HOW MULTILATERAL RESISTANCE AND FIRM HETEROGENEITY AFFECT TRADE ELASTICITIES*

Alberto Behar† Benjamin D. Nelson‡

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PRELIMINARY AND INCOMPLETE: COMMENTS WELCOME!

Abstract

We derive, conduct comparative statics, estimate and perform simulations on a gravity equation that unites two recent strands of the literature on international trade: that accounting for multilateral resistance, and that accounting for firm heterogeneity and country-selection into trade. Ignoring either can lead to erroneous results. Key implications include: (a) If all countries reduce their trade frictions, the impact of multilateral resistance is so strong that bilateral trade falls in many cases. (b) Elasticities at the extensive margin of trade are larger for smaller countries, which undermines the positive relationship between country size and trade responses suggested by multilateral resistance effects alone. Comparative statics must therefore account for both multilateral resistance and firm heterogeneity for multilateral changes in trade costs. (c) For isolated bilateral changes in trade frictions, multilateral resistance effects are small for most countries' elasticities.

JEL Classifications: F10, F12, F14, F17
Key Words: Gravity models, multilateral resistance, firm heterogeneity

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†CSAE and Department of Economics, University of Oxford. Email: alberto.behar@economics.ox.ac.uk
‡Nuffield College and Department of Economics, University of Oxford. Email: ben.nelson@nuffield.ox.ac.uk

1 Introduction

For almost half a century, economists have used gravity models to explain bilateral trade flows. Such models typically specify exports from one country to another as proportional to GDP and inversely proportional to trade frictions. Frictions typically include geographical features like distance or policy variables like free-trade agreements. Gravity models continue to be employed to estimate the elasticity of trade flows with respect to frictions, the approach having been subject to a number of theoretical and empirical refinements. In this paper, we derive a gravity equation that unites two recent strands of the literature: that stressing the importance of multilateral resistance and that stressing the importance of firm heterogeneity. To understand their impact on trade elasticities, we use the model to conduct comparative statics. After estimating the model parameters, we perform simulations to illustrate the comparative static results with real data.

The first strand on which this paper draws is that following Anderson and van Wincoop (2003) (henceforth AvW). In their paper, the authors solve the so-called 'border puzzle' – the apparently large negative effect of the US-Canadian border on trade between US states and Canadian provinces highlighted by McCallum (1995).[1] AvW do this by showing that traditional gravity equations, while empirically 'successful', consider only the impact of *bilateral* trade costs on trade flows, omitting the fact that countries operate in a *multilateral* world. As such, traditional estimates suffer from omitted variable bias. This is the failure to control for theoretically motivated price index terms, which they call *multilateral resistance* (MR). AvW show that bilateral trade flows depend on bilateral trade costs *relative to those incurred by trading with the rest of the world*, which is captured by MR. In general, a failure to account for MR effects leads one to overstate the importance of changes in trade barriers on bilateral trade flows.

In particular, for small countries, which typically trade large fractions of their output internationally, external trade barriers have a large impact on their multilateral resistance terms. Hence an increase in external trade costs increases a small country's MR by relatively more. This *dampens* the negative effect of an external border on bilateral trade flows for a small country. The reason is that the increase in bilateral trade costs *relative to MR* resulting from the border is smaller. For comparative statics to be valid, modelling the general equilibrium effects of MR is therefore necessary *a priori*.

The second strand of the literature on which this paper draws is Helpman, Melitz and Rubinstein (2008) (henceforth HMR). As reviewed in Bernard, Jensen, Redding & Schott (2007), heterogeneous firm productivities within countries means not all firms may engage in exporting in the presence of fixed costs of trade.[2] One simple reason for this is that not all firms will have a productivity level high enough to generate profits sufficient to cover the fixed costs of exporting. It further follows that, if fixed costs are high enough, *no* firms in a given country may find it profitable to export. Hence in the presence of fixed costs of trade, 'zeros' naturally arise in the trade data; these are countries for which exports to a particular destination are zero.[3] In HMR's sample, the proportion of countries that do not trade with each other or trade in only one direction is around half of all observations.

A corollary of this is that, in addition to making existing exporters export more, a fall in trade costs can also make ex ante non-exporters choose to export ex post. These two effects are referred to as the *intensive* and the *extensive* margins respectively. HMR argue that failure to account for firm heterogeneity causes standard estimates of gravity equation parameters to be erroneous.

Using bilateral (country-level) trade data, their remedy is to estimate the predicted probability that at least one firm will export from one country to another (i.e. that country-level exports are positive). They then use this estimate to construct two controls; one for the selection of country pairs into trading, and another to control for the proportion of firms in a country that export (the extensive margin). Together with fixed effects, which control for MR in estimation, including these two controls allows HMR to obtain consistent estimates of the coefficients in their gravity equation.

We argue that one needs to take both AvW and HMR's findings into account, otherwise interpretation of the effects of trade frictions on trade flows will be misleading. We therefore unite these two strands of the literature. We derive a theoretically grounded gravity equation and then extend a method of approximating

[1] AvW obtain an estimate of this effect of between 20-50%, much more plausible than the previous finding that the border led to trade between Canadian provinces that was a factor of 22 times (2,200%) larger than that between Canadian provinces and US states.

[2] Heterogeneity in productivity has traditionally referred to efficiency, but can also be interpreted as quality (see Johnson, 2008; Baldwin & Harrigan, 2007).

[3] High fixed costs of exporting from country j to country i do not imply that the absence of trade in the opposite direction, from i to j. These *asymmetric* trade flows are also a salient feature of the data that can be accommodated by modeling firm heterogeneity.

MR terms, developed by Baier and Bergstrand (2009) (henceforth BB), to the case of firm heterogeneity. This approach uses a first-order Taylor expansion to approximate the relevant price index (or MR) terms, and results in an expression for a trader's multilateral resistance which is a linear function of its trade frictions with each of its trading partners, weighted by the partner's GDP share. This allows us to work with tractable MR terms, together with HMR's heterogeneity variable, for comparative statics.

We replicate the estimation exercise in HMR (providing a step-by-step guide to implementation) and begin with a partial equilibrium simulation exercise which does not account for MR. For all our observations, traditional linear estimates bias *downwards* the effect of observable trade barriers on *country-level* trade flows. This difference, rather than the firm-level bias in the opposite direction highlighted by HMR, is arguably more relevant for policy. Furthermore, larger countries have smaller elasticities at the extensive margin and hence lower country-level responses.

To analyze the general equilibrium effects of MR, we use the data to conduct simulations in which (i) all countries reduce their trade frictions (multilateral changes) and (ii) only two countries do so (bilateral changes). Consistent with AvW's 'Implication 1', larger countries have larger *firm-level* elasticities of bilateral trade in response to multilateral changes in trade costs. However we show that, once firm entry into trade is accounted for, this is no longer unambiguously true in theory for overall elasticities at the country level. Moreover, on balance we find a *negative* correlation in the data between country size and bilateral trade elasticities once changes in the extensive margin are accounted for. This is because smaller countries tend to have larger elasticities at the extensive margin, which we find empirically to be sufficient to override the larger dampening effect experienced by smaller countries resulting from MR. We explore theoretically the reason for this interaction between country size and both the effects of MR and the effects of changes in trade costs through the extensive margin.

Furthermore, consistent with AvW, we find the effects of ignoring MR to be dramatic for multilateral changes in trade costs. After accounting for MR, bilateral trade responses are much lower. Many elasticities are negative, which means that the general equilibrium effects are so strong that many country pairs reduce their bilateral trade after a multilateral reduction in trade frictions. Because the results are so dramatic and because the interactions between heterogeneity and MR are idiosyncratic, accounting for both is of paramount importance.

We also consider bilateral changes in trade costs. In such a case, for two average size countries liberalizing trade in isolation, it is intuitive that the general equilibrium effects will be much smaller. We show that for average size countries active in international trade, this is indeed the case, it being safe to ignore MR in comparative statics. By contrast, for big country pairs, within the G7 perhaps, MR has a material impact. An implication of firm heterogeneity and zeros in the trade data however is that a country's MR should be defined over the *active set of traders* with that country, not all traders in the world. This implies that it is difficult to generalize the *a priori* intuition that general equilibrium effects will be small for most country pairs. In particular, for a small exporter with few export destinations, a big country could have a "very" big GDP as a share of all export destinations. This would apply for example when a poor, remote African country enters a trade deal with a large country like the US or Japan, or region like the EU. We demonstrate how it is theoretically and practically appropriate to account for MR in cases such as these.

Table 1 acts as a reference point for our main results and places our contributions in context. The body of the paper proceeds as follows: In the next section we outline the model, before deriving tractable linear multilateral resistance terms in section 3. We present the comparative statics analytically in section 4. In section 5, we describe and implement the estimation strategy of HMR before conducting a simulation based on a fall in distance in the absence of MR. Section 6 shows the effects of allowing for MR and section 7 concludes.

2 Theory

Adopting the exposition in HMR, there are J countries, $j = 1, ..., J$. Within each country are monopolistically competitive firms which produce a continuum of differentiated products. Consumers have a 'taste for variety', embodied in standard CES preferences, given by

$$u_j = \left[\int_{l \in B_j} x_j(l)^{\alpha} dl \right]^{\frac{1}{\alpha}} \tag{1}$$

where $x(l)$ is consumption of variety l, contained in the set of varieties available in j, B_j. Let $\sigma \equiv 1/(1-\alpha)$ be the elasticity of substitution. With income in j of Y_j, firms face demand of

$$x_j(l) = \frac{Y_j}{P_j^{1-\sigma}} p_j(l)^{-\sigma} \tag{2}$$

where $p_j(l)$ is the price of variety l in j and P_j is j's ideal price index, given in the standard way to be $P_j = \left[\int_{l \in B_j} p_j(l)^{1-\sigma} dl\right]^{\frac{1}{1-\sigma}}$. Note that here it is defined over the set of available goods B_j, which constitutes the set of goods imported by j from active exporters.

Each country produces N_j varieties, with one variety per firm. As introduced by Melitz (2003), the unit cost of production by a firm with unit input requirement a is $c_j a$. a is firm specific, while c_j reflects the cost of factors of production in country j. Assume firms draw a independently from the identical distribution function $G(a)$ with support $[a_L, a_H]$, such that a_L is the lower bound on possible unit input requirement draws, while a_H is the upper bound. We can identify each firm's unique variety l with its cost level a.

We assume two types of cost of exporting. The first is an 'iceberg' variable trade cost $t_{ij} > 1$. The second is a fixed cost of exporting $f_{ij} > 0$, $f_{ii} = 0$. Taken together, a firm in j exporting to i producing q_{ij} units of output has a cost function given by

$$C_{ij}(a) = a t_{ij} c_j q_{ij} + c_j f_{ij} \tag{3}$$

Given demand and costs, each firm chooses price so as to maximize its profits. This gives the standard price and profit function for a firm exporting from j to i as

$$p_j(a) = \frac{t_{ij} c_j a}{\alpha} \tag{4}$$

$$\pi_{ij}(a) = (1-\alpha)\left[\frac{t_{ij} c_j a}{\alpha P_i}\right]^{1-\sigma} Y_i - c_j f_{ij} \tag{5}$$

Sales by firms in country j are only profitable in country i if $\pi_{ij}(a) > 0$. Hence we define a productivity cut-off a_{ij} by $\pi_{ij}(a_{ij}) = 0$, which is the inverse productivity level (or cost level) below which it is profitable to export. Firms with $a > a_{ij}$ do not generate profits high enough to cover the fixed costs of exporting f_{ij}. Using an exporting firm's profit function above then gives us the cut-off as

$$a_{ij} = \left[\frac{Y_i(1-\alpha)}{f_{ij} c_j}\right]^{\frac{1}{\sigma-1}} \frac{\alpha P_i}{c_j t_{ij}} \tag{6}$$

This gives us the *extensive margin* of trade. When a_{ij} is higher, the extensive margin is greater, implying a larger subset of firms export. It rises as the income of the importing country rises, and as both fixed and variable costs of trade fall. Whenever $a_{ij} < a_H$, there will be *firm selection into exporting*. In particular, firms with the highest costs will choose not to export.

The total value of imports by country i from country j is given by $M_{ij} = \int_{a_L}^{a_{ij}} p_j q_i N_j dG(a)$. Substituting in for prices and quantities, we obtain

$$M_{ij} = \left[\frac{t_{ij}}{\alpha P_i}\right]^{1-\sigma} N_j Y_i \int_{a_L}^{a_{ij}} a^{1-\sigma} dG(a) \tag{7}$$

We then define $V_{ij} \equiv \int_{a_L}^{a_{ij}} a^{1-\sigma} dG(a)$ as a term capturing the firm selection effect. Note that as a_{ij} rises, indicating that the cost level above which firms find it unprofitable to export rises, V_{ij} rises. In other words, as this export cut-off rises, a larger set of firms exports. Using this, we have bilateral exports from j to i given by

$$M_{ij} = \left[\frac{c_j t_{ij}}{\alpha P_i}\right]^{1-\sigma} N_j Y_i V_{ij} \tag{8}$$

V_{ij} in equation (8), which is the same as in HMR, forms the basis for accounting for firm heterogeneity. It is the omission of this term which leads to erroneous estimates of the impact of trade barriers on firm level trade.

For some countries, trade costs will be high enough to prohibit exporting to particular locations. This arises naturally in a model with fixed costs of exporting. It means however that it becomes useful to define two types of set of countries. First, we define J_i as the set of all exporters to i. In particular, if country j exports to i, we say that it is contained in the set J_i. Similarly, we define I_j as the set of all importers from j. So analogously, if i imports from j, it is contained in I_j. Importantly, we allow for asymmetries in trade flows by allowing for $J_i \neq I_j$. This says that the set of exporters to country i is not necessarily equal to the set of importers from country j. In particular, we allow for $J_k \neq I_k$, which says that the set of exporters to country k is not necessarily equal to the set of importers from country k; k could import from j, but not export back.

An important implication of this is that the price indices are defined over these sets. More explicitly, for country i it is given by $P_i^{1-\sigma} = \sum\limits_{j \in J_i} \int\limits_{a_L}^{a_{ij}} p(a)^{1-\sigma} dG(a)$. Using $p(a) = c_j a t_{ij}/\alpha$ the price index can be written

$$P_i^{1-\sigma} = \sum_{j \in J_i} (c_j t_{ij}/\alpha)^{1-\sigma} N_j V_{ij} \tag{9}$$

such that it is defined over the set of exporters to i. AvW interpret the price indices in the gravity equation as multilateral resistance terms, which we discuss further below. It is important to note however that firm heterogeneity and hence possible country selection into trade imply that multilateral resistance terms must be defined over the set of active traders.

To get closer to AvW's system, we assume trade balance in order to achieve general equilibrium closure. Specifically, assume output equal to income for each country, such that $Y_j = \sum\limits_{i \in I_j} M_{ij}$. In the appendix, we show that using this in (8) allows one to write

$$M_{ij} = \frac{Y_i Y_j}{Y^{I_j}} \left(\frac{t_{ij}}{P_i \widehat{P}_j} \right)^{1-\sigma} V_{ij} \tag{10}$$

where P_i is country i's *importing* multilateral resistance term, $\widehat{P}_j$ is country j's *exporting* multilateral resistance term, and $Y^{I_j} \equiv \sum_{k \in I_j} Y_k$ is the total output of the set of importers from j, I_j. In arriving at this equation, using bilateral trade balance allows one to write these price indices as

$$P_i^{1-\sigma} = \sum_{j \in J_i} \left(\frac{t_{ij}}{\widehat{P}_j} \right)^{1-\sigma} s_j^{J_i} V_{ij} R_{ij} \tag{11}$$

$$\widehat{P}_j^{1-\sigma} = \sum_{i \in I_j} \left(\frac{t_{ij}}{P_i} \right)^{1-\sigma} s_i^{I_j} V_{ij} \tag{12}$$

We use the 'hat' to denote *exporting* multilateral resistance, in order to distinguish it from *importing* multilateral resistance[4]. In these price index equations, the s_k^X terms represent country k's GDP as a share of the total output of all the countries in set X. That is, we define $s_j^{J_i} \equiv Y_j / \sum_{k \in J_i} Y_k$ as exporter j's GDP as a share of the total output of all exporters to i. Similarly, $s_i^{I_j} \equiv Y_i / \sum_{k \in I_j} Y_k$ is importer i's GDP as a share of the total output of all importers from j.

In arriving at the above gravity equation, we have defined Y^{I_j} as the total output of all importers from country j. It is intuitive that as this quantity rises, it becomes relatively less attractive for firms in j to export to i, so that bilateral trade falls. Finally, $R_{ij} \equiv Y^{J_i}/Y^{I_j}$ is the output of exporters to i relative to that of importers from j. When it is high, relatively many exporters trade with i, indicating an increase in product variety, and hence a reduction in importer i's price index.

The inclusion of two price terms makes system (10)-(12) resemble that of AvW, with the crucial difference that it allows for firm heterogeneity and country selection[5]. A corollary of this is that the multilateral resistance terms in AvW's equation become asymmetric in our equation. Nonetheless, the point made by

[4] Anderson (2009) refers to these as indices of sellers' and buyers' trade cost incidence respectively. This distinction is necessary given asymmetries in trade flows.

[5] It also preserves the potential for asymmetries trade flows, which is precluded by the 'decomposability assumption' made by HMR in deriving a similar equation contained in their appendix.

AvW that, when performing comparative statics, multilateral resistance must be accounted for, still stands. In particular, reductions in trade costs affect both the numerator and the denominator of the gravity equation. For example, when t_{ij} falls, bilateral trade increases, but this increase can be dampened if the reduction in t_{ij} also affects P_i and/or $\widehat{P}_j$. Where a reduction in t_{ij} also reduces either or both of these multilateral resistance terms, the resulting increase in bilateral trade will be smaller than in the absence of changes in multilateral resistance, all else equal.

We wish to include these general equilibrium multilateral resistance effects in our simulations. One way to do this is to construct the system of nonlinear price index equations and estimate the system following AvW. This method is computationally demanding however, especially when considering more than 150 countries, allowing for asymmetries, and for GDP shares that are specific to sets of active traders. We therefore follow BB in taking a first order Taylor approximation of the multilateral resistance terms, and use these to compute comparative statics. This approach has a number of advantages. First, as noted by BB, the parameters of the multilateral resistance terms, once approximated, are observable. Second, we can use (linear) fixed effects for estimation, but use our Taylor approximations for comparative statics. Third, the Taylor expanded multilateral resistance terms are highly tractable, and so offer a good intuition for the effects obtained from the comparative static exercises we subsequently consider. Against these advantages, of course, we should remember that what we obtain are linear approximations of non-linear price index equations.

3 Multilateral Resistance

In approximating the terms P_i and $\widehat{P}_j$, we use a first order Taylor expansion around a world of symmetric trade frictions. In this symmetric world, we imagine that all countries trade with each other. In this context, the real world, represented by our data, is a perturbation from this centre. For estimation and comparative statics, we will work with the log form of the gravity equation (10). This requires us to derive an expression for $\ln P_i + \ln \widehat{P}_j$. In the appendix, we show that this procedure allows us to obtain the expressions

$$\ln P_i = -\sum_{j \in J_i} s_j^{J_i} \ln \widehat{P}_j + \sum_{j \in J_i} s_j^{J_i} \ln t_{ij} V_{ij}^{\frac{1}{1-\sigma}} R_{ij}^{\frac{1}{1-\sigma}} \tag{13}$$

and

$$\ln \widehat{P}_j = -\sum_{i \in I_j} s_i^{I_j} \ln P_i + \sum_{i \in I_j} s_i^{I_j} \ln t_{ij} V_{ij}^{\frac{1}{1-\sigma}} \tag{14}$$

The first of these, equation (13), is an approximation of importer i's multilateral resistance and, as above, is defined over the set of exporters to i, J_i. The first term on the right hand side of (13) is *world* exporter MR, that is, a weighted average of the MRs faced by all exporters to i. When all exporters face high MR, implying that exporting is hard in general, exporting to i is replaced by domestic trade. Hence when the first term is large, P_i is small, lowering bilateral trade flows *ceteris paribus*. The second term on the right hand side of (13) is importer i's MR, that is, a weighted average of importer i's trade frictions incurred in importing from every exporter in J_i. When this term is large, i faces high import barriers from all exporters, and so P_i is also large. This implies exports from j to i are relatively attractive, giving larger bilateral trade flows. That is, when importing from all destinations is hard, importing from a country j becomes *relatively* more attractive, all else equal. The interpretation of (14) is exactly analogous, being a function of both world importer MR, and exporter j's MR.

In the appendix, we eliminate the endogenous variables from the right hand sides of (13) and (14). For the generic importer country 1 and the generic exporter country 2, we obtain

$$\ln P_1 + \ln \widehat{P}_2 = -\sum_{j \in J_1} s_j^{J_1} \sum_{i \in I_j} s_i^{I_j} \ln t_{ij} V_{ij}^{\frac{1}{1-\sigma}} + \sum_{j \in J_1} s_j^{J_1} \ln t_{1j} V_{1j}^{\frac{1}{1-\sigma}} R_{1j}^{\frac{1}{1-\sigma}} + \sum_{i \in I_2} s_i^{I_2} \ln t_{i2} V_{i2}^{\frac{1}{1-\sigma}}$$

which is similar to BB's approximation, but has differences between export and import price indices and accommodates firm selection through the V_{ij} terms. The expression says that, for the pair 1 and 2, multilateral resistance can be thought of as being approximated by three terms. The first term on the right hand side captures general 'world' multilateral resistance. This can be thought of as the general difficulty with which international trade is conducted. In particular, when it is high, international trade is relatively difficult, which lowers bilateral trade between 1 and 2 in favour of domestic trade. Our construction renders it 1-specific. The next two terms work against this. The second term is 1's importing multilateral resistance, defined over the set of exporters to 1, J_1. When 1 faces high costs of importing from all other countries in its import set, importing from 2 is relatively more attractive. Accordingly, bilateral exports from 2 to 1 are higher, all else equal. This term is also 1-specific. The 2-specific third term is 2's exporting multilateral resistance, defined over the set of importers from 2, I_2. When it is high, such that 2 faces high costs of exporting to all other destinations, exporting to 1 is relatively attractive. Bilateral exports from 2 to 1 are therefore higher, all else equal[6]. Before turning to estimation however, we first consider some comparative statics.

4 Comparative Statics

When we come to estimate our gravity equation, we will work with (10) in log form. Using our expression for multilateral resistance, we can write the log of exports from country 2 to country 1 as

$$\begin{aligned} m_{12} &= -y^{I_2} + y_1 + y_2 - (\sigma-1)\ln t_{12} + w_{12} \\ &+[-\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j}((\sigma-1)\ln t_{ij} - w_{ij}) + \sum_{j\in J_1} s_j^{J_1}((\sigma-1)\ln t_{1j} - w_{1j}) \\ &+\sum_{i\in I_2} s_i^{I_2}((\sigma-1)\ln t_{i2} - w_{i2})] \end{aligned} \tag{15}$$

where $y = \ln Y$ and w is the log of the ij specific component of V_{ij}[7]. Further specifying variable trade costs t_{ij} as a function of bilateral distance D_{ij} and unobservable trade barriers u_{ij} such that $t_{ij}^{\sigma-1} = D_{ij}^{\gamma} e^{u_{ij}}$ gives

$$\begin{aligned} m_{12} &= -y^{I_j} + y_1 + y_2 - \gamma d_{12} + w_{12} \\ &+ \left[-\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j}(\gamma d_{ij} - w_{ij}) + \sum_{j\in J_1} s_j^{J_1}(\gamma d_{1j} - w_{1j}) + \sum_{i\in I_2} s_i^{I_2}(\gamma d_{i2} - w_{i2}) \right] + u_{12} \end{aligned} \tag{16}$$

where $d = \ln D$. We then consider the impact of changes in trade costs, proxied by changes in distance, on the flow of exports from country 2 to country 1. There are three channels in our model through which changes in trade costs affect trade flows. These are the intensive margin, the extensive margin and through MR. Changes affecting the intensive margin straightforwardly give rise to an elasticity γ, from (16). We consider the other channels in turn, starting with effects through MR.

4.1 Multilateral Resistance effects: multilateral versus bilateral changes

AvW show that, because small countries will in general trade a larger proportion of their output internationally, small countries' MR responses to *multilateral* changes in trade costs will be larger. In the appendix, we confirm this to be the case in our model which uses the Taylor approximation method. Denoting the elasticity of importer MR by $\frac{\partial \ln P_i}{\partial d} \equiv \varepsilon_i^P$, we show that, for multilateral changes, it is the case that

$$\frac{\partial \varepsilon_i^P}{\partial s_i} < 0 \qquad \text{(Multilateral changes)}$$

for given exporter MR.

We show in the appendix that the reverse is true however for *bilateral* changes in trade costs: for these changes, smaller countries experience *smaller* price index (MR) elasticticies, such that

$$\frac{\partial \varepsilon_i^P}{\partial s_i} > 0 \qquad \text{(Bilateral changes)}$$

[6] The term $\sum_{j\in J_1} s_j^{J_1} \ln R_{ij}^{\frac{1}{1-\sigma}}$ is 1-specific. For the purposes of estimation, it is therefore captured by a country 1 fixed effect.

[7] V_{ij} is also a function of a constant, which stores parameters governing the assumed distribution of firm productivity and preferences. Together with y^{I_j}, this is captured by the constant in regressions and can be safely ignored.

What is the reason for this reversal? When multilateral changes occur, a large fraction of a small country's total trade is affected, such that its price index falls by relatively more. By contrast, for multilateral changes, large countries' smaller proportion of internationally traded output implies that their MR terms are affected proportionately less. This is reversed when trade costs change only bilaterally. For a pair of large countries, each 'counts for a lot' in the other's MR term, since MR reflects weighted average trade costs in our approximation. For a pair of small countries, the reverse is true: they 'count for relatively little' in determining each other's MR. Then bilateral changes in trade costs have a bigger effect on the MR terms for larger country pairs. So while larger country pairs experience smaller effects through MR when trade costs change *multilaterally*, they experience larger effects through MR when trade costs change *bilaterally*. We summarize this in the following Lemma, which we use later:

Lemma 1 *The elasticity of importer price indices (MR terms) is*

(a) *decreasing in country size for multilateral changes in trade costs;*

(b) *increasing in country size for bilateral changes in trade costs.*

4.2 Trade costs and the extensive margin

The impact of trade costs on the extensive margin comes through in our model via the V_{ij} term. How do we expect the elasticity of the extensive margin to vary across countries? Can we say anything systematic from theory? First, from $V_{ij} = \int_{a_L}^{a_{ij}} a^{1-\sigma} dG(a)$, it is clear that changes in trade costs affect the V_{ij} term through changes in a_{ij}, the cut off cost level above which firms do not find it profitable to export given by (6). We can explore this further by writing the elasticity of the extensive margin with respect to trade costs as (see appendix)

$$\frac{\partial V_{ij}}{\partial t_{ij}} \frac{t_{ij}}{V_{ij}} = \frac{a_{ij}^{2-\sigma} g(a_{ij})}{V_{ij}} [\varepsilon_i^P(t_{ij}) - 1] \tag{17}$$

where $\varepsilon_i^P(t_{ij})$ is the elasticity of importer i's price index, or MR, with respect to variable trade costs from exporter j. For $\frac{\partial V_{ij}}{\partial t_{ij}} \frac{t_{ij}}{V_{ij}} < 0$ we require that $\varepsilon_i^P(t_{ij}) < 1$, which will naturally hold whenever a country trades at least some output domestically. When this is the case, increases in trade costs reduce the extensive margin, as we would expect. The condition on the elasticity of the price index is required in order that increases in trade costs acting through the MR term do not have effects large enough to offset completely the direct effect of changes in trade costs on the extensive margin. This can be seen in equation (6): for an increase in t_{ij} to reduce a_{ij}, the extensive margin, the rise in t_{ij} must not be offset completely by a concomitant rise in P_i, the importer's MR term.

We show in the appendix that the elasticity of the extensive margin with respect to trade costs given by (17) will in general be larger for smaller country pairs[8]. The intuition is as follows. First, exporters trading with small, distant importers, with low GDPs (low values of Y_i) and high trade costs (high values of t_{ij}, f_{ij}) will have lower cut-off cost levels a_{ij}, indicating that the range of firms that exports from j is smaller for a given lower bound on the distribution of costs, a_L. This low level of a_{ij} of course implies that V_{ij} is small, such that few firms export from j to i. This initially small extensive margin of firms then raises the elasticity of the extensive margin with respect to trade costs given by (17), all else equal.

Second, using the result in Lemma 1 (b) and that $\varepsilon_i^P(t_{ij}) - 1 < 0$, small countries have smaller offsetting MR effects in response to bilateral changes in trade costs, which implies that the direct effect of a trade cost change has a larger net impact on the extensive margin elasticity. With $\varepsilon_i^P(t_{ij})$ smaller for small countries in this case, the absolute value of the right hand side of (17) is larger, indicating a larger absolute value of the elasticity of the extensive margin with respect to variable trade costs for small countries, all else equal.

Finally, if the elasticity of substitution σ is sufficiently large[9], the term $a_{ij}^{2-\sigma} g(a_{ij})$ is decreasing in a_{ij}, such that the effect of changes in size on this term reinforces that of V_{ij} and ε^P. When this condition holds, smaller countries have larger absolute elasticities at the extensive margin with respect to variable trade costs, due to a combination of their small initial extensive margins, and their reduced sensitivity to *bilateral* changes in international trade costs through their MR terms.

[8] In the appendix we derive sufficient conditions for this to be the case.

[9] The sufficient condition we require is that $\sigma > 2 - [g'(a_{ij})/g(a_{ij})]$.

Lemma 2 *Smaller countries have larger elasticities of bilateral exports at the extensive margin in response to bilateral changes in trade costs owing to*

(i) *the initially small range of firms that exports and*

(ii) *their reduced sensitivity to bilateral changes in trade costs through their multilateral resistance terms.*

We use this in what follows next, where we consider the overall effect of changes in trade costs on the elasticities of bilateral trade.

4.3 Bilateral changes in trade costs

It is intuitive that the general equilibrium effects captured by MR will be less material when considering changes in a small subset of countries. To investigate this, we consider the special case of two countries reducing their frictions, but nobody else doing so. For practical purposes, can one effectively 'ignore' MR when considering changes for sufficiently small countries? In the appendix, we find sufficient conditions such that

Proposition 3 *For bilateral changes in trade costs, bilateral trade elasticities are decreasing in country size owing to*

(i) *smaller elasticities at the extensive margin and*

(ii) *a larger offsetting effect, through multilateral resistance, of the direct effects of changes in trade costs.*

Corollary 4 *Accounting for the effects of trade costs through multilateral resistance increases in importance as importer size increases relative to the set of active traders.*

The bilateral elasticity of exports from 2 to 1 when they reduce distance between each other is given by $\xi_{12}^{B} \equiv -\frac{\partial m_{12}}{\partial d_{12,21}}$. In the appendix, we show this can be written:

$$\xi_{12}^{B} = (\gamma - w_{12}') + s_1^{J_1} s_2^{I_1}(\gamma - w_{21}') + s_2^{J_1} s_1^{I_2}(\gamma - w_{12}') - s_2^{J_1}(\gamma - w_{12}') - s_1^{I_2}(\gamma - w_{12}') \tag{18}$$

Note that, by $w_{ij}' < 0$, it is the case that $\gamma - w_{ij}' > 0$. This implies that, in the absence of MR (which is captured by the terms involving country shares, the s's), the bilateral trade elasticity is positive. Including MR however adds some ambiguity to this theoretical statement. To get a sense of the typical impact of MR, suppose that $w_{21}' = w_{12}'$ and define $\hat{\xi}_{12}^{B} \equiv (\gamma - w_{12}')$, the *gross bilateral elasticity,* as that *ignoring* MR. It is given by the first term in parentheses on the right hand side of (18). Then consider the size of the net elasticity given by (18) relative to the gross elasticity $\hat{\xi}_{12}^{B}$. The ratio of the former to the latter, which we call the *multiplier*, is

$$\frac{\xi_{12}^{B}}{\hat{\xi}_{12}^{B}} \approx 1 - s_2^{J_1} - s_1^{I_2} + s_1^{J_1} s_2^{I_1} + s_2^{J_1} s_1^{I_2} \tag{19}$$

(19) is always positive, and captures in a simple way the *dampening effect* multilateral resistance has on bilateral trade elasticities. It has a maximum value of 1. The further below 1 is the multiplier, the greater the extent to which MR affects the net elasticity.

Further, we can see that the multiplier decreases further below unity as country size increases. Given that bigger countries have lower elasticities at the extensive margin (Lemma 2), and that bigger countries have larger MR elasticities in response to bilateral changes in trade costs (Lemma 1 (b)) we arrive at Proposition 3. Here, the impact of MR goes in the same direction as the impact of firm heterogeneity. In particular, as well as having smaller elasticities at the extensive margin, larger countries 'count for more' in the MR terms of their respective trade partners. When these big trade partners reduce their trade barriers against each other, their MR terms fall by relatively more; this provides a larger MR dampening effect for larger countries, which reinforces their smaller extensive margin elasticities.

Country shares are typically small, so the value of the multiplier will usually be a value close to unity. This indicates that the impact of bilateral changes in trade costs on multilateral resistance will have a small impact on the net effect of bilateral changes in trade costs overall. One might infer therefore that it is only important to account for MR when considering two big countries. However, in the presence of firm heterogeneity, which causes 'zeros' and asymmetries in trade flows, it is the size of a country's GDP *relative to other active traders* that matters for comparative statics. This is highlighted in Corollary 4. To see this, consider a trade deal between a small African country and the US or Japan. In this example, (a) $s_1^{I_2}$ is large; (b) $s_2^{J_1}$ is very small; and (c) $s_2^{I_1}$ is very small. (a) states that the importing country accounts for a large share of the combined output of all importers from 2. This is the case if 2 exports to very few countries, of which 1 is the largest. (b) states that the share of country 2 in the total output of exporters to 1 is very small. This is the case, say, for a small developing country exporting to the US, which in turn imports from lots of other large countries. Finally, (c) states that the share of country 2 is small as a proportion of the total output of importers from 1. Under these conditions, letting $s_2^{I_1}, s_2^{J_1} \approx 0$ gives

$$\frac{\xi_{12}^B}{\hat{\xi}_{12}^B} \approx 1 - s_1^{I_2} \tag{20}$$

which clearly decreases at the importer (country 1) accounts for more and more of the total output of importers from the exporter (country 2). Again, the multiplier (20) captures in a simple way the likely dampening effect of multilateral resistance in the case of a small exporter trading with a large importer. For country pairs such as these, ignoring MR, even for bilateral changes in trade costs, may not be innocuous, and increasingly so as the importer becomes larger[10].

4.4 Multilateral changes in trade costs

We now analyze the elasticity of bilateral exports from 2 to 1 given a multilateral change in trade costs $\xi_{12}^M \equiv -\partial m_{12}/\partial d$. Intuitively, MR terms are likely to be crucial in determining correct comparative static effects as the general equilibrium effects of trade cost changes are likely to be more significant when those changes are multilateral in nature. In the appendix, we show that the export elasticity when all countries get closer is given by

$$\xi_{12}^M = \gamma - w_{12}' + \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} (\gamma - w_{ij}') - \sum_{j \neq 1, j \in J_1} s_j^{J_1} (\gamma - w_{1j}') - \sum_{i \neq 2, i \in I_2} s_i^{I_2} (\gamma - w_{i2}') \tag{22}$$

The restrictions that $j \neq 1$ and $i \neq 2$ under the summation signs reflect the fact that we consider changes in international trade costs, with domestic trade costs held constant.[11] It can easily be verified that, were we to allow for falls in internal distance, $\xi_{12} = 0$. This confirms that it is international trade costs *relative* to domestic trade costs that influence international trade flows. We then use (22) to state the following proposition (for a derivation, see appendix):

Proposition 5 *For multilateral changes in trade costs*

(a) *after accounting for effects through multilateral resistance, it is the case that*

$$\xi_{12}^M \gtrless 0 \tag{23}$$

such that the sign of the elasticity of bilateral trade is ambiguous in theory. In particular, for some country pairs, it could be negative.

[10] This can also be seen by considering the difference $\Delta \equiv \xi_{12}^B - \hat{\xi}_{12}^B$ given by

$$\Delta = s_1^{J_1} s_2^{I_2} (\gamma - w_{21}') + s_2^{J_1} s_1^{I_2} (\gamma - w_{12}') - s_2^{J_1} (\gamma - w_{12}') - s_1^{I_2} (\gamma - w_{12}')$$

Then if $s_2^{I_2}, s_2^{J_1} \approx 0$, we have

$$\Delta \approx -s_1^{I_2} (\gamma - w_{12}') \tag{21}$$

which is unambiguously negative, and larger the bigger is the importer as a proportion of all importers from 2, and the larger is the response at the extensive margin of exporter country 2.

[11] Note also that the assumption of $f_{ii} = 0$ implies that there is no change in the extensive margin of domestic firms serving the domestic market. The mechanism identified in Melitz (2003) is thereby shut down in this model.

(b) (i) *If the extensive margin does not change,*

$$\frac{\partial \xi_{12}^M}{\partial s_1} > 0 \tag{24}$$

$$\frac{\partial \xi_{12}^M}{\partial s_2} > 0 \tag{25}$$

under reasonable conditions (see appendix), such that larger trading partners have larger firm-level responses to multilateral trade liberalizations.

(b) (ii) *If the extensive margin changes,*

$$\frac{\partial \xi_{12}^M}{\partial s_1} \gtrless 0 \tag{26}$$

$$\frac{\partial \xi_{12}^M}{\partial s_2} \gtrless 0 \tag{27}$$

such that the relationship between size and the country-level elasticity is equivocal in theory.

First, to get a sense of the impact of MR in isolation, consider the firm-level effect only (such that changes in the extensive margin are precluded). In this case, the multiplier is given by

$$\frac{\xi_{12}^M}{\hat{\xi}_{12}^M} \approx s_1^{J_1} + s_2^{I_2} - \sum_{j \in J_1} s_j^{J_1} s_j^{I_j} \tag{28}$$

In contrast to the bilateral case (19), the multiplier in the multilateral case can be *positive or negative* for the typical country, consistent with Proposition 5, part (a). If positive, it is likely to be far from unity. The extra complexity introduced by the extensive margin affects the multiplier in idiosyncratic ways, but does not affect Proposition 5, part (a). In particular, the ambiguity of the sign in part (a) is driven by considering the impact of MR alone[12]. The theoretical origin of this result is the 'endowment economy' nature of the model studied here; changes in trade costs serve to reallocate output from one activity (e.g. domestic trade) to another (e.g. international trade). When viewed in this way, it seems perfectly natural for exports to one destination to be redirected towards another destination in response to a trade liberalization, in order that incomes and expenditures be balanced. This 'endowment economy effect' gives rise to the possibility of negative export elasticities described above.

(b) part (i) of Proposition 5 repeats AvW's 'Implication 1' using our Taylor approximation. It states that larger country pairs have larger elasticities of bilateral trade when multilateral trade liberalization takes place. The result follows from Lemma 1 (a): larger countries typically trade a larger fraction of their output domestically for a given external tariff. An implication of this is that, for large countries, a smaller proportion of their total (i.e. domestic plus international) trade is affected by tariff changes. This effect is captured by the multilateral resistance terms. It follows that a multilateral reduction in trade barriers reduces larger countries' multilateral resistance by less. Recall that when multilateral resistance falls, bilateral trade falls in favour of other trading destinations. Then because this dampening effect is smaller for larger countries, the effect of a tariff reduction on bilateral trade is bigger. As shown in the appendix, AvW's result holds under reasonable conditions in our case, and can also be seen by inspection of (28).

Proposition 5 (b) part (ii) however states that this theoretical relationship no longer holds when the extensive margin responds to multilateral trade liberalizations as well. We show that

$$\begin{aligned} \frac{\partial \xi_{12}^M}{\partial s_1} = {} & \gamma(1 - s_1^{I_1} - s_1^{J_1}) \\ & - \sum_{i \neq 1, i \in I_1} s_i^{I_1} w'_{i1} - \sum_{j \neq 1, j \in J_1} s_j^{J_1} w'_{1j} + w'_{12} \\ & - w'_{12,s} - \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} w'_{ij,s} + \sum_{j \neq 1, j \in J_1} s_j^{J_1} w'_{1j,s} + \sum_{i \neq 2, i \in I_2} s_i^{I_2} w'_{i2,s} \end{aligned} \tag{29}$$

[12] C.f. AvW, their footnote number 15.

where we denote $\frac{\partial w'_{ij}}{\partial s_i} \equiv w'_{ij,s}$, and in which we identify three effects. The first line corresponds to the effect of a change in country size on the *weighting* of trade frictions at the intensive margin. The second line is this same weighting effect, but at the extensive margin. Finally, the third term corresponds to the *direct* effect on the extensive margin of a change in country size, highlighted by Lemma 2.

Whenever the importer's output shares are small relative to the total output of the relevant trading set, the first line is positive, corresponding to part (b) (i). The second line is also typically positive, for which a sufficient condition is that the {1,2}-pair specific effect at the extensive margin is less than the sum of the weighted averages given by $-\sum_{i\neq 1, i\in I_1} s_i^{I_1} w'_{i1} - \sum_{j\neq 1, j\in J_1} s_j^{J_1} w'_{1j}$. The sign of the third line is ambiguous. While it could amplify the positive derivative in Proposition 5 (b) part (i), it could "flip" the relationship, making it negative. This would be the case where the larger response at the extensive margin for smaller countries (Lemma 2) is sufficient to offset the greater dampening effect resulting from multilateral resistance experienced by small countries given multilateral changes (Lemma 1 (a)). In this way, Lemma 1 (a) and Lemma 2 work *against* each other in determining the effect of country size on bilateral trade elasticities given multilateral changes in trade costs. Their competing effects suggest a potentially non-monotonic relationship between country size and bilateral trade elasticities, and explain the ambiguity at the heart of Proposition 5 parts (a) and (b) (ii).

5 Estimation

This section begins with a brief description of the methodology. It follows with an account of the replication of the HMR regression results. True to their treatment, we illustrate the trade elasticity effects using the example of distance.

5.1 Method

We follow a two stage procedure in which the first stage generates two controls for inclusion in the second. The first control is for selection into trading – captured by the inverse Mills ratio. The second control is for the proportion of firms exporting to a particular destination. Because we are approximating the non-linear price system, all multilateral resistance terms are captured in the constant and by fixed effects. Accounting for multilateral resistance therefore has no implications for estimation conditional on these fixed effects being included.

A step-by-step description of the procedure is provided in the appendix. In the first stage, we estimate a probit model for the probability that j exports to i, denoted ρ_{ij}. At least one firm exports if the most productive firm can do so profitably. The most productive firm's profit is is captured by an unobserved latent variable z_{ij}. The most productive firm in j exports to i when $z_{ij} > 0$. We let $z_{ij} = z(\chi_j, \chi_i, t_{ij}, f_{ij}) + (u_{ij} + v_{ij})$ where $z(.)$ is log-linear. The χ's capture country-specific effects, t_{ij} is observed bilateral variable trade costs and f_{ij} is a bilateral observed fixed trade cost. u_{ij} is unobserved variable trade costs and v_{ij} is unobserved fixed trade costs. The composite error term $u_{ij} + v_{ij} = \eta_{ij} \sim N[0, \sigma_\eta]$ where $\sigma_\eta = \sigma_u^2 + \sigma_v^2$. We cannot observe z_{ij} but can observe bilateral exports. Letting T_{ij} be unity when exports from j to i are observed and zero otherwise, we can write

$$\rho_{ij} = \Pr(T_{ij} = 1|\text{Observables}) \tag{30}$$

Probit estimates can be used to generate predicted values for z_{ij}, $\hat{z}^*_{ij}$, where the '*' reflects the normalization of the coefficient estimates by σ_η[13]. It is well known that the predicted probability $\hat{\rho}_{ij}$ can be used to estimate the inverse Mills ratio $\widehat{\overline{\eta}}^*_{ij}$. This is a consistent estimate of the expected value of the unobserved trade frictions, given that trade takes place. Including $\widehat{\overline{\eta}}^*_{ij}$ in the second stage thus controls for the country selection effect. Furthermore, HMR show how $\hat{z}^*_{ij}$ and $\widehat{\overline{\eta}}^*_{ij}$ can be used to account for firm selection. Define $\widehat{x}_{ij} \equiv \hat{z}^*_{ij} + \widehat{\overline{\eta}}^*_{ij}$, which we call the "propensity to export". This *estimate* of the latent variable z is a function of both observable and (an estimate of) unobservable trade frictions. The propensity to export can enter our estimating equation in a number of ways. Following HMR, re-introducing distance explicitly together with assuming Pareto distributed (inverse) productivities leads to the estimating equation

$$m_{ij} = \psi_0 + \psi_i + \psi_j - \gamma d_{ij} + \log(e^{\delta \widehat{x}_{ij}} - 1) + \widehat{\overline{\eta}}^*_{ij} + \epsilon_{ij}, \tag{31}$$

[13] The error terms are therefore distributed according to a unit normal distribution after this transformation.

which we refer to as the Non-Linear ('NL') specification because this requires a non-linear estimator[14]. $\epsilon_{ij} \sim N[0, \sigma_e]$ is now orthogonal to the variables. The constant ψ_0 includes $-y^{I_j}$, the invariant components of the MR terms and the constant parameters affecting firm selection. $\psi_i + \psi_j$ are country-specific fixed effects including GDP and MR terms. Under more flexible assumptions on the distribution of productivity, HMR offer an alternative estimating equation in the form of

$$m_{ij} = \psi_0 + \psi_i + \psi_j - \gamma d_{ij} + \lambda_1 \widehat{x}_{ij} + \lambda_2 \widehat{x}_{ij}^2 + \lambda_3 \widehat{x}_{ij}^3 + \widehat{\overline{\eta}}_{ij}^* + \epsilon_{ij}, \tag{32}$$

which we refer to as the polynomial specification ('poly') and can be estimated by linear methods. $\log(e^{\delta \widehat{x}_{ij}} - 1)$ or $\lambda_1 \widehat{x}_{ij} + \lambda_2 \widehat{x}_{ij}^2 + \lambda_3 \widehat{x}_{ij}^3$, together with the exporter and importer specific fixed effects, control for firm selection *and* multilateral resistance and therefore generate consistent estimates of the parameters.

5.2 Regression results

We begin with a reproduction of the coefficients in HMR, using data taken from Elhanan Helpman's web-site, which is explained in the appendix.

Table 2 produces the results. Column 1 presents the standard linear OLS regression results. Column 2 presents the first stage (probit) results. Column 3 is the polynomial results while column 4 presents the Pareto results estimated with non-linear least squares. For comparison with HMR, we have included the relevant page references at the bottom of the table, together with HMR's maximum likelihood estimates in column 5.[15]

As found in HMR, $|\hat{\gamma}^{OLS}| > |\hat{\gamma}^{NL}|$ because linear OLS estimates conflate effects at the intensive and extensive margin, and γ is only the intensive (firm-level) effect in the theoretical setup. $\hat{\gamma}^{OLS}$ can be biased in the opposite direction because of the omission of the inverse Mills ratio. However, this *country* selection effect is shown to be smaller than the *firm* selection effect for this particular data.

Note that the variable indicating similarity of religion is used for identification in the second stage of the estimation procedure. Denote this variable H_{ij}. Significance in the probit stage indicates religious similarity affects the fixed costs of exporting. Its non-significance and consequent exclusion from the second stage, conditional on inclusion of controls for country and firm selection, is required for identification. Not including these controls in a one-stage OLS regression does not produce a significant coefficient for religion in column 1.[16] This means that one would conclude, erroneously, that a fixed cost like religion does not matter for trade quantities, when in fact it does. First, our theoretical analysis emphasizes how fixed costs like religious dissimilarity affect the probability of trade between two countries. Second, fixed costs also affect the quantity of trade between two countries via the extensive margin. This can be seen from the expression for $\frac{\partial m_{ij}}{\partial H_{ij}}$. Whenever $\frac{\partial \widehat{x}_{ij}}{\partial H_{ij}} \neq 0$, we have that $\frac{\partial m_{ij}}{\partial H_{ij}}|_{P_i, \widehat{P}_j} = -\frac{\partial \widehat{x}_{ij}}{\partial H_{ij}} \frac{\delta e^{\delta \widehat{x}_{ij}}}{e^{\delta \widehat{x}_{ij}} - 1} \neq 0$.

6 Simulations

This section begins with an analysis of the intensive and extensive margins in the absence of MR, replicating the simulations in HMR. It then accounts for multilateral resistance in the case of a bilateral reduction in distance, followed by an example of a multilateral reduction in distance.

[14] The term $\log(e^{\delta \hat{x}_{ij}} - 1)$ is exactly that of HMR, and can be derived from letting the latent variable $Z_{ij} = e^{z_{ij}}$ denote the ratio of variable profits to fixed costs of the most productive firm in j. Exports only occur when this value is greater than 1 (equivalently $z_{ij} > 0$). If productivities are Pareto distributed, then $V_{ij} = [\text{constant}] * W_{ij}$, where $W_{ij} = \max[(a_{ij}/a_L)^{k-\sigma+1} - 1, 0]$ or $W_{ij} = \max[Z_{ij}^{\delta} - 1, 0]$, $\delta \equiv \sigma_\eta (k - \sigma + 1)/(\sigma - 1)$, where k is the Pareto parameter. Then $w_{ij} = \log[e^{\delta z_{ij}} - 1]$ used in estimation.

[15] We have reproduced the polynomial specification (32). We have not managed to replicate the NL specification in HMR08, but get close to the NL specification in HMR07, despite the fact that, as in HMR08, we use non-linear least squares and that HMR07 use maximum likelihood. In both the polynomial and Pareto (NL) cases, these regressions lead to the same simulation results as HMR08.

[16] It is possible for a fixed cost to show up as significant erroneously because of the omission of the aforementioned controls.

6.1 No multilateral resistance

First, excluding the effects of MR, and concentrating on those countries already trading,[17] differentiation of (31), the NL (Pareto) case, yields

$$\frac{\partial m_{ij}}{\partial d_{ij}}|_{P_i,\widehat{P}_j} = -\gamma + \frac{\partial \widehat{x}_{ij}}{\partial d_{ij}} \frac{\delta e^{\delta \widehat{x}_{ij}}}{e^{\delta \widehat{x}_{ij}} - 1} \tag{33}$$

The first term is the intensive margin while the second is the extensive margin. For all countries, the intensive margin is estimated to be $\hat{\gamma} = 0.799$. Probit estimates imply $\frac{\partial \widehat{x}_{ij}}{\partial d_{ij}} = -0.66$ for all countries.[18] We can calculate the value of the second term $\frac{\delta e^{\delta \widehat{x}_{ij}}}{e^{\delta \widehat{x}_{ij}}-1}$ using each country-pair's estimated propensity to export, $\widehat{x}_{ij}$. As noted in HMR, this is the source of cross-country variation in export elasticities in their application. In Figure 1, we map the elasticities for an infinitesimal change in distance against $\widehat{x}_{ij}$. 'Linear OLS' gives conventional estimates of the country-level effect (1.176). 'Firm-level' elasticities capture the rise in exports at the intensive margin ($\hat{\gamma} = 0.799$) while 'Overall' accounts for both the intensive and extensive margins. Allowing for some movement at the extensive margin produces a larger country-level elasticity for all countries relative to Linear OLS. The higher the predicted propensity to export, the lower the absolute value of the elasticity. It follows that bigger countries will have lower gross elasticities, confirming our Lemma 2[19].

HMR record the response elasticities for a 10% fall in distance. We reproduce these results in Table 3. The polynomial elasticities have a higher mean than the NL elasticities. The polynomial elasticities also have a higher standard deviation. The simulated results presented allow for a substantial effect at the extensive margin, even if $\hat{\rho}$ is close to unity.[20] Figure 1b reproduces the negative relationship between the gross elasticity and the propensity to export for the polynomial case. These results are also consistent with those in HMR, where elasticities are lower for pairs of developed countries than for developing countries. Furthermore, while the theoretical analysis indicated the potential for heterogeneity in elasticities, we have demonstrated that empirically it can be substantial.

Together with Figure 1, Table 3 shows that overall elasticities exceed those from OLS. It is important to emphasize this because HMR's point that the *firm* effect is overestimated by OLS does not imply the effect on *country-level* trade is overestimated by OLS. In our case OLS provides an *underestimate* for all country pairs in the sample, conditional on their trading. Despite the fact that this can be gleaned from the comparative statics in HMR, we feel this point needs to be allocated more prominence to avoid misinterpretation of HMR's findings. The country-level effect is arguably more relevant for policy.

6.2 Simulations with multilateral resistance

The simulations have thus far ignored multilateral resistance. We refer to those elasticities as *gross* elasticities, the variable $\hat{\xi}$ in our theoretical section above. We now turn to net elasticities, ξ, which *do* take multilateral resistance effects into account.

6.2.1 Bilateral changes

Figures 2a and 2b plot the net elasticities generated in our sample against the gross elasticities. The (red) dots act as a reference line while the blue dots reflect the actual transformations after accounting for MR effects. The overwhelming majority of cases produce net elasticities close to the red line, in support of the

[17] This is consistent with our theoretical setup. One implication of this is we do not differentiate the inverse Mills ratio with respect to distance.

[18] A strict theoretical interpretation of the latent variable as the potential profit prescribes $\gamma = \frac{\partial \hat{x}_{ij},}{\partial d_{ij}}$, but we have chosen to display the probit estimate because this is what generates the simulations in HMR.

[19] By differentiating (33) with respect to $\widehat{x}_{ij}$, it is easy to see that the elasticity will be lower (less negative) for higher values of $\widehat{x}_{ij}$. Because $\widehat{x}_{ij}$ is a positive function of country size, the extensive margin is smaller for bigger countries. This illustrates Lemma 2 when Pareto distributed costs are assumed.

[20] This raises the issue of what to do with predicted probabilities exceeding 0.9999999. A value of $\rho = 1$ would not allow for the proportion of firms to be identified and, akin to all firms exporting already, generates elasticities approaching $\hat{\gamma} = 0.799$. Alternatively, and seemingly preferred by HMR, the elasticities for $\rho > 0.9999999$ are set equal to those for $\rho = 0.9999999$. This means the proportion of firms is fixed based on this value of ρ and hence the minimum extensive margin is fixed, and hence the minimum elasticity is calculated to be 1.2832. This is what generates the substantial effect at the extensive margin for the highest values of ρ.

idea that bilateral changes in trade costs should be subject to small MR effects. In Table 4, we produce some examples for illustration. The first two rows give the mean and median values for GDP shares and elasticities. It also includes the multiplier[21] $\frac{\xi^B_{12}}{\hat{\xi}^B_{12}}$. The median multiplier is 0.9930 while the mean is 0.9721, which suggests accounting for MR has a small material effect for the average country.

When we take two small countries - Nigeria and Burkina Faso make a small contribution to world GDP - we see the multiplier is very close to 1. In contrast, as the world's two largest economies, the USA and Japan comprised 45% of world GDP in 1986. That they trade with almost everyone is reflected in the various share measures in the table, which account for these countries' active trading partners. In this case, we see multipliers of about 0.64. This suggests that MR is important for big countries. Mexico and Spain were the 10th and 11th biggest countries in the world, but they each had less than 2% of world GDP. Their multiplier of 0.96 is still sufficiently close to unity to suggest it is only "very" big countries for which MR matters. Empirically, this is a stronger assertion than that made in Corollary 4. More generally, bigger countries, which have lower gross elasticities through the extensive margin *and* lower multipliers through multilateral resistance, will have lower net elasticities overall (cf Proposition 3).

However, while the USA and Japan are the biggest in terms of world GDP shares, their multiplier is not the lowest and neither is their net elasticity. As shown in Table 5, the lowest multipliers are generated by small (and distant) countries exporting to the largest countries, as we illustrated in Section 4.1. These cases, of which many are islands, do not export to many destinations. Therefore, importers like the US or Japan comprise a disproportionately large share of export destination GDP. In the case of Bhutan, which exports to only nine countries, Japan's share is 72% and the multiplier is consequently 0.28. As we suggested before, below a threshold, the size of the exporter's GDP does not matter; the key effect is the magnitude of the importer. Here, the multiplier is effectively $1 - s_i^{Ij}$ (equation (19))[22]. The importers in Table 5 are either the US or Japan. More generally, there are 618 multipliers below 0.85. Of these, only four do not have the USA or Japan as one of the trading partners. As one might expect for these pairs, the gross elasticity is roughly the sample average (see tables 3 and 4) and higher than for the US trading with Japan, through their extensive margin effects. Therefore, despite lower multipliers, the net elasticities ξ^B_{12} are not necessarily lower than for the US and Japan. In our case, the Bhutan-Japan example produces the lowest net elasticity.

In summary, lower elasticities and lower multipliers work together to make bigger countries have lower responses to bilateral reductions in frictions. We have illustrated empirically how the presence of selection into exporting affects which country pairs do or do not get materially affected by MR in the bilateral case. Countries of average size are largely unaffected, but we need to be cautious regarding large country pairs or small exporters with few export destinations if studying their exports to a big importer.

6.2.2 Multilateral changes

Figures 3a and 3b demonstrate the dramatic effects of accounting for MR when *all* countries reduce frictions. Net elasticities are substantially lower than gross elasticities — typically by 1.6 units. Approximately half of the net elasticities are negative, which demonstrates that Proposition 5 (a) is not just a theoretical possibility, but relevant empirically. This implies that the general equilibrium effects are so strong that many country pairs reduce their bilateral trade after reductions in their frictions, and trade is redirected elsewhere.

Can we characterize the source of the MR effects and see where it is strongest? Figure 4 plots multipliers from the Pareto estimates, given by the y-axis, against country-pair size.[23] Referring to Proposition 5 (a) again, the (maroon) crosses give the multipliers for the firm level effect and indicate that the majority of values are below zero. In fact, only 3% are positive; 305 of the 310 positive multipliers occur when the USA or Japan export. Thus, outside of these two countries, the typical firm reduces its bilateral exports. The crosses also illustrate Proposition 5 (b, i) — bigger countries have higher multipliers and it follows that bigger countries have higher firm-level elasticities. The correlation coefficient between the size of the country pair and the *firm-level* elasticity is 0.85.

The (blue) dots are the multipliers for the *country-level* elasticities. These allow for the extensive margin and generate substantial variation in the multipliers. Smaller country-pairs display particularly large variation. In two-thirds of our observations, the country-level multiplier is higher than the firm-level multiplier.

[21] We construct this to account for the asymmetry at the extensive margin, from which we abstracted in the theoretical discussion above.

[22] In terms of the difference (equation (21)), $\Delta \approx -1.08$.

[23] Although many of our analytical results refer to derivatives with respect to particular country share definitions, it is just as informative to refer to the size of the country pair, not adjusting for trading partners, in our graphical illustrations.

This interaction effect between MR and the extensive margin means that about half the elasticities are positive, instead of just 3% when the extensive margin is shut down. For bigger countries, the interaction with the extensive margin tends to magnify rather than mitigate the effects of MR. This is illustrated by the USA & Japan (except when they trade with each other on the far right of the graph) and by a batch of European G7 countries at around 0.05 on the x-axis. For these countries, the blue dots (which include the extensive margin effect) lie close to the maroon crosses (which exclude the extensive margin)[24].

In Figure 4, the positive association between the multiplier and pair sizes is no longer clearly present once the extensive margin effects are included. For overall elasticities, we have two contrary forces. Bigger pairs have generally lower elasticities at the extensive margin, tending to reduce gross elasticities for these countries. Contrary to this, bigger pairs tend to have higher firm-level multipliers after accounting for MR effects. The potentially non-monotonic effect arising from the combination of these two forces, highlighted in our theory section above, is therefore illustrated in Figure 4.

Figures 5a and 5b plot the overall elasticities against pair size. Some of the pattern from Figure 4 is visible. Beyond a threshold, the derivative of the elasticity with respect to country size is positive.[25] However, below that threshold, including the average trading pair, the derivative is negative. The overall correlation coefficients are -0.1 for Pareto estimates and -0.27 for polynomial estimates. Thus, as stated in Proposition 5 (b, ii), the positive relationship between the firm-level elasticity and country size has been overturned by the extensive margin, such the relationship is becomes negative overall. This reverses AvW's 'Implication 1'.

Table 6 illustrates some of these points with specific observations. Considering the effect at the firm level under NL estimation, we see the average net elasticity is -0.0958. The average multiplier is -0.1199, indicating that the effect of MR is to turn the average firm level elasticity negative. Including the extensive margin, the average at the country level is almost zero and the average multiplier is -0.0226. In the polynomial case, the firm-level effect is the same (-0.1034), but the country-effect has a substantially different value (0.2036). Unlike the NL estimate, it is positive, indicating that the net country level elasticity is positive when we do not impose a Pareto distribution on firm productivities. Consistent with the NL estimate, it is higher (along the real line) than for the respective intensive margin effect.

At the intensive margin, we see that the multiplier is higher for the USA - Japan than for Mexico - Spain, which in turn is higher than for Burkina Faso - Nigeria. This is consistent with Proposition 5 (b, i). It is negative for two of those three cases. For the African pair however, the extensive margin effect is large and positive; in fact it is strong enough to convert a negative firm-level effect to a positive country-level effect.

Compare with this the negative net elasticity for Mexico-Spain. In doing so we see that the net elasticity is greater when Burkina Faso exports to Nigeria than when Mexico exports to Spain, indicating the dominant effect of the extensive margin for country sizes in this range. However comparing the Mexico - Spain net elasticity with that for USA - Japan, we observe the dominant effect of MR for country sizes in this range. These three country pairs therefore illustrate the ambiguity described in Proposition 5 (b, ii).

In our theoretical section we discussed how negative bilateral responses could be the outcome of diversion from some export destinations to others. Given the endowment economy model studied here, negative export elasticties with some destinations should have offsetting positive elasticities with others. For consistency with theory therefore, it is necessary for each exporter to have *at least one* import destination with which it has a positive export response. For our Pareto estimates, the Netherlands, Spain and Belgium-Luxembourg are the only countries where this does not happen. Canada, Denmark, France, Germany, Italy, Sweden Switzerland and the UK have solitary positive responses, which are with the US, Bhutan or Kiribati. Our more flexible polynomial estimates have a solitary positive elasticities for exports from the Netherlands to the US. Kiribati and Bhutan featured in our discussion of low bilateral multipliers, in part because they have few trading partners.[26] Otherwise, our findings suggest that countries with very many negative trade responses are those who typically increase their exports to the US[27].

The idiosyncratic nature of the interactions between country size and the extensive margin amplify the importance of accounting for MR when analyzing multilateral changes in trade frictions. This is imperative,

[24] The large-country exception is the USSR at a value of about 0.075 on the x-axis. Whether this is a feature of the Soviet economy or an artifact of poor data is moot.

[25] This threshold is quite high - about 0.05 - and excludes trade involving the USSR. This could be an artifact of the GDP data, which was constructed by summing the values for the (now) former Republics.

[26] Of course, this could be attributable to devious data.

[27] This is consistent with the view that many relatively small countries receive fewer products, since more of them become redirected to the US.

given the dramatic empirical illustration of Proposition 5 (a).

7 Conclusion

We have built a gravity model that accounts explicitly for firm selection into exports and for general equilibrium price effects acting through multilateral resistance terms. Furthermore, we have shown how, despite using fixed effects for estimation, a Taylor approximation along the lines of Baier & Bergstrand (2009) can be used to allow for multilateral effects when conducting comparative statics, showing the approach can be extended to include firm heterogeneity.

Taking the contribution of Helpman, Melitz & Rubinstein (2008) as a starting point, we have emphasized that overestimates of the firm-level distance coefficient do not imply an overestimate of the country-level effect. Our results show that, for all countries that already trade, failing to account for firm heterogeneity underestimates the effects of a fall in trade frictions. The intensive margin makes up approximately half the effect on average, but the extensive margin is smaller for bigger country pairs. We attribute the greater effect at the extensive margin for smaller countries to their initially small set of exporting firms - an effect re-enforced by their greater sensitivity to changes in trade frictions through Anderson & van Wincoop (2003) price index (multilateral resistance) terms.

We have shown reductions in frictions by two countries (or by a small subset of countries) can for practical purposes be analyzed without factoring in MR for the average country. For all but the largest trading pairs, such inaccuracies are in all likelihood small compared to, say, errors in the trade data. However, the simultaneous modelling of MR and heterogeneity identifies small exporters — with few trading partners and who export to a large importer — as additional candidates for which MR effects are important under bilateral changes in trade costs. In general, we have shown that bigger country pairs have lower net elasticities when bilateral trade costs change in isolation.

In contrast to the bilateral case, our simulations for multilateral changes in trade costs show that MR wipes out a large proportion of the comparative static effect found when MR is ignored; in some cases, bilateral trade falls. The firm-level response is greater for bigger countries, but the extensive margin effect is bigger for smaller traders. Net elasticities are potentially non-monotonic in country size, and it is therefore essential to account for MR in the case of multilateral changes in frictions. In our sample, we find that the extensive margin effects are large enough to give rise to a negative correlation between net elasticities and country size, which is opposite to that found by AvW when the extensive margin is ignored. The Taylor method we have extended provides a straightforward way for accounting for these effects.

Nonetheless, the Taylor approach is an approximation. In BB's sample of 88 countries, Monte Carlo simulations revealed only 8% of simulation results differed from the AvW system by 20% or more. The biggest inaccuracy was a 38% deviation from the 'true' value. BB note that the largest inaccuracies involved countries in the European Economic Area (EEA) who were relatively small and relatively close to their largest trading partners. Extending beyond the EEA in our case, some North African countries may be in the same category for our 1986 data, while some small East Asian countries may now be susceptible because of the emergence of China and India. Bhutan, for example, borders both, but we have otherwise, we have been careful not to base our examples on any of these countries.

Furthermore, BB propose an iterative method which allows the Taylor approximation to converge on the results produced by the AvW system. This certainly is an approach that can be taken when it comes to implementation, but our main message is that to employ a first-order adjustment is better than to use no adjustment for MR at all. In fact, as a further simplification, some applications may permit the use of unadjusted country shares of world GDP — rather than measures adjusting for active traders — without a serious loss of accuracy. This would be true when considering the effects of multilateral reductions in trade frictions on trade between those countries in the world who have many trading partners. Extensions beyond such cases should be treated with caution however.

The number of negative responses we recorded have not allowed for the possibility that some countries would no longer trade. More generally, we have not considered the formation of new export relationships between two countries with zero trade. While HMR argue that very little of the expansion of world trade seen over the last few decades is attributable to new trading pairs, another fruitful area for research would be the use of the empirical framework to examine this possibility.

Finally, our analysis has concentrated on bilateral trade. Further work is needed to see the effects of combining firm heterogeneity and multilateral resistance on analyses of a country's total exports, and on world trade. The endowment economy nature of the class of model studied here suggests that our results are best thought of as static effects.

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A Deriving the Gravity Equation

Using bilateral trade balance in (8) gives

$$\sum_{i\in I_j} M_{ij} = \sum_{i\in I_j}\left(\frac{c_j t_{ij}}{\alpha P_i}\right)^{1-\sigma} Y_i N_j V_{ij} \tag{34}$$

$$Y_j = \left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j \sum_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij} \tag{35}$$

$$\left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j = \frac{Y_j}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} \tag{36}$$

Use in (8) to obtain

$$M_{ij} = \left(\frac{c_j t_{ij}}{\alpha P_i}\right)^{1-\sigma} Y_i N_j V_{ij} \tag{37}$$

$$= Y_i Y_j \frac{1}{P_i^{1-\sigma}} \frac{t_{ij}^{1-\sigma}}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} V_{ij} \tag{38}$$

Then

Define: Y^{I_j} as the total GDP of all importers from j, or $Y^{I_j} \equiv \sum\limits_{i\in I_j} Y_i$.

Dividing top and bottom by this gives

$$M_{ij} = \frac{Y_i Y_j}{Y^{I_j}} \frac{1}{P_i^{1-\sigma}} \frac{t_{ij}^{1-\sigma}}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} \frac{Y_i}{Y^{I_j}} V_{ij}^{\sigma-1}} V_{ij} \tag{39}$$

Further:

Define: $s_i^{I_j}$ as the share of i's GDP in the set of all importers from j, or $s_i^{I_j} \equiv \frac{Y_i}{Y^{I_j}} = \frac{Y_i}{\sum\limits_{i\in I_j} Y_i}$.

Then it follows that $\sum\limits_{i\in I_j} s_i^{I_j} = \sum\limits_{i\in I_j} \frac{Y_i}{\sum\limits_{i\in I_j} Y_i} = 1$. We write the gravity equation as

$$M_{ij} = \frac{Y_i Y_j}{Y^{I_j}} \frac{1}{P_i^{1-\sigma}} \frac{t_{ij}^{1-\sigma}}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} s_i^{I_j} V_{ij}} V_{ij} \tag{40}$$

or

$$M_{ij} = \frac{Y_i Y_j}{Y^{I_j}} \frac{1}{P_i^{1-\sigma}} \frac{t_{ij}^{1-\sigma}}{\sum\limits_{h\in I_j}\left(\frac{t_{hj}}{P_h}\right)^{1-\sigma} s_h^{I_j} V_{hj}} V_{ij} \tag{41}$$

Now

Define: The exporter's MR term $\widehat{P}_j^{1-\sigma} \equiv \sum\limits_{h\in I_j}\left(\frac{t_{hj}}{P_h}\right)^{1-\sigma} s_h^{I_j} V_{hj}$.

Note that, intuitively, it is defined over the set of importers from j, I_j. Using this in our earlier equation

$$\left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j = \frac{Y_j}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} \tag{42}$$

$$= \frac{Y^{I_j}}{Y^{I_j}} \frac{Y_j}{\sum\limits_{i\in I_j}\left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} Y_i V_{ij}} \tag{43}$$

$$= \frac{Y_j}{Y^{I_j}} \frac{1}{\widehat{P}_j^{1-\sigma}} \tag{44}$$

Note that $\sum\limits_{j\in J_i} \frac{Y_j}{Y^{I_j}} = \sum\limits_{j\in J_i} \frac{Y_j}{\sum\limits_{i\in I_j} Y_i} \neq 1$. This is the case when $I_j \neq J_i$, or that the set of all importers from j is not equal to the set of all exporters to i, as is the case with asymmetric trade flows. From (44), perform

$$\left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j = \frac{Y_j}{Y^{I_j}} \frac{1}{\widehat{P}_j^{1-\sigma}} \frac{\sum\limits_{j\in J_i} Y_j}{\sum\limits_{j\in J_i} Y_j} \tag{45}$$

$$= \frac{Y_j}{\sum\limits_{j\in J_i} Y_j} \frac{1}{\widehat{P}_j^{1-\sigma}} \frac{\sum\limits_{j\in J_i} Y_j}{Y^{I_j}} \tag{46}$$

$$= \frac{Y_j}{\sum\limits_{j\in J_i} Y_j} \frac{1}{\widehat{P}_j^{1-\sigma}} \frac{\sum\limits_{j\in J_i} Y_j}{\sum\limits_{i\in I_j} Y_i} \tag{47}$$

Define: $s_j^{J_i}$ is the share of j's GDP in the set of all exporters to i, such that $s_j^{J_i} \equiv \frac{Y_j}{\sum\limits_{j\in J_i} Y_j}$.

Then we have the convenient property that $\sum\limits_{j\in J_i} s_j^{J_i} \equiv \sum\limits_{j\in J_i} \frac{Y_j}{\sum\limits_{j\in J_i} Y_j} = 1$. We are left with the object $\sum\limits_{j\in J_i} Y_j / \sum\limits_{i\in I_j} Y_i$ to work with. It is the ratio of the total output of all exporters to i to that of all importers from j. It is defined over the sets I_j and J_i, and as such is an "ij" variable. Then

Define: $R_{ij} \equiv \frac{\sum\limits_{j\in J_i} Y_j}{\sum\limits_{i\in I_j} Y_i}$ as the ratio of total output for all exporters to i to that of all importers from j.

Using it we can write

$$\left(\frac{c_j}{\alpha}\right)^{1-\sigma} N_j = \frac{s_j^{J_i}}{\widehat{P}_j^{1-\sigma}} R_{ij} \tag{48}$$

and so the price index for the importer as

$$P_i^{1-\sigma} = \sum_{j\in J_i} (c_j t_{ij}/\alpha)^{1-\sigma} N_j V_{ij} \tag{49}$$

$$= \sum_{j\in J_i} \left(\frac{t_{ij}}{\widehat{P}_j^{1-\sigma}}\right)^{1-\sigma} s_j^{J_i} V_{ij} R_{ij} \tag{50}$$

The terms we wish to approximate, which give multilateral resistance, are therefore

$$\widehat{P}_j^{1-\sigma} \equiv \sum_{i \in I_j} \left(\frac{t_{ij}}{P_i}\right)^{1-\sigma} s_i^{I_j} V_{ij} \tag{51}$$

$$= \sum_{i \in I_j} \left(\frac{T_{ij}}{P_i}\right)^{1-\sigma} s_i^{I_j} \tag{52}$$

$$P_i^{1-\sigma} = \sum_{j \in J_i} \left(\frac{t_{ij}}{\widehat{P}_j^{1-\sigma}}\right)^{1-\sigma} s_j^{J_i} V_{ij} R_{ij} \tag{53}$$

$$= \sum_{j \in J_i} \left(\frac{T'_{ij}}{\widehat{P}_j^{1-\sigma}}\right)^{1-\sigma} s_j^{J_i} \tag{54}$$

where we have **defined** $T_{ij}^{1-\sigma} = t_{ij}^{1-\sigma} V_{ij}$ and **defined** $T_{ij}'^{1-\sigma} = t_{ij}^{1-\sigma} V_{ij} R_{ij}$. The gravity equation then looks like that in the text (10).

B Multilateral Resistance: approximations

Consider an approximation via a first order Taylor expansion around a world of symmetric trade frictions, such that $t_{ij}^{1-\sigma} V_{ij} \equiv T_{ij}^{1-\sigma} = T^{1-\sigma}$ for all i, j and that $R_{ij} = R = 1$ by $I_j = J_i$ in this symmetric configuration. This further implies that $T' = T$. These statements are tantamount to assuming a centre for our Taylor approximation in which trade frictions are symmetric but all countries trade. In this centre, $P_i = P_j = P$ and $\widehat{P}_i = \widehat{P}_j = \widehat{P}$. So

$$P^{1-\sigma} = \sum_{j \in J_i} \left(\frac{T}{\widehat{P}}\right)^{1-\sigma} s_j^{J_i} \tag{55}$$

$$= \left(\frac{T}{\widehat{P}}\right)^{1-\sigma} \sum_{j \in J_i} s_j^{J_i} \tag{56}$$

$$= \left(\frac{T}{\widehat{P}}\right)^{1-\sigma} \tag{57}$$

$$P = \frac{T}{\widehat{P}} \tag{58}$$

And

$$\widehat{P}^{1-\sigma} = \sum_{h \in I_j} \left(\frac{T}{P}\right)^{1-\sigma} s_h^{I_j} \tag{59}$$

$$= \left(\frac{T}{P}\right)^{1-\sigma} \sum_{h \in I_j} s_h^{I_j} \tag{60}$$

$$= \left(\frac{T}{P}\right)^{1-\sigma} \tag{61}$$

$$\widehat{P} = \frac{T}{P} \tag{62}$$

Following AvW (p. 175) a solution to this system is $P = \widehat{P}$ and $P = T^{1/2}$. The solution that $P = \widehat{P}$ makes sense in a world with symmetric trade costs (which is the centre for our approximation). Take logs and exponents of the price index equations to gives

$$e^{(1-\sigma)\ln P_i} = \sum_{j\in J_i} e^{\ln s_j^{J_i}} e^{(1-\sigma)\ln T'_{ij}} e^{(\sigma-1)\ln \widehat{P}_j} \tag{63}$$

$$e^{(1-\sigma)\ln \widehat{P}_j} = \sum_{i\in I_j} e^{\ln s_i^{I_j}} e^{(1-\sigma)\ln T_{ij}} e^{(\sigma-1)\ln P_i} \tag{64}$$

We work first on (63). We use that an approximation of $f(x_i)$ around x is given by $f(x_i) \approx f(x)+\frac{df(x)}{dx}(x_i-x)$. Expanding the left hand side around $\ln P$ and the right hand side around $\ln T'$and $\ln \widehat{P}$, and using that $\frac{de^{(1-\sigma)\ln P_i}}{d\ln P_i} = (1-\sigma)e^{(1-\sigma)\ln P_i}$, we obtain

$$P^{1-\sigma} + (1-\sigma)P^{1-\sigma}[\ln P_i - \ln P] = \sum_{j\in J_i} s_j^{J_i}\left(\frac{T'}{\widehat{P}}\right)^{1-\sigma} + (1-\sigma)\sum_{j\in J_i} s_j^{J_i}\left(\frac{T'}{\widehat{P}}\right)^{1-\sigma}[\ln T'_{ij} - \ln T'] \tag{65}$$

$$+(\sigma-1)\sum_{j\in J_i} s_j^{J_i}\left(\frac{T'}{\widehat{P}}\right)^{1-\sigma}[\ln \widehat{P}_j - \ln \widehat{P}] \tag{66}$$

Next, on the RHS we use that $\left(\frac{T'}{\widehat{P}}\right)^{1-\sigma} = P^{1-\sigma}$, and that $\ln T' = \ln T = 2\ln \widehat{P}$ from $T = \widehat{P}^2$. On the LHS we use that $P = \widehat{P}$ under symmetry, all of which taken together gives

$$P^{1-\sigma} + (1-\sigma)P^{1-\sigma}[\ln P_i - \ln \widehat{P}] = \sum_{j\in J_i} s_j^{J_i} P^{1-\sigma} + (1-\sigma)\sum_{j\in J_i} s_j^{J_i} P^{1-\sigma}[\ln T'_{ij} - 2\ln \widehat{P}] \tag{67}$$

$$+(\sigma-1)\sum_{j\in J_i} s_j^{J_i} P^{1-\sigma}[\ln \widehat{P}_j - \ln \widehat{P}] \tag{68}$$

Then divide both sides by $P^{1-\sigma}$, use $\sum_{j\in J_i} s_j^{J_i} = 1$, and subtract $1-(1-\sigma)\ln \widehat{P}$ from both sides to arrive at

$$\ln P_i = -\sum_{j\in J_i} s_j^{J_i}\ln \widehat{P}_j + \sum_{j\in J_i} s_j^{J_i}\ln T'_{ij} \tag{69}$$

an approximation of importer i's multilateral resistance. It is defined over the set of exporters to i, J_i.The first term on the RHS is *world* exporter MR, that is, a weighted average of the MRs faced by all exporters to i. When all exporters face high MR, implying that exporting is hard in general, exporting to i is replaced by domestic trade. Hence when the first term is large, P_i is small, lowering bilateral trade flows. The second term is importer i's MR, that is, a weighted average of importer i's trade frictions incurred in importing from every exporter in J_i. When this term is large, i faces high import barriers from all exporters, and so P_i is also large. This implies exports from j to i are relatively attractive, giving larger bilateral trade flows. That is, when importing from all destinations is hard, importing from a country j becomes *relatively* more attractive.

Likewise, from (64), performing analogous procedures, one obtains

$$\ln \widehat{P}_j = -\sum_{i\in I_j} s_i^{I_j}\ln P_i + \sum_{i\in I_j} s_i^{I_j}\ln T_{ij} \tag{70}$$

the interpretation of which is exactly analogous to (69). In particular, the first term is *world* importer MR, while the second term is j's exporting MR.

The log form of the gravity equation requires us to derive an expression for $\ln P_i + \ln \widehat{P}_j$. For this, we wish to eliminate the endogenous variables from the right hand sides of (69) and (70).

Then for (70) perform the following

$$s_j^{J_i}\ln\widehat{P}_j = -s_j^{J_i}\sum_{i\in I_j}s_i^{I_j}\ln P_i + s_j^{J_i}\sum_{i\in I_j}s_i^{I_j}\ln T_{ij} \tag{71}$$

$$\sum_{j\in J_i}s_j^{J_i}\ln\widehat{P}_j = -\sum_{j\in J_i}s_j^{J_i}\sum_{i\in I_j}s_i^{I_j}\ln P_i + \sum_{j\in J_i}s_j^{J_i}\sum_{i\in I_j}s_i^{I_j}\ln T_{ij} \tag{72}$$

$$= -\sum_{i\in I_j}s_i^{I_j}\ln P_i + \sum_{j\in J_i}s_j^{J_i}\sum_{i\in I_j}s_i^{I_j}\ln T_{ij} \tag{73}$$

In the text we consider a generic importer 1 and a generic exporter 2. When this is the case, the last line should be written

$$\sum_{j\in J_1}s_j^{J_1}\ln\widehat{P}_j = -\sum_{i\in I_2}s_i^{I_2}\ln P_i + \sum_{j\in J_1}s_j^{J_1}\sum_{i\in I_j}s_i^{I_j}\ln T_{ij}$$

(Note that the very last term, $\sum_{i\in I_j}s_i^{I_j}\ln T_{ij}$, gives an object that is j-specific. The subsequent summation of this term over all $j\in J_1$ gives an object that is importing country 1-specific.) Adding (69) and (70) together for importer 1 and exporter 2 and using the above gives

$$\ln P_1 + \ln\widehat{P}_2 = -\sum_{j\in J_1}s_j^{J_1}\ln\widehat{P}_j + \sum_{j\in J_1}s_j^{J_1}\ln T'_{1j} - \sum_{i\in I_2}s_i^{I_2}\ln P_i + \sum_{i\in I_2}s_i^{I_2}\ln T_{i2} \tag{74}$$

$$= -\left(-\sum_{i\in I_2}s_i^{I_2}\ln P_i + \sum_{j\in J_1}s_j^{J_1}\sum_{i\in I_j}s_i^{I_j}\ln T_{ij}\right) \tag{75}$$

$$+\sum_{j\in J_1}s_j^{J_1}\ln T'_{1j} - \sum_{i\in I_2}s_i^{I_2}\ln P_i + \sum_{i\in I_2}s_i^{I_2}\ln T_{i2} \tag{76}$$

$$= -\sum_{j\in J_1}s_j^{J_1}\sum_{i\in I_j}s_i^{I_j}\ln t_{ij}V_{ij}^{\frac{1}{1-\sigma}} + \sum_{j\in J_1}s_j^{J_1}\ln t_{1j}V_{1j}^{\frac{1}{1-\sigma}}R_{1j}^{\frac{1}{1-\sigma}} + \sum_{i\in I_2}s_i^{I_2}\ln t_{i2}V_{i2}^{\frac{1}{1-\sigma}} \tag{77}$$

Where $T_{ij} = t_{ij}V_{ij}^{\frac{1}{1-\sigma}}$ from our above definition. If $V_{ij} = KW_{ij}$, define $w = \ln W$, we can write the term $\sum_{j\in J_i}s_j^{J_i}\ln t_{1j}V_{1j}^{\frac{1}{1-\sigma}}R_{1j}^{\frac{1}{1-\sigma}} = \sum_{j\in J_i}s_j^{J_i}(\ln t_{1j} - \frac{1}{\sigma-1}w_{1j} + \ln R_{1j}^{\frac{1}{1-\sigma}})$.

Examine the R term. We can write this as

$$\begin{aligned}
\sum_{j\in J_1}s_j^{J_1}\ln R_{1j}^{\frac{1}{1-\sigma}} &= \frac{1}{1-\sigma}\sum_{j\in J_1}s_j^{J_1}\ln R_{1j} \\
&= \frac{1}{1-\sigma}\sum_{j\in J_1}s_j^{J_1}\ln\frac{\sum\limits_{j\in J_1}Y_j}{\sum\limits_{i\in I_j}Y_i} \\
&= \frac{1}{1-\sigma}\sum_{j\in J_1}s_j^{J_1}\left(\ln\sum_{j\in J_1}Y_j - \ln\sum_{i\in I_j}Y_i\right) \\
&= \frac{1}{1-\sigma}\sum_{j\in J_1}s_j^{J_1}\left(\ln Y^{J_1} - \ln Y^{I_j}\right) \\
&= \frac{1}{1-\sigma}\left(\sum_{j\in J_1}s_j^{J_1}\ln Y^{J_1} - \sum_{j\in J_1}s_j^{J_1}\ln Y^{I_j}\right) \\
&= \frac{1}{1-\sigma}\left(\ln Y^{J_1} - \sum_{j\in J_1}s_j^{J_1}\ln Y^{I_j}\right)
\end{aligned}$$

which is a 1-specific variable. It is therefore captured by our 1-specific fixed effect.

For comparative statics purposes therefore, we can use the MR term

$$
\begin{aligned}
\ln P_1 + \ln \widehat{P}_2 &= -\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} \ln t_{ij} V_{ij}^{\frac{1}{1-\sigma}} + \sum_{j\in J_1} s_j^{J_1} \ln t_{1j} V_{1j}^{\frac{1}{1-\sigma}} + \sum_{i\in I_2} s_i^{I_2} \ln t_{i2} V_{i2}^{\frac{1}{1-\sigma}} \\
&= -\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} (\ln t_{ij} - \frac{1}{\sigma-1} w_{ij}) + \sum_{j\in J_1} s_j^{J_1} (\ln t_{1j} - \frac{1}{\sigma-1} w_{1j}) + \sum_{i\in I_2} s_i^{I_2} (\ln t_{i2} - \frac{1}{\sigma-1} w_{i2}) \\
&= -\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} (\frac{\sigma-1}{\sigma-1} \ln t_{ij} - \frac{1}{\sigma-1} w_{ij}) + \sum_{j\in J_1} s_j^{J_1} (\frac{\sigma-1}{\sigma-1} \ln t_{1j} - \frac{1}{\sigma-1} w_{1j}) \\
&\quad + \sum_{i\in I_2} s_i^{I_2} (\frac{\sigma-1}{\sigma-1} \ln t_{i2} - \frac{1}{\sigma-1} w_{i2}) \\
&= \frac{1}{\sigma-1} \left(\begin{array}{c} -\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} ((\sigma-1)\ln t_{ij} - w_{ij}) + \sum_{j\in J_1} s_j^{J_1} ((\sigma-1)\ln t_{1j} - w_{1j}) \\ + \sum_{i\in I_2} s_i^{I_2} ((\sigma-1)\ln t_{i2} - w_{i2}) \end{array} \right)
\end{aligned}
$$

In particular, the gravity equation can be written in log form fully as follows. Following the above convention, let the importer in question be country 1 and the exporter be country 2, then we have exports from 2 to 1 as

$$
\begin{aligned}
m_{12} &= \lambda + \lambda_1 + \lambda_2 - (\sigma-1)\ln t_{12} + w_{12} \\
&\quad + \left[-\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} ((\sigma-1)\ln t_{ij} - w_{ij}) + \sum_{j\in J_1} s_j^{J_1} ((\sigma-1)\ln t_{1j} - w_{1j}) + \sum_{i\in I_2} s_i^{I_2} ((\sigma-1)\ln t_{i2} - w_{i2}) \right]
\end{aligned} \tag{78}
$$

Note that under the assumption that $t_{ij}^{\sigma-1} = D_{ij}^{\gamma}$, then $(\sigma-1)\ln t_{ij} = \gamma d_{ij}$. Then

$$
\begin{aligned}
m_{12} &= \lambda + \lambda_1 + \lambda_2 - \gamma d_{12} + w_{12} \\
&\quad + \left[-\sum_{j\in J_1} s_j^{J_1} \sum_{i\in I_j} s_i^{I_j} (\gamma d_{ij} - w_{ij}) + \sum_{j\in J_1} s_j^{J_1} (\gamma d_{1j} - w_{1j}) + \sum_{i\in I_2} s_i^{I_2} (\gamma d_{i2} - w_{i2}) \right]
\end{aligned} \tag{79}
$$

C Comparative Statics

C.1 Lemma 1: MR elasticities

Consider the term $\ln P_i = -\sum_{j\in J_i} s_j^{J_i} \ln \widehat{P}_j + \sum_{j\in J_i} s_j^{J_i} \ln t_{ij} V_{ij}^{\frac{1}{1-\sigma}}$, our approximated MR term for importer i. The elasticity of this price index with respect to a change in trade costs, for given exporter MR terms, is given by $\frac{\partial \ln P_i}{\partial \ln t_{ij}} \equiv \varepsilon_i^P$. Consider two cases

1. $\partial \ln t_{ij} = \partial \ln t$ for all i, j, for $i \neq j$. This is a *multilateral* change in trade costs. Then, for given exporter MR terms

$$
\varepsilon_i^P(t_{ij}) = \frac{\partial \ln P_i}{\partial \ln t} = \sum_{j\neq i, j\in J_i} s_j^{J_i} \left(1 - \frac{1}{\sigma-1} w'_{ij} \right)
$$

using $\frac{\partial \ln t_{ij}}{\partial \ln t} = 1$. Note that the subscript under the summation $j \neq i$ indicates that internal trade costs have not changed. The right hand side of this equation is decreasing as $s_i^{J_i}$ increases i.e. as country i trades more with itself, *for a given* w'_{ij}. In the extreme case in which i does not trade at all internationally, it is obviously the case that $s_i^{J_i} = 1$ and that $\frac{\partial \ln P_i}{\partial \ln t} = 0$. Thus as $s_i^{J_i}$ increases from 0 to 1, or as country i trades more with itself but less internationally, the elasticity of its price index, or importing MR term, with respect to *multilateral* changes in trade costs falls:

$$
\frac{\partial \varepsilon_i^P}{\partial s_i} < 0 \qquad \text{(Multilateral case)}
$$

AvW's 'Implication 1' result therefore holds with our approximation.

2. $\partial \ln t_{ij} = \partial \ln t_{ji} > 0$, and $\partial \ln t_{ij} = 0$ for all country pairs except ij. This is a *bilateral* change in trade costs. Then we can write, for given exporter MR terms, that

$$\varepsilon_i^P(t_{ij}) = \frac{\partial \ln P_i}{\partial \ln t_{ij,ji}} = s_i^{J_i}\left(1 - \frac{1}{\sigma - 1}w'_{ji}\right) + s_j^{J_i}\left(1 - \frac{1}{\sigma - 1}w'_{ij}\right)$$

which is *increasing* as the exporter j and importer i get larger *for given* w'_{ij}. This is in contrast to the multilateral case in (a). It shows that, for *bilateral* changes in trade costs, *larger* country pairs have *larger* price index elasticities all else equal. Intuitively, when trade is liberalized bilaterally between two large countries, they 'count for more' in each others' MR terms. That is, bigger countries are bigger determinants of multilateral resistance. Hence when countries such as these liberalize trade bilaterally, they experience larger general equilibrium through MR than smaller countries:

$$\frac{\partial \varepsilon_i^P}{\partial s_i} > 0 \qquad \text{(Bilateral case)}$$

C.2 Lemma 2: Extensive margin elasticity

We derive the expression in the text for the elasticity of the extensive margin with respect to variable trade costs, $\frac{\partial V_{ij}}{\partial t_{ij}}(t_{ij}/V_{ij})$. In the definition of V_{ij}, define $h(a) \equiv a^{1-\sigma}g(a)$ where $g(a) = G'(a)$ is the density function for a such that $g(a)da = dG(a)$. Then $V_{ij} = \int_{a_L}^{a_{ij}} h(a)da$. Straightforwardly, we have that $\frac{\partial V_{ij}}{\partial a_{ij}} = h(a_{ij})$. Then write

$$\frac{\partial V_{ij}}{\partial t_{ij}}\frac{t_{ij}}{V_{ij}} = \frac{\partial V_{ij}}{\partial a_{ij}}\frac{\partial a_{ij}}{\partial t_{ij}}\frac{t_{ij}}{V_{ij}}$$

and writing $a_{ij} = \bar{a}P_i/t_{ij}$ obtain

$$\begin{aligned} \frac{\partial a_{ij}}{\partial t_{ij}} &= \frac{\bar{a}}{t_{ij}}\left(\frac{\partial P_i}{\partial t_{ij}} - \frac{P_i}{t_{ij}}\right) \\ &= \frac{\bar{a}}{t_{ij}}\frac{P_i}{t_{ij}}\left(\frac{\partial P_i}{\partial t_{ij}}\frac{t_{ij}}{P_i} - 1\right) \\ &= \frac{\bar{a}}{t_{ij}}\frac{P_i}{t_{ij}}[\varepsilon_i^P(t_{ij}) - 1] \end{aligned} \tag{80}$$

where $\varepsilon_i^P(t_{ij}) \equiv \frac{\partial P_i}{\partial t_{ij}}\frac{t_{ij}}{P_i}$ is the elasticity of i's price index (importing MR) with respect to variable trade costs. We expect that $\frac{\partial a_{ij}}{\partial t_{ij}} < 0$, or that increases in variable trade costs lower the cost level below which firms find it profitable to export. By inspection of (6), this will be the case when increases in t_{ij} do not have large offsetting increases in P_i. In other words, the elasticity of the price index P_i with respect to trade costs must not be 'too large'. Equivalently, in (80), we require that $\varepsilon_i^P(t_{ij}) < 1$ such that it is indeed the case that $\frac{\partial a_{ij}}{\partial t_{ij}} < 0$. Consistent with our log-linear gravity equation, let the ij component of $\ln V_{ij}$ be given by w_{ij}. Then the elasticity can be written $\frac{\partial \ln V_{ij}}{\partial \ln t_{ij}} = \frac{\partial w_{ij}}{\partial \ln t_{ij}} \equiv w'_{ij}$. Using the above, we write

$$w'_{ij} = \frac{\partial V_{ij}}{\partial t_{ij}}\frac{t_{ij}}{V_{ij}} = h(a_{ij})\frac{\bar{a}}{t_{ij}}\left(\frac{\partial P_i}{\partial t_{ij}} - \frac{P_i}{t_{ij}}\right)\frac{t_{ij}}{V_{ij}}\frac{a_{ij}}{a_{ij}} \tag{81}$$

$$= \frac{a_{ij}h(a_{ij})}{V_{ij}}\frac{t_{ij}}{P_i}\left(\frac{\partial P_i}{\partial t_{ij}} - \frac{P_i}{t_{ij}}\right) \tag{82}$$

$$= \frac{a_{ij}h(a_{ij})}{V_{ij}}\left(\frac{\partial P_i}{\partial t_{ij}}\frac{t_{ij}}{P_i} - 1\right) \tag{83}$$

$$= \frac{a_{ij}h(a_{ij})}{V_{ij}}[\varepsilon_i^P(t_{ij}) - 1] \tag{84}$$

used in the text. By $\varepsilon_i^P(t_{ij}) < 1$ we have that $w'_{ij} < 0$, such that increases in trade costs reduce the extensive margin.

In order to think about how w_{ij} varies with country size, we consider the effects of size on the three terms $a_{ij}h(a_{ij})$, V_{ij} and $\varepsilon_i^P(t_{ij})$.

1. A small country (small Y_i) with high trade costs (high t_{ij}, f_{ij}) will in general have a low a_{ij}, indicating that the cost cut-off for trading is low, such that few firms export for a given lower bound on the support of (inverse) productivities, a_L. This implies that V_{ij} is low, raising the elasticity of the extensive margin with respect to trade costs for small countries (raising the value of the right hand side of (84)).

2. Using the results in Lemma 1 (b), larger countries experience larger MR elasticities $\varepsilon_i^P(t_{ij})$ than smaller countries when undertaking *bilateral* trade liberalization. Then the term $[\varepsilon_i^P(t_{ij}) - 1] < 0$ is increasing in country size for bilateral changes in trade costs, or $\left|\varepsilon_i^P(t_{ij}) - 1\right|$ is decreasing in country size. This implies that, ceteris paribus, larger countries will experience smaller values of $|w_{ij}|$, the absolute value of the elasticity of the extensive margin, given bilateral changes in trade costs.

3. Finally, for a sufficiently large elasticity of substitution σ, the term $a_{ij}h(a_{ij}) = a_{ij}^{2-\sigma}g(a_{ij})$ is decreasing in a_{ij}, such that smaller countries have larger values of $a_{ij}h(a_{ij})$, again raising the elasticity of the extensive margin with respect to trade costs. The sufficient condition for this to be the case is that $\sigma > 2 - a_{ij}[g'(a_{ij})/g(a_{ij})]$. (Note that Anderson & van Wincoop (2004) conclude that estimates of σ range from 5 to 10. Further, it makes sense to think of $g'(a_{ij}) > 0$, or that the density of firms is higher at higher cost levels.)

We summarize this by saying that, for bilateral changes in trade costs, it is the case that smaller countries have larger elasticities of the extensive margin, such that

$$\frac{\partial w'_{ij}}{\partial s_i} \equiv w'_{ij,s} > 0$$

C.3 MR terms

In general, the MR terms for country 1 and 2 can be written

$$\begin{aligned} \sum_{j \in J_1} s_j^{J_1} X_{1j} &= s_1^{J_1} X_{11} + s_2^{J_1} X_{12} + ... + s_n^{J_1} X_{1n} \\ \sum_{i \in I_2} s_i^{I_2} X_{i2} &= s_1^{I_2} X_{12} + s_2^{I_2} X_{22} + ... + s_n^{I_2} X_{n2} \end{aligned}$$

where $X_{ij} \equiv \gamma d_{ij} - w_{ij}$. The world resistance (WR) term can be written

$$\begin{aligned} \sum_{j \in J_1} \sum_{i \in I_j} s_j^{J_1} s_i^{I_j} X_{ij} &= \sum_{j \in J_1} s_j^{J_1} [s_1^{I_j} X_{1j} + s_2^{I_j} X_{2j} + ...] && (85) \\ &= s_1^{J_1} [s_1^{I_1} X_{11} + s_2^{I_1} X_{21} + ...] && (86) \\ &\quad + s_2^{J_1} [s_1^{I_2} X_{12} + s_2^{I_2} X_{22} + ...] && (87) \\ &\quad + ... && (88) \end{aligned}$$

C.4 Proposition 3: Bilateral changes

Recall

$$\begin{aligned} m_{12} &= \lambda + \lambda_1 + \lambda_2 - X_{12} && (89) \\ &\quad + \left[- \sum_{j \in J_1} s_j^{J_1} \sum_{i \in I_j} s_i^{I_j} X_{ij} + \sum_{j \in J_1} s_j^{J_1} X_{1j} + \sum_{i \in I_2} s_i^{I_2} X_{i2} \right] \end{aligned}$$

and consider bilateral changes, such that d_{12} and d_{21} change. In general

$$\begin{aligned} \frac{\partial m_{12}}{\partial d_{12,21}} &= -X'_{12} + \left[-(s_1^{J_1} s_2^{I_1} X'_{21} + s_2^{J_1} s_1^{I_2} X'_{12}) + s_2^{J_1} X'_{12} + s_1^{I_2} X'_{12} \right] \\ &= -[X'_{12} + s_1^{J_1} s_2^{I_1} X'_{21} + s_2^{J_1} s_1^{I_2} X'_{12} - s_2^{J_1} X'_{12} - s_1^{I_2} X'_{12}] \end{aligned}$$

Such that **defining** $\xi_{12}^B \equiv -\frac{\partial m_{12}}{\partial d_{12,21}}$ gives

$$\xi_{12}^B = X_{12}' + s_1^{J_1} s_2^{I_1} X_{21}' + s_2^{J_1} s_1^{I_2} X_{12}' - s_2^{J_1} X_{12}' - s_1^{I_2} X_{12}'$$

Consider a change in the size of country 1 such that $\partial s_1^{J_1} = \partial s_1^{I_2} = \partial s_1$. Then

$$\begin{aligned}\frac{\partial \xi_{12}^B}{\partial s_1} &= X_{12,s}'(1 + s_2^{J_1} s_1^{I_2} - s_2^{J_1} - s_1^{I_2}) + X_{21,s}' s_1^{J_1} s_2^{I_1} \\ &\quad + s_2^{I_1} X_{21}' + s_2^{J_1} X_{12}' - X_{12}'\end{aligned}$$

This equation gives the impact of a change in country size on the elasticity of bilateral trade with respect to a bilateral change in trade costs. The first line describes the impact on this of changes in country size through the extensive margin. The second line describes the impact of country size through MR. Consider the first line in isolation. Re-writing the X's in terms of their underlying elasticities $X_{ij,s}' = -w_{ij,s}'$, we have

$$\begin{aligned}& X_{12,s}'(1 + s_2^{J_1} s_1^{I_2} - s_2^{J_1} - s_1^{I_2}) + X_{21,s}' s_1^{J_1} s_2^{I_1} \\ = \;& -w_{12,s}'(1 + s_2^{J_1} s_1^{I_2} - s_2^{J_1} - s_1^{I_2}) - w_{21,s}' s_1^{J_1} s_2^{I_1}\end{aligned}$$

Then use that $w_{ij,s}' > 0$ to argue that

$$-w_{12,s}'(\underbrace{1 + s_2^{J_1} s_1^{I_2} - s_2^{J_1} - s_1^{I_2}}_{\geq 0}) - \underbrace{w_{21,s}' s_1^{J_1} s_2^{I_1}}_{>0} < 0$$

This says that, *as country size increases, the impact through changes in the extensive margin is to reduce bilateral trade elasticities in response to bilateral changes in trade costs.* So changes at the extensive margin, which are smaller for larger countries, contribute to smaller bilateral trade elasticities for larger countries when trade costs change bilaterally.

Consider the second line $s_2^{I_1} X_{21}' + s_2^{J_1} X_{12}' - X_{12}'$ in isolation. Re-writing again gives

$$\begin{aligned}& s_2^{I_1} X_{21}' + s_2^{J_1} X_{12}' - X_{12}' \\ = \;& s_2^{I_1}(\gamma - w_{21}') + s_2^{J_1}(\gamma - w_{12}') - (\gamma - w_{12}') \\ = \;& s_2^{I_1}(\gamma - w_{21}') + (s_2^{J_1} - 1)(\gamma - w_{12}')\end{aligned}$$

which is negative iff

$$\frac{s_2^{I_1}}{1 - s_2^{J_1}} < \frac{\gamma - w_{12}'}{\gamma - w_{21}'} \tag{90}$$

which says that country 2 must be sufficiently small. As a benchmark, consider the case in which $w_{12}' = w_{21}'$ and $J_1 = I_1$. Then this requires that

$$s_2 < \frac{1}{2}$$

such that 2 accounts for less than 50% of output of the relevant trading group.

In sum then, when (90) holds, it is the case that

$$\frac{\partial \xi_{12}^B}{\partial s_1} < 0$$

which says that *the elasticity of bilateral trade with respect to a bilateral change in trade costs is falling in country size.* (One can readily show the analogous case for changes in the size of the exporter 2.) This establishes Proposition 3.

C.5 Proposition 5: Multilateral Changes

Suppose there is a multilateral change in some variable Y such that $\frac{\partial X_{ij}}{\partial Y} = X'_{ij}$. Then for the MR and WR terms respectively

$$\frac{\partial}{\partial Y} \sum_{j \in J_1} s_j^{J_1} X_{1j} = \sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} \tag{91}$$

$$\frac{\partial}{\partial Y} \sum_{i \in I_2} s_i^{I_2} X_{i2} = \sum_{i \neq 2, i \in I_2} s_i^{I_2} X'_{i2} \tag{92}$$

$$\frac{\partial}{\partial Y} \sum_{j \in J_1} \sum_{i \in I_j} s_j^{J_1} s_i^{I_j} X_{ij} = \sum_{j \in J_1} \sum_{i \neq j, i \in I_2} s_j^{J_1} s_i^{I_j} X'_{ij} \tag{93}$$

As above, recall our gravity equation

$$\begin{aligned} m_{12} &= \lambda + \lambda_1 + \lambda_2 - X_{12} \\ &+ \left[- \sum_{j \in J_1} s_j^{J_1} \sum_{i \in I_j} s_i^{I_j} X_{ij} + \sum_{j \in J_1} s_j^{J_1} X_{1j} + \sum_{i \in I_2} s_i^{I_2} X_{i2} \right] \end{aligned} \tag{94}$$

where as above $X'_{ij} = \gamma - w'_{ij} > 0$ by $w'_{ij} < 0$.

Then consider the multilateral change ∂Y above where $\partial Y = \partial d$, where d is international trade costs such that $\frac{\partial X_{ij}}{\partial d} = X'_{ij}$ but internal trade costs d_{ii} do not change such that $X'_{ii} = 0$. Then

$$\begin{aligned} \frac{\partial m_{12}}{\partial \ln t} &= -X'_{12} + \left[- \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} X'_{ij} + \sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} + \sum_{i \neq 2, i \in I_2} s_i^{I_2} X'_{i2} \right] \\ &= - \left[X'_{12} + \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} X'_{ij} - \sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} - \sum_{i \neq 2, i \in I_2} s_i^{I_2} X'_{i2} \right] \end{aligned}$$

Define the Multilateral (M) elasticity as the negative of this such that $\xi_{12}^M \equiv -\frac{\partial m_{12}}{\partial \ln t}$, then

$$\xi_{12}^M = X'_{12} + \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} X'_{ij} - \sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} - \sum_{i \neq 2, i \in I_2} s_i^{I_2} X'_{i2} \tag{95}$$

Now consider the following.

(a) Suppose we ignore MR effects. Then

$$\xi_{12}^M = X'_{12} > 0$$

such that the bilateral trade elasticity is positive. Including MR effects however makes this ambiguous, which can be seen by signing the MR terms in (95). In particular

$$\underbrace{\sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} X'_{ij}}_{>0} \underbrace{- \sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} - \sum_{i \neq 2, i \in I_2} s_i^{I_2} X'_{i2}}_{<0} \gtrless 0$$

such that when we *do* account for MR effects we have that

$$\xi_{12}^M \gtrless 0$$

This raises the possibility that $\xi_{12}^M < 0$ when trade is liberalized multilaterally. This will be the case when importer and exporter MR terms (given by $-\sum_{j \neq 1, j \in J_1} s_j^{J_1} X'_{1j} - \sum_{i \neq 2, i \in I_j} s_i^{I_2} X'_{i2}$) fall sufficiently to offset the rise in trade that is encouraged by the combined fall in bilateral and world trade resistance (given by $X'_{12} + \sum_{j \in J_1} s_j^{J_1} \sum_{i \neq j, i \in I_j} s_i^{I_j} X'_{ij}$).

(b) (i) Suppose the extensive margin is held constant. This could be thought of as the AvW case. Then the elasticity can be written as

$$\xi_{12}^M = \gamma\left[1 + \sum_{j\in J_1} s_j^{J_1} \sum_{i\neq j, i\in I_j} s_i^{I_j} - \sum_{j\neq 1, j\in J_1} s_j^{J_1} - \sum_{i\neq 2, i\in I_2} s_i^{I_2}\right]$$

Consider the impact of an increase in the size of country 1 such that $\partial s_1^{J_1} = \partial s_1^{I_2} = \partial s_1$. Then

$$\begin{aligned}
\frac{\partial \xi_{12}^M}{\partial s_1} &= \gamma\left[\sum_{i\neq 1, i\in I_1} s_i^{I_1} + \sum_{j\neq 1, j\in J_1} s_j^{J_1} - 1\right] \\
&= \gamma\left[(1 - s_1^{I_1}) + (1 - s_1^{J_1}) - 1\right] \\
&= \gamma\left[1 - s_1^{I_1} - s_1^{J_1}\right]
\end{aligned}$$

which is positive iff $s_1^{I_1} + s_1^{J_1} < 1$. In the benchmark case in which $I_1 = J_1$ this requires that $s_1 < 1/2$, or that country 1 accounts for less than 50% of output of the relevant trading group. When this is the case, we have AvW's result that

$$\frac{\partial \xi_{12}^M}{\partial s_1} > 0$$

or that bilateral trade elasticties are increasing in country size given a multilateral change in trade costs.

(b) (ii) Second, suppose the elasticity at the extensive margin *is* allowed to change. The relevant bilateral elasticity is then given by (95) where $X'_{ij} = \gamma - w'_{ij}$. Consider the impact of an increase in the size of country 1 such that $\partial s_1^{J_1} = \partial s_1^{I_2} = \partial s_1$. When allowing the extensive margin to change, there are two additional effects arising from this experiment to consider. First, as for the intensive margin, a change in s_1 affects the *weighting* given to country 1's trade frictions with its various trade partners. Second, as indicated in Lemma 2, a change in country size will also affect the extensive margin elasticity *directly*. Accordingly, we obtain

$$\begin{aligned}
\frac{\partial \xi_{12}^M}{\partial s_1} &= \gamma(1 - s_1^{I_1} - s_1^{J_1}) \\
&\quad - \sum_{i\neq 1, i\in I_1} s_i^{I_1} w'_{i1} - \sum_{j\neq 1, j\in J_1} s_j^{J_1} w'_{1j} + w'_{12} \qquad (96)\\
&\quad - w'_{12,s} - \sum_{j\in J_1} s_j^{J_1} \sum_{i\neq j, i\in I_j} s_i^{I_j} w'_{ij,s} + \sum_{j\neq 1, j\in J_1} s_j^{J_1} w'_{1j,s} + \sum_{i\neq 2, i\in I_2} s_i^{I_2} w'_{i2,s}
\end{aligned}$$

in which (1) the first line is the effect of a change in country size on the weighting of trade frictions at the intensive margin, (2) the second line is the effect of a change in country size on the weighting of trade frictions at the extensive margin, and (3) the third line is the *direct effect* of a change in country size on the extensive margin itself. Compared with the case in which the extensive margin does not change in (b) (i), effects (2) and (3) are new. The second line will typically be positive, by $w'_{ij} < 0$, for which a sufficient condition is that $-\sum_{i\neq 1, i\in I_1} s_i^{I_1} w'_{i1} - \sum_{j\neq 1, j\in J_1} s_j^{J_1} w'_{1j} > -w'_{12}$, or intuitively that the {1,2}-pair effect is less twice the weighted average effect. By $w'_{ij,s} > 0$, in the third line we have that

$$-w'_{12,s} - \sum_{j\in J_1} s_j^{J_1} \sum_{i\neq j, i\in I_j} s_i^{I_j} w'_{ij,s} < 0$$

but that

$$\sum_{j\neq 1, j\in J_1} s_j^{J_1} w'_{1j,s} + \sum_{i\neq 2, i\in I_2} s_i^{I_2} w'_{i2,s} > 0$$

If changes in the extensive margin are particularly big, such as those for *small* countries, the former effect can dominate the latter. In other words, the direct effect of trade liberalizations on the extensive margin may be so large for small countries that the result in (b) (i) is reversed. Theory therefore suggests that the relationship between trade elasticities and country size given multilateral changes in trade costs is *ambiguous*, such that

$$\frac{\partial \xi_{12}^{M}}{\partial s_1} \lesseqgtr 0$$

In this sense, adjustments at the extensive margin open the potential for AvW's result to be reversed.

D Description of variables

The dependent variable is the log of exports in 1986 measured in constant (2000) US dollars. The main variables used are

- distance: the log of distance in km between the importer and exporter
- border: a binary variable indicating whether the country pair shares a common physical boundary
- island: a binary variable indicating whether at least one country is an island (in HMR, this is described as an indicator of whether both countries are an island)
- landlocked: a binary variable indicating whether at least one country is landlocked (in HMR, this is described as an indicator of whether both countries are landlocked)
- legal: a binary variable indicating whether or not the country pair share the same legal origin
- language: a binary variable indicating whether the country pair share a common language[28]
- colonial ties: a binary variable indicating whether one country every colonized the other
- FTA: a binary variable indicating whether or not the country pair formed a regional trade agreement
- religion: a variable constructed by HMR indicating how similar the religious composition is in the country pair (% Protestants in j multiplied by % Protestants in i + % Catholics in j multiplied by % Catholics in i + % Muslims in j multiplied by % Muslims in i).
- exporter and importer dummies to capture fixed effects

To construct GDP Shares, GDP data are sourced from the World Bank's World Development Indicators. Where necessary, WDI data were combined to meet the country definitions in the trade data (for example combining Belgium and Luxembourg). Where possible, missing observations (for the former Soviet and Yugoslav Republics for example) were supplemented with data from the United Nations Common Database (UNCDB). Otherwise, a GDP of 0.1 was inputted manually. The denominator is manually constructed based on the sum of the individual countries' GDPs. Shares are based on the subset of countries the importer imports from or the exporter exports to. Except in the example of a small exporter with few trading partners exporting to a large exporter, we have achieved similar results simply by using the share of world GDP, where world GDP is the value provided by the WDI.

E Description of empirical methodology

Here, we describe the methodology used for estimation and the comparative statics exercises. Our empirics were run on *Stata 10.0 MP*.

[28] HMR do not explain the construction of this variable, so it is not clear how this is defined.

E.1 Estimation

1. Estimate a probit model for the probability of positive exports from j to i. The first stage includes the bilateral variables listed in the data appendix together with importer and exporter dummies. The set of bilateral variables must include a variable which will be omitted from the second stage so that identification does not rely on assumptions of joint normality in the errors in the first and second stage.[29] Theoretically, such a variable should affect the fixed costs of exporting, but not variable costs. For the full sample, HMR use religion for this purpose, but argue the common language variable produces similar results. Some countries export to everyone else or import from everyone else. The fixed effects are thus perfect predictors and cannot be included in the probit and they therefore do not feature in the subsequent stages.[30]

2. Generate predicted probabilities ($\hat{\rho}$)of positive exports from country j to i. Many of these are close to unity. Values predicted to exceed 0.9999999 can be indistinguishable from unity in some statistical packages.[31] The Inverse Mills Ratio (see point 3 below) would be undefined and the terms capturing firm heterogeneity (see point 4) would be unidentified. The approach in HMR converts all $\hat{\rho} > 0.9999999$ to values of $\hat{\rho} = 0.9999999$.

3. Predict the Inverse Mills Ratio $\widehat{\overline{\eta}}_{ij}^{*} = \frac{\Phi^{-1}\phi(\hat{\rho})}{\hat{\rho}}$, where ϕ is the the standard normal density function and Φ is the standard normal distribution function.

4. Generate the predicted controls for firm heterogeneity. Together with unidentified terms in the constant, these should reflect the proportion of firms exporting. We describe two of the three approaches used by HMR. The non-linear function $\hat{\omega} = \log(e^{\delta\hat{x}} - 1)$, where log is the natural logarithm, requires no preparation prior to estimation. For the polynomial approximation, generate $\hat{x} = \Phi^{-1}(\hat{\rho}) + \widehat{\overline{\eta}}_{ij}^{*}$ as well as $\hat{x}^2$ and $\hat{x}^3$ such that $\hat{\omega} \approx \lambda_1\hat{x} + \lambda_2\hat{x}^2 + \lambda_3\hat{x}^3$ can be estimated.

5. Estimate the second stage

   (a) For the polynomial approximation, estimate using OLS, including the exporter and importer fixed effects, bilateral variables (except religion or some other valid variable), $\widehat{\overline{\eta}}_{ij}^{*}$ and the polynomial in $\hat{x}$.

   (b) For the non-linear approach, HMR07 use maximum likelihood while HMR08 use non-linear least squares. We follow HMR08. In principle, the default interactive version of the *nl* command should be sufficient. However, the large number of dummies (over 300) generates an error message. Following Carrayol (2006), we use the function evaluator program version of the nl command.

6. Generate predicted values for trade $\hat{m}$ for each specification chosen, including a predicted value $\hat{\omega} = \log(e^{\hat{\delta}\hat{x}} - 1)$.

E.2 Simulations

Generate an alternative measure of the variable of interest. In the case of distance, the new measure is 10% lower than the original.

1. Estimate a new probit model with the alternative distance measure instead of the true one.

2. Generate the new predicted probabilities $\hat{\rho}'$, setting values above 0.9999999 equal to 0.9999999 as before.

3. Keep the originally estimated Inverse Mills Ratio $\widehat{\overline{\eta}}_{ij}^{*}$. The original estimate of unobserved trade frictions, conditional on the same countries trading, is still the prediction based on the original values.

[29] In the polynomial specification, the $\hat{x}$ term is a linear function of the observables, so exclusion for identification is even more important. This may provide an argument for excluding not one but two variables.

[30] HMR omit such variables from the study. However, an alternative option might be to generate predicted probabilities equal to 0.9999999, as done in step 2 for a number of countries, and continue from there.

[31] In Stata 10, the storage format for the predicted probability (and all other predicted variables) is "double", which allows for the highest number of decimal places.

4. Generate new predictions $\hat{x}'$, using the same value for $\widehat{\bar{\eta}}_{ij}^{*}$ but the new predicted probabilities. Use the estimate $\hat{\delta}$ and $\hat{x}'$ to generate an alternative prediction of the non-linear term $\hat{\omega}' = \log(e^{\hat{\delta}\hat{x}'} - 1)$.

5. Generate the alternative predicted values for trade $\hat{m}'$ based on the new distance values and $\hat{\omega}'$ or the polynomial in $\hat{x}'$.

6. Calculate the elasticity $\frac{\hat{m}' - \hat{m}}{|\log 0.9|}$. For country-pairs with $\hat{\rho}' > 0.9999999$, assign the elasticity generated by a value of $\hat{\rho}' = 0.9999999$. For the polynomial approximation, this elasticity is 1.283 and for the non-linear specification, the number is 1.294. By doing this, HMR are taking a probability of 0.9999999 to imply a certain proportion of firms exporting. An alternative would have been to assume $\hat{\rho} > 0.9999999$ implies all firms export such that no effect occurs at the extensive margin. Then, the minimum elasticity would be that given by γ.

Table 1: Summary and contextualization of results

Question	Their method	Our method	Their results	Our results
Helpman, Melitz & Rubinstein (2008): How does firm heterogeneity affect estimates of gravity models using bilateral trade data?	• Generate consistent estimate of proportion of firms exporting for inclusion in gravity equation. • (Fixed effects control for multilateral resistance in estimation and generate consistent parameter estimates, but simulations ignore MR)	• Replicate HMR estimation procedure	1) Ignoring firm heterogeneity leads to overestimate of gravity model parameters > overstatement of the impact of reductions in trade frictions on bilateral trade	1.i) While the firm-level effect is overestimated, the country-level effect is *under*-estimated 1.ii) Bigger countries have smaller country-level effects (abstracting from multilateral resistance)
Anderson & van Wincoop (2003): How does accounting for multilateral trade frictions (“Multilateral Resistance” or MR) affect estimation and interpretation of gravity models using bilateral trade data?	• Estimate system of multilateral prices • (No controls for firm heterogeneity, which in many applications leads to inconsistent parameter estimates)	• Model MR effects using Taylor Approximation, drawing on Baier & Bergstrand (2009)	2) Ignoring MR leads to dramatic overstatement of the impact of multilateral reductions in trade frictions on bilateral trade 3) Overstatement is more severe for smaller countries -> bigger countries have larger trade responses to trade frictions	2.i) MR effects are so dramatic that many bilateral trade responses to multilateral reductions in trade frictions are negative 2.ii) For bilateral reductions in trade frictions, MR effects are usually immaterial 3.) The relationship between country size and trade response is non-monotonic

	1	2	3	4	5
	Linear	Probit	Poly	Pareto	Pareto (HMR07)
Dependent variable: Bilateral Exports					
distance	-1.176***	-0.660***	-0.862***	-0.799***	-0.801
	0.0269	*0.0206*	*0.0382*	*0.0352*	
border	0.458***	-0.382***	0.786***	0.832***	0.831
	0.118	*0.0941*	*0.149*	*0.124*	
island	-0.391***	-0.345***	-0.2	-0.17	-0.171
	0.107	*0.0751*	*0.12*	*0.0972*	
landlocked	-0.561***	-0.181	-0.482***	-0.447***	-0.448
	0.164	*0.105*	*0.135*	*0.128*	
legal	0.486***	0.0964**	0.385***	0.388***	0.388
	0.043	*0.0297*	*0.0437*	*0.0393*	
language	0.176**	0.284***	0.0454	0.0232	0.024
	0.0556	*0.0381*	*0.066*	*0.0593*	
colonial ties	1.299***	0.325	1.038***	1.003***	1.003
	0.132	*0.292*	*0.0919*	*0.0906*	
currency union	1.364***	0.492***	1.106***	1.024***	1.026
	0.216	*0.132*	*0.249*	*0.269*	
FTA	0.759***	1.985***	0.457***	0.378**	0.386
	0.162	*0.314*	*0.108*	*0.144*	
religion	0.102	0.261***			
	0.09	*0.061*			
inv. Mills ratio			1.131***	0.392***	0.399
			0.137	*0.0491*	
x			3.602***		
			0.348		
x^2			-0.782***		
			0.112		
x^3			0.0641***		
			0.0112		
δ				0.716***	0.716
				0.0494	
constant	3.168***	3.813***	4.197***	517.2	
	0.788	*0.371*	*0.517*	*764.8*	
Reference	HMR08:471	HMR07:37	HMR08:467	HMR07:37	
Observations	11146	24649	11146	11146	11146
R^2	0.709		0.721	0.718	0.709

Table 2: Regression results. Significance at 0.001, 0.01 & 0.05 levels denoted with ***, ** & *. Standard errors in italics (based on 50 bootstrap replications in columns 3 & 4).

variable	Pairs	mean	median	Std dev	max	min
Polynomial estimates	11146	1.8532	1.8464	0.5177	2.9955	1.1415
NL estimates	11146	1.5635	1.4682	0.2886	3.7771	1.2832

Table 3: Summary statistics for elasticity estimates

Bilateral

Countries		Shares				NL			Poly		
Exporter	Importer	share_iJi	share_iIj	share_jIj	share_jJi	Gross	Net	Multiple	Gross	Net	Multiple
Sample mean		0.0121	0.0140	0.0128	0.0141	1.5635	1.52462	0.9721	1.8532	1.8119	0.9721
Sample median		0.0012	0.0012	0.0019	0.0019	1.4682	1.450682	0.9930	1.8464	1.8183	0.9930
USA	Japan	0.1576	0.1575	0.2925	0.2926	1.2832	0.8257	0.6435	1.2940	0.8279	0.6398
Burkina Faso	Nigeria	0.0013	0.0017	0.00010	0.00008	1.9732	1.9697	0.9982	2.5621	2.5576	0.9982
Mexico	Spain	0.0172	0.0173	0.0174	0.0172	1.2929	1.2491	0.9661	1.1527	1.1136	0.9661

Table 4: Effect of MR for bilateral reductions in frictions. Each entry evaluated at sample mean/median. For example, the ratio of the mean Net : mean Gross elasticity is 0.975

Countries		Size			NL Elasticities		
Exporter	Importer	gdpexp	share_jIj	share_iIj	Gross	Net	Multiple
Bhutan	Japan	1.95E+08	4.21E-05	0.7234	1.4896	0.4121	0.2766
Equatorial Guinea	USA	2.27E+08	2.05E-05	0.5610	1.5010	0.6589	0.4390
Kiribati	USA	28432340	2.14E-06	0.4678	1.6451	0.8756	0.5322
Solomon Islands	USA	2.22E+08	1.62E-05	0.4538	1.5628	0.8536	0.5462
French Guiana	USA	2.37E+09	0.000163	0.4276	1.4085	0.8063	0.5724
Lao	USA	8.02E+08	5.41E-05	0.4191	1.5094	0.8767	0.5809
Chad	USA	9.70E+08	6.36E-05	0.4077	1.5172	0.8986	0.5923

Table 5: The lowest multipliers for bilateral reductions in trade frictions. Gdpexp gives the actual size of the exporter's GDP, while the shares account for trading partners.

Multilateral

		Intensive NL			Gross NL			Intensive Poly			Gross Poly		
Exporter	Importer	Intensive	Net	Multiple	Gross	Net	Multiple	Intensive	Net	Multiple	Gross	Net	Multiple
Sample mean		0.7987	-0.0958	-0.1199	1.5635	-0.0014	-0.0226	0.8622	-0.1034	-0.1199	1.8532	0.2036	0.0535
Sample median		0.7987	-0.1028	-0.1287	1.4682	-0.0852	-0.0582	0.8622	-0.1109	-0.1287	1.8464	0.1794	0.0972
USA	Japan	0.7987	0.1510	0.1891	1.2832	0.2521	0.1965	0.8622	0.1630	0.1891	1.2940	0.2804	0.2167
Burk F	Nigeria	0.7987	-0.1045	-0.1308	1.9732	0.2672	0.1354	0.8622	-0.1128	-0.1308	2.5621	0.4810	0.1877
Mexico	Spain	0.7987	-0.0883	-0.1105	1.2929	-0.1934	-0.1496	0.8622	-0.0953	-0.1105	1.1527	-0.3724	-0.3231

Table 6: Effects of MR for multilateral reductions in frictions. Each entry evaluated at sample mean/median.

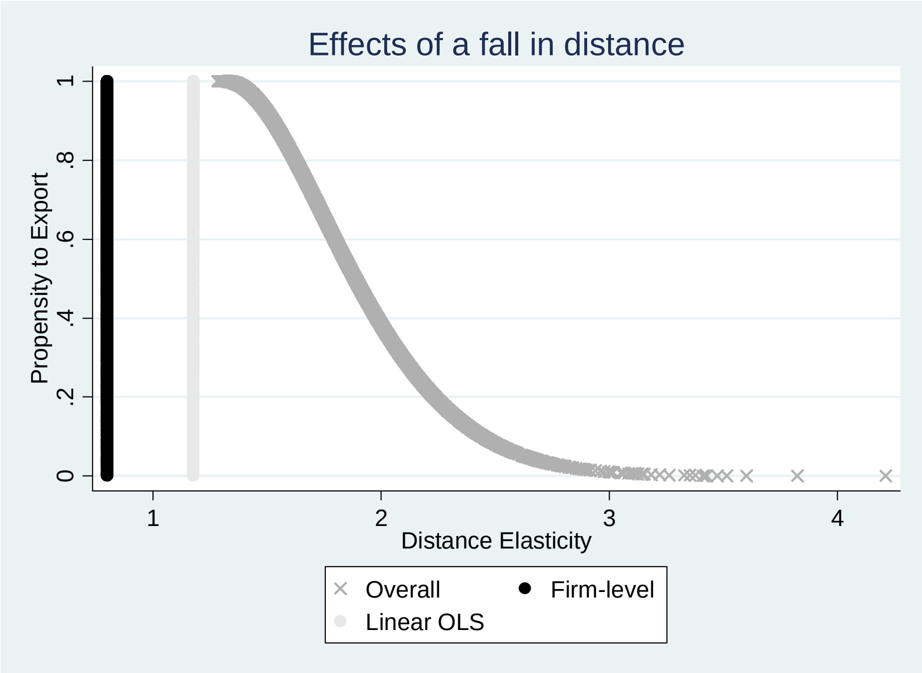

Figure 1a: Elasticity estimates for 11146 country pairs that trade, based on NL estimates. Our focus is the x-axis, which gives the absolute value of the elasticity of trade with respect to distance. Linear OLS estimate gives conventional estimates of the country-level effect (1.176). The other two estimates come from methods accounting for firm heterogeneity. Firm-level elasticities capture the rise in exports at the intensive margin (0.799 for the NL estimates) while Overall accounts for both the intensive and extensive margins.

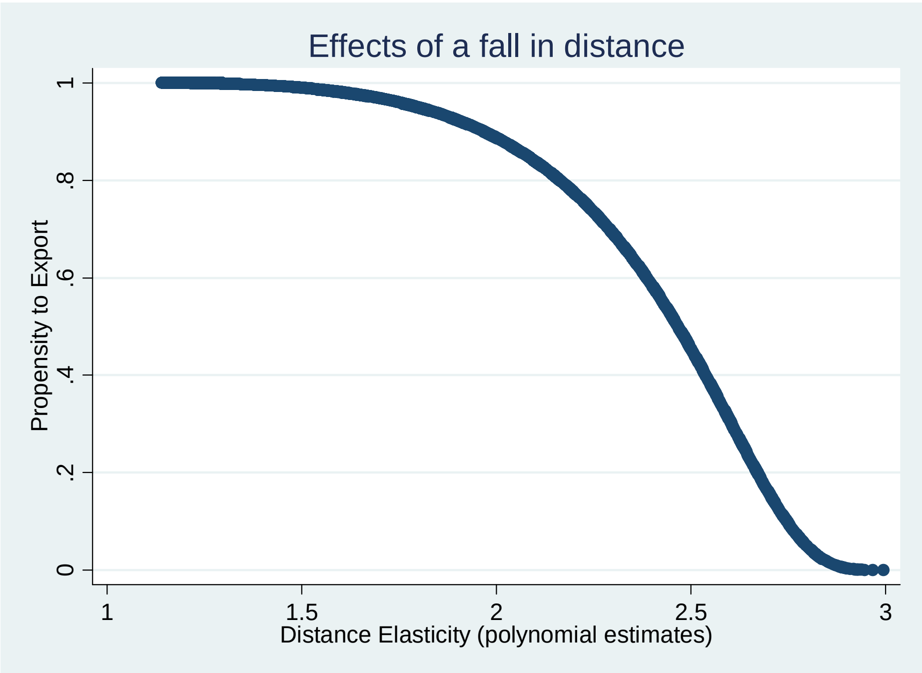

Figure 1b: The negative relationship between the propensity to export and the gross elasticity (polynomial estimates)

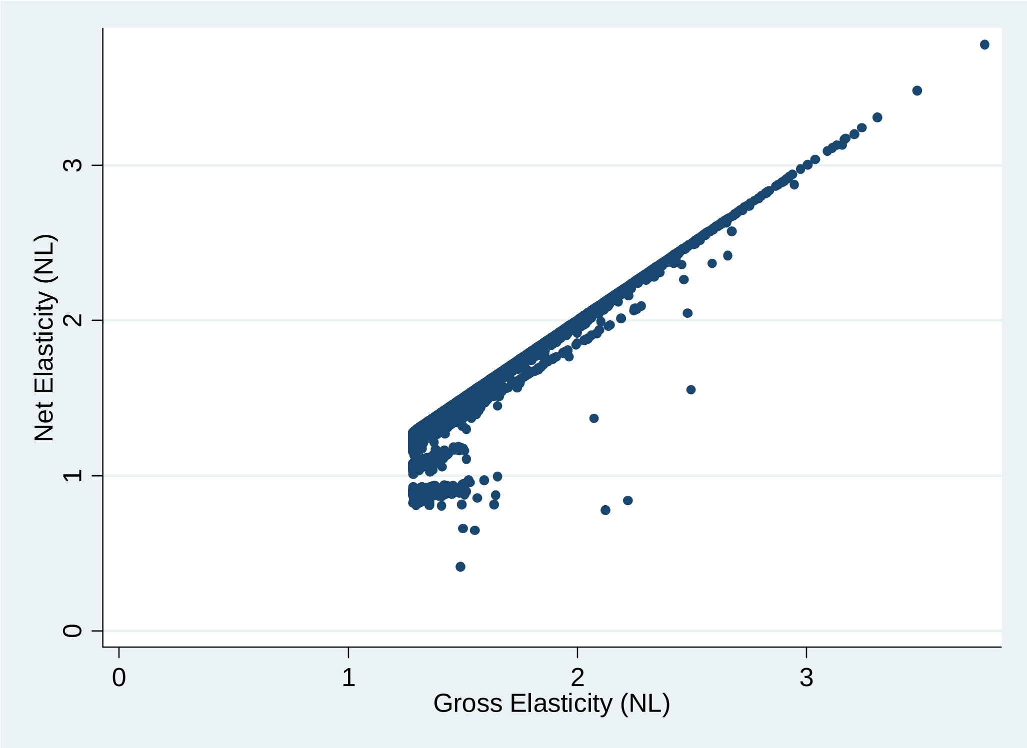

Figure 2a: Effects of MR for bilateral changes in distance (Pareto estimates)

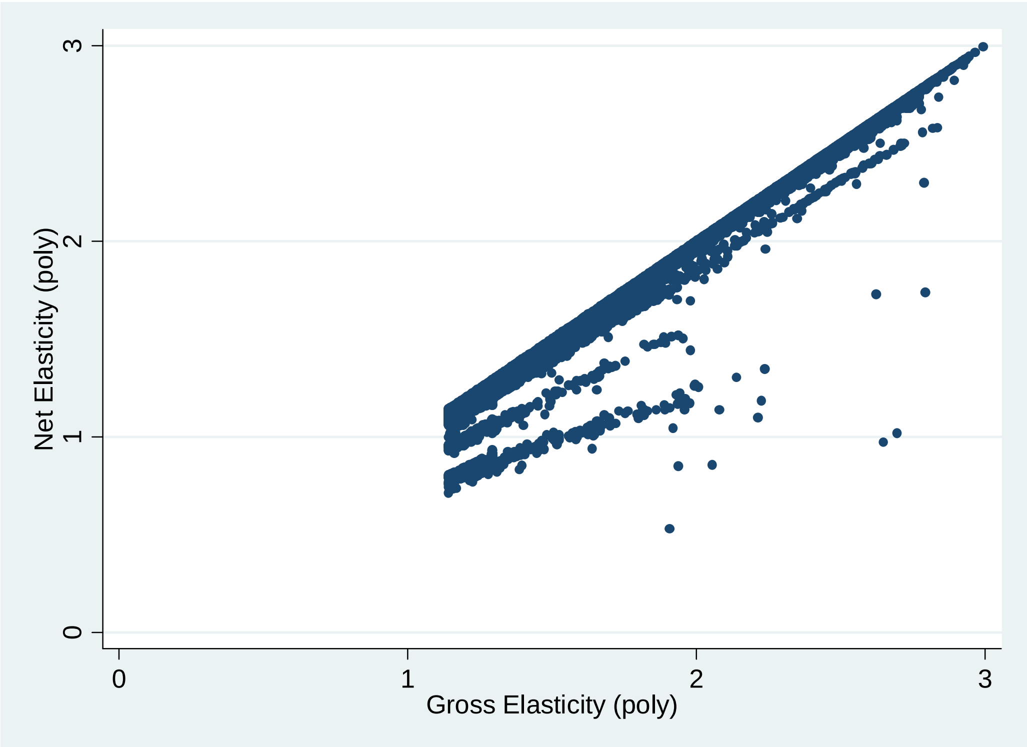

Figure 2b: Effects of MR for bilateral changes in distance (polynomial estimates)

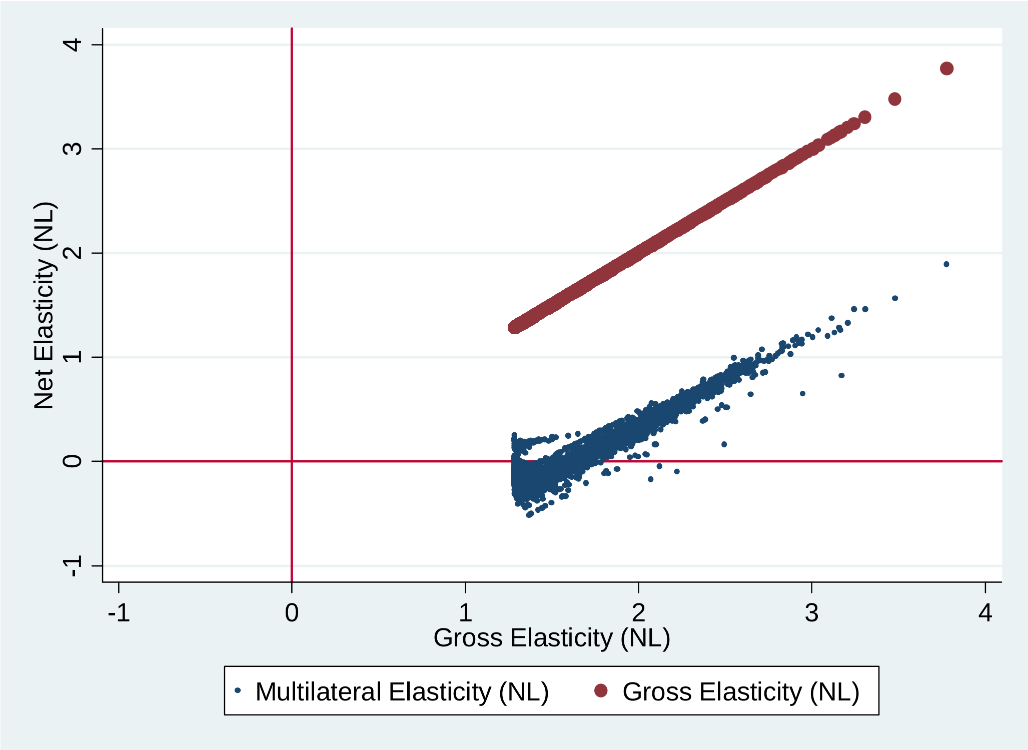

Figure 3a: Effects of MR when all countries reduce frictions (Pareto estimates)

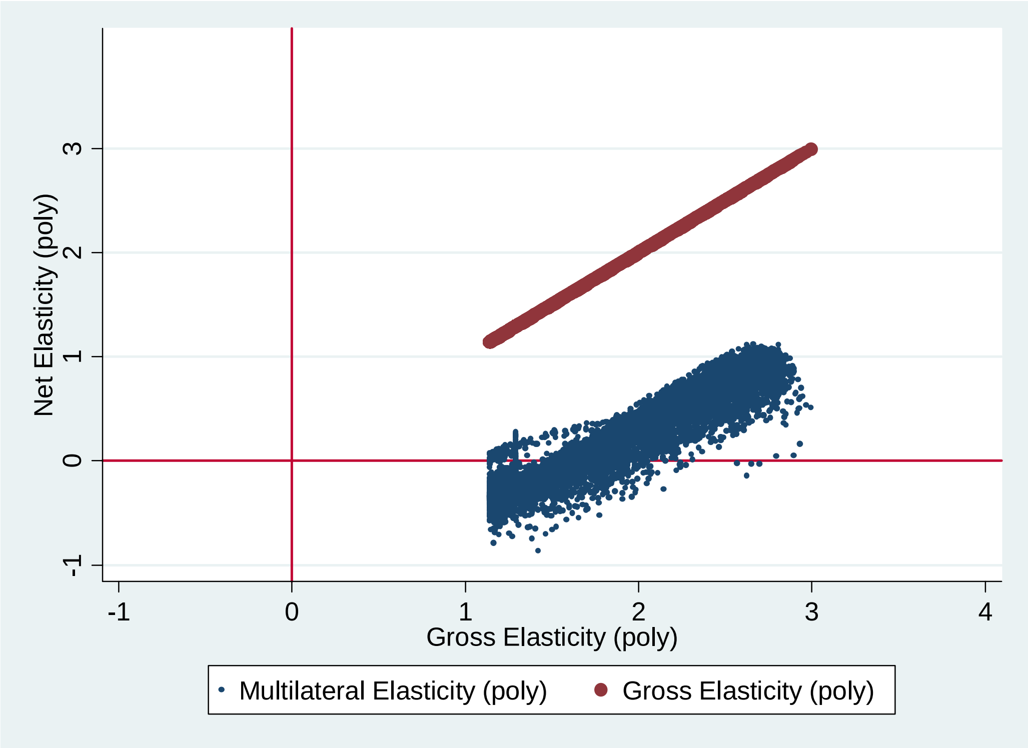

Figure 3b: Effects of MR when all countries reduce frictions (polynomial estimates)

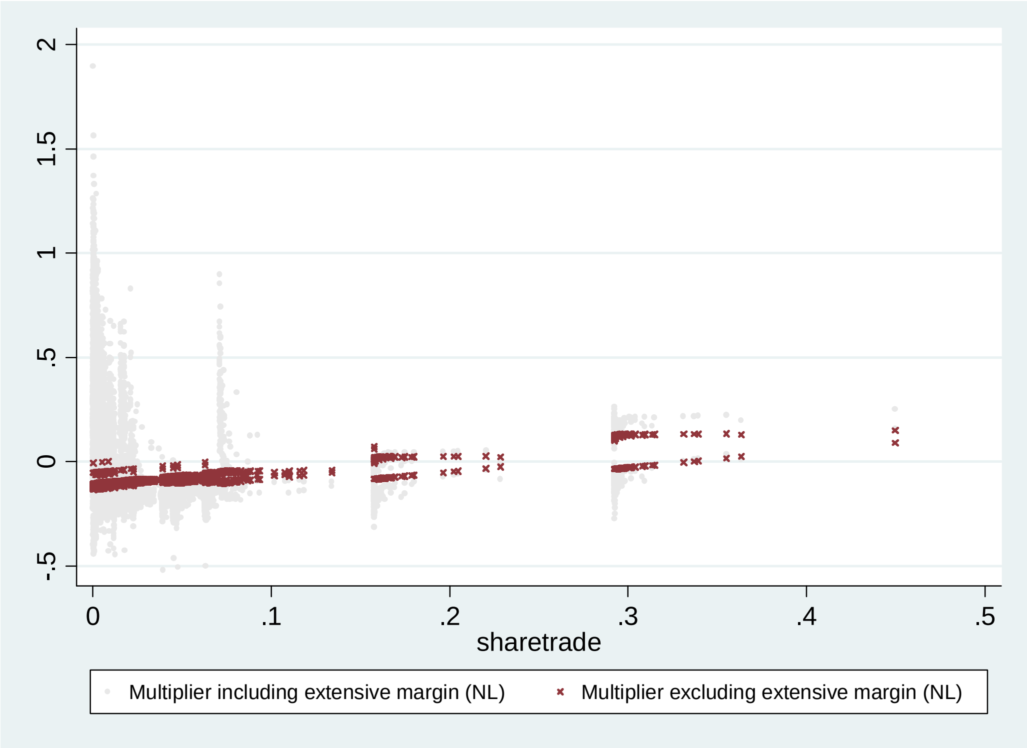

Figure 4: Multipliers and country size (NL)

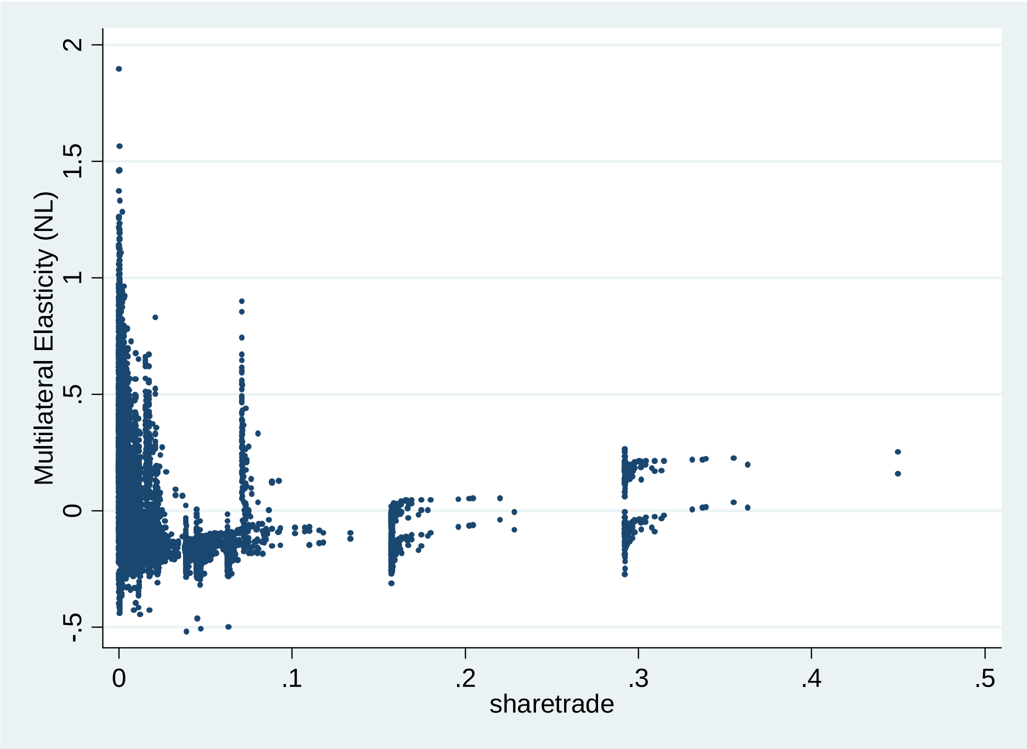

Figure 5a: Net elasticity when all countries reduce frictions (Pareto estimates)

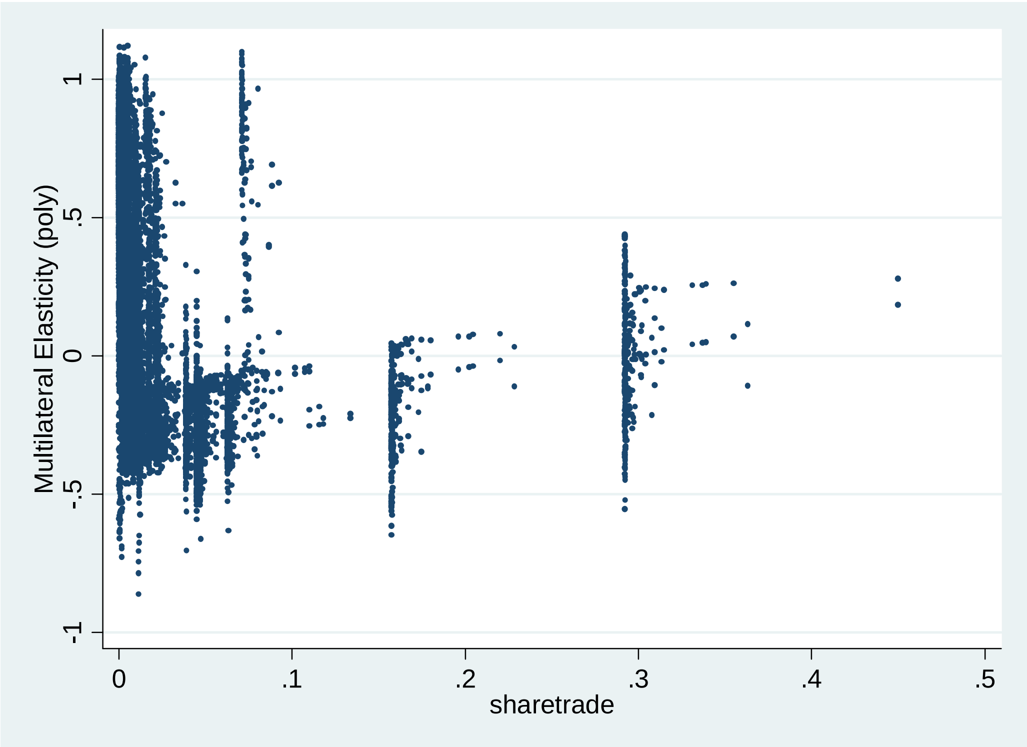

Figure 5b: Net elasticity when all countries reduce frictions (polynomial estimates)