Growth and Offshoring*

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Abstract

This paper investigates the relationship between growth and offshoring in a small economy. The movement of jobs abroad is of concern for many economies. I show, that growth is enhanced by offshoring in an economy that does offshore. Further, offshoring is seen as a potential for countries that receive offshoring. Therefore analysing a small economy that receives the offshoring I show that there are adverse effect under some circumstance.

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JEL Classification: F11;F43;F16

1 Introduction

This paper looks at the effect of offshoring on growth. Both topics have important implications for the welfare of an economy. On the one hand, growth is widely linked to positive effects: growth increases the production possibilities of a country. On the other hand, offshoring is seen as a more controversial phenomenon, especially outside academia. It seems therefore of importance to take a closer look at the link between the two topics, with special regard to the question whether offshoring increases economic growth.

This paper employs tools from growth theory and international trade. A wide literature exists on the topic of economic growth. More recent models emphasize

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the importance of endogenous growth. They assume that growth is produced in an economy by allocating recourses to an innovative sector. Economists have adapted these model to a trade environments.

In the existing literature countries attain higher growth due to increased specialization. The specialization is in terms of final goods or intermediate inputs. In the framework I develop, a country specializes in either research or production of tasks. A task, in this paper, is an action performed by labour aimed at producing one unit of output. Each action is distinct from other actions. For one unit of output to be made, a certain number of tasks must be performed. However, not all tasks must be performed at the production facility and can therefore be offshored. This notion of offshoring follows the one developed in Grossman and Rossi-Hansberg (2006).

In the model I develop offshoring affects the allocation of workers between an innovative sector and a production sector. The idea is similar to Rivera-Batiz and Romer (1991) who show that an integrated economy has a higher growth rate because more resources available for research. I show that in an endogenous growth framework developed by Aghion and Howitt (1992) that a counrty that offshores unambiguously increases growth. For a counrty that receives the offshored task I show that the growth rate might fall or increase. The reason for the latter result is, that resources are taken away from the research sector.

2 The Model

I start the discussion of offshoring and growth with the country that offshores tasks. In this next section I analyse an economy that receives the offshored tasks. The set up of the both economies are the same. In each case the economy is assumed to be small relative to the rest of the world. It therefore takes world wide prices as given. I further focus on one industry in the economy.

2.1 Set Up

The small economy has three sectors. Firstly, a sector that assembles a final good from an intermediate input. Secondly, an intermediate sector that produces the intermediate from a continuum of tasks. It is possible to offshore tasks in the intermediate sector. Thirdly, a research sector where the innovation takes place. I now discuss each sector in detail before deriving the equilibrium conditions.

2.1.1 Consumers

The consumers are assumed to care only about consumption in each period. The utility function therefore takes the form $U = \int_0^\infty e^{-r\kappa} c_\kappa d\kappa$, where r is discount factor which is the interest rate in the economy. Consumption is measured over a period of real time κ. It is important to keep in mind, that there is a difference between real time period and the time that elapses between two innovations. Consumers are not able to save any over their consumption.

The economy is endowed with labour L which is allocated to the production of the intermediate L_x^D and innovation n.

2.1.2 Final Output

The final output sector is assumed to be perfectly competitive. The final output is manufactured using an intermediate input which is produced by the holder of a patent in period t, where period refers to the time a patent hold is the incumbent in the market. In each period the final good is produced according to the production function

$$y_t = A_t x_t^\alpha \tag{1}$$

where x_t denotes the state-of-the-art intermediate input and A_t denotes the current level of technology. With each innovation, the current level of technology is improved by $\gamma > 1$. The technological progress takes the following form. The research sector has access to the state-of-the-art technology which must be improved to become the new incumbent producer. Therefore, the post-innovation productivity is $A_{t+1} = \gamma A_t$.

2.1.3 Research

With each innovation production possibilities are expanded. Discoveries of improved technologies are made By allocating labour to a research sector the existing technology is improved. The innovations are assumed to occur at random time intervals. The stochastic arrival time is assumed to be Poisson distributed with an (average) arrival rate of λn. The λ is a country's productivity in research and n is the number of workers employed in research. The intuition for this formulation is a constant return technology in research. I further assume that the incumbent producer exits the market with the next innovation.

2.1.4 Intermediate Production

The intermediate input is produced by a continuum of task that are performed by labour. I assumed that the measure of the tasks is unity. Each task needs a_x units of labour regardless of where the task is performed. Not all tasks have to be executed by domestic labour. If a task is offshored it is subject to at a transport cost $\tau(j) > 1$, where j indicates the task. This assumption was introduced by Grossman and Rossi-Hansberg (2008). The idea is, that not all tasks are equally offshorable[1]. Some tasks for instance need to be performed in proximity to the domestic production facility whereas other tasks can be located somewhere else at low costs. Accordingly, it would be very costly - even prohibitively costly - to offshore some tasks. Further, the costs of offshoring are not necessarily correlated to skill intensity. For example, accounting could be easily offshored whereas a janitors job cannot be offshored. I further assume that all tasks are ordered in a non-decreasing way, which implies $\tau'(j) > 0$, and that $\tau(j)$ is a continuous function.

Offshoring follows a cost savings motive: all tasks that can be produced cheaper abroad are offshored. The latter point is illustrated by

$$w \geq w^* \tau(j) \tag{2}$$

where w is the domestic wage and asterisk indicates the rest of the world. As long as the domestic labour is more expensive in the production of task j than the import price of a tasks, the task is offshored. A marginal task J exists if the domestic labour costs are equal to the import price of the task. Given that the transport costs increase in j, there might exist a task for which the economy is indifferent between domestic production or offshoring. I assume that the marginal tasks is offshored.

The existence of a marginal tasks is endogenous. For the proceeding discussion I assume that a marginal task exists, such that equation (2) holds with equality. However, I will show, that a marginal tasks does not always exists. The unit cost of an intermediate producer is

$$c_m = w(1-J)a_x + w^* a_x \int_0^J \tau(j)dj. \tag{3}$$

The first term on the right-hand-side (rhs) is the compensation of domestic labour for the tasks it performs. The second term on the rhs is the compensation paid for foreign tasks which includes the workers compensation and trade costs. Using the

[1] An excellent summary of offshorability is included in Baldwin and Robert-Nicoud (2007).

cutoff I rewrite the cost function as follows

$$c_m = w a_x \Omega(J) \tag{4}$$

where $\Omega(J) \equiv 1 - J + \frac{\int_0^J \tau(j)dj}{\tau(J)}$ which is smaller than one. $\Omega(J)$ is interpreted as the cost savings potential in terms of domestic units[2]; the smaller $\Omega(J)$ the higher are the cost savings potential. Figure 1 gives a graphical interpretation of cost savings potential. $1 - \Omega(J)$, the costs savings to the left of the curved line and the vertical axis. If there exist no cost saving by offshoring $\Omega(0)$ equals one. If the marginal task is $J = 1$ all tasks are offshored. The figure depicts a case where a fraction of J tasks is offshored.

[Figure 1 about here]

A different interpretation of $\Omega(J)$ is possible. Grossman and Rossi-Hansberg point out, that offshoring is analogous to labour augmenting technological change. It is possible to interpret $a_x\Omega(J)$ as effective unit labour input requirements. With an increase in J - an increase in the offshoring share - the labour needed to produce one unit of output falls. Accordingly, the same amount of labour can produce more of the intermediate good which implies that labour has become more productive.

The demand for the intermediate is

$$p_t = \alpha A_t x_t^{\alpha-1} \tag{5}$$

which is derived from the final good sector. Given that each incumbent intermediate producer is a monopolist she is maximizing her profits. Therefore, the incumbent sets an output such that

$$x_t = \arg\max_{x_t} [\alpha A_t x_t^{\alpha} - w_t a_x \Omega(J) x_t] \tag{6}$$

$$= \left(\frac{w_t a_x \Omega(J)}{\alpha^2 A_t} \right)^{\frac{1}{\alpha-1}} \tag{7}$$

The output of the intermediate depends negatively on the effective wage, $\omega_t \equiv \frac{w_t}{A_t}$ and on the cost savings. Therefore, a higher costs savings potential implies a higher output.

[2]Grossman and Rossi-Hansberg call this the productivity effect of offshoring. They, however, use look at changes in $\Omega(J)$ as J changes.

2.1.5 Equilibrium Conditions

There are three equilibrium conditions in this model. Firstly, labour must be fully employed. Therefore we have

$$L = L_x^D + n \tag{8}$$

The labour demand from the intermediate sector is the labour used to perform the domestic tasks, $L_x^D = (1-J)a_x x_t$. In the latter equation $1-J$ measures the number of tasks performed at home times the labour needed to perform the task times the number of intermediated that are produced. As the households do not value leisure, each period must be characterized by full employment.

Secondly, there is a no-arbitrage condition. In the research sector, the investment in innovation is choosen profit maximizing. Successful research yields the discounted profits V_{t+1}. The probability of success is λn, where the probability increases in the number of researchers employed. Researchers must be paid the market wage rate w_t. The resulting first order condition of the profit maximization[3] is $\lambda V_{t+1} = w_t$. In words this says that expected discounted profits must be equal the wage paid per worker. The reason is, that a worker can decide to work in the intermediate sector or to engage in research. In order for the labour market to be in an equilibrium the worker must be indifferent between the two sectors. This will be the case if the expected discounted return from research is equal to the current wage rate.

Further, the discounted profits are governed by a asset equation, $rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} V_{t+1}$. The latter says that rent created by an investment in bonds of size V_{t+1} (left hand side) must be equal to the rent created by research (right hand side). The research rent is made up of two parts. Firstly, the profits that are obtained by the $(1+t)$th innovation. Secondly, the profit stream lost when the innovation is replaced by its successor, which happens with a arrival rate of λn_{t+1}. This constitues the process of creativ destruction. If the research rent is larger than the one on the capital market, profitable opportunities in research are exploited by increasing the employment in research which lowers the expected rent of research until both rents are equated.

Rewriting the asset price equation and substituting it into the no-arbitrage condition results in the expanded no-arbitrage condition

$$w_t = \frac{\lambda \pi_{t+1}}{r + \lambda n_{t+1}}. \tag{9}$$

[3]Strictly speaking the first order condition must hold with a weak inequality. Nevertheless, equality is used in this model for convenience.

The current wage rate depends therefore on the profits of the next successful innovator and the number of researchers that aim for replacing the next innovator.

The third equilibrium condition is the equation for the marginal task (2). The latter equation governs the number of tasks that are offshored. If the economy could still realize cost savings the economy is not in an equilibrium.

2.1.6 Equilibrium

The decision the economy faces in each period is how to allocate labour to research and intermediate production. In this subsection I solve for possible equilibria. I show, that there exists a steady state growh rate, which is defined as a constant growth of all variables. To solve for the equilibrium decision I substitute the full employment equation into the no-arbitrage condition and solve for n. I skip the index t because I discuss steady states. The equilibrium employment condition in the research sector is

$$\hat{n} = \frac{\Theta(J)L - r}{\lambda + \Theta(J)} \tag{10}$$

where $\Theta \equiv \gamma\lambda\frac{1-\alpha}{\alpha}\left(1 + \frac{\int_0^J \tau(j)dj}{\tau(J)(1-J)}\right)$.

The equilibrium employment condition in research depends on the marginal tasks J. In order to determine the optimal J I have to look again on the equation (2), which determines the marginal task. Rewriting the latter equation in logs and differentiating it with respect to time t I obtain

$$\frac{\partial J}{\partial t} = \frac{\tilde{w} - \tilde{w}^*}{\tilde{\tau}(J)} \tag{11}$$

where the tilde represents a percentage change[4]. The economy takes the evolution of the wage in the rest of the world, $\tilde{w}^*$, as given. The sign of the derivative is determined by the relative wage gap in the small economy and the rest of the world. In the following I distinguish between three cases (I) divergence in the wage gap, (II) constant wage gap and (III) convergence in the wage gap.

(I) Divergence ($\tilde{w} > \tilde{w}^*$)

Divergence in the wage gap implies that the home country growths faster than the outside world. The reason is, that the increase in wage in the small economy is driven by the technological improvements.

[4] $\tilde{w}$ and $\tilde{w}^*$ are percentage changes over time and $\tilde{\tau}(J)$ represents a percentage change of the trade cost of task J with an increase in J.

What is the pattern of offshoring? In the case under consideration it holds that $\frac{\partial J}{\partial t} > 0$ and therefore the marginal tasks increases until all tasks are offshored. Accordingly, the small economy has a comparative advantage in research which it specialized in.

It follows that the steady state employment in research is

$$\hat{n} = L \tag{12}$$

(II) Constant wage gap ($\tilde{w} = \tilde{w}^*$)

A constant wage gap implies that both - the small economy and the rest of the world - grow at the same rate. If the wage gap is sufficiently small the marginal tasks has an interor solution, $J \in (0, 1)$ and stays constant over time. Therefore the small economy remains incompletely specialized.

It follows that the steady state employment in research is

$$\hat{n} = \frac{\Theta(J)L - r}{\lambda + \Theta(J)} \tag{13}$$

(III) Convergence ($\tilde{w} < \tilde{w}^*$)

In this case, the wage gap between the small economy and the rest of the world decreases. Accordingly, the wage gap closes or even reverses. I limit my discussion to the former case and turn to the latter case when I analyze a small economy that exports tasks. Accordingly, no tasks are eventually offshored as the cost savings incentive vanishes.

It follows that the steady state employment in research is

$$\hat{n} = \frac{\lambda\gamma\frac{1-\alpha}{\alpha}L - r}{\lambda(1 + \gamma\frac{1-\alpha}{\alpha})} \tag{14}$$

2.2 An Offshoring Economy

2.2.1 Growth

I now look at the average growth rate of the economy. Growth is defined as the average change in final output over period of real time, $g \equiv E(\ln y_{\kappa+1} - \ln y_\kappa)$. I firstly look at the growth rate of an economy that does offshore and is incompletely specialized with $J < 1$. Subsequently, I look at an economy that is completely specialized in research.

Growth in the economy is driven by improvements in the productivity of the final output production. From the Poisson arrival rate I know that the time period between two innovations decreases with λn. But λn is the average number of innovations in one period of real time[5]. Each time an innovation is introduced to the market the parameter A_t is increased by γ. Accordingly, I can write the log change of output with each innovation as $\ln y_{t+1} - \ln y_t = \ln \gamma$. Further, this increase in output per period happens λn times in an interval of real time. Accordingly the growth rate is

$$g = \lambda \hat{n} \ln \gamma. \tag{15}$$

It follows that growth and employment in research are positively correlated: an economy that allocates more labour towards research in a steady state has a higher growth rate. This can be understood as follows. The more labour is in the research sector the shorter is the average period between two innovations. Therefore, the number of innovations in one interval of real time has increased as well. Of course, the employment in research depends on the parameters of the economy under consideration.

I now turn to the case where the rest of the world has a lower growth rate in wages. Contrary to the previous case of an constant wage gap, the output in the intermediate sector increases as $\frac{w_t^*}{A_t}$ decreases over time. This implies, that the effective wage paid to foreign workers decrease[6]. Substituting the expression for the output of the intermediate (6) into the production function for the final output and taking logs of the resulting expression yields

$$\Delta_t \ln Y_t = \frac{1}{1-\alpha} \ln \gamma - \frac{\alpha}{1-\alpha} \Delta_t \ln w_t^*$$

where Δ_t is the difference between to innovations. The latter expression is larger than zero which implies that the small economy experiences positive growth. The growth rate of the small economy therefore must take into account the change in the wage of the rest of the world. The small economy will experience growth from innovation as before and additionally growth from a declining effective import costs.

[5]The equivalence is derived from the Poisson distribution. Intuitively, the unit inverval of real time can be divided in subsegments of length $(\lambda n)^{-1}$. Accordingly there are λn intervals in one unit real time interval.

[6]Despite that the wage paid to the foreign workers grows.

Therefore the increase of final output with an innovation is

$$g = \frac{1}{1-\alpha}\lambda L \ln\gamma - \frac{\alpha}{1-\alpha}\tilde{w}^* \tag{16}$$

where I assume that the small economy is completely specialized in research. A slower growth of the rest of the world increases the growth rate of the small economy because it can utilize the growth of the intermediate input.

Corollary 1. *The growth rate with complete specialization of the small country is higher then with incomplete specialization.*

Proof. To prove the above corollary, I need to show that $\lambda\hat{n}\ln\gamma < \frac{1}{1-\alpha}\lambda L\ln\gamma - \frac{\alpha}{1-\alpha}\tilde{w}^*$. But rearranging the latter expression yields $\lambda\ln\gamma(\hat{n}-L) < \alpha(\lambda\hat{n}\ln\gamma - \tilde{w}^*)$. The left hand side of the inequality is negative. The right hand side is positive assumption which proofes the corollary. □

The intuition behind this result is, that if the wage gap increases, the small economy's income grows faster than the effective costs of the imported tasks. Accordingly, it realizes gains from increased use of the intermediate in production. This corollary is used later to generealize a result.

2.2.2 Gains from Trade

In this section I compare the steady states of the small economy with and without offshoring. The case of no offshoring is denoted by a. I focus on the cases of a constant wage gap. The results can be extended to the case of an increasing wage gap by referring to corollary 1.

Proposition 1. *The small country gains from offshoring from a reallocation effect, which increases the growth rate.*

A formal proof of the proposition is found in the Appendix. The reallocation effect increases the fraction of workers in research. Despite an increase in final demand, the labour demand in the production sector falls as a share of the tasks is offshored to the rest of the world. Accordingly, jobs get relatively scares in the intermediate sector which decreases the wage. A lower wage opens profitable opportunities in the research sector which are exploited by workers switching to research. Therefore growth increases, as more worker do research.

Offshoring increases the output in the final good sector as well. This effect is a static, because it occurs additional to the increase in growth. An intermediate producer can reduce its costs of production by offshoring. This fall in marginal costs implies an increase in the output of the intermediate good[7] given the monopoly behaviour of the intermediate producer. The intermediate producer is able to substitute foreign workers for domestic workers in the production due to offshoring. The intuition is similar to the labour supply effect in Grossman and Rossi-Hansberg (2008): the effective labour endowment in the economy is increased by offshoring.

The effect of offshoring on the wage paid to workers is ambiguous. On the one hand the wage decreases as tasks are moved abroad because workers now compete for less jobs. On the other hand, due to the reallocation effect, less workers compete for the tasks.

2.3 An Receiving Economy of Offshoring

In this section I analyze a small economy whose wage is below the worldwide wage. Accordingly, the small economy receives the offshored tasks and exports finished tasks. I assume that the recipient country produces the tasks with a unit input requirements a_x. I do not have to make any assumptions on the technology of task production in the rest of the world, as the worldwide technology has no direct impact on the small country. This might change if offshoring includes technology transfers from the rest of the world to the small economy.

I build on the previous discussion of the model for the source country. The mechanics of the model are the same. I therefore skip the detailed derivation of the equations and focus on introducing the differences between the models.

Proposition 2. *Let the small economy have a lower wage than in the rest of the world. If the wage gap is suffciently large, the country with the lower wage specializes in the production of tasks which it exports.*

Firstly I should state what I mean by "sufficiently large wage gap". A sufficiently large wage gap implies that $w^* \geq \tau(J = 1)w_s \geq w$, where w_s is the prevailing wage under specialization. The reason why a country would specialize under this condition is as follows. I assumed that the wage in the rest of the world is w^*. Therefore labour in the small economy can sell tasks to the rest of the world at a price that is slightly below the wage in the rest of the world. In that case firms in the rest of the world would still be willing to offshore tasks. However, labour is only willing to completely specialize in the production of tasks if it does not earn less by doing so. The domestic wage is linked to the return in the research sector by the no-abitrage condition in (9). Therefore, if the return of research is sufficiently

[7] Given the production structure of the final good the output of the latter increases as well.

low, labour can earn a higher income in the export sector and therefore the country specializes completely. This proofs the proposition.

I now analyse how receiving offshored tasks affects the growth with incomplete specialization. The main influence of offshoring in the recipient country is that its domestic labour is diverted from the production process for domestic purpose. Therefore the labour market clearing function is

$$L = L_x^* + L_x + n \tag{17}$$

where L_x^* denotes the labour used to do the tasks that are offshored. Using actual labour demands and rearranging the equation I get

$$x^* = \frac{L - n}{a_x} - a_x x^* \int_0^J \tau(j)dj. \tag{18}$$

Substituting the latter equation into the no-arbitrage condition yields

$$\hat{n} = \frac{\psi L - r}{\lambda + \psi} - \frac{\psi a_x x^* \int_0^J \tau(j)dj}{\lambda^* + \psi} \tag{19}$$

where $\psi \equiv \lambda\gamma\frac{1-\alpha}{\alpha}$. The first term on the rhs is the autarky employment in research. The second term indicates the labour that is diverted from research into the production of the tasks for the source country.

As growth depends positively on the number of people employed in research the recipient country experiences a fall in its growth rate if it is opened up, given the countries are symmetric.

2.3.1 Growth

Analyzing growth I can distinguish between three scenarios. Firstly, if the small economy grows faster than the rest of the world, the catch up process of the small economy is slowed down as the growth rate falls. Secondly, the small economies growth rate is or falls below the growth rate in the rest of the world. In that case the small economy eventually specializes fully. Thirdly, the growth rate in the small economy jumps in par with the world wide growth rate which implies an interior solution with respect to offshoring.

Corollary 2. *If the small economy is specialized in the production of tasks it grows at the same rate as the rest of the world. Growth in this instance is measured in income growth.*

The proof of the corollary follows from the wage setting under complete specialization. The wage in the rest of the world grows at a rate $\tilde{w}^*$. For simplicity I assume that labour in the small economy has set a wage $w_s = \frac{w^*}{\tau(J=1)}$. Log linearizing the latter equation and differentiating it with respect to time yields

$$g = \tilde{w}_s = \tilde{w}^*. \tag{20}$$

The reason is, that if the wage in the rest of the world increase, labour can demand a higher wage without risking to loose jobs. Accordingly, consumption possibilities increase with the wage.

I now look at the growth rate of the small economy with incomplete specilization. The derivation of the growth rate of the small country that does export tasks is similar to country that offshores: I have to measure the increase in output with every innovation and then multiply it by the number of average number of innovations occurring in one period of real time. As I did previously I assume that the wages in the small economy and the rest of the world grow at the same rate. Together with a sufficiently small wage gap this implies incomplete specialization. The growth rate is

$$g = \lambda \hat{n} \ln \gamma. \tag{21}$$

Given that equilibrium employment has declined compared to autarky, growth is harmed in the small economy. The reason is, that offshoring reallocates labour from research towards the production of the tasks. Therefore, less discoveries are made in the research sector in one unit of real time.

In the case of convergence of wages the growth rate is increasing until wages are equalized with the rest of the world. If then the small economy is still growing at a faster rate it starts to offshore to the rest of the world which was analysed previously.

3 Discussion

In this paper I did show, that a country that offshores gains in terms of a higher growth rate. The reason is, that it can utilize labour in the rest of the world to perform some or all of the tasks needed to produce the final good. The small economy can therefore specialize in research which boosts growth.

I did further show that a small economy that receives the offshored tasks can gain from offshoring, but only if it completly specialices in the production of tasks.

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Appendix

Proof of Proposition 1

To proof the proposition I firstly show that $\hat{n} - \hat{n}_a > 0$, where a indicates autarky. Then I show that the output in the final good sector increases as well.

The growth rate of an economy which offshores tasks is higher, if it has a higher employment in research. Therefore,

$$\begin{aligned} \hat{n} - \hat{n}_a &> 0 \\ \frac{\chi\Lambda(J)L - r}{1 + \chi\Lambda} - \frac{\chi L - r}{1 + \chi} &> 0 \\ (\Lambda - 1)(L + r) &> 0 \\ \frac{\int_0^J \tau(j)dj}{\tau(J)(1 - J)}(L + r) &> 0 \end{aligned}$$

where $\chi \equiv \lambda\gamma\frac{1-\alpha}{\alpha}$ and $\Lambda \equiv 1+\frac{\int_0^J \tau(j)dj}{\tau(J)(1-J)}$. The first expression on the right hand side is always positive for $J \in [0, 1)$. In case of $J = 1$ all labour moves to the research sector. This shows that an economy that offshores has a hgiher growth rate.

I now show that the output of the final good increases in every periode as well. The proof is developed by comparing the steady state of the economy with and without offshoring. In order to proof that the output of the intermediate good increase I look at the no-arbitrage condition and rewrite it as follows:

$$r + \lambda n = \frac{\lambda\gamma\tilde{\pi}(\omega)}{\omega}$$

I define the difference between a steady state variable with offshoring and without offshoring as Δ. Using the expression for the profits and differencing the two steady states yields

$$\Delta n = \gamma a_x \frac{1-\alpha}{\alpha}\left(\Omega(J)x - x^a\right) \tag{22}$$

I know from the first part of the proof that the left hand side is positive and hence the right hand side must be positive as well. To see that the right hand side is positive the expression in brackets must be positive. Therefore, $\Omega(J)x > x^a$. Rewriting the latter inequality I obtain the following series of inequalities, where the first inequality sign is shown in the text, $1 > \Omega(J) > \frac{x^a}{x}$. Accordingly the final result follows that the output must increase with offshoring, $x > x^a$.

Figures

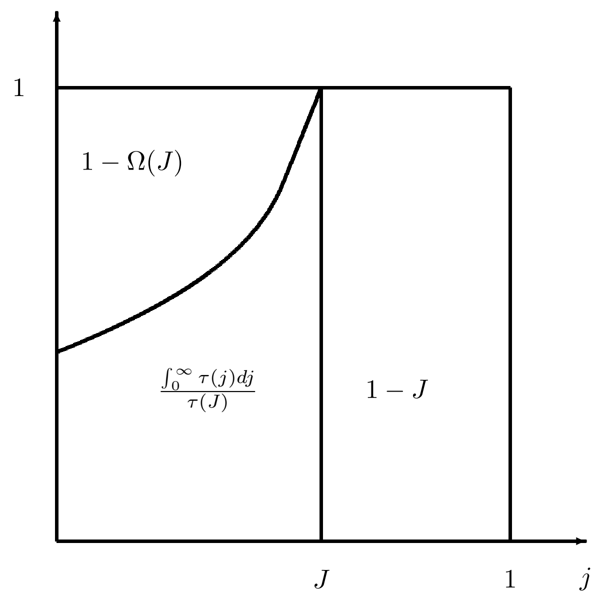

Figure 1: Cost Savings Potential