Contagious Bank Failures in a Dynamic Bayesian Context: The role of informational spillovers? *

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Abstract

This paper models banking panic transmission as an equilibrium phenomenon in a dynamic bayesian setting. There are two banks with cohorts of Diamond-Dybvig(1983) depositors, who sequentially receive noisy information of their bank's idiosyncratic fundamentals. There is a macroeconomic fundamental (not publicly observed), to which both banks are commonly exposed. The features of the banks' investment technology impose a natural restriction on the coordination possibilities of depositors of each bank. Given the dynamic bayesian setting of the game, each depositor must coordinate his actions with depositors of his own bank (contemporaneous complementarities) as well as with depositors of the other bank (dynamic complementarities). This affects the way depositors in both banks respond to informational spillovers. We show that, for the global games approach to work in this setting and a contagious informational channel to explain the spread of failures across banks, necessary restrictions need to be placed on relative complementarities. Doing so enables us to pin down the perfect bayesian equilibrium of this game as a unique monotone equilibrium and examine the features of contagious bank runs. Our results are insightful in that, in addition to being able

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to make probabilistic assessments of the likelihoods of contagious failures and correlated failures, we can crucially distinguish between the two.

Contagion: ***1.a.*** *Disease transmission by direct or indirect contact;* ***b.*** *A disease that is or may be transmitted by direct or indirect contact; a contagious disease;* ***c.*** *The direct cause, such as a bacterium or virus, of a communicable disease;* ***2.*** *Psychology: The spread of a behaviour pattern, attitude, or emotion from person to person or group to group through suggestion, propaganda, rumor, or imitation;* ***3.*** *A harmful, corrupting in! uence: "feared that violence on television was a contagion affecting young viewers;"* ***4.*** *The tendency to spread , as of a doctrine, in! uence , or emotional state."*
- American Heritage Dictionary

Quoted from Forbes and Rigobon (2002)

1 Introduction

Financial systems[1] play a fourfold-role in the economy (Allen and Gale (2003)): They channel savings from households and corporate sector to those in need; they allow for intertemporal smoothing of consumption by households and expenditure by &rms; they enable households and &rms share risks; they allow for the efficient &nancing of pro&table investment projects. Ever since the special critique of Fama (1980) about the specialness of banks or &nancial intermediaries as to their relevance in an Arrow-Debreu setup, a huge body of the literature has surged, validating the role of banks by stressing on their role in alleviating different forms of market imperfections (Freixas and Rochet (2002))[2]. As dealers in non-marketable &nancial contracts of different forms, the nature of a bank 's activities[3] exposes it to panics or runs, which occur mainly when depositors, fearing that the bank will be unable to meet its contractual obligations, decide to withdraw their funds from the bank. Bank runs remain an accute issue today. While Europe and the United States have experienced a large number of bank runs in the 19th century and beginning of the 20th, many emerging markets have experienced severe episodes of banking crisis in recent years. Latin America seems to suffer from these episodes once every decade (Chile (1980s), Argentina (mid 1980s, 2002), Mexico (mid 1980s). Other spectacular accounts of banking crises include the South East Asian ! u (1997) and the banking distress that plagued the Eastern European countries (Baltic countries (1992), Bulgaria (1997)). As Gorton and Winton (2002) note in a recent survey on &nancial intermediaries, even countries that have never experienced bank runs strive hard to pre-empt the likelihood of a banking crisis from developing by adopting tough

[1] Financial markets and &nancial intermediaries

[2] See Freixas and Rochet, Microeconomics of Banking (chapter 2) for more details.

[3] Here, we have in mind qualitative features such as Asset-Liability maturity and liquidity mismatch, high gearing (low capitalisation), inverse relationship between liquidity and pro&tability on the asset side.

lines on regulatory measures, the costs of banking crises in terms of loss output, dis-intermediation and dismantling of the settlement system, being too high[4].

In this paper, we are concerned with a wider issue surrounding bank runs: the spread of a banking crisis from one bank to another or &nancial contagion[5]. As widely documented in the literature, individual bank runs may be severe enough to warrant the failure of other banks, making otherwise healthy banks temporarily illiquid and insolvent. The theoretical literature considers a number of channels that may explain why and how a crisis may spill over to other institutions. This literature may be divided into two categories: real contagion models which stress direct channels connecting banks and pure contagion channels which stress on informational changes, as principal driving cause of multiple bank collapse.

Real contagion models purport that banks are directly connected and that contagious runs ! ow through these channels. Banks may be connected through the interbank market, either through the exchange of interbank deposits or through the exchange of interbank loans. Alternatively, banks may be connected to a common macroeconomic fundamental. A nice example can illustrate the latter example: two banks that accept deposits in one currency and give loans in a different currency, may be exposed to the risk of exchange rate changes. A natural conundrum that arises, when we consider the issue of a common exposure, is the distinction between contagion and correlation. Did bank A fail because of the failure of bank B or because their performance is commonly driven by the deterioration of some common fundamental? The theoretical literature is not clear about this distinction. Nonetheless, the models we review here, ignore the issue of common exposure.

Allen and Gale (2000) focus on the network architecture connecting banks as the main driver of &nancial contagion[6]. Banks cross-hold deposits as insurance against regional liquidity shocks. While the interbank deposits provide insurance, they also create a pattern of overlapping interbank claims that can easily propagate a crisis from one bank to another. This is what happens in the

[4] The optimal level of bank regulation is subject to much debate though! A new &eld in microeconomic theory of banks is that banking panics are viewed as a natural consequence of a banking system ful&lling its fourfold allocational roles. Thus, any attempt to deal with banking crises, will inevitably impinge on the ability of the &nancial system to perform its four roles efficiently. Whether banking regulation is desirable or not, crucially depends on the bene&ts of such regulation exceeding the cost of so-doing. Models that stress on this trade-off, focus on the need to assess the legitimacy of regulation, from a welfare-theoretic point.

[5] The concept of &nancial contagion described here is a restricted version of a more general issue surrounding Systemic Risk. Here, we are just concerned with banks and there is no &nancial market. Another way of modelling Systemic Risk would be to show the interraction between a bank and a &nancial market in an incomplete market setup, which leads to excess price volatility for the asset the bank holds. This may mean that the bank is unable to meet its contractual arrangements and fails, dragging the &nancial market down with it.

[6] Another paper that focuses on the network architecture is Freixas, Parigi and Rochet (2000)

presence of aggregate liquidity shocks. Allen and Gale (2000) stress that the particular form of connectedness matters for the occurrence of financial contagion. A complete network contains implicit mechanism for checking contagious flows whereas an incomplete network is susceptible to flow of crisis across the system.

Dasgupta (2004) considers an identical model to Allen and Gale (2000), but adopts the global games approach to characterise contagion and examine its properties. The model does not rely on the presence of aggregate liquidity shocks as trigger for a banking crisis. Rather, bad fundamentals on the bank's asset side are the initiators of a banking crisis, and the existence of an interbank market for deposits, acts as propagator of the crisis across banks. Using the global games approach has a number of appealling features: Financial contagion occurs as a unique equilibrium phenomenon and there is an equilibrium in which one bank fails only because the other bank has failed. Furthermore the probability of contagion arises endogeneously and is positive. This feature better places it at characterising the optimal level of interbank deposits, by trading off the benefits of extra regional liquidity insurance with the costs of greater contagious risk. As a result, the insurance is less than complete. Furthermore, Dasgupta (2004) results are robust in that the occurrence of contagion does not rely on network architecture connecting banks. Even under a complete network, financial contagion arises with positive probabiity.

Rochet and Tirole (1996) consider the benefits of contagious risks as providers of incentives for peer monitoring among banks, in a setup involving moral hazard and lack of contractibility between debt holders and bank managers on manager efforts. The model considers banking regulation as the interraction between interbank lending and peer monitoring in the interbank market. It focuses on an optimal regulatory system as being one that can minimise the risks of contagious risks ex-post, while being able to preserve the incentives for peer monitoring ex-ante. Banks that have lent to others, should have their survival tied to the performance of the borrowing banks, and should be closed if the borrowing banks become insolvent/illiquid. Contagion should be allowed in order to provide incentives for banks to monitor each other ex-ante. This nonetheless limits the practical relevance of monitoring, since allowing too many banks to fail, is not a credible policy for a fully committed central bank ex-post.

Models of pure contagion stress on the different uses of information, as possible channel explaining why a failure may propagate from one bank to another, even though banks are not directly linked through fundamentals[7]. The basic mechanism propagating shocks across banks is the same. Agents are assumed to

[7] Some papers focus on contagion between unrelated countries, mainly from the vantage point of wealth effect from the investors. A failure of investment in one country reduces wealth of investors and forces them to consider optimal portfolio reallocation. This leads to declines in security prices in other countries and results in contagious effects.

be Bayesians and use Bayesian updating to reassess their own bank's position, in light of the occurrence of an event in another bank.

Chen (1999) considers the interplay between negative payoff externalities (due to sequential service constraints) and informational externalities, as critical in affecting the way depositors use and react to information. In the paper, uninformed depositors of bank A react to noisy information about bank B's performance. Knowing this, the informed agents of the bank A anticipate that, thanks to the &rst-come-&rst-serve rule enshrined in the demand eposit contract, it is optimal for them to withdraw as well, rather than having to wait for arrival of precise information. Thus, contagious runs occur when uninformed depositors interpret liquidity withdrawal shocks as (pessimistic) informational shocks. Panics occur in other banks because of the need for depositors to respond early to noisy information, due to the presence of negative payoff externailities.

Acharya and Yorulmazer (2002) analyse the interraction between contagion on the liability side of banks and the ex-ante correlation on the asset side of banks. Contagion occurs ex-post when bad news in a bank raises the cost of borrowing for depositors in another bank and makes the other bank illiquid. Correlation arises endogeneously ex-ante, since banks have an incentive to invest in common investment technologies, so as to maximise the likelihood of joint common survival. The rationale for this ex-ante behaviour is that, for an individual bank, individual bank failure is costlier than multiple bank failures. Thus, for banks that are perceived to be linked, their degree of asset correlation is high as well.

Vaugirard (2005) considers a case of multiple bank attacks in a setup similar to Chang and Velasco (2000). The storyline is similar to Chen (1999). In his paper, home depositors are assumed to have an informational advantage over foreign lenders regarding the liquidation costs of assets. A bank run in one country leads foreigners to reassess the liquidation yields in that country and in other countries as well (the likelihood of bank failure increases with the liquidation yield taking a low value.) As a result of the reassessment, banks in another country becomes illiquid as well and more prone to bank runs. In Vaugirard (2005), cross country correlation between yields and Bayesian reassessment of liquidation yields are critical in explaining banking panic spreads.

Our paper is a hybrid of Dasgupta (2004) and informational channels of contagion. The model is brie! y outlined as follows:

There are two banks in the economy, each of which spans a particular region of the economy. At $t = 0$, depositors in both regions invest their endowment in the bank of their region[8]. These depositors face liquidity shocks of the

[8] For the sake of simplicity, the rationale for depositors investing speci&cally in the bank of their region is ignored. We thus rule out arguments of the Hotelling's or Salop's type, which would investigate the reason for depositors to deposit money in a nearby bank to economise,say, on shoe-leather costs.

Diamond-Dybvig (1983) type, and can consume early or late. There is no aggregate uncertainty about liquidity shocks in the model. In return for accepting deposits, banks offer depositors demand deposit contracts, allowing depositors to withdraw either at $t = 1$ or $t = 2$, depending on the realisation of the liquidity shock (which is only known at the beginning of period $t = 1$). Both banks invest in the **same** investment technology at $t = 0$. The performance of the investment technology depends on each bank's idiosyncratic fundamental (e.g the quality of the bank's management), a common macroeconomic fundamental to which both banks are exposed, through some exogeneous correlation structure (known at beginning of $t = 1$) and the proportion of depositors withdrawing early in **each** bank. There is no correlation between each bank's idiosyncratic fundamental.

Each bank's idiosyncratic fundamental and the common macroeconomic fundamental, are not common knowledge, although their probability distributions are. Depositors in each bank noisily observe their bank's idiosyncratic fundamental, through some *private signal structure*. Thus, for each depositor of a given bank, this private information contains strategic information on the behaviour of other depositors of the **same** bank. For the sake of simplicity, we shall denote this coordination game between depositors, as $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$, for bank A and B respectively. Furthermore, in the spirit of dynamic Bayesian games, nature picks up at random the &rst movers of the game. We shall be assuming that depositors in bank A move &rst and depositors in bank B move second. The latter cohort also observe a noisy public information about the event in bank A. The precision of the public information is assumed to vary monotonically with the exogenous correlation structure. For depositors of a given bank, the public information contains strategic information about behaviour of depositors of the **other** bank.

As such, depositors in each bank face **two** coordination failure problems. Since the performance of the investment technology of each bank depends, amongst other factors, on the proportion of depositors withdrawing prematurely in each bank, depositors are actually coordinating their decision with other depositors of the same bank as well as with depositors of the other bank. That part of the coordination which is with depositors of the same bank is dubbed *contemporaneous strategic complementarities*. That part which is with depositors of the other bank is dubbed *dynamic strategic complementarities*. These complementarities mean that the incentive for a depositor to withdraw, increases with the proportion of depositors in the same bank as well as those in the other bank, taking the same action. Given a common level of exposure to the macroeconomic fundamental, the relative importance of contemporaneous complementarities v/s dynamic complementarities in depositors' decision sets may give rise to various theoretical equilibrium possibilities. Crucially, this tradeoff between different forms of complementarities affect the way depositors respond to informational spillovers. The reasoning is as follows: In equilibrium, depositors of the second bank play a best response, after observing their private signals about their own bank's fundamentals and the event in the first bank.The event in the first bank actually leads them to update their beliefs about the state of the common macroeconomic fundamental. Depositors of the first bank anticipate that, upon observing their action (which has led to the event in the first bank), those in the second bank will react in the way described above, and will, in turn, play a best response. The crucial factor that makes the payoffs of depositors dependent on the actions of each other within and across banks, is the investment technology whose returns depend on the actions of depositors in all banks. The way the investment technology affects the payoffs, is through these relative complementarities. In turn, relative complementarities affect the way depositors in the second bank respond to information. Throughout the paper, we will assume that depositors are bayesians and that the flow of information across banks take the form of bayesian updating of beliefs about the state of the common macroeconomic fundamental.

Our model differs from those in the theoretical literature in two main ways:

First, all informational contagion models highlighted above, consider a Diamond-Dybvig (1983) environment, in which multiple equilibria is not precluded. The presence of multiple equilibria represents a major impediment in the sound theoretical foundation of a financial contagion model. Even though bank fundamentals may be sound, depositors' beliefs in one bank may follow a self-fulfilling nature: actions follow beliefs and, in turn, validate the state of the world postulated by those beliefs. Self-fulfilling beliefs of depositors are equally consistent with a good outcome (in which no bank run occurs) and a bad outcome (in which it occurs). Bank Runs occur when the completely unpredictable choice among Pareto-ordered Nash equilibrium falls on the Pareto-dominated (bad) outcome. However, there is nothing in the model that tells when and why these runs occur in the first instance. There is thus indeterminacy in the model. This lack of predictability as to which equilibrium will prevail makes it difficult to study how a bank failure may spread from one bank to another. Put differently, if a model can predict that, depending on depositors' beliefs, any outcome of Bank A can be an equilibrium but it remains silent about beliefs, it is hardly able to predict how the outcome of bank A could affect Bank B. Similar problems arise in any international financial crisis model with a strong element of self-fulfilling beliefs. The existence of multiple equilibria makes it very difficult to examine individual bank runs, which compounds the difficulty involved in isolating contagious effects in a multi-bank setting. Quoting from Vaugirard (2005), "*...indeed the key sticking point when trying to display pure contagion in models of financial crises with multiple equilibria and based solely on self-fulfilling beliefs, is that the mechanism for jumps between equilibria, is not articulated. Therefore, these models fail to rigorously capture contagious effect, in which a crisis in one country (i.e the particular outcome among the set of possible equilibria) affects the likelihood of a crisis in another country....*" There are two theoretical ways out of the conundrum: (a) identify a particular channel pinning down the cause-effect relationship, out of the whole set of possible multiple outcomes; (b) use global games methodology pioneered by Carlsson and vanDamme (1993) , and reformulated by Morris and Shin (1998) to a model of speculative currency attack; Theoretical models of informational contagion, as highlighted above, use the &rst approach. We use the second approach, in that it eliminates uncertainty regarding the cause-effect relationship.

Despite the appealling features of the global games paradigm, adopting it in an informational model is not straightforward. In global games models such as Dasgupta(2004), each bank has its own investment technology and the performance of the investment technology does not depend on performance of other banks. This simpli&es the analysis in that it rules out the need for depositors to coordinate across banks. Furthermore, it facilitates the use of the global games approach in that, it can be adapted to each bank independent of what happens to other banks. Put in another way, the absence of a game between depositors of the two banks means that the equilibrium threshold in each bank can be assessed in a similar way as if only that bank existed. To quote from Dasgupta (2004), "*....we adopt the global games approach of Goldstein and Pauzner(2003)..... but we extend it to our more complicated payoff structure...*" In our paper, the existence of a game between depositors within a bank and across banks, complicates analysis. Given the different possible uses of public information and the dramatic impact that such information could have on the balance between strategic and fundamental uncertainty, a number of unappealling equilibria features may pop up. We nevetheless show that, by imposing certain restrictions on relative complementarities for depositors in each bank, we can still adapt the global games methodology in our setting.

Second, in Dasgupta (2004), the propagator of shocks across banks, is the balance sheet connection that arises due to interbank cross holdings of deposits. Absent this connection, &nancial contagion would not arise. This has a major limitation: empirical evidence on contagious bank failures tend to unearth two stylized facts[9]: Firstly, it is widely documented that contagion sometimes occurs even among banks that are not tied through explicit &nancial contracts. This &nding seriously limits the rationale for using interbank market contract as possible connection among banks and, thereby, lends credence to the validity of the informational channel. Secondly, a bank failure is more likely to affect banks that share the same fundamentals as the crisis catalyst bank. Thus, contagion appears to be "clustered" among identical banks. Our model, by stressing on the informational channel, helps to explain how a crisis spreads across banks, even in the absence od some form of connection. Thus, we purport to build a theoretical model that can, through the global games methodology, allow us to examine the properties of contagion and, at the same token, satisfy the stylized facts.

The rest of the paper is organised as follows: Section 2 introduces the model in details. Section 3 explicates the underlying signal structure. Section 4 ex-

[9]For detailed explanation of these two facts, please see section 8.

plains the strategy profiles. Section 5 applies the global games methodology and characterises the unique equilibrium. Section 6 describes financial contagion and makes the fundamental distinction between contagion and correlation. Section 7 discusses some practical relevance of our paper and throws some light on policy recommendations. Section 8 concludes. All graphical analysis used throughout this paper, can be found in the appendix.

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2 The Model

The economy is divided into two ex-ante identical regions, A and B. The regional structure can be a spatial metaphor. There are three periods, $t = 0, 1, 2$. Each region contains one commercial bank which accepts deposits of money from consumers and invest the proceedings in different technologies. There is a continuum of risk-averse consumers, with each consumer i[10] of measure 1, such that $i \in M = [0, 1]^i$, having strictly concave and monotonic preference functions that satisfy inada-style conditions, and, being depositors in the bank of their region. Each agent lives for three periods only and is endowed with one unit of a homogeneous good at $t = 0$ and deposits his endowment in the bank of his region at $t = 0$. Formally, we model the initial set of endowments for each agent as a non-empty singleton set ϖ_i,such that $\varpi_i = \{1\}$, with the set of all endowments being $\times_{i \in M} \varpi_i \in \{\varpi : M \to \Re_+\}$ or $\varpi_{i \in M} \in \int_0^1 \varpi_i \; di$,for some finite Euclidean Space. We assume that there is no Central Bank and no financial markets in the model and that only banks have a comparative advantage in providing liquidity[11].

2.0.1 Returns Stucture and Bank's Investment Technologies

Each bank $i \in (A, B)$ can either invest in a safe-and-liquid technology or in a risky-and-illiquid technology. One unit deposited at t yields exactly one unit at $t + 1$ under the safe-and-liquid technology. Interpret the technology as representing cash reserves that the banks keep to meet demand for early withdrawals. The returns structure under the risky-and-illiquid technology is more extricate: if the investment is liquidated in the interim period to meet, say, the demand for early withdrawals, it yields a return of $r(< 1)$ - meaning that there are costs to early liquidation. Conversely, if the investment project is carried out till period

[10] Throughout this paper, we shall refer to customer/depositor i as being a typical depositor of bank i.

[11] In this sense, we rule out arguments of the Jacklin (1987) type, which compare the risk-sharing arrangement provided by demand-deposit economies as opposed to that provided by equity economies. Our purpose in the model is not to look at risk-sharing agreements.

$t = 2$, then the returns will be contingent on a number of structural parameters of the model. As mentioned in section 1.1, the risky investment technology is common to both banks and the returns to this technology at time $t = 2$ depends on the idiosyncratic fundamental of **each** bank, the common macroeconomic fundamental and the proportions of early withdrawals from **each** bank. In the latter case, the returns can be 0, $R_{\max}, \tilde{R}(\theta_i, \theta_{-i}, u_i^j, \delta_i, \delta_{-i})$. This is nicely summed up in table 1:

Table 1: Returns Structure of the Risky-and-illiquid Investment Portfolio for Bank i $r < 1$ at time $t = 1$ if investment is liquidated prematurely

(If investment is carried on till time $t = 2$)

$R_{\max}$	if $\theta_i > u_i^j + z_i\delta_i$
$0 < \tilde{R}(\theta_i, \theta_{-i}, u_i^j, \delta_i, \delta_{-i}) < R_{\max}$	if $\theta_i = u_i^j + z_i\delta_i$
0	if $\theta_i < u_i^j + z_i\delta_i$

Interpretation:

Let $j = \{G, Bad\}$ denote $\{Good\ State,\ Bad\ State\}$ and $i = \{A, B\}$ denote $\{Bank\ A,\ Bank\ B\}$.

(a) We distinguish between two fundamentals that are relevant for our analysis: bank i's *idiosyncratic fundamental* and a *macroeconomic fundamental* that is common to both banks. Parameter θ_i simply denotes bank i's idiosyncratic fundamental. We assume that it is drawn randomly from some *uniform density* on a unit interval. Each depositor in bank i can only noisily observe θ_i but the underlying probability distribution supporting θ_i is common knowledge to all depositors. We also make the important assumption that, once a value for θ_i is realised at $t = 0$, it does not change throughout the whole experiment. We return to a more formal analysis of each bank's idiosyncratic fundamental in section 3.1.

(b) Parameter u_i^j ,where $\{j = \{G, Bad\}, i = \{A, B\}\}$,represents the state of some macroeconomic fundamental that affects each bank $i \in \{A, B\}$. It affects bank i's investment returns, but not bank i's idiosyncratic fundamental, θ_i. The two distinguishing features of u_i^j are as follows:

(i) u_i^j represents either a Good (G) or Bad (B) macroeconomic state that affects bank i. The exact realisation of the state of the common macroeconomic fundamental is not observed by depositors but the (prior) probability distribution underlying the binary states is common knowledge. For simplicity, we assume that $P(u_i^{Bad}) = 1 - P(u_i^G) = k$, with $u_i^{Bad} > u_i^G \quad i = \{A, B\}$. We denote the probability space containing all possible states of the common macroeconomic fundamental as ζ . ζ is assumed to follow a *Bernoulli distribution*, with $\zeta \in \{i = \{A, B\};\ \exists\ k \text{ s.t } P(u_i^{Bad}) = 1 - P(u_i^G) = k;\ k \in [0, 1]\}$. The common macroeconomic fundamental is realised at $t = 0$ and we assume that its realisation (which is never observed) remains stationary throughout the experiment.

(ii) Conditional on the state of the common macroeconomic fundamental, u_i^j affects each bank i in a similar or dissimilar fashion, with varying strength of exposure. The strength of exposure is captured by the correlation coefficient parameter between u_A^j and u_B^j, $Corr(\ u_A^j, u_B^j)$,where, conditional on a state i being realised, $-1 \leq Corr(\ u_A^j, u_B^j) \leq +1$. Interpret the correlation space as follows: As $Corr(\ u_A^j, u_B^j) \rightarrow +1$, the greater is the likelihood that both banks are connected in a similar way and the stronger is their strength of exposure. Conversely, as $Corr(\ u_A^j, u_B^j) \rightarrow -1$, the more likely it is that the banks are connected in a dissimilar way but the stronger is their common exposure. Similar interpretations can be sought out for intermediate values of the correlation coefficient. For the sake of tractability, we shall focus on a truncated part of the correlation spaceonly - that which is non-negative. Also,interpret $Corr(\ u_A^j, u_B^j)$ henceforth as ρ where $0 \leq \rho < +1$.

(c) Parameter $z_i$denotes the loss caused by early withdrawals of deposits from the bank. The greater z_i is, the greater the disruption caused and the greater is the likelihood that θ_i is low relative to $\{u_i^j + z_i\delta\}$. Note that, by adopting the speci&cation as in Table 1, we are implicitly endogenising the returns of the risky-and-illiquid project; for extreme values of the idiosyncratic fundamental θ_i,the returns to the long asset depend exclusively on the value of the idiosyncratic fundamental θ_i.Before moving further, we make the following structural assumptions about parameter values: [1] $u_i^G > 0$, [2] $u_i^{Bad} + z_i < 1$, [3] $u_i^{Bad} < u_i^G + z_i$, [4] $P(u_i^{Bad}) = 1 - P(u_i^G) = k$, [5] $P(u_i^{Bad}) > P(u_i^G)$ with $u_i^{Bad} > u_i^G$. Consider the following scenarios:

2.0.2 Dominance Regions and the θ_i−space

De&ne a worst case scenario as one in which the state of the common macroeconomic fundamental is bad (u_i^B) and everybody withdraws money from the bank ($\delta_i = 1$); if θ_i is high enough that it exceeds $\{u_i^{Bad} + \delta_i\}$, then table 1 suggests that the returns to the investment project should be $R_{\max}$. This suggests that even in the worst case scenario, θ_i is strong enough to be dominant (i.e determines long term returns.) In the *best case* scenario (i.e one in which the state of the common fundamental is good (u_i^G) and nobody withdraws, the project may still fail if the value of θ_i is so low that it lies below u_i^G. These case scenarios depict an important result for the returns structure of the risky-and-illiquid technology: Regions $\{\theta_i : [\theta_i > u_i^{Bad} + z_i] \cup [\theta_i < u_i^G]\} \subset [0, 1]$ depict those segments of the $\theta - space$ for which θ_i is strictly dominant i.e can always ruin or save the risky project and become the overriding determinant of the risky technology. The intermediate region $\left\{\theta_i : u_i^G \leq \theta_i \leq u_i^{Bad} + z_i\right\} \subset [0,1]$ rules out any possibility of θ_i dominance and an interraction between different model parameters will determine the outcome of the project. This is represented by &gure 2 and by the following de&nitions:

Figure 1: Segregation of the θ_i-space into Strict and Weak dominance regions[12]

(*Insert Figure 1 here from Appendix*)

(In all cases, $0 \leq \theta_i < 1$)

De&nition 1 ***(Strict Lower Dominance Region (SLDR))*** *(Fundamental-Based Bank Failure)* $\left\{\theta_i : \min\{\ [\theta_i < u_i^G],\ [\theta_i < u_i^{Bad}]\}\right\} \equiv \left\{\theta_i : [0 \leq \theta_i < u_i^G]\right\}$ *$\Rightarrow$ Region of the $\theta_i -$ space, for which bank i fails with probability 1, no matter what the state of the common macroeconomic fundamental is. Associated with the idiosyncratic fundamental being "Too Low To Succeed".*

De&nition 2 ***(Weak Lower Dominance Region (WLDR))*** $\left\{\theta_i : u_i^G \leq \theta_i < u_i^{Bad}]\ \right\} \Rightarrow$ *Region of the θ_i-space for which, contingent on the state of the common fundamental being bad, bank i fails irrespective of the behaviour of its patient depositors*

De&nition 3 ***(Strict Upper Dominance Region (SUDR))*** *(Fundamental-Based Bank Success)* $\left\{\theta_i : \max\left\{[\theta_i > u_i^G + z_i],\ [\theta_i > u_i^{Bad} + z_i]\right\}\right\} \equiv \left\{\theta_i : [\theta_i > u_i^{Bad} + z_i]\right\}$ *$\Rightarrow$ Region of the $\theta_i -$ space, for which bank i fails with probability* 0*, no matter what the state of the common macroeconomic fundamental is. Associated with the idiosyncratic fundamental being "Too Large To Fail".*

De&nition 4 ***(Weak Upper Dominance Region (WUDR))*** $\left\{\theta_i :\ u_i^G + z_i \leq \theta_i < u_i^{Bad} + z_i]\right\} \Rightarrow$ *Region of the θ_i-space for which, contingent on the occurence of state of the common fundamental being good, bank i succeeds, irrespective of the behaviour of its patient depositors.*

All four regions put powerful assumptions on the role of θ_i as a driver of bank i's performance. The only difference lies in the interpretation. For SLDR and SUDR, the macroeconomic state variable does not matter. For SLDR (respectively SUDR), θ_i is so low (respectively high) that the bank is guaranteed to

[12] Different papers in the literature have emphasised this tripartite classi&cation. See for example, Morris and Shin (1998), Goldstein and Pauzner (2002), Dasgupta (2003), Boonprakaikawe and Ghosal (2000)

fail (respectively to succeed). On the other hand side, with WLDR and WUDR, the state of the common fundamental does matter. For example, suppose that the state of the common fundamental is bad. Any $\theta_i \in \left[u_i^G, u_i^{Bad}\right]$ would be classified as part of the "lower dominance region". If the state of the fundamental was good, $\theta_i \in \left[u_i^G, u_i^{Bad}\right]$ would be part of the segment of θ_i , for which the bank's behaviour would depend on the behaviour of patient depositors. On the other hand side, any $\theta_i \in \left[0, u_i^G\right]$ would be classified as part of the "lower dominance region", irrespective of the state of the common fundamental. Thus, $\theta_i \in \left[0, u_i^G\right]$ is strictly lower dominant, because it does not depend on the state of the common fundamental. A similar analysis can explain the rationale for WUDR and SUDR.

Given assumptions $[1] - [5]$ above, we summarise the following features of $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})$ for any bank i: $[a]$ $\forall \theta_i < u_i^G$, $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i}) = 0$, $[b]$ $\forall \theta_i > u_i^B + z_i$, $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i}) = R_{\max}$, $[c]$ $\forall \theta_i$ s.t $\left\{ u_i^j \leq \theta_i \leq u_i^j + z_i \right\}$, $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})$has the following properties: $[c.1]$ $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial u_i^j} < 0$, $[c.2]$ $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial \theta_i} > 0$ for a given j, $[c.3]\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial z_i} < 0$ for a given state j, $[c.4]\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial \delta_i} > 0$; furthermore, for a given state j, as $\delta_i \to 0, \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i}) \to 0$ iff $\theta_i \to u_i^j$; as $\delta_i \to 1$, $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i}) \to R_{\max}$ iff $\theta_i \to u_i^j + z_i$. The assumption that $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial u_i^j} < 0$ presents some slight abuse of notation - what it is saying is that, for some bank i, moving from a good state (u_i^G) to a bad one (u_i^B) will lower returns. The assumption that $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial z_i} < 0$ tells us that, for some bank i and given some **specific** state j, $\tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})$ will rotate downwards with a change in z_i[13]. Assumptions $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial \theta_i} > 0$ and $\frac{\partial \tilde{R}(\theta_i, \theta_{-i,} u_i^j, \delta_i, \delta_{-i})}{\partial \delta_i} > 0$ are intuitive and hold true for some bank i and given some **specific** state j. Figure 2 shows the relationship between the returns structure of bank i's risky technology, bank i's idiosyncratic fundamental and the common macroeconomic fundamental.

Figure 2: the relationship between idiosyncratic fundamental, common macroeconomic fundamental and (risky) returns technology for a typical bank

(*Insert Figure 2 here from Appendix*)

[13] Again, there is some slight abuse here: we present it this way for notational simplicity.

2.1 Payoff structure to depositors in each bank

In return for investing depositors money in different investment technologies, each bank offers *demand deposit contracts*[14] to depositors. These deposit contracts simply convert deposits into cash at par on demand in period 1, conditional on there being sufficient cash available in the reserves. If there is no sufficient cash available, the bank is compelled to liquidate its risky asset prematurely and to divide the proceeds of the liquidated asset equally among those who have chosen to withdraw early[15]. For those who remain, the bank pays a stochastic amount which is dependent on the structural parameters of the returns structure.

As in all models of bank runs, we assume that depositors in each bank i face *Liquidity Preference Shocks* i.e each of the depositors can consume early (i.e at $t = 1$) with probability λ and late(i.e at $t = 2$) with probability $1-\lambda$. There is a privately observed uninsurable risk of being patient or impatient, with there being no aggregate liquidity uncertainty in the economy. The probability distribution of liquidity preference shocks is assumed to be common knowledge. Ex-ante, each depositor has an equal and independent chance of being of impatient type. It is only at $t = 1$ that depositors learn their type.

While depositors face uncertainty ex-ante about their liquidity needs, banks do not face such uncertainty. The liquidity needs for depositors are mutualised, so that, by the law of large numbers, the banks can reasonably expect a fraction λ of depositors to withdraw early and a fraction $1-\lambda$ to withdraw late. Due to absence of uncertainty about the proportion of early withdrawals, each bank can earmark a fraction λ to its liquid asset and a fraction $1-\lambda$ to its illiquid asset.

We are now well equipped to characterise the payoff structure of depositors in bank i. Since each bank is otherwise identical, the design of the payoff structure to depositors applies equally to depositors of each bank. For notational simplicity, we shall be assuming that we are dealing with the general case of bank i, with no speci&c reference as to which region it belongs to.

Following the previous discussion, a proportion λ of depositors in bank i is impatient. Suppose that a proportion δ_i of the remaining patient depositors want to withdraw at $t = 1$. The total demand for liquidity that bank i faces is thus $\{\lambda + \delta_i(1-\lambda)\}$ .Where does the bank draw its supply of liquidity to meet high early demand? It has λ in the liquid technology. It may also draw upon its illiquid technology and use the resulting proceeds to meet high demand for early withdrawals. The total supply of liquidity is thus $\{\lambda + r(1-\lambda)\}$. If the

[14] We are simply taking the contracts as given in our paper.

[15] Since there is no &nancial market in the setup (and no interbank market), the price at which the risky asset trades, is determined exogenously.

total demand for early withdrawals exceed the available pool of assets that the bank can make available, then the bank is technically bankrupt at $t = 1$. This helps us characterise the bankruptcy threshold of the bank.

Definition 5 ***(Bankruptcy Threshold)*** *The threshold that separates the Bankruptcy Condition (BC) from the No-Bankruptcy Condition (NBC). Bank i stops being a going-concern at* $t = 1$ *if and only if* $\{\lambda + \delta_i(1-\lambda)\} > \{\lambda + r(1-\lambda)\}$ *i.e if* $\delta_i > r$ *and carries on operations if* $\delta_i \leq r$.

The importance of the bankruptcy threshold is that it determines the "allocation rule" for depositors at $t = 1$ and $t = 2$. Suppose that $\delta_i > r$ (i.e *Bankruptcy condition*). Depositors who choose to withdraw early appropriate the whole proceeds that the bank can generate at $t = 1$. Each depositor gets an amount $\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}$,with utility $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right]$. Since $\delta_i > r$, clearly, $\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)} < 1$. Utility functions, being an increasing function of payoffs, this implies $U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] > U(1)$. The depositor is worse off than when he received his full endowment back. Those patient depositors who do not choose to imitate the impatient ones and who have chosen to withdraw at $t = 2$, get a payoff of zero, with utility $U(0)$.

Suppose now that $\delta_i < r$ (i.e *No-Bankruptcy condition*). The whole measure of depositors who claim early withdrawals get their whole endowment back, with utility U(1). With this condition, to satisfy the demand for early withdrawals, the proportion of illiquid assets that has to be liquidated is $\frac{\delta_i(1-\lambda)}{r}$. The leftover of illiquid assets that is carried on till $t = 2$ to finance the withdrawals of patient depositors is thus: $\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i, \theta_{-i}, u_i^j, \delta_i, \delta_{-i})$. Each of the patient depositors shares this leftover, appropriated by the exact proportion of depositors who are claiming this leftover. Each depositor thus gets $\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]$,with utility $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i})}{(1-\lambda)(1-\delta_i)}\right]$.

To summarise, the payoff structure for each depositor i[16] takes the form of a mapping : $U_i : \{\} \times A \to \Re$ where

• For impatient depositors and the proportion of depositors who choose to withdraw early:

$$U(\theta_i, \delta_i, t = 1) = \left\{ \begin{array}{ll} U(1) & \delta_i \leq r \\ U\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] & \delta_i > r \end{array} \right\} \quad (1)$$

• For the proportion of patient depositors who withdraw late:

[16] By depositor i, we mean a typical depositor of bank i. Ostensibly, put aside liquidity risk, all depositors are otherwise homogeneous.

$$U(\theta_i,\delta_i,t=2)=\left\{\begin{array}{lr} U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] & \delta_i \leq r \\ U(0) & \delta_i > r \end{array}\right\} \tag{2}$$

Table 2 summarises the relationship between the net payoff to staying for depositor i as a function of the Bankruptcy and the No-Bankruptcy Threshold:

	No-Bankruptcy Condition(NBC) $\delta_i \leq r$	**Bankruptcy Condition(I** $\delta_i > r$
Payoff to Staying	$U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]$	$U_i(0)$
Payoff to Withdrawing	$U_i(1)$	$U_i\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right]$
Net Payoff to Staying	$U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]-U_i(1)$	$U_i(0)-U_i\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right]$

2.2 Structural Parameter Restrictions and qualitative features of the Payoff Structure

(A.1) **(Liquidity and Pro&tability)** Given that the deposit contract is taken as given in our set-up, there is a circularity between liquidity and pro&tability of bank i: bank i is insolvent if and only if it is illiquid; it is illiquid if and only if depositors expect it to be insolvent.

(A.2)Under the **Bankruptcy-Condition (BC)** with $\delta_i > r$, $U_i\left[\frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)}\right] > U_i(0)$, $i \in [0,1]$. This result holds sway because of the feature that $0 \leq \frac{\lambda+r(1-\lambda)}{\lambda+\delta_i(1-\lambda)} \leq 1$. The net payoff to staying as opposed to withdrawing is therefore negative in the BC threshold.

(A.3) Under the **No-Bankruptcy-Condition (NBC)** with $\delta_i \leq r$, the relationship between $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]$ and $U_i(1)$, $i \in [0,1]$, depends on the location of δ_i in the NBC segment. More precisely, there exists a $\delta^{\#}$ $\left(\text{equal to } \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}\right)$, at which $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] = U_i(1)$. For $0 \leq \delta < \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}$, $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] > U_i(1)$. Thus, it is strictly preferable to stay. For $\frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r} \leq \delta < 1$, $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] <$

$U_i(1)$. Here, it is strictly preferable to withdraw[17]. The relationship between the payoff to staying and payoff to withdrawing, can be shown as follows, in &gure 4:

Figure 3: Depositor Payoff Structure: Payoff to staying v/s Payoff to withdrawing

(*Insert Figure 3 here from Appendix)*

2.3 Taxonomy of the Dynamic Bayesian Game

Armed with the conceptual pillars we have developed in the previous section and subsections, we are now ready to provide an illuminating synopsis of the sequential game that is being played between depositors of the 2 banks. Some additional assumptions are follow the discussion.

An important part of the sequential game with incomplete information is who determines the &rst-mover of the game Since both banks are otherwise completely identical to each other ans each stage payoffs are stationary, it makes no difference as to which bank shall move &rst. In line with good economic theory and not to abuse the literature of sequential move games with incomplete information, we shall be assuming that nature chooses at random and, with equal probability, the &rst mover of the game. This probability distribution is common knowledge and the structure of th dynamic game is also common knowledge. Lets assume that depositors in bank A are chosen to act &rst[18].

[17] Here is the proof: Since $U[.]$ is concave and strictly increasing, condition $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] = U_i(1)$ implies that $\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)} = 1$. Making δ_i subject of formula, will lead to the following: $\delta^{\#} = \frac{r\left(\widetilde{R}-1\right)}{\widetilde{R}-r}$. Since $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]$ is decreasing in δ_i, it follows that for $\delta_i < \delta^{\#}$, $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] > U_i(1)$. A similar analysis will show that $U_i\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right] < U_i(1)$ if $\delta_i > \delta^{\#}$

[18] Given the features of the payoff structure of each bank and the assumption of complete homogeneity, it does not matter which cohort of depositors move &rst. For ease of exposition, we simple label the &rst-mover bank as bank A and the second-mover bank as bank B. Issues like "First-Mover Advantages" are not present in our set-up. They could be present, though, in

The stage game that represents this is $\Gamma_{A,t=1}$. Then depositors of bank B are chosen to act in $\Gamma_{B,t=1}$. Note that although each stage game is represented by time 1, they actually take place *at different points* in time 1.

Table 3 - The Dynamic Bayesian Game

· **Period 0**

- Consumer i, $\forall i \in [0,1]^i$, invests in the bank of his region

- The Bank invests each unit of endowment, ϖ_i,in either a safe or risky-and-illiquid technology

- Realisations of θ_i or u_i^j, $i = \{A, B\}$, $j = \{G, B\}$, occur. These fundamentals are randonly drawn from commonly observed probability distributions but the actual realisations are not observed. Also, these variables are stationary throughout the experiment.

- Which group of depositors will be called upon to act &rst becomes publicly known (say, Bank A)

· **Period 1**

- Impatient depositors have a dominant strategy of withdrawing early

- $(\Gamma_{A,1})$ Patient depositor i , $\forall i \in [0,1]^A$,receives information about his bank's idiosyncratic fundamental, θ_A, in the form of private signals

- Those patient depositors who demand early payment are paid, contingent on there being sufficient cash available to meet withdrawals demands.

- The event in bank A becomes Public Knowledge and is commonly observed by depositors of bank B with some noise

- $(\Gamma_{B,1})$ Patient depositor i of bank B, $\forall i \in [0,1]^B$, receives his private signal about bank B's idiosyncratic fundamental, θ_B. In addition, they noisily observe an endogeneously derived Public Signal, encapsulating events in bank A

- Those patient depositors who demand early payment are paid, contingent on there being sufficient cash available to meet withdrawals demands.

·**Period 2**

-Investment technology returns are realised

- Depositors who chose to stay rather than withdraw in bank i, $i \in [A, B]$, get their due back.

where $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ denote two coordination games that are also known as "Games of Regime Change"[19]. Because we do not focus on how changes in θ_i and u_i^j affect dynamic equilibria, we take their (unobserved) realisations as being static throughout the experiment. This enables us focus on how, given the private informational structure for depositors in each bank, the ! ow of Public

models in which the banks are directly connected to each other through the interbank market (in deposits or loans). In this case, regional liquidity shocks would mean that one bank is a debtor and the other bank is a creditor at a given period of time. See Dasgupta (2003) for more.

[19] Angeletos, Hellwig, Pavan (2003)

Information *by itself* can affect dynamics of coordination at each stage game, $\Gamma_{i,t=1}$ and spread a &nancial crisis contagiously from one bank to another.

Assumptions (*contd):*

(A.4) **(No "direct-link" &nancial contracts acting as intertemporal linkages)** Since each bank is homogeneous and faces the same operating environment ex-ante with the same structural features, the only variable that links the payoffs for each stage game is the common macroeconomic fundamental. De&ne a stage game as $\Gamma_{i,t}$, where $\Gamma_{i,t}$ represents a (coordination) game among depositors of bank i , with $i \in [A, B]$,at time t, with $t \in [t_1, t_2]$[20]. We abstract from any other form of (direct) intertemporal linkages in the payoff structure of banks. In a richer model with regional liquidity shocks and the existence of some sort of contingency plan provided by the interbank market (in deposits or in loans), there would have been this sort of intertemporal link in the payoff structure. By assuming that payoffs in $\Gamma_{i,t}$ are only tied by some perception of the common fundamental and, by removing any overlapping network of &nancial contracts that would have connected the banks, we can focus on how the dynamics of (public)information ! ow affect the dynamics of coordination in each bank[21].

(A.5) (**Intertemporal Payoff Dependence)** Whilst the computation of the payoff has been derived from bank i's balance sheet, it follows that there does exist "indirect" linkages through depositors' payoff functions, due to (A.4). Each depositor in bank i, conditional on $\delta_i \leq r$, has a payoff to staying, depicted as $U\left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\}\tilde{R}(\theta_i,\theta_{-i},u_i^j,\delta_i,\delta_{-i}))}{(1-\lambda)(1-\delta_i)}\right]$. Following on from the earlier discussion on returns to the risky technology, it follows that $\tilde{R}(\theta_i, \theta_{-i}, u_i^j, \delta_i, \delta_{-i})$ depends, among other factors, on both, δ_i and δ_{-i}. Subsequently, it becomes obvious that (1) any action that a depositor in bank i takes, will affect, not only, other depositors in the same bank but also depositors in the other bank; (2) any action that depositors in the other bank take, even though it may be taken at a different point of time, will affect the payoff of depositors in bank i. The rationale for this intertemporal payoff dependence is the existence of a common risky technology for both banks and the returns of that technology, being contingent on parameters affecting both banks. We shall see the implication of this intertemporal payoff dependence for the Perfect Bayesian Equilibrium later on. This form of

dependence both, within and across different banks, has a twin implication for payoff supermodularities: this is subsumed in (A.6) and (A.7):

(A.6)(**Contemporaneous Strategic Complementarities/Supermodularities in** $\Gamma_{i,t=1}$) There exists (partial) supermodularities in the payoffs of depositors

[20] The reason why we focus on times 1 and 2 will become clear in the following section when we give an expositional view of the whole dynamic bayesian game between depositors of the two banks

[21] Angeletos, Hellwig and Pavan (2004) use the same approach but their model is not con&ned to the speci&city of the banking sector.

of bank i. Consider table 2: if the banruptcy constraint is not violated (i.e $\delta_i \leq r$),the incentive for each depositor i in $\Gamma_{i,t=1}$, $\forall i \in [0,1]^i$ to withdraw increases as more and more depositors withdraw. This NBC threshold is hence characterised by supermodularities in payoffs - actions of depositors are "*strategic complements*". On the other hand side, if the bankruptcy constraint is violated and the bank is insolvent (i.e $\delta_i > r$),the incentive for depositor i in $\Gamma_{i,t}$, $\forall i \in [0,1]^i$, to withdraw decreases as more and more depositors withdraw. Actions of depositors are thus "*strategic substitutes*"in the BC threshold.[22]

(A.7) (**Dynamic Strategic Complementarities between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$**)

Coming soon

3 Informational structure

3.1 Private Signal structure

As mentioned before, we assume that depositors cannot observe the idiosyncratic fundamental of their bank and do not observe the actual realisation of the common macroeconomic fundamental. While impatient depositors have a *dominant strategy* of withdrawing in period $t = 1$, patient depositors are confronted to a non-trivial problem: They face a coordination problem in period $t = 1$ as regards their decision of whether to stay or withdraw. Their decision is motivated by their informational endowment at the time of acting. We model formally the action space available to depositors as follows: for each patient depositor i, $a_i \in A_i = \{W, S\}$, $\forall i \in [0,1]^i$, $A_i \in A$ and $A = \times_{i \in [0,1]^i} A_i$.

Each patient depositor i noisily observes the idiosyncratic fundamental of his bank, θ_i. A depositor's private signal can be viewed as his private heterogeneous information available to him regarding his opinion about the long term viability of the bank's investment project. Each agent i receives a (bounded) signal $s_i \in S = [s_L, s_U] \subset \Re$, where s$_L$ denotes the lower bound of the signal space and s_U denotes the upper bound. Formally, let ξ denote the set of all "lower bound" θ, where $\xi = \{u^G, u^B\} \subset [0,1]$. Since $u^B > u^G$, the greatest lower

[22] In the NBC threshold for $\Gamma_{i,t}$, as more and more depositors withdraw (i.e as δ_i rises), depositor i has a clear incentive to withdraw as well because if δ_i is growing sufficiently high that it exceeds r, then the depositor reasons that he will get only $\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)}$if he stays as opposed to 1 if he withdraws. Since $\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)} < 1$ when the BC holds, it is in depositor i's interest to imitate other depositors. Thus actions are strategic complements only in the BC space.

bound is the realisation of θ that corresponds to state u^G. Let s_L denote the signal that relates to that state. Similarly, we de&ne $\xi\prime$ as the set of upper bound θ, where $\xi\prime = \left\{u^G + z,\ u^B + z\right\} \subset [0,1]$. Since $u^B + z > u^G + z$, the greatest upper bound is the realisation of θ relating to $u^B + z$. We let s_U denote the signal that corresponds to that value of θ. Thus, formally, we could represent s_L and s_U as: $s_L \equiv \inf\{s : \Pr ob(\theta < \theta_L \mid s) < 1\}$ where $\theta_L = u^G$and $s_U \equiv \inf\{s : \Pr ob(\theta < \theta_U \mid s) > 0\}$ where $\theta_U = u^B + z$. The point behind such formalisation is that it enables us highlight, which segment of $\theta - space$ the behaviour of depositors can be anticipated for sure, *irrespective of the realisation of the common macroeconomic fundamental.* Each agent s signal s_i is assumed to be independent and identically distributed, conditional on θ_i. We assume that, for each depositor i, s_i is his type and is distributed according to some continuous density function $f(.)$ which satis&es the *Monotone Likelihood Ratio Property (MLRP).* In particular, for ease of illustration, assume that, following the uniform distribution of θ_i on $[0,1]$, there exists a threshold θ^* such that $(\theta^*, 1]$ constitutes the upper segment of the idiosyncratic fundamental space and $[0, \theta^*)$ constitutes the lower segment. For any $\theta^H \in (\theta^*, 1]$ and $\theta^L \in [0, \theta^*)$, the private signal structure satis&es the property that $\frac{f(s_i|\theta^H)}{f(s_i|\theta^L)}$ is strictly increasing in s_i. Since s_i has bounded support, then the Monotone Likelihood Ratio $\frac{f(s_i|\theta^H)}{f(s_i|\theta^L)}$ is also bounded[23]. To keep the analysis simple bearing in mind the above features, we shall model the relationship between s_i and θ_i as follows:

$$s_i = \theta_i + \varepsilon_i \tag{3}$$

where ε_i denotes the noise technology. We assume that the noise technology is common knowledge and is uniformly distributed on a closed interval $[-\varepsilon, +\varepsilon]$. Each ε_i is independent of θ and ε_m, $\forall m \neq i$. The existence of a bounded support for the distributions of θ_i and ε_i means that the lower and upper bounds for the private signal are respectively: $[u^G - \varepsilon,\ u^B + z + \varepsilon]$ [24]. There exists a tripartite classi&cation of the $s - space$ (i.e the signal space) such that $s \in \{s : s_{untable} \cup s_{\mathrm{mod}\,erate} \cup s_{stable}\}$ where $s_{untable} = \left\{s : 0 < s < u^G - \varepsilon\right\}$, $s_{\mathrm{mod}\,erate} = \{s : u^G - \varepsilon \leq s \leq u^B + z + \varepsilon\}$, $s_{stable} = \left\{u^B + z + \varepsilon < s < 1\right\}$. The interpretation of that tripartite classi&cation is self-explanatory: $s_{untable} = \left\{s : 0 < s < u^G - \varepsilon\right\}$ denotes the (unstable) region in which the depositors always withdraw, no matter what; $s_{stable} = \left\{u^B + z + \varepsilon < s < 1\right\}$ denotes the (stable) region in which the depositors always stays. $s_{\mathrm{mod}\,erate} = \{s : u^G - \varepsilon \leq s \leq u^B + z + \varepsilon\}$ denotes the middle ground, at which the bank is sound but

[23] For proof, see Dasgupta(2000) Social Learning with Payoff Complementarities

[24] The Lower,Upper and Middle bounds on the Private Signal structure can be traced directly from our earlier discussion on best case and worst case scenarios for depicting limits on the idiosyncratic fundamental space (see section 1.1, pp)

is vulnerable to a large attack that triggers a regime change. We make the following remarks[25] about the choice of s in the signal range:

Remark 6 ***(No-Dominance signal segment)*** *Attention will be restricted to the segment of the signal space in which there is strategic interraction (i.e Dominance is ruled out.) This means that s lies in interval $[s_{L,}s_U]$, where $s_L \equiv \inf\{s : \Pr ob(\theta < \theta_L \mid s) < 1\}$ and $s_U \equiv \inf\{s : \Pr ob(\theta < \theta_U \mid s) > 0\}$. The arguments for ruling out dominance were enunciated in section 2.1.2*

Remark 7 ***(Uniformity of prior and posterior distribution)*** *While the prior distribution of the idiosyncratic fundamental is common knowledge and follows the uniform distribution law, the posterior distribution of the idiosyncratic fundamental, through certain restrictions on the degree of precision of the signals, will also follow the uniform districution law. The necessary and sufficient condition for that restriction on the noise structure is: $2\varepsilon \leq u^G$.*

Proof. It only suffices to impose sufficient structure on the noise technology in order to be assured of uniformity in the prior and posterior estimates of the idiosyncratic fundamental. We know that the error technology is uniformly distributed on $[-\varepsilon, +\varepsilon]$, with density rate $\frac{1}{2\varepsilon}$. In order to guarantee that the posterior distribution of θ_i, conditional on observing the private signal s_i, is uniform, we need to ensure that the support of θ_i, conditional on s_i, namely $\left[u^G - \varepsilon,\ u^B + z + \varepsilon\right]$, lies exactly within $[0, 1]$.

(1) Restriction $\left[u^G - \varepsilon\right] \geq 0$ implies that $\varepsilon \leq u^G$. Furthermore, the assumption that $s_i > s_L$ is implied by setting $s_i > \inf\{s : \Pr ob(\theta < \theta_L \mid s) < 1\}$. Also, we require that $\min\left[s_i - \varepsilon,\ s_i + \varepsilon\right] \geq 0$. Thus, we are left with a restriction that $\varepsilon \leq s_i$. However, $s_i > \inf\{s_i : \Pr ob(\theta < \theta_L \mid s_i) < 1\} \Leftrightarrow s_i \geq u^G - \varepsilon$. The fact that $\varepsilon \leq s_i \Rightarrow \varepsilon \leq u^G - \varepsilon$. Thus, $2\varepsilon \leq u^G$.

(2) Restriction $\left[u^B + z + \varepsilon\right] \leq 1$ implies $[s_i + \varepsilon] \leq 1$ ($\Rightarrow \varepsilon \leq 1 - s_i$). Furthermore, the assumption that $s_i < s_U$ is implied by setting $s_U \equiv \inf\{s_i : \Pr ob(\theta < \theta_U \mid s_i) > 0\}$. However, $s_U \equiv \inf\{s_i : \Pr ob(\theta < \theta_U \mid s_i) > 0\} \Leftrightarrow s_i \leq u^B + z + \varepsilon$. Since $\varepsilon \leq 1 - s_i$, we can rewrite the whole expression as $\varepsilon \leq 1 - \left(u^B + z + \varepsilon\right) \Rightarrow 2\varepsilon \leq 1 - u^B - z$.

Thus, restriction $\left[u^G - \varepsilon\right] \geq 0$ and restriction $\left[u^B + z + \varepsilon\right] \leq 1$ imply that $2\varepsilon \leq \min[u^G, 1 - u^B - z]$. By assumptions [1] , [2] and [3] on Pp5 (section 2.11), we know that $0 \leq u^G < u^B + z \leq 1$, implying that $u^G < 1 - u^B - z$. Thus, restriction $2\varepsilon < u^G$ is a necessary and sufficient condition for the uniform law to be applicable to posterior distribution ■

It is important to note that, in our framework, it is impossible for depositors of bank i to meet, share their information and learn the true value of θ_i through the Law of Large Numbers(LLN).

[25] These follow from Morris and Shin (1998). We adapt them in the context of our model here

3.2 Public Information Structure

For patient depositors acting in $\Gamma_{B,t=1}$, in addition to their private signal s_B about their bank's idiosyncratic fundamental θ_B, they observe a (non-empty) set of (historical) events that took place in $\Gamma_{A,t=1}$. Let Ω^A be the space of events in bank A. Ω^A comprises a (non-empty)set of k events, where $k = \{1, ..., n\}$, with each event denoted as Φ_k , $\Phi_k \in \{\Phi_1, ..., \Phi_n\} = \Omega^A$. We assume that the folllowing properties hold: $P(\cup_{k=1}^n \Phi_k) = 1$ and $P(\cap_{k=1}^n \Phi_k) = 0$ i.e the events are *mutually exclusive* and *collectively exhaustive*. In our setting, the events spanning $\Gamma_{A,t=1}$ can be either a Success (S^A) or Failure (F^A). Thus, $k = 2$ and $\Omega^A = \{\Phi_1, \Phi_2\}$, with $\Phi_1 = S^A$ and $\Phi_2 = F^A$.The event $\Omega_A = \{S^A, F^A\} \equiv \{Success\ of\ Bank\ A, Failure\ of\ Bank\ A\}$ is commonly observed by all depositors who act in $\Gamma_{B,t=1}$, and forms part of their informational endowment. Upon observing $\Omega^A = \{S^A, F^A\}$, depositors in $\Gamma_{B,t=1}$ will form a re-assessement of the probability distribution of the state of the common macroeconomic fundamental.In addition to their private signal about their bank's idiosyncratic fundamental, depositors in $\Gamma_{B,t=1}$ (i.e the second-mover bank) have a public signal as part of their informational endowment, when it is their turn to act in $\Gamma_{B,t=1}$: the event that takes place in bank A, as a result of $\Gamma_{A,t=1}$. We formally model the public signal as follows:

(1) It is endogeneously derived. In particular, which event, S^A or F^A, in Ω^A occurs is not selected at random, but rather, is the result of solving the model and characterising the Perfect Bayesian Equilibrium (PBE) of the dynamic game. We return to this characteristic more thoroughly in the next section when we define strategies for depositors in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$and discuss the properties of the PBE. The endogeneously derived Public Signal can be used by depositors in $\Gamma_{B,t=1}$in two main ways: (a) It can be used to update

beliefs about the state of the common macroeconomic fundamental (*Bayesian Updating role); (b)* It can act as a coordinating device among depositors in $\Gamma_{B,t=1}$. We shall later show that the specific use of the Public Signal will be vital in determining the nature of the equilibrium of the dynamic game.

(2)All depositors in $\Gamma_{B,t=1}$observe the public signal independently of each other. The public signal is identical for all depositors in bank B and confers the same qualitative information about the event that took place in bank A. Furthermore, all depositors in $\Gamma_{B,t=1}$ are assumed to observe the public event $\Omega_A = \{S^A, F^A\}$ *with some noise.* The precision of the Public Signal varies monotonically with the strength of correlation. More formally, if $0 \leq Corr(u_A^i, u_B^i) \equiv \rho \leq +1$ denote the strength of common exposure in the truncated correlation space, then the precision of the Public Signal takes the form, $F\left[\Omega_A(\rho)\right]$. The precision of the Public Signal varies as thus: As $\rho \to 0$, $F\left[\Omega_A(\rho)\right] \to 0$; as $\rho \to +1$, $F\left[\Omega_A(\rho)\right] \to \Omega_A$. In other words, as correlation tends to 0, the precision of the public signal shrinks to 0 (i.e becomes totally uninformative). As $\rho \to 1$, $F\left[\Omega_A(\rho)\right] \to \Omega_A$ (meaning fulkl revelation of the event Ω_A).In the extreme case when it is equal to 0, depositors in $\Gamma_{B,t=1}$ do not observe the event in $\Gamma_{A,t=1}$ at all.

(3)The event space, $\Omega^A = \{S^A, F^A\}$, is a *sufficient statistic* for the actions of depositors in $\Gamma_{A,t=1}$. Since events in bank A are triggered essentially as a coordinated response by depositors who act in $\Gamma_{A,t=1}$, they are perfectly informative of the (coordinated) actions of depositors in $\Gamma_{A,t=1}$. Hence events perfectly communicate (coordinated) actions in our set-up. If bank A fails (F^A is observed), then it is clear to successors that all patient depositors in $\Gamma_{A,t=1}$had chosen to withdraw (W) early rather than Stay (S). *There thus exists a one-to-one mapping from the space characterising the predecessors event to action space of predecessors.*

(4)Even though events perfectly communicate actions of predecessors, they do not tell anything about what caused such actions. Did bank A fail because of low realisations of its idiosyncratic fundamental, θ_A,or because the state of the common macroeconomic fundamental was bad? Depositors of bank B are thus presented with a *statistical inference* problem at hand.

(5)Before taking an action, all depositors in $\Gamma_{B,t=1}$update their beliefs in the same way. We assume that all depositors in our model are Bayesians (or at least, can perform some Bayesian Statistical Modelling) and can update their beliefs about the state of the common macroeconomic fundamental in the same way.

Subsequently, the private signals for each depositor in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$,characterise the incompleteness of information *within* each coordination game, $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ respectively. Beliefs that each depositor in $\Gamma_{i,t=1}$, $i = \{A, B\}$,has about his peers in $\Gamma_{i,t=1}$,are driven essentially by his private signal. The noisy public signal characterises the incompleteness of information *across* the two coordination games, $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ respectively. It drives the beliefs of each depositor in $\Gamma_{A,t=1}$(respectively, $\Gamma_{B,t=1}$) has on what others in $\Gamma_{B,t=1}$(respectively, $\Gamma_{A,t=1}$) will do (respectively, have done). Thus, it is safe to say that there are two parts to the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$: (1) two static counterparts with coordination and beliefs (about idiosyncratic fundamental) being driven by depositors private signals and (2) a dynamic counterpart with beliefs (about common macroeconomic fundamental) being driven by the noisy public signal. With assumptions (A.6) and (A.7) in mind, we can refer to *Contemporaneous Strategic Complementarities,* as characterising each static coordination game $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$, and *Dynamic Strategic Complementarities,* as characterising the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. The interraction between *Contemporaneous Strategic Complementarities* and *Dynamic Strategic Complementarities* will determine the precise outcome of the game. We shall allow this outcome to be a function of a varying exogeneous correlation structure at the background.

4 Informational endowments: Private Signals and Dynamic Equilibrium characterisation

We start this sub-section with a proposition by modelling the strategy profiles in the coordination games, $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$. Here, we assume that depositor i in $\Gamma_{B,t=1}$ does not ignore any public signal[26] that is available when he moves and we obtain the unique characterisation of the equilibrium in $\Gamma_{i,t=1}, i = \{A, B\}$. We proceed as follows: first, we represent the strategies available to depositors in the monotone/switching form; the payoff structure for the "marginal depositor" in each bank is then derived. The Perfect Bayesian Equilibrium (PBE) is then defined and formally related to our model. One interesting result is that the PBE satisfies the monotone equilibrium. That simplifies the analysis greatly and enables us to focus on monotone/switching equilibrium throughout. We shall later show, how, by varying the informational assumptions, the concept of a Perfect Bayesian Equilibrium as defined in this section, will change the nature of equilibrium and we may get unique equilibrium, multiple equilibria and Herding outcomes, depending on how strongly both banks are perceived to be connected to the common macroeconomic fundamental.

4.1 Strategy Profiles

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In this section, we will characterise the strategy profiles of depositors in banks A and B. First-mover depositors (i.e depositors in bank A) do not observe a history of past events, when they are called upon (randomly by nature) to move in $\Gamma_{A,t=1}$. Their informational endowment when they act in $\Gamma_{A,t=1}$,thus consists of their private signal (which denotes their type), the common (prior) probability distribution of the state of the macroeconomic fundamental and the history set depicting the set of action profiles by predecessors, which in this case, is equal to the null set. Formally, let $\Theta^A_{i,t=1}$denote the informational endowment of a patient depositor i in bank A at $\Gamma_{A,t=1}$. Then, conditional on $\Gamma_{A,t=1}$, $\Theta^A_{i,t=1} = \{s_A,\ \zeta,\ H^{\Gamma_{A,t=1}}\}$ where s_A denotes the private signal of the depositor about θ_A (with all the associated features of the private signal as discussed before), $\zeta \in \left\{\forall i \in \{A, B\}\,; \exists\ k\ s.t\ P(u^B_i) = 1 - P(u^G_i) = k;\ k \in [0, 1]\right\}$ is the prior probability distribution over the common macroeconomic states and $H^{\Gamma_{A,t=1}} = \{\phi\}$ denotes the history of actions for depositors in $\Gamma_{A,t=1}$. So, for each depositor i acting in $\Gamma_{A,t=1}$,the equilibrium strategy profile takes the following mapping: $\sigma_i : \Theta^A_{i,t=1} \rightarrow a_i \in A_i = \{W, S\}\,, \forall i \in [0, 1]^A$. We will be focusing on *Monotone* or *Switching Strategies* throughout the analysis, which we define as follows[27]:

[26] We shall explicitly model the public signal in the next section.

[27] This definition relates to the one used by Dasgupta(2000), (2001) and is now standard in the modelling of strategies and action profiles in coordination games with incomplete information.

De&nition 8 *(**Monotone Strategy for depositors in** $\Gamma_{A,t=1}$) Depositor i, when acting in $\Gamma_{A,t=1}$, is said to be following a Monotone or Trigger Strategy if he changes his action pro&le, depending on whether the private signal he receives is below or above a signal threshold, s^*. If $\sigma_i : \Theta^A_{i,t=1} \to a_i \in A_i = \{W, S\}$, $\forall i \in [0,1]^A$ holds, then a Monotone Strategy will take the following form:*

$$\sigma_i(\Theta^A_{i,t=1}) = \left\{ \begin{array}{ll} W & if\ s_i \leq s^* \\ S & if\ s_i \geq s^* \end{array} \right\}$$

Upon observing $\Omega^A = \{S^A, F^A\}$, depositors in $\Gamma_{B,t=1}$ know that the returns to the long technology, $E[\tilde{R}(\theta_A, \theta_B, u^j_i)]$ will be affected and that will directly affect their payoffs. At the same time, they will form a re-assessement of the probability distribution of the state of the common macroeconomic fundamental . The updated (posterior) probability distribution spanning the state of the common macroeconomic fundamental is denoted as ζ'[28]. Thus, formally, if $\Theta^B_{i,t=1}$denotes the informational endowment of depositors who move in $\Gamma_{B,t=1}$, then $\Theta^B_{i,t=1} = \{s_B, \zeta', H^{\Gamma_{B,t=1}}\}$ where s_B denotes the private signal on θ_B, $\zeta' \in \left\{P(u^B_i), P(u^G_i)\right\}'$,and $H^{\Gamma_{B,t=1}} = \{\Omega^A\} = \{S^A, F^A\}$. In a similar line of reasoning as for depositors in $\Gamma_{A,t=1}$, we argue that strategies for those acting in $\Gamma_{B,t=1}$take the following mapping: $\sigma_i : \Theta^B_{i,t=1} \to a_i \in A_i = \{W, S\}$, $\forall i \in [0,1]^B$ and that all depositors follow monotone strategies around some signal threshold. Trigger Strategy for those acting in $\Gamma_{B,t=1}$ are de&ned in an analoguous way to that of de&nition 8, except that here, we should be augmenting the informational attributes of depositors in order to account for updated re-assessment of common probability distributions and inclusion of a non-empty historical set. Here is the formal de&nition:

De&nition 9 *(**Monotone Strategy for depositors in** $\Gamma_{B,t=1}$) Depositor i, when acting in $\Gamma_{B,t=1}$, is said to follow a trigger strategy with the following mapping, $\sigma_i : \Theta^B_{i,t=1} \to a_i \in A_i = \{W, S\}$, $\forall i \in [0,1]^B$, if his behaviour is de&ned as follows:*

$$\sigma_i(\Theta^B_{i,t=1}) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s_i \leq s^*(u^{Bad})) \\ S & if & (\ \Omega^A = \{S^A\}) \cap (\ s_i \geq s^*(u^{Good})) \\ S\ or\ W & if & \left\{ \begin{array}{l} either\ \left((\ \Omega^A = \{S^A\}) \cap (\ s_i < s^*(u^{Good}))\right) \\ \quad or\ \left((\ \Omega^A = \{F^A\}) \cap (\ s_i > s^*(u^{Bad}))\right) \end{array} \right\} \end{array} \right\}$$

with $s^*(u^{Bad}) > s^*(u^{Good})$

This de&nition of monotone strategy for depositors in $\Gamma_{B,t=1}$ provides a straightforward characterisation of the behaviour of these depositors. Depositors stay if they observe the public information of the success of bank A (i.e $\Omega^A = \{S^A\}$) and their private signals exceed a certain threshold in their private information space (i.e $s_i \geq s^*(u^{Good})$). With the reverse ordering, they

[28] In a later subsection, we shall see that this re-assessment of the state of the common macroeconomic fundamental is basically at the heart of the whole mechanics of the Bayesian Learning process in the model

will choose to withdraw. It is also important to note that, here, the public event itself has an impact on the threshold of the private signal e.g cases of $\Omega^A = \{S^A\}$ will always lead depositor in $\Gamma_{B,t=1}$, to stochastically infer that the state of the macroeconomic fundamental is good ($u = u^{Good}$). The reverse is true for cases involving failure of bank A. The behaviour of depositors in $\Gamma_{B,t=1}$, will be indeterminate if event $\left[(\ \Omega^A = \{F^A\}) \cap (s_i \leq s^*(u^{Bad}))\right]^C \cap \left[(\ \Omega^A = \{S^A\}) \cap \ (\ s_i \geq s^*(u^{Good}))\right]^C$ holds i.e the events which constitute the basis of their decision sets, form part of the complementary sets of the events that trigger withdrawal or stay[29]. One of such possibility is the occurrence of, say, event $\left((\ \Omega^A = \{F^A\}) \cap (\ s_i > s^*(u^{Bad}))\right)$. Here, observing the failure of bank A is likely to bias the depositor's decision towards withdrawing but a strong private signal is likely to have the opposite effect. In this case, the decision as to whether to stay or withdraw, will depend on comparison of the payoff to staying with the payoff to withdrawing..

4.1.1 Perfect Bayesian Equilibrium

Upon observing $H^{\Gamma_{B,t=1}}$, no depositor in $\Gamma_{B,t=1}$can detect out-of-equilibrium play i.e all strategies in the continuation game can be reached with positive probability in the equilibrium pro&le of the game. This occurs because $H^{\Gamma_{B,t=1}}$ is compatible with any strategies in the equilibrium pro&le of the game[30]. In addition, because we have abstracted from explicit institutional arrangements that connect the banks through &nancial contracts (i.e the interbank market), it follows that the payoffs of depositors acting in $\Gamma_{A,t=1}$, are only related to the payoffs of depositors acting in $\Gamma_{B,t=1}$, through an **informational channel** (different perceptions of u_i^j) (payoffs in $\Gamma_{A,t=1}(\Gamma_{B,t=1})$ are directly related to payoffs in $\Gamma_{B,t=1}(\Gamma_{A,t=1})$ through $\tilde{R}(\theta_A, \theta_B, u_i^j)$). Subsequently, in the presence of such interdependent payoff structure, strategies for depositors in $\Gamma_{i,t=1}$, $i = \{A, B\}$,are sequentially rational if they maximise the payoffs in $\Gamma_{i,t=1}$, $i = \{A, B\}$, given the strategies played by all other players in the game. Beliefs about revised assessments of macroeconomic fundamentals, are pinned down by Bayes rule throughout the whole pathway prescribed by the dynamic equilibrium concept.

We &rst provide a formal de&nition of the Perfect Bayesian Equilibrium and relate it to parameters of our model. For simplicity, we emphasise on the Perfect

[29] It is obvious that event $\left[(\ \Omega^A = \{F^A\}) \cap (s_i \leq s^*(u^{Bad}))\right]^C \cap \left[(\ \Omega^A = \{S^A\}) \cap \ (\ s_i \geq s^*(u^{Good}))\right]$ is tantamount to the occurrence of event $\left((\ \Omega^A = \{S^A\}) \cap \ (\ s_i < s^*(u^{Good}))\right)$ and of event $\left((\ \Omega^A = \{F^A\}) \cap (\ s_i > s^*(u^{Bad}))\right)$,which we have shown in the de&nition

[30] Recall that bank A may fail due to coordinated attack by depositors in $\Gamma_{A,t=1}$.But it may also fail for extremely low realisations of θ_A,irrespective of the actions of depositors in $\Gamma_{A,t=1}$.

Bayesian Equilibrium concept[31].

Definition 10 ***(Perfect Bayesian Equilibrium)*** *A Perfect Bayesian Equilibrium (PBE) in the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$, is an assessment of strategy profiles $\{\sigma_i : \Theta^A_{i,t=1} \to a_i \in A_i = \{W, S\}, \forall i \in [0,1]^A$ in $\Gamma_{A,t=1}$ and $\sigma_i : \Theta^B_{i,t=1} \to a_i \in A_i = \{W, S\}, \forall i \in [0,1]^B$ in $\Gamma_{B,1}\}$ and beliefs system $\{\{\zeta, \zeta\prime\} = \{i = \{A, B\}; \{P(u^B_i), P(u^G_i)\}, \{P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$ such that:*

(1) Given the beliefs system $\{\zeta, \zeta\prime\} = \{i = \{A, B\}; \{P(u^B_i), P(u^G_i)\}, \{P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$, and after every possible history $H^{\Gamma_i, t=1}$, each depositor strategy is rational (i.e is a best-response to any possible moves by all depositors of the same bank as well as to all depositors of the other bank);

(2) If $H^{\Gamma_i, t=1}$ occurs with positive probability, then the beliefs system $\{\zeta, \zeta\prime\} = \{i = \{A, B\}; \{P(u^B_i), P(u^G_i)\}, \{P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$ should be optimal given $\sigma_i(\Theta^A_{i,t=1}), \forall i \in [0,1]^A$ and given $\sigma_i(\Theta^B_{i,t=1}), \forall i \in [0,1]^B$. This means that $\zeta\prime = \{i = \{A, B\}; P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$ is derived from $\zeta = \{i = \{A, B\}; P(u^B_i), P(u^G_i)\}$ using Bayes Rule.

In our model, the above formal definition of the PBE in the game between $\Gamma_{A,t=1}$and $\Gamma_{B,t=1}$translates into the following criteria/requirements[32]:

Criterion 11 ***(Beliefs Formation)****At each information set, the depositor with the move must have a belief (represented by some probability distribution) over which node of his information set has been reached.*

Since it is never common knowledge as to which state of the common macroeconomic fundamental has been realised, each cohort of depositors in $\Gamma_{i,t=1}, i = \{A, B\}$, has some prior beliefs about the state of the common fundamental. The beliefs-formation mechanism is represented as $\{\zeta, \zeta\prime\} = \{i = \{A, B\}; \{P(u^B_i), P(u^G_i)\}, \{P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$, where $\zeta = \{i = \{A, B\}; \{P(u^B_i), P(u^G_i)\}$ represents the prior beliefs and $\zeta\prime = \{P(u^B_i \mid \Omega_A), P(u^G_i \mid \Omega_A)\}$ represent the posterior beliefs, after observing $\Omega_A = \{F_A, S_A\}$.

Criterion 12 ***(Sequential Rationality)*** *At each of the possible nodes(characterising his prior beliefs) in his information set, each depositor's strategy must maximise his payoffs, given his beliefs about what depositors of his own bank and those of the other bank will do.*

[31] That avoids the need of modelling any out-of-equilibrium beliefs since they do not arise in the model.

[32] See Gibbons, "A primer of Game Theory" for more details. We have adopted their definition within the context of our model.

This idea of rationality needs more elaboration, given the complex nature of our payoff function and given that, unlike most sequential move games with incomplete information, we do not have one individual moving at a time, but a continuum of individuals doing so. The analysis will also provide us with a way of characterising Perfect Bayesian Equilibrium in sequential games with cohorts of individuals moving at different times.

Suppose bank A fails. The failure of bank A conveys negative information about the state of the common macroeconomic fundamental to depositors playing in $\Gamma_{B,t=1.}$ Given this information, depositors playing in $\Gamma_{B,t=1}$, will update their beliefs[33] about the state of the common macroeconomic fundamental, and this posterior change in beliefs will adversely affect returns from the investment portfolio. Thus, given perceived bad news about macroeconomic fundamental, each depositor in $\Gamma_{B,t=1}$,lowers his perceptions of the expected returns on the investment portfolio, $E\left[\tilde{R}(\theta_A, \theta_{B,}u_i^j)\right]$, and chooses to withdraw. Bank B is almost likely to share the same fate as bank A, merely from an informational channel. Because $\tilde{R}(\theta_A, \theta_{B,}u_i^j)$ depends on a parameter that commonly links both banks, u_i^j and is realised after a decision has been made in both banks, any event in bank B triggering a change in $\tilde{R}(\theta_A, \theta_{B,}u_i^j)$ will also affect the payoff of depositors playing in $\Gamma_{A,t=1}$. Each depositor in $\Gamma_{A,t=1}$, must anticipate that, conditional upon observing him (and other depositors in $\Gamma_{A,t=1}$) withdraw, bank B will fail. Due to lower expected returns on the investment portfolio, $E\left[\tilde{R}(\theta_A, \theta_{B,}u_i^j)\right]$, that the failure of bank B will bring, each depositor in $\Gamma_{A,t=1}$, will have an incentive to withdraw. He is also aware that all other depositors playing in $\Gamma_{A,t=1}$, are also aware of this. As a result, by anticipating the lower returns that will result due to the effects of their own actions on depositors of bank B, all depositors playing in $\Gamma_{A,t=1}$ withdraw and bank A fails. This circularity between event triggered in bank A and event triggered in bank B, shows that each depositor playing in $\Gamma_{i,t=1}, i = \{A, B\}$, must coordinate his decision with depositors of the same bank, as well as with depositors of the other bank. His decision as to whether to stay or withdraw, stems from the entirely rational decision of accounting for what his action will have on depositors across banks and the feedback effect resulting from these depositors actions, on depositors of his own bank.

This characterisation enables us capture the importance of contemporaneous complementarities and dynamic complementarities in depositors decision sets.

Each depositor i in $\Gamma_{i,t=1}$faces a uniform posterior belief over θ_i, conditional on observing his private signal $s_{i.}$ Thus, we can model that posterior belief formally as : $\theta_i \mid s_i \sim Uniform[s_i - \varepsilon,\, s_i + \varepsilon]$, $\varepsilon \leq s_i \leq 1 - \varepsilon$. Assuming that all other depositors in $\Gamma_{i,t=1}$ play the game around the threshold θ^*, then the proportion of early withdrawals can be modelled as:

[33] The speci&c form of the change in (posterior) beliefs, will be spelt out later

$$\delta_i[\theta,\theta^*] = \left\{ \begin{array}{lr} 1 & \theta < \theta^* - \varepsilon \\ \frac{1}{2} + \frac{(\theta^*-\theta)}{2\varepsilon} & \theta^* - \varepsilon \leq \ \theta < \theta^* + \varepsilon \\ 0 & \theta \geq \theta^* + \varepsilon \end{array} \right\}$$

Given that dynamics in the model are driven by reassessments of beliefs about u_i^j, we can represent the payoff structure in simple algebraic terms.

In particular, the "net payoff to staying" for depositor i in $\Gamma_{i,t=1}$: $\left\{ U_i \left[\frac{\left\{(1-\lambda)-\frac{\delta_i(1-\lambda)}{r}\right\} E[\tilde{R}(\theta_A,\theta_B,u_i^j)]}{(1-\lambda)(1-\delta_i)} \right] - U_i$

can be represented as: $\Pi_i(s_i,\theta^*) = \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta,\delta_i[\theta,\theta^*])\, d\theta$, where $\pi(\theta,\delta_i[\theta,\theta^*])$ tells us how the net payoff varies with $\delta_i[\theta,\theta^*]$. First, we move with the characterisation of the Perfect Bayesian Equilibrium (PBE) of the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$,by starting with the decision problem of depositor

i in $\Gamma_{B,t=1}$, $i \in [0,1]^B$. With the payoff structure denoted as $\Pi^B(s^B,s^*) = \int_{s_i^B-\varepsilon}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))\ d\theta^B$and $\Theta_{i,t=1}^B = \{s_B,\zeta\prime,H^{\Gamma_{B,t=1}}\}$, depositor i in $\Gamma_{B,t=1}$, $i \in [0,1]^B$ observes the actions of his predecessors (i.e depositors in $\Gamma_{A,t=1}$) through parameter $H^{\Gamma_{B,t=1}}$ and he adjusts his beliefs of the probability of the common macroeconomic fundamental from ζ to $\zeta\prime$. As a result, he plays a best response to the strategies of his predecessors, whilst updating his beliefs about the state of the common fundamental. His expected utility to staying as opposed to withdrawing would depend on this posterior belief $\zeta\prime$, his private signal s^B and the strategy of successors in the continuation game. Since the withdrawal game ends after $\Gamma_{B,t=1}$, there are no successors in this game and the strategy set for successors is the null set. Formally, the expected utility to staying as opposed to withdrawing is modelled as: $EU_{i\in[0,1]^B}[s_B,\zeta\prime = \left\{P(u_i^B),P(u_i^G)\right\}\prime] = P_i \int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B + (1-P_i)\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B$

where, P_i denotes the probability that bank B succeeds, given the strategies pursued by depositors in bank A, or formally, $\Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$, $0 < \Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.)) \leq 1$, and as before, $\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))$ $d\theta^B$ denotes the positive part of the net payoff to staying and $\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))$ $d\theta^B$ denotes the negative part. Since $\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))\, d\theta^B = -\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))$ $d\theta^B$,then $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$ can be re-written as :

$$EU_{i\in[0,1]^B}[s_B,\zeta\prime] = [2P_i - 1] \int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B, \delta_B(s^*,\theta^B))d\theta^B \tag{4}$$

The expression we give for $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$ is very intuitive. The expected utility to staying for any depositor playing in $\Gamma_{B,t=1}$, does not depend on his own action but the actions of other depositors in $\Gamma_{B,t=1}$, as well as on the actions of depositors in $\Gamma_{A,t=1}$, in the circularity spirit outlined earlier. Whether bank B survives or fails, does not depend on the action of only one depositor but on the collective action of all depositors playing in $\Gamma_{B,t=1}$. The parameter $\Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$ encapsulates the action of other depositors of other depositors in $\Gamma_{B,t=1}$ on the expected utility of a typical depositor playing in $\Gamma_{B,t=1}$. To be more speci&c, P (reduced version of $\Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$) denotes the probability of bank B surviving due to joint proportion of other depositors withdrawing, $\delta_B(s^*,\theta^B)$, being less than the critical bankruptcy threshold, r. Similarly, $1-P$ denotes the probability of bank B failing due to $\delta_B(s^*,\theta^B)$ being above the bankruptcy threshold. The associated payoffs $\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B, \delta_B(s^*,\theta^B))d\theta^B$ and $\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B, \delta_B(s^*,\theta^B))d\theta^B$, respectively depict the ex-post payoffs to the depositor in $\Gamma_{B,t=1}$, when bank B survives and fails respectively. Thus, one can see that $\delta_B(s^*,\theta^B)$ affects not only P, but also expected payoffs $\Pi_i(s_i,\theta^*)$ $(= \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta, \delta_i[\theta,\theta^*])\, d\theta)$. For example, when $\delta_B(s^*,\theta^B)$ is sufficiently strong, the payoff to staying for a depositor with a monotone strategy, will be given by $\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B, \delta_B(s^*,\theta^B))d\theta^B$ (<0). Conversely, for low proportion of early withdrawals, $\delta_B(s^*,\theta^B)$, the ex-post payoff to staying is positive i.e $\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B, \delta_B(s^*,\theta^B))d\theta^B > 0$. Thus, each depositor i in in $\Gamma_{B,t=1}$, $i \in [0,1]^B$, maximises $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$, given his endowment $\Theta^B_{i,t=1} = \{s_B, \zeta\prime, H^{\Gamma_{B,t=1}}\}$. Taking into account the beliefs updating process as well as the best reponse of his predecessors, we de&ne the Best-Response function for depositor i in $\Gamma_{B,t=1}$as:.

$$\Psi^B_{i\in[0,1]^B}(.) = \max_{a_i \in A_i} [2P_i[\Psi^A_{i\in[0,1]^A}(.)] - 1] \int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B, \delta_B(s^*,\theta^B))\mu(u^j_B \mid \Theta^B_{i,t=1})d\theta^B \tag{5}$$

Each depositor in $\Gamma_{B,t=1}$is assumed to have the same best-response function, as de&ned in $\Psi^B_{i\in[0,1]^B}(.)$.

Each depositor i in $\Gamma_{A,t=1},\ i \in [0,1]^A$, will anticipate that, no matter what action he takes, his successors (i.e depositor i in $\Gamma_{B,t=1},\ i \in [0,1]^B$) will always be playing a best-response, as dictated by $\Psi^B_{i\in[0,1]^B}(.)$. Since the whole structure of the game, as seen in section 1.3, table 3, is common knowledge, each depositor i in $\Gamma_{A,t=1}, i \in [0,1]^A$, will take $\Psi^B_{i\in[0,1]^B}(.)$ into consideration when tracing out his best-response.

Thus, with $\Theta^A_{i,t=1} = \{s_{A,}\ \zeta,\ H^{\Gamma_{A,1}}\}$ and the payoff structure expressible as $\Pi^A(s^A, s^*) = \displaystyle\int_{s_i^A-\varepsilon}^{s_i^A+\varepsilon} \pi^A(\theta^A, \delta_A(s^*, \theta^A))\ d\theta^A$, depositor i in $\Gamma_{A,t=1},\ i \in [0,1]^A$, has an expected utility which is dependent on his private signal s^A, his prior belief ζ (with an empty history set $H^{\Gamma_{A,t=1}} = \{0\}$) and strategy of successors in the continuation game (i.e depositors in $\Gamma_{B,t=1}$), as captured by the best response function $\Psi^B_{i\in[0,1]^B}(.)$. Each depositor in $\Gamma_{A,t=1}$knows that, upon any decision he takes, depositors in $\Gamma_{B,t=1}$, will observe the event generted and will attempt to play a best response to it. In a way analoguous to the analysis carried out for depositors in $\Gamma_{B,t=1}$, we de&ne the best response function for those in $\Gamma_{A,t=1}$ as:

$$\Psi^A_{i\in[0,1]^A}(.) = \max_{a_i\ \in\ A_i} [2P_i(\Psi^B_{i\in[0,1]^B}(.)) - 1] \int_{\theta^*}^{s_i^A+\varepsilon} \pi^A(\theta^A, \delta_A(s^*, \theta^A))\mu(u^j_A \mid \Theta^A_{i,t=1})d\theta^A \tag{6}$$

where P_i simply denotes $\Pr\left(\delta_A(s^*, \theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right)$, $0 < \Pr\left(\delta_A(s^*, \theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right) \leq 1$.

Criterion 13 ***(Bayes Updating Process)*** *At an information set which lies on the equilibrium path (i.e can be reached with positive probability given the equilibrium strategy of the game), the beliefs updating process is undertaken through Bayes rule.*

Denote $\mu(u^j_B \mid \Theta^B_{i,t=1})$ as the process of of updating beliefs about u^j_B from their prior state $\zeta = \{i = \{A, B\}\,; \{P(u^{Bad}_B), P(u^G_B)\}$ to the posterior state $\zeta\prime = \{P(u^{Bad}_B \mid \Omega_A), P(u^G_B \mid \Omega_A)\}$, for depositors in $\Gamma_{B,t=1}$. Similarly, for depositors in $\Gamma_{A,t=1}$, $\mu(u^j_A \mid \Theta^A_{i,t=1})$ denotes that updating process. Since depositors in

$\Gamma_{A,t=1}$ move &rst and their information set, $\Theta^A_{i,t=1}$,contains an empty historical set, it is not hard to realise that $\mu(u^j_A \mid \Theta^A_{i,t=1})$ would be the same as $\mu(u^j_A)$, the prior probability over states denoted as $\zeta = \{i = \{A, B\}\,; \{P(u^B_i), P(u^G_i)\}$. We focus on the exact mechanics of the updating process in the next subsection. For the moment, it just suffices to believe that, with no information set being off the equilibrium path given the equilibrium strategies of the game, any updating process that conforms with Bayes rule will still keep us along the trajectory pathway as prescribed by the Perfect Bayesian Equilibrium concept.

This mutual best-response functional relationship for strategies of depositors in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ is in conformity with the circularity that exists between strategies and beliefs formation in a Perfect Bayesian Equilibrium - that strategies are optimally derived from beliefs and that beliefs are consistent with those strategies. In models of dynamic coordination games where the Perfect Bayesian Equilibrium is modelled, it is not uncommon to restrict attention to a speci&c class of equilibria that greatly simplify the analysis. Such modelling also enables us to capture the idea of complementarities in games $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ respectively. In bank B, for instance, $\Pr(\delta_B(s^*, \theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$ is decreasing in $\delta_B(s^*, \theta^B)$. Similarly, in bank A, $\Pr\left(\delta_A(s^*, \theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right)$ is decreasing in $\delta_A(s^*, \theta^A)$. The idea is quite intuitive: as proportion of depositors who withdraw early rises, the probability of bank surviving such a run decreases. This has important implications for the ex-post payoff to depositors in the bank. We can thus, without loss of generality, deduce that, for a depositor playing in $\Gamma_{B,t=1}$, as $\delta_B(s^*, \theta^B) \to 1$, $\Pr(\delta_B(s^*, \theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.)) \to 0$ and $EU_{i\in[0,1]^B}[.] \to \displaystyle\int_{s^B_i-\varepsilon}^{\theta^*} \pi^B(\theta^B, \delta_B(s^*, \theta^B))d\theta^B$ (< 0). A similar conclusion will be drawn for depositors in $\Gamma_{A,t=1}$. Interpret it as follows: as proportion of depositors running on the bank increases, the probability that the bank will resist that run (or survive) decreases and the expected utility for a depositor of staying in the bank becomes negative. Thus, the depositor has less incentive to stay in the bank and, instead, chooses to run. This is what we dubbed **contemporaneous strategic complementarities.** We have just shown that, by coordinating each depositors action with the action of other depositors of the same , the payoff of each individual depositor is characterised by contemporaneous complementarities.

In a similar way, it can be shown that each depositors action is increasing in action of depositors in the other bank. This is what we dub **dynamic strategic complementarities.** In particular, we want to show that, for each depositor in $\Gamma_{A,t=1}$, as $\delta_B(s^*, \theta^B) \to 1$, $\Pr\left(\delta_A(s^*, \theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right) \to 0$ and $EU_{i\in[0,1]^B}[.] \to \displaystyle\int_{s^A_i-\varepsilon}^{\theta^*} \pi^A(\theta^A, \delta_A(s^*, \theta^A))d\theta^A$ (< 0). Similarly, for depositor in $\Gamma_{B,t=1}$, as $\delta_A(s^*, \theta^A) \to 1$, $\Pr\left(\delta_B(s^*, \theta^B) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right) \to 0$ and

$EU_{i\in[0,1]^B}[.] \rightarrow \int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B$ (<0). In other words, each individual depositor's incentive to withdraw increases as more and more depositors across the bank withdraw. It is not hard to prove dynamic strategic complementarities. The existence of the returns to the investment technology, $E\left[\tilde{R}(\theta_A,\theta_B,u_i^j)\right]$, and most importantly, its timing, acts as the common link between $\delta_A(s^*,\theta^A)$ and $\delta_B(s^*,\theta^B)$.

We next turn to that specification: in particular, we want to show that all equilibrium profiles that satisfy the PBE concept must also satisfy the Trigger Equilibrium concept[34]. This will enable us simplify the analysis of the dynamic equilibrium pathway considerably and to focus attention on trigger equilibria throughout the whole experiment

Proposition 14 *If the Public Signal is used for Bayesian Updating only, then any Perfect Bayesian Equilibrium of the dynamic game between* $\Gamma_{A,t=1}$ *and* $\Gamma_{B,t=1}$ *is a Monotone Equilibrium. This holds true irrespective of the correlation structure characterising the link between the two banks. On the other hand, if the Public Signal is used for purposes other than Bayesian Updating (e.g acting as possible coordination device), then the set of equilibrium traced by the Perfect Bayesian Concept, is much wider than the class that just includes Monotone Equilibrium*

Proof. (use of **Intermediate Value Theorem)**

Let assessments $\{\ \sigma_i(\Theta^A_{i,t=1}), \forall i \in [0,1]^A, \sigma_i(\Theta^B_{i,t=1}), \forall i \in [0,1]^B,\ \zeta = [i = \{A,B\}\,; \{P(u_i^B), P(u_i^G)\}],\ \zeta\prime = [P(u_i^B \mid \Omega_A),\ \ P(u_i^G \mid \Omega_A)]\ \}$ denote the Perfect Bayesian Equilibrium of the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. Any depositor in $\Gamma_{i,t=1}$, $i=\{A,B\}$,will play a best-response to actions of predecessors and successors (where applicable), with the best response function defined by $\Psi^A_{i\in[0,1]^A}(.)$ $= \max[2P_i(\Psi^B_{i\in[0,1]^B}(.)) - 1]\int_{\theta^*}^{s_i^A+\varepsilon} \pi^A(\theta^A,\delta_A(s^*,\theta^A))\mu(u_A^j \mid \Theta^A_{i,t=1})d\theta^A$ and $\Psi^B_{i\in[0,1]^B}(.) = \max_{a_i\, \in\, A_i}[2P_i[\Psi^A_{i\in[0,1]^A}(.)] - 1]\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))\mu(u_B^j \mid \Theta^B_{i,t=1})d\theta^B$, depending on whether he plays in $\Gamma_{A,t=1}$ or in $\Gamma_{B,t=1}$.

For a depositor in $\Gamma_{A,t=1}$, for example, the expected utility, $EU_{i\in[0,1]^A}[s_A,\zeta\prime] = [2P_i - 1]\int_{\theta^*}^{s_i^A+\varepsilon} \pi^A(\theta^A,\delta_A(s^*,\theta^A))d\theta^A$,varies continuously and monotonically with s^A. High values of s^A are associated with low value of proportion of early withdrawals, $\delta_A(s^*,\theta^A)$. Thus, payoff to staying, $\pi^A(\theta^A,\delta_A(s^*,\theta^A))$ is high and probability of staying, $\Pr\left(\delta_A(s^*,\theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right)$, takes a high

[34] The same trick was used by Dasgupta (2001) in his study of herding in investment portfolio choices

value as well. These explain why, for high values of s^A, $EU_{i\in[0,1]^A}[s_A,\zeta\prime] > 0$. For low realisations of s^A,we have a reverse ordering: $\delta_A(s^*,\theta^A)$ is high and $\pi^A(\theta^A,\delta_A(s^*,\theta^A)$ and $\Pr\left(\delta_A(s^*,\theta^A) < r \mid \Psi^B_{i\in[0,1]^B}(.)\right)$ take low realisations. Thus, $EU_{i\in[0,1]^A}[s_A,\zeta\prime] \leq 0$ for low realisations of s^A. Thus, generalising the argument to any depositor i (no matter to which game he belongs to), we can argue that he will stay if $EU_{i\in[0,1]^A}[s_A,\zeta\prime] \geq 0$ and withdraw if $EU_{i\in[0,1]^A}[s_A,\zeta\prime] < 0$ (as per proposition 3). Since $EU_{i\in[0,1]^A}[s_A,\zeta\prime]$ is continuous and monotonically increasing in s_i, then by the intermediate value theorem, $\exists\ s^*$ such that $\forall s_A > s^*$, $EU_{i\in[0,1]^A}[s_A,\zeta\prime] \geq 0$ and $\forall s_i < s^*$, $EU_{i\in[0,1]^A}[s_A,\zeta\prime] < 0$. In line with the existence of $\Psi^A_{i\in[0,1]^A}(.)$, $\exists\ \sigma_A(\Theta_{A,t=1})$ such that:

$$\sigma_A(\Theta_{A,t=1}) = \left\{ \begin{array}{ll} W & if\ s_A \leq s^* \\ S & if\ s_A > s^* \end{array} \right\}$$

which corresponds exactly to the notion of Monotone Equilibrium that we stated in proposition 2.

For depositor in $\Gamma_{B,t=1}$, the expected utility is given by expression: $EU_{i\in[0,1]^B}[s_B,\zeta\prime] = [2P_i - 1]\displaystyle\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B$. When bank A survives , $\Omega = \{S_A\}$,and s_B is high ($> s^*$), $\delta_B(s^*,\theta^B)$ is low by the logic inherent in analysis in pp 35. Thus, $\Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$ becomes high and $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$ has relatively more of the $\displaystyle\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B$ (> 0) component and relatively less of the $\displaystyle\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B (< 0)$. Thus, $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$ > 0 when $s_B > s^*$ and $\Omega = \{S_A\}$.

When $\Omega = \{F_A\}$ and s_B is low ($< s^*$), $\delta_B(s^*,\theta^B)$ is high, $\Pr(\delta_B(s^*,\theta^B) < r \mid \Psi^A_{i\in[0,1]^A}(.))$ is low and $EU_{i\in[0,1]^B}[s_B,\zeta\prime]$ has relatively less of the $\displaystyle\int_{\theta^*}^{s_i^B+\varepsilon} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B$ (> 0) component and relatively more of the $\displaystyle\int_{s_i^B-\varepsilon}^{\theta^*} \pi^B(\theta^B,\delta_B(s^*,\theta^B))d\theta^B (< 0)$. Thus, $EU_{i\in[0,1]^B}[s_B,\zeta\prime] < 0$ when $s_B < s^*$and $\Omega = \{F_A\}$. Thus, the perfect bayesian equilibrium concept, as defined above, will lead depositors in $\Gamma_{B,t=1}$, to follow a strategy along the following lines:

$$\sigma_i(\Theta^B_{i,t=1}) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s_i \leq s^*) \\ S & if & (\ \Omega^A = \{S^A\}) \cap (\ s_i \geq s^*) \\ S \text{ or } W & if & \left\{ \begin{array}{c} either\ \left((\ \Omega^A = \{S^A\}) \cap (\ s_i < s^*)\right) \\ or\ \left((\ \Omega^A = \{F^A\}) \cap (\ s_i > s^*)\right)\right) \end{array} \right\} \end{array} \right\}$$

which is exactly the trigger equilibrium we defined above. ■

An interesting extension of our result would be to see how the result would vary as we tighten the informational assumptions of the model. In the most extreme case in which depositors only consider their private signals and nothing else, there is a θ^*[35] for each game $\Gamma_{i,t=1}$, $i = \{A,B\}$, and θ^* would consti-

[35] Since each bank starts with the same endowment ex-ante, then the same value for θ^*

tute the unique Bayesian-Nash equilibrium for each $\Gamma_{i,t=1}$, $i = \{A, B\}$. This is summarised in the following corollary:

Corollary 15 *If the event $\Omega^A = \{S^A, F^A\}$ is completely ignored by depositors in $\Gamma_{B,t=1}$ (e.g when $\rho \to 0$ or more precisely, $\digamma(\Omega_A(\rho) \to 0)$ and is not used for Bayesian updating about the state of u_i^j, the resulting parametric restrictions break the dynamic game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ into two static-coordination games. u_i^j becomes a payoff-irrelevant variable. The same realisation of θ^* would characterise the equilibrium of each static game, $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. Provided that depositors in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ are playing according to their monotone strategies as defined in section 4.1, θ^* is also the unique Bayesian-Nash equilibrium of each static game, with no dynamic elements linking the θ^* of each bank .*

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This result is quite intuitive. If depositor i in $\Gamma_{i,t=1}$, $i = \{A, B\}$ uses only his private signal and he completely ignores the whole process of Bayesian updating about state u_i^j, the vital link that drives the dynamics of beliefs in the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ gets severed. Depositor i in $\Gamma_{A,t=1}$, would simply be maximising his payoffs given his type and given his beliefs about other depositors' types (all in $\Gamma_{A,t=1}$). Actions of depositors in the continuation game will be ignored. Similarly, each depositor in $\Gamma_{B,t=1}$ will be playing a best response to other depositors' actions in $\Gamma_{B,t=1}$, while ignoring the past history of events. The major weakness about such informational restrictions is that it pre-empts the major purpose for which this paper has been set: to explain how dynamics of information flows affect dynamics of coordination in $\Gamma_{i,t=1}$, $i = \{A, B\}$. In addition, contagion cannot be modelled in this set-up because the unique θ^* is static and does not vary at all with parameters of the model.

5 Strict Private Informational Dominance and the Perfect Bayesian Equilibrium

Claim 16 *For sufficiently 'low'[36] degree of exposure to the common macroeconomic fundamental, $0 \leq \rho < \acute{\rho}$, the Private Signals stochastically dominate the Public Signal in depositors' decision set. Three observations can be made:*

(1) There exists an equilibrium trigger in $\Gamma_{i,t=1}, i = \{A, B\}$

(2) The equilibrium trigger is Unique (Non-Monotone equilibria are precluded)

would apply to each bank.

[36] How 'low' this refers to, will become clear in a later section when we reconcile the notion of signal ordering, with the notion of strategic complementarities

(3) The equilibrium trigger in each bank is dependent on the common macroeconomic fundamental and varies inversely with each other.

It is to an analysis of this that we now turn to:

Proposition 17 ***(Existence of a Trigger Equilibrium in*** $\Gamma_{i,t=1}$ *) There exists a threshold* θ^* *in the idiosyncratic fundamental space and* s^* *in the private signal space, such that the bank fails if* $\theta < \theta^*$ *(i.e everybody withdraws with* $s < s^*$*) and succeeds if* $\theta > \theta^*$ *(i.e everybody stays with* $s > s^*$*)*

Proof. [37] Each depositor i in $\Gamma_{i,1}$faces a uniform posterior belief over θ_i,

conditional on observing his private signal $s_{i.}$ Thus, we can model that posterior belief formally as : $\theta_i \mid s_i \sim Uniform[s_i - \varepsilon,\ s_i + \varepsilon]$, $\varepsilon \leq s_i \leq 1 - \varepsilon$. Assuming that all other depositors in $\Gamma_{i,1}$ play the game aroung the threshold θ^*, then the proportion of early withdrawals can be modelled as:

$$\delta_i[\theta, \theta^*] = \left\{ \begin{array}{ll} 1 & \theta < \theta^* - \varepsilon \\ \frac{1}{2} + \frac{(\theta^* - \theta)}{2\varepsilon} & \theta^* - \varepsilon \leq\ \theta < \theta^* + \varepsilon \\ 0 & \theta \geq \theta^* + \varepsilon \end{array} \right\}$$

Considering table 2 (section 1.2), the net payoff to staying as opposed to withdrawing for each depositor i in $\Gamma_{i,1}$can be re-parameterised in terms of θ and θ^*as follows:

$$\Pi_i(s_i, \theta^*) = \int_{s_i - \varepsilon}^{s_i + \varepsilon} \pi(\theta, \delta_i[\theta, \theta^*])\ d\theta$$

In the *Bankruptcy Condition Space,* $\delta_i[\theta, \theta^*] > r \implies \{[\ \frac{1}{2} + \frac{(\theta^* - \theta)}{2\varepsilon}] > r\} \implies \{\theta < \theta^* + \varepsilon(1 - 2r)\}$

In the *No-Bankruptcy Condition Space,* $\delta_i[\theta, \theta^*] < r \implies \{[\ \frac{1}{2} + \frac{(\theta^* - \theta)}{2\varepsilon}] < r\} \implies \{\theta < \theta^* + \varepsilon(1 - 2r)\}$

Thus, we may partition $\Pi_i(s_i, \theta^*) = \displaystyle\int_{s_i - \varepsilon}^{s_i + \varepsilon} \pi(\theta, \delta_i[\theta, \theta^*])\ d\theta$ into the *Bankruptcy Condition Space* and the *No-Bankruptcy Condition Space* as follows:

$$\Pi_i(s_i, \theta^*) = \int_{\theta^* + \varepsilon(1-2r)}^{\theta^* + \varepsilon} \{U_i \left[\frac{\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\} R(.)}{(1-\lambda)(1-\delta_i)}\right] - U_i(1)\}\ d\theta + \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon(1-2r)} \{U_i(0) - U_i\left[\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)}\right]\}\ d\theta$$

$$= \int_{\theta^* + \varepsilon(1-2r)}^{\theta^* + \varepsilon} U_i \left[\frac{\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\} R(.)}{(1-\lambda)(1-\delta_i)}\right] d\theta - \int_{\theta^* - \varepsilon}^{\theta^* + \varepsilon(1-2r)} U_i \left[\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)}\right] d\theta + \{U_i(0)[2\varepsilon(1 - r)] - U_i(1)[2\varepsilon r].$$

Now, let $U_i \left[\frac{\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\} R(.)}{(1-\lambda)(1-\delta_i)}\right]$ be denoted as $\eta(\theta, \theta^*)$, let $U_i \left[\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)}\right]$ be denoted as $\lambda(\theta, \theta^*)$, let $\{U_i(0)[2\varepsilon(1 - r)] - U_i(1)[2\varepsilon r]$ be a constant, χ. A much simpler expression for $\Pi_i(s_i, \theta^*)$ would be as follows:

[37] The proof for existence of equilibrium in coordination games can be found in Goldstein and Pauzner (2003) and Dasgupta (2003). We follow Dasgupta (2003) version here.

$$\Pi_i(s_i,\theta^*) = \int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \eta(\theta,\theta^*)\ d\theta + \int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \lambda(\theta,\theta^*)\ d\theta\ + \chi$$

An important feature that arises is that, since $\delta_i[\theta,\theta^*]$ is monotonic in θ^*, and $\pi(\theta,\delta_i[\theta,\theta^*])$ itself, is monotonic in θ^*, then $\Pi_i(s_i,\theta^*)$ is necessarily monotonic in θ^*. We are interested in establishing how $\Pi_i(s_i,\theta^*)$ varies with θ^*. Take derivatives with respect to θ^* throughout:

$$\frac{\partial}{\partial\theta^*}\Pi_i(s_i,\theta^*) = \frac{\partial}{\partial\theta^*}\int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \eta(\theta,\theta^*)\ d\theta + \frac{\partial}{\partial\theta^*}\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \lambda(\theta,\theta^*)\ d\theta$$

$$\frac{\partial}{\partial\theta^*}\int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \eta(\theta,\theta^*)\ d\theta = [\eta(\theta,\theta^*)]_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} + \int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \frac{\partial}{\partial\theta^*}[\eta(\theta,\theta^*)]$$
$d\theta$

$$\frac{\partial}{\partial\theta^*}\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \lambda(\theta,\theta^*)\ d\theta = [\lambda(\theta,\theta^*)]_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} + \int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \frac{\partial}{\partial\theta^*}[\lambda(\theta,\theta^*)]$$
$d\theta$

The following properties hold for $\eta(\theta,\theta^*)$ and $\lambda(\theta,\theta^*)$: (1)$\frac{\partial\eta(\theta,\theta^*)}{\partial\delta_i} < 0$, (2)$\frac{\partial\lambda(\theta,\theta^*)}{\partial\delta_i} < 0$, (3) $\frac{\partial\delta_i(\theta,\theta^*)}{\partial\theta^*} > 0$, (4) $\frac{\partial\delta_i(\theta,\theta^*)}{\partial\theta} < 0$, (5) $\left|\frac{\partial\delta_i(\theta,\theta^*)}{\partial\theta^*}\right| = \left|\frac{\partial\delta_i(\theta,\theta^*)}{\partial\theta}\right|$. By (1) and (3), $\frac{\partial\eta(\theta,\theta^*)}{\partial\theta^*} < 0$. By (1) and (4), $\frac{\partial\eta(\theta,\theta^*)}{\partial\theta} > 0$. This gives rise to the important property that: $\int_{\theta^*+\varepsilon(1-2r}^{\theta^*+\varepsilon} \frac{\partial}{\partial\theta}[\eta(\theta,\theta^*)]d\theta$,as represented by $[\eta(\theta,\theta^*)]_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon}$, exceeds $\int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \frac{\partial}{\partial\theta^*}[\eta(\theta,\theta^*)]\ d\theta$. This implies that $\frac{\partial}{\partial\theta^*}\int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \eta(\theta,\theta^*)\ d\theta > 0$.

Repeating the same exercise for $\eta(\theta,\theta^*)$, we can see that by (6) $\frac{\partial\lambda(\theta,\theta^*)}{\partial\theta^*} < 0$, (7) $\frac{\partial\lambda(\theta,\theta^*)}{\partial\theta} > 0$. Given (5),(6) and (7), it can be established that $\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \frac{\partial}{\partial\theta^*}[\lambda(\theta,\theta^*)]$ $d\theta = -\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \frac{\partial}{\partial\theta}[\lambda(\theta,\theta^*)]\ d\theta$. Thus, $\frac{\partial}{\partial\theta^*}\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \lambda(\theta,\theta^*)\ d\theta = 0$. Through the values of $\frac{\partial}{\partial\theta^*}\int_{\theta^*+\varepsilon(1-2r)}^{\theta^*+\varepsilon} \eta(\theta,\theta^*)\ d\theta$ and $\frac{\partial}{\partial\theta^*}\int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon(1-2r)} \lambda(\theta,\theta^*)$ $d\theta$, we can establish that $\frac{\partial}{\partial\theta^*}\Pi_i(s_i,\theta^*) > 0$. Thus, there exists a value of θ^* that solves the model for any $\Pi_i(s_i,\theta^*) = k$. ■

Given the existence of a value of θ^*that solves the model for any $\Pi_i(s_i,\theta^*) = k$, we can now turn to the existence of θ^* that supports the Monotone Strategy as in de&nition 2. In particular, since notice that when $s_i = \theta^*, \Pi_i(\theta^*,\theta^*) = 0$. This exactly partitions $\Pi_i(\theta^*,\theta^*)$ into two separate spaces spanning the Bankruptcy and No-Bankruptcy conditions. We summarise the arguments in the following proposition:

Proposition 18 ***(Single-crossing property of Payoff structure)*** *The assumption of monotonicity in the payoff structure of depositors and the subsequent existence of a trigger equilibrium θ^* in $\Gamma_{i,t=1}, i = \{A,B\}$, mean that the* *payoff structure of each bank satisfies the single-crossing property with the following properties: (1) For* $\theta > \theta^*$, $\Pi^i(s^i, s^*) = \int_{s^i-\varepsilon}^{s^i+\varepsilon} \pi^i(\theta^i, \delta_i(s^*, \theta^i))\ d\theta^i > 0$ *(No-Bankruptcy condition); (2) For* $\theta < \theta^*$, $\Pi^i(s^i, s^*) = \int_{s^i-\varepsilon}^{s^i+\varepsilon} \pi^i(\theta^i, \delta_i(s^*, \theta^i))$ $d\theta^i < 0$ *(Bankruptcy condition); (3)The existence of (1) and (2) means that there should be a point at which* $\Pi^i(s^i, s^*) = \int_{s^i-\varepsilon}^{s^i+\varepsilon} \pi^i(\theta^i, \delta_i(s^*, \theta^i))\ d\theta^i = 0$*. This point is at* θ^**. The existence of* θ^* *and the single-crossing property imply that* $\int_{\theta^*}^{s^i+\varepsilon} \pi^i(\theta^i, \delta_i(s^*, \theta^i))\ d\theta^i = -\int_{s^i-\varepsilon}^{\theta^*} \pi^i(\theta^i, \delta_i(s^*, \theta^i))\ \ d\theta^i$.

This leads us to another important result which we relate to parameters of the model:

Proposition 19 ***(Uniqueness of*** θ_i^* ***and no non-monotone equilibria)*** *If* θ_i^* *exists, then it is the unique Bayesian-Nash equilibrium (Monotone equilibrium) of* $\Gamma_{i,t=1}, i = \{A, B\}$*. Furthermore, there exists no non-monotone equilibria in the model.*

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We show this proof by establishing 3 lemmas first, each with sub-proofs:

Lemma 20 *For each depositor i, whenever* $\delta_i > r$, $U_i\left[\frac{\lambda + r(1-\lambda)}{\lambda + \delta_i(1-\lambda)}\right] > U_i[0]$*. Thus, if* $\delta_i(.) > \delta_i'(.)$*, then* $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i \geq \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i'[\theta_i, \theta^*])$ $d\theta_i$*. Similarly, whenever* $\delta_i < r$, $U_i\left[\frac{\left\{(1-\lambda) - \frac{\delta_i(1-\lambda)}{r}\right\} R(.)}{(1-\lambda)(1-\delta_i)}\right] > U_i[1]$*. If* $\delta_i(.) > \delta_i'(.)$*,then it implies that* $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i \leq \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i'[\theta_i, \theta^*])\ d\theta_i$

The above Lemma is simply asserting that the absence of global supermodularities in the depositors' withdrawal game. We know (from the discussion of section 2.3) that whenever $\delta_i > r$, the bankruptcy threshold is met and incentives (for any one depositor) to withdraw are decreasing proportions of the proportion of other depositors withdrawing. The No-Bankruptcy threshold (i.e when $\delta_i < r$) is one with the exact opposite effect[39].

[38] See also Goldstein and Pauzner (2002), Dasgupta(2003) and Vaugirard (2004) for other illustrations of this proof

[39] Given that $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i$ denotes the net payoff to staying for depositors and

Lemma 21 *If all depositors are following a switching / monotone strategy around* s^**, then the expected net payoff to staying, given by* $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])$ $d\theta_i$*, is continuous, monotonic and strictly increasing in* s^* *and satisfies the single-crossing property.*

Proof. See propositions 8 and 9 of the main text ■

Lemma 22 *If there exists a threshold for* s^**, then, it is unique*

Proof. We use the "Intermediate Value Theorem" to prove this result. See Appendix, section B, pp

Here, we are concerned with a particular application of the Intermediate Value Theorem to our setting. What we need to show is that since $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])$ $d\theta_i$ is continuous in θ^*, then for $\left\{\theta_i : \theta^{\inf} \equiv\right\}$ and $\{\theta_i : \theta^{\sup} \equiv\}$, where $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\sup}, \delta_i[\theta, \theta^*])$ $d\theta_i > \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\inf}, \delta_i[\theta, \theta^*])\ d\theta_i$, then there exists a particular value of θ^* such that $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta, \theta^*])\ d\theta_i \equiv k$. Clearly, $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\inf}, \delta_i[\theta, \theta^*])\ d\theta_i < k < \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\sup}, \delta_i[\theta, \theta^*])\ d\theta_i$.

From the discussion in section 2.1.2, we know that $u^G < u^B < u^G + z < u^B + z$. Let the Weak Lower Dominance Region (WLDR) constitute the lower boundary on the threshold space and the Weak Upper Dominance Region (WUDR) constitute the upper boundary on the space. Define $\alpha = \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WLDR}, \delta_i[\theta, \theta^*])$ $d\theta_i$ (where $\theta^{WLDR} \equiv \left\{\theta_i \leq u^G\right\}$) and $\beta = \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WUDR}, \delta_i[\theta, \theta^*])\ d\theta_i$ (where $\theta^{WUDR} \equiv \left\{\theta_i > u^B + z\right\}$). It is obvious that $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WUDR}, \delta_i[\theta, \theta^*])\ d\theta_i >$

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that $\pi(\theta_i, \delta_i[\theta_i, \theta^*])$ is the payoff to staying for any particular depositor, let $-\pi(\theta_i, \delta_i[\theta_i, \theta^*]) \equiv \Lambda(\theta_i, \delta_i[\theta_i, \theta^*])$ denote the payoff to withdrawing for any depositor. Thus, for the case when $\delta_i > r$, if $\delta_i\,[.] > \delta\prime_i\,[.]$, $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \Lambda(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i < \int_{s_i-\varepsilon}^{s_i+\varepsilon} \Lambda(\theta_i, \delta_i'[\theta_i, \theta^*])\ d\theta_i \Leftrightarrow \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i > \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i'[\theta_i, \theta^*])\ d\theta_i$. Similarly, whenever $\delta_i < r$, if $\delta_i\,[.] > \delta\prime_i\,[.]$, $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \Lambda(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i > \int_{s_i-\varepsilon}^{s_i+\varepsilon} \Lambda(\theta_i, \delta_i'[\theta_i, \theta^*]) \Leftrightarrow \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i[\theta_i, \theta^*])\ d\theta_i < \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta_i, \delta_i'[\theta_i, \theta^*])\ d\theta_i$.

$\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WLDR}, \delta_i[\theta,\theta^*])\, d\theta_i$. Since $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\sup}, \delta_i[\theta,\theta^*])\, d\theta_i \geq \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WUDR}, \delta_i[\theta,\theta^*])$ $d\theta_i$ and $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\inf}, \delta_i[\theta,\theta^*])\ d\theta_i \leq \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WLDR}, \delta_i[\theta,\theta^*])\ d\theta_i$, it follows that $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WLDR}, \delta_i[\theta,\theta^*])\ d\theta_i \leq k \leq \int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{WUDR}, \delta_i[\theta,\theta^*])\ d\theta_i$. Follow Morris and Shin (1998), if we relax the de&nition of $\theta^{\inf}$ and $\theta^{\sup}$ and give the following de&nition: let $\theta^{\inf}$ be the least value of idiosyncratic fundamental of bank i for which at least one depositor stays and let $\theta^{\sup}$ be the maximum value of the fundamental for which at least one depositor withdraws, then we have the following condition: $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\inf}, \delta_i[\theta,\theta^*])\ d\theta_i \geq k \geq$ $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\sup}, \delta_i[\theta,\theta^*])\ d\theta_i$. This contradicts what we just observed about the implication of $\theta^{\inf}$, $\theta^{\sup}$ and θ^* for the relationship between $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\inf}, \delta_i[\theta,\theta^*])$ $d\theta_i$, k and $\int_{s_i-\varepsilon}^{s_i+\varepsilon} \pi(\theta^{\sup}, \delta_i[\theta,\theta^*])\ d\theta_i$. Thus, the only way to reconcile this apparently contrasting result is to posit that $\theta^{\inf} = \theta^* = \theta^{\sup}$ ■

The above proposition helps in characterising the set of monotone equilibria that de&ne the Perfect Bayesian Equilibrium of the model. Without loss of generality, we can argue that there exists a threshold θ_i^* in $\Gamma_{i,t=1}, i = \{A, B\}$, such that bank i fails for $\theta_i < \theta_i^*$ and survives if $\theta_i \geq \theta_i^*$. But the above derivations did not speci&cally explicate how θ^* varies with structural changes in parameters that characterise the returns structure of the illiquid-and-risky technology. We summarise these qualitative features as follows:0

Proposition 23 ***(Features of*** θ_i^**) By Propositions (17) and (19), there exists a threshold* θ_i^* *in* $\Gamma_{i,t=1}$*,* $i = \{A, B\}$*. Given the features of each bank's illiquid-and-risky investment returns structure, the location of* θ_i^* *in the uniformly distributed* $\theta-$*space varies with* u_i^j *as follows:* $\theta_i^*(u_i^{Bad}) > \theta_i^*(u_i^G)$ *with* $u_i^{Bad} > u_i^G$*,* $i = \{A, B\}$

Proof. The analysis is for a marginal depositor in $\Gamma_{i,t=1}$,who observes $s_i = s^*$ and who believes that all other depositors in $\Gamma_{i,t=1}$, will follow a monotone strategy around s^*. For any particular realisations of the state of the common macroeconomic fundamental, u_i^j, $j = \{G, B\}, i = \{A, B\}$, there exists a critical value of θ that ensures that, from the returns technology in table 1 :

$$\theta_i^{crit} = u_i^j + z\delta_i[\theta,\theta^*],\ \text{where}\ \delta_i[\theta,\theta^*] = \left\{ \begin{array}{lr} 1 & s^* > \theta + \varepsilon \\ \frac{1}{2} + \frac{(s^*-\theta)}{2\varepsilon} & \theta - \varepsilon \leq\ s^* \leq \theta + \varepsilon \\ 0 & s^* < \theta - \varepsilon \end{array} \right\}$$

This ensures that the returns structure of bank i, $i = \{A, B\}$,takes a random realisation, $\tilde{R}(\theta_i, \theta_{-i}, u_i^j, \delta_i, \delta_{-i})$, which is dependent on realisations of u_i^j, δ, z. It can be observed that if $\theta > \theta_i^{crit}$ ($\Rightarrow \theta_i > u_i^j + z\delta_i[\theta,\theta^*]$), then the bank's project succeeds whereas if $\theta < \theta_i^{crit}(\Rightarrow \theta_i < u_i^j + z\delta_i[\theta, \theta^*])$, then the bank's project fails. Generalising the argument for any bank i, we have the following:

Using the expression for $\delta_i[\theta, \theta^*]$ and bearing in mind that, for bank i, $u \in \{u_i^G, u_i^B\}$, θ can be expressed as:

$$\theta = \left\{ \begin{array}{lr} u_i^j + z & s^* > \theta + \varepsilon \\ u_i^j + \frac{z}{2\varepsilon}\{(\theta^* - \theta) + \varepsilon\} & \theta - \varepsilon \leq s^* \leq \theta + \varepsilon \\ u_i^j & s^* < \theta - \varepsilon \end{array} \right\}$$

The above expression applies for general values of θ. For $\theta_i^{crit}(.)$, following on θ, we characterise $\theta_i^{crit}(.)$ in terms of parameters that in! uence the bank's illiquid-and-risky technology:

$$\theta_i^{crit}(u_i^j) = \left\{ \begin{array}{lr} u_i^j + z & s^* > u_i^j + z + \varepsilon \\ \frac{z(s^*+\varepsilon)+2\varepsilon u_i^j}{z+2\varepsilon} & u_i^j - \varepsilon \leq s^* \leq u_i^j + z + \varepsilon \\ u_i^j & s^* \leq u_i^j - \varepsilon \end{array} \right\}$$

With the above expression for $\theta_i^{crit}(u_i^j)$ and given that $\theta \mid s_i \sim Uniform$ $[s_i - \varepsilon, s_i + \varepsilon]$, $\varepsilon \leq s_i \leq 1 - \varepsilon$, we can express the decision of depositor i in $\Gamma_{i,t=1}$ to stay or withdraw in *probabilistic terms*. Let $P(s^*, \delta_i, u_i^j) = Prob(\theta > \theta_i^{crit}(u_i^j) \mid s_i = s^*)$ denote the probability for any depositor i in $\Gamma_{i,t=1}$ staying as opposed to withdrawing. $P(s^*, \delta_i, u_i^j) = Prob(\theta > \theta_i^{crit}(u_i^j) \mid s_i = s^*)$ can be expressed as thus:

$$P(s^*, \delta_i, u_i^j) = \left\{ \begin{array}{lr} 1 & s^* > u_i^j + z + \varepsilon \\ \frac{(s^* - u_i^j + \varepsilon)}{2\varepsilon + z} & u_i^j - \varepsilon < s^* \leq u_i^j + z + \varepsilon \\ 0 & s^* \leq u_i^j - \varepsilon \end{array} \right\}$$

with $P(s^*, \delta_i, u_i^j)$ being strictly *monotonically increasing* in s^* in $[u_i^j - \varepsilon, u_i^j + z + \varepsilon]$ and having the property that, as $s^* \longrightarrow u_i^j + z + \varepsilon$, $P(s^*, \delta_i, u_i^j) \longrightarrow +1$ and as $s^* \longrightarrow u_i^j - \varepsilon$, $P(s^*, \delta_i, u_i^j) \longrightarrow 0$. Notice that, with parameters s^*, ε and z, $P(s^*, \delta_i, u_i^B) < P(\theta^*, \delta_i, u_i^G)$, $u_i^B > u_i^G$. This is simply saying that, with a bad realisation of the common macroeconomic fundamental, the probability that depositor i in $\Gamma_{i,t=1}$ will choose to stay as opposed to withdrawing decreases. But the exact probability value from the probability function $P(s^*, \delta_i, u_i^j)$ characterising this decision cannot be known ex-ante because it depends on variable u_i^j, the actual value of which is not common knowledge. Since we are interested in the $\{u_i^j - \varepsilon < s^* \leq u_i^j + z + \varepsilon\}$ space with probability function being $\frac{(s^* - u_i^j + \varepsilon)}{2\varepsilon + z}$, it can be seen that, *for any given realisation of* δ_i and of u_i^j *which gives rise to probability* $P^* \subset P(\theta^*, \delta_i, u_i^j) = \frac{(\theta^* - u_i^j + \varepsilon)}{2\varepsilon + z}$, θ^* is higher with u_i^{Bad} rather than with u_i^G.

In particular, for the good and bad states, we have the following:

$$P(s^*, \delta_i, u_i^{Bad}) = \left\{ \begin{array}{lr} 1 & s^* > u_i^{Bad} + z + \varepsilon \\ \frac{(s^* - u_i^{Bad} + \varepsilon)}{2\varepsilon + z} & u_i^{Bad} - \varepsilon \leq s^* \leq u_i^{Bad} + z + \varepsilon \\ 0 & s^* > u_i^{Bad} - \varepsilon \end{array} \right\}$$

for bad state

and

$$P(s^*,\delta_i,u_i^G)=\left\{\begin{array}{ll} 1 & s^*>\ u_i^G+z+\varepsilon \\ \frac{(s^*-\ u_i^G\ +\ \varepsilon)}{2\varepsilon+z} & u_i^G\text{ - }\varepsilon\ \leq s^*\leq\ u_i^G+z+\varepsilon \\ 0 & s^*>\ u_i^G\text{- }\varepsilon \end{array}\right\}$$

for the Good state

Let $P(s^*,\delta_i)=kP(s^*,\delta_i,u_i^{Bad})+(1-k)\,P(s^*,\delta_i,u_i^G)$. Then, for particular values of s^*,we have the following corresponding values for $P(s^*,\delta_i)$:

u_i^G- $\varepsilon\leq s^*<u_i^{Bad}$- ε , $\quad P(s^*,\delta_i)\ =(1-k)\left[\frac{(s^*-\ u_i^G\ +\ \varepsilon)}{2\varepsilon+z}\right]+k\,[0]$ $=(1-k)\left[\frac{(s^*-\ u_i^G\ +\ \varepsilon)}{2\varepsilon+z}\right]$

u_i^{Bad}- $\varepsilon\leq s^*<u_i^G+z+\varepsilon$, $\quad P(s^*,\delta_i)\ =(1-k)\left[\frac{(s^*-\ u_i^G\ +\ \varepsilon)}{2\varepsilon+z}\right]+$ $k\left[\frac{(s^*-\ u_i^{Bad}\ +\ \varepsilon)}{2\varepsilon+z}\right]\ =\frac{s^*+e+\left(k\left(u^G-u^{Bad}\right)-u^G\right)}{2\varepsilon+z}$

$u_i^G+z+\varepsilon\leq s^*<u_i^{Bad}+z+\varepsilon,\quad P(s^*,\delta_i)\ =(1-k)\,[1]+k\left[\frac{(s^*-\ u_i^{Bad}\ +\ \varepsilon)}{2\varepsilon+z}\right]$ $=k\left[\frac{(s^*-\ u_i^{Bad}\ +\ \varepsilon)}{2\varepsilon+z}\right]+(1-k)$

It is easy to verify that, for any particular realisation of s^* in interval $\left[u_i^G-\varepsilon,\quad u_i^{Bad}+z+\varepsilon\right]$, $\frac{\partial P(s^*,\delta_i)}{\partial s^*}>0$. In particular,

u_i^G- $\varepsilon\leq s^*<u_i^{Bad}$- ε , $\quad \frac{\partial P(s^*,\delta_i)}{\partial s^*}=\frac{1-k}{2\varepsilon+z}>0$

u_i^{Bad}- $\varepsilon\leq s^*<u_i^G+z+\varepsilon$, $\quad \frac{\partial P(s^*,\delta_i)}{\partial s^*}=\frac{1}{2\varepsilon+z}>0$

$u_i^G+z+\varepsilon\leq s^*<u_i^{Bad}+z+\varepsilon,\quad \frac{\partial P(s^*,\delta_i)}{\partial s^*}=\frac{k}{2\varepsilon+z}>0$

Thus, the location of s_i^*, in the uniformly-distributed $\theta-space$, is in! uenced the the state of the common macroeconomic fundamental. For a bad realisation of the common fundamental, s_i^* moves closer to 1 in the $[0,1]$ space - meaning that a greater section of the tail of s_i^* in $[0,1]$ space is exposed - bank i has a greater probability of failing, for a given realisation of its idiosyncratic fundamental, θ_i . This completes the proof. ■

Theorem 24 ***(Unique characterisation of*** $\{x_A^*(.)\,,\theta_A^*(.)\}$ *in* $\Gamma_{A,t=1}$ *and of* $\{x_B^*(.)\,,\theta_B^*(.)\}$*in* $\Gamma_{B,t=1}$)

Given $\sigma_i(\Theta_{i,t=1}^A)\rightarrow a_i\ \epsilon\ A_i=\{W,S\}$ *and* $\sigma_i(\Theta_{i,t=1}^B)\rightarrow a_i\ \epsilon\ A_i=\{W,S\}$ *for depositors in* $\Gamma_{A,t=1}$*and in* $\Gamma_{B,t=1}$ *respectively, and, given the de©nition of perfect bayesian equilibrium we adopt as above, we can summarise the algorithm that traces the equilibrium values of* $\{x_A^*(.)\,,\theta_A^*(.)\}$ *and of* $\{x_B^*(.)\,,\theta_B^*(.)\}$ *as follows:*

Algorithm tracing equilibrium values of $x_A^*(.)\,,\theta_A^*(.)\,,x_B^*(.)\,,\theta_B^*(.)$:

For depositors in $\Gamma_{A,t=1}$, $\sigma_i(\Theta_{i,t=1}^A)=\left\{\begin{array}{ll} W & if\ \ s_i<s^* \\ S & if\ \ s_i\geq s^* \end{array}\right\}$

$$\textit{and}\ \theta_A^*(.)\ \textit{solves}\left\{\begin{array}{l} \Pi_A(\theta_A^*(.)\,,\theta_B^*(.)\,,...)=0 \\ \textit{and}\ \delta_i[\theta,\theta^*]=\left\{\begin{array}{ll} 1 & \theta<\theta^*-\varepsilon \\ \frac{1}{2}+\frac{(\theta^*-\theta)}{2\varepsilon} & \theta^*-\varepsilon\leq\ \theta<\theta^*+\varepsilon \\ 0 & \theta\geq\theta^*+\varepsilon \end{array}\right\} \end{array}\right\}$$

where $\Pi_A(\theta_A^*(.)\,,\theta_B^*(.)\,,...)=0$ *is the expected utility for staying and is given by:* $\Pi_A(\theta_A^*(.)\,,\theta_B^*(.)\,,...)$ =

$$For\ depositors\ in\ \Gamma_{B,t=1},\ \sigma_i(\Theta^B_{i,t=1}) = \left\{ \begin{array}{lll} W & if & (\ \Omega^A = \{F^A\}) \cap (\ s_i \leq s^*) \\ S & if & (\ \Omega^A = \{S^A\}) \cap (\ s_i \geq s^*) \\ S\ or\ W & if & \left\{ \begin{array}{c} either\ \left((\ \Omega^A = \{S^A\}) \cap (\ s_i < s^*)\right) \\ or\ \left((\ \Omega^A = \{F^A\}) \cap (\ s_i > s^*)\right)\right) \end{array} \right\} \end{array} \right\}$$

$$and\ \theta^*_B\,(.)\ solves \left\{ \begin{array}{c} \Pi_B(\theta^*_A\,(.)\,, \theta^*_B\,(.)\,, ...) = 0 \\ and\ \delta_B[\theta, \theta^*] = \left\{ \begin{array}{ll} 1 & \theta < \theta^* - \varepsilon \\ \frac{1}{2} + \frac{(\theta^* - \theta)}{2\varepsilon} & \theta^* - \varepsilon \leq\ \theta < \theta^* + \varepsilon \\ 0 & \theta \geq \theta^* + \varepsilon \end{array} \right\} \end{array} \right\}$$

$$where\ \Pi_B(\theta^*_A\,(.)\,, \theta^*_B\,(.)\,, ...) = 0\ is\ given\ by$$

The derivation of the unique threshold for each bank can also be found in other models in the literature. Dasgupta(2003) obtains similar results, albeit with a more complex payoff structure. The existence of the overlapping networks structure of &nancial contracts that tie the banks together (through the interbank market in deposits) can explain contagion as a unique phenomenon. The failure of bank A means that depositors in $\Gamma_{B,t=1}$,suffer a loss of claims due to them. As a result, their behaviour changes. Other papers in the literature do get the uniqueness result: Goldstein and Pauzner (2003) endogenise the probability of bank runs and relate that probability to the features of the demand-deposit contract. In their paper, as second-best solution, the optimal contract is featured by a trade-off between risk-sharing (efficiency) and the (endogeneous) probability of bank runs (instability).

The novelty of these papers is that they rationalise the case for unique equilibrium in the coordination game facing depositors, even in the absence of global strategic complementarities. The uniqueness result of Carlsson and Van-Damme(1993) and Morris and Shin (1998),(1999), necessarily rely on the existence of (global) strategic complementarities/supermodularities in coordination games. As we have seen in section 2.3, assumption (A.5), banking models are not featured by supermodularities in the payoff structure - above some threshold, decisions become strategic substitutes. Nonetheless, the innovative approach of Dasgupta(2003) and of Goldstein-Pauzner(2003) models is that they show that through the existence of (1) single-crossing property in the payoff structure and (2) an error technology that satis&es the Monotone-Likelihood Ratio Property(MLRP), a unque result can exist even in the absence of strategic complementarities. We have formalised their approach and adopted it within the speci&c con&nes of our model. Others...

5.1 Mechanics of Beliefs updating

The re-assessment of the beliefs system from the prior distribution $\zeta = \left\{i = \{A, B\}\,; P(u_i^{Bad}), P(u_i^G)\right\}$ to the posterior distribution $\zeta\prime = \left\{i = \{A, B\}\,; P(u_i^{Bad} \mid \Omega_A), P(u_i^G \mid \Omega_A)\right\}$ takes place along the equilibrium pathway depicted by the Perfect Bayesian Equilibrium that we formally explicated in section 4.3. The analysis was however constrained to some general form of Bayesian updating process, without explicit reference to the intrinsic stochastic properties of the updating process. In this section, we will add statistical structure to the updating process, elaborate on the stochastic properties of the resulting informational generating process. We focus on the Bayesian Learning process between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$ as the ! ow of *public information* that leads to a revised assessment of the state of the common macroeconomic fundamental. The learning process does not focus on depositors private signals because each depositor i in $\Gamma_{i,t=1}$ receives his private signal only once in $\Gamma_{i,t=1}$and there is no evolution of private signals over time[40]. Furthermore, by the assumption that $2\varepsilon \leq \min\ [u^G, 1-u^G-z]$, each depositor i has a private signal which is of minimal precision.

The updating mechanism thus concerns only parameter u_i^j , $i = \{A, B\}$, $j = \{G, B\}$. The actual realisation of u_i^j is not *apriori* known to depositors in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$. But upon observing $\Omega_A = \{S_A, F_A\}$, depositors in $\Gamma_{B,t=1}$ have an extra information on u_i^j. Since they do not observe what is triggering the event in the space Ω_A, they face a *statistical inference* problem. Any revised version of the state u_i^j, conditional upon observing the event in the space Ω_A, constitutes the learning process in our set-up.

To keep the model analytically tractable[41], we shall place a few restrictions on the *apriori* distribution, $\zeta = \left\{i = \{A, B\}\,; P(u_i^B), P(u_i^G)\right\}$: Let $P(u_i^{Bad}) = k$ and $P(u_i^G) = 1-k$. The other properties of $\zeta = \left\{i = \{A, B\}\,; P(u_i^{Bad}), P(u_i^G)\right\}$ still hold, as elaborated in section 3.2. De&ne the partitioned space events, S_A and F_A as follows: $S_A : \{(\theta_A, u) : \theta_A \geq \theta_A^*(u)\}$ and $F_A : \{(\theta_A, u) : \theta_A < \theta_A^*(u)\}$ with the qualitative features of $\{S_A, F_A\}$ being as in section 3.2. Since θ_A is uniformly distributed on $[0, 1]$, it follows that $Prob(\theta_A > \theta_A^*(u)) = 1 - \theta_A^*(u)$ and that $Prob(\theta_A \leq \theta_A^*(u)) = \theta_A^*(u)$. With the property that, if $u_i^{Bad} > u_i^G$, then $\theta_i^*(u_i^{Bad}) > \theta_i^*(u_i^G)$, $i = \{A, B\}$, we can argue that $Prob(\theta_A \leq \theta_A^*(u^B)) = \theta_A^*(u^{Bad}) > Prob(\theta_A \leq \theta_A^*(u^G)) = \theta_A^*(u^G)$. The following probability assessments subsequently hold:

$$
\begin{aligned}
\Pr ob(F_A \quad &| \quad u = u^B) = \theta_A^*(u^{Bad}) \\
\Pr ob(F_A \quad &| \quad u = u^G) = \theta_A^*(u^G) \\
\Pr ob(S_A \quad &| \quad u = u^B) = 1 - \theta_A^*(u^{Bad}) \\
\Pr ob(S_A \quad &| \quad u = u^G) = 1 - \theta_A^*(u^G)
\end{aligned}
$$

[40] In this sense, the learning process embodied in our model differs from that of Angeletos, Hellwig and Pavan (2002) in that they focus on how the evolution of both, the private and the public signal, will affect the relative precisions of the signals.

[41] Note that, throughout the analysis, we will be focusing on the equilibrium path traced out by the Perfect Bayesian Equilibrium concept.

with $\theta_A^*(u^{Bad}) > \theta_A^*(u^G)$ and $1 - \theta_A^*(u^{Bad}) < 1 - \theta_A^*(u^G)$. What is the updating process for each depositor i in $\Gamma_{B,t=1}$? Using Bayes rule, we have the following revision estimates conditional upon observing an event in Ω_A :

$$\Pr ob(u = u^{Bad} \mid F_A) = \frac{P(F_A \mid u = u^{Bad})P(u = u^{Bad})}{P(F_A \mid u = u^{Bad})P(u = u^{Bad}) + P(F_A \mid u = u^G)P(u = u^G)} \tag{7}$$

$$= \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad}) + (1-k)\theta_A^*(u^G)}$$

Similarly,

$$\Pr ob(u = u^{Bad} \mid S_A) = \frac{P(S_A \mid u = u^{Bad})P(u = u^{Bad})}{P(S_A \mid u = u^{Bad})P(u = u^{Bad}) + P(S_A \mid u = u^G)P(u = u^G)} \tag{8}$$

$$= \frac{k.(1 - \theta_A^*(u^{Bad}))}{k.(1 - \theta_A^*(u^{Bad})) + (1-k)(1 - \theta_A^*(u^G))}$$

Analoguously, $Prob(u = u^G \mid S_A) = 1 - \Pr ob(u = u^{Bad} \mid S_A) = \frac{(1-k)(1-\theta_A^*(u^{Bad}))}{(1-k)(1-\theta_A^*(u^{Bad})) + k(1-\theta_A^*(u^G))}$ and $Prob(u = u^G \mid F_A) = 1 - \Pr ob(u = u^{Bad} \mid F_A) = \frac{(1-k)\theta_A^*(u^G)}{(1-k)\theta_A^*(u^G) + k\theta_A^*(u^{Bad})}$. The unconditional probability of bad state is given as: $\Pr ob(u = u^{Bad}) = k = 1 - \Pr ob(u = u^G)$. This yields a proposition:

Proposition 25 *Upon observing the failure of bank A, the probability that the common macroeconomic fundamental was in its bad state is more likely than unconditionally. Thus, (1)* $\Pr ob(u = u^{Bad} \mid F_A) > \Pr ob(u = u^{Bad}) > \Pr ob(u = u^{Bad} \mid S_A)$ *Similarly, conditional on observing the success of bank A, the probability that the common macroeconomic fundamental was in its good state is more likely than unconditionally. Thus, (2)* $\Pr ob(u = u^G \mid S_A) > \Pr ob(u = u^G) > \Pr ob(u = u^G \mid F_A)$

Proof. (1)With the all-important property that, if $u^{Bad} > u^G$, then $\theta_A^*(u^{Bad}) >$

$\theta_A^*(u^G)$, it can be inferred that $\theta_A^*(u^{Bad}) > k.\theta_A^*(u^{Bad}) + (1-k).\theta_A^*(u^G)$, $0 \leq k < 1$. Thus, $\frac{\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)} > 1 \Rightarrow \frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)} > k.$

This implies that: $\Pr ob(u=u^{Bad}\mid F_A)=\frac{k.\theta_A^*(u^{Bad})}{k.\theta_A^*(u^{Bad})+(1-k).\theta_A^*(u^G)}>k$. Subsequently, $\Pr ob(u=u^{Bad}\mid F_A)>\Pr ob(u=u^{Bad})$ where $\Pr ob(u=u^{Bad})=k$. Similarly, if $u^{Bad}>u^G$, then $1-\theta_A^*(u^{Bad})<1-\theta_A^*(u^G)$. Thus, it must be the case that $1-\theta_A^*(u^{Bad})<k.[1-\theta_A^*(u^{Bad})]+(1-k).[1-\theta_A^*(u^G)]$. Thus, $\frac{1-\theta_A^*(u^{Bad})}{k.[1-\theta_A^*(u^{Bad})]+(1-k).[1-\theta_A^*(u^G)]}<1\Longrightarrow\frac{k.[1-\theta_A^*(u^{Bad})]}{k.[1-\theta_A^*(u^{Bad})]+(1-k).[1-\theta_A^*(u^G)]}<k$. Subsequently, $\Pr ob(u=u^{Bad}\mid S_A)<\Pr ob(u=u^{Bad})$ where $\Pr ob(u=u^{Bad})=k$. This establishes the general result that: $\Pr ob(u=u^{Bad}\mid F_A)>\Pr ob(u=u^{Bad})>\Pr ob(u=u^{Bad}\mid S_A)$

(2) can be proved in a similar way. With $\theta_A^*(u^G)<\theta_A^*(u^{Bad})\Rightarrow(1-k)\theta_A^*(u^G)<(1-k)\theta_A^*(u^{Bad})$. We can express $\theta_A^*(u^G)$ as a linear function: $\theta_A^*(u^G)<k\theta_A^*(u^G)+(1-k)\theta_A^*(u^{Bad})$. This implies that $\frac{\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^B)}$ <1. Multiply both sides by $(1-k)$ yields: $\frac{(1-k)\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^{Bad})}<(1-k)$. But $\frac{(1-k)\theta_A^*(u^G)}{k\theta_A^*(u^G)+(1-k)\theta_A^*(u^{Bad})}=\Pr ob(u=u^G\mid F_A)$ and $(1-k)=\Pr ob(u=u^G)$. This therefore suggests that $\Pr ob(u=u^G\mid F_A)<\Pr ob(u=u^G)$. With $\theta_A^*(u^G)<\theta_A^*(u^{Bad})\Rightarrow 1-\theta_A^*(u^G)>1-\theta_A^*(u^{Bad})\Rightarrow(1-k)[1-\theta_A^*(u^G)]>(1-k)[1-\theta_A^*(u^{Bad})]$. In turn, $[1-\theta_A^*(u^G)]>k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]$, which implies that $\frac{[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]}>1$. Multiplying both sides by $(1-k)$ yields $\frac{(1-k)[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]}>(1-k)$. As derived above, $\frac{(1-k)[1-\theta_A^*(u^G)]}{k\left[1-\theta_A^*(u^G)\right]+(1-k)[1-\theta_A^*(u^{Bad})]}=\Pr ob(u=u^G\mid S_A)$ and $(1-k)=\Pr ob(u=u^G)$. This suggests that $\Pr ob(u=u^G\mid S_A)>\Pr ob(u=u^G)$. We have therefore proved the general result for (2), that, $\Pr ob(u=u^G\mid S_A)>\Pr ob(u=u^G)>\Pr ob(u=u^G\mid F_A)$ ■

This means that, conditional on observing the success of bank A, rational depositors in $\Gamma_{B,t=1}$ infer that the state of the common fundamental is more likely to be in its good state than unconditionally. Thus, upon observing the performance of bank A, depositors in $\Gamma_{B,t=1}$ adjust their expectations of the likelihood of the state of the common macroeconomic fundamental, such that the good state is more likely to be associated with the good event. The above proposition gives way to a yet more important proposition - one that contains the stochastic attributes of the informational system and that will be used throughout the rest of this paper for analysing the different forms in which dynamic equilibria may manifest themselves into:

Corollary 26 ***(Stochastic Properties of Beliefs Updating Mechanism)*** *Since* $\Pr ob(u=u^{Bad}\mid F_A)>\Pr ob(u=u^{Bad})>\Pr ob(u=u^{Bad}\mid S_A)$, *it can be seen that* $\frac{\Pr ob(u=u^{Bad}|S_A)}{\Pr ob(u=u^{Bad}|F_A)}<1$. *Let* $L^B(u^{Bad},\Omega_A)=\frac{\Pr ob(u=u^{Bad}|S_A)}{\Pr ob(u=u^{Bad}|F_A)}$. *It follows that* $L^B(u^{Bad},\Omega_A)=\frac{\Pr ob(u=u^{Bad}|S_A)}{\Pr ob(u=u^{Bad}|F_A)}$ *follows the Monotone Likelihood Ratio*

Property (MLRP), since it is decreasing in u. This implies that an event in space Ω_A has some important stochastic informational content - the event is partially informative of the state of the common macroeconomic fundamental. Depositors in $\Gamma_{B,t=1}$ can learn on the state of the common macroeconomic fundamental, upon observing an event in space Ω_A. A similar property can be derived for the case in which $u = u^G$. Here, $L^G(u^G, \Omega_A) = \frac{\Pr ob(u=u^G|F_A)}{\Pr ob(u=u^G|S_A)}$,would be strictly decreasing in u. The MLRP is also satisfied with $u = u^G$.

5.2 An analytically-solvable version of the model

The findings of the previous sections can describe the Perfect Bayesian Equilibrium of the game between $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$as a set of monotone equilibria

such that (1) Each depositor i finds it optimal to withdraw in $\Gamma_{i,t=1}, i = \{A, B\}$ if $s_i < s^*$ and stays if $s_i \geq s^*$, (2) bank i fails in $\Gamma_{i,t=1}, i = \{A, B\}$ if $\theta < \theta^*(u)$ and does not fail if $\theta \geq \theta^*(u)$.

We can easily see, from the above, that the different possibilities of an event in bank A being associated with an event in bank B can be represented by a set of set of equations that characterise the probability of the events taking place . If we represent $\{F_A, F_B, S_A, S_B, \Theta^B_{i,t=1}\}$ analoguously to what we have done before in the previous section, then we may represent the probability of a failure in bank A being associated with a failure in bank B as follows: $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,}) = \Pr(\theta_B < \theta^*_B(u) \mid \Theta^B_{i,t=1}, \theta_A \geq \theta^*_A(u))$, where $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,})$ denotes the probability of bank B failing, given the informational endowment of depositors in $\Gamma_{B,t=1}$ and given the observed public event in bank A. This can be represented as follows: $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,}) = \Pr(F_B \mid \{u = u^{Bad}\} \cap F_A) \Pr(\{u = u^{Bad}\} \mid F_A) + \Pr(F_B \mid \{u = u^G\} \cap F_A) \Pr(\{u = u^G\} \mid F_A)$. Since we know the values of $\Pr(\{u = u^{Bad}\} \mid F_A)$ and $\Pr(\{u = u^G\} \mid F_A)$, we can replace these values in the above expression and get a much simplified version of $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,})$: $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,}) = \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) + (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) + (1-k)\theta^*_A(u^G)} \right\}$. Similarly, $Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,}) = \Pr(\theta_B < \theta^*_B(u) \mid \Theta^B_{i,t=1}, \theta_A < \theta^*_A(u))$, where $Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,})$ denotes the probability that bank B fails, given that it is observed that bank A has survived an attack before. Similarly, $Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,}) = \Pr(F_B \mid \{u = u^{Bad}\} \cap S_A) \Pr(\{u = u^{Bad}\} \mid S_A) + \Pr(F_B \mid \{u = u^G\} \cap S_A) \Pr(\{u = u^G\} \mid S_A)$. We can thus represent it as follows: $Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,}) = \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad}) + (1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1-k\theta^*_A(u^{Bad}) - (1-k)\theta^*_A(u^G)} \right\}$. Events $Pr(S_B \mid \Theta^B_{i,t=1}, F_{A,})$ and $Pr(S_B \mid \Theta^B_{i,t=1}, S_{A,})$ can be derived analoguously in terms of parameters of our model. For simplicity, $Pr(S_B \mid \Theta^B_{i,t=1}, F_{A,}) = 1 - Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,}) = 1 - \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) + (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) + (1-k)\theta^*_A(u^G)} \right\}$. Likewise, $Pr(S_B \mid$

$\Theta^B_{i,t=1}, S_{A,}) = 1 - Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,}) = 1 - \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad}) + (1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1 - k\theta^*_A(u^{Bad}) - (1-k)\theta^*_A(u^G)} \right\}$.
To arrive at equilibrium values of s* and θ^*, we need to relate the probability of events (as described above) with the critical mass of depositors needed to trigger an attack in $\Gamma_{i,t=1}, i = \{A, B\}$, where the critical mass was previously de&ned as the point in the idiosyncratic fundamental space where $\theta^*_i = \delta(\theta^*_i)$. We summarise the set of equilibrium conditions and relate them to the events happening in banks A and B as follows:

Claim 27 [42] *For events taking place in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$, the respective sets of equilibrium for $\{s^*_A, s^*_B\}$ and $\{\theta^*_A, \theta^*_B\}$ can be traced out as a function of the probability of these events taking place. In each case, the equilibrium is found by solving either of the following events:*

(1) $Pr(F_B \mid \Theta^B_{i,t=1}, F_{A,}) = \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) + (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) + (1-k)\theta^*_A(u^G)} \right\}$

(2) $Pr(F_B \mid \Theta^B_{i,t=1}, S_{A,}) = \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad}) + (1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1 - k\theta^*_A(u^{Bad}) - (1-k)\theta^*_A(u^G)} \right\}$

(3) $Pr(S_B \mid \Theta^B_{i,t=1}, F_{A,}) = 1 - \left\{ \frac{k\theta^*_A(u^{Bad})\theta^*_B(u^{Bad}) + (1-k)\theta^*_A(u^G)\theta^*_B(u^G)}{k\theta^*_A(u^{Bad}) + (1-k)\theta^*_A(u^G)} \right\}$

(4) $Pr(S_B \mid \Theta^B_{i,t=1}, S_{A,}) = 1 - \left\{ \frac{k(1-\theta^*_A(u^{Bad}))\theta^*_B(u^{Bad}) + (1-k)(1-\theta^*_A(u^G))\theta^*_B(u^G)}{1 - k\theta^*_A(u^{Bad}) - (1-k)\theta^*_A(u^G)} \right\}$

with the critical mass of depositors, $\theta^*_i = \delta(\theta^*_i)$, $i = \{A, B\}$, in $\Gamma_{A,t=1}$ and $\Gamma_{B,t=1}$

6 Financial Contagion

There is no □one-size-&ts-all□ de&nition of &nancial contagion given by the literature. The existence of a common macroeconomic fundamental in our model, nonetheless, complicates matters. There may be multiple bank failures due to adverse macroeconomic fundamental, to which both banks are commonly exposed to. But that does not necessarily mean that one bank failure is actually causing the other. For instance, if the two banks have assets denominated in one currency and liabilities denominated in another currency, a currency change will affect both banks together in a similar way. This common failure is merely due to common exposure to the exchange rate, and is not what we are primarily concerned here.

To be able to de&ne &nancial contagion appropriately within the setup we have adopted, it is important to stress on the cause-effect relationship that

[42] The unconditional probabilities are as follows: $P(F_A) = P(F_A \mid u = u_{Bad})P(u = u_{Bad}) + P(F_A \mid u = u_G)P(u = u_G)$

$P(F_B) = P(F_B \mid F_A)P(F_A) + P(F_B \mid S_A)P(S_A)$

underpins the concept. Heuristically, we could view financial contagion as "*an event that occurs when the failure of bank A causes bank B to fail, when bank B would not have failed otherwise*". Note the importance of the second part of the statement "*...when bank B would not have failed otherwise......*" This implies that, in our definition, without bank A, bank B could fail for other reasons (e.g its idiosyncratic fundamental is too low) or it could possibly not fail at all. What the statement is really saying, is that, the performance of bank A, by itself, will increase the likelihood of failure of bank B over what could possibly have happened, without its presence. Before moving on further, we must first elicit the conditions under which this would hold true. Then we shall formalise the concept of contagion, through appropriate use of diagrams. Consider Figure 4:

Figure 4 - Idiosyncratic thresholds of Banks A and B

(*Insert Figure 4 here from Appendix*)

Figure 4 highlights the unique threshold in each bank. For the moment, let us forget about the dynamics that would cause θ_A^* and $\theta_B^*(\ )$to vary and, attempt to situate what we have learned in the previous topic, in the above diagram. Thus, initially, we set $\theta_A^* = \theta_B^*(\ )$ and, with slight abuse of the language, shall refer to this as the *autarky situation*[43].

Quadrants 3 and 2 show similar results in both banks. Quadrant 3 depicts the phenomenon of both banks failing (i.e $\theta_A < \theta_A^*$, $\theta_B < \theta_B^*\ (\ )$) while quadrant 2 shows both banks succeeding or "not failing" (i.e $\theta_A \geq \theta_A^*$, $\theta_B \geq \theta_B^*\ (\ )$). Quadrants 1 and 4 show mixed result. The former depicts the success of bank B but failure of bank A (i.e $\theta_A < \theta_A^*$, $\theta_B \geq \theta_B^*\ (\ )$) while the latter shows the reverse effects (i.e $\theta_A \geq \theta_A^*$, $\theta_B < \theta_B^*\ (\ )$).

How would our concept of financial contagion fit into the diagram? Could we possibly argue that contagion is an event that occurs in quadrant 3? Doing so would merely show the joint occurrence of failures of bank A and B, but there is nothing to tell us about the causation of the crises. Any permutation would be possible in that quadrant. Bank B could fail for reasons other than failure of bank A and vice versa. To get a proper representation of financial contagion, we abstract from what may commonly be driving the performance of both banks. This is done by controlling for the level of the common macroeconomic

[43] Autarky typically refers to absence of trade but here, it means that there is no interraction among the banks. Depositors of each bank behave as if the other bank did not exist. Due to identical endowments and similar returns structure, it is obvious that $\theta_A^* = \theta_B^*(\ )$.

fundamental. The aim is to assess, mathematically, how the failure of bank A, by itself, can cause the failure of bank B, after controlling for the common fundamental. Thus, we must show that, whenever bank A fails (i.e $\theta_A < \theta_A^*$), the probability of bank B failing, for a *given* level of macroeconomic fundamental, would be higher than θ_B^* ( ). For each of the possible two realisations of the common macroeconomic fundamental, this probability can be assessed. What extra feature does the failure of bank A has on bank B's threshold ? It

was previously shown that, upon failure of bank A, the trigger of bank B is adjusted in such a way that depositors in bank B are most likely to share a similar fate to those of bank A, with the position of the threshold being dependent on the realisation of the common macroeconomic fundamental. Taking the level of the common macroeconomic fundamental as given, the cause-effect relationship between failure of bank A and failure of bank B can be represented as events Pr $(\theta_B \leq \theta_B^*\ (.) \mid \theta_A \leq \theta_A^* \cap u = u_{Bad})$ and Pr $(\theta_B \leq \theta_B^*\ (\ ) \mid \theta_A \leq \theta_A^* \cap$

$u = u_G)$ for the bad state and the good state of the common macroeconomic fundamental respectively. More specifically, recall that $\Pr(\theta_B \leq \theta_B^*(\ ) \mid \theta_A \leq \theta_A^* \cap u = u_{Bad}) \equiv \theta_{B,u_{Bad}}^{F_A}$. We referred to this as the threshold for bank B but computed with conditional probability, $\Pr(u = u_{Bad} \mid F_A)$, which we gave earlier as $\frac{k\ \theta_A^*(u_{Bad})}{k\ \theta_A^*(u_{Bad})\ +\ (1-k)\theta_A^*(u_{Bad})}$. Clearly, $\theta_{B,u_{Bad}}^{F_A} > \theta_B^*$ ( ), where θ_B^* ( ) is computed as in the autarky case. Similarly, we computed the event that bank B fails, conditional on success of bank A and the state of the common fundamental being bad as $\Pr(\theta_B \leq \theta_B^*(\ ) \mid \theta_A > \theta_A^* \cap u = u_{Bad}) \equiv \theta_{B,u_{Bad}}^{S_A}$. This refers to the threshold

of bank B, computed with conditional probability $\Pr(u = u_{Bad} \mid S_A)$,which we gave earlier as $\frac{k\ (1-\theta_A^*(u_{Bad}))}{k\ (1-\theta_A^*(u_{Bad}))\ +\ (1-k)(1-\theta_A^*(u_G))}$. Clearly, $\theta_{B,u_{Bad}}^{S_A} \leq \theta_B^*$ ( ), where θ_B^* ( ) is computed as in the autarky case. We present the autarky thresholds θ_A^*, θ_B^* ( ), $\theta_{B,u_{Bad}}^{S_A}$ and $\theta_{B,u_{Bad}}^{F_A}$ in the following diagram:

Figure 5 - thresholds θ_A^*, θ_B^* ( ), $\theta_{B,u_{Bad}}^{S_A}$, $\theta_{B,u_{Bad}}^{F_A}$ and financial contagion

(*Insert Figure 5 here from Appendix*)

Figure 6 in the Appendix gives us the analoguous representation for financial contagion in case the state of the common macroeconomic fundamental is good.The representation in the diagram enables us formalise the definition of financial contagion as follows:

De&nition 28 ***(Financial Contagion)*** *For the part of the game between* $\Gamma_{A,t=1}$ *and* $\Gamma_{B,t=1}$ *characterised by Strict Private Informational Dominance (SPID) and the existence of a unique threshold in the depositors□game, &nancial contagion is said to occur when:*

Either (1) event $\left\{ \theta_A \in \left[\underline{\theta} \ , \ \overline{\theta} \right] \ , \ \theta_B \in \left[\underline{\theta} \ , \ \overline{\theta} \right] : \{ \ \theta_A \leq \theta_A^* \} \ \cap \ \{ \ \theta_B^*(.) \leq \ \theta_B \leq \theta_{B,u^j}^{F_A} \} \right\}$ *for a given macroeconomic state,* u^j

The probability of contagion is a weighted average of the above event, with each weight corresponding to a particular state of the macroeconomic fundamental and is given as:

$$P(Contagion) = k \left(\theta_{B,u_{Bad}}^{F_A} - \theta_B^*(u^{Bad}) \right) \left(\theta_A^* - \underline{\theta} \right) + (1-k) \left(\theta_{B,u_{Bad}}^{F_A} - \theta_B^*(u^G) \right) \left(\theta_A^* - \underline{\theta} \right)$$

Or (2) event $\left\{ \theta_A \in \left[\underline{\theta} \ , \ \overline{\theta} \right], \ \theta_B \in \left[\underline{\theta} \ , \ \overline{\theta} \right] : \{ \ \theta_A > \theta_A^* \ \} \cap \ \{ \theta_{B,u^j}^{S_A} \leq \theta_B \leq \theta_B^*(.) \} \right\}$ *for a given macroeconomic state,* u^j

The probability of contagion is a weighted average of the above event, with each weight corresponding to a particular state of the macroeconomic fundamental and is given as:

$$P(Contagion) = k \left(\theta_B^*(u^{Bad}) - \theta_{B,u^{Bad}}^{S_A} \right) \left(\overline{\theta} - \theta_A^* \right) + (1-k) \left(\theta_B^*(u^G) - \theta_{B,u^G}^{S_A} \right) \left(\overline{\theta} - \theta_A^* \right)$$

Notice that, in &gures 5 and 6, &nancial contagion can be represented as the two shaded segments of the graphs. It is only in these two segments that we can reasonably have a cause-effect relationship. For instance, assume that the state of the common macroeconomic fundamental is bad. We have argued that, when bank A fails, the trigger of bank B is revised upwards, taking into account the fact that bad news have raised the trigger from $\theta_B^*(\ )$ to $\theta_{B,u_{Bad}}^{F_A}$. This extra increase in the trigger due to the event in bank A is what the shaded segment on the left of &gure 5 is all about. Here, bad news about bank A, have altered the behaviour of depositors in bank B, such that, given the level of the common macroeconomic fundamental, bank B fails for a wider range of its own fundamentals. The difference $\theta_{B,u_{Bad}}^{F_A} - \theta_B^*(u^{Bad})$ represents this cause-effect relationship. Point M in &gure 5, shows a case where failure of bank A can cause bank B to fail. Notice that points below the horizontal (dotted) line $\theta_B^*(.)$,represent failure of bank B, even though bank A does not exist. Point N, thus cannot represent &nancial contagion, because, even though both banks A and B fail, bank B would have failed anyway, even without bank A□s presence. A□s presence (and in particular, the event that befalls it e.g failure) have raised the threshold of bank B, given that the common macroeconomic fundamental is assumed to be in bad state, to $\theta_{B,u_{Bad}}^{F_A}$. Any realisation of the fundamental, lying between $\theta_B^*(.)$ and $\theta_{B,u_{Bad}}^{F_A}$, represent contagion. In the same token, success of bank A will lower the trigger of bank B from $\theta_B^*(\ )$ to $\theta_{B,u_{Bad}}^{S_A}$. That extra fall in the trigger of bank B due to the event of bank A, also depicts &nancial contagion (shown as the right hand shaded segment of &gure 5). All arguments we have put forward, so far in this section, concern the case when the state of the common macroeconomic fundamental is in the bad state (i.e $u = u_B$). Ostensibly, the arguments also run through if the common fundamental was in the good state (i.e $u = u_G$). It simply suffices to compute thresholds $\theta^{F_A}_{B,u_G}$ and $\theta^{S_A}_{B,u_G}$ with probabilities $\Pr(u = u_G \mid F_A)$ and $\Pr(u = u_G \mid S_A)$ [44] respectively and all arguments will run through.

7 Practical Relevance and applications

Though the model of banking panic transmission highlighted in this paper is, admittedly highly theoretical and has been drawn up with no particular real-world example in mind, the model nonetheless has practical relevance and can (hopefully) cast light into new ways that Central Banks and international institutions such as the International Monetary Fund (IMF), should design the regulatory structure. We present the applicability of this paper with respect to the following: demystifying important puzzles in the literature and policy implications to improve on the regulatory setting of banks activities.

7.1 Demystifying important puzzles

Surveying the empirical literature on &nancial contagion helps unearth two puzzles about &nancial contagion, which are inextricably linked to one another:

Puzzle 1: (Zero-Link issue) *Why does the failure of one &nancial intermediary sometimes lead to the failure of another intermediary when there is no apparent physical or direct link between them?*

Puzzle 2: (Clustering issue) *Why does &nancial contagion not arbitrarily spread from one institution to another, but rather seems to affect identical institutions only?*

Models of &nancial contagion that focus on direct link (Allen and Gale (2000) and Dasgupta (2004)) do not explain the zero-link issue.The importance of that issue cannot be understated though. The essence of these models of contagion is the existence of a direct link itself that lies at the heart of spreading a crisis from one bank to another. For example,in Allen and Gale (2000), the existence of a network of overlapping interbank claims provides the key propagator channel, such that a bank failure means that another bank will

[44] We computed $\Pr(u = u_G \mid F_A)$ as $\frac{k\ \theta^*_A(u_G)}{k\ \theta^*_A(u_G)\ +\ (1-k)\theta^*_A(u_G)}$ and $\Pr(u = u_G \mid S_A)$ as $\frac{k\ (1-\theta^*_A(u_G))}{k\ (1-\theta^*_A(u_G))\ +\ (1-k)(1-\theta^*_A(u_G))}$

surely suffer a loss of interbank claims. Hence, it is more likely to suffer from the same fate as the first bank. If there were no financial contracts provided by the interbank market for deposits as a way of insuring against regional liquidity shocks, then there would no banking panic transmission. Dasgupta (2004) also has the interbank market as the direct link between banks, although he also allows sequential moves across banks. Nonetheless, the global games paradigm he adopts, endogenises the probability of bank runs by showing the existence of a unique equilibrium in the banks' idiosyncratic fundamental. Hence, absent the direct link, then contagion cannot be defined. These weaknesses provided by the "direct link" theories are not present in our model. We have showed that, *even though,* there is no apparent direct link between banks, *contagion may still occur in equilibrium.* Our model, which comes closer to the "*pure contagion theories*" hence represent a Pareto improvement over the "*Direct Link theories*", as far as explanation of the zero-link issue is concerned.

Puzzle 2 has been widely documented by Aharony and Swary (1996), who conducted a study of 33 US banks in the mid 1990s and found that the extent of negative impact of contagion is greater for banks that are similar to the failed bank. Likewise, Ahluwalia(2000) shows that, for a sample of 19 countries and three episodes of crises, a country's vulnerability to contagious crises depends on the visible similarities between this country and the country actually experiencing the crisis. The models of Allen and Gale (2000) and Dasgupta (2004) cannot explain the clustering issue: they both focus on identical banks (though heterogeneous because of liquidity shocks) and the strength of connection provided by the direct link is same for all banks. In our model, identical banks are those that are exposed to some common macroeconomic fundamental. Thus,because of this common exposure, the dynamics of informational spillover flow is such that, depositors of the second bank rationally update their beliefs about the state of the common macroeconomic fundamental - meaning that the second bank is more likely to suffer the same fate as the first. If banks were not linked to the common fundamental (i .e were not identical), depositors of the second bank would not have adjusted their beliefs in that way. We are thus, able to capture that, only identical banks are likely to suffer the same fate. For non-identical banks, the relationship between events happening at banks is not that straightforward.

7.2 Regulatory Mechanism Design - Microprudential v/s Macroprudential regulations

A great part of the literature on banking regulation (or the design of optimal regulatory framework for banking) tends to focus on the specific means to preempt the likelihood of financial contagion. Whilst microprudential regulation has received much attention and theoretical support, macroprudential regulation has often been ignored in debates over what the appropriate regulatory framework should be.

Microprudential regulation concerns all the preventive measures taken at individual bank level, designed to ward off the possibility of a bank failure bieing transmitted to the whole banking and financial system. It consists mainly of "one-sided" policy measures[45] either intended to protect the depositors of the bank or as a general safety net designed to maintain the confidence of all stakeholders in the banking system. Deposit Insurance schemes characterise the former set. Suspension-Of-Convertibility (SOC) and Lender-Of-Last-Resort (LOLR) characterise the latter set. In "direct link" models of contagion, microprudential regulatory means would work in pre-empting the spread of a banking panic. The commonly help syndrome "Help one, Save all" works. For example, if due to excess regional liquidity shocks, a bank faces a higher than normal proportion of early withdrawals, the Central Bank, through the LOLR agency, will intervene and earmark some emergency fund to help bridge the bank's temporary illiquidity problem[46]. By preventing the loss of claims in the other bank, these one-sided measures help to ward off the possibility of a systemic risk. The intuition is simple: if the interbank market is the main propagator of financial crises, then it is the channel through which the crisis is prevented, when the first bank receives funding through LOLR. However, note the following points: (1) For the interbank market to work, it is necessary for the regional liquidity shock to be negatively correlated across regions. If the regional liquidity shock is positively correlated (i.e all banks face high premature early withdrawals at the same time), then LOLR does not work; (2) Microprudential measures do not work effectively if the main reason for bank failure is some commonly based fundamental that links both banks. For example, suppose two banks have received financial contracts (lent) in dollars and have issued financial contracts (borrowed from depositors) in euros. A depreciation of the dollar against the euro, could negatively affect the balance sheet of both banks and lead to premature withdrawals by depositors in each bank. In this case, the interbank market is to no avail; one-sided measures will also not likely work. What is needed is some policy measures to target the common macroeconomic fundamental that is commonly driving both banks' performance e.g limit the fluctuation of the dollar against the euro by designing some form of explicit exchange rate arrangement that will achieve this goal of currency stability. In the South East Asian crisis of 1997, the banking panic throughout the region occured because of the banks' exposure to extreme exchange rate changes, which softened their balance sheets and made them much more vulnerable and prone to bank runs. In instances such as these, macroprudential regulation should be given the overriding concern..

[45] We use the term "One-Sided" measure because we shall be assuming that the policy applies only to the bank facing the crisis. There is no randomisation among the banks (i.e good banks or bad banks) and no economy-wide safety net

[46] Technically, a LOLR agency would intervene if (1) it reckons that the bank's problem is just temporary illiquidity but otherwise, is solvent in the long term; (2) it fears that the failure of the bank could lead to the collapse of the whole banking and financial system. This has prompted debate in the literature about whether the size of the bank matters for LOLR. Rochet and Tirole (1996) argue that it is the amplitute of financial connections that matter, not the bank size.

In our paper, there is no initial unhedged liquidity shock as trigger of original crisis in a bank. Rather, we focus on adverse information on bank's portfolio of returns as being prime catalyst, prompting a change in depositors' behaviour. As in models of financial crisis using the global games methodology, increased transparency could help. Since beliefs of depositors are driven by the idiosyncratic fundamentals of each bank, conditional on the state of the common macroeconomic fundamental, regulators should focus on what is driving the triggers of the two banks: the common macroeconomic fundamental in order to ward off a crisis or to minimise the probability of contagion. The second bank will suffer the same fate as the first bank, with bayesian updating agents. The main driver of this tale of common fates is the extra-stong exposure of the banks to the common fundamental. In this case, controlling the common macroeconomic variable would be helpful in minimising the possibility of multiple bank failure. If information is not available about the state of common macroeconomic fundamental, regulators or the central bank should disclose this information readily for more informed judgements.

The novel approach of our paper is that, by allowing for informational spillovers in a sequential move game and by imposing restrictions on underlying coordination problems, we can see implications of the new structure for regulatory measures used in the literature of bank runs. As in the global games approach, bank runs are still caused by depositors withdrawing for fear of other depositors withdrawing early. What is coordinating depositors' beliefs is their bank's fundamental. Microprudential regulatory measures still seem best at pre-empting the likelihood of a crisis from existing in the first instance, by effectively acting as a mechanism that coordinates the beliefs of depositors on the right equilibrium. However, because of the feedback mechanism implicit in the informational spillover channel, we conjecture something much stronger: microprudential measures (e.g LOLR) work best if they send a positive signal on coordination possibilities to depositors of the second bank (i.e if they contain an informational element that enables depositors of the second bank coordinate on an outcome that prevents bank failure from happening at the second bank.) In turn, a favourable outcome at the second bank, will make depositors of the first bank less willing to withdraw. As a result, the coordinating behaviour of depositors of the first bank, actually complements the institution of microprudential measure, in favouring a good outcome at the first bank and, at the same token, a good outcome at the second bank.

Contrast between the following two banking economies: one in which Suspension-Of-Convertibility (SOC) is adopted by the first bank and another in which LOLR is the prime external funding source. We conjecture that these one-sided measures play a vital signalling role that may radically alter the behaviour of depositors in the second bank. A LOLR banking economy does better at eliminating financial contagion because the measure may send a positive or negative signal to depositors of the second bank[47]. On the other side, a SOC banking economy

[47] The signal could be described as thus: if depositors of the second bank observe the first

may always send the wrong signals to second mover depositors[48]. Thus, even though it prevents a failure at the bank that was initially facing a crisis, *it actually creates a channel of contagion of its own*. Thus , we may conclude the following as far as microprudential measures : *Microprudential measures may work as a pivotal mechanism that coordinates the expectations of depositors on the right outcome if and only if they send the right signals to depositors of the second mover bank. In case they send the wrong signals, then the "Help one, Save all" syndrome is broken: they may, on their own, help create an additional channel of financial contagion.*

This new implication for microprudential policy design is important, because it tells us that, in sequential games with informational spillovers, there are different ways of interpreting the implementation of that measure: instead of acting as a coordination mechanism for depositors of the same bank, these measures need to coordinate the expectations and beliefs of depositors across different banks on the correct equilibrium. For that, it is imperative that positive signals are sent.

8 Conclusion

In this paper, we have attempted to build a theoretical model of contagious bank runs, which uses the informational spillover channel to explain the spread of failures from one bank to another. The nature of the investment technology complicates the coordination problem that depositors in each bank face. Focusing on informational spillover channel, endogeneously introduces two forms of coordination in each bank: each depositor must coordinate his action with other depositors of the same bank (contemporaneous complementarities) but also, with depositors of the other bank (dynamic complementarities). In this case, the interplay between fundamental uncertainty and strategic uncertainty, which is crucial for global games approach to work in a coordination problem, is seriously affected and leads to structural complications and obvious methodological problems. Nonetheless, we show that, by using the theoretical argument

bank receiving financial aid in the form of LOLR, they may interpret the information in two ways: (1) Something is wrong about the first bank: Since the two banks are perceived to be connected to the macro fundamental in varying degrees of connection strength, the second bank may meet the same fate. So, they decide to withdraw now (Negative Signal)

(2) Confidence is being maintained in the first bank through LOLR and its temporary illiquidity problem is being solved. Therefore, nothing is going wrong (Positive Signal).

Notice that, in the case of the negative signal, the LOLR, being one-sided, has actually created a channel of contagion of its own

[48] The Negative Signal associated with SOC comes from the fact that depositors of the second bank may interpret the information in the following way: if something is wrong in the first bank and depositors wishing to withdraw are not getting their dues, they may also not get their dues, if their bank meets the same fate tomorrow. Thus, their best response is to withdraw now. Just by suspending convertibility in one bank to try to limit multiple banking failure, has led to a run on the second bank!

centered around putting restrictions on relative complementarities, we can use the global games methodology to analyse contagious bank runs with informational spillovers.Along the path dictated by the perfect bayesian equilibrium concept, depositors need to coordinate their actions within their own bank and across banks. Doing so enables us capture how the ! ow of information from one bank to another affects the possibility of a coordinated attack in each bank and contagiously spread a crisis across banks. Our analysis shows that Dasgupta (2004)'s results are robust and intuitively very appealling: the probability of occurrence of contagion is positive and contagious failures manifest themselves as a unique equilibrium of a dynamic game with incomplete information. The institutional setup we have adopted also enables us to go further: the intrinsic features of contagious probabilities enable us to distinguish between contagious bank failures and correlated bank failures in equilibrium. Doing so also has the appealling features of explaining stylized facts of contagious bank failures, which former papers in the literature seemed to sideline.

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APPENDIX

Figure 1 : Segregation of the θ_i - space into Strict and Weak dominance regions

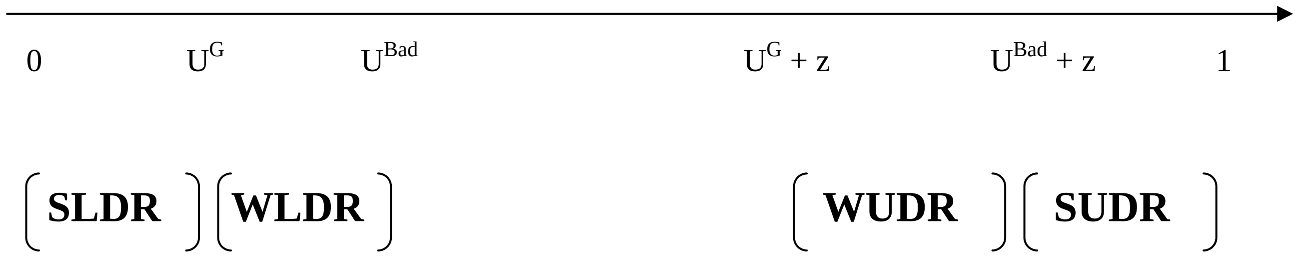

Figure 2 : The relationship between idiosyncratic fundamental, common macroeconomic fundamentals and (risky) returns technology for a bank

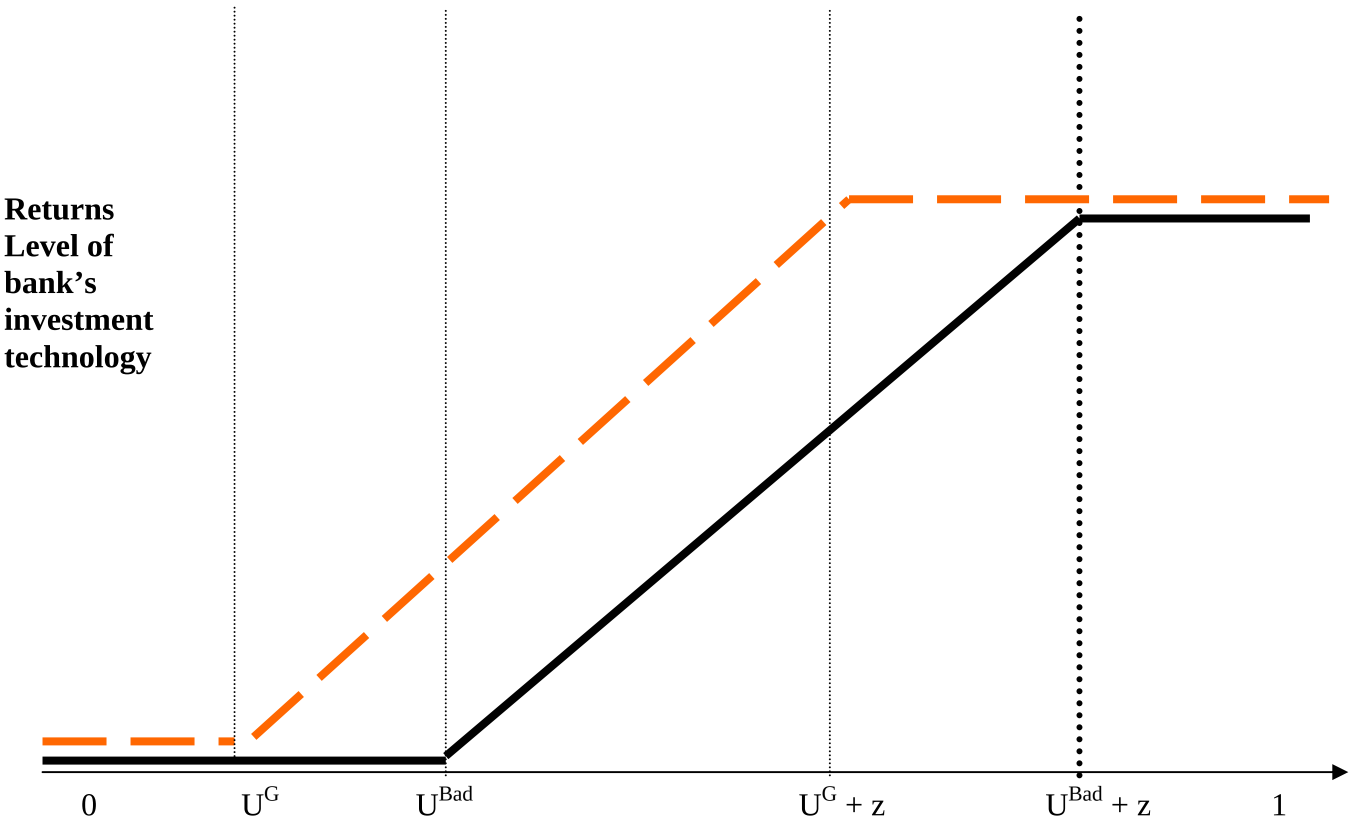

Figure 3 (a) : Payoff to Staying v/s Payoff to withdrawing for a depositor

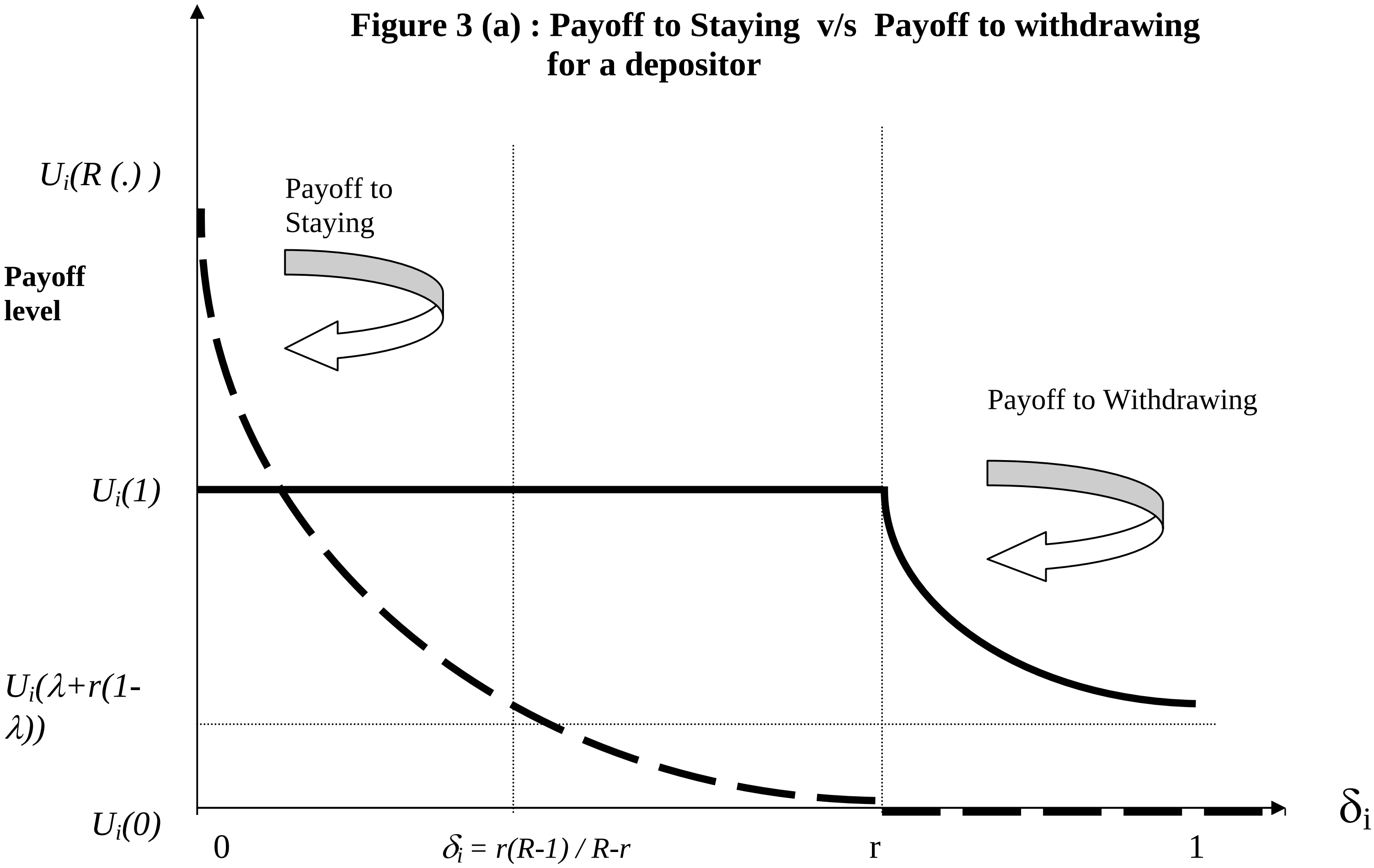

Figure 3 (b) : Net Payoff to Staying for a patient depositor

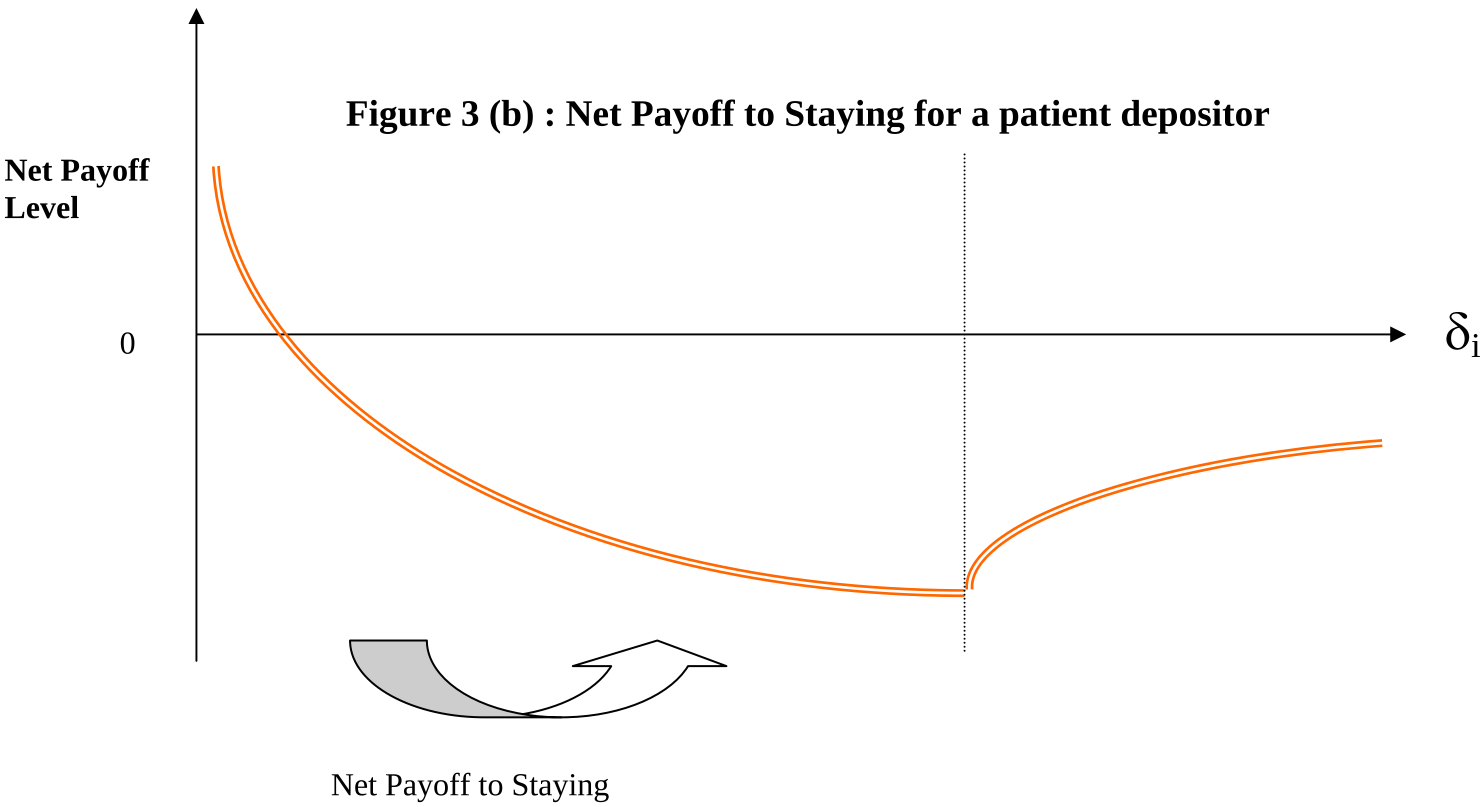

Figure 4: Idiosyncratic fundamentals ('Autarky' case)

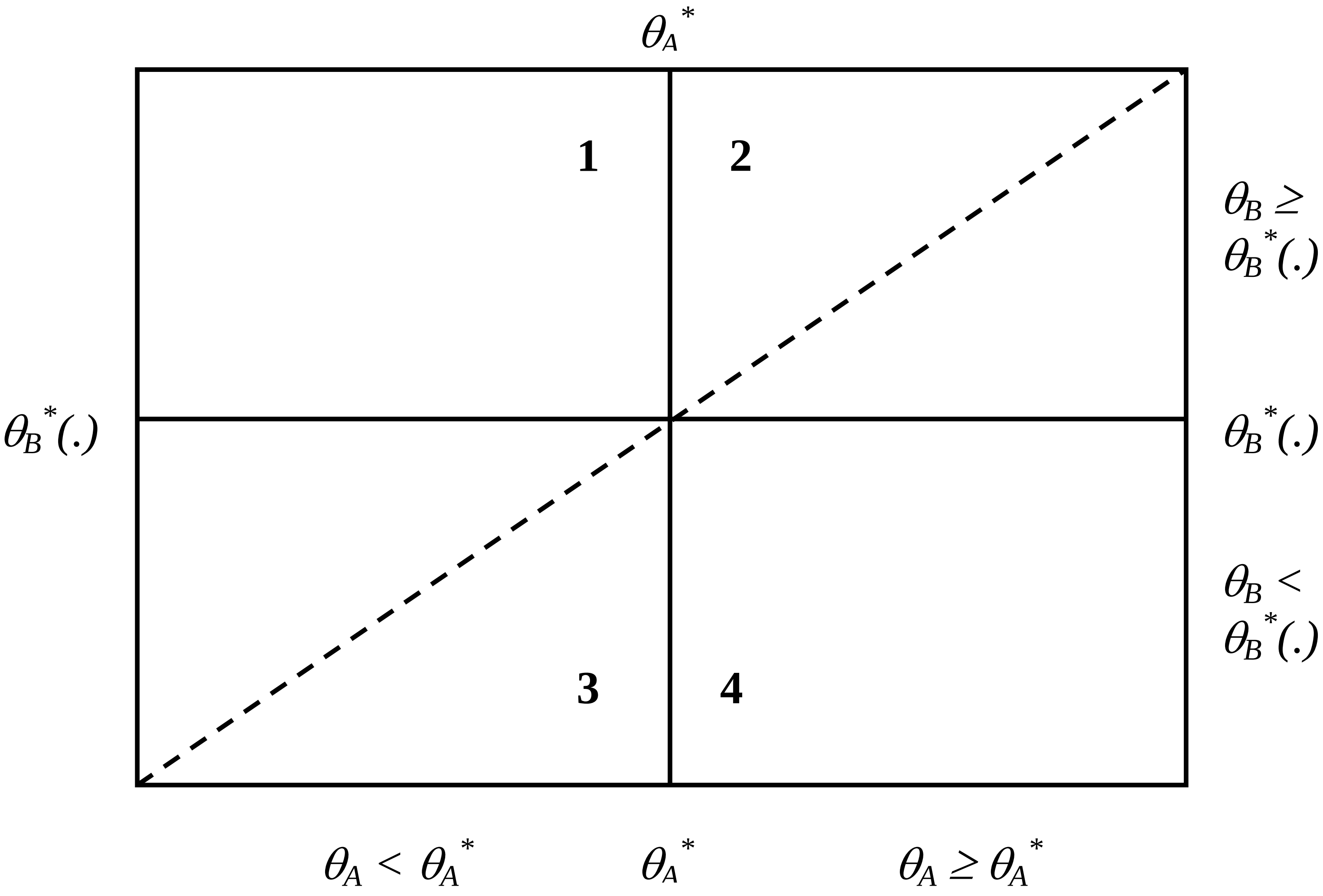

Figure 5 : Idiosyncratic fundamentals for θ_A^*, $\theta_B^*(.)$, $\theta_B(F_A, u_{Bad})$, $\theta_B(S_A, u_{Bad})$

'Financial Contagion' as a unique equilibrium - Case when macroeconomic fundamental is in Bad state

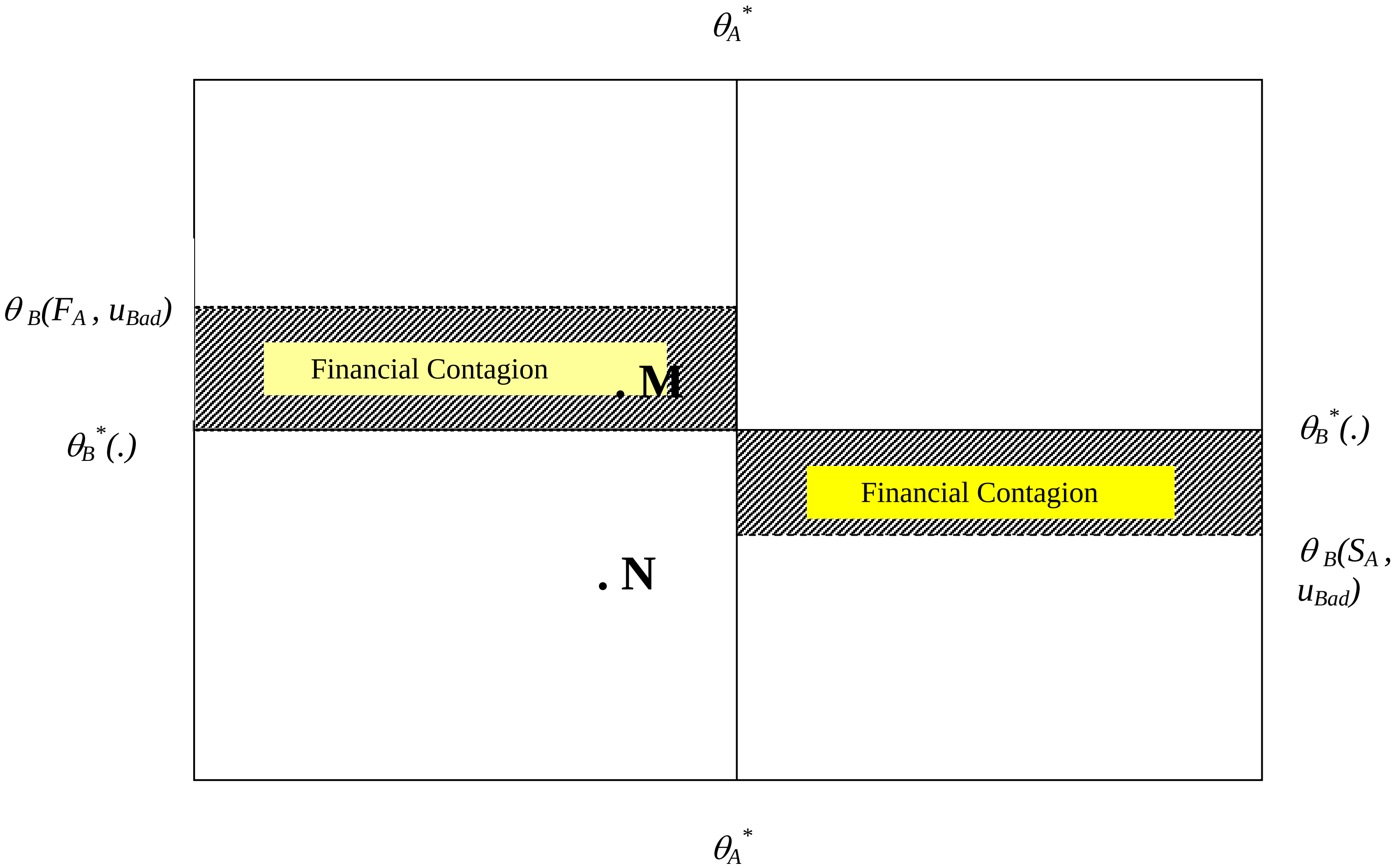

Figure 6 : Idiosyncratic fundamentals for θ_A^*, $\theta_B^*(.)$, $\theta_B(F_A, u_G)$, $\theta_B(S_A, u_G)$

'Financial Contagion' as a unique equilibrium - Case when macroeconomic fundamental is in Good state