity for notational simplicity; total income in society is normalized to one. Let $I_i > 0$ denote the income of individual i; so that $I_{-i} = 1 - I_i$. The two individuals play a standard Cournot game of voluntary contributions to the public good, as in Itaya *et al.* (1997). Thus, for individual $i \in \{1,2\}$, the optimization problem is:

$$\underset{x_i, y_i}{Max}\, u(x_i, y_i + \eta y_{-i}) \text{ subject to:}$$

$$x_i + y_i = I_i;$$

$$x_i, y_i \geq 0.$$

Denoting the Nash equilibrium utility levels by u_1^*, u_2^*, aggregate welfare in equilibrium is given by some social welfare function that is symmetric, separable and concave in individual utilities:

$$W = g(u_1^*) + g(u_2^*);$$

where $g' > 0, g'' < 0$. Since equilibrium utilities depend on the income distribution, so does social welfare. Income redistribution affects social welfare by changing individual utilities in equilibrium.

[4] Our analysis can be easily generalized to more than two individuals, without altering our conclusions.

3. Results

Does maximization of social welfare *necessarily* rule out income equality? Lacking knowledge of individual preferences, can one identify some positive magnitude of inequality in the status quo as socially inadequate, in that a marginal rise in inequality will *necessarily* improve social welfare? We now proceed to answer these questions.

Proposition 1. *Suppose the public good is impure ($\eta \in (0,1)$). Then there exist utility functions under which maximization of social welfare necessitates income equalization.*

Proof: See the Appendix.

Proposition 1 implies that, given any symmetric, separable and concave social welfare function, voluntary provision of public goods does not, by itself, negate the case for income equalization. Indeed, maximization of social welfare may necessitate exact equality, even though greater inequality elicits greater public good contribution from the rich. There do exist cases where maximization of social welfare precludes exact equality, but, unless the public good is pure, there can be no *a priori* presumption that this will indeed be so, irrespective of individual preferences.

As Itaya *et al.* (1997) have shown, such a presumption, independent of individual preferences, is valid when the public good is pure. Nevertheless, even in this case, no amount of wealth inequality, however small, can be presumed to be socially optimal or inadequate, irrespective of individual preferences. This is so because any given magnitude of wealth inequality is socially excessive under particular preference configurations. Thus, without knowledge of individual preferences, one cannot dismiss the claim that a marginal redistribution from the rich to the poor will improve social welfare, regardless of how small inequality is in the status quo. We conclude by formally stating this result.

Proposition 2. *Suppose the public good is pure ($\eta = 1$). Then, given any unequal income distribution, there exist utility functions that imply a marginal redistribution from the rich to the poor will improve social welfare.*

Proof: See the Appendix.

Appendix

Proof of Proposition 1. Let preferences be given by: $u_i = \lambda \ln x_i + \ln(y_i + \eta y_{-i})$; $\eta \in \left(0, \frac{1}{2}\right]$, $\lambda \in \Re_{++}$. Without loss of generality, suppose $I_1 \equiv \theta < \frac{1}{2}$. Consider any social welfare function: $W = g(u_1) + g(u_2)$; $g' > 0; g'' \leq 0$. Two cases are possible.

Case 1: $\theta < \frac{\lambda\eta}{1+\lambda+\lambda\eta} < \frac{1}{2}$.

It can be checked that Nash equilibrium consumption is given by: [$x_2^* = (1-\theta)\left(\frac{\lambda}{1+\lambda}\right)$, $y_2^* = (1-\theta)\left(\frac{1}{1+\lambda}\right)$, $x_1^* = \theta$]. Furthermore, $x_1^* < x_2^*$, implying $u_1^* < u_2^*$. Thus, $g'(u_1^*) \geq g'(u_2^*)$. Now, [$u_1^* \equiv \lambda \ln\theta + \ln\eta + \ln(1-\theta) - \ln(1+\lambda)$] and [$u_2^* = (\lambda+1)\ln(1-\theta) + \lambda\ln\lambda - (1+\lambda)\ln(1+\lambda)$]. Thus, we get: [$\frac{\partial u_1^*}{\partial\theta} \equiv \frac{\lambda(1-\theta)-\theta}{\theta(1-\theta)}$ and $\frac{\partial u_2^*}{\partial\theta} = -\frac{(\lambda+1)\theta}{(1-\theta)\theta}$], implying [$\frac{\partial u_1^*}{\partial\theta} + \frac{\partial u_2^*}{\partial\theta} \leq 0$ iff $\theta \geq \frac{\lambda}{2(1+\lambda)}$]. However, by assumption, $\theta < \frac{\lambda}{\frac{1}{\eta}+\frac{\lambda}{\eta}+\lambda} < \frac{\lambda}{2(1+\lambda)}$ (since $\eta \leq \frac{1}{2}$). Thus, $\frac{\partial u_1^*}{\partial\theta} + \frac{\partial u_2^*}{\partial\theta} > 0$. Noting that $\frac{\partial u_2^*}{\partial\theta} < 0$, and $g'(u_1^*) \geq g'(u_2^*) > 0$, we therefore get: $\frac{\partial W}{\partial\theta} > 0$.

Case 2: $\frac{\lambda\eta}{1+\lambda+\lambda\eta} \leq \theta < \frac{1}{2}$

In this case, Nash equilibrium consumption is given by: [$x_2^* = \left[(1-\theta)+\eta y_1^*\right]\left(\frac{\lambda}{1+\lambda}\right)$; $x_1^* = \left[\theta+\eta y_2^*\right]\left(\frac{\lambda}{1+\lambda}\right)$; $y_2^* = \frac{(1-\theta)}{1+\lambda} - \frac{\eta y_1^*\lambda}{1+\lambda}$ and $y_1^* = \frac{\theta}{1+\lambda} - \frac{\eta y_2^*\lambda}{1+\lambda}$] Solving, we get: $y_1^* = \frac{\theta(1+\lambda)-(1-\theta)\eta\lambda}{(1+\lambda)^2-\eta^2\lambda^2}$ and $y_2^* = \frac{(1-\theta)(1+\lambda)-\theta\eta\lambda}{(1+\lambda)^2-\eta^2\lambda^2}$. Thus, $y_2^* > y_1^*$, implying (since $\eta < 1$); [$y_2^* + \eta y_1^*$] > [$y_1^* + \eta y_2^*$]. Noting that, for all $i \in \{1,2\}$, [$x_i^* = \lambda(y_i^* + \eta y_{-i}^*)$], we thus get $x_2^* > x_1^*$, and, therefore, $u_2^* > u_1^*$, implying $g'(u_2^*) \leq g'(u_1^*)$. Notice now that [$u_i^* = (\lambda+1)\ln r_i^* + \lambda\ln\lambda - (\lambda+1)\ln(1+\lambda)$], where $r_i^* \equiv I_i + \eta y_{-i}^*$. Using the solutions for individual contributions, we get: $r_2^* = \left[(1-\theta) + \frac{\theta\eta(1+\lambda)-(1-\theta)\eta^2\lambda}{(1+\lambda)^2-\eta^2\lambda^2}\right]$; $r_1^* = \left[\theta + \frac{\eta(1-\theta)(1+\lambda)-\theta\eta^2\lambda}{(1+\lambda)^2-\eta^2\lambda^2}\right]$, $r_2^* > r_1^*$ and [$\frac{\partial r_2^*}{\partial\theta} = -\frac{\partial r_1^*}{\partial\theta} < 0$]. As $r_2^* > r_1^*$, we have $\frac{\partial u_2^*}{\partial r_2} < \frac{\partial u_1^*}{\partial r_1}$. Recalling $g'(u_2^*) \leq g'(u_1^*)$, $\frac{\partial r_1^*}{\partial\theta} = -\frac{\partial r_2^*}{\partial\theta} > 0$, it follows that $\frac{\partial W}{\partial\theta} > 0$. ◊

Proof of Proposition 2. Let preferences be given by: $u_i = \lambda \ln x_i + \ln(y_i + y_{-i})$; $\lambda \in \Re_{++}$. Without loss of generality, suppose $I_1 \equiv \theta < \frac{1}{2}$; and let $\lambda > \frac{2\theta}{(1-2\theta)}$. Then, in the Nash equilibrium, [$x_2^* = (1-\theta)\left(\frac{\lambda}{1+\lambda}\right)$, $y^* = (1-\theta)\left(\frac{1}{1+\lambda}\right)$, and $x_1^* = \theta$]. We thus get the equilibrium utilities: [$u_1^* = \lambda \ln \theta + \ln(1-\theta) - \ln(1+\lambda)$, and $u_2^* = (\lambda+1)\ln(1-\theta) + \lambda \ln \lambda - (1+\lambda)\ln(1+\lambda)$]. Notice now that $x_2^* \leq x_1^*$ iff $\frac{\lambda}{1+\lambda} \leq \frac{\theta}{1-\theta}$; i.e., iff $\lambda \leq \frac{\theta}{(1-2\theta)}$, which violates our prior assumption $\lambda > \frac{2\theta}{(1-2\theta)}$. Hence, $x_2^* > x_1^*$, and thus $u_2^* > u_1^*$. Consider any social welfare function: $W = g(u_1) + g(u_2)$; $g' > 0; g'' \leq 0$. Since $u_2^* > u_1^*$, $g'(u_1^*) \geq g'(u_2^*)$. Furthermore, given $\lambda > \frac{2\theta}{(1-2\theta)}$, [$\frac{\partial u_1^*}{\partial \theta} + \frac{\partial u_2^*}{\partial \theta} = \frac{\lambda(1-2\theta) - 2\theta}{\theta(1-\theta)} > 0$]. Hence, noting $\frac{\partial u_2^*}{\partial \theta} < 0$, $\frac{\partial W}{\partial \theta} > 0$. ◊

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