Trade, Human Capital and Inequality

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1 The Model

There are two countries: A and B. Each country is populated by a continuum of agents of measure 1. Each agent (i for country A and j for country B) is endowed with one unit of labor and some level of human capital, $h_i\,(h_j)$, randomly drawn from the the interval $[1, h_{MAX}]$. Let f_A and f_B denote the density functiions and F_A and F_B the corresponding human capital distribution functions of countries A and B respectively.

There are two goods X and Y. Good X is a primary commodity and each unit produced requires one unit of labor. In contrast, good Y is a hich-tech product and each unit produced requires, in addition, one unit of human capital.

All agents derive utility from the consumption of both goods and they have identical preferences.

1.1 Autarky

In this section, we derive the equilibrium under autarky. Without any loss of generality we concentrate on country A.

1.1.1 Production Possibilities Frontier

The maximum amount of good X that can be produced is equal to 1. Each agent uses her single labor unit endowment to produce one unit of the primary good. The PPF's slope at the point where it intersects the x axis is equal to h_{MAX} as efficiency requires specialization according to comparative advantage. As production of the high-tech product increases the slope of the PPF decreases because the new producers have lower endowments of human capital. The maximum amount of good Y that the economy can produce,

$\hat{h}_A$ is attained when all the agents produce good Y, hence, is equal to the average endowment of human capital. Thus,

$$\hat{h}_A = \int_1^{h_{MAX}} h_i f_A(h) dh$$

At the point where the PPF crosses the y axis its slope is equal to 1.

1.1.2 Equilibrium

Define as p_A the relative price (i.e. the price of good X measured in units of good Y), $q_A(X)$ the quantity produced of good X and $q_A(Y)$ the corresponding quantity of good Y. Then,

Proposition 1 *Equilibrium under Autarky:*

The equilibrium price satisfies $1 < p_A < h_{MAX}$ *and there exists a critical level of human capital endowment,* h_A^**, such that* $p_A = h_A^*$*, all agents with* $h_i < h_A^*$ *produce good* X*, all agents with* $h_i > h_A^*$ *produce good* Y*,* $q_A(X) = \int_1^{h_A^*} f_A(h)dh$*, and* $q_A(Y) = \int_{h_A^*}^{h_{MAX}} hf_A(h)dh$*.*

Notice that agents with human capital endowments equal to h_A^* are indifferent between producing X or Y.

1.1.3 Income Distribution

In order to measure incomes we need a numeraire. Any welfare comparisons before and after a change in relative prices will be affected by the choice of numeraire. However, as long as we are interested in changes in inequality the choice of numeraire is incosequential. Therefore, without any loss of generality we use good Y as the numeraire. Then, for each type of equilibrium we can derive the corresponding income distribution of the economy. Let y_i denote the income of agent i. Then,

Proposition 2 *Income Distribution under Autarky:*

$y_i = h_A^*$ *for all* i *such that* $h_i \leqslant h_A^*$*, and* $y_i = h_i > h_A^*$ *for all* i *such that* $h_i > h_A^*$*. The proportion of agents with income exactly equal to* h_A^* *is given by* $F_A(h_A^*)$ *and the proportion of agents with income less than* h_i $(h_A^* < h_i < h_{MAX})$ *is given by* $1 - F_A(h_A^*)$*.*

Example 1 *Suppose that preferences are described by the utility function:* $U(X,Y) = \gamma \ln X + \delta \ln Y$. *For a given price* p_A*, those agents with* $h_i \geqslant p_A$ *(producers of* Y*) maximize the above utility subject to the budget constraint:* $h_i - p_A X - Y = 0$ *that yields the following demand functions:*

$$X = \frac{h_i}{p_A}\frac{\gamma}{\gamma+\delta},\ Y = h_i\frac{\delta}{\gamma+\delta}$$

while those agents with $h_i \leqslant p_A$ *(producers of* X*) maximize the same utility subject to the budget constraint:* $p_A - p_A X - Y = 0$ *that yields the demand functions:*

$$X = \frac{\gamma}{\gamma+\delta},\ Y = p_A\frac{\delta}{\gamma+\delta}$$

The equilibrium price h_A^* *is such that the supply of* X *(demand for* Y*) is equal to the demand for* X *(supply of* Y*); in other words it satisfies the following equality:*

$$\int_1^{h_A^*} \frac{\delta}{\gamma+\delta} f_A(h)dh = \int_{h_A^*}^{h_{MAX}} \frac{h_i}{h_A^*}\frac{\gamma}{\gamma+\delta} f_A(h)dh$$

Notice that each producer of X *produces* 1 *unit, consumes* $\frac{\gamma}{\gamma+\delta}$ *units and supplies* $\frac{\delta}{\gamma+\delta}$ *units of* X*. Simplifying the above expression we get:*

$$h_A^* = \frac{\gamma}{\delta}\frac{\int_{h_A^*}^{h_{MAX}} h_i f_A(h)dh}{F_A(h_A^*)} = \frac{\gamma}{\delta}\frac{q_A(Y)}{q_A(X)} \tag{1}$$

1.2 Trade

The only way that the two countries differ is in their distributions of human capital endowments. In general, this implies that $h_A^* \neq h_B^*$. Different relative prices imply that there are opportunities for trade.

1.2.1 Global PPF

Using each country's human capital endowments and their identical technologies we can construct a global PPF. The construction of the global frontier follows the same rules as those followed for the costruction of each individual frontier using the global distribution of human capital endowments. The latter distribution denoted by G is given by:

$$G(h) = \frac{F_A(h) + F_B(h)}{2}$$

The maximum quantity of X that the two countries can produce is equal to 2 and the corresponding quantity of good Y is equal to $\hat{h}_A + \hat{h}_B$.

1.2.2 Global Equilibrium

The global equilibrium depends on the relative prices of the two countries.

Proposition 3 *The country with the higher relative price ratio will export good Y and import good X.*

Proof. Without any loss of generality, consider the case where $p_A > p_B$ ($h_A^* > h_B^*$). Consider an agent in country A with endowment $h_A^* - \varepsilon$ (ε small). Under autarky that agent produces one unit of X and consumes any linear combination of one unit of X and h_A^* units of Y. When trade is allowed the same agent will produce $h_A^* - \varepsilon$ units of Y, trades them for $(h_A^* - \varepsilon)\frac{1}{p_B} > 1$ units of X and consumes any linear combination of $(h_A^* - \varepsilon)\frac{1}{p_B} > 1$ units of X and $(h_A^* - \varepsilon)p_A/p_B = (h_A^* - \varepsilon)\frac{1}{p_B}h_A^* > h_A^*$ units of Y. A similar argument shows that an agent in country B with endowment $h_B^* + \varepsilon$ who produced good Y under autarky produces good X under trade. At the global equilibrium there is a cut-off level of human capital endowment, h_G^* ($h_B^* < h_G^* < h_A^*$), such that all agents in both countries with endowments higher than h_G^* produce good Y and all agents with endowmments less than h_G^* produce good X. ■

Example 2 *Suppose that preferences are described by the utility function: $U(X, Y) = \gamma \ln X + \delta \ln Y$. The global realtive price, p^*, is equal to h_G^* and is given by:*

$$h_G^* = \frac{\gamma}{\delta} \frac{\int_{h_G^*}^{h_{MAX}} h_i g(h) dh}{G(h_G^*)}$$

where g is the density of G.

1.2.3 Size

Up to this point, we have assumed that the two countries are of equal size. However, in our model it is simple to derive the global trade equilibrium when the sizes of the two countries are not equal. In this section, we change the population of country B so that it is n times larger than the one of country A. The human capital distributions are the same as above. This implies that now the measure of country B's distribution is equal to n. Notice, as there are not any economies of scale in our model, that this change has no effect on country B's autarky price. It is as if n countries with exactly the same human capital distribution and of equal size attempt to trade. However, there are not any gains from trade because all of them have the same autarky price.

Our analysis in the previous section suggests that in order to find the new global trade equilibrium price we need to derive the global human capital distribution. We denote this new distribution by $G^n(h)$ and is given by:

$$G^n(h) = \frac{F_A(h) + nF_B(h)}{n+1}$$

Lemma 1 *As* $n \longrightarrow \infty$, $G^n(h) \longrightarrow F_B(h)$.

The above lemma states that as the difference in size gets larger the global trade equiliobrium price approaches the atarky price of the larger country. At the limit the small country cannot influence the global trade equilibrium price.

1.2.4 Patterns of Trade

Suppose that the two countries are of equal size and their human capital distributions have the same mean which implies that their aggregate endowmnets are equal. Generally, if the two human capital distributions are different then the autarky prices will be different. This argument demonstrates that aggregate endowments are not accurate predictors of the patterns of trade. Then one might want to know under what conditions the country that has a higher aggregate endowment in human capital will export the human capital intensive good. The answer is complicated and it depends on the specifications of preferences and human capital distribution functions. However, we can prove the following general result.

Proposition 4 *Suppose that preferences are Cobb-Douglas, the sizes of the two countries are equal and let* $F_B(h)$ *dominate* $F_A(h)$ *in the sense of first-order stochastic dominance. Then country* B*, that is the human capital abundant country, will export the human capital intensive good.*

Proof. We need to show that $h_A^* < h_B^*$. First-order stochastic dominance implies that $F_A(h) \geqslant F_B(h)$ and $\int_{h'}^{h_{MAX}} h_i f_A(h)dh < \int_{h'}^{h_{MAX}} h_i f_B(h)dh$ for every h'. Then the inequality follows directly from the autarky price equilibrium condition (1). ■

Next, we present two examples where the human capital abundant country does not export the human capital intensive good. In both cases what matters for trade patterns is not differences in aggregate endowments but differences in the tails of the two distributions. In the first example, country B has a higher aggregate endowment of human capital but country A's distribution has a higher variance that implies that the agents with very high human capital endowments live in country A. Furthermore, there is a strong preference for the primary good. The combination of the above restrictions in the two distributions and the preferences imply that relatively a small amount of resources will be devoted for the production of the high-tech product and the majority will be provided by country A.

Example 3 *Suppose that agents in both countries have preferences described by the utility function used in example 1 with* $\frac{\gamma}{\delta} = 2$. *In addition, let the human capital distribution functions of both countries have a uniform density. The support of country A's distribution is the interval* $[1, 2]$ *while the support for country B's distribution is the interval* $[1.3, 1.8]$. *Notice that* $\hat{h}_A = 1.5 < 1.55 = \hat{h}_B$, *so that country B has a higher human capital aggregate endowment than country A. Using the equilibrium condition under autarky given by equation (1), we find that* $h_A^* = 1.69 > 1.64 = h_B^*$. *Then proposition 3 implies that the human capital abundant country will not export the human capital intensive good.*

In the second example, country A has a higher aggregate endowment and its distribution has a higher variance that implies that the agents with very low human capital endowments live in country A. Furthermore, there is a strong preference for the high-tech product. The combination of the above restrictions in the two distributions and the preferences imply that relatively a small amount of resources will be devoted for the production of the primary good and the majority will be provided by country A.

Example 4 *Suppose that agents in both countries have preferences described by the utility function used in example 1 with* $\frac{\gamma}{\delta} = \frac{1}{2}$. *In addition, let the human capital distribution functions of both countries have a uniform density. The support of country A's distribution is the interval* $[1, 2]$ *while the support for country B's distribution is the interval* $[1.2, 1.7]$. *Notice that*

$\hat{h}_A = 1.5 > 1.45 = \hat{h}_B$, *so that country A has a higher human capital aggregate endowment than country B. Using the equilibrium condition under autarky given by equation (1), we find that* $h^*_A = 1.21 < 1.29 = h^*_B$. *Again, proposition 3 implies that the human capital abundant country will not export the human capital intensive good.*

1.2.5 Trade and Inequality

We compare the income distributions of each country under autaky and after trade. In this section, we are only interested in changes in inequality and thus the choice of numeraire does not matter.

Proposition 5 *Trade increases inequality in the country that exports the high-tech product and reduces inequality in the country that exports the primary commodity.*

Proof. Without any loss of generality, assume that $h^*_B < h^*_G < h^*_A$. therefore at the global equilibrium country A exports the high-tech product while country B exports the primary commodity. Using again good Y as the numeraire, we observe that, after trade, country A's income distribution is as follows: $y_i = h^*_G$ for all i such that $h_i \leqslant h^*_G$, and $y_i = h_i > h^*_G$ for all i such that $h_i > h^*_G$. Comparing this distribution to the corresponding one obtained under autarky we find that all agents with $h_i \leqslant h^*_G$ (proportion equal to $F_A(h^*_G)$) have experienced a decrease in income equal to $h^*_A - h^*_G$, those agents with $h^*_G < h_i < h^*_A$ (proportion equal to $F_A(h^*_A) - F_A(h^*_G)$) have experienced a decrease in income equal to $h^*_A - h_i$, while the income of the rest of the agents (proportion equal to $1 - F_A(h^*_A)$) has remained the same. Therefore, the poor have experienced the greatest relative loss in income, the loss of the middle-income group has been more moderate, while the incomes of those agents in the high-income group has remained unchanged. Similarly, comparing country B's after trade income distribution to the corresponding one obtained under autarky we find that the income of the poor (proportion equal to $F_B(h^*_B)$) has increased by $h^*_G - h^*_B$, the income of those in the middle income group (proportion equal to $F_B(h^*_G) - F_B(h^*_B)$) has increased by $h^*_G - h_i$, while the incomes of those agents in the high-income group (proportion equal to $1 - F_B(h^*_G)$) has remained unchanged. ■

The World Income Distribution Under the assumption that the populations of the two countries are equal the world income distribution under autarky is given by:

Proportion of Agents	Income
$\frac{F_B(P_B)}{2}$	P_B
$\frac{F_B(P_A)-F_B(P_B)}{2}$	$P_B < h_i < P_A$
$\frac{F_A(P_A)}{2}$	P_A
$1 - \frac{F_B(P_A)+F_A(P_A)}{2}$	$P_A < h_i < h_{MAX}$

The post trade world income distribution is given by:

Proportion of Agents	Income
$\frac{F_B(P^*)+F_A(P^*)}{2}$	P^*
$1 - \frac{F_B(P^*)+F_A(P^*)}{2}$	$P^* < h_i < h_{MAX}$

1.2.6 Trade and Welfare

In this section, using an example, we are going to demonstrate that trade does not necessarily enhance social welfare. We are going to measure welfare by using the following social welfare function:

$$W(h,p) = \int_1^{h_{MAX}} U(X(h,p), Y(h,p))f(h)dh \tag{2}$$

Let p^a denote the equilibrium relative price under autarky. If trade is welfare improving then the following must be true:

$$p^a = \arg\min \left\{ \int_1^{h_{MAX}} U(X(h,p), Y(h,p))f(h)dh \right\}$$

We can prove the following result:

Proposition 6 *Suppose that the preferences of agents are described by the utility function $U(X,Y) = \gamma \ln X + \delta \ln Y$ (with $\gamma + \delta = 1$). Then there exists a set of prices that are less than p^a such that if the country trades at those prices its welfare will decrease.*

Proof. Using (2), social welfare is given by:

$$\int_1^p \gamma \ln \gamma + \delta \ln(\delta p) f(h)dh$$
$$+ \int_p^{h_{MAX}} \left(\gamma \ln \left(\gamma \frac{h}{p} \right) + \delta \ln (\delta h) \right) f(h)dh$$

The f.o.c. condition for a minimum is given by:

$$F(p) - \gamma = 0 \tag{3}$$

Notice that the s.o.c. is also satisfied. (3) in (1) we find that if the two prices are the same then the following equality must be true:

$$p^a = \frac{1}{\delta} \int_{p^\alpha}^{h_{MAX}} h_i f_A(h) dh$$

However $\delta < 1$ which implies that the price that minimizes social welfare is less than p^a. Then, by continuity there exists a number p' such that if the global equilibrium price belongs to the interval (p', p^α) then trade will reduce welfare. ■

Example 5 *Suppose that the preferences of agents are described by the utility function used in example 1 with $\gamma = \delta = 1$. In addition, let its human capital distribution function have a uniform density with support in the interval $[1, 2]$. Using the equilibrium condition under autarky given by equation (1), we find that $h_A^* = 1.53518$. Using the derivations in example 1, we find that*

$$X(h) = \frac{1}{2},\ Y(h) = \frac{1.53518}{2} \quad for\ h \leqslant 1.53518$$

and

$$X(h) = \frac{1}{2}\frac{h}{1.53518},\ Y(h) = \frac{h}{2} \quad for\ h \geqslant 1.53518$$

Substituting the above solutions to the social welfare function we find that under autarky social welfare is equal to:

$$\begin{aligned} & \int_1^{1.53518} \left(\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1.53518}{2}\right)\right) dh \\ & + \int_{1.53518}^{2} \left(\ln\left(\frac{1}{2}\frac{h}{1.53518}\right) + \ln\left(\frac{h}{2}\right)\right) dh \\ = \ & -0.82927 \end{aligned}$$

From (3) we find that social welfare is minimized when the global price, $p^* = 1.5$. In fact, at this equilibrium price social welfare is equal to:

$$\begin{aligned} & \int_1^{1.5} \left(\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1.5}{2}\right)\right) dh \\ & + \int_{1.5}^{2} \left(\ln\left(\frac{1}{2}\frac{h}{1.5}\right) + \ln\left(\frac{h}{2}\right)\right) dh \\ = \ & -0.83011 \end{aligned}$$

Notice that the social welfare under autarky is higher than the social welfare under trade at a global equilibrium price equal to 1.5.[1]

The following result is a direct consequence of proposition 6.

Corollary 1 *If trade reduces welfare then it increases inequality.*

1.2.7 Gains of Trade

In the previous section we have demonstrated that moving from autarky to trade without compensating those whose welfare is reduced by such a move can be welfare reducing. Of course, we would expect that if trade is accompanied by an appropriate income redistribution that it will be not only welfare enhancing but also Pareto-improving. In this section, we are going to show these gains for the case of Cobb-Douglas preferences.

Proposition 7 *If preferences are Cobb-Douglas then trade always Pareto dominates autarky if it is accompanied by an appropriate income redistribution.*

Proof. Let p_A and h_A denote the equilibrium price and income under autarky and p_T and h_T the equilibrium price and income under trade. We are going to prove the proposition for the case $p_A > p_T$. The proof when $p_A < p_T$ is similar. Let γ denote the share of income spent on the primary commodity. Then we can write the indirect utility function under autarky as

[1] Similar results can be obtained for other specifications of preferences. For example, suppose that the preferences of agents are described by the following CES utility function: $U(X,Y) = X^{1/2} + Y^{1/2}$. Then, those agents with $h_i < p_A$ have demand functions $X = \frac{p_A}{1+p_A}$ and $Y = \frac{(p_A)^2}{1+p_A}$, while those agents with $h_i > p_A$ have demand functions $X = \frac{h_i}{p_A(1+p_A)}$ and $Y = \frac{h_i p_A}{1+p_A}$. The equilibrium price under autarky is given by the solution of

$$F(p_A) = \frac{1}{(p_A)^2} \int_{p_A}^{h_{MAX}} h_i f(h) dh$$

while the price that minimizes social welfare is given by the solution of

$$F(p) = \frac{1}{(p)^{\frac{3}{2}}} \int_{p_A}^{h_{MAX}} (h_i)^{1/2} f(h) dh$$

When the human capital distribution has a uniform density with support in the interval $[1,2]$, $p_A = 1.45054$ while the price that minimizes social welfare is equal to 1.43279.

$$V_A = h_A p_A^{-\gamma} \tag{4}$$

and the indirect utility function under trade as:

$$V_T = h_T p_T^{-\gamma} \tag{5}$$

Since preferences are homothetic a change in income at any given price level will not affect the shares of income spent on each good. Suppose that after trade we adjust each agent's income so that their post-adjustment indirect utility is equal to their indirect utility under autarky. Let $\tau(h_T)$ denote the tax (subsidy if negative) imposed on an agent whose post-trade income is equal to h. By definition the tax (subsidy) must satisfy the following equality:

$$V_T - \tau(h_T) = V_A, \forall h$$

Substituting (4) and (5) in the above expression and rearranging we find that the tax (subsidy) must satisfy

$$\tau(h_T) = h_T - h_A \left(\frac{p_A}{p_T}\right)^{-\gamma} \tag{6}$$

In order to prove the proposition we need to show that

$$\int_1^{h_{MAX}} \tau(h_T) f(h) dh > 0 \tag{7}$$

In words aggregate tax revenues must be higher than aggregate subsidy expenditures which implies that the tax revenues that can be raised from those agents whose welfare improves under trade is higher than the total amount of subsidies offered to those agents whose welfare deteriorates. Substituting (6) in the left-hand side of (7) we get:

$$\begin{aligned}
& \int_1^{p_T} \left(p_T - p_A \left(\frac{p_A}{p_T}\right)^{-\gamma}\right) f(h) dh + \int_{p_T}^{p_A} \left(h - p_A \left(\frac{p_A}{p_T}\right)^{-\gamma}\right) f(h) dh \\
& + \int_{p_A}^{h_{MAX}} \left(h - h \left(\frac{p_A}{p_T}\right)^{-\gamma}\right) f(h) dh \\
= \ & p_T F(p_T) + \int_{p_T}^{h_{MAX}} h f(h) dh - p_A \left(\frac{p_A}{p_T}\right)^{-\gamma} F(p_A) \\
& - \left(\frac{p_A}{p_T}\right)^{-\gamma} \int_{p_A}^{h_{MAX}} h f(h) dh
\end{aligned}$$

Next we multiply the last expression by $(p_T)^{-\gamma}$, which does not affect its sign, and get

$$(p_T)^{-\gamma}\left(F\left(p_T\right)+\int_{p_T}^{h_{MAX}} hf(h)dh\right)-(p_A)^{-\gamma}\left(F\left(p_A\right)+\int_{p_A}^{h_{MAX}} hf(h)dh\right)$$

The proof will be complete if we can show that the function

$$p^{-\gamma}\left(F\left(p\right)+\int_{p}^{h_{MAX}} hf(h)dh\right)$$

achieves a minimum at $p = p_A$. The f.o.c. requires that

$$p^{-\gamma}\left(\left(1-\gamma\right)F\left(p\right)-\gamma\frac{1}{p}\int_{p}^{h_{MAX}} hf(h)dh\right)=0$$

Since $p > 0$ the expression inside the brackets must be equal to 0 and (1) implies that this is indeed the case when $p = p_A$. The s.o.c. evaluated at $p = p_A$ is equal to

$$(p_A)^{-\gamma}\left(\left(1-\gamma\right)f(p_A)+\gamma(p_A)^{-2}\int_{p_A}^{h_{MAX}} hf(h)dh\right)+\gamma f(p_A)>0$$

which completes the proof. ■

1.2.8 Trade and Politico-Economic Equilibrium

Up to this point we have assumed that there is no redistribution of income to compensate those agents that their utility under trade is lower than their utility under autarky. In this section, we are going to demonstrate that such policies might be ruled out in a politico-economic equilibrium. We assume that a rule of majority voting decides both (a) the choice between autarky and trade, and (b) any redistribution policies.

We begin with country A where the equilibrium price under autarky is higher than the global equilibrium price under trade. We know that the welfare of all those agents with human capital endowments such that $h_i > h_A^*$ is higher under trade and the welfare of all agents with human capital endowments such that $h_i < h_G^*$ is lower under trade. Since utility is wealky monotonic in endowments it implies that for those agents, with human capital endowments such that $h_G^* < h_i < h_A^*$ there exists a threshold level of endowment $\tilde{h}$ such that the welfare of all agents with human capital endowments such that $h_G^* < h_i < \tilde{h}$ is lower under trade and the welfare of all agents with h uman capital endowments such that $\tilde{h} < h_i < h_A^*$ is higher under trade.

Let h^m denote the human capital endowment of the median voter; i.e. $F_A(h^m) = 0.5$. Then, we can show the following:

Proposition 8 *Characterization of Politico-Economic Equilibria for $h_A^* > h_G^*$:*

Type I: $\tilde{h} > h^m$ *(Trade with redistribution)*
Type II: $\tilde{h} < h^m$ *(Trade without redistribution)*

Proof. Type I: The welfare of the majority of voters is reduced under trade.

Type II: The welfare of the majority of voters is improved under trade.
■

Next, we consider country B where the equilibrium price under autarky, h_B^* is lower than the global equilibrium price under trade. in this case, we know that the welfare of all those agents with human capital endowments such that $h_i < h_B^*$ is higher under trade and the welfare of all agents with human capital endowments such that $h_i > h_G^*$ is lower under trade. Again since utility is weakly monotonic in endowments it implies thta for those agents, with human capital endowments such that $h_G^* > h_i > h_B^*$ there exists a threshold level of endowment $\tilde{h}$ such that the welfare of all agents with human capital endowments such that $h_G^* > h_i > \tilde{h}$ is lower under trade and the welfare of all agents with human capital endowments such that $\tilde{h} > h_i > h_B^*$ is higher under trade.

Proposition 9 *Characterization of Politico-Economic Equilibria for $h_B^* < h_G^*$:*

Type I: $\tilde{h} > h^m$ *(Trade without redistribution)*
Type II: $\tilde{h} < h^m$ *(Trade with redistribution)*

Proof. Type I: The welfare of the majority of voters is improved under trade.

Type II: The welfare of the majority of voters is reduced under trade. ■

Example 6 *Suppose that the preferences of agents are described by the utility function used in example 1. In this example $h^m = 1.5$. Next, we find $\tilde{h}$.*

An agent with $h_i = \tilde{h}$ must be indifferent between autarky (where she produces the primary good) and trade (where she produces the high-tech product). Therefore, $\tilde{h}$ must satisfy:

$$\gamma \ln \left(\frac{\gamma}{\gamma + \delta} \right) + \delta \ln \left(h_A^* \frac{\delta}{\gamma + \delta} \right) = \gamma \ln \left(\frac{\tilde{h}}{h_G^*} \frac{\gamma}{\gamma + \delta} \right) + \delta \ln \left(\tilde{h} \frac{\delta}{\gamma + \delta} \right)$$

(notice that the right-hand side that denotes welfare under trade is increasing in $\tilde{h}$) which implies that

$$\delta \ln \frac{h_A^*}{\tilde{h}} = \gamma \ln \frac{\tilde{h}}{h_G^*}$$