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The Simple Analytics of Trade and Labor Mobility I: Homogeneous Workers[1]

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1 Introduction.

This paper presents a model of labor mobility incorporated into a simple trade model. The goal is to provide a framework that is rich enough to capture the main empirical features of labor mobility in practice and yet simple enough to be tractable within a general equilibrium model. We thus hope that this framework can become a useful part of a trade economist's toolkit.

All aspects of an economy's response to trade policy, and particularly the distributional aspects, depend crucially on how labor adjusts, and the costs workers face in doing so (see Slaughter (1998) and Magee (1989) for extensive discussions of the distributional implications). However, for the most part trade theory has ignored the question of labor mobility, either assuming frictionless mobility or complete immobility. Moreover, most of the few existing theoretical treatments of labor mobility are hard to reconcile with key empirical features of labor mobility, in particular with the evidence on gross flows. Here, we set up a model that takes on these questions head on.

A number of approaches have been suggested for analyzing labor adjustment in trade models. A traditional static approach used as a benchmark by many trade economists is to think of a model with 'specific factors,' factors that cannot adjust at all, as a good model of the short run, but a model with fricionless adjustment as a good model of the long run. Mussa (1974), for example, uses the 'Ricardo-Viner' model for the short run, which features instantaneous adjustment for labor but no adjustment for any other factors, and the Heckscher-Ohlin model, which assumes no mobility costs for any factor, for the long-run. Of course, this approach offers no insight into dynamics, and assumes away mobility costs for workers.

Explicit dynamics emerge in Mussa's (1978) seminal model of intersectoral capital adjustment as an intertemporal optimization problem within a general equilibrium model. In that model, adjustment is gradual because of convex adjustment costs for capital. A similar approach has sometimes been adopted for labor, by stipulating convex retraining costs, as in Karp and Paul (1994) and Dehejia (1997). A feature of these models is that marginal adjustment costs are assumed to be zero when net movements of labor are zero, and so the long-run steady-state is the same as in a model with no adjustment costs (as in Hechscher-Ohlin).

Dixit & Rob (1994) present a model with a constant cost to switching sectors, equal for all workers, in a stochastic environment. Feenstra and Lewis (1994) use a one-period model of worker adjustment to study compensation policies. Topel (1986) allows for moving costs that vary across workers. Matsuyama (1992) models intergenerational labor adjustment.

All of the above papers have the property that all workers who switch sectors move in the same direction: Gross flows always equal net flows. This is a difficulty, since empirically gross flows of labor tend to be an order of magnitude larger than net flows (see Jovanovic and Moffitt(1990), Table 1). Few trade models provide theories that allow for gross flows in excess of net flows. An important exception is Davidson and Matusz (2001), in which exogenous separations of workers from jobs cause some exogenous intersectoral flows.[2] Another paper that has a close relationship is Jovanovic and Moffitt (1990)'s job-search-matching model, although it does not deal with trade.

What we do in this paper is to develop a dynamic stochastic rational expectations model of labor flows within a simple trade model, the key innovation in which is that we allow workers to face time-varying idiosyncratic moving costs. This is a simplified and more tractable version of a more general model presented in Cameron, Chaudhuri and McLaren (2002). The general version can accomodate any number of industries and geographic regions within the country under study, and has the virtue that its main parameters can be econometrically estimated, as described in that paper. Here, we focus on the implications of the model for trade theory in a simple two-sector special case with one type of labor. A companion paper (Chaudhuri and McLaren (2002)) extends the analysis to the case of skilled and unskilled labor.

The main results are: (i) Gross flows of labor always exceed net flows, so there are always workers moving across sectors, even in the steady state.

(ii) Equilibrium is unique, and there is no hysteresis, despite the presence of an unavoidable fixed cost to switching sectors.

(iii) Wage differentials persist across locations or sectors even in the long run, despite the fact that there are always some workers changing sectors. In particular, cancellation of a sector's tariff protection leads not only to a short-run drop in that sector's wage relative to the other sector, but also to a (smaller but still positive) drop in its relative wage in the long run. Thus, *a frictionless model is not a good predictor for the steady state behavior of the model.*

(iv) The economy adjusts only gradually to changes in policy. This is consistent with findings by Topel (1986), Blanchard & Katz (1992), and others. This complicates empirical work on trade and wages, however, because changes in wages and labor allocation will continue long after the change in policy has occurred.

(v) The economy begins to adjust to a policy change as soon as it is anticipated. Evidence of this has been noted in data on trade agreements (Freund and McLaren (1999)). This complicates empirical work even more, because it means that adjustment begins *before* the policy change takes effect, and in fact changes in wages can reverse their direction when the policy change actually is executed.

(vi) The incidence of trade policy needs to be analyzed on the basis of lifetime utility, taking worker mobility into account, and not simply on the basis of wage levels. We show how to do this. The correction is important: It is theoretically possible, for example, for a policy change to lower real wages in a sector both in the short run and in the long run, and yet for the workers in the sector to be better off as a result of it.

(vii) Announcing a trade liberalization in advance tends to reduce the difference between its effect on export-sector workers and its effect on import-competing workers. This can have the effect in the limit of making all workers beneficiaries of trade liberalization or of making them all net losers from it. Thus, the common presump-

[2] This is the paper that is closes in spirit to this project. Some advantages of the Davidson and Matusz approach are that it allows both an analytic solution for the transition path, and the estimation of aggregate losses from worker adjustment costs, which the authors find to be large. Disadvantages are that rates of turnover are exogenous, and there is no evident way to estimate the structural parameters econometrically.

tion that a phased-in liberalization helps to cushion the blow to affected workers can be exactly wrong in some cases. We offer a simple condition that determines which way it will go.

Plan of paper: The following section lays out the basic model. Then one section presents results on the steady state of the model, and the following section presents results on dynamic adjustment. A further section discusses the incidence of policy changes in the model, taking lifetime utility into account. A final section summarizes.

2 The Basic Model.

There are two sectors, X and Y, each with a large number of competitive employers, who combine a sector-specific fixed (latent) factor with labor for production. The two sectors may be located in two separate regions and may require different skills, making it costly for workers to move between them. Without any costs of moving between sectors, and without any idiosyncratic shocks to workers, the economy would be a Ricardo-Viner model (Jones, 1971). We shall see that its equilibrium is very different from that model, both in its dynamic character and in its steady state.

The economy's workers form a continuum of measure L.

2.1 Production.

The quantity of aggregate output in sector i in period t is given by:

$$q_t^i = Q^i(L_t^i, s_t),$$

where L_t^i denotes labor used in sector i in period t, s_t denotes a vector of state variables, e.g., tariffs, technology shocks, world prices. Assume that s_t follows a first-order Markov process on a compact state space $S \subset \Re^k$, and that $Q^i(\cdot, s_t)$ is increasing, continuously differentiable, strictly concave in L_t^i, and continuous in s_t.

The domestic price of good i, p^i, is exogenous, but may be affected by the state s_t. A central example, which will be treated explicitly, is of a small open economy in which the domestic price is equal to the given world price plus a tariff (export subsidy) for a good that is imported (exported).

The wage in sector i in time t is competitively determined:

$$\widetilde{w}_t^i = \widetilde{w}^i(L_t^i, s_t) \equiv p^i(s_t)\frac{\partial Q^i(L_t^i, s_t)}{\partial L_t^i},$$

where $\widetilde{w}_t^i$ denotes the nominal wage paid by sector i at time t and p^i denotes the domestic price of good i. Thus, the competitive employers in each sector each take the wage as given and maximize profit; the wage adjusts so that this wage just clears the spot labor market in that sector.

2.2 Labor Mobility.

The cost to a worker ω, who was in i during period t, of moving from i to $j \neq i$ at the end of t is:

$$\varepsilon^i_{\omega,t} - \varepsilon^j_{\omega,t} + C,$$

where $C \geq 0$ is the deterministic component of mobility costs, common to all workers (such as retraining or relocation costs, or psychic costs of moving to a new occupation), and $[\varepsilon^i_{\omega,t} - \varepsilon^j_{\omega,t}]$ is the idiosyncratic stochastic component of mobility costs. The variable $\varepsilon^i_{\omega,t}$ is the non-pecuniary benefit —independent of wages—to ω of being in i between the end of t and the end of $t+1$. For example, a worker who is recently divorced may have a desire for a new location for non-pecuniary reasons ($\varepsilon^i_{\omega,t} - \varepsilon^j_{\omega,t} < 0$), and a worker with a child in the final year of high school may have a non-pecuniary reason to stay in the current location ($\varepsilon^i_{\omega,t} - \varepsilon^j_{\omega,t} > 0$).

We assume that the $\varepsilon^i_{\omega,t}$ are independently and identically distributed across workers and sectors and over time, with cdf, $F(.)$, pdf $f(.)$, full support:

$$f(\varepsilon) > 0 \forall \varepsilon,$$

and mean zero:

$$E(\varepsilon) = \int \varepsilon f(\varepsilon) d\varepsilon = 0.$$

Further, we make the boundedness assumption:

$$E(\max\{\varepsilon^X, \varepsilon^Y\}) = 2 \int \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon < \infty.$$

This ensures that the worker's problem is meaningful; if it was violated, the worker could ensure infinite utility simply by choosing the sector with the higher value of ε in each period.

Since what is important for workers' decisions is the difference between ε^i and ε^j, we can simplify notation by defining:

$$\mu^i_t = \varepsilon^i_t - \varepsilon^j_t,$$

for a worker currently in sector i, where μ^i_t is symmetrically distributed around mean zero, with cdf, $G(.)$ and pdf $g(.)$.

The transition equations governing the allocation of labor are:

$$m^{ii}_t L^i_t + m^{ji}_t L^j_t = L^i_{t+1} \qquad i = X, Y \; ; \; j \neq i,$$

where m^{ji}_t denotes the fraction of labor force in i at the beginning of t that moves to j by the end of t, or in other words, the gross flow from i to j.

The timing of events can be summarized thus:

$$\begin{array}{cccccccc}
 & & \downarrow s_t & & \downarrow \varepsilon_t & & & \\
 & L^X_t & \longrightarrow & w^X_t & \longrightarrow & m^{XX}_t, m^{XY}_t & & L^X_{t+1} \\
\overset{t\mapsto}{\Longrightarrow} & & & & & & \overset{t+1\mapsto}{\Longrightarrow} & \\
 & L^Y_t & \longrightarrow & w^Y_t & \longrightarrow & m^{YY}_t, m^{YX}_t & & L^Y_{t+1}
\end{array}$$

The stock of workers in each sector in each period is determined by events in the previous period. Once the value of the state variable for this period is realized, that together with the current labor alloctions determines wages through spot labor-market clearing. Then each worker learns her ε's and decides whether to remain in her current sector or move. In the aggregate, these decisions determine the following period's labor allocation.

2.3 Preferences and Expectations.

All agents are risk neutral, have rational expectations and have a common discount factor $\beta < 1$. Further, all workers have identical and homothetic prferences, which allows us to identify a common cost-of-living index. Letting good X be the numeraire, let the cost-of-living index be denoted $\phi(p^Y)$, an increasing function. Thus, the real wage w_t^i received by a worker in sector i at date t is given by:

$$w_t^i(L_t^i, s_t) \equiv \widetilde{w}_t^i/\phi(p^Y) = \frac{p^i(s_t)}{\phi(p^Y(s_t))}\frac{\partial X^i(L_t^i, s_t)}{\partial L_t^i}.$$

Note that an increase in s_t that raises p^Y without affecting the production functions will shift w_t^Y down as a function of L_t^i and shift w_t^Y up as a function of L_t^Y.

Each worker makes a location decision in each period to maximize the expected present discounted value of real wages, net of common (C) and idiosyncratic (μ) moving costs. Let $u^i(L_t, s_t, \varepsilon_t)$ denote the (maximized) value to a worker of being in i given $L_t = (L_t^X, L_t^Y)$, s_t, and idiosyncratic shocks $\varepsilon_t = (\varepsilon_t^X, \varepsilon_t^Y)$ realized by the worker. Then $v^i(L_t, s_t) \equiv E_\varepsilon(u^i(L_t, s_t, \varepsilon_t))$ gives the expected value of u^i before idiosyncratic shocks are realized, but conditional on (L_t, s_t).

Since the worker is optimizing, $u^i(L_t, s_t, \varepsilon_t)$ can be written:

$$\begin{aligned} u^i(L_t, s_t, \varepsilon_t) &= w_t^i + \max\{\varepsilon_t^i + \beta E_t v^i(L_{t+1}, s_{t+1}), \varepsilon_t^j - C + \beta E_t v^j(L_{t+1}, s_{t+1})\} \\ &= w_t^i + \beta E_t v^i(L_{t+1}, s_t) + \varepsilon_t^i + \max\{0, \overline{\mu}_t^i - \mu_t^i\}, \end{aligned}$$

where

$$\overline{\mu}_t^i = \beta[E_t v^j(L_{t+1}, s_{t+1}) - E_t v^i(L_{t+1}, s_{t+1})] - C, \tag{1}$$

and $i \neq j$. The expression $\overline{\mu}_t^i$ is the common value of the net benefit of moving from i to j. If this is greater than the idiosyncratic cost μ, the worker will move; otherwise, the worker will stay.

Taking expectations with respect to the ε's (and hence the μ's):

$$v^i(L_t, s_t) = w_t^i + \beta E_t v^i(L_{t+1}, s_t) + \Omega(\overline{\mu}_t^i), \tag{2}$$

where:

$$\Omega(\overline{\mu}) = E_\mu \max\{0, \overline{\mu} - \mu\} = G(\overline{\mu})\overline{\mu} - \int_{-\infty}^{\overline{\mu}} \mu dG(\mu). \tag{3}$$

In other words, the value, v^i, of being in i is the sum of: (i) the wage, w_t^i, that is received; (ii) the base value, $\beta E_t v^i(L_{t+1}, s_t)$, of staying on in i; and (iii) the additional value, $\Omega(\overline{\mu}_t^i)$, of having the option to move. The expression $\Omega(\overline{\mu}_t^i)$ is thus interpreted as representing option value.

2.4 Key Equilibrium Conditions.

An equilibrium is a moving rule characterized by a value of $(\overline{\mu}_t^X, \overline{\mu}_t^Y)$ each period (where a worker in i moves if and only if $\mu < \overline{\mu}_t^i$), such the aggregate movements of workers induced by that rule generate a time path for wages in each sector that make the proposed moving rule optimal. Here we derive a key equation that is useful in characterizing equilibrium.

From (1), together with (2) applied to period $t+1$, we know that

$$\begin{aligned} C + \overline{\mu}_t^i &= \beta E_t \Big([w_{t+1}^j - w_{t+1}^i] + \beta E_{t+1}(v_{t+2}^j - v_{t+2}^i) \\ &\quad + \Omega(\overline{\mu}_{t+1}^j) - \Omega(\overline{\mu}_{t+1}^i) \Big), \end{aligned}$$

but using (1) applied to period $t+1$, this becomes:

$$\begin{aligned} C + \overline{\mu}_t^i &= \beta E_t \Big([w_{t+1}^j - w_{t+1}^i] + C + \overline{\mu}_{t+1}^i \\ &\quad + \Omega(\overline{\mu}_{t+1}^j) - \Omega(\overline{\mu}_{t+1}^i) \Big). \end{aligned} \tag{4}$$

This is an important relationship for characterising the equilibrium behavior of the model. The interpretation is as follows. The cost of moving $(C + \overline{\mu}_t^i)$ for the marginal mover from i to j equals the expected future benefits of being in j instead of i at time $t+1$. This has three components: (i) the expected wage differential next period, $[w_{t+1}^j - w_{t+1}^i]$; (ii) the difference in expected continuation values, captured by the expected cost borne by the marginal mover from i to j at time $t+1$, $C + \overline{\mu}_{t+1}^i$; and (iii) the difference in option values associated with being in each sector, $\Omega(\overline{\mu}_{t+1}^j) - \Omega(\overline{\mu}_{t+1}^i)$.

Note that from (1), $\overline{\mu}^X$ and $\overline{\mu}^Y$ are related:

$$\overline{\mu}_t^X = -\overline{\mu}_t^Y - 2C.$$

Given this and the symmetry of the distribution of μ, the equilibrium reallocations of labor are given by the following relationships:

$$\begin{aligned} m_t^{ij} &= G(\overline{\mu}_t^i) \quad ; \quad m_t^{ii} = G(-\overline{\mu}_t^i) \\ m_t^{ji} &= G(\overline{\mu}_t^j) = G(-\overline{\mu}_t^i - 2C) \quad ; \quad m_t^{jj} = G(\overline{\mu}_t^i + 2C). \end{aligned} \tag{5}$$

As a result, the intersectoral allocation of labor follows the following law of motion:

$$G(-\overline{\mu}_t^i)L_t^i + G(-\overline{\mu}_t^i - 2C)(L - L_t^i) = L_{t+1}^i. \tag{6}$$

3 Characteristics of the steady state

For now, suppose that $s_t = s\ \forall t$, so that there is no aggregate uncertainty and we may speak meaningfully of a steady state associated with s. Here we will derive properties of the steady state, deferring discussion of the path to the steady state until the next section.

In discussing steady states, we will naturally drop time subscripts. The steady-state level of L^X will be denoted $\overline{L}^Y$. In addition, we will let $\overline{\mu}$ stand for the steady-state value of $\overline{\mu}^X$, and so the steady-state value of $\overline{\mu}^Y$ is given by $-\overline{\mu} - 2C$. Then $\overline{L}^X$ can be derived from (6) as a function of $\overline{\mu}$, and written $\overline{L}^X(\overline{\mu})$.

The first result is a uniqueness property:

Proposition 1 *There is a unique steady-state level of $\overline{\mu}$ and L^X associated with a given constant s.*

Proof. Because the $\overline{m}^{ij}$ and $\overline{L}^i$ derive uniquely from $\overline{\mu}$, it suffices to prove uniqueness of the threshold $\overline{\mu}$ given s. The value of $\overline{\mu}$ is implicitly defined by the equilibrium condition, which comes directly from (4):

$$\begin{aligned}\overline{\mu} + C &= \frac{\beta}{1-\beta}\left[w^Y(L - \overline{L}^X(\overline{\mu}); s) - w^X(\overline{L}^X(\overline{\mu}); s)\right] \\ &\quad + \frac{\beta}{1-\beta}\left[\Omega(-\overline{\mu} - 2C) - \Omega(\overline{\mu})\right].\end{aligned} \tag{7}$$

Since

$$\frac{\partial \Omega(\overline{\mu})}{\partial \overline{\mu}} = G(\overline{\mu})$$

and

$$\overline{L}^X = \frac{\overline{m}^{YX}}{\overline{m}^{YX} + \overline{m}^{XY}} L = \frac{G(-\overline{\mu} - 2C)}{G(-\overline{\mu} - 2C) + G(\overline{\mu})} L,$$

it is easily shown that the right-hand side of (7) is continuous and strictly decreasing in $\overline{\mu}$. Since the left-hand side is continuous and increasing on $\overline{\mu}$ on $(-\infty, \infty)$, there is a unique solution for $\overline{\mu}(s)$. Q.E.D.

Although it is very simple, this result demonstrates the striking difference that the idiosyncratic effects make for the behavior of the model. If there were only common moving costs ($C > 0$) with no idiosyncratic shocks ($\mu \equiv 0$), then there would be a range of steady states. Any allocation of labor such that $|w^X - w^Y| < C(1-\beta)$ would then be a steady state, and this would be a non-degenerate interval of values of L^X. This is illustrated in Figure 1, which is the standard Ricardo-Viner diagram adapted to our model. The length of the box is L, the downward-sloping curve is the marginal value product of labor in X, where the quantity L^X is measured from the left axis, and the upward-sloping curve is the marginal value product of labor in Y, where the quantity L^Y is measured from the right axis. The middle panel shows the range of labor allocations for which it would be unprofitable for workers to move.

Thus, in such a model, there would be hysteresis: The value of L^X at which the system comes to rest would be determined by the initial conditions. However, in the present model with gross flows, every year a trickle of people moves from one cell to another, and this constant stirring of the pot eventually removes the effect of initial conditions.

A second result concerns wage differentials.

Proposition 2 *In the steady state, the larger sector must have a higher wage:* $\overline{L}^X \lesseqgtr \overline{L}^Y \Rightarrow \overline{w}^X \lesseqgtr \overline{w}^Y$.

Proof. Suppose $\overline{L}^X < \overline{L}^Y$ but $\overline{w}^X > \overline{w}^Y$. Then:

$$\begin{array}{rcll} \overline{m}^{YX} & < & \overline{m}^{XY} & \text{since } \overline{m}^{XY}\overline{L}^X = \overline{m}^{YX}\overline{L}^Y \text{ in steady state; so} \\ \overline{\mu}^X & > & \overline{\mu}^Y & \text{since } \overline{m}^{YX} = G(\overline{\mu}^Y) < G(\overline{\mu}^X) = \overline{m}^{XY}\text{; so} \\ \beta[\overline{v}^Y - \overline{v}^X] & > & 0 & \text{since } \overline{\mu}^X = \beta[\overline{v}^Y - \overline{v}^X] - C\text{; so} \\ \overline{v}^Y & > & \overline{v}^X. & \end{array}$$

But:

$$\begin{array}{rcl} \overline{v}^X & = & \dfrac{\overline{w}^X}{1-\beta} + \dfrac{1}{1-\beta}\Omega(\overline{\mu}^X) \\ & > & \dfrac{\overline{w}^Y}{1-\beta} + \dfrac{1}{1-\beta}\Omega(\overline{\mu}^Y) \\ & = & \overline{v}^Y \qquad \text{since } \Omega(\overline{\mu}) \text{ is increasing in } \overline{\mu}, \end{array}$$

which contradicts the initial assumption. Q.E.D.

Note that this implies persistent wage differentials, even in long-run equilibrium, and even though in each period some fraction of the workers in each sector move to the other. The point is that if a given sector is to be larger that the other in the steady state, it must have a lower rate of worker exit than the other sector does. In order for that to be the case, it must have a higher wage. Put differently, suppose for the sake of argument that X and Y had the same wage and $\overline{\mu}^X = \overline{\mu}^Y$ while $L^X > L^Y$. Then the rate of exit from each sector would be the same, so a larger group of X workers would arrive in Y each period than the group leaving Y. This would put downward pressure on the wage in Y, opening up a wage differential in favor of X, the larger sector.

There is also an unambiguous relationship between the steady state of this model and the Ricardo-Viner equilibrium (which, recall, is the equilibrium of the model with $C, \mu \equiv 0$). Specifically, the steady-state intersectoral allocation of workers always lies somewhere between the Ricardo-Viner model and equal division of workers between the sectors.

To see this, first let the allocation of workers to the X sector in the Ricardo-Viner model, L^X_{RV}, be defined implicitly by:

$$w^X(L^X_{RV}, s) = w^Y(L - L^X_{RV}, s).$$

Proposition 3 *The following inequalties must hold in the steady state:*

$$\begin{aligned} L^X_{RV} &< \frac{1}{2} \Rightarrow L^X_{RV} < \overline{L}^X < \frac{1}{2} \\ L^X_{RV} &= \frac{1}{2} \Rightarrow L^X_{RV} = \overline{L}^X = \frac{1}{2} \\ L^X_{RV} &> \frac{1}{2} \Rightarrow L^X_{RV} > \overline{L}^X > \frac{1}{2} \end{aligned}$$

Proof. The result follows directly from the demand curves being downward sloping and Proposition 2. Suppose $L^X_{RV} > \frac{1}{2}$, which means the demand curves cross to the right of the midway point as shown in Figure 1. If $\overline{L}^X$ were, in contradiction to the claim, to lie to the right of L^X_{RV}, i.e., $\frac{1}{2} < L^X_{RV} < \overline{L}^X$ then $\overline{w}^X$ would have to be less than $\overline{w}^Y$. This follows from the definition of L^X_{RV} and the fact that the labor demand curves are downward sloping. (Look at the diagram.) But that would contradict the earlier result that the larger sector (in this case X) has to have the higher wage in steady state.Q.E.D.

An interpretation of this result is that the stochastic idiosyncratic moving costs keep any one sector from becoming too large—so, for instance, attempts to distort relative sectoral sizes through tariffs will be partially offset.

3.1 Steady-state impact of policy changes.

Let us assume that sector Y is the import-competing sector, and that it is initially protected by a tariff that raises the domestic price above the world price. The following two results analyze what happens to steady-state labor allocations and wages as a result of a change in the tariff.

First, the steady-state impact on labor allocations goes in the same direction as in a model with no mobility costs.

For concreteness, consider two models, model 0 and model 1, identical except that in model 1 the tariff, and hence p^Y, is lower than in model 0. Denote the steady-state sector-X employment in the two models by $\overline{L}^X_0$ and $\overline{L}^X_1$ respectively, and denote the Ricardo-Viner equilibrium of the two models by $\widehat{L}^X_0$ and $\widehat{L}^X_1$ respectively. Then, of course, $\widehat{L}^X_1 > \widehat{L}^X_0$. We will show that the direction of the steady-state impact of a labor demand shock is the same in our model as it is in the model with no mobility costs:

Proposition 4 *Under the stated assumptions,* $\overline{L}^X_1 > \overline{L}^X_0$.

Proof. Suppose instead that $\overline{L}_1^X < \overline{L}_0^X$. Then:

$$\begin{aligned}
\overline{\mu}_0 &< \overline{\mu}_1 \qquad \text{because } \frac{\partial \overline{L}^X(\overline{\mu})}{\partial \overline{\mu}} < 0. \text{ This implies:} \\
\overline{\mu}_0 + C &< \overline{\mu}_1 + C \\
&= \frac{\beta}{1-\beta}\left\{w_1^Y(L-\overline{L}_1^X) - w^X(\overline{L}_1^X) + \Omega(-\overline{\mu}_1 - 2C) - \Omega(\overline{\mu}_1)\right\} \\
&< \frac{\beta}{1-\beta}\left\{w_0^Y(L-\overline{L}_1^X) - w^X(\overline{L}_1^X) + \Omega(-\overline{\mu}_1 - 2C) - \Omega(\overline{\mu}_1)\right\} \\
&\quad \text{(since by definition of a labor demand shock in } Y,\ w_0^Y(L^Y) > w_1^Y(L^Y)) \\
&< \frac{\beta}{1-\beta}\left\{w_0^Y(L-\overline{L}_0^X) - w^X(\overline{L}_0^X) + \Omega(-\overline{\mu}_1 - 2C) - \Omega(\overline{\mu}_1)\right\} \\
&\quad \text{(since we assumed } \overline{L}_1^X < \overline{L}_0^X < \widehat{L}_0^X) \\
&< \frac{\beta}{1-\beta}\left\{w_0^Y(L-\overline{L}_0^X) - w^X(\overline{L}_0^X) + \Omega(-\overline{\mu}_0 - 2C) - \Omega(\overline{\mu}_0)\right\} \\
&\quad \text{(since } \Omega(-\overline{\mu} - 2C) - \Omega(\overline{\mu}) \text{ is decreasing in } \overline{\mu} \text{ and } \overline{\mu}_1 > \overline{\mu}_0) \\
&= \overline{\mu}_0 + C \qquad \text{(which contradicts the initial inequality).}
\end{aligned}$$

Finally, as a last comparative static result, we note that the greater the size differential between the two sectors, the larger the wage differential. Consequently, if the tariff on Y is removed, then from Proposition 4, the steady-state real wage in Y will fall relative to the wage in X. (Note: in the following, we focus without loss of generality on the case in which sector X is the larger of the two. Keep in mind, therefore, that $\overline{L}_k^X > \frac{1}{2}$ implies $\overline{w}_k^Y < \overline{w}_k^X$.)

Proposition 5 *Suppose* $\overline{L}_1^X > \overline{L}_0^X > \frac{1}{2}$. *Then* $[\overline{w}_1^X - \overline{w}_1^Y] > [\overline{w}_0^X - \overline{w}_0^Y]$.

Proof. Suppose not. Then:

$$\begin{aligned}
[\overline{w}_1^Y - \overline{w}_1^X] &> [\overline{w}_0^Y - \overline{w}_0^X], \text{ so from (7)} \\
\frac{1-\beta}{\beta}[\overline{\mu}_1 + C] + \Omega(\overline{\mu}_1) - \Omega(-\overline{\mu}_1 - 2C) &> \frac{1-\beta}{\beta}[\overline{\mu}_0 + C] + \Omega(\overline{\mu}_0) - \Omega(-\overline{\mu}_0 - 2C), \\
\text{which implies } \overline{\mu}_1 &> \overline{\mu}_0.
\end{aligned}$$

But our initial assumption that $\overline{L}_1^X > \overline{L}_0^X$ implies that $\overline{\mu}_0 > \overline{\mu}_1$, which yields a contradiction. Q.E.D.

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4 Dynamic adjustment.

4.1 Preliminaries.

For the time being, we continue to assume that $s_t = s\ \forall t$, so that there is no aggregate uncertainty and we can drop the state variable from the production function. In analyzing the model's dynamics, it is extremely useful to know that the *ex ante* value of being in X instead of Y at any given moment, $v^X - v^Y$, is a strictly decreasing function of L^X. That will be demonstrated here in three steps. First, equilibrium in the model solves a particular dynamic programming problem. Second, the value function from that optimization problem is strictly concave with respect to the state variable L^X. Third, its derivative with respect to L^X is equal to $v^X - v^Y$.

A general rule for labor allocation in this model could be characterized by two functions. The function $d^{XY}(L^X, \mu)$ gives the probability that a worker in X will move to Y in the current period, given the current stock L^X of workers in X and the worker's idiosyncratic cost μ^X of moving from X to Y. The function $d^{YX}(L^X, \mu)$ gives the probability that a worker in Y will move to X in the current period, given L^X and the worker's idiosyncratic cost μ^Y of moving from Y to X. These functions define a feasible allocation rule if and only if $d^{ij} \in [0, 1]$ over the whole domain. For any given L_0^X, these functions induce a sequence L_t^X for $t = 1, \infty$ and a sequence π_t^{iX} giving the probability that a worker who was in i at time 0 will be in X at time t. (Clearly, $\pi_0^{iX} = 1$ if $i = X$ and 0 if $i = Y$.)

Proposition 6 *Any equilibrium maximizes:*

$$\begin{array}{c} \sum_{t=0}^{\infty} \beta^t [p^X Q^X(L_t^X) + p^Y Q^Y(L - L_t^X) \\ -L_t^X \int (\mu + C) d^{XY}(L_t^X, \mu) g(\mu) d\mu - L_t^Y \int (\mu + C) d^{YX}(L_t^X, \mu) g(\mu) d\mu] \end{array} \tag{8}$$

within the class of feasible allocation rules d^{XY}, d^{YX}.[3]

Proof. Suppose that $\widetilde{d}^{XY}$ and $\widetilde{d}^{YX}$ are an equilibrium allocation rule, with induced sequences $\widetilde{L}_t^X$ for L_t^X and $\widetilde{\pi}_t^{iX}$ for π_t^{iX}. Consider any alternative feasible rule $\widehat{d}^{XY}$, $\widehat{d}^{YX}$ with induced $\widehat{L}_t^X$ and $\widehat{\pi}_t^{iX}$.

From worker optimization, we must have:

$$\begin{array}{c} \sum_{t=0}^{\infty} \beta^t [\widetilde{\pi}_t^{iX} w^X(\widetilde{L}_t^X) + (1 - \widetilde{\pi}_t^{iX}) w^Y(L - \widetilde{L}_t^X) \\ - \left(\widetilde{\pi}_t^{iX} \int (\mu + C) \widetilde{d}^{XY}(\widetilde{L}_t^X, \mu) g(\mu) d\mu + (1 - \widetilde{\pi}_t^{iX}) \int (\mu + C) \widetilde{d}^{YX}(\widetilde{L}_t^X, \mu) g(\mu) d\mu \right)] \\ \\ \geq \sum_{t=0}^{\infty} \beta^t [\widehat{\pi}_t^{iX} w^X(\widetilde{L}_t^X) + (1 - \widehat{\pi}_t^{iX}) w^Y(L - \widetilde{L}_t^X) \\ - \left(\widehat{\pi}_t^{iX} \int (\mu + C) \widehat{d}^{XY}(\widetilde{L}_t^X, \mu) g(\mu) d\mu + (1 - \widehat{\pi}_t^{iX}) \int (\mu + C) \widehat{d}^{YX}(\widetilde{L}_t^X, \mu) g(\mu) d\mu \right)] \end{array} \tag{9}$$

for $i = X, Y$. In other words, each worker maximizes her lifetime utility, taking the time-path of wages as given.

At the same time, by spot-market clearing, the incomes of employers are maximized with respect to L_t^i in each sector in each period taking the wage as given, so we also have:

$$\begin{array}{c} \sum_{t=0}^{\infty} \beta^t [p^X Q^X(\widetilde{L}_t^X) - w^X(\widetilde{L}_t^X)\widetilde{L}_t^X + p^Y Q^Y(L - \widetilde{L}_t^X) - w^Y(L - \widetilde{L}_t^X)(L - \widetilde{L}_t^X)] \\ \geq \sum_{t=0}^{\infty} \beta^t [p^X Q^X(\widehat{L}_t^X) - w^X(\widetilde{L}_t^X)\widehat{L}_t^X + p^Y Q^Y(L - \widehat{L}_t^X) - w^Y(L - \widetilde{L}_t^X)(L - \widehat{L}_t^X)]. \end{array} \tag{10}$$

[3]It may be helpful to offer a reminder that we are defining μ as $\varepsilon^a - \varepsilon^b$ for workers in a and as $\varepsilon^b - \varepsilon^a$ for workers in b, so that in each case it represents the idiosyncratic cost of leaving one's sector. Thus, it has the first form in the first integral of (8) and the second form in the second integral.

Multiplying (9) by L_0^i and adding (9) for $i = X, Y$ to (10), noting that $L_0^X \pi_t^{Xi} + L_0^Y \pi_t^{Yi} = L_t^i$, we have that (8) is larger for $\widetilde{d}^{XY}$, $\widetilde{d}^{YX}$ than for $\widehat{d}^{XY}$, $\widehat{d}^{YX}$. Q.E.D.

Therefore, studying optimization problem (8) can tell us about the equilibrium. Note that this is a dynamic analogue to the revenue function maximized by equilibrium in a static neoclassical trade model (see Dixit and Norman 1980, Ch. 2). Let $W(L^X)$ denote the maximized value of the objective function (8) from an initial value L^X of workers in X.

Since the problem is stationary, the usual dynamic programming logic will apply, and the function W can be computed by solving a Bellman equation. For any bounded function $\widetilde{W}$ on $[0, L]$, define the operator T by:

$$\begin{aligned} T(\widetilde{W})(L^X) \quad \equiv \quad & \max_{\{d^{XY}, d^{YX}\}} [p^X Q^X(L_t^X) + p^Y Q^Y(L - L_t^X) \qquad (11) \\ & - L_t^X \int (\mu + C) d^{XY}(L_t^X, \mu) g(\mu) d\mu \\ & - (L - L_t^X) \int (\mu + C) d^{YX}(L_t^X, \mu) g(\mu) d\mu \qquad (12) \\ & + \beta \widetilde{W}(L_{t+1}^X)], \end{aligned}$$

where $L_{t+1}^X = \int \{(1 - d^{XY}(L_t^X, \mu)) L_t^X + d^{YX}(L_t^X, \mu)(L - L_t^X)\} g(\mu) d\mu$. The Bellman equation is then $T(W) = W$. It is easy to show that T satisfies the usual Blackwell properties, and that as a result T has a unique fixed point, which can be found as the unique uniform limit of iterations on the Bellman equation starting from any bounded candidate function.

The following observation is helpful in characterizing the system's dynamics.

Proposition 7 *The function W is strictly concave.*

Proof. Consider any candidate concave value function W^0. By the contraction mapping property, $T^k(W^0)$ will converge uniformly to the unique solution W as $k \to \infty$. Now consider two different values for L_t^X, denoted $\widetilde{L}_t^X$ and $\widehat{L}_t^X$, $\widetilde{L}_t^X \neq \widehat{L}_t^X$, and any number $\alpha \in (0, 1)$. Let $(\widetilde{d}^{XY}, \widetilde{d}^{YX})$ denote an optimal choice for allocation rule when $L_t^X = \widetilde{L}_t^X$ and let the optimized value of the right-hand side of the Bellman equation be $\widetilde{W} \equiv T(W^0)(\widetilde{L}_t^X)$. Let $(\widehat{d}^{XY}, \widehat{d}^{YX})$ denote an optimal choice when $L_t^X = \widehat{L}_t^X$, and let the optimized value of the right-hand side of the Bellman equation be $\widehat{W} \equiv T(W^0)(\widehat{L}_t^X)$. Clearly, when $L_t^X = \overline{L}_t^X \equiv \alpha \widetilde{L}_t^X + (1 - \alpha) \widehat{L}_t^X$, then a feasible choice of allocation rule is $(\overline{d}^{XY}, \overline{d}^{YX}) \equiv ((\alpha \widetilde{d}^{XY} + (1 - \alpha) \widehat{d}^{XY}, \alpha \widetilde{d}^{YX} + (1 - \alpha) \widehat{d}^{YX})$. If W^0 is concave, then it is easy to confirm that when $L_t^X = \overline{L}_t^X$, setting the allocation rule equal to $(\overline{d}^{XY}, \overline{d}^{YX})$ will result in a value of the maximand greater than $\alpha \widetilde{W} + (1 - \alpha) \widehat{W}$ (strict inequality follows from the strict concavity of X^i). Therefore, the strict concavity of $T(W^0)$ follows. By the same reasoning, $T^k(W^0)$ is strictly concave for all k, and so W is concave. Finally, since $W = T(W)$, the argument just given ensures that the concavity of W is strict. Q.E.D.

This result tells us that the equilibrium is unique, since in strictly concave optimization problems the optimum is unique. It also tells us that the function $W'(L^X)$ is strictly decreasing in L^X. The following tells us that the value of this function is always equal to $v^X - v^Y$, which then gives us the result we were looking for.

Proposition 8 *In equilibrium at each date and in each state, $v^X - v^Y$ is equal to W'.*

Proof. First, note that in the optimization it is never optimal to have workers moving from i to j and at the same time other workers with lower values of μ who are remaining in i. (In that case, an equal number of workers from the two groups could have their actions reversed, leaving the future allocation of workers unchanged but reducing aggregate idiosyncratic moving costs.) Therefore, in an optimal allocation there is for each value of L^X at each date a number $\overline{\mu}^X$ such that $d^{XY}(L^X, \mu) = 1$ if $\mu < \overline{\mu}^X$ and $d^{XY}(L^X, \mu) = 0$ if $\mu > \overline{\mu}^X$. Simlarly, there is for each value of L^X at each date a number $\overline{\mu}^Y$ such that $d^{YX}(L^X, \mu) = 1$ if $\mu < \overline{\mu}^Y$ and $d^{XY}(L^X, \mu) = 0$ if $\mu > \overline{\mu}^Y$. This means that we can rewrite the Bellman equation as follows.

$$
\begin{aligned}
& W(L_t^X) \\
= \quad & \max_{\{\overline{\mu}^X, \overline{\mu}^Y\}} [p^X Q^X(L_t^X) + p^Y Q^Y(L - L_t^X) \\
& - \left(L_t^X \int_{-\infty}^{\overline{\mu}_t^X} (\mu + C) g(\mu) d\mu + (L - L_t^X) \int_{\infty}^{\overline{\mu}_t^Y} (\mu + C) g(\mu) d\mu \right) \\
& + \beta W((1 - G(\overline{\mu}_t^X)) L_t^X + G(\overline{\mu}_t^Y)(L - L_t^X))].
\end{aligned}
$$

The first-order conditions for this with respect to $\overline{\mu}^X$ and $\overline{\mu}^Y$ are

$$
\begin{aligned}
\overline{\mu}_t^X + C &= -\beta W(L_{t+1}^X)' \text{ and} \\
\overline{\mu}_t^Y + C &= \beta W(L_{t+1}^X)'.
\end{aligned}
\tag{13}
$$

But then from (1), the result follows. Q.E.D.

This tells us that in equilibrium, the attractiveness of either sector relative to the other is a strictly decreasing function of the number of workers who are located in that sector. Now we can use this to analyze the economy's dynamics.

4.2 Gradual adjustment to unanticipated changes.

The preceding analysis can be used to show a number of properties of the model's dynamic adjustment. First, labor market adjustments to any change, such as terms of trade shocks or policy changes, will be sluggish. In particular, suppose that the economy is in a steady state associated with an initial $s = s_I$. If a one-time shock occurs that results in a new $s = s_N$, the economy will not reach the steady state associated with s_N in finite time.

To see this, consider Figure 2. This illustrates the first-order condition (1), or equivalently, (13), for choice of $\overline{\mu}_t^Y$ and hence of L_{t+1}^X given the current value of L_t^X. The solid upward sloping curve indicates the locus of points $(L_{t+1}^X, \overline{\mu}_t^Y + C)$ such that $L_{t+1}^X = G(\overline{\mu}_t^Y)(L - L_t^X) + (1 - G(-\overline{\mu}_t^Y - 2C))L_t^X$. One can, thus, interpret it as the marginal cost curve for the supply of X-workers: the height is the moving cost for the marginal worker moving to X, given that the total number who wind up in X at the end of this period is equal to L_{t+1}^X. The downward sloping curve gives

$v^X_{t+1} - v^Y_{t+1} = W'(L^X_{t+1})$. This can be interpreted as the marginal benefit of moving a worker from Y to X. Given L^X_t, the values of L^X_{t+1} and of $\overline{\mu}^Y_t$ are determined as the intersection of these two curves.

Now, note that if $C > 0$, increasing L^X_t by Δ units shifts the marginal cost curve to the right at each point by an amount strictly between 0 and Δ. Since the marginal benefit curve is strictly decreasing, this implies an increase in L^X_{t+1} that lies strictly between 0 and Δ. This can be summarized in Figure 3, which shows the transition function that gives L^X_{t+1} as a function of L^X_t. Provided that $C > 0$, this curve must be strictly increasing, with a slope strictly less than 1. Thus, there is a unique steady state, and if the system begins at a point other than the steady state, it will move toward it without ever reaching or overshooting it.

This provides the result. For example, if the system is initially at a steady state with a high tariff that is expected to continue permanently, and then the tariff is suddenly removed never to be restored, then the system will move toward the new steady state each period, attaining it only in the limit.

Gradual labor-market adjustment to external shocks and policy changes, and the persistent wage differentials that they imply, have been documented empirically by, among others, Topel (1986), Blanchard & Katz (1992) and Rappaport (2000). They appear in the model of Davidson and Matusz (2001), due to re-training and search delays and exogenous rates of individual job separation. They can also be rationalized in theory by or convex training costs for labor (as in Karp and Paul (1994) or Dehejia (1997), in analogy with convex adjustment costs for capital as in Mussa (1978)). Here, they result from the presence of time-varying idiosyncratic shocks to workers. Even if a worker is suffering low wages as a result of loss of protection to that worker's sector, it will often be in that worker's interest to wait until her personal moving costs are sufficiently low before leaving the sector.

4.3 Anticipatory adjustment to pre-announced changes.

Another feature of the model's dynamics is anticipatory adjustment. Suppose the economy is in an initial steady state s_I. At time $t = 0$, the government announces a surprise policy change—elimination of a tariff that had been protecting sector Y—starting at some date $t^* > 0$. We will see that anticipatory net outflows of labor from sector Y will begin immediately from the time of the announcement.

Clearly, we need to reintroduce the state variable s. Let $s_t = p'$ for $t < t^*$ and $s_t = p''$ for $t \geq t^*$, with $p'' < p'$. Then let $p^Y_t = s_t \forall t$. Note that $w^X_2(L^X, s) < 0$ and $w^Y_2(L - L^X, s) > 0$. (The denominator in each case is a cost-of-living index that increases with an increase in p^Y, with an elasticity less than 1.)

Lemma 1 *Assume that $C > 0$. (i) Consider p' and p'', with $p'' < p'$, and let $\widetilde{W}$ be a bounded, concave, differentable function on $[0, L]$. Then $\partial T(\widetilde{W})(L^X, p'')\partial L^X > \partial T(\widetilde{W})(L^X, p')\partial L^X$. (ii) Let $\widetilde{W}$ and $\widehat{W}$ be bounded, concave, differentable functions on $[0, L]$, with $\widetilde{W}' > \widehat{W}'$ everywhere. Then $\partial T(\widetilde{W})(L^X, p)\partial L^X > \partial T(\widehat{W})(L^X, p)\partial L^X$ for any p.*

Proof. By the envelope theorem, we have:

$$\begin{aligned}\partial T(\widetilde{W})(L_t^X, s_t)/\partial L_t^X &= w^X(L_t^X, s_t) - w^Y(L - L_t^X, s_t) \\ &\quad - \int_{-\infty}^{\overline{\mu}^X} (\mu + C) g(\mu) d\mu + \int_{\infty}^{\overline{\mu}^Y} (\mu + C) g(\mu) d\mu \\ &\quad + \beta W'(L_{t+1}^X, s_{t+1})[1 - G(\overline{\mu}_t^X) - G(\overline{\mu}_t^Y)].\end{aligned}$$

Using (13) and (3) and rearranging, this becomes the following.

$$\partial T(\widetilde{W})(L_t^X, s_t)/\partial L_t^X = w^X(L_t^X, s_t) - w^Y(L - L_t^X, s_t) + \Omega(-\overline{\mu}_t^Y - 2C) - \Omega(\overline{\mu}_t^Y) + \overline{\mu}_t^Y + C. \tag{14}$$

(i) If we replace $s_t = p'$ with $s_t = p''$, the only terms that change are the first two, with a rise in the real wage in X and a fall in the real wage in Y. This proves the result. (ii) If we replace $\widehat{W}$ with $\widetilde{W}$, then in the first-order condition for the optimization in $T(\widetilde{W})(L^X, p)$, the marginal benefit curve shifts up (recall Figure 2). Thus, for any L_t^X, we have a rise in the value of $\overline{\mu}_t^Y$ that is chosen. The first two terms of (14) are unchanged. The derivative of the last three terms with respect to $\overline{\mu}_t^Y$ is equal to $-G(-\overline{\mu}_t^Y - 2C) - G(\overline{\mu}_t^Y) + 1$, which by the symmetry of the distribution of μ is equal to $G(\overline{\mu}_t^Y + 2C) - G(\overline{\mu}_t^Y) > 0$. Therefore, the value of (14) has gone up, proving the result. Q.E.D.

Now, consider a model in which $p_t^Y \equiv p'$ and call it model I (for 'initial'), with value function W^I and steady-state value of L^X equal to L_I^X. Consider in the same way a model in which $p_t^Y \equiv p''$ and call it model N (for 'new'), with value function W^N and steady-state value of L^X equal to L_N^X. The two comparative statics results just derived, applied to the recursions on the Bellman operator T, imply that $W_1^I < W_1^N$ and $L_I^X < L_N^X$.

Now, return to the problem of the model with the announced policy change at time t^*. From time t^* on, the trasition function and the value function will be exactly as they are in model N. Consider date $t^* - 1$. For a given value of L^X, the value function for date $t = t^* - 1$ is given by $T(W^N)(L^X, p')$ (the next–period value function is W^N, but the current value of p^Y is p'). By the lemma, we can conclude:

$$W_1^I(L^X, p') < \partial T(W^N)(L^X, p')/\partial L_t^X < W_1^N(L^X, p'').$$

The first inequality results from part (ii) of the Lemma because $W^I = T(W^I)$, and $W_1^I < W_1^N$. The second inequality results from part (i) of the Lemma because $p' > p''$.

However, referring again to the first-order condition for the choice of L_{t+1}^X given L_t^X (see Figure 2), we see that (**??**) implies that the transition function for period $t^* - 1$ lies strictly in between the transition function for model I and that for model N. This is illustrated in Figure 4. By the same logic, the transition function for period $t^* - 2$ must lie strictly between that for $t^* - 1$ and that for model I, and so on. Then if we were already in a steady state of model I and it was announced (to everone's surprise) at date 0 that the tariff would be removed at date t^*, the dynamics of the system would follow the path indicated in Figure 4, drawn for the assumption that $t^* = 3$. Adjustment toward the new steady state would begin immediately at date 0, and would continue permanently, always moving toward the new steady state but never reaching it.

Of course, anticipatory labor adjustments also imply anticipatory wage changes. In particular, anticipatory outflows from Y prior to the actual tariff removal will progressively raise w^Y up to the time of the policy change, at which point it will fall discretely, and will progressively push down w^X up to the time of the policy change, at which point it will rise discretely due to the drop in the consumer's price index. This has obvious implications for empirical analyses of the impact of trade liberalization on wages.

In particular, suppose that a researcher obtains data on wages and employment levels at dates $t^* - 1$ and $t^* + 1$, and compares the values before and after the liberalization. If the differences thus observed are interpreted to be the effects of the liberalization, then because the anticipatory effects are omitted, the study will greatly overestimate the wage effects and underestimate the sectoral employment effects.

5 Welfare and Incidence.

A large part of the reason for studying the workings of a model with labor mobility costs is to refine our understanding of who gains and who is hurt from a change in trade policy. Here we will look at two different angles of this question. First, a simple envelope result shows how the effect on a given worker can be expressed in terms of flow probabilities and changes in wages only. This result shows the importance of gross flows in analyzing incidence. Second, we derive some results on how delayed trade liberalization can affect who gains and who loses from a reform.

5.1 An envelope result.

Return again to the case with constant $s_t = p^Y$. Consider again an unannounced and permanent change in tariff, which changes the value of p^Y once and for all. Returning to (2), we can see that the change in the utility of a worker in sector i would be:

$$dv_t^{i*}/dp^Y = dw_t^{i*}/dp^Y + \beta dv_{t+1}^{i*}/dp^Y + \Omega'(\overline{\mu}_t^i)d\overline{\mu}_t^{i*}/dp^Y,$$

where w_t^{i*}, v_t^{i*} and $\overline{\mu}_t^{i*}$ denote the equilibrium values of the wage, worker's utility, and moving threshold in sector i and at date t respectively. Noting that $\Omega'(\mu) = G(\mu)$, this means:

$$dv_t^{i*}/dp^Y = dw_t^{i*}/dp^Y + \beta dv_{t+1}^{i*}/dp^Y + G(\overline{\mu}_t^i)\beta[dv_{t+1}^{j*}/dp^Y - dv_{t+1}^{i*}/dp^Y], \text{ or}$$

$$dv_t^{i*}/dp^Y = dw_t^{i*}/dp^Y + \beta dv_{t+1}^{i*}/dp^Y(1 - G(\overline{\mu}_t^i)) + \beta G(\overline{\mu}_t^i)dv_{t+1}^{j*}/dp^Y.$$

Recalling (5) and following the recursive logic forward, this becomes:

$$dv_t^{i*}/dp^Y = \sum_{n=0}^{\infty} \beta^n \sum_{k=X,Y} \pi_{t+n}^{ik} dw_{t+n}^{k*}/dp^Y,$$

where as before π_t^{ik} is the probability that a worker who was in i at time 0 will be in k at time t. Thus, despite the moving costs, a properly constructed discounted sum of wages alone, using gross flows to average between sectors, is sufficient for evaluating incidence.

Note again that this has implications for empirical work. It suggests that looking at the level of sectoral wages either in the short run or in the long run is not in general the right approach. In particular, it is quite possible to construct examples in which a drop in tariff lowers wages in sector Y in the short run and in the long run, and yet every worker in the economy, benefits, including those in Y.[4] The reason is that real wages in X rise by enough, and the economy is fluid enough, that current Y workers expect to make up in future employment in X for what they have lost in Y. This also stands in stark contrast to the convex-adjustment cost approach (Karp and Paul, 1994; Dehejia, 1997); in equilibrium in those models, a Y-worker must be indifferent between leaving Y and remaining there permanently, and so would definitely be worse off if Y wages were to fall permanently.

5.2 The effects of policy delay on incidence.

Trade liberalization measures are usually phased in over time. For example, the elimination of the Multi-Fibre Arrangement in the Uruguay round was scheduled to be phased in over ten years, with most of the reduction loaded at the end of the phase-in period. One reason for this is to soften the effects of the reform on workers likely to be hurt by it, giving them time to adjust and perhaps removing an incentive to oppose the reform politically. This motive is studied by Dehejia (2003), in a Heckscher-Ohlin model with convex adjustment costs for workers (and zero steady-state flows of workers). In that model, it is shown that gradual phase-in can bring all workers behind a trade liberalization that would otherwise have been opposed by import-competing workers, making the reform politically feasible.

We will here study the effect of delayed trade liberalization in the present model. To study a simple version of the problem that makes the mechanisms clear, we focus on a stark liberalization that brings the economy from autarchy to free trade. Beginning from an autarchic steady state, the opening of trade may either occur immediately or be announced (with full commitment) to occur at a later date. In contrast to Dehejia, we find that delay does not, in general, soften the blow of trade liberalization to workers. What it does do, if the period of delay is long enough, is to unite all workers, so that they all either benefit from trade or lose from it. To anticipate, it turns out that if an increase in labor supply increases the economy's long-run relative supply of export goods, then delay fosters unification of workers *behind* free trade, but if if an increase in labor supply decreases long-run relative

[4] For example, consider a model with Ricardian technology. If $Q^i(L_t^i, s_t) \equiv A^i L_t^i$ for positive constants A^i, $i = X, Y$, then $w^X = A^X/\phi(p^Y)$ and $w^Y = p^Y A^Y/\phi(p^Y)$. Suppose that the two goods are perfect substitutes in consumption so that $\phi(p) \equiv \min\{1, p\}$. Then for $p^Y > 1$, $dw^X/dp^Y = 0$ and $dw^Y/dp^Y = A^Y$. Further, as $p^Y \to \infty$, $\pi_t^{iY} \to 1 \forall t > 0$. Thus, the difference in the welfare of an X worker resulting from a rise in the price of Y from p' to $p'' > p'$, $\int_{p'}^{p''} dv_t^{X*}/dp^Y dp^Y$, can be made arbitrarily large by making p'' sufficiently large (the integrand approaches a positive limit as $p'' \to \infty$). Thus, if the initial price of Y is below unity and there is enough of an increase in it, X sector workers are made better off even though X-sector real wages fall in the short run and in the long run.

supply of export goods, delay fosters unification of workers *in opposition* to free trade.

Consider an economy that is as of period 0 in an autarchic steady-state with a relative price of good Y given by $p = p'$. At date 0 it is announced that the economy will be opened up to free trade as of date T (which could be equal to zero, representing the case of unanticipated liberalization). Suppose that the world relative price of good Y is given by $p = p''$. It is useful to rewrite the period-t value function for the planner's problem that the equilibrium solves as $W^t(L_t^X, L_t^Y; p', p'', T)$. (Of course, the value function is not stationary before T, although it will be afterward, hence the time superscript for the value function.) Let $T = \infty$ represent the case in which no trade liberalization is announced.

It is straightforward to check from the envelope theorem applied to the Bellman equation for the planner's problem that the payoff to an individual worker in the X sector at the beginning of period 0, just after the policy announcement, is equal to $v_0^X = W_1^0(L_t^X, L_t^Y; p', p'', T)$, or the marginal value of an X worker from the planner's point of view. Similarly, the payoff to a Y worker is given by $v_0^Y = W_2^0$. Given that, the question of the value of advance notice is essentially the question of whether or not an increase in T changes the sign of $W_i^0(L_t^X, L_t^Y; p', p'', T) - W_i^0(L_t^X, L_t^Y; p', p'', \infty) = W_i^0(L_t^X, L_t^Y; p', p'', T) - W_i^0(L_t^X, L_t^Y; p', p', \infty)$ for $i = 1, 2$. Call this difference an i worker's 'willingness to consent.' It is difficult to obtain general results on this, but if one can find results on the cross derivative $W_{i4}^0(L_t^X, L_t^Y; p', p', T)$, or the derivative of a worker's payoff with respect to the world price, evaluated at a value of the world price equal to the domestic autarchic price, then (since $W_i^0(L_t^X, L_t^Y; p', p', T) - W_i^0(L_t^X, L_t^Y; p', p', \infty) = 0$) one has signed the i worker's willingness to consent in an interval for p'' that includes p'. That is the approach taken here.

Denote the equilibrium employment in sector i at time t, as a function of initial labor stocks, by $L_t^i(L_0^X, L_0^Y)$. Denote the steady state employment similarly by $L_\infty^i(L_0^X, L_0^Y)$. Define $q_t^i(L_0^X, L_0^Y)$ and $q_\infty^i(L_0^X, L_0^Y)$ respectively as output in the i sector at time t and in the steady state. (In general these functions would be conditioned on p', p'', and T as well as L_0^X, L_0^Y, but we will need to evaluate these functions only at the point $p'' = p'$, so these additional arguments will be suppressed.) The following will be useful.

Lemma. Assume that the value function is twice continuously differentiable in

(L_t^X, L_t^Y). Then:

(i) The functions L_t^X and L_t^Y are differentiable in (L_0^X, L_0^Y).

(ii) The derivatives $L_{t1}^X \equiv \partial L_t^X(L_0^X, L_0^Y)/\partial L_0^X$ and $L_{t1}^Y \equiv \partial L_t^Y(L_0^X, L_0^Y)/\partial L_0^X$ are all non-negative, and $L_{t1}^X + L_{t1}^Y = 1 \forall t > 0$. Further, L_{t1}^X is decreasing in t.

(iii) The derivatives L_{t2}^X and L_{t2}^Y are all non-negative, and $L_{t2}^Y + L_{t2}^X = 1 \forall t > 0$. Further, $L_{t2}^X$is increasing in t.

(iv) $L_{t1}^i \to L_{\infty 1}^i$ as $t \to \infty$, for $i = X, Y$.

Parts (i) and (iv) are technical preliminaries. The result in (iv) requires proof, since even when a series of functions converges uniformly to a limit function, in general the sequence of derivatives does not converge to the derivative of the limit series. In this case the result follows because the functions are solutions to an optimisation problem. Parts (ii) and (iii) follow directly from the model's dynamics. For example, if one was to add some workers to X in period 0, that would result in more X workers in each period, but the number of X workers would fall over time, as workers reallocate toward the Y sector. With a slight abuse of notation, we will write $L_{01}^X = 1$, $L_{02}^X = 0$, $L_{01}^Y = 0$, and $L_{01}^Y = 1$.

Henceforth, we will assume that the planner's value function is twice continuously differentiable in all arguments.

With this notation, the effect on the payoff of a worker in X is:

$$\frac{\partial v_0^X}{\partial p''}|_{p''=p'} = W_{14}^0(L_t^X, L_t^Y; p', p', T) = W_{41}^0(L_t^X, L_t^Y; p', p', T)$$

by Young's theorem, which by the envelope theorem becomes:

$$\frac{\partial}{\partial L_0^X}\sum_{t=T}^{\infty}\beta^t\left[\frac{\partial}{\partial p''}\left(\frac{1}{\phi(p'')}\right)q_t^X + \frac{\partial}{\partial p''}\left(\frac{p''}{\phi(p'')}\right)q_t^Y\right]|_{p''=p'}$$

$$= \frac{1}{\phi(p')}\sum_{t=T}^{\infty}\beta^t\left[(1-\alpha)q_{t1}^Y - \frac{\alpha}{p'}q_{t1}^X\right]$$

$$\equiv \frac{1}{\phi(p')}\sum_{t=T}^{\infty}\beta^t B_t,$$

where $\alpha \equiv p'\phi'(p')/\phi(p')$ is the share of good Y in autarchic consumption expenditure.[5] Note that this provides a simple way of analyzing the effects of delay; the *only* effect of an increase in T on this exression is to eliminate some to the terms from the summation. This is the key to the results that follow.

Since q_{t1}^X is decreasing in t and q_{t1}^Y is increasing in t, B_t is increasing in t. Since $\frac{\alpha}{p'(1-\alpha)}$ is the autarchic steady-state ratio of Y consumption to X consumption (and hence the ratio of production as well), $B_\infty \equiv \lim_{t\to\infty} B_t > 0$ if and only if an increase in the economy's total labor supply would increase q_∞^Y/q_∞^X at a fixed world price of $p'' = p'$, leading to exports of Y, and $B_\infty < 0$ if an increase in labor supply would lead to imports of Y. In addition, $q_{01}^Y = 0$, since an exogenous increase in the stock of labor in one sector cannot have a contemporaneous effect on output in the other sector. Thus, $B_0 < 0$.

As a result, if $B_\infty < 0$, then $B_t < 0$ for all t, and $\frac{\partial v_0^X}{\partial p''}|_{p''=p'} < 0$. However, if $B_\infty > 0$, then B_t is positive for t below some threshold and negative for t above the threshold. The implication is that $\exists \bar{t}$ such that if $T \geq \bar{t}$, $\frac{\partial v_0^X}{\partial p''}|_{p''=p'} > 0$.

Analogously, it can be seen that

$$\frac{\partial v_0^Y}{\partial p''}|_{p''=p'} = \frac{1}{\phi(p')}\sum_{t=T}^{\infty}\beta^t D_t,$$

where $D_t \equiv (1-\alpha)q_{t2}^Y - \frac{\alpha}{p'}q_{t2}^X$. Clearly, since $q_{t2}^X = 0$, q_{t2}^X is increasing in t, and q_{t2}^Y is decreasing in t, while q_{t2}^X and q_{t2}^Y are non-negative for all t, $D_0 > 0$ and D_t is

[5] The switch in the order of differentiation is analogous to the demonstration that in a static trade model derivatives of factor prices with respect to output derivates is dual to the derivative of output with respect to factor prices, as in Dixit and Norman (1980, pp.54-55).

decreasing in t. Further, by part (iv) of the lemma and the uniqueness of the steady state which implies that $L^X_{\infty 1} = L^X_{\infty 2}$ and $L^Y_{\infty 1} = L^Y_{\infty 2}$, we conclude that $D_\infty = B_\infty$. This all implies that $B_t < D_t$ for all t. Given that $B_0 < 0 < D_0$, we conclude that $\frac{\partial v_0^X}{\partial p''}|_{p''=p'} < \frac{\partial v_0^Y}{\partial p''}|_{p''=p'}$.

By extension of the logic just used, if $D_\infty < 0$, then $\exists \bar{t}$ such that if $T \geq \bar{t}$, $\frac{\partial v_0^Y}{\partial p''}|_{p''=p'} < 0$. Further, if $B_\infty = D_\infty = 0$, then $B_t < 0 < D_t$ for all t, and $\frac{\partial v_0^X}{\partial p''}|_{p''=p'} < 0 < \frac{\partial v_0^Y}{\partial p''}|_{p''=p'}$ regardless of T.

In addition, since $B_0 < 0 < D_0$, it is clear that increasing T from 0 to 1, by chopping off a term that is negative in the first case and positive in the second, increases $\frac{\partial v_0^X}{\partial p''}|_{p''=p'}$ and decreases $\frac{\partial v_0^Y}{\partial p''}|_{p''=p'}$. Thus, workers in the export sector are hurt by a bit of delay, while workers in the import competing sector benefit from a bit of delay.

This can all be summarized as follows. The first proposition treats the case with $B_\infty \neq D_\infty$, and the second treats the case with $B_\infty = D_\infty$.

Proposition 9 *Suppose that, at the autarchy relative price* p', *steady state relative supply of the export good is increased [decreased] by an increase in labor supply. Then, there is an open interval containing* p' *such that if the world price* p'' *is in that interval:*

(i) Workers in the export [import-competing] sector as of period 0 *will benefit from [be hurt by] immediate unanticipated opening of trade.*

(ii) The net benefit to workers in the import-competing sector from the opening up of trade is strictly less than the benefit to export sector workers, whether the opening is delayed or not.

(iii) A one-period delay hurts export-sector workers and benefits import-competing workers.

(iv) A sufficiently long delay before the opening of trade will make workers in both sectors net beneficiaries [net losers] from trade.

Proposition 10 *Suppose that, at the autarchy relative price* p', *steady state relative supply of the export good is unchanged at the margin by an increase in labor supply. Then there is an open interval containing* p' *such that if the world price* p'' *is in that interval:*

(i) Workers in the export sector as of period 0 *will benefit from the opening of trade, whether it is delayed or not.*

(i) Workers in the import-competing sector as of period 0 *will be hurt by the opening of trade, whether it is delayed or not.*

(iii) A one-period delay hurts export-sector workers and benefits import-competing workers.

Some examples. Four simple special cases illustrate the different effects delay can have. For all examples, assume that $p'' > p'$, so that Y is the export good, and that the difference between the world price and the domestic autarchy price is small enough that the propositions apply.

(i) *Ricardian technology.* If $Q^i(L^i_t, s_t) \equiv A^i L^i_t$ for positive constants A^i, $i = X, Y$, an increase in total labor supply will result in an equiproportionate increase in output of both sectors. (Note that w^X and w^Y are both independent of labor supplies, and so by (7) $\overline{\mu}$ is unchanged by a change in labor supply.) Therefore, $B_\infty = 0$, and in this case delay has no role: export sector workers benefit from trade and import-competing workers are hurt by it, regardless of T.

(ii) *Inelastic import-competing labor demand.* Suppose that both X and Y are produced by a sector-specific asset in fixed supply (K^X and K^Y respectively) together with labor. Suppose further that the production function in X is Leontieff: $Q^X(K^X, L^X) = \min\{K^X, L^X\}$, and that the production function for Y is CES: $Q^Y(L^Y, K^Y) = ((L^Y)^\rho + (K^Y)^\rho)^{1/\rho}$, with $\rho \in (-\infty, 0)$ (implying an elasticity of substitution equal to $1/(1-\rho)$). In this case, $q^X_{t1} = q^X_{t2} = 0 \forall t$. Therefore, $B_t, D_t \geq 0 \forall t$, with strict inequality for $t > 0$, so both groups of worker will benefit from trade with or without delay. The interpretation is that the increased labor demand in the export sector forces wages up in the import-competing sector because of its inelastic labor demand, benefitting all workers.

(iii) *Inelastic export-sector labor demand.* Now, reverse the production functions for the two sectors. In this case, $q^Y_{t1} = q^Y_{t2} = 0 \forall t$. Therefore, $B_t, D_t < 0 \forall t$, with strict inequality for $t > 0$, so both groups of worker will be hurt by trade with or without delay. The interpretation is that the reduced labor demand in the import-competing sector forces wages down in the export sector because of its inelastic labor demand, hurting all workers. In this example, all of the gains from trade are captured by the owners of the fixed factors.

(iv) *An example in which delay tips the scales in favor of trade.* Now consider a general CES specification, in which $Q^i(L^i, K^i) = ((L^i)^{\rho^i} + (K^i)^{\rho^i})^{1/\rho^i}$, $i = X, Y$, with $\rho^i \in (-\infty, 0)$. Example (ii) shows that with any finite ρ^Y and with ρ^X sufficiently large and negative, import-competing workers strictly benefit from an unannounced liberalization. Example (iii) shows that with any finite ρ^X and with ρ^Y sufficiently large and negative, import-competing workers are strictly hurt by an unannounced liberalization. Choose any two such parameter pairs, and connect them with a curve in (ρ^X, ρ^Y) space. There must be a point on the curve at which import-competing workers are indifferent between a sudden trade opening and the autarchic steady state. For this parameter pair, by part (ii) of Proposition 9, export workers are strict beneficiaries of sudden trade. In addition, export workers will remain net beneficiaries of trade if the opening is delayed (D_∞ must be positive, because in order for X workers to be indifferent, B_∞ must be positive; but then $D_t > 0 \forall t$). Furthermore, since $B_0 < 0$ and B_t is increasing in t, any delay will make the X workers strict beneficiaries of trade. Thus, an immediate liberalization would benefit Y workers but not X workers, but a delayed liberalization would strictly benefit both classes of worker. (A slight perterbation of the (ρ^X, ρ^Y) pair could then make the X workers stictly hurt by immediate trade opening, and strict beneficiaries of delayed opening.)

The interpretation of this result is as follows. An unexpected trade opening pushes down the real wages of X workers immediately, while pushing up the real wages of Y workers. The X workers move only gradually to take advantage of the higher Y wages because of the moving costs. However, if the trade opening is announced in advance, X workers who happen to have low moving costs at the moment begin moving to Y *in anticipation.* This makes X workers more scarce, pushing up wages for those who do not move (while pushing Y wages down), even though no change in output prices has yet occurred. If the elasticity of labor demand in the X sector is sufficiently low, this anticipatory wage increase is the dominant effect.

(v) *An example in which delay tips the scales against trade.* Exactly the same logic as in (iv) can be used to construct a case in which *export* sector workers are indifferent between immediate trade and the autarchic steady state. In this case, by part (ii) of Proposition 9, import-competing workers are strictly hurt by trade whether it is delayed or not (since B_∞ must be negative, because in order for Y workers to be indifferent, D_∞ must be negative; but then $B_t < 0 \forall t$). Furthermore, since $D_0 > 0$ and D_t is decreasing in t, any delay will make the Y workers strictly harmed by trade. Thus, in the event of an immediate liberalization, workers would be divided over trade, while delay would unite all workers in their opposition. (Again, a slight perturbation of the example can make Y workers strict beneficiaries of immediate trade.)

The interpretation of this example is as follows. From the point of view of the Y workers, immediate trade has three effects. It lowers real wages in the X sector; it provides an immediate increase in real wages in the Y sector; and it pushes a certain number of X workers out of the X sector and into the Y sector over time, pushing Y wages down. If labor demand in Y is sufficiently inelastic, the Y wage will be lower in the long run than it was under autarchy. Thus, in this example, for a Y worker, the benefit from free trade is short-lived, and must be weighed against a long-run cost. On the other hand, if the trade opening is announced in advance, X workers begin moving into Y right away, pushing Y wages down *even before output prices change*. Thus, Y workers lose the benefit of the short-run wage increase that they would have enjoyed under unanticipated trade, and jump immediately to the long-run cost of increased competition from former X workers.

These examples serve to illustrate the range of possible outcomes. Except in the knife-edge case with $B_\infty = D_\infty = 0$, delay tends to make workers unanimous in their stance toward trade, but whether it is a positive or negative stance depends on the relative responsiveness of labor demand in the two sectors. Of course, whether delay in any given real world liberalization event is likely to turn workers into beneficiaries or victims of trade is an empirical matter. Simulation of a less simplified version of the model with parameters calibrated to data can help; this is left to later work.

6 Conclusion.

We have studied a simple trade model with costly labor mobilty and idiosyncratic moving-cost shocks. The model is shown to have non-trivial dynamics, including gradual adjustment and anticipatory effects, resulting solely from fixed moving costs plus time-varying idiosyncratic moving cost shocks to individual workers. A key conclusion is that the steady state does not resemble a model with frictionless labor mobility. Specifically, intersectoral wage differentials persist permanently, making workers more expensive in larger sectors. In addition, trade liberalization lowers the long-run wage in the import-competing sector relative to the export sector.

Thus, the trade economist's habit of assuming that a frictionless model will be a good predictor of long-run trade effects is called into question. A companion paper (Chaudhuri and McLaren (2002)) makes a similar point in a two-factor version of the model, where the two factors are skilled and unskilled labor, and where the points of conflict between the frictionless model and the steady state of the costly-mobility model are richer and more numerous.

7 Appendix.

Lemma. Assume that the value function is twice continuously differentiable in (L_t^X, L_t^Y).

Then:

(i) The functions L_t^X and L_t^Y are differentiable in (L_0^X, L_0^Y).

(ii) The derivatives $L_{t1}^X \equiv \partial L_t^X(L_0^X, L_0^Y)/\partial L_0^X$ and $L_{t1}^Y \equiv \partial L_t^Y(L_0^X, L_0^Y)/\partial L_0^X$ are all non-negative, and $L_{t1}^X + L_{t1}^Y = 1 \forall t > 0$. Further, L_{t1}^X is decreasing in t.

(iii) The derivatives L_{t2}^X and L_{t2}^Y are all non-negative, and $L_{t2}^Y + L_{t2}^X = 1 \forall t > 0$. Further, $L_{t2}^X$is increasing in t.

(iv) $L_{t1}^i \to L_{\infty 1}^i$ as $t \to \infty$, for $i = X, Y$.

Proof. First, note that since $\overline{\mu}_t^X = -\overline{\mu}_t^Y - 2C$, once $\overline{\mu}_t^X$ has been specified, $\overline{\mu}_t^Y$ can be computed from it, and then L_{t+1}^X and L_{t+1}^Y can be computed from $\overline{\mu}_t^X$, L_t^X and L_t^Y by (6). We can thus write L_{t+1}^X as a function of $\overline{\mu}_t^X$, conditional on L_t^X and L_t^Y. Note that it is a strictly decreasing function, allowing us to define its inverse: $\overline{\mu}^X(L_{t+1}^X; L_t^X, L_t^Y)$, which is clearly differentiable.

Part (i) can be seen as follows. We can write the planner's first-order condition (13) in this form:.

$$\overline{\mu}^X(L_{t+1}^X; L_t^X, L_t^Y) + C = \beta[V_1(L_{t+1}^X, L_{t+1}^Y) - V_2(L_{t+1}^X, L_{t+1}^Y)], \text{ or}$$

$$\begin{aligned} & \overline{\mu}^X(L_{t+1}^X; L_t^X, L_0^X + L_0^Y - L_t^X) + C \\ = \; & \beta[V_1(L_{t+1}^X, L_0^X + L_0^Y - L_{t+1}^X) - V_2(L_{t+1}^X, L_0^X + L_0^Y - L_{t+1}^X)]. \end{aligned}$$

The differentiability of L_1^X can be inferred by applying the Implicit Function Theorem to this equation for $t = 0$. Given the differentiability of $L_{t'}^X$, the differentiability of $L_{t'+1}^X$ can be inferred by applying the same logic to the equation for $t = t'$. Thus, the result follows by induction.

Parts (ii) and (iii) simply follow from our results on the dynamics of adjustment, plus the requirement that $L_t^X + L_t^Y = L_0^X + L_0^Y \forall t$.

To see (iv), differentiate the planner's first-order condition with respect to L_0^X to get:

$$\overline{\mu}_1^X L_{t+1,1}^X + \overline{\mu}_2^X L_{t1}^X + \overline{\mu}_3^X L_{t1}^Y = \beta[V_{11} L_{t+1,1}^X + V_{12} L_{t+1,1}^Y - V_{21} L_{t+1,1}^X - V_{22} L_{t+1,1}^Y], \text{ or}$$

$$(\overline{\mu}_2^X - \overline{\mu}_3^X) L_{t1}^X = (\beta[(V_{11} - V_{12} - V_{21} + V_{22})] - \overline{\mu}_1^X) L_{t+1,1}^X + \beta(V_{12} - V_{22}) - \overline{\mu}_3^X,$$

where the derivatives of $\overline{\mu}_1^X$ are evaluated at $(L_{t+1}^X; L_t^X, L_t^Y)$ and the derivatives of the value function are evaluated at (L_{t+1}^X, L_{t+1}^Y). Now, since L_{t1}^X is positive but decreasing in t, it must take a limit, say, η. Taking limits of this equation as $t \to \infty$, we find:

$$\eta = \frac{\beta(V_{12} - V_{22}) + \overline{\mu}_3^X}{\overline{\mu}_2^X - \overline{\mu}_3^X - \beta[(V_{11} - V_{12} - V_{21} + V_{22})},$$

where the derivatives of $\overline{\mu}^X$ are evaluated at $(L^X_\infty; L^X_\infty, L^Y_\infty)$ and the derivatives of the value function are evaluated at (L^X_∞, L^Y_∞). Now, noting that the steady-state values must satisfy

$$\overline{\mu}^X(L^X_\infty; L^X_\infty, L^Y_\infty) + C = \beta[V_1(L^X_\infty, L^Y_\infty) - V_2(L^X_\infty, L^Y_\infty)],$$

we can differentiate this, and, using $L^X_\infty + L^Y_\infty = L^X_0 + L^Y_0$ (and thus $L^X_{\infty 1} + L^Y_{\infty 1} = 1$), solve for $L^X_{\infty 1}$. But this then yields $L^X_{\infty 1} = \eta$. **Q.E.D.**

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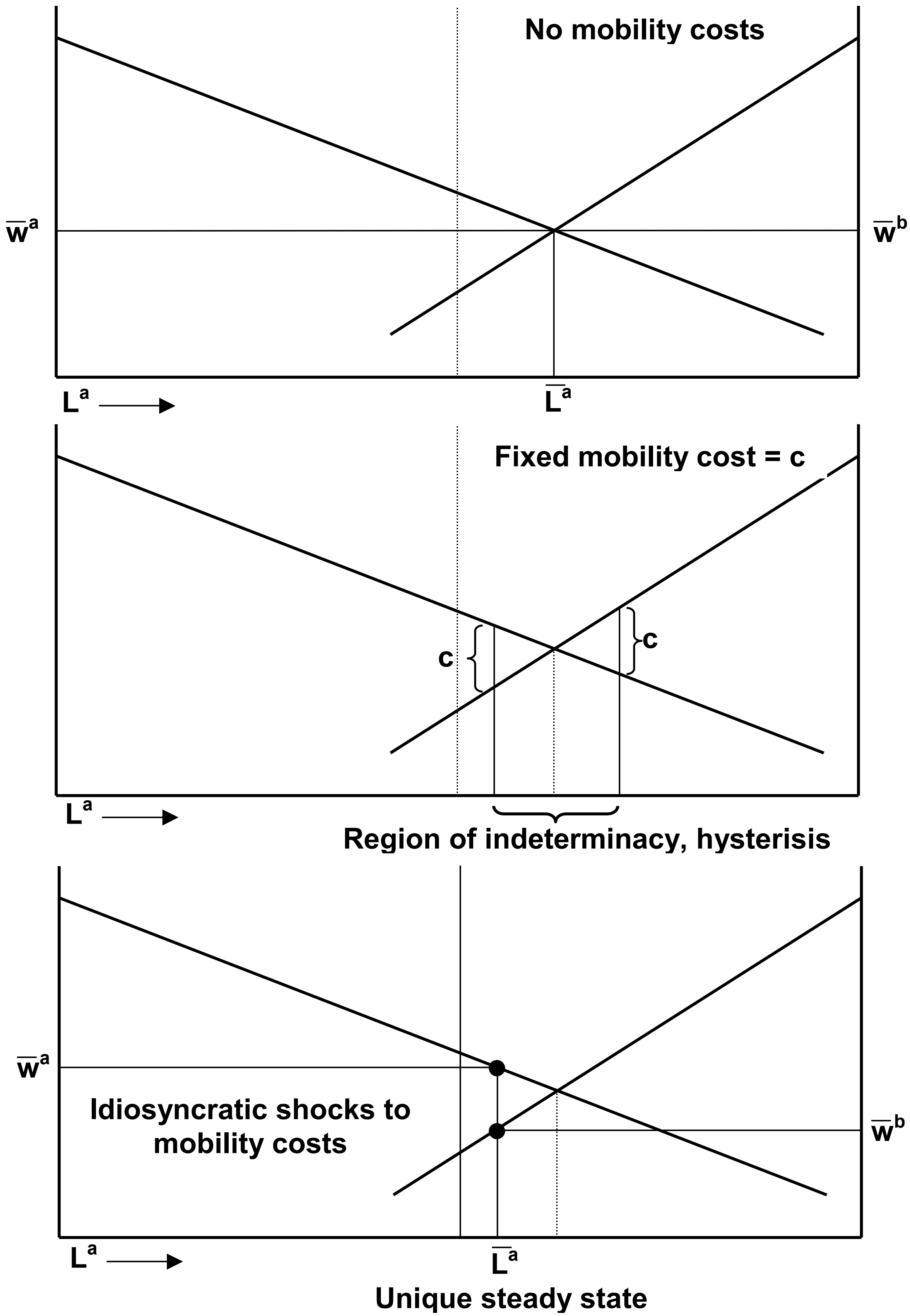
Steady state labor allocations under alternative assumptions about mobility costs



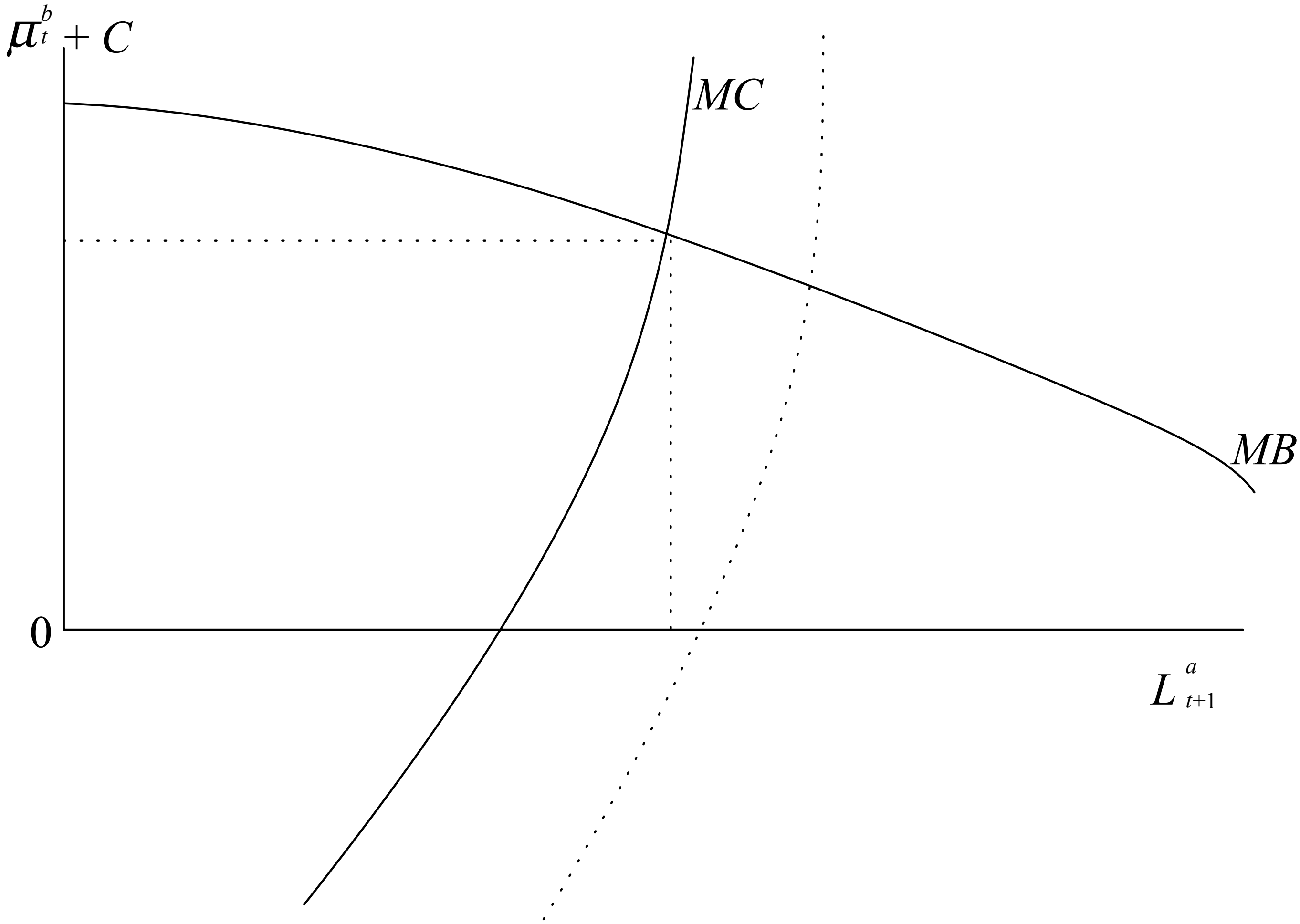


Figure 2: The First-order Condition.

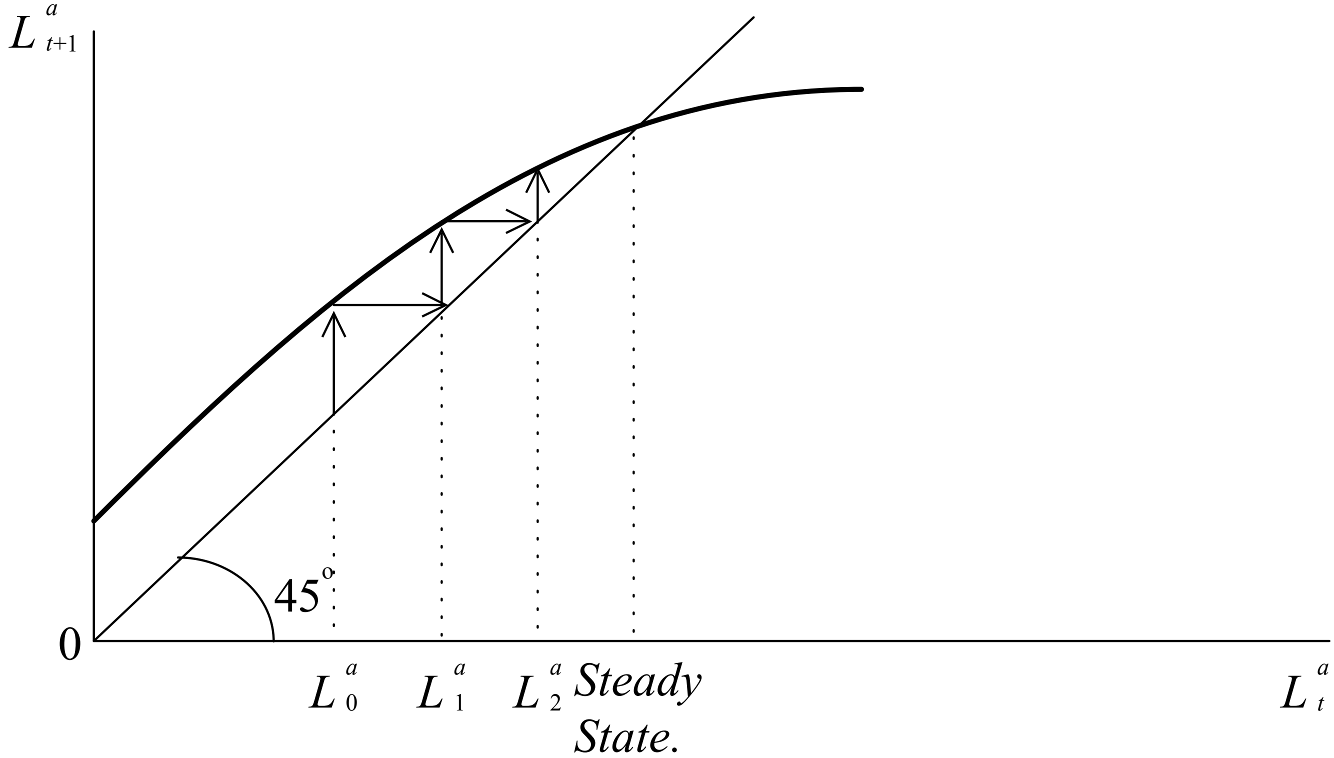


Figure 3: The Transition Function.

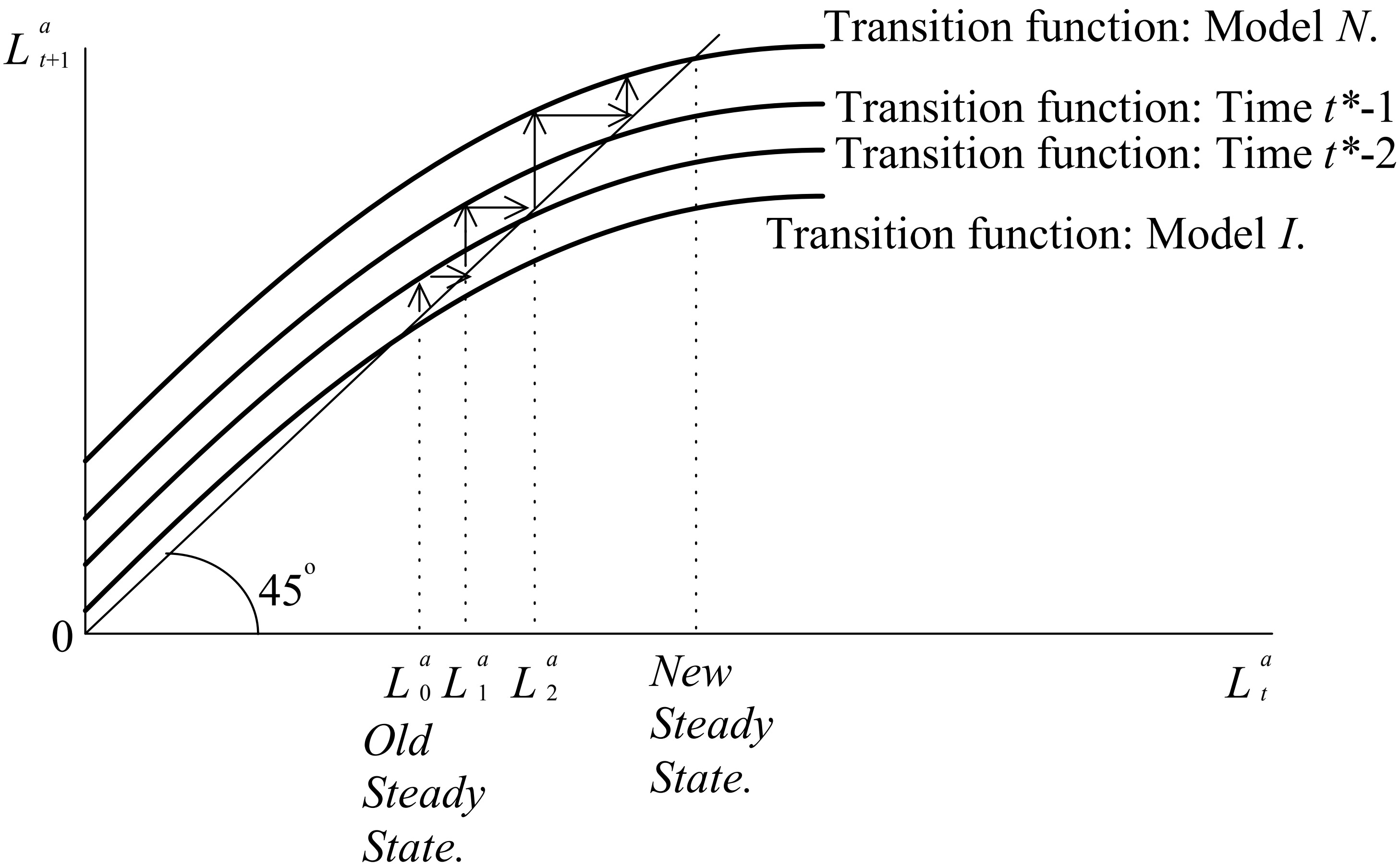


Figure 4: Anticipated Trade Liberalization..