Bootstrap-based Bias Corrected Within Estimation of Threshold Regression Models in Dynamic Panels*

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Abstract

Recently, Shin (2007) proposes new estimation procedure to analyse asymmetric threshold effects in a threshold autoregressive model in dynamic panels with unobserved individual effects when the number of time periods is fixed by combining time series techniques on nonlinear threshold modelling with the existing FD-GMM estimation techniques. This paper follows the bias corrected within estimator approach introduced by Everaert and Pozzi (2006) in linear dynamic panels and advances a bias corrected estimation procedure for the threshold dynamic panel data model based on an iterative bootstrap. Monte Carlo simulation exercises confirm the validity of our proposed approach. In an application to the dynamic threshold version of Tobin's Q investment function using the company panel data set examined by many researches and using different variables resepctively as a transition variable, we are able to find strong evidence in favor of nonlinear dynamic threshold effects.

JEL Classification: C12, C13, C15, C33.
Key Words: Threshold Regression Models in Dynamic Panels, Bootstrap-based Bias Corrected Within Estimator, Testing for Threshold Effects, Monte Carlo Simulations

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1 Introduction

In recent time series literature there have been many studies that examine the implications of the existence of a particular kind of nonlinear asymmetric dynamics. Examples are Markov-Switching, Smooth Transition and Threshold Autoregression Models. The popularity of these models is that they allow us to draw inferences about the underlying data generating process or to yield reliable forecasts in a manner that is not possible using linear models.

Increasing availability of large panel data sets, in conjunction with various developments in time series analysis, has prompted more rigorous econometric analyses of dynamic heterogeneous panels. One area of importance is the literature on the panel-based unit root and cointegration tests with and without cross-section dependence, see the survey papers by Choi (2004) and Breitung and Pesaran (2005). Most of these approaches require either T (the number of time periods) or both N (the number of cross section individuals) and T being large.

Until recently most econometric analysis has stopped short of studying the issues of nonlinear asymmetric dynamic mechanisms explicitly within a panel data context. Hansen (1999) develops the panel threshold regression model where regression coefficients can take on a small number of different values, depending on the value of other exogenous stationary variable. González, Teräsvirta and van Dijk (2005) generalise this approach and develop a panel smooth transition regression model which allows the coefficients to change gradually from one regime to another. See also Fok, van Dijk and Franses (2005). In a broad context these models are a specific example of the panel data approach that allows coefficients to vary randomly over time and across cross-sectional units as surveyed by Hsiao (2003, Chapter 6). Both approaches are static in nature, though they can be applied to the conventional panel data with large N and fixed T.

In general, there have been a rather small number of studies to adopt these time-series technique into the dynamic panel data model with large N and fixed T, though there is a huge literature on GMM estimation of linear dynamic panels with heterogeneous individual effects, e.g., Arellano and Bond (1991), Ahn and Schmidt (1995), Arellano and Bover (1995), Blundell and Bond (1998), Blundell, Bond and Windmeijer (2000), Alvarez and Arellano (2003) and Hayakawa (2006).

Further, there is no rigorous single study investigating the important issue of nonlinear asymmetric dynamic mechanism in this context. Recently, Shin (2007) proposes new estimation procedure to analyse asymmetric threshold effects in a threshold autoregressive model in dynamic panels with unobserved individual effects when the number of time periods is fixed by combining time series techniques on nonlinear threshold modelling with the existing FD-GMM estimation techniques.

This paper aims to provides an alternative estimation procedure which is obtained as the bias corrected within estimator. Everaert and Pozzi (2006) introduce the bootstrap-based bias correcting algorithm in linear dynamic panels and demonstrate that it can reduce the bias of the within estimator substantially whist it maintains its higher efficiency relative to GMM estimators. Here we follow Everaert and Pozzi (2006) and develop a bias corrected estimation procedure for the threshold dynamic panel data model procedure based on an iterative bootstrap procedure.

Monte Carlo simulation exercises confirm the validity of our proposed approach. In an application to the dynamic threshold version of Tobin's Q investment function using the company panel data set examined by many researches and using different variables respectively as a transition variable, we are able to find strong evidence in favor of nonlinear dynamic threshold effects. However, the evidence on the cash flow sensitivity of investment is somewhat mixed. When leverage is used as the transition variable, the results do not support the hypothesis that cash flows available should be more relevant for financially constrained firms. With Tobin's Q and cash flows adopted as the transition variable, the evidence is somewhat supportive of the investment-cash flow sensitivity although our discussion shows that these two variables may not be a good indicator of financial constraints.

The plan of the paper is as follows: Section 2 discusses the models and the assumptions. Sections 3 and 4 develops the bootstrap-based bias corrected within estimation methodologies and the associated asymptotic theory. Section 5 discusses consistent estimation of threshold parameter, the bootstrap-based inference for the presence of threshold effects and further modelling issues. Section 6 evaluates the small sample performance of the proposed estimators by stochastic simulations. Section 7 presents an empirical application. Section 8 concludes. Mathematical proofs are collected in an Appendix.

2 Model and Assumptions

Consider the following dynamic panel threshold regression model:

$$y_{it} = (\phi_1 y_{it-1} + \boldsymbol{\beta}_1' \mathbf{x}_{it}) 1 (q_{it} \leq \gamma) + (\phi_2 y_{it-1} + \boldsymbol{\beta}_2' \mathbf{x}_{it}) 1 (q_{it} > \gamma) + \varepsilon_{it}, \tag{2.1}$$

for $i = 1, ..., N;\ t = 1, ..., T$, where y_{it} is a a scalar stochastic dependent variable, $\mathbf{x}_{it}$ is a $k \times 1$ vector of weakly exogenous variables, $1(\cdot)$ is an indicator function, q_{it} is the transition variable with γ being a threshold parameter, ϕ_i and $\boldsymbol{\beta}_i$ for $i = 1, 2$ are the corresponding heterogeneous parameters associated with two different regimes, and ε_{it} consists of the error components,[1]

$$\varepsilon_{it} = \alpha_i + v_{it},$$

where α_i is an unobserved individual effect and v_{it} is a zero mean idiosyncratic random disturbance. This is a panel extension of the dynamic threshold regression model in time series.

We make the following assumptions:

Assumption 1. $\{v_{it}\}$ are iid and independent of η_{it} and y_{i1} with $E(v_{it}) = 0$, $Var(v_{it}) = \sigma^2$ and have the finite 4th moment.

Assumption 2. α_i are iid with $E(\alpha_i) = 0$, $Var(\alpha_i) = \sigma_\alpha^2$ and have the finite 4th moment.

Assumption 3. y_{it} is geometrically ergodic and the initial observations satisfy the mean stationarity condition. Stability condition, $|\phi_i| < 1$ for $i = 1, 2$ or global stability condition, $|\phi_1 + \phi_2| < 2$??

Assumption 4. Exogenous variables, x_{it} are either I(0) or I(1), correlated with α_i but not correlated with v_{it}.The threshold variable, q_{it} is stationary and exogenous or predetermined uncorrelated with α_i and v_{it}.

Assumption 5. N is large and T is fixed.

All these assumptions are fairly standard in the literature, e.g. Alvarez and Arellano (2003) and Hansen (1999).

3 Bootstrap-based Bias Corrected Within Estimator

To simplify the notations we write (2.1) as

$$y_{it} = \boldsymbol{\delta}_1' \mathbf{z}_{1,it}(\gamma) + \boldsymbol{\delta}_2' \mathbf{z}_{2,it}(\gamma) + \varepsilon_{it} = \boldsymbol{\delta}' \mathbf{z}_{it}(\gamma) + \varepsilon_{it}, \tag{3.1}$$

where

$$\mathbf{z}_{1,it}(\gamma) = \begin{bmatrix} y_{i,t-1} \\ \mathbf{x}_{it} \end{bmatrix} \times 1(q_{it} \leq \gamma);\ \mathbf{z}_{2,it}(\gamma) = \begin{bmatrix} y_{i,t-1} \\ \mathbf{x}_{it} \end{bmatrix} \times 1(q_{it} > \gamma);$$

[1] The extension to the panels with the time-specific dummy effects is straightforward.

$$\mathbf{z}_{it}(\gamma)=\begin{bmatrix}\mathbf{z}_{1,it}(\gamma)\\ \mathbf{z}_{2,it}(\gamma)\end{bmatrix};\ \boldsymbol{\delta}_1=\begin{bmatrix}\phi_1\\ \boldsymbol{\beta}_1\end{bmatrix};\ \boldsymbol{\delta}_2=\begin{bmatrix}\phi_2\\ \boldsymbol{\beta}_2\end{bmatrix};\ \boldsymbol{\delta}=\begin{bmatrix}\boldsymbol{\delta}_1\\ \boldsymbol{\delta}_2\end{bmatrix}.$$

Taking the within transformation of (3.1), we obtain

$$\tilde{y}_{it}=\boldsymbol{\delta}'\tilde{\mathbf{z}}_{it}(\gamma)+\tilde{v}_{it}, \tag{3.2}$$

where

$$\tilde{y}_{it}=y_{it}-\bar{y}_i;\ \tilde{\mathbf{z}}_{it}(\gamma)=\mathbf{z}_{it}(\gamma)-\bar{\mathbf{z}}_i(\gamma);\ \tilde{v}_{it}=v_{it}-\bar{v}_i;$$
$$\bar{y}_i=T^{-1}\sum_{i=1}^{T}y_{it};\ \bar{\mathbf{z}}_i(\gamma)=T^{-1}\sum_{i=1}^{T}\begin{pmatrix}\mathbf{z}_{1,it}(\gamma)\\ \mathbf{z}_{2,it}(\gamma)\end{pmatrix}.$$

Next, we write (3.2) in the matrix form:

$$\tilde{\mathbf{y}}=\tilde{\mathbf{Z}}(\gamma)\boldsymbol{\delta}+\tilde{\mathbf{v}}, \tag{3.3}$$

where

$$\tilde{\mathbf{y}}=\begin{bmatrix}\tilde{\mathbf{y}}_1\\ \vdots\\ \tilde{\mathbf{y}}_N\end{bmatrix}_{NT\times 1},\ \tilde{\mathbf{Z}}(\gamma)=\begin{bmatrix}\tilde{\mathbf{z}}_1(\gamma)\\ \vdots\\ \tilde{\mathbf{z}}_N(\gamma)\end{bmatrix}_{NT\times 2(k+1)},\ \tilde{\mathbf{v}}=\begin{bmatrix}\tilde{\mathbf{v}}_1\\ \vdots\\ \tilde{\mathbf{v}}_N\end{bmatrix}_{NT\times 1},$$
$$\tilde{\mathbf{y}}_i=\begin{bmatrix}\tilde{y}_{i1}\\ \vdots\\ \tilde{y}_{iT}\end{bmatrix}_{T\times 1},\ \tilde{\mathbf{z}}_i(\gamma)=\begin{bmatrix}\tilde{\mathbf{z}}'_{i1}(\gamma)\\ \vdots\\ \tilde{\mathbf{z}}'_{iT}(\gamma)\end{bmatrix}_{T\times 2(k+1)},\ \tilde{\mathbf{v}}_i=\begin{bmatrix}\tilde{v}_{i1}\\ \vdots\\ \tilde{v}_{iT}\end{bmatrix}_{T\times 1}.$$

For given γ and for large N and large T, $\boldsymbol{\delta}$ can be consistently estimated by the following within estimator:

$$\hat{\boldsymbol{\delta}}(\gamma)=\left(\tilde{\mathbf{Z}}(\gamma)'\tilde{\mathbf{Z}}(\gamma)\right)^{-1}\tilde{\mathbf{Z}}(\gamma)'\tilde{\mathbf{y}}. \tag{3.4}$$

Suppose that the consistent estimator of the threshold parameter is available and denoted, $\hat{\gamma}$. Then inference on $\boldsymbol{\delta}$ can proceed if $\hat{\gamma}$ were true value. Hence,

$$\hat{\boldsymbol{\delta}}(\gamma)\overset{a}{\sim}N(\boldsymbol{\delta},\boldsymbol{\Omega}),$$

where the consistent estimate of $\boldsymbol{\Omega}$ can be obtained either by

$$\hat{\boldsymbol{\Omega}}=\hat{\sigma}^2\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{\mathbf{z}}_{it}(\hat{\gamma})\tilde{\mathbf{z}}_{it}(\hat{\gamma})'\right)^{-1},$$

if the errors are assumed to be iid or by

$$\hat{\mathbf{V}}=\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{\mathbf{z}}_{it}(\hat{\gamma})\tilde{\mathbf{z}}_{it}(\hat{\gamma})'\right)^{-1}\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{\mathbf{z}}_{it}(\hat{\gamma})\tilde{\mathbf{z}}_{it}(\hat{\gamma})'\widehat{\tilde{v}}^2_{it}(\hat{\gamma})\right)\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\tilde{\mathbf{z}}_{it}(\hat{\gamma})\tilde{\mathbf{z}}_{it}(\hat{\gamma})'\right)^{-1},$$

where $\widehat{\tilde{v}}^2_{it}(\hat{\gamma})=\tilde{y}_{it}-\hat{\boldsymbol{\delta}}'(\hat{\gamma})\tilde{\mathbf{z}}_{it}(\gamma)$, if the errors are allowed to be conditional heteroskedastic.

However, it is well-established in the linear dynamic panels with fixed T that the fixed effects estimator of the autoregressive parameter is biased downward, e.g. Nickell (1981). To deal with the correlation of the regressors with individual effects in (2.1), Shin (2007) adopt the GMM approach by Arellano and Bond (1991) and Blundell and Bond (1998) and propose various GMM estimation methodologies, namely the FD-GMM, the Level-GMM and the System-GMM estimators.

Here we follow Everaert and Pozzi (2006) and develop a bias corrected estimation procedure based on an iterative bootstrap procedure. Various simulation studies on linear dynamic panel data models show that the GMM-IV estimators have a relatively large standard errors and are subject to substantial small sample biases when T is relatively large to N such that too many moment conditions are available, see Ziliak (1997). The bootstrap-based bias correcting algorithm proposed by Everaert and Pozzi (2006) aims to reduce the bias of the FE estimator while it maintains its higher efficiency relative to GMM estimators. In particular, one of main advantages of the bootstrapping algorithms is that this can be applied in a straightforward manner to the complex model where its analytic corrections are not easily available.

For convenience we define $\hat{\boldsymbol{\delta}}(\gamma) = \hat{\boldsymbol{\delta}}$ without loss of generality. When T is fixed, it is easily seen that

$$E\left(\hat{\boldsymbol{\delta}}\right) \neq \boldsymbol{\delta}.$$

Suppose that using the repeated sampling experiment we are able to generate a sequence of J biased estimates, $\hat{\boldsymbol{\delta}}_1^*(\boldsymbol{\delta}), ..., \hat{\boldsymbol{\delta}}_J^*(\boldsymbol{\delta})$. Then, it follows that

$$E\left(\hat{\boldsymbol{\delta}}\right) = \lim_{J\to\infty} \frac{1}{J}\sum_{j=1}^{J} \hat{\boldsymbol{\delta}}_j^*(\boldsymbol{\delta}).$$

It is then clear that $\bar{\boldsymbol{\delta}}$ will be an unbiased estimator of $\boldsymbol{\delta}$ if the following condition holds:

$$\hat{\boldsymbol{\delta}} = \lim_{J\to\infty} \frac{1}{J}\sum_{j=1}^{J} \hat{\boldsymbol{\delta}}_j^*\left(\bar{\boldsymbol{\delta}}\right). \tag{3.5}$$

In other words, if we would sample repeatedly from a population with parameters $\bar{\boldsymbol{\delta}}$ and calculate the estimate $\hat{\boldsymbol{\delta}}_j^*\left(\bar{\boldsymbol{\delta}}\right)$ in each sample, $\bar{\boldsymbol{\delta}}$ is an unbiased estimator of $\boldsymbol{\delta}$ if the average of $\hat{\boldsymbol{\delta}}_j^*\left(\bar{\boldsymbol{\delta}}\right)$, $j = 1, ..., J$ corresponds to the FE estimate, $\hat{\boldsymbol{\delta}}$ based on the original data. See also Tanazaki (2004).

A bias corrected estimate of $\boldsymbol{\delta}$ can be obtained by searching over the parameter space until $\bar{\boldsymbol{\delta}}$ is found that satisfies (3.5). This search is implemented using the following iterative bootstrap algorithm: The core of this algorithm consists of a bootstrap procedure which simulates the distribution of the FE estimator when sampling from (3.1) with the initial values of a parameter vector, denoted $\tilde{\boldsymbol{\delta}}_{(0)}$.

1. Estimate the individual effects by

$$\tilde{\boldsymbol{\alpha}} = (T-1)^{-1}\mathbf{D}'\left(\mathbf{y} - \mathbf{Z}\tilde{\boldsymbol{\delta}}_{(0)}\right),$$

where $\tilde{\boldsymbol{\alpha}} = (\tilde{\alpha}_1, ..., \tilde{\alpha}_N)'$, $\mathbf{D} = \mathbf{I}_N \otimes \mathbf{e}_{T-1}$, $\mathbf{I}_N$ is an $N \times N$ identity matrix and $\mathbf{e}_{T-1} = (1, ..., 1)'$ is the $(T-1) \times 1$ vector of ones. Then estimate the residual vector by

$$\tilde{\mathbf{v}} = \tilde{\mathbf{y}} - \tilde{\mathbf{Z}}\tilde{\boldsymbol{\delta}}_{(0)},$$

or rescale them as (see MacKinnon, 2002)

$$\tilde{v}_{it}^* = \sqrt{\frac{T-1}{T-2}}\left(\frac{\tilde{v}_{it}}{\sqrt{m_{it}}} - \frac{1}{T-1}\sum_{s=2}^{T}\frac{\tilde{v}_{is}}{\sqrt{m_{is}}}\right),$$

where m_{it} is the ith diagonal element of the idempotent matrix, $\mathbf{M} = \mathbf{I}_{N(T-1)} - \tilde{\mathbf{Z}}'\left(\tilde{\mathbf{Z}}'\tilde{\mathbf{Z}}\right)^{-1}\tilde{\mathbf{Z}}'$.

2. We generate the bth bootstrap samples for $b = 1, ..., B$ as follows:

   (a) Draw the bth bootstrap sample residual vector, denoted $\tilde{\mathbf{v}}^{(b)}$, from either $\tilde{\mathbf{v}}$ or $\tilde{\mathbf{v}}^*$ and generate a bootstrap sample by

   $$\mathbf{y}^{(b)} = \mathbf{Z}^{(b)}\tilde{\boldsymbol{\delta}}_{(0)} + \mathbf{D}\tilde{\boldsymbol{\alpha}} + \tilde{\mathbf{v}}^{(b)},$$

   where $\mathbf{z}_{it}^{(b)}(\gamma) = \left(z_{1,it}^{(b)}(\gamma), z_{2,it}^{(b)}(\gamma)\right) = \left\{y_{i,t-1}^{(b)} 1(q_{it} \leq \gamma), y_{i,t-1}^{(b)} 1(q_{it} > \gamma)\right\}$ and we condition on the initial value, y_{i1} such that $y_{i1}^{(b)} = y_{i1}$.

   (b) Compute the FE estimator by (3.4), namely

   $$\tilde{\boldsymbol{\delta}}^{(b)}\left(\tilde{\boldsymbol{\delta}}_{(0)}\right) = \left(\tilde{\mathbf{Z}}^{(b)\prime}\tilde{\mathbf{Z}}^{(b)}\right)^{-1}\tilde{\mathbf{Z}}^{(b)\prime}\tilde{\mathbf{y}}^{(b)}. \quad (3.6)$$

3. Repeat the step 2 B times and calculate the empirical mean by

   $$\bar{\boldsymbol{\delta}}_{(0)} = B^{-1}\sum_{b=1}^{B}\tilde{\boldsymbol{\delta}}^{(b)}\left(\tilde{\boldsymbol{\delta}}_{(0)}\right).$$

   Define the difference between $\hat{\boldsymbol{\delta}}$ and $\bar{\boldsymbol{\delta}}_{(0)}$ by

   $$\mathbf{b}_{(0)} = \hat{\boldsymbol{\delta}} - \bar{\boldsymbol{\delta}}_{(0)}.$$

   (a) If $\mathbf{b}_{(0)} = \mathbf{0}$, $\tilde{\boldsymbol{\delta}}_{(0)}$ will be an unbiased estimator of $\boldsymbol{\phi}$ by the condition, (3.5).

   (b) Otherwise update

   $$\tilde{\boldsymbol{\delta}}_{(k+1)} = \tilde{\boldsymbol{\delta}}_{(k)} + \mathbf{b}_{(k)}, \;\; k = 0, 1, 2, ...$$

   and iterate the bootstrap procedures outlined in 1-3 until the condition, (3.5) is satisfied.

Following Everaert and Pozzi (2006), we will set the number of bootstrap samples B equal to 1000 and use the convergence criterion $\left|\mathbf{b}_{(k)}\right| < 0.005$ and set the upper bound on the number of iterations equal to 50. We also use the FE estimator as $\tilde{\boldsymbol{\delta}}_{(0)}$.

We now discuss how to generate the residual vector, $\tilde{\mathbf{v}}^{(b)}$ in details. Resampling $\tilde{\mathbf{v}}^{(b)}$ in a nonparametric way has the advantage that it does not require an explicit distributional assumption for $\mathbf{v}$. As we may also allow for temporal dependence in $\mathbf{v}$ we consider two alternative resampling schemes. First, when v_{it} is assumed to be iid across i and over t, we resample from

$$\tilde{\mathbf{v}}_i^{(b)} = \left(\tilde{v}_{i_1,t_2}^*, ..., \tilde{v}_{i_N,t_T}^*\right); \; i = 1, ..., N,$$

where the vectors of indices $(i_1, ..., i_N)$ and $(t_2, ..., t_T)$ are obtained by drawing with replacement randomly from $(1, ..., N)'$ and $(2, ..., T)'$, respectively. Second, if ε_{it} exhibits temporal dependence, e.g. conditional heteroskedasticity, we will use the wild bootstrap (Goncalves and Kilian, 2004) and resample from

$$\tilde{\mathbf{v}}_i^{(b)} = \left(\tau_{i2}\tilde{v}_{j2}^*, ..., \tau_{iT}\tilde{v}_{jT}^*\right); \; i = 1, ..., N,$$

where the index j is drawn with replacement from $(1, ..., N)'$ and τ_{it} is a binomial random variable with mean 0 and variance 1 that takes on value -1 and 1 respectively with probability 1/2. The advantage is that it is asymptotically valid for either T, N or both grow large. We then collect all the resampled residual vectors in

$$\tilde{\mathbf{v}}^{(b)} = \left(\tilde{\mathbf{v}}_1^{(b)\prime}, ..., \tilde{\mathbf{v}}_N^{(b)\prime}\right).$$

4 Estimation of and Testing for Threshold Effects

We have developed the optimal estimation procedure for the threshold autoregressive model in dynamic panels under the implicit assumption that the value of the threshold parameter, γ is given. This section will address consistent estimation of γ and develop the bootstrap-based testing procedure for the null of no threshold effects in dynamic panels.

We follow Chan (1993) and Hansen (1999) and obtain the consistent estimator of γ in by

$$\hat{\gamma} = \arg\min_{\gamma} Q_1(\gamma), \tag{4.7}$$

where $Q_1(\gamma)$ is the generalised minimum distance measure given by

$$Q_1(\gamma) = \hat{\mathbf{v}}(\gamma)' \hat{\mathbf{v}}(\gamma), \tag{4.8}$$

where $\hat{\mathbf{v}}(\gamma) = \tilde{\mathbf{y}} - \tilde{\mathbf{Z}}(\gamma) \hat{\boldsymbol{\delta}}^B(\gamma)$ and $\hat{\boldsymbol{\delta}}^B(\gamma)$ is the bootstrap-bias corrected estimator given γ. Once $\hat{\gamma}$ is obtained, we obtain

$$\hat{\boldsymbol{\delta}}^B = \hat{\boldsymbol{\delta}}^B(\hat{\gamma})\,;\ \hat{\mathbf{v}} = \hat{\mathbf{v}}(\hat{\gamma})\,;\ \hat{\sigma}^2 = \frac{1}{N(T-2)} Q_1(\hat{\gamma}). \tag{4.9}$$

Since $Q_1(\gamma)$ depends only on γ through the indicator function, this is a step function with most $N(T-2)$ steps with the steps occurring at distinct values of the observed threshold variable q_{it}. Thus the minimisation problem can be reduced to searching over the values of γ equalling the distinct values of q_{it} in the sample. In practice we need to truncate the smallest and largest 10% for example. The remaining values constitute the values of γ which can be searched for $\hat{\gamma}$. For each of these values regression are estimated yielding the SSE and the smallest value yields the estimate $\hat{\gamma}$ and $\hat{\boldsymbol{\delta}}^B = \hat{\boldsymbol{\delta}}^B(\hat{\gamma})$.

We follow Hansen (1996) and develop a bootstrap procedure to simulate the asymptotic distribution of the LR test statistic for the null hypothesis of no threshold:

$$H_0 : \boldsymbol{\delta}_1 = \boldsymbol{\delta}_2.$$

Under the null of no threshold, the model (2.1) reduces to

$$y_{it} = \phi_1 y_{it-1} + \boldsymbol{\beta}_1' \mathbf{x}_{it} + \varepsilon_{it} = \boldsymbol{\delta}_1' \mathbf{z}_{it}^* + \varepsilon_{it}, \tag{4.10}$$

where $\mathbf{z}_{it}^* = (y_{it-1}, \mathbf{x}_{it}')'$. Taking the within transformation, we have

$$\tilde{y}_{it} = \boldsymbol{\delta}_1' \tilde{\mathbf{z}}_{it}^* + \tilde{v}_{it}, \tag{4.11}$$

where $\tilde{\mathbf{z}}_{it}^* = \mathbf{z}_{it}^* - T^{-1} \sum_{i=1}^{T} \mathbf{z}_{it}^*$, from which we obtain the bootstrap-bias corrected estimator $\tilde{\boldsymbol{\delta}}_1^B$, and the corresponding generalised minimum distance measure:

$$Q_0 = \tilde{\tilde{\mathbf{v}}}' \tilde{\tilde{\mathbf{v}}}, \tag{4.12}$$

where $\tilde{\tilde{\mathbf{v}}} = \left\{ \tilde{\tilde{v}}_{it} \right\}$ with $\tilde{\tilde{v}}_{it} = \tilde{y}_{it} - \tilde{\boldsymbol{\delta}}_1^{B\prime} \tilde{\mathbf{z}}_{it}^*$. The LR test statistic is then given by

$$LR = \frac{Q_0 - Q_1(\hat{\gamma})}{\hat{\sigma}^2}. \tag{4.13}$$

We take the residuals, $\hat{\mathbf{v}}_i = \hat{\mathbf{v}}_i(\hat{\gamma})$ (see (4.8)), group them by individual $i = 1, ..., N$, namely $\hat{\mathbf{v}}_i = (\hat{v}_{i2}, ..., \hat{v}_{iT})'$ and treat $(\hat{\mathbf{v}}_1, ..., \hat{\mathbf{v}}_N)$ as the empirical distribution to be used for bootstrapping. We then draw (with replacement) a sample of size N from the empirical distribution and use these errors to create a bootstrap sample under H_0 and under the alternative, separately. Treating the initial value y_{i1} as given, we use the bootstrap sample and estimate the model under the null and under the alternative and calculate the bootstrap value of the LR test at each replication. We repeat this procedure a large number of times, e.g. 1000 times and calculate the percentage of draws for which the simulated statistic exceeds the actual. This is the bootstrap estimate of the asymptotic p-value for LR under H_0. Hansen (1996) shows that a bootstrap procedure attains the first-order asymptotic distribution, so p-values are asymptotically valid.

Hansen (1999, 2000) also suggests that the confidence interval for γ be constructed by forming the no-rejection region using the LR statistic for $H_0 : \gamma = \gamma_0$, where the LR statistic is assumed to weakly converge in distribution. These procedures can be easily adapted to balanced panels, but it will be quite complicated to use in unbalanced panels.

4.1 Subsampling-based Inference

Alternatively, we suggest the subsampling approach that will be valid under weaker conditions. Additional advantage is that it can be easily applied in unbalanced panels. Under the probability law P, let

$$J_N(x, P) = Pr\left\{\tau_N \left|\hat{\gamma}_N - \gamma\right| \leq x\right\}, \tag{4.14}$$

where $\tau_N = N^{\varrho}$ is a normalizing sequence for some positive real number ϱ. Subsampling approximation to $J_N(x, P)$ is defined by

$$\hat{L}_{N,b_i}(x) = \frac{1}{N - b_i - 1} \sum_{h=1}^{N-b_i+1} 1\left\{\hat{\tau}_{b_i} \left|\hat{\gamma}_{b_i,h} - \hat{\gamma}_N\right| \leq x\right\}, \ i = 1, ..., I, \tag{4.15}$$

where the integer $1 < b_i < N$ is the block size of the ith subsample, $\hat{\gamma}_{b_i,h}$ is consistent estimator of γ obtained using the subsample block size, b_i, and $\hat{\tau}_{b_i}$ is the associated consistent estimator of the convergence rate. Hence, the (symmetric) subsampling confidence interval is then given by

$$\widehat{CI}(\hat{\gamma}_N) = \hat{\gamma}_N \pm \tau_N^{-1} c_N (1 - \alpha), \tag{4.16}$$

where $\hat{c}_N(1 - \alpha)$ is an $(1 - \alpha)$ quantile of $\hat{L}_{N,b_i}$. The confidence interval of (**??**) has asymptotic coverage probability of $1 - \alpha$ under weak conditions.

Since $\tau_N \left|\hat{\gamma}_N - \gamma\right|$ converges to a non-degenerate distribution, $\left|\hat{\gamma}_N - \gamma\right|$ converges to the point with measure zero at rate τ_N. Therefore, one can consistently estimate convergence rate, ϱ by comparing a number of subsampling distributions of non-scaled $\left|\hat{\gamma}_N - \gamma\right|$ derived for each of the block sizes, $b_1, \ldots, b_I$. We follow Gonzalo and Wolf (2005) and define

$$K_{N,b_i}(x) = \frac{1}{N - b_i + 1} \sum_{h=1}^{N-b_i+1} 1\left\{\left|\hat{\gamma}_{b_i,h} - \gamma\right| \leq x\right\}, \ i = 1, ..., I. \tag{4.17}$$

Next, denote an α_j-quantile of K_{N,b_i} by $K_{N,b_i}^{-1}(\alpha_j)$ with $\alpha_j \in (0.5, 1)$, $j = 1, \ldots, J$, and let $b_i = [N^{\kappa_i}]$ for $0 < \kappa_1 < \cdots < \kappa_I$. Then ϱ can be consistently estimated by[2]

$$\hat{\varrho} = -\frac{\sum_{i=1}^{I} (\bar{r}_{i\cdot} - \bar{r}) \left(\log b_i - \overline{\log b}\right)}{\sum_{i=1}^{I} \left(\log b_i - \overline{\log b}\right)^2}, \tag{4.18}$$

where

$$r_{i,j} = \log\left(K_{N,b_i}^{-1}(\alpha_j)\right), \ \bar{r}_{i\cdot} = \frac{1}{J}\sum_{j=1}^{J} r_{i,j}, \ \bar{r} = \frac{1}{IJ}\sum_{i=1}^{I}\sum_{j=1}^{J} r_{i,j}, \ \overline{\log b} = \frac{1}{I}\sum_{i=1}^{I} \log b_i. \tag{4.19}$$

[2]See Gonzalo and Wolf (2005) for a proof of $\hat{\varrho}_{I,J} = \varrho + o_P\left((\log N)^{-1}\right)$.

Consider the LR statistic for the null of symmetry, which is assumed to weakly converges to (unknown) non-degenerate distribution. Define

$$G_N(x, P) = \Pr\{LR_N \leq x\}$$

The distribution of LR_N is then approximated by the subsampling counterpart:

$$\hat{G}_{N,b_i}(x) = \frac{1}{N - b_i + 1} \sum_{h=1}^{N-b_i+1} 1\{LR_{b_i,h} \leq x\}, \ i = 1, ..., I, \tag{4.20}$$

where $LR_{b,i}$ is the LR statistic obtained using the subsample block size b_i. Hence, the associated critical value is obtained as the $1 - \alpha$ quantile of $\hat{G}_{N,b_i}$, which is given by

$$g_N(1 - \alpha) = \inf\left\{x : \hat{G}_{N,b_i}(x) \geq 1 - \alpha\right\} \tag{4.21}$$

and we reject the null if and only if $LR_N > g_N(1 - \alpha)$.

5 Monte Carlo Studies

To be filled.

6 Empirical Application: A Dynamic Threshold Model of Investment

An important research question in the investment literature is whether capital markets imperfections affect the firm's investment behaviour. Farazzi *et al.* (1988) examine the relationship between investment expenditures and internal funds, and show that investment spending by firms with low dividend payments is strongly affected by the availability of cash flows, rather than just by the availability of positive NPV projects. They identify firms that retain most of their earnings and pay low dividends as those which are unable to raise external finance, and face some sorts of financial constraints. Their empirical findings support the hypothesis that cash flow has a significantly positive effect on investment for financially constrained firms, and concludes that the sensitivity of investment to cash flows is an indicator of the degree of financial constraints facing the firm.

This strand of research is however subject to two criticisms. First, it is questionable as how firms are classified as financially "most constrained" and "least constrained". Kaplan and Zingales (1997) adopt a different approach to identifying financial constraints and find that the relationship between cash flows and investment is not monotonic with respect to financial constraints. The evidence on the investment to cash flow sensitivity depends crucially on the selected financing constraints criterion. Consequently, a large body of the recent literature seeks to address the question of what measures can be used to classify firms as 'financially constrained' and 'unconstrained'. Several criteria have been suggested, including size, age, leverage, financial slack (available cash flows), market to book value (growth opportunities), dividend payout and bond rating (e.g. Hovikimian and Titman, 2006).

One of the main methodological problems facing the conventional investment literature is that the distinction between constrained and unconstrained firms is routinely based on an arbitrary threshold level of the measure used to split the sample. Furthermore, once the split or grouping is made, firms are not allowed to change groups over time since the split-sample is fixed for the complete sample period. Hence we apply a threshold model of investment in dynamic panels to address this problem. Most popular investment model takes the form of a Tobin's Q model in which the expectation of future profitability is captured by the forward-looking stock market valuation;

$$I_{it} = \beta_0 + \beta_1 Q_{it-1} + \beta_2 CF_{it-1} + \varepsilon_{it} \tag{6.1}$$

where I_{it} stands for investment, Q_{it} for Tobin's Q and CF_{it} for cash flows. To be consistent with previous empirical models, e.g Hansen (1999), we use the lagged values of Q and CF in order to avoid the potential endogenous regressor problem. The coefficient β_2 is of a main importance since it represents the cash flow sensitivity of investment. If firms are not financially constrained, external finance can be raised to fund future investments without the use of internal finance. In this case, cash flows are least relevant to investment spending and β_2 is expected to be equal or close to zero. In contrast, if firms were to face certain financial constraints, β_2 would be expected to be significantly positive. Extensions of this Tobin's Q model also involve an additional financing variables such as leverage to control for the effect of capital structure on investment (Lang *et al.*, 1996) as well as lagged investment to capture the accelerator effect of investment in which past investments have a positive effect on future investments (Aivazian *et al.*, 2005a,b). Therefore, we consider the following augmented dynamic investment model:

$$I_{it} = \beta_0 + \delta I_{it-1} + \beta_1 Q_{it-1} + \beta_2 CF_{it-1} + \beta_3 L_{it-1} + \varepsilon_{it}, \tag{6.2}$$

where L_{it} represents leverage and ε_{it} consists of the one-way error components, $\varepsilon_{it} = \alpha_i + v_{it}$.[3] We then extend (6.2) into the threshold dynamic panel framework developed in the paper:

$$\begin{aligned} I_{it} = {} & (\delta_1 I_{it-1} + \beta_{11} Q_{it-1} + \beta_{21} CF_{it-1} + \beta_{31} L_{it-1}))1\{q_{it} \le \gamma\} \\ & + (\delta_2 I_{it-1} + \beta_{12} Q_{it-1} + \beta_{22} CF_{it-1} + \beta_{32} L_{it-1}) 1\{q_{it} > \gamma\} + \varepsilon_{it}. \end{aligned} \tag{6.3}$$

For estimation of (6.3) we employ the same data set used in Hansen (1999) and Gonzalez *et al.* (2005). This data set is a balanced panel of 565 US firms over the period 1973-1987, which is extracted from an original data set constructed by Hall and Hall (1993), and is similar to the panel of firms used by Fazzari *et al.*, (1998). Hence this study will make it allowing for comparisons with the investment literature. Following Gonzalez *et al.* (2005), we exclude five companies with extreme data values, and consider a final sample of 560 companies with 7840 company-year observations. An exact definition of the variables is as follow: Investment is measured by investment to the book value of assets, Tobin's Q the market value to the book value of assets, leverage long-term debt to the book value of assets, cash flow is cash flow to the book value of assets. For convenience descriptive statistics of these variables are provided in Table 1, which are consistent with those reported in Gonzalez *et al.* (2005).

Table 1. Descriptive Statistics

Variables	Mean	Median	Std Dev
I_{it}	0.088	0.076	0.059
CF_{it}	0.239	0.215	0.197
Q_{it}	1.051	0.692	1.128
L_{it}	0.239	0.208	0.219

Table 2 summarises the estimation results for the dynamic threshold model of investment, (6.3), with leverage, Tobin's Q and cash flow used as the transition variable, which are expected to proxy the certain degree of financial constraints. As discussed below this choice

[3] We have also estimated the model with the two-way error components by including the time-specific dummies. The results, available upon request, are qualitatively similar.

may be somewhat ad hoc but is broader than Hansen (1999) who considers only leverage as the transition variable and Gonzalez *et al.* (2005) who employ both leverage and market to book value (Tobin's Q). In each case we report the (biased) fixed effects estimator and the two bootstrap-based bias corrected estimators, where BSTR1 uses the iid bootstrap and BSTR2 uses the wild block bootstrap as described in the previous section. The estimation results are reported respectively in the low and high regimes. We mainly focus on the results obtained by the first bootstrapping estimation method, which seems provide more plausible coefficient estimates. Columns (1)-(3) present the estimation results for the case where the transition variable is Tobin's Q. The estimate of the threshold parameter (obtained by all the three estimators) is 2.325, which is relatively high as compared to mean Tobin's Q (1.051). So the majority of the observations fall into the low regime (7119 firm-year observations). We find that past investment has a stronger positive effect on current investment for firms with high Tobin's Q (0.380), as compared to firms with low Tobin's Q (0.215). This suggests that firms with higher growth opportunities are able to take more growth options in order to continue past investment strategies, and maintain a higher accelerator effect. The coefficient estimate on Tobin's Q in the low regime is higher than that in the high regime (0.019 vs. 0.001). This indicates that firms with low growth options respond more strongly to changes in their investment opportunities than those with high growth options. This finding is consistent with Gonzalez *et al.*, (2005), but not with Hu and Schiantarelli (1998), who, nevertheless, consider multiple indicators to measure financial constraints and the degree of asymmetric information and use a different switching-type model. The sensitivity of investment to cash flow is also relatively higher for high-growth firms (0.067) than low-growth firms (0.056). This finding, therefore, does not support the hypothesis that cash flow should be more relevant for firms with limited prospects and potentially high financial constraints. This is also inconsistent with Gonzalez *et al.* (2005), who find that cash flow has a stronger effect on investment for firms with low Tobin's Q. Note, however, that low-growth opportunities may not indicate a high degree of financial constraints since low-growth firms may have low demand for finance so that they may be less liquidity constrained. Furthermore, it can be also argued that firms in established and mature industries may face limited growth options but are not necessarily financially-constrained. This finding suggests a possibility that high-growth firms require large financing resources to fund their future investments so they may even need to rely more on available cash flows. Finally, the results reveal a negative relationship between investment and leverage, but significantly only for low-growth firms. This is consistent with Lang *et al.* (1996) and the argument that when growth options cannot be anticipated sufficiently early, highly-levered firms may face the typical underinvestment problem whereby positive NPV projects may go unfunded (Myers, 1977).

In columns (4)-(6), leverage is used as the transition variable. The results show that the threshold estimate obtained by the fixed effects and the two bootstrapping methods is 0.338 and 0.325, respectively. This threshold is higher than the mean leverage (0.239) and more than 6000 observations fall into the low-leverage regime. We find that past investment has a smaller positive impact on current investment for highly-levered firms, suggesting that firms with high leverage experience some forms of financial constraints that prevent them from responding to growth options quickly, hence a lower accelerator effect. The effect of Tobin's Q on investment is smaller for lowly-levered firms as compared to highly-levered firms (0.002 vs. 0.010). This does not support the argument that by lowering the risky "debt overhang" to control underinvestment incentives *ex ante*, firms are able to take more growth opportunities and make more investments *ex post*. The coefficient estimate on cash flow is slightly lower for firms in the high-leverage regime, a finding inconsistent with the prediction that cash flow should be more relevant and have a stronger effect on the level of investment for financially constrained firms. Notice however Hansen (1999) also fails to find strong and conclusive evidence in favor of this prediction by estimating a non-dynamic three-regime threshold model of investment. Finally, the leverage has a significantly negative effect on investment for both constrained and unconstrained firms, and stronger for constrained firms, a fining consistent with the overinvestment hypothesis about the role of leverage as a disciplining device that prevents firms from over-investing in negative NPV projects (e.g. Jensen, 1986).

The last three columns contain the results for the model with cash flow used as a transition variable. The coefficient estimate on lagged investment is slightly higher for firms with high cash flow (0.298 vs. 0.251), suggesting that the accelerator effect of investment is slightly stronger for cash-rich firms that can use the available internal finance in order to fund past investment spending. The coefficient estimate on Tobin's Q reveals an unexpected finding that firms respond to growth opportunities more quickly when they are cash-constrained (0.002) than when they are unconstrained (0.001). In terms of the results for the cash flow variable, the sensitivity of investment to cash flow is higher for cash-constrained firms than for cash-rich firms (0.090 vs. 0.075). Firms with limited cash resources are likely to face some forms of financial constraints (Kaplan and Zingales, 1997; Kashap *et al.*, 1994); hence cash flow resources available, no matter how limited they are, of important relevance for those firms. This finding supports the evidence for the role of financial constraints in the investment-cash flow sensitivity. It should be noted, however, that limited cash flow is not a perfect measure of financial constraints. Some research suggests that high cash flows may be an indication of high financial constraints since financially constrained firms have more incentives to hold large cash balances (Fazzari *et al.*, 2000). Lastly, leverage and investment exhibit a negative relationship for both highly-levered and lowly-levered firms, but the effect is more pronounced for cash-rich firms. This is consistent with the overinvestment hypothesis, according to which firms with available cash flows may mitigate the principal-agent problem by pre-committing themselves to a high-leverage policy in order to prevent the managers from over investing in risky projects or spending excessively on perks (Jensen and Meckling, 1976; Jensen, 1976).

Table 2. Estimation Results of Threshold Models of Investment

Independent variables	Tobin's Q as transition variable			Leverage as transition variable			Cash Flows as transition variable		
	FE	BSTR1	BSTR2	FE	BSTR1	BSTR2	FE	BSTR1	BSTR2
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Low regime									
I_{it-1}	0.127	0.215	0.245	0.208	0.284	0.290	0.165	0.251	0.207
	(0.012)	(0.012)	(0.012)	(0.013)	(0.014)	(0.014)	(0.012)	(0.012)	(0.015)
Q_{it-1}	0.021	0.019	0.020	0.002	0.001	0.002	0.003	0.002	0.002
	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
CF_{it-1}	0.059	0.056	0.053	0.075	0.071	0.071	0.092	0.090	0.041
	(0.006)	(0.006)	(0.006)	(0.004)	(0.005)	(0.005)	(0.006)	(0.006)	(0.010)
L_{it-1}	-0.031	-0.029	-0.032	-0.049	-0.049	-0.047	-0.020	-0.020	-0.022
	(0.004)	(0.004)	(0.004)	(0.008)	(0.008)	(0.008)	(0.004)	(0.004)	(0.004)
High regime									
I_{it-1}	0.319	0.380	0.404	0.118	0.192	0.189	0.225	0.298	0.251
	(0.022)	(0.023)	(0.023)	(0.016)	(0.016)	(0.016)	(0.022)	(0.022)	(0.014)
Q_{it-1}	0.001	0.001	0.001	0.010	0.009	0.010	0.001	0.001	0.001
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
CF_{it-1}	0.068	0.067	0.065	0.065	0.065	0.064	0.084	0.083	0.075
	(0.005)	(0.005)	(0.005)	(0.008)	(0.008)	(0.009)	(0.006)	(0.006)	(0.005)
L_{it-1}	-0.006	-0.004	-0.004	-0.029	-0.029	-0.026	-0.073	-0.071	-0.030
	(0.007)	(0.007)	(0.008)	(0.004)	(0.004)	(0.005)	(0.008)	(0.008)	(0.005)

In summary, the results on the relationships between investment and past investment, as well as Tobin's Q and leverage are generally consistent with theory and prior empirical evidence. However, the evidence on the cash flow sensitivity of investment is somewhat mixed. When leverage is used as the transition variable, the results do not support the hypothesis that cash flows available should be more relevant for financially constrained firms. With Tobin's Q and cash flows adopted as the transition variable, the evidence is somewhat supportive of the investment-cash flow sensitivity although our discussion shows that these two variables may not be a good indicator of financial constraints. Methodologically, our results clearly demonstrate that the bootstrapping method has reduced the size of the bias associated with the fixed effects estimator, though the magnitude of correction size is relatively small, which is acceptable when the true autoregressive coefficients are not very persistent as in the current data set.

This paper examines a dynamic threshold panel estimation of Tobin's Q model of investment by using the lagged Tobin's Q, leverage and cash flow as a possible transition variable. But there are a few potential limitations. First, Tobin's Q is often difficult to measure: the replacement value of assets is generally unreported so book value of total assets are used in the denominator instead. But, this practice may render a potential estimation problem in which intangible assets are not properly measured and accounted for. Second, it has been widely argued that cash flows, in addition to reflecting information about internal liquidity, may also capture information about future investment prospects that are not captured by measures of Tobin's Q. In this case, it will be extremely difficult to disentangle the effect of growth opportunities from the one of financial constraints in the coefficient estimate of the cash flow variable. Chirinko and Schaller (1995) further suggest that average Tobin's Q is flawed as it reflects the average return on a company's total capital whereas it is the marginal return on capital that is relevant. Finally and more importantly, the use of Tobin's Q, leverage and cash flows as the transition variable may have caveats since these variables are imperfect measures of financial constraints. Tobin's Q implies that high-growth firms have greater demand for external financing and may be more liquidity constrained whilst a high Tobin's Q as measured by the market to book ratio indicates that the firm's growth prospects are highly perceived by the market, suggesting that firms with a high Tobin's Q may have easier access to external funds. Leverage is likely to be a more useful indicator of financial constraints since highly-levered firms are generally classified as most constrained due to potential liquidity problems. In contrast, firms adopting a low-leverage policy are least constrained because the debt capacity allows them to raise external finance when growth options are available. Free cash flows or financial slack may be an indicator of either a high or low degree of financial constraints. Firms with ample cash reserves are regarded as potentially financially constrained, since constrained firms have more incentive to hold large cash balances (Fazzari *et al.*, 2000). It can be argued however that firms with a high level of financial slack are not necessarily constrained since their investment is not limited by a lack of finance (Kaplan and Zingales, 1997). An alternative better approach would be to use indices computed to control for financial constraints, e.g. Whited and Wu (2006). Nonetheless, all these issues are beyond the scope of this paper.

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