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International factor mobility and long-run economic growth

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Preliminary

Abstract

Long-run economic growth is analysed in a global model with many small countries prone to national level *total factor productivity* shocks. The possibility of precautionary saving or dissaving is a function of the higher-order moments and the *cross-moments* of the factor income distributions, which in turn depend on the global regime governing factor mobility. International capital mobility generates precautionary saving by eliminating interest uncertainty and by increasing earnings uncertainty, while international labour mobility reduces saving by achieving the opposite. If firms operate under a learning-by-doing investment externality, these effects then translate into long-run growth outcomes. However, besides these uncertainty effects on saving, there are also effects from aggregation, factor price determination and labour supply, which together show that international capital mobility unambiguously promotes economic growth above the autarky level but that the growth consequences of international labour mobility are less clear cut though conflicting effects.

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1. Introduction

There is a large and well established body of literature built on the initial insights of Romer (1986), (1990) and of Lucas (1988) that the feature of non-decreasing social returns to capital causes household saving to be a key determinant of long-run economic growth. A greater part of this work has utilised the common assumption that households save only in order to smooth consumption over the life-cycle in an environment of certainty. However, parallel research into the analysis of individual behaviour has concluded that a considerable part of household saving - possibly as much as 60% - is made for precautionary purposes as a way of dealing with uncertainty. See, for example, the papers by Skinner (1988), Cabellero (1990), Dardanoni (1991), Hubbard, Skinner and Zeldes (1994), Carroll and Samwick (1998).

In response to this, a number of papers have been developed to investigate the macroeconomic implications of uncertainty and with particular reference to the issue of long-run growth. Rodriguez (1999) incorporated idiosyncratic uncertainty into a macroeconomic model and replicated the finding of pronounced effects, while others, Deveureux and Gregor Smith (1994), Obstfeld (1994), R. Todd Smith (1996), Ghosh and Ostry (1997), Rankin (1998) and Hek (1999)[3], have considered macroeconomic sources of uncertainty. This present paper is closest in intention to the papers by Deveureux and Smith and by Obstfeld in attempting to determine economic growth within an open economy by considering the effects of international factor mobility on macroeconomic uncertainty. In application, however, it differs in five main areas.

First, it makes an explicit consideration of the early insight of Sandmo (1970) that uncertainty has an ambiguous effect on saving, depending on whether it pertains to earnings income or to interest income. [4][5] It is established that a positive third

[3] While these have uncovered some interesting comparative static implications, quantifying the possible effects of pure macroeconomic risk on household saving has produced disappointing results.
[4] The paper sets the scene by quoting two contradictory statements from Boulding (1966) and Marshall (1920) relating, respectively, to earnings and interest uncertainty .
[5] The countervailing effects of these two kinds of uncertainty may help to explain why researchers have been unable to find a consistent sign for the correlation between volatility and growth.

derivative for the utility function is necessary and sufficient for earnings uncertainty to generate precautionary saving. There is also a general awareness that the effects of interest uncertainty can cause saving to respond in either direction. A positive third derivative is then necessary but not sufficient. These two points are reiterated in *Appendix A*, which has been included in order to make two additional but possibly lesser known points. These are, first, that the mean value of earnings income is also important for determining the sign of the effect of interest income uncertainty on saving and, secondly, that the sign and the magnitude of *covariance* between earnings and interest is also relevant to the overall effects on saving.

Secondly, the main motivation of this paper is the idea that the international factor mobility regime is crucial to the balance between these two different kinds of uncertainty, with implications for economic growth. International capital mobility is modelled as a regime that eliminates interest uncertainty at the expense of increasing the uncertainty of earnings through the obverse effect of unleashing international capital flows. On the other hand, international labour mobility, which is given an analogous treatment, alters the pattern of factor price volatility and consequently reverses the results for precautionary saving. Thus, the paper extends the ground covered by Deveureux and Smith (1994) and of Obstfeld (1994) by considering the implications for economic growth of the international integration of labour markets as well as capital markets. A third departure from these two papers is the consideration of an overlapping-generations framework with finite lives, which allows the model to incorporate the intergenerational redistribution effects precluded by representative-agent, infinite-horizon models.

Fourthly, our assumption that the learning-by-doing technology is global rather than national - or, in other words, that *knowledge* is perfectly internationally mobile – is a source of international linkage even where the factors of production are completely immobile internationally. This has two important implications. One is that it opens the door to the property of convergence: *in expectation* countries will converge to the same long-run growth rate that is driven by the growth of freely disseminated knowledge. The second is that – with constant returns to capital overall – national wages and savings are concave in both own (and global) capital, which means that international factor market integration, whether relating to capital or to labour, causes favourable aggregation effects.

Fifthly, in the last part of the analysis, we consider endogenous labour supply in terms of the variation in hours supplied or the length of the period worked by the older generation of the working population. Variable labour supply may interact with overall saving through a variety of channels, affecting life-cycle saving by altering the mean values of the factor price distributions and affecting precautionary saving by altering the higher-order moments and cross-moments of these distributions. The issues are whether *ex post* labour supply adjustment ameliorates the problem of uncertainty and the implications for the factor mobility regime.

Throughout the analysis uncertainty derives from the unpredictability of *total factor productivity* (TFP). The first main result is that under autarky - in conjunction with the assumption that household utilities are logarithmic functions of consumption - volatility has no effect on saving, since (i) the positive effect of earnings uncertainty, (ii) the negative effect of interest uncertainty and (ii) the negative impact of the positive covariance between these two exactly cancel out. Although saving remains at the certainty-equivalent level, there is a negative aggregation effect, since the wages and, hence, savings are concave functions of the country-level shocks. Thus, *TFP* volatility reduces the long-run of global economic growth under autarky without affecting saving.

Turning from autarky to perfect capital mobility, defined where the whole world can borrow and save at a single interest rate, we find that volatility unambiguously raises growth above the autarky level. This is because of an increase in precautionary saving as well as the elimination of the adverse aggregation effect. The law of large numbers implies that country-specific shocks aggregate to zero, so removing interest uncertainty. The obverse of the unleashing of international capital movements in response to stochastic country differences in *TFP* raises earnings uncertainty, causing a greater rise in precautionary saving. In addition, since international capital mobility brings about a linear dependence of national capital on global capital, the adverse aggregation effect under autarky is lost.

A numerical evaluation of these unambiguous qualitative gains has led to two conclusions. First, a fairly heroic assumption about the size of the shock variance - although alongside a possibly low value of unity for the intertemporal elasticity of substitution in consumption - generates a modest gain in growth. Secondly, and more specific to this particular model, we find that the saving effects are of a comparable magnitude to the aggregation effects. Based on the assumption of a binomial shock distribution, where the two realizations would imply a difference in the annual growth rates of 3%, we find that the effect of moving from autarky to PCM would be raise the global average growth rate from 1.80% to 2.16%. Of this 0.36% gain, 0.16% can be attributed to a rise in precautionary savings increase and the remaining 0.20% to an aggregation effect.

Following this analysis, perfect labour mobility (PLM) is modelled in an analogous way where an implicit migration arbitrages away any potential differences in country-level wages, where capital, instead, is deemed to be immobile. The consequence of PLM is an increased volatility in interest rates, as these now bear the full brunt of the shocks. As may be deduced, switching from autarky to PLM entails a fall in precautionary saving, because of the removal of earnings uncertainty and because of the amplification of interest uncertainty. However, this negative effect on growth is offset by a positive aggregation, found to be of a comparable magnitude, effect *vis-a-vis* autarky, since a common world wage also implies that saving and investment levels are the same in all countries. *A priori*, the sign of the combined effect cannot be determined, but the chosen parameter values suggest that PLM is consistent with an annual growth rate of 1.66% compared with the autarkic equivalent of 1.80%.

In the final part of the analysis, some allowance is made for an endogenous labour supply insofar as the old choose the number of hours they supply or the timing of their retirement. The qualitative results are interesting, although the numerical results, based on an arguably low elasticity of labour supply, are undramatic: there is an increase in the annual growth rates by about 0.04% for each of the two factor mobility regimes. The model is specified such that labour supply responds positively to changes in the wage, but negatively to a rise in interest income. Consequently, an endogenous labour supply is consistent with an increased variance of earned income and introduces a negative covariance between earned and unearned income.

The rest of the paper is organized as follows Section 2 presents a general framework that is fundamental to the whole analysis. Profit-maximizing firms are introduced along with expected-utility-maximizing households that generally face both earnings and interest uncertainty. Sections 3, 4 and 5 separately consider each of the regimes of autarky, PCM and PLM. Section 6 then re-evaluates each of these three in the light of an endogenous labour supply. Section 7 finally provides a brief summary of the whole analysis and some further discussion.

2. The basic model

The model is an open-economy version of the Diamond (1965) overlapping generations model. It consists of many small countries, where each, indexed z, also contains many small, identical firms. Each firm produces the single world good under conditions of perfect competition, according to the following technology,

$$y_t^z = (1+\rho_t^z)^{1-\alpha} B_t^z {k_t^z}^{\alpha}, \quad B_t^z = \left(A_t^z k_t\right)^{1-\alpha}$$

$$A_t^z = (1+\sigma_t^z)A \qquad E(\sigma_t^z \sigma_t^{z'}) = 0 \text{ for } z \neq z'$$

$$\sigma_t^z = -\sigma, \sigma \ , \qquad 0 < \sigma < 1 \text{ and}$$

$$pr(\sigma_t^z = +\sigma) = pr(\sigma_t^z = -\sigma) = 1/2, \text{ so } E(\sigma_t^z) = 0 \qquad (1)$$

Output in a representative firm in county z is y_t^z, where $1+\rho_t^z$ and k_t^z are its labour and capital inputs. The terms B_t^z represents *total factor productivity*, which depends on a productivity shock, A_t^z, which has a mean and standard deviation, A and σ_t^z, at values common to all countries.

There are constant returns to scale with respect to the firm's own factors, so that the production function may be presented per capita in terms of its own young workers. Even so, we assume there are two generations at work, the young and the old, who are equally productive and also equally numerous in the assumed absence of population growth and presence of full employment. With respect to production, they differ to the extent that the young supply an inelastic *unit* of labour, while the old supply a *fraction,* ρ_t^z , $0<\rho_t^z<1$. This fraction is initially regarded as a parameter but is later determined endogenously. Either way, it may be regarded as indicative of part-time but continued working by the old, or else the related term $(1-\rho_t^z)/2$ may be interpreted as the proportion of the adult life spent in retirement. The total labour supply for the representative firm in country z, and also for each country, under the assumption of a unit measure of firms, is $1+\rho_t^z$.

In addition to this labour input, production requires capital, k_t^z , and there is also a *total factor productivity* (*TFP*) parameter, B_t^z , which transforms the services of the two factor inputs into an output, y_t^z. *TFP* is determined both by a country-specific productivity shock, A_t^z , and a knowledge of the production technology, which is acquired through a process of learning-by-doing, following Arrow (1962), which is then freely disseminated to the rest of the economy. The knowledge spill-over is assumed to be global rather than national in extent, so that an international measure of the capital stock, the global average, $k_t = E(k_{t+1}^x)$, is specified as the relevant variable for embodying technology. This amounts to the assumption that *knowledge* is always perfectly, internationally mobile, whether or not the factors of production are. It also ensures that, in expectation, countries converge to the same long-run growth rate irrespective of factor mobility. Another implication of global knowledge - in conjunction with the growth-generating assumption of constant-returns-to-scale in overall capital - is that country-specific, technology shocks are a source of aggregation effects as will give rise to global aggregation effects as well as uncertainty.

The sources of uncertainty are country-specific, stochastic shocks to *TFP*, σ_t^z. These are independently and identically distributed. The assumption of a symmetric and binomial shock distribution is made to obtain analytic – at best, linear and, at worst, quadratic – solutions for the savings functions in the presence of uncertainty. The feature of very, many small countries allows us to appeal throughout the analysis to *the law of large numbers*; and we may begin here by stating that there is an absence of global uncertainty, because country-level shocks sum to zero with the existence of very, many small countries.

As noted, each firm in each country produces the single, world good under the condition of perfect competition, so that profits are zero. Marginal cost-product equalization then leads to the following inverse factor demands for the firm,

$$R_t^z = \alpha\left((1+\rho_t^z)(1+\sigma_t^z)Ak_t\right)^{1-\alpha}\left(k_t^z\right)^{\alpha-1}, \tag{2}$$

$$w_t^z = (1-\alpha)(1+\rho_t^z)^{-\alpha}\left((1+\sigma_t^z)Ak_t\right)^{1-\alpha}\left(k_t^z\right)^{\alpha} \tag{3}$$

The marginal cost of investment comprises both the interest rate and the depreciation rates. Since the latter is 100%, according to the usual assumption for a half-life period, the marginal cost is in effect the interest *factor*, R_t^z. The interest factor and the wage are each potentially stochastic through their common dependence on the *TFP* shock, σ_t^z, although their actual properties will be shown to depend on the factor mobility regime in place.

Households live, work and consume for two periods but save only in the first. Saving is undertaken not only to smooth consumption, but also as a possible response to factor income uncertainty. Saving is decided after the first-period wage has been determined, so that it responds to the first-period *TFP* shock realisation in addition to being formulated in anticipation of the second-period *TFP* shock distribution.[6]

Second-period uncertainty implies the following expectational form for the household's utility function,

$$E\left(U_t^z\right) = \ln\left(w_t^z - s_t^z\right) + \beta E\left(\ln\left(\rho_{t+1}^z w_{t+1}^z + R_{t+1}^z s_t^z\right)\right)$$

This logarithmic specification, although standard, in the present context enables us to obtain both tractable solutions to the model and an interesting benchmark case. The assumptions of a two-period utility function and of binomial and symmetric shocks produce the following simplified first-order condition for the household maximization in the presence of uncertainty,

[6] An alternative to this assumption is considered in *Appendix A*.

$$\frac{-1}{w_t^z - s_t^z} + \frac{\beta E}{2}\left(\frac{R_{t+1}^z(+)}{\rho_{t+1}^z w_{t+1}^z(+) + R_{t+1}^z(+)s_t^z} + \frac{R_{t+1}^z(-)}{\rho_{t+1}^z w_{t+1}^z(-) + R_{t+1}^z(-)s_t^z}\right) = 0, \tag{4}$$

where the two outcomes for the shock are indexed $(+)$ and $(-)$.

It is worth noting that the logarithmic utility function renders expectations of the *ratio* of future factor prices, w_{t+1}^z / R_{t+1}^z, to be of relevance rather than separate expectations of each of these two variables. This means that under autarky, where each country's factor prices are jointly and multiplicatively dependent on the same level of *TFP*, the factor price ratio is unaffected by the shock, and, hence, saving is determined at the certainty-equivalent level.

This particular case, where both factors of production, capital and labour, are completely immobile at the global level, is the first to be considered. Following this, autarky will be compared with perfect capital mobility (where labour is deemed to be immobile) and then with that perfect labour mobility (also in the absence of capital mobility). Finally, the analysis will make an allowance for an endogenous second-period labour supply under each of the three factor mobility regimes.

3. Autarky

Under autarky, equations (2) and (3) suffice as the solutions for each country's interest factor and wage. The two main characteristics of autarky, are, pertaining first to the capital market, that each country's, z, capital stock is determined by its own saving,

$$k_{t+1}^z = s_t^z, \tag{5}$$

and that, with reference to both factor markets, as mentioned, shocks to *TFP*, σ_{t+1}^z - or merely the levels of TFP, B_{t+1}^z- have no impact on relative factor prices, $w_{t+1}^z / R_{t+1}^z = (1-\alpha)k_{t+1}^z / \alpha(1+\rho_{t+1}^z)$.

The future capital stock is determined by current saving according to equation (5), and as this is known[7], so too is the future relative factor price. Thus, with a logarithmic utility function, saving is unaffected by the anticipation of uncertainty, and is solved as

$$s_t^z\Big|_{AUT} = \frac{\beta w_t^z - E\left(\rho_{t+1}^z w_{t+1}^z \big/ R_{t+1}^z\right)}{1+\beta} \tag{6}$$

Result One: *Saving under autarky is at the certainty-equivalent level, if the source of uncertainty is shocks to total factor productivity and if the utility functions are logarithmic.*

The result that uncertainty has no effect on saving under autarky may be explained in terms of its implications for the wage and interest factor distributions, the details of which are given in *Appendix A*. First, the independent variance of future wage income is a source of precautionary saving, since the logarithmic utility function has a positive third derivative, the necessary and sufficient condition. Secondly, a unitary elasticity of intertemporal substitution in conjunction with a positive mean value for future earnings implies that the anticipated variance of interest income reduces saving. Thirdly, the positive covariance between wage income and interest income under autarky, caused by TFP shocks under autarky, reduces saving further. These last two negative effects happen to exactly cancel the first positive effect, so that saving is unaffected by uncertainty, thus remaining at the life-cycle level.

Combining equations (2), (3), (5) and (6) gives capital accumulation in country z as

$$k_{t+1}^z\Big|_{AUT} = s_t^z = \Omega\left((1+\sigma_t^z)k_t\right)^{1-\alpha} k_t^{z\,\alpha}$$

$$\Omega \equiv \frac{\beta(1-\alpha)(1+\rho)^{-\alpha} A^{1-\alpha}}{1+\beta+\rho\mu}\,,\ \mu \equiv \frac{(1-\alpha)}{\alpha(1+\rho)} \tag{7}$$

The composite parameter Ω is the growth factor that would emerge in the absence of *TFP* volatility, $\sigma_t^z = 0$, $\forall t$, since this, with the symmetry assumption, also implies $k_t^z = k_t$, $\forall z$.

[7] We are working within the usual *rational expectations* paradigm of parameter certainty whereby individuals know the aggregate outcomes of their decisions.

Note that the time-series, k^z_{t+j}, $j=1,2,...$, where σ^z_{t+j}, $j=1,2,...$ would generally consists of positive and negative values, produces a negative correlation between volatility and country-level growth. This result provides a possible a rationale for the empirical finding of a negative correlation between growth and volatility in the papers by Ramey and Ramey (1995) and by Kneller and Young (2001), even though this relates to the cross-sectional data.[8] What might seem to present a *prima facie* case for stabilization policy merely reflects the fact that an irreducible stochastic term, σ^z_t, is being raised to the power of $1-\alpha$, where $0<1-\alpha<1$, in the equation for k^z_{t+1}.[9]

Equation (7) may also be aggregated spatially to obtain a solution for global economic growth. This admits an additional concavity, since the implicit equation for k_{t+1} contains k^z_t raised to the power of α, where $0<\alpha<1$. In obtaining a solution, we see that under autarky k^z_t is predetermined by the previous level of own-country saving, s^z_{t-1}, so that it is necessarily uncorrelated with the current shock σ^z_t. Global growth may then be approximated by

$$G\big|_{AUT} \equiv \frac{k_{t+1}}{k_t} = \Omega\left(1-\frac{\alpha(1-\alpha)^2}{2(1+\alpha)}\operatorname{var}\big(\ln(1+\sigma)\big)\right)\left(\frac{1+\sigma)^{1-\alpha}+(1-\sigma)^{1-\alpha}}{2}\right)$$

$$\Omega \equiv \frac{\beta(1-\alpha)(1+\rho)^{-\alpha}A^{1-\alpha}}{1+\beta+\rho\mu},\ \mu \equiv \frac{(1-\alpha)}{\alpha(1+\rho)} \qquad (8)$$

[The details are given in *Appendix B.*]

It is evident that *TFP* volatility, $|\sigma|>0$, reduces global economic growth below the volatility-free factor Ω, entirely because of aggregation effects. However, for a substantial magnitude of these shocks, the effect on aggregate growth in equation (8) is quite small.

[8] This holds even though here the shock variance is assumed to be common to all countries.
[9] An alternative possibility is investigated by Blackburn and Pelloni (2005).

If, for example, $|\sigma| = 0.5$ and $\alpha = 0.33$, according to equation (1), the difference between a positive shock ($\sigma = 0.5$) and a negative shock ($\sigma = -0.5$) is 3% growth per annum over a designated twenty-five year period. This level of volatility would reduce the global average annualized growth rate by about one-fifth of a percentage point.[10] We now consider the regime of perfect capital mobility.

4. Perfect capital mobility (PCM)

It is straightforward to model international capital mobility in its absolute or perfect (PCM) form as the case where all households save and where all firms borrow at a single, world interest rate, $R_t = R_t^z,\ \forall z$. The property of constant social returns to capital and the assumption that technology is international rather than national, respectively, imply that the relevant interest rate is pinned down as a parameter and that this parameter is a composite of international parameters.

Inverting equation (2) gives an expression for each country z's investment demand as a function of this single interest factor, R_t,

$$k_t^z = \left(\alpha / R_t\right)^{1/(1-\alpha)} (1+\rho_t^z)(1+\sigma_t^z) A k_t, \quad \forall z$$

Aggregation across all countries then gives

$$k_t = E(k_t^x) = \left(\alpha / R_t\right)^{1/(1-\alpha)} E\left(1 + E(\rho_t^x) + \mathrm{cov}(\rho_t^x, \sigma_t^x)\right) A k_t$$ [11]

Combining these two equations gives

$$k_t^z = \left(\frac{(1+\rho_t^z)(1+\sigma_t^z)}{1 + E(\rho_t^x) + \mathrm{cov}(\rho_t^x \sigma_t^x)} \right) k_t, \quad \forall z$$

Each country's capital stock relative to the global average relates positively to its marginal productivity *vis a vis* the rest of the world. Moreover, not only are capital-rich countries those subject to favourable *TFP* shocks, but each country's capital stock is a *linear* function of this shock, which has implications for aggregation.

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[10] The first calculation is $\left((1+0.5)/(1-0.5)\right)^{(1-1/3)1/25} - 1 = 2.97\%$ and the second, which produces 0.204%, derives from the fact that $\mathrm{var}\left(\ln(1+\sigma)\right) = 0.302$.

[11] $E\left((1+\rho_t^x)(1+\sigma_t^x)\right) = 1 + E(\rho_t^x) + \mathrm{cov}(\rho_t^x \sigma_t^x)$ as $E(\sigma_t^x) = 0$.

Next eliminating the common factor, k_t, in the aggregated form above and then re-inverting the equation gives the solution for the world interest factor under PCM,

$$R_t\big|_{PCM} = \alpha\left(1 + E(\rho_{t+1}^x) + \mathrm{cov}(\rho_{t+1}^x \sigma_{t+1}^x)\right)^{1-\alpha} A^{1-\alpha} \tag{9}$$

While the global demand for capital is indeterminate, its quantity is supply-determined by aggregated world saving.

From the competitive-market, zero profit condition, the wage for each country is then determined as

$$w_t^z\big|_{PCM} = (1-\alpha)\left(1 + E(\rho_{t+1}^x) + \mathrm{cov}(\rho_{t+1}^x \sigma_{t+1}^x)\right)^{-\alpha} A^{1-\alpha} k_t (1+\sigma_t^z), \quad \forall z \tag{10}$$

The obverse of a single world interest rate under PCM is that international capital movements amplify the effects on wages of *TFP* shocks. More precisely, the elasticity of the wage with respect to the shock, $\partial \ln w_t^z \big/ \partial \ln(1+\sigma_t^z)$, is $1-\alpha$ in equation (3) for autarky but unity in equation (10) for PCM.

Initially, we fix labour supply and at a level common to all countries, $\rho_t^x = \rho$, $\forall x$, so that equations (9) and (10) are rewritten as

$$R_t\big|_{PCM} = \alpha(1+\rho)^{1-\alpha} A^{1-\alpha} \tag{11}$$

$$w_t^z\big|_{PCM} = (1-\alpha)(1+\rho)^{-\alpha} A^{1-\alpha} k_t (1+\sigma_t^z) \tag{12}$$

We may draw the following conclusions.

Result Two: *The effect of PCM is to (a) reduce the variance of the interest factor to zero, (b) raise the variance of the wage, (c) raise the global average of the wage and (d) lead to no change in the global average of the discounted future wage (with an ambiguous effect on the global average of the interest factor).* [12]

See *Appendix C* for the details and proof.

While part (a) merely constitutes our definition of PCM and part (b) reflects a general implication of it, the remaining parts (c) and (d) are specific to this particular model

[12] Parts (c) and (d) at first glance appear contradictory, but are not, because the mean of the ratio of the wage to the interest factor differs from the ratio of the mean wage to the mean interest factor.

and follow from the assumptions of a global technology along with constant social returns to capital.

Next considering saving, the PCM restriction $R_{t+1}\big|_{PCM} = R^z_{t+1}(+) = R^z_{t+1}(-)$ is applied to the first-order condition (4), which generates a quadratic form for the saving function,

$$s^z_t\big|_{PCM} = \frac{\beta\left(w^z_t + \rho E(w_{t+1}/R)\right)}{2(1+\beta)} - \rho\, E(w_{t+1}/R_{t+1})$$

$$+\sqrt{\left(\frac{\beta\left(w^z_t + \rho E\,(w_{t+1}/R)\right)}{2(1+\beta)}\right)^2 + \frac{\left(\rho\, E(w_{t+1}/R_{t+1})\right)^2}{(1+\beta)}\sigma^2}\ , \quad \frac{\partial s^z_t\big|_{PCM}}{\partial \sigma^2} > 0$$

where $E\left(w_{t+1}/R_{t+1}\right) \equiv \left((1-\alpha)/\alpha(1+\rho)\right)k_{t+1}$ (13)

Note that the elimination of the variance term, σ^2, causes the solution to revert to the linear form in equation (6) .

Corollary to Result Two: *Saving is higher under PCM than under autarky.*

Parts (a) and (b) of *Result Two* imply that there is an additional, precautionary element to saving under PCM – for unchanged mean values of factor prices, because of both an increase in the volatility of future earnings income and the elimination of interest uncertainty, which is captured by the presence of σ^2 in equation (13) where $\partial s^z_t \big/ \partial \sigma^2 > 0$. Parts (c) and (d) also suggest that there will also be an incremental rise in life-cycle saving, because the mean values of factor prices *do* change and in way that is favourable - with a rise in the mean value of current earnings and a fall in that of discounted future earnings.

In comparison with autarky in equation (5), the capital stock of any country is now determined by global rather than national saving – plus its own *TFP* shock,

$$k^z_{t+1}\big|_{PCM} = (1+\sigma^z_t)s_t\,,$$

Global capital accumulation is

$$k_{t+1} = s_t\,, \qquad (14)$$

A solution for growth is obtained by adding up equation (13) across all countries. This entails another positive aggregation effect, as $E\sqrt{f(w_t^z,..)} > \sqrt{f\left(E(w_t^z,..)\right)}$. This convex effect is favourable to capital accumulation under PCM, and thus supports the thrust of the results so far, but is discarded in unduly complicating the solution for this case. Consequently, a simpler form for the solution is obtained,

$$G\big|_{PCM} = \frac{\beta(1-\alpha)(1+\rho)^{-\alpha} A^{1-\alpha}}{1+\beta+\rho\mu-\dfrac{(\rho\mu)^2}{1+\rho\mu}\sigma^2}, \quad \frac{\partial G\big|_{PCM}}{\partial\sigma^2} > 0 \text{ where } \mu \equiv \frac{(1-\alpha)}{\alpha(1+\rho)}, \quad (15)$$

which bears closer comparison with equation (8), providing the first major result.

Proposition One: *(i) Global economic growth is higher under perfect capital mobility than autarky, (ii) and there is a positive relationship between volatility and global economic growth.*

This follows from *Results One, Two* and the *Corollary* and is also evident from a comparison of equations (8) and (15), since $\sigma^2 > 0$ implies that $G\big|_{PCM} > \Omega > G\big|_{AUT}$.

There are two reasons why PCM will raise the international economic growth rate. One is that it gives rise to precautionary saving; the second is that aggregation effects from capital market integration lead to an increase in global life-cycle saving. It is also worthy of note that while the correlation between volatility and growth is negative under autarky, it is positive under PCM, which suggests that, in general, the sign of the correlation may depend on the degree of capital market integration.

Obstfeld (1994) obtains the same basic result through a portfolio effect within a infinite-lived, representative-agent where only interest risk an issue. For each country there are two investment strategies, involving a high risk but high return production technology and a low risk and safer alternative. The benefit of international financial integration is in facilitating greater risk-diversification, which encourages households to invest more in the riskier but more productive technologies. Deveureux and Smith (1994) provide a similar model in which the main part of their analysis centres instead on earnings uncertainty from endowment shocks.

International integration then provides insurance against exogenous country-specific shocks, which actually causes saving and growth to fall under PCM through removing the motive for precautionary saving.[13] Their secondary application coincides with our own in considering *TFP* shocks that jointly earnings and interest uncertainty. However, their application of an infinite-lived, representative-agent model means their results depend on the sign of the interest rate response and, hence, on the specification of the utility function. Within the general CRRA class, the logarithmic function provides a borderline, neutral case where international capital market integration has no effect; and uncertainty will raise/lower growth in their model, if the elasticity of intertemporal substitution is greater/less than unity. In contrast with our own consideration of a finite-horizon model, PCM raises economic growth above the autarky level for *any value of the CRRA*, while the logarithmic case remains an interesting benchmark case, but for an entirely different reason that there *TFP* volatility has no effect on saving *under autarky*.

Again to evaluate the results, we select the values, $\beta = 0.8$, $\rho = 0.8$, in additional to the previous ones, $\alpha = 0.33$ and $|\sigma| = 0.5$. First, we find that under PCM the annualized average growth rate is 0.16% higher above the volatility-free case, which is due to precautionary saving, since aggregation effects are absent by construction. However, the economically meaningful comparison is to be made between autarky and PCM for the given level of volatility, where aggregation effects appear. Accordingly, as we have already found that autarky reduces the annualized growth rate by 0.20%, it is a matter of deduction that PCM will raise the annual growth rate by 0.36% above the autarky level, of which just under a half may be attributed to precautionary saving and the rest to aggregation. In terms of the accumulated effect over time, our fairly generous assumption about the degree of *TFP* volatility implies that the world would on average be 9.5% richer after twenty-five years of full capital market integration.

5. Perfect labour mobility (PLM)

[13] They also draw attention to the point that integration may reduce welfare, despite the fact it allows for insurance, because investment is already at an inefficiently low level in the presence of an externality and is reduced even further.

Our attention now turns to international labour market integration. Although some measure of global labour mobility is clearly observable, the limiting case of perfect labour mobility (PLM) is much less plausible because of relative costs. Nevertheless, it is considered in order to reveal the flavour of the results for a more limited form of labour mobility and a comparative case of interest. It is modelled analogously, where implicit population movements would arbitrage away any potential country earnings differences until a single world wage is obtained. This would require that some part of the labour force, we assume the young generation, is able to migrate without incurring any cost.[14] In order to isolate this particular case, we abstract from the possibility that there is any parallel mobility in capital.[15] The basic result is then that shocks to *TFP* will lead to increased volatility in the interest rates at which households and firms save and borrow, but that wages remain at a certain and global level.

The total labour supply in each country is now the variable, $L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z}$, where the number of young workers, $L_t^{Y,z}$, is determined currently and where the number of old workers, who were previously the young, $L_{t-1}^{Y,z}$, was determined previously. The demand for labour in each country as a function of a common world wage, w_t, is

$$L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z} = \frac{(1-\alpha)^{1/\alpha}\left((1+\sigma_t^z)Ak_t\right)^{1/\alpha-1} K_t^z}{w_t^{1/\alpha}} \quad \forall z \tag{16}$$

The assumption that only the young migrate conveniently means that that σ_t^z and $L_{t-1}^{Y,z}$ are uncorrelated - and so too are ρ_t^z and $L_{t-1}^{Y,z}$, when endogenous labour in terms of hours is considered further below. Furthermore, since σ_t^z and K_t^z are also uncorrelated, following from the assumed absence of any capital mobility, equation (16) may be aggregated spatially as

[14] More properly, where young workers do not subsequently return to their countries of birth, we should assume that the migration decision should ideally be based on an evaluation of relative life-time utilities rather than of current wages. However, the migration problem becomes convoluted, where there is no capital mobility and hence a continuous distribution of interest rates causing a continuous distribution of life-time utilities both for the countries of birth and for the potential host countries.

[15] If both capital and labour are both assumed to be perfectly mobile, the further assumption of constant returns to scale then implies that negative country-level TFP shocks would eliminate whole economies.

$$L + E(\rho_t^x)L = \frac{(1-\alpha)^{1/\alpha} E\left((1+\sigma_t^x)^{1/\alpha - 1}\right)(Ak_t)^{1/\alpha - 1} K_t}{w_t^{1/\alpha}}. \tag{17}$$

This is interpreted as the global labour market equilibrium condition, where L, $L = E(L_t^{Y,x}), E(L_{t-1}^{Y,x})$, is the international average supply of workers from each generation. It may then be inverted, using $k_t \equiv K_t / L_t$, to give the wage solution,

$$w_t = \frac{(1-\alpha)\left(E\left((1+\sigma_t^x)^{1/\alpha - 1}\right)\right)^{\alpha} A^{1-\alpha} k_t}{\left(1 + E(\rho_t^x)\right)^{\alpha}} \tag{18}$$

This is then returned to equation (16) to determine the labour supply per head of its own young population for each country,

$$1 + \rho_t^z\, L_{t-1}^{Y,z} / L_t^{Y,z} = \frac{(1+\sigma_t^z)^{1/\alpha - 1}}{E(1+\sigma_t^x)^{1/\alpha - 1}} \frac{k_t^z}{k_t} \left(1 + E(\rho_t^x)\right), \qquad \forall z$$

which bears comparison with $1 + \rho_t^z$ for the initial formulation in equation (1) where labour is immobile.

Each country's interest factor is then solved from the zero profit condition as

$$R_t^z\big|_{PLM} = \frac{\alpha\left(1 + E(\rho_t^x)\right)^{1-\alpha} A^{1-\alpha} (1+\sigma_t^z)^{1/\alpha - 1}}{\left(E\left((1+\sigma_t^x)^{1/\alpha - 1}\right)\right)^{1-\alpha}}, \qquad \forall z \tag{19}$$

National shocks to *TFP* now affect only interest rates, which means a greater elasticity of response, $\partial \ln R / \partial \ln \sigma$: at $1/\alpha - 1$ in equation (19) for PLM compared with $1-\alpha$ in equation (2) for autarky, a three-fold increase if $\alpha = 1/3$.

Initially, as we are imposing an exogenous labour supply in terms of hours for all countries, $\rho_t^z = \rho$, $\forall z$, so that equations (18) and (19) are slightly modified to

$$w_t = \frac{(1-\alpha)\left(E\left((1+\sigma_t^x)^{1/\alpha - 1}\right)\right)^{\alpha} A^{1-\alpha} k_t}{(1+\rho)^{\alpha}} \tag{20}$$

$$R_t^z\big|_{PLM} = \frac{\alpha(1+\rho)^{1-\alpha} A^{1-\alpha} (1+\sigma_t^z)^{1/\alpha - 1}}{\left(E\left((1+\sigma_t^x)^{1/\alpha - 1}\right)\right)^{1-\alpha}} \tag{21}$$

We can conclude the following.

Result Three: *The effect of PLM is to (a) reduce the variance of the wage to zero, (b) raise the variance of the interest factor, (c) raise the global average wage, (d) raise the global average of the future discounted wage (with an ambiguous effect on the global average interest factor).*

See *Appendix D* for the details and proof.

Again Part (a) is by definition of PLM and the other parts are by implication; (b) is a more general implication, while (c) and (d) are specific to this model.

Corollary to Result Three: *Saving is lower under PLM than under autarky provided that the effect of a higher wage in part (c) does not dominate the others.*

As for parts (a) and (b) of *Result Three*, the reduction in earnings uncertainty and the rise in earnings uncertainty must each cause dissaving relative to the autarky case. The factor price effects in parts (c) and (d) should impinge on life-cycle saving, but the combined effects are not clear.

The saving function for each country becomes

$$s_t^z\big|_{PLM} = \frac{\beta w_t - (2+\beta)\lambda_{t+1}\rho E\left(w_{t+1}/R_{t+1}^z\right)}{2(1+\beta)}$$

$$+\sqrt{\left(\frac{\beta(w_t + \lambda_{t+1}\rho E\left(w_{t+1}/R_{t+1}^z\right)}{2(1+\beta)}\right)^2 + \frac{\left(\rho E\left(w_{t+1}/R_{t+1}^z\right)\right)^2 \varepsilon_{t+1}{}^2}{1+\beta}}$$

$$\lambda_{t+1} \equiv \left(\frac{1}{R_{t+1}^z(+)} + \frac{1}{R_{t+1}^z(-)}\right)\frac{E(R_{t+1}^z)}{2} > 1,$$

$$\varepsilon_{t+1}{}^2 \equiv \left(\frac{1}{R_{t+1}^z(+)} - \frac{1}{R_{t+1}^z(-)}\right)^2\left(\frac{E(R_{t+1}^z)}{2}\right)^2 > 0, \quad \forall z \tag{22}$$

Since wages and the *expected future distribution* of interest rates are the same for each country, it follows that $s_t^z = s_t, \ \forall z$; so that equation (22) and the previous two give, after some manipulation, the following expression for global economic growth under PLM.

$$G\big|_{PLM} = \frac{\dfrac{\beta(1-\alpha)}{(1+\rho)^{\alpha}}\left(\phi + \dfrac{(1-\sigma^2)^{1/\alpha-1}}{\left(E\left((1+\sigma^x)^{1/\alpha-1}\right)\right)^2}\right)A^{1-\alpha}\left(E\left((1+\sigma^x)^{1/\alpha-1}\right)\right)^{\alpha}}{\phi^2 + (2+\beta)\phi + (1+\beta)\dfrac{(1-\sigma^2)^{1/\alpha-1}}{\left(E\left((1+\sigma^x)^{1/\alpha-1}\right)\right)^2}}$$

where $\phi \equiv \left(\dfrac{1-\alpha}{\alpha}\right)\left(\dfrac{\rho}{1+\rho}\right)$ (23)

This gives rise to the second major result.

Proposition Two: *(i) Global economic growth is lower (higher) under international labour mobility than autarky if α is large (small) in which case (ii) there will be a negative relationship between volatility and growth under international labour mobility.*

Proof: See *Appendix E.*

Qualitatively, we cannot be sure of the effect of PLM compared with autarky. However, if we apply the same parameter values as before, we find that moving from autarky to PLM reduces the annual growth rate from 1.80% to 1.66%. This is partly constituted by a fall in precautionary saving, since earnings uncertainty is eliminated and interest uncertainty is amplified, but is offset by a positive aggregation effect caused by wage equalization. Since the volatility-free case of 2.00% precludes both uncertainty and aggregation effects, we may deduce that these two opposing effects are respectively responsible for -0.34% and +0.20% of the -0.14% change. These relative magnitudes depend, of course, on the numerical values, especially that of α, which has been set at $1/3$. We can summarize all the results obtained so far, where the labour supplies have been fixed at a common parameter, in the following table.

Table One: Economic Growth where labour supply is exogenous

International regime	**Annual growth rate**
No volatility (benchmark)	2.00% (assigned)
Autarky	1.80%
Perfect capital mobility	2.16%

Perfect labour mobility	1.66%

6. Endogenous labour supply

6.1 Household optimization revised

Given the logarithmic form of the utility function, the presence of interest rate effects on saving is due to the expected presence of future earnings, since households supply labour in the second-period of their lives. This supply up to now has been treated as exogenous in order to limit the scope of the analysis, but now we extend it to focus on the interaction between *TFP* volatility to see how this may modify the results for the relationship between the factor mobility regime and economic growth.

An increase in the mean value of second-period hours, for example, would reduce first-period earnings through an indirect effect of total labour supply on the wage and raise second period earnings through the dominance of the direct effect. Both of effects of would raise life-cycle saving through consumption-smoothing, while a fall in average labour supply would, of course, have the opposite effect. Another question is how will the possibly stochastic nature of labour supply interact with precautionary saving?

In order to answer this, we augment the household utility function with a term that represents the disutility of second-period labour supply. Maintaining a general logarithmic form, the extra term, $1-\rho_{t+1}^{z}$, is included to capture the *extra* leisure to be enjoyed in the second period,

$$U_t^z = \ln\left(w_t^z - s_t^z\right) + \beta\left(\ln\left(\rho_{t+1}^z w_{t+1}^z + R_{t+1}^z s_t^z\right) + \eta \ln(1-\rho_{t+1}^z)\right)$$

The new parameter η is the (semi-) elasticity of utility with respect to leisure.[16]

[16] We should add that Flodén (2006) has already provided an analysis of the relationship between labour supply and saving in the midst of uncertainty, which is more general since there is no specification of functional forms but also more specialized to the extent that earnings uncertainty is considered to the exclusion of interest uncertainty.

In the second-period where s_t^z has been already been determined, the household chooses its labour supply, $\rho_{t+1}^{z,i}$, to maximize $\ln\left(\rho_{t+1}^z w_{t+1}^z + R_{t+1}^z s_t^z\right) + \eta \ln(1-\rho_{t+1}^z)$:

$$\rho_{t+1}^{z,i} = \frac{1-\eta s_t^{z,i} R_{t+1}^z \big/ w_{t+1}^z}{1+\eta}, \quad \frac{\partial \rho_{t+1t+1}^{z,i}}{\partial s_{t+1}^{z,i}} < 0, \quad \frac{\partial \rho_{t+1t+1}^{z,i}}{\partial w_{t+1}^z} > 0, \quad \frac{\partial \rho_{t+1t+1}^{z,i}}{\partial R_{t+1}^z} < 0 \quad (24)$$

Labour supply responds positively to wage changes, because the specification is such that the substitution effect dominates the income effect, but negatively to changes in interest rates with only an income. Consequently, an endogenous labour supply exacerbates the effect of earnings uncertainty - because the demand for labour is elastic, but dampens the effect of interest uncertainty by introducing a negative correlation between unearned and earned sources of income. Each of these two outcomes means that labour supply variability is consistent with a rise in precautionary saving.

In solving the predetermined level of first-period saving, the probabilistic response of second-period labour supply must be accounted for by the household and, so, it is endogenized by incorporating equation (24) into the *ex ante* utility function,

$$U_t^z = \ln\left(w_t^z - s_t^z\right) + E\left(\beta(1+\eta)\left(\ln(w_{t+1}^z + R_{t+1}^z s_t^z) - \ln(1+\eta)\right) + \beta\eta\left(\ln\eta - \ln w_{t+1}^z\right)\right)$$

The first-order condition, hitherto given in equation (4), is modified to

$$\frac{-1}{w_t^z - s_t^z} + \frac{\beta(1+\eta)}{2}\left(\frac{R_{t+1}^z(+)}{w_{t+1}^z(+) + R_{t+1}^z(+)s_t^z} + \frac{R_{t+1}^z(-)}{w_{t+1}^z(-) + R_{t+1}^z(-)s_t^z}\right) = 0 \quad (25)$$

In terms of calculating a solution - rather than with respect to the intuition - $\beta(1+\eta)$ now appears instead of β in equation (4) and *unity* replaces the exogenous fraction, ρ. Economic growth under each of the three main regimes of autarky, PCM and PLM may now be considered where, notationally, the respective outcomes with labour supply endogeneity are designated as AUT*, PCM* and PLM*.

6.2 Autarky with endogenous labour supply

The first main finding is that under autarky, because of the assumed logarithmic utility *from consumption*, shocks to *TFP* do not affect labour supply, because the ratio of earnings to interest income remains unaltered. Individual labour supply, although endogenous, is invariant and determined from equations (2), (3), (5) and (24) as

$$\rho_{t+1}^{z,i}\big|_{AUT*} = \frac{1-\eta\left(\alpha(1+\rho_{t+1}^{z}\big|_{AUT*})\big/(1-\alpha)\right)}{1+\eta}$$

Result Four: *Endogenous labour supply is constant under autarky.*

In the equation above we have distinguished between the individual i's labour supply, $\rho_{t+1}^{z,i}\big|_{AUT*}$, and the country-level, $\rho_{t+1}^{z}\big|_{AUT*}$, because the latter indirectly affects the former through country-level wage determination - although not *vice versa*. Symmetry across individuals, $\rho_{t+1}^{z,i} = \rho_{t+1}^{z}$, $\forall i$, implies

$$\rho_{t+1}\Big|_{AUT^*} = \rho^* = \frac{1-\alpha(1+\eta)}{1-\alpha+\eta} \quad \text{where} \quad \eta \le (1-\alpha)/\alpha \tag{26}$$

The departure from the previous analysis of an exogenous labour supply under autarky would seem to be trivial, because whereas it was previously defined as being parametric at ρ, it is now derived as a deterministic function of two parameters, the newly specified, η, and the existing, α. Consequently, the main analytical interest must relate to how labour supply endogeneity might modify the results for the two factor mobility regimes. In order to isolate these effects, we set the optimal, albeit constant, autarky level of labour supply, $\rho_{t+1}\Big|_{AUT^*}$ into equality with the parametric level, ρ. This is equivalent to placing the following restriction on η, the elasticity of leisure,

$$\eta = \frac{(1-\alpha)(1-\rho)}{\alpha+\rho} \tag{27}$$

6.2 PCM with endogenous labour supply

Ex post labour supply under PCM according to equations (9), (10), (14) into (26) is

$$\rho_{t+1}^{z} = (1+\eta)^{-1}\left(1-\eta\frac{\alpha}{1-\alpha}\frac{\left((1+E(\rho_{t+1}^{x})+\operatorname{cov}(\rho_{t+1}^{x},\sigma_{t+1}^{x})\right)}{1+\sigma_{t+1}^{z}}\frac{s_t^z}{s_t}\right) \tag{28}$$

It is apparent that the covariance, $\operatorname{cov}(\rho_{t+1}^{z},\sigma_{t+1}^{z})\big|_{PCM}$, is positive, since $\partial\rho_{t+1}^{z}\big/\partial\sigma_{t+1}^{z}>0$, but the less obvious details of the solution are assigned to *Appendix F*, which shows that a manipulation of equation (28) leads to

$$E(\rho\big|_{PCM}*)=(1+\eta)^{-1}\left(1-\frac{\eta\alpha(2+\eta)}{(1+\eta-\alpha)(1-\sigma^{2})}\right) \tag{29}$$

Equation (26) into (29) gives

$$E(\rho\big|_{PCM}*)=\rho-\left(\frac{\alpha(1-\rho^{2})}{\alpha+\rho}\right)\frac{\sigma^{2}}{1-\sigma^{2}}, \text{ where } \frac{\partial E(\rho\big|_{PCM}*)}{\partial\sigma^{2}}<0$$

Thus, under PCM, volatility reduces the expected value of second-period labour supply, because of the concave response of labour supply to shocks to earnings. However, the fall is quite small: if under autarky, $\rho=\rho\big|_{AUT}=0.8$, under our maintained parameter values, $\alpha=0.33$ and $|\sigma=0.5|$, then $E(\rho\big|_{PCM}*)=0.765$. Furthermore, for the average country for which $s_{t}^{z}=s_{t}$, *ex post* second period-labour supply is,

$$\rho_{t+1}^{z}=(1+\eta)^{-1}\left(1-\frac{\eta\alpha(2+\eta)}{1+\eta-\alpha}\left(\frac{1}{1+\sigma_{t+1}^{z}}\right)\right)$$

The assigned values imply an outcome of 0.832 in the event of a positive *TFP* shock and of 0.705 for a negative one.

The factor prices are then solved as

$$R_{t}\big|_{PCM}*=\alpha\left(1+E\left(\rho\big|_{PCM}*\right)\right)^{1-\alpha}A^{1-\alpha} \tag{30}$$

$$w_{t}^{z}\Big|_{PCM^{*}}=(1-\alpha)(1+\sigma_{t}^{z})\left(1+E\left(\rho\big|_{PCM}*\right)\right)^{-\alpha}A^{1-\alpha}k_{t} \tag{31}$$

The remaining implications of PCM with endogenous labour are now summarized in the following two results.

Result Five: *If labour supply is endogenous, PCM introduces (a) a positive covariance between the wage and labour supply, and causes (b) a fall in the mean value of labour supply, and (c) a rise in the current wage, (d) a rise in the interest* *factor, (d) a fall in the present value of future earnings income, (e) a rise in the normalized variance of future earnings.*

See *Appendix F.*

Result Six: *If labour supply is endogenous, (i) perfect capital mobility causes growth to be even higher above the autarky level and (ii) the positive relationship between volatility and growth is strengthened further.*

From *Result Five*.

Equation (25) with wage shocks alone under PCM* implies that savings is

$$s_t^z\big|_{PCM}* = \frac{\beta(1+\eta)\left(w_t^z + E(w_{t+1}/R)\right)}{2\left(1+\beta(1+\eta)\right)} - E(w_{t+1}/R_{t+1})$$
$$+\sqrt{\left(\frac{\beta\left(w_t^z + E(w_{t+1}/R)\right)}{2\left(1+\beta(1+\eta)\right)}\right)^2 + \frac{\left(\rho E(w_{t+1}/R)\right)^2}{1+\beta(1+\eta)}\sigma^2}\ , \qquad (32)$$

Again there is a favourable but complicating aggregation effect, which is excluded; and equations (14) and (29)-(32) are combined to give

$$G\big|_{PCM}* = \frac{\beta(1+\eta)(1-\alpha)(1+\hat{\rho})^{-\alpha} A^{1-\alpha}}{1+\beta(1+\eta)+\mu\big|_{PCM}* - \dfrac{(\mu\big|_{PCM}*)^2}{1+\mu\big|_{PCM}*}\sigma^2}$$

where $\mu\big|_{PCM}* \equiv \dfrac{(1-\alpha)}{\alpha(1+\rho\big|_{PCM}*)}$, $\qquad \eta \equiv \dfrac{(1-\alpha)(1-\rho)}{\alpha+\rho} \qquad (33)$

All of the marginal effects of an endogenous labour supply reinforce the result of Section 4 that PCM raises long-run economic growth above the autarky level. The effect of a "procyclical" labour supply increases earnings volatility and, so, precautionary saving and the change in average factor prices is such as to promote a rise in life-cycle saving. However, the size of the change is quite small. The maintained values of $\alpha = 0.33$, $\rho = 0.8$, $\beta = 0.8$ imply $\rho\big|_{PCM}* = 0.765$, $\mu = 1.\dot{1}$, $\mu\big|_{PCM}* = 1.133$ and $\eta = 0.1175$, and with $|\sigma| = 0.5$ $A = 31.92$, as before, the effect of an endogenous labour supply is for the annual world economic growth rate to rise from 2.16%, to 2.20% under PCM.

6.3 PLM with endogenous labour supply

Equation (22) with (18), (19) and (5) give the labour supply of an individual j in country z as

$$\rho_{t+1}^{z,j}\Big|_{PLM*} = (1+\eta)^{-1}\left(1-\eta\frac{\alpha}{1-\alpha}\frac{(1+\rho_{t+1}^{z})\left(1+\sigma_{t+1}^{z}\right)^{1/\alpha-1}}{E\left((1+\sigma_{t+1}^{x})^{1/\alpha-1}\right)}\frac{s_t^{z,j}}{s_t}\right)$$

Symmetry across all individuals in country z and $s_t^z = s_t, \forall z$, implies that for country z,

$$\rho_{t+1}^{z}\Big|_{PLM*} = \frac{(1-\alpha)E\left((1+\sigma_{t+1}^{x})^{1/\alpha-1}\right)-\eta\alpha(1+\sigma_{t+1}^{z})^{1/\alpha-1}}{(1+\eta)(1-\alpha)E\left((1+\sigma_{t+1}^{x})^{1/\alpha-1}\right)+\eta\alpha(1+\sigma_{t+1}^{z})^{1/\alpha-1}},$$

so that globally,

$$\rho\Big|_{PLM*} = \frac{(1+\eta)(1-\alpha)^2-\eta^2\alpha(1-\alpha)-(\eta\alpha)^2\dfrac{\left(1-\sigma^2\right)^{1/\alpha-1}}{\left(E\left((1+\sigma)^{1/\alpha-1}\right)\right)^2}}{(1+\eta)^2(1-\alpha)^2+2\eta(1+\eta)\alpha(1-\alpha)+(\eta\alpha)^2\dfrac{\left(1-\sigma^2\right)^{1/\alpha-1}}{\left(E\left((1+\sigma)^{1/\alpha-1}\right)\right)^2}}$$

$$\frac{\partial\rho_{t+1}^{z}\Big|_{PLM*}}{\partial|\sigma|} > 0 \tag{34}$$

TFP volatility raises the mean value of labour supply under PLM above the autarkic level, because of the convexity of the response to the shocks, although this effect turns out to trivial, merely 0.803 instead of 0.8. The remaining results are summarized as follows.

Result Seven: *If labour supply is endogenous, PLM causes (a) a negative covariance between the labour supply and the TFP shock, (b) a rise in the mean value of labour supply, and (c) a fall in current earnings, (d) a rise in the interest factor, (e) a rise in the present value of future earnings.*

Result Eight: *If labour supply is endogenous, (i) perfect labour mobility causes economic growth to be lower than where it is exogenous, provided that any possible* *rise in precautionary saving is of a low magnitude, in which case (ii) the correlation between volatility and growth becomes more negative.*

This follows from *Result Six* and from equations (18), (19) and (34).

The solution is straightforward, as saving remains the same for each country, and we find equation (25) with the shocks to interest rates in isolation leads to the following modification of equation (22),

$$s_t^z\Big|_{PLM}* = \frac{\beta(1+\eta)w_t - \left(2+\beta(1+\eta)\right)\lambda_{t+1}E\left(w_{t+1}/R_{t+1}^z\right)}{2\left(1+\beta(1+\eta)\right)}$$

$$+\sqrt{\left(\frac{\beta(1+\eta)(w_t+\lambda_{t+1}E\left(w_{t+1}/R_{t+1}^z\right)}{2\left(1+\beta(1+\eta)\right)}\right)^2+\frac{\left(E\left(w_{t+1}/R_{t+1}^z\right)\right)^2 {\varepsilon_{t+1}}^2}{1+\beta(1+\eta)}}$$

$$\lambda_{t+1} \equiv \left(\frac{1}{R_{t+1}^z(+)}+\frac{1}{R_{t+1}^z(-)}\right)\frac{E(R_{t+1}^z)}{2} > 1,$$

$${\varepsilon_{t+1}}^2 \equiv \left(\frac{1}{R_{t+1}^z(+)}-\frac{1}{R_{t+1}^z(-)}\right)^2\left(\frac{E(R_{t+1}^z)}{2}\right)^2 > 0, \quad \forall z \tag{35}$$

The modified implied factor price equations,

$$R_t\Big|_{PLM}* = \alpha\left(1+E\left(\rho\big|_{PLM}*\right)\right)^{1-\alpha} A^{1-\alpha}, \tag{36}$$

$$w_t^z\Big|_{PLM}* = (1-\alpha)(1+\sigma_t^z)\left(1+E\left(\rho\big|_{PLM}*\right)\right)^{-\alpha} A^{1-\alpha}k_t, \tag{37}$$

and the same reasoning in *Section Five* imply the growth factor is

$$G\Big|_{PLM}* = \frac{(1-\alpha)\beta(1+\eta)(1+\rho\big|_{PLM}*)^{-1/3}\left(\mu\big|_{PLM}*+\left(\frac{1-\sigma^2}{1+\sigma^2}\right)^2\right)(1+\sigma^2)^{1/3}A^{2/3}}{\left(\mu\big|_{PLM}*\right)^2+\left(2+\beta(1+\eta)\right)\mu\big|_{PLM}*+\left(1+\beta(1+\eta)\right)\left(\frac{1-\sigma^2}{1+\sigma^2}\right)^2}$$

where $\mu\big|_{PLM}* \equiv \dfrac{(1-\alpha)}{\alpha(1+\rho\big|_{PLM}*)}$, $\qquad \eta \equiv \dfrac{(1-\alpha)(1-\rho)}{\alpha+\rho}$ (38)

The negative covariance between the *TFP* shock and labour supply means there is now some measure of earnings uncertainty, which raises saving. However, as in the previous case of PCM, the overall effect of an endogenous labour supply is small, raising the annualized growth rate to 1.70%.

The following table replicates the results from *Table One* and includes those for this present *Section*.

Table Two: Economic Growth

International regime	**Annual international growth rate**	
	Exogenous Labour	Endogenous Labour
No volatility (benchmark)	2.00% (assigned)	2.00% (assigned)
Autarky	1.80%	1.80%
Perfect capital mobility	2.16%	2.20%
Perfect labour mobility	1.66%	1.70%

7. Summary and discussion

The purpose of this analysis has been to consider the implications of more general forms of uncertainty within a model of overlapping generations. Its application was to an international macro-model with the consequence that the form of international factor mobility regime was found to be important for household saving and for global economic growth. PCM was shown to raise saving by reducing interest uncertainty at the expense of increasing earnings uncertainty; an analogous consideration of PLM had the opposite effect. On top of this, the specification of a global technology alongside an effective *AK* growth mechanism implied that each form of factor market integration led to a positive aggregation effect. This caused PCM to be unambiguously growth-enhancing, while the impact of PLM depended on countervailing effects.

Other authors, notably Obstfeld (1994) and Deveureux and Smith (1994), have also considered the effects of PCM on precautionary saving, albeit within alternative, representative-agent, infinite-horizon models. Obstfeld found also that PCM raised saving but through the slightly different and more complex mechanism of asset diversification, which made households more willing to save at risky rates of return rather than facilitating a general reduction in risk. For the case of TFP shocks in Deveureux and Smith, which matched our own concern, the different representative-agent, infinite-horizon specification caused their results to hinge on an interest rate effect and, thus, critically depended on the form of the utility function. To our knowledge to date, no authors have given a similar treatment to the effects of PLM nor have integrated an analysis of labour supply responses to uncertainty.

As usual, there are a number of caveats. One is that a quantitative evaluation of the results confirmed the result in Smith (1996) for a closed-economy that macroeconomic uncertainty has a small quantitative effect on saving. In mitigation, this could be due to imposing – for analytical ease – a unitary and so a possibly too low value of unity for the elasticity of intertemporal substitution in consumption. One possible resolution for might be to consider models with unemployment in which the uncertainty of employment has a more powerful effect on household saving than the mere, associated uncertainty of income. The truism remains, however, that relatively small improvements in growth will have a compounding effect on output that will eventually come to dwarf the magnitude of any cyclical movements.

There is still more to be done in modelling the interaction between labour supply, following Flodén (2006). Apart from the usual life-cycle considerations, the scope to be able to vary hours *ex post* in response to any kind of shock may have important implications for saving. We have considered this in a restricted way, although in our view without undermining the relative implications of the various factor mobility regimes. One obvious extension is to consider how an endogenous labour supply in all periods would affect the results. However, the general area of labour supply affects is, of course, as potentially wide as the range of theories governing the labour market.

Another point is that the effects of international capital and labour mobility have been investigated separately and not simultaneously. This was hostage to the assumption of internal constant returns to scale, since joint factor movements with the requirement of non-negative profits, would entail the loss of firms - and - and with the symmetry assumption of firms within countries – the disappearance of whole countries. For this reason, further research should turn towards considering decreasing returns to scale assumption, where the numbers of firms can be determined and where foreign direct investment in the sense of the migration of firms would become an issue. The analysis to date suggests that the effects on growth through precautionary saving of movements in both capital and labour can go either way, because of uncertain effects on the anticipated price distributions.

A final point is that the quantified growth gains from factor market integration have probably been understated by our assumption that the global transfer of technology is independent of any accompanying movement in the factors of production, especially, labour. This is at odds with the findings in Keller (2004) who showed that technology diffusion and economic integration are essentially complements: ideas as blueprints may be transmitted electronically, but their full, practical implementation may also require that people move with them, so that the growth-enhancing effects of labour mobility, at least, have probably been understated.

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Appendices

Appendix A: Saving under uncertainty

A.1 Utility

Consider the following two-period expected utility function,

$$U = u(w_0 - s) + E(\beta u(w + Rs))$$

where $u(.)$ is the single period utility function, β is a subjective discount factor and E is the expectations operator. The term w_0 is predetermined earnings, s is the choice variable of saving, while w and R are realisations from the distributions for future earnings and for the interest factor over which expectations must be formed. The Euler equation is

$$F \equiv \frac{\partial U}{\partial s} = -u_1(w_0 - s) + \beta E(u_1(w + Rs)R) = 0 \qquad \text{(A1)}$$

A Taylor series expansion of $E(u_1(w + Rs)R)$ around the mean of w, $\overline{w}$, gives

$$u_1(w + Rs)R = u_1(\overline{w} + Rs)R + u_2(\overline{w} + Rs)R(w - \overline{w}) + \frac{u_3(\overline{w} + Rs)R(w - \overline{w})^2}{2}$$
$$+ \frac{u_4(\overline{w} + Rs)R(\overline{w} - w)^3}{6} + .. \qquad \text{(A2)}$$

where $u_1 > 0$, $u_2 < 0$. For CRRA and CARA $u_3 > 0$, $u_4 < 0$; for quadratic utility $u_3 = 0$, $u_4 = 0$.

Further Taylor series expansions around the mean of R, $\overline{R}$, for each of the right-hand terms in (A2) give

$$u_1(\overline{w} + Rs)R = \overline{u}_1\overline{R} + \left(\overline{u}_1 + \overline{u}_2 s\overline{R}\right)(R - \overline{R}) + \frac{\left(2\overline{u}_2 s + \overline{u}_3 s^2\overline{R}\right)(R - \overline{R})^2}{2}$$
$$+ \frac{\left(3\overline{u}_3 s^2 + \overline{u}_4 s^3\overline{R}\right)(R - \overline{R})^3}{6} + .. \qquad \text{(A3.1)}$$

$$u_2(\overline{w}+Rs)R(w-\overline{w}) = \begin{pmatrix} \overline{u}_2\overline{R} + \left(\overline{u}_2 + \overline{u}_3 s\overline{R}\right)(R-\overline{R}) + \dfrac{\left(2\overline{u}_3 s + \overline{u}_4 s^2\overline{R}\right)(R-\overline{R})^2}{2} \\ + \dfrac{3\overline{u}_4 s^2 + \overline{u}_5 s^3 (R-R)^3}{6} + .. \end{pmatrix}(w-\overline{w})$$

(A.3.2)

$$\frac{u_3(\overline{w}+Rs)R(w-\overline{w})^2}{2} = \begin{pmatrix} \overline{u}_3\overline{R} + \left(\overline{u}_3 + \overline{u}_4 s\overline{R}\right)(R-\overline{R}) + \dfrac{\left(2\overline{u}_4 s + \overline{u}_5 s^2\overline{R}\right)(R-\overline{R})^2}{2} \\ + \dfrac{3\overline{u}_5 s^2 + \overline{u}_6 s^3 (R-R)^3}{6} + .. \end{pmatrix}\frac{(w-\overline{w})^2}{2}$$

(A.3.3)

$$\frac{u_4(\overline{w}+Rs)R(w-\overline{w})^3}{6} = \begin{pmatrix} \overline{u}_4\overline{R} + \left(\overline{u}_4 + \overline{u}_5 s\overline{R}\right)(R-\overline{R}) + \dfrac{\left(2\overline{u}_5 s + \overline{u}_6 s^2\overline{R}\right)(R-\overline{R})^2}{2} \\ + \dfrac{3\overline{u}_6 s^2 + \overline{u}_7 s^3 (R-R)^3}{6} + .. \end{pmatrix}\frac{(w-\overline{w})^3}{6}$$

(A.3.4)

Combining the above terms and taking expectations gives,

$$\begin{aligned} E\left(u_1(w+Rs)R\right) &= \overline{u}_1\overline{R} + \frac{\left(2\overline{u}_2 s + \overline{u}_3 s^2\overline{R}\right)E(R-\overline{R})^2}{2} + \frac{\left(3\overline{u}_3 s^2 + \overline{u}_4 s^3\overline{R}\right)E(R-\overline{R})^3}{6} + .. \\ &+ \left(\left(\overline{u}_2 + \overline{u}_3 s\overline{R}\right)E\left((R-\overline{R})(w-\overline{w})\right) + \frac{\left(2\overline{u}_3 s + \overline{u}_4 s^2\overline{R}\right)E\left((R-\overline{R})^2(w-\overline{w})\right)}{2} + \frac{\overline{u}_5 s^3 E\left((R-R)^3(w-\overline{w})\right)}{6} + .. \right) \\ &+ \frac{1}{2}\begin{pmatrix} \overline{u}_3\overline{R}E\left((w-\overline{w})^2\right) + \left(\overline{u}_3 + \overline{u}_4 s\overline{R}\right)E\left((R-\overline{R})(w-\overline{w})^2\right) + \dfrac{\left(2\overline{u}_4 s + \overline{u}_5 s^2\overline{R}\right)E\left((R-\overline{R})^2(w-\overline{w})^2\right)}{2} \\ + \dfrac{3\overline{u}_5 s^2 E(w-\overline{w})^2 + \overline{u}_6 s^3 E\left((R-R)^3(w-\overline{w})^2\right)}{6} + .. \end{pmatrix} \\ &+ \frac{1}{6}\begin{pmatrix} \overline{u}_4\overline{R}E(w-\overline{w})^3 + \left(\overline{u}_4 + \overline{u}_5 s\overline{R}\right)E\left((R-\overline{R})(w-\overline{w})^3\right) + \dfrac{\left(2\overline{u}_5 s + \overline{u}_6 s^2\overline{R}\right)E\left((R-\overline{R})^2(w-w)^3\right)}{2} \\ + \dfrac{3\overline{u}_6 s^2 E(w-\overline{w})^3 + \overline{u}_7 s^3 E\left((R-R)^3(w-\overline{w})^3\right)}{6} + .. \end{pmatrix} \end{aligned}$$

where $\overline{u}_i \equiv u_i\left(\overline{w}+\overline{R}s\right), \quad i = 1,2,3,..$ (A.4)

$$E\big(u_1(w+Rs)R\big)=\bar{u}_1\bar{R}+\frac{\big(2\bar{u}_2 s+\bar{u}_3 s^2\bar{R}\big)E(R-\bar{R})^2}{2}+$$
$$+\Big(\big(\bar{u}_2+\bar{u}_3 s\bar{R}\big)E\big((R-\bar{R})(w-\bar{w})\big)+\Big)+\frac{1}{2}\Big(\bar{u}_3\bar{R}E\big((w-\bar{w})^2\big)\Big)$$

Overlooking the higher-order moments and cross-moments above the variance and the covariance gives

$$E\big(u_1(w+Rs)R\big)=\bar{u}_1\bar{R}+\bar{u}_2\Big(E(R-\bar{R})^2 s+E\big((R-\bar{R})(w-\bar{w})\big)\Big)$$
$$+\frac{\bar{u}_3}{2}\bar{R}\Big(E\big((R-\bar{R})^2\big)s^2+2E\big((R-\bar{R})(w-\bar{w})\big)s\bar{R}+E(w-\bar{w})^2\Big) \qquad \text{(A.5)}$$

The effect of risk-inversion ($\bar{u}_2<0$) is through interest uncertainty alone and any effect from the convexity of the marginal utility $\bar{u}_3(\geq 0)$ works through the uncertainty to consumption caused by both interest and wage uncertainty, since

$$\Big(E\big((R-\bar{R})^2\big)s^2+2E\big((R-\bar{R})(w-\bar{w})\big)s+E(w-\bar{w})^2\Big)=E(c-\bar{c})^2$$

Using Kimball's (1990) definition of *prudence*[17], defined as $\theta(c)\equiv -u_3/u_2$ as analogous to the Arrow-Pratt concept of *absolute risk-aversion*, $\phi(c)\equiv -u_2/u_1$, we can re-present

$$E\big(u_1(w+Rs)R\big)=\bar{u}_1\left[\bar{R}-\phi(c)\left[\begin{array}{l}\big(E(R-\bar{R})^2 s+E(R-\bar{R})(w-\bar{w})\big)-\\ \dfrac{\theta(c)\bar{R}}{2}\Big(E(R-\bar{R})s^2+2E(R-\bar{R})(w-\bar{w})s+E(w-\bar{w})^2\Big)\end{array}\right]\right]$$

(A.6)

A.2 The Euler equation

Consider the maximization of the following two-period expected utility function,

$$U=u\big(w_0-s\big)+E\big(\beta u(w+Rs)\big)$$

The first-order condition is

$$F\equiv\partial F/\partial s=-u_1\big(w_0-s\big)+\beta E\big(u_1(w+Rs)R\big)=0$$

A Taylor series expansions of w around $E(w)$ is taken; and then for the each term involving R, further Taylor series expansions around $E(R)$. Excluding all higher order terms beyond the variance and covariance gives

[17] Kimball also describes *prudence* as "the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to "risk aversion," which is how much one dislikes uncertainty and would turn away from uncertainty if possible."

$$F \equiv -u_1(w_0 - s) + \beta \overline{u}_1 \overline{R} + \overline{u}_2 \left(E(R-\overline{R})^2 s + E\left((R-\overline{R})(w-\overline{w})\right)\right)$$

$$+ \beta \frac{\overline{u}_3}{2} \overline{R} \left(E\left((R-\overline{R})^2\right) s^2 + 2E\left((R-\overline{R})(w-\overline{w})\right) s + E(w-\overline{w})^2 \right)$$

and using Arrow-Pratt's definition of *absolute risk-aversion*, $\phi(c) \equiv -u_2/u_1$, and Kimball's (1990) analogous concept of *prudence*[18], defined as $\theta(c) \equiv -u_3/u_2$, we obtain

$$F \equiv -u_1(w_0 - s) + \beta \overline{u}_1 \left[\overline{R} - \phi(c) \left[\left(\sigma_R{}^2 s + \sigma_{RW} \right) - \frac{\theta(c) E(R)}{2} \left(\sigma_R{}^2 s^2 + 2\sigma_{RW} + \sigma_W{}^2 \right) \right] \right]$$

where $\sigma_W{}^2 \equiv E(w-\overline{w})^2$, $\sigma_R{}^2 \equiv E(R-\overline{R})^2$, $\sigma_{RW} \equiv E(R-\overline{R})(w-\overline{w})$

$$E\left(u_1(w + Rs)R\right) = \overline{u}_1 \left[\overline{R} - \phi(c) \left[\left(\sigma_R{}^2 s + \sigma_{RW} \right) - \frac{\theta(c) E(R)}{2} \left(\sigma_R{}^2 s^2 + 2\sigma_{RW} s + \sigma_W{}^2 \right) \right] \right]$$

An interior solution requiting $\partial^2 U / \partial s^2 < 0$, implies that $\partial F / \partial s < 0$ according to the definition of F, so that an implication of the *implicit function theorem* is that $sign(\partial s/\partial x) = sign(\partial F/\partial x)$. We may consider various cases of the Euler equation in F.

A.3 Earnings uncertainty alone

$$F \equiv -u_1(w_0 - s) + \beta \overline{u}_1 \left[\overline{R} + \phi(c) \left[\frac{\theta(c) E(R)}{2} \sigma_w{}^2 \right] \right]$$

$$\frac{\partial s}{\partial \sigma_W{}^2} > 0, \quad if\ \theta(c) > 0$$

As known, it is necessary and sufficient that that $u_3 > 0$ or $\beta(c) > 0$ in order for earnings uncertainty to generate precautionary saving. Earnings uncertainty reduces the expected value of future consumption leading to an increase in saving as a compensating factor in obtaining a higher expected value.

A.4 Interest uncertainty alone

[18] Kimball also describes *prudence* as "the propensity to prepare and forearm oneself in the face of uncertainty, in contrast to "risk aversion," which is how much one dislikes uncertainty and would turn away from uncertainty if possible."

$$F \equiv -u_1(w_0 - s) + \beta \overline{u}_1 \left[\overline{R} - \phi(c) \left[1 - \frac{\theta(c)E(R)}{2} s \right] \sigma_R{}^2 s \right]$$

$$\frac{\partial s}{\partial \sigma_R{}^2} > 0, \quad \textit{if } \theta(c) > \frac{2}{E(R)s}$$

Again it is necessary for precautionary saving $u_3 > 0$ or $\theta(c) > 0$, but it is no longer sufficient because of interest risk. Although, as before, a rise in saving increases the expected value of future consumption as a compensating factor for income uncertainty, saving is also the source of income risk, so that there is a trade-off between expected value and variance leading to a generally ambiguous savings response.

This case is often considered with reference to a *constant rate of relative risk-aversion* (CRRA) utility function, $U = c^{1-\phi} / 1 - \phi$. The relationship between CRRA and prudence is

$\theta(c) = \dfrac{1+\phi}{\phi c} = \dfrac{1+\phi}{\phi\left(E(w) + E(R)s\right)}$, so that the above condition for this cases becomes

$$(\phi - 1)E(R)s > 2\phi E(w)$$

In the absence of a second-period income, $E(w) = 0$, precautionary saving only requires $\phi > 1$. Here saving is unaffected by interest uncertainty in the logarithmic case ($\phi = 1$), but reduced if $\phi < 1$ (in Marshall's). However, if there is a second period income, $E(w) > 0$, Marshall's case becomes more likely as, precautionary saving requires $\left(E(R)s - 2E(w)\right)\phi > E(R)s$, for which it is necessary but not sufficient that $E(R)s > 2E(w)$.

A.5 Quadratic utility

$$F \equiv -u_1(w_0 - s) + \beta \overline{u}_1 \left[\overline{R} - \phi(c) \left[\left(\sigma_R{}^2 s + \sigma_{RW} \right) \right] \right]$$

It is generally believed that quadratic utility is synonymous with the case of certainty equivalence. However, this is strictly correct only for the case of pure earnings uncertainty, where $\sigma_W{}^2$ drops out of the Euler equation. In general, interest uncertainty also affects saving, and this remains so for the case of a quadratic utility function. The effect will be even be positive, constituting precautionary saving, if there is a negative covariance between earnings and interest income of sufficient magnitude.[19] Generally,

$$sign(\partial s/\partial\sigma_{RW}) = -sign(\sigma_{RW} + \sigma_R{}^2 s) \text{ if } \sigma_{RW} + \sigma_R{}^2 s < 0,$$

Furthermore, if s is sufficiently small, $\partial s/\partial\sigma_{RW} > 0$ for a negative covariance of any magnitude.

A.6 The general effect of covariance

More generally, the covariance between earning and interest uncertainty is important for saving. There will be precautionary saving (dissaving) if

$$\frac{\partial s}{\partial\sigma_{RW}} > (<) 0 \text{ if } \theta(c) > (<) (E(R)s)^{-1}$$

Again for a CRRA, ϕ, where $\theta(c) = (1+\phi)/(E(w) + E(R)s)$

$sign(\partial F/\partial\sigma_{RW}) = sign(\phi E(R)s - E(w))$, so that a more positive covariance between interest income and earnings income raises precautionary saving only if the interest component of future income is large relative to the earnings component or, if the utility function is logarithmic, simply larger.

A.7 First period income uncertainty

Finally, there is also the hypothetical case that the saving decision is made prior to the realization of the first period income[20], in which case the Euler condition would become

$$F \equiv E(u_1(w_0 - s)) + \beta E(u_1(w + Rs)R) = 0$$

As $E(u_1) = \bar{u}_1 + \frac{\bar{u}_3}{2}\sigma^2_{w0}$, $\frac{\partial s}{\partial\sigma^2_{w0}} < (>) 0$ **if** $\bar{u}_3 > (<) 0$

We see that for the standard case where $\bar{u}_3 > 0$, the uncertainty of first-period income – however it is comprised – will reduce saving.

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19 This basic result has also recently been discovered by Menegatti (2009) in the slightly different context of (earnings) *income risk* with *background risk*.

20 There must be precommitment to the initial level of saving with no possibility of subsequent borrowing or additional saving in response to the realization of income. This suggests both a contractual rigidity and an absence of supplementary loan and saving markets.

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These basic considerations underlie the following analysis of a global model of household saving, capital accumulation and economic growth. Various international regimes have different implications for the factor income distributions, namely, the values of ${\sigma_W}^2$, ${\sigma_R}^2$ and σ_{RW}, which in turn impacts upon these three main variables.

Appendix B: Derivation of equation (8).

This is based on aggregating equation (7) across countries, using the approximations,

$$E\left({k_t^z}^\alpha \right) \approx \left(1 - \frac{\alpha(1-\alpha)}{2} \frac{\mathrm{var}\left(k_t^z\right)}{{k_t}^2} \right) k_t \quad \text{where } k_t = E\left(k_t^z\right) \tag{B1}$$

and

$$\mathrm{var}\left(\ln k_t^z\right) \approx \frac{\mathrm{var}(k_t^z)}{{k_t}^2} \tag{B2}$$

Equation (7) is in a logarithmic form may be expanded recursively to give

$$\ln k_t^z = \frac{\ln \Omega}{1-\alpha} + (1-\alpha) \sum_{i=0}^{\infty} \alpha^j \left(\ln(1 + \sigma_{t-1-j}^z) + \ln k_{t-1-j} \right), \tag{B3}$$

depending on an infinite past of stochastic shocks. Their i.i.d. property implies

$$\mathrm{var}\left(\ln k_t^z\right) = \left(\frac{(1-\alpha)^2}{1-\alpha^2} \right) \mathrm{var}\left(\ln\left(1+\sigma\right)\right) = \left(\frac{1-\alpha}{1+\alpha} \right) \mathrm{var}\left(\ln(1+\sigma)\right) \tag{B4}$$

so that

$$E\left({k_t^z}^\alpha \right) \approx \left(1 - \frac{\alpha(1-\alpha)^2}{2(1+\alpha)} \mathrm{var}\left(\ln(1+\sigma_t^z\right) \right) {k_t}^\alpha \tag{B5}$$

Appendix C: Details and proof for Result Two

(a) Equations (2) and (11) show that a movement from autarky to PCM causes the variance of the logarithm of the interest factor to fall from $(1-\alpha)^2 \sigma^2$ to zero.

(b) Likewise, equations (3) and (12) show the variance of the logarithm of the wage rises from $(1-\alpha)^2 \sigma^2$ to σ^2. (c) The mean wage under PCM is

$E\left(w_t^z \middle|_{PCM}\right) = (1-\alpha)(1+\rho)^{-\alpha} A^{1-\alpha} k_t$, which exceeds

$E\left(w_t^z\big|_{AUT}\right) = (1+\rho)^{-\alpha}(1-\alpha)(Ak_t)^{1-\alpha} E\left((1+\sigma_t^z)^{1-\alpha}\right) E(k_t^{z\,\alpha})$ as

$k_t^{\alpha} > E\left((1+\sigma_t^z)^{1-\alpha}\right) E\left(k_t^{z\,\alpha}\right)$ as $0<\alpha<1$.

(d) $E\left(R_t^z\big|_{AUT}\right) = \alpha(1+\rho)^{1-\alpha}(Ak_t)^{1-\alpha} E\left((1+\sigma_t^z)^{1-\alpha}\right) E\left(k_t^{z\,\alpha-1}\right)$ and

$R_t\big|_{PCM} = \alpha(1+\rho)^{1-\alpha} A^{1-\alpha}$. While $k_t^{1-\alpha} E\left(k_t^{z\,\alpha-1}\right) > 1$, $E\left((1+\sigma_t^z)^{1-\alpha}\right) < 1$.

However, $E\left(w_{t+1}^z / R_{t+1}^z\big|_{PCM}\right) = E\left(w_{t+1}^z / R_{t+1}^z\big|_{AUT}\right) = \left((1-\alpha)/\alpha(1+\rho)\right)k_t$

Appendix D: Details and proof for Result Three

(a) Comparing equations (3) and (15), the variance of the logarithm of the wage falls from approximately $(1-\alpha)^2\sigma^2$ to zero. (b) Equations (2) and (16) show that the variance of the logarithm of the interest factor rises from approximately $(1-\alpha)^2\sigma^2$ to $(1/\alpha-1)^2\sigma^2$.

(c) The common wage under PLM, given by

$w_t\big|_{PLM} = w_t^z(+) = w_t^z(-) = (1-\alpha)(1+\rho)^{-\alpha}\left(E\left((1+\sigma_t^z)^{1/\alpha-1}\right)\right)^{\alpha} A^{1-\alpha}k_t$,

which exceeds

$E\left(w_t^z\big|_{AUT}\right) = (1+\rho)^{-\alpha}(1-\alpha)(Ak_t)^{1-\alpha} E\left((1+\sigma_t^z)^{1-\alpha}\right) E(k_t^{z\,\alpha})$ as

$\left(E\left((1+\sigma_t^z)^{1/\alpha-1}\right)\right)^{\alpha} > E\left((1+\sigma_t^z)^{1-\alpha}\right)$ and $k_t^{\alpha} > E(k_t^{z\,\alpha})$

(d) The mean interest factor under PLM is

$E\left(R_t^z\big|_{PLM}\right) = \alpha(1+\rho)^{1-\alpha} E\left((1+\sigma_t^z)^{1/\alpha-1}\right)^{\alpha} A^{1-\alpha}$ in comparison with

$E\left(R_t^z\big|_{AUT}\right) = \alpha(1+\rho)^{1-\alpha}(Ak_t)^{1-\alpha} E\left((1+\sigma_t^z)^{1-\alpha}\right) E\left(k_t^{z\,\alpha-1}\right)$, and

while $E\left((1+\sigma_t^z)^{1/\alpha-1}\right)^{\alpha} > E\left((1+\sigma_t^z)^{1-\alpha}\right)$, also $k_t^{1-\alpha} E\left(k_t^{z\,\alpha-1}\right) > 1$

$E\left(w_{t+1}^z / R_{t+1}^z\big|_{PLM}\right) = \left((1-\alpha)/\alpha(1+\rho)\right) E\left((1+\sigma_t^z)^{1/\alpha-1}\right) E\left((1+\sigma_t^z)^{1-1/\alpha}\right) k_{t+1}$

As $E\left((1+\sigma_t^z)^{1/\alpha-1}\right) E\left((1+\sigma_t^z)^{1-1/\alpha}\right) > 1$

$E\left(w_{t+1}^z / R_{t+1}^z\big|_{PCM}\right) > E\left(w_{t+1}^z / R_{t+1}^z\big|_{AUT}\right)$

Appendix E: The proof for Proposition Two

Without any volatility in TFP where $\sigma = 0$,

$$G\Big|_{PLM,\sigma=0} = \frac{\beta\alpha(1-\alpha)(1+\rho)^{-\alpha} A^{1-\alpha}}{(1+\beta)\alpha + \dfrac{\rho}{1+\rho}(1-\alpha)} = \Omega .$$

with maximal volatility in TFP where $\sigma = 1$,

$$G\Big|_{PLM,\sigma=1} = \frac{\beta\alpha(1-\alpha)(1+\rho)^{-\alpha} 2^{1-\alpha} A^{1-\alpha}}{(2+\beta)\alpha + \dfrac{\rho}{1+\rho}(1-\alpha)}$$

Volatility in TFP raises (lowers) growth under PLM, $\partial G\big|_{PLM} / \partial\sigma > 0 (< 0)$, if

$$(2^{1-\alpha} - 1)\left((1+\beta)\alpha + \frac{\rho}{1+\rho}(1-\alpha) \right) > (<)\alpha$$

As equation (8) gives $G\Big|_{AUT,\sigma=0} = \Omega$ and $\partial G\big|_{AUT} / \partial\sigma < 0$,

$(2^{1-\alpha} - 1)\left((1+\beta)\alpha + \dfrac{\rho}{1+\rho}(1-\alpha) \right) > \alpha$ implies $G\big|_{PLM} > G\big|_{AUT}$ where $\sigma > 0$.

Appendix F: The effect of perfect capital mobility on the labour supply distribution

Equation (24) gives labour supply as

$$\rho_{t+1}^{z} = (1+\eta)^{-1}\left(1 - \eta \frac{\alpha}{1-\alpha} \frac{\left((1 + E(\rho_{t+1}^{x}) + \text{cov}(\rho_{t+1}^{x}, \sigma_{t+1}^{x})\right)}{1+\sigma_{t+1}^{z}} \frac{s_t^z}{s_t} \right)$$

As s_t^z is uncorrelated with σ_{t+1}^{z} and ρ_{t+1}^{z} and the shock is binomial, we can calculate

$$\text{cov}(\rho_{t+1}^{x}, \sigma_{t+1}^{x}) = \frac{\eta}{1+\eta} \frac{\alpha}{1-\alpha} \left(1 + E(\rho_{t+1}^{x}) + \text{cov}(\rho_{t+1}, \sigma_{t+1})\right) \frac{\sigma^2}{1-\sigma^2} \quad \text{or}$$

$$\text{cov}(\rho_{t+1}^{x}, \sigma_{t+1}^{x}) = \frac{\eta\alpha\left(1 + E(\rho_{t+1}^{x})\right)\sigma^2}{(1+\eta)(1-\alpha)(1-\sigma^2) - \eta\alpha\sigma^2} \tag{F1}$$

And so

$$1+E(\rho_{t+1}^{x})+\operatorname{cov}(\rho_{t+1}^{x},\sigma_{t+1}^{x})=\frac{(1+\eta)(1-\alpha)\left(1+E(\rho_{t+1}^{x})\right)(1-\sigma^{2})}{(1+\eta)(1-\alpha)(1-\sigma^{2})-\eta\alpha\sigma^{2}} \qquad \text{(F2)}$$

The mean value of (24) is

$$E(\rho_{t+1}^{x})=(1+\eta)^{-1}\left(1-\eta\frac{\alpha}{1-\alpha}\frac{\left((1+E(\rho_{t+1}^{x})+\operatorname{cov}(\rho_{t+1}^{x},\sigma_{t+1}^{x})\right)}{1-\sigma^{2}}\right) \qquad \text{(F3)}$$

Equations (F2) and (F3)

$$E(\rho_{t+1}^{x})=(1+\eta)^{-1}\left(1-\frac{\eta\alpha(2+\eta)}{(1+\eta-\alpha)(1-\sigma^{2})}\right) \qquad (27)$$

which into (F1) gives

$$\operatorname{cov}(\rho_{t+1}^{x},\sigma_{t+1}^{x})=\left(\frac{2+\eta}{1+\eta}\right)\left(\frac{\alpha\eta}{1+\eta-\alpha}\right)\frac{\sigma^{2}}{1-\sigma^{2}}$$

and into (F2)

$$1+E(\rho_{t+1}^{x})+\operatorname{cov}(\rho_{t+1}^{x},\sigma_{t+1}^{x})=\frac{(1-\alpha)\left(2+\eta\right))}{(1+\eta-\alpha)} \qquad \text{(F4)}$$

This with equation (24) for a country for which, $s_t^z = s_t$, gives

$$\rho_{t+1}^{z}=(1+\eta)^{-1}\left(1-\frac{\eta\alpha(2+\eta)}{1+\eta-\alpha}\left(\frac{1}{1+\sigma_{t+1}^{z}}\right)\right)$$

Finally, earnings is given by

$$\frac{\rho_{t+1}w_{t+1}^{z}\Big|_{PCM,EN-L}}{k_{t+1}}=\frac{\left((1-\alpha)(1+\sigma_{t+1}^{z})(1+\rho)^{-\alpha}-\eta\alpha\frac{s_t^{z}}{s_t}(1+\rho)^{1-\alpha}\right)A^{1-\alpha}}{1+\eta} \qquad \text{(F5)}$$

whereas before with exogenous laboour

$$\frac{\rho w_{t}^{z}\Big|_{PCM,EX-L}}{k_{t+1}}=\rho(1-\alpha)(1+\sigma_{t}^{z})(1+\rho)^{-\alpha}A^{1-\alpha} \qquad \text{(F6)}$$

The variance of earnings increases by the factor $\left(1-\alpha/(1+\eta)\right)/\left(1-\alpha(1+\eta)\right)^{2}>1$ as $\eta>0$.

Appendix G: The effect of perfect labour mobility on the labour supply distribution

The total labour supply in country z is $L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z}$ where $L_{t-1}^{Y,z}$ is predetermined and so for a given $L_t^{Y,Z}$

$$Y_t^z = \left(\left(L_t^{Y,z} + (1+\rho_t^z)L_{t-1}^{Y,z}\right)(1+\sigma_t^z)Ak_t\right)^{1-\alpha} K_t^{z\,\alpha} \tag{G1}$$

$$w_t = (1-\alpha)\left(L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z}\right)^{-\alpha}\left((1+\sigma_t^z)Ak_t\right)^{1-\alpha} K_t^{z\,\alpha} \tag{G2}$$

$$L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z} = \frac{(1-\alpha)^{1/\alpha}\left((1+\sigma_t^z)Ak_t\right)^{1/\alpha-1} K_t^z}{w_t^{1/\alpha}} \tag{G3}$$

As both ρ_t^z and $L_{t-1}^{Y,z}$ are uncorrelated and σ_t^z and K_t^z are uncorrelated,

$$L + E(\rho_t^z)L = \frac{(1-\alpha)^{1/\alpha} E\left((1+\sigma_t^z)^{1/\alpha-1}\right)(Ak_t)^{1/\alpha-1} K_t}{w_t^{1/\alpha}},$$

$$L = E(L_t^{Y,z}), E(L_{t-1}^{Y,z}) \tag{G4}$$

This gives

$$w_t = \frac{(1-\alpha)\left(E\left((1+\sigma_t^z)^{1/\alpha-1}\right)\right)^{\alpha} A^{1-\alpha} k_t}{\left(1+E(\rho_t^z)\right)^{\alpha}} \tag{G5}$$

And

$$\frac{L_t^{Y,z} + \rho_t^z L_{t-1}^{Y,z}}{\left(1+E(\rho_t^z)\right)L} = \frac{(1+\sigma_t^z)^{1/\alpha-1}}{E\left((1+\sigma_t^z)^{1/\alpha-1}\right)} \frac{K_t^z}{K_t} \tag{G6}$$

$$R_t^z = \frac{\alpha\left(1+E(\rho_t^z)\right)^{1-\alpha} A^{1-\alpha}(1+\sigma_t^z)^{1/\alpha-1}}{\left(E\left((1+\sigma_t^z)^{1/\alpha-1}\right)\right)^{1-\alpha}} \tag{G7}$$

$$\frac{R_t^z}{w_t} = \frac{\alpha}{1-\alpha} \frac{\left(1+E(\rho_t^z)\right)(1+\sigma_t^z)^{1/\alpha-1}}{E\left((1+\sigma_t^z)^{1/\alpha-1}\right)k_t} \tag{G8}$$

$$\rho_{t+1}^z = (1+\eta)^{-1}\left(1-\eta\frac{\alpha}{1-\alpha}\frac{\left(1+E(\rho_{t+1}^z)\right)(1+\sigma_{t+1}^z)^{1/\alpha-1}s_t^z}{E\left((1+\sigma_{t+1}^z)^{1/\alpha-1}\right)k_{t+1}}\right) \tag{G9}$$

As σ_{t+1}^z and s_t^z are uncorrelated

$$E(\rho_{t+1}^{z}) = (1+\eta)^{-1}\left(1-\eta\frac{\alpha}{1-\alpha}\left(1+E(\rho_{t+1}^{z})\right)\right) = \frac{1-\alpha(1+\eta)}{1+\eta-\alpha} \qquad \text{(G10)}$$