A Search and Learning Model of Export Dynamics

(Preliminary and Incomplete)

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1 Introduction

In developing countries, policy makers often strive to establish new export markets for their country's products. By doing so they hope to create jobs, augment demand for their currency, and—in many cases—further their industrial sector development. Well-known success stories from Latin America include Brazilian regional jets, Chilean wines, and Colombian cut flowers.

Despite widespread interest in the determinants of industry-specific export surges, economists have yet to develop an analytical framework that reliably explains this phenomenon. Traditional gravity models—which relate bilateral trade flows to the GDPs of trading partners and the distances between them—focus on long run determinants of aggregate bilateral export flows, and are poorly-suited for the analysis of short-term industry-specific export dynamics.[1] Sunk-cost hysteresis models—which emphasize the start-up costs that new exporters face—help us explain patterns of foreign market entry and exit by individual firms (Dixit, 1989; Baldwin and Krugman, 1989; Das, et al, 2007). But aside from exchange rate effects, they treat the export volumes of firms that successfully penetrate foreign markets as determined outside the model. They thus provide little guidance as to why some exporters are able to rapidly expand their foreign sales while others struggle.

This paper explores the conjecture that export booms reflect a process of search and learning in foreign markets. That is, producers who are interested in a particular foreign market devote resources to identifying potential buyers there. When they find one, they learn something (receive a noisy signal) about the appeal of their products in this market. They

[1] Recent contributions to the gravity literature include Helpman et al (2008) and Anderson and van Wincoop (2003). Deardorff (1998) provides a survey of the earlier literature.

also learn about the scope for profits by observing the experiences of rival sellers of similar products in the same foreign market. Taking stock of the available information, home-market firms update their beliefs concerning the scope for export profits, and they adjust the intensity of their search efforts accordingly, attempting to maximize their net expected profit streams. Export booms take place when home-market firms receive positive early signals about the scope for profits—both from their own experiences and from the experiences of rivals—and they intensify their search/marketing efforts, adding quickly to their foreign client base.

The motivation for this paper comes from descriptive analysis of a decade's worth of individual merchandise shipments from Colombia to the United States. We begin by reviewing the stylized facts that come out of this analysis, including a number of finding that we have not reported in our earlier work (Eaton et al, forthcoming). Then we introduce our model, discuss its calibration, and demonstrate that, by adopting the assumptions mentioned in the previous paragraph, we are able to explain the basic features of the shipments data.

2 Firm-Level Trade: Transaction Level Evidence

The emprical motivation for our model comes from two sources. The first is a comprehensive data set that describes all shipments from Colombia to the United States (and elsewhere) that passed through Colombian customs during the period 1996-2005. Each customs record includes a date, the US dollar value of the product shipped, a 6-digit harmonized system product code (augmented by addition product information), a quantity index, a seller ID code, and the location of the buyer.[2] The second data base provides analogous information for the period 1992-2005. However it is based on U.S. Customs records, and it describes imports by buyers in the United States from Colombian exporters (as well as other sources). Critically, in addition to providing all of the information contained in the Colombian records, the U.S. customs data include ID codes for both sellers and buyers. It therefore allows us to identify the formation and dissolution of business relationships between individual buyers in the U.S. and sellers in Colombia.

2.1 Evidence from Colombian Customs Records

Following Brooks (2006) and Eaton et al. (2008), Table 1 provides various annual measures of Colombian exports to the United States for the years 1996-2007.[3] Each column follows an exporting cohort—i.e., a group of firms that began exporting in a particular year, after at least one year of no exporting—from the year of its appearance through time. (Since we don't know the history of firms before 1996, the 1996 "cohort" consists of all firms present that year regardless of when they began exporting.) The panels of the Table report number of exporters, total exports, and exports per firm, respectively.

[2] Because we use the same data that are used for official statistics, the merchandise exports in our data set aggregate to within one percent of total merchandise exports reported by the Colombian Bureau of Statistics (Departamento Administrativo Nacional de Estadística or DANE). The deviation is due to mistakes in the records of tax identifiers. Since following firms over time is central to our analysis, our database includes only records of transactions in which the tax identifier has the appropriate format. Not satisfying this requirement is a clear indication that the firm is not correctly identified in the record.

[3] Similar tables for Colombian exports to all destinations combined appear in Eaton, et al, 2008.

Table 1a: Number of Exporting Firms, by Entry Cohort

	Year of entry into U.S. market									
Year	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
1996	10,517	0	0	0	0	0	0	0	0	0
1997	4,414	6,049	0	0	0	0	0	0	0	0
1998	3,306	1,002	3,389	0	0	0	0	0	0	0
1999	2,718	617	938	2,492	0	0	0	0	0	0
2000	2,539	552	761	938	2,847	0	0	0	0	0
2001	2,418	523	700	735	1,113	3,348	0	0	0	0
2002	2,260	484	632	621	833	1,156	3,116	0	0	0
2003	2,200	465	578	553	697	903	1,048	3,655	0	0
2004	2,089	435	528	519	637	759	859	1,131	4,377	0
2005	2,051	420	362	407	505	568	578	769	1,000	5,060

Table 1b: Value of Exports, by Entry Cohort (millions of $US)

	Year of entry into U.S. market									
Year	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
1996	10,651	0	0	0	0	0	0	0	0	0
1997	11,182	369	0	0	0	0	0	0	0	0
1998	10,053	361	477	0	0	0	0	0	0	0
1999	10,514	421	392	241	0	0	0	0	0	0
2000	11,723	475	335	377	207	0	0	0	0	0
2001	10,373	483	296	395	525	233	0	0	0	0
2002	10,049	422	286	362	406	240	136	0	0	0
2003	10,651	490	358	381	546	228	222	251	0	0
2004	13,547	442	409	342	600	366	269	329	427	0
2005	16,207	725	451	588	891	435	295	349	585	665

Table 1c: Exports per Firm, by Entry Cohort (thousands of $US)

	Year of entry into U.S. Market									
Year	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**
1996	1013	0	0	0	0	0	0	0	0	0
1997	2533	61	0	0	0	0	0	0	0	0
1998	3041	360	141	0	0	0	0	0	0	0
1999	3868	683	418	97	0	0	0	0	0	0
2000	4617	861	440	402	73	0	0	0	0	0
2001	4290	923	423	537	471	70	0	0	0	0
2002	4446	872	452	584	487	208	44	0	0	0
2003	4841	1053	620	689	783	252	212	69	0	0
2004	6485	1016	776	658	942	482	313	291	98	0
2005	7902	1725	1247	1444	1764	766	510	454	585	131

Consider panel 1a first. Firms can drop out of the U.S. market and later reappear, which is why the number of firms in a cohort occasionally rises. But generally, each cohort's membership falls as it matures. Note that there is very high attrition the first year, with at least half and up to three-fourths of firms dropping out. Conditional on making it to the second year, the survival probability is much higher, however, with an average attrition rate around 20 percent. Thus, in terms of numbers, the most recent cohort is always larger than any previous one (excepting the 1996 cohort, which is a special case). Note that by the end of the period, firms that were around in 1996 constitute only about one in seven of firms exporting to the United States.

Panels 1b shows that, despite the rapid initial decline in its membership, the total sales of a cohort tends to rise over time, although quite unevenly. By the end of the period the 1996 cohort contributes about 76 percent of total sales, despite their relatively small number. The 2005 cohort contributes the second largest share.

The decline in number of firms per cohort along with their increasing contribution to total sales means, of course, that sales per firm are growing substantially (panel 1c). In fact, export sales for young survivors in each cohort tend to grow far more rapidly than total export sales, so that cohorts' market shares tend to rise *despite* rapid attrition during their early years. Finally, note that cohort size and success (in terms of survival and sales) vary substantially across entry years. For example, the 2005 cohort appears very robust both in terms of number of exporters and exports per firm, with 2006 weak by comparison. This suggests that entry selection mechanisms vary over time in response to market-wide forces.

2.2 Evidence from U.S. Customs records

Individual buyers and sellers are identified in the transaction level data collected by the United States Census Bureau. Accordingly, this data set allows us to keep track of how many buyers each Colombian exporter is shipping to, and to see when buyers are dropped or added. We next use these data to characterize the buyer-seller matchings that took place during our sample period of 1992-2005. The Table below provides some summary statistics:

Table 2

	Colombian Exporters	U.S. Importers	Exporter-Importer Pairs
start	3,742	1,265	5,297
end	5,297	2,214	8,046

The number of Colombian exporters appearing in the sample grew from 3,742 in 1992 to 5,297 in 2005, a growth of 3.5 while the number of U.S. importing firms grew by 4.4 percent. The number of Colombian exporter-U.S. importer pairs (representing at least one transaction between them in a year) grew at an annual rate of 3.3 percent. A typical Colombian exporter was involved in around 1.4 relationships with U.S. firms while a U.S. buyer was involved in around 4 relationships with Colombian firms. Both figures declined slightly over the period.

Most relationships are very short-lived. Of the buyer-seller matches that existed at the beginning of the period, 47 percent didn't make it to the 1993. But of those that survived into that year, almost 70 percent made it into the next year. Similarly, of the relationships that existed in 2005, 48 percent started that year, but of those that started the previous year, 65 percent had been around at least 3 years before. Of the matches present in 1992, only 85 endure (are present every year) throughout the period.

2.2.1 Transition Probabilities

The following Table reports the probability with which a Colombian firm participating in certain number of relationships with buyers transits into different number of relationships the following year. This table reports the annual average for 1992-1997 across all industries. Numbers for later periods are very similar. Thus, of firms not exporting to the United States at all in year t but that do export in year $t+1$, 92.5 percent sell to only one U.S. firm, etc. Of those that cell to one U.S. buyer in a year, 63 percent don't export the next year,while only about 6 percent go on to establish a larger number of relationships. For firms with two relationships in a year, about 14 percent enter into a larger number of relationships, etc. Hence there is an enormous amount of churning at the lower end. Even for firms with a large number of relationships the most likely outcome is to have fewer the next year.

Transition Probabilites: Number of Clients

t+1\t	**0**	**1**	**2**	**3**	**4**	**5**	**6-10**	**11-25**
0	**0.000**	0.630	0.265	0.153	0.050	0.024	0.039	0.000
1	0.925	**0.310**	0.344	0.246	0.131	0.079	0.039	0.000
2	0.056	0.046	**0.244**	0.222	0.202	0.211	0.087	0.000
3	0.012	0.010	0.096	**0.186**	0.223	0.168	0.082	0.000
4	0.004	0.003	0.031	0.116	**0.165**	0.184	0.117	0.000
5	0.002	0.000	0.012	0.045	0.108	**0.105**	0.169	0.380
6-10	0.002	0.000	0.004	0.016	0.113	0.205	**0.429**	0.620
11-25	0.000	0.000	0.004	0.016	0.009	0.024	0.039	**0 000**
sum	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

We can ask what this pattern of entry and growth implies about the ergodic distribution of relationships. If we assume that the number of entrants in a year replace exiters to the extent that the overall number of firms rises by 3.5 percent a year, the ergodic distribution implied by this transition matrix is given by:

Table 4

	1	2	3	4	5	6 to 10	11 to 25
ergodic	.809	.109	.039	.019	.010	.013	.002
period average	.800	.114	.041	.020	.010	.012	.003

For purposes of comparison the average annual share of Colombian firms in each group is reported as well in the period is reported as well. Note that the ergodic distribution implied by the transition matrix is very close to the distribution in the data.

3 A Model of Exporting at the Transactions Level

The tables above describe exports from a very different perspective than the standard one. Rather than considering exports as a flow over some interval of time, we are observing individual buyer-seller relationships. Existing trade models, including those emanating from the emerging literature on export activity by individual firms, model sales as flows to specific markets rather than as a discrete set of business relationships. (Arkolakis (2008) is an exception.)

We propose a model that is consistent with the patterns documented in the previous section, and that provides new micro foundations for export booms. It explains firm-specific export adjustments on three margins: clients (buyers) per destination market, per-period sales per client, and duration of the buyer-seller relationship.The model is consistent with four key patterns documented above: (1) many new exporting firms appear each period; (2) most new exporters sell tiny amounts and disappear from export markets in the following period; (3) those exporters who survive expand their export volume very rapidly over the following period, often accumulating additional buyers; (4) firms that sell more initially are more likely to survive into the following period.[4] It also explains (5) inter-temporal fluctuations in the size of the entering cohort, and (6) market-wide and relationship-specific fluctuations in per-period sales volumes.

The model builds on existing models of firm heterogeneity and exporting. As in Melitz (2003) and Bernard et al. (2003), firms are heterogeneous in terms of their underlying efficiency, with more efficient firms having greater incentive to overcome trade costs to sell in foreign markets. As in Das et al. (2007) and Irarrazabal and Opromolla (2007) firms experience shocks to their efficiency that lead them to switch into or out of exporting. As in Arkolakis (2008), by incurring a larger fixed cost a firm can increase the number of buyers it can reach. Finally, as in Rauch and Watson (2003), firms are initially uncertain about how their product will be received in an export market.

What we add to these models is the following structure of decision making and learning. Suppose that before it enters an export market a firm is unsure of the appeal that its product has to buyers there. However, the firm can invest in activities that bring its product to the attention of individual buyers, such as advertising, participation in trade fairs, and maintenance of a foreign sales office. The more a firm spends on these activities, the more likely it will encounter a foreign buyer per unit of time. And when a match does occur, its sale not only generates a profit for the firm, it conveys information to the firm about its product's appeal in that market. On the basis of this information the firm updates its beliefs about its product's ultimate chances for success in that market. Good news means that future matches are likely to be more profitable, so it strengthens its efforts to encounter buyers, while bad news discourages the firm from putting in so much effort.

[4] See EEKT (2008).

3.1 Profits

To characterize the profit flow, consider firm j with an efficiency φ_{jt} (taking into account transport costs) at time t. This efficiency is known to the firm and evolves over time with idiosyncratic shocks. Given that it pays a wage (or more generally, unit input price) w_t it can produce at cost w_t/φ_{jt} in terms of local currency. If the exchange rate is e_t, its unit cost in the foreign market is $e_t w_t/\varphi_{jt}$. So assuming that all foreign buyers have Dixit-Stiglitz preferences with known demand elasticity η, seller j offers price:

$$p_{ijt} = \frac{\eta}{\eta - 1} \frac{e_t w_t}{\varphi_{jt}} \tag{1}$$

to any foreign buyer i with whom it matches.[5]

If potential buyer i is confronted with an opportunity to purchase firm j's product, that is, if j matches with i, j's period t sales to i are:

$$X_{ijt} = \exp(z_j + \epsilon_{ij}) \left(\frac{p_{ijt}}{P_t}\right)^{1-\eta} \overline{X_t}. \tag{2}$$

Here z_j is a product appeal index that is common to all foreign buyers but is initially unknown to the firm. Whatever its true product appeal we assume that every firm starts out with a prior that is distributed $N(0, \sigma_z^2)$. The term ϵ_{ij} is an idiosyncratic appeal/spending shock that is distributed i.i.d. $N(0, \sigma_\epsilon^2)$ across foreign buyers, $\overline{X_t}$ is the average spending level among potential buyers, and P_t is the relevant price index for all competing products in the export

[5] For simplicity we assume that the firm makes a take-it-or-leave-it price offer. An alternative specification would introduce bilateral bargaining between buyer and seller, although the seller's uncertainty about the buyer's evaluation of the product renders this second approach substantially more complicated.

market.[6]

The flow profit in exporter currency implied by (1) and (2) is:

$$\pi(P_t, P_{Ct}, e_t, z_j, \epsilon_{ij}, \varphi_{jt}) = \frac{1}{\eta}\frac{\overline{X}_t}{e_t P_{Ct}} \exp(z_j + \epsilon_{ij}) \left(\frac{e_t w_t \eta/(\eta-1)}{\varphi_{jt} P_t}\right)^{1-\eta}, \tag{3}$$

where P_{Ct} is the price level in the home country. Or, combining all the aggregate variables and constants:

$$\pi(X_t, z_j, \epsilon_{ij}, \varphi_{jt}) = X_t \exp(z_j + \epsilon_{ij}) \varphi_{jt}^{\eta-1} \tag{4}$$

where:

$$X_t = \frac{1}{\eta}\frac{\overline{X}_t}{e_t P_{Ct}} \left(\frac{e_t w_t \eta/(\eta-1)}{P_t}\right)^{1-\eta}.$$

The term X_t summarizes all the macroeconomic information about the export market (i.e., information that applies to any seller j when matched to any buyer i). We can thus characterize the aggregate state of demand in the export market with X_t. We assume that X and φ evolve over time according to a Markov process, so that given (X_t, φ_{jt}) in period t, the period $t+1$ values have a joint distribution $G(X', \varphi' | X_t, \varphi_{jt})$. The difference between X and φ is that the first applies to all firms while the second is idiosyncratic to a specific firm. Hence the first generates behavior that is correlated across firms while the second is independent.

For purposes of the dynamic optimization problem to be introduced below, it will be convenient to define $\widetilde{\pi}(X_t, z_j, \varphi_{jt})$ as the expected present value of firm j's entire profit profit stream associated with a match as perceived at time t. That is, $\widetilde{\pi}(X_t, z_j, \varphi_{jt})$ is the discounted

[6] Not all buyers necessarily face the same range of goods and hence the same aggregate price index P_t. We treat idiosyncratic components of the price index as P_t as reflected in ϵ_{ijt}.

expected future value of the $\pi(X_t, z_j, \epsilon_{ij}, \varphi_{jt})$ trajectory, with expectations taken over ϵ_{ij} and the future trajectory of (X_t, φ_{jt}). The value of $\widetilde{\pi}(X_t, z_j, \varphi_{jt})$ depends on the firm's discount rate r and the hazard of separating from a particular buyer, which we treat as occuring at the exogenous rate δ. Thus $\widetilde{\pi}(X_t, z_j, \varphi_{jt})$ solves:

$$\widetilde{\pi}(X_t, z_j, \varphi_{jt}) = X_t E_\epsilon \left[\exp(z_j + \epsilon_{ij})\right] \varphi_{jt}^{\eta-1} + \frac{1}{r+\delta} \int_{X'.} \int_{\varphi'} \widetilde{\pi}(X', z_j, \varphi') dG(X', \varphi' | X_t, \varphi_{jt}). \quad (5)$$

This expression gives the value to firm j of matching with a buyer at time t, conditioned on macro conditions, X_t, the firm's product appeal index, z_j, and productivity, φ_{jt} Note that, having met this buyer and imputed $z_j + \epsilon_{ij}$ (as discussed below), there is nothing more for firm j to learn from her about product appeal. Nevertheless its sales to this buyer will continue to fluctuate in response to macroeconomic shocks X and shocks to the firm's efficiency φ.

3.2 Information about product appeal

Firm j knows neither z_j nor ϵ_{ij} individually, but each time the firm matches with another buyer it learns more about z_j. That is, knowing the macro state $(\overline{X}_t)$, the price index (P_t) and its own price (p_{ijt}), firm j can compute the signal:

$$s_{ij} = z_j + \epsilon_{ij}.$$

of its product appeal in foreign markets. Combining this signal with the signals it inferred from earlier matches, and with the prior beliefs $N(0, \sigma_z^2)$ it held about z_j before any matches occurred, firm j calculates the posterior distribution of z_j to be $N(\widehat{z}_j^n, \sigma_n^2)$, where:

$$\widehat{z}_j^n = z_j^0 \frac{\sigma_z^{-2}}{\sigma_z^{-2} + n\sigma_\varepsilon^{-2}} + \overline{s}_j^n \frac{n\sigma_\epsilon^{-2}}{\sigma_z^{-2} + n\sigma_\epsilon^{-2}}, \quad (6)$$

and:

$$\sigma_n = \left(\sigma_z^{-2} + n\sigma_\epsilon^{-2}\right)^{-1/2}, \tag{7}$$

n is the number of matches the firm has experienced, and $\overline{s}_j^n = n^{-1}\sum_{i=1}^n s_{ij}$.

This characterization of Bayesian learning generalizes to allow for correlation between a firm's product appeal in the foreign market and (1) its product appeal at home (d), or (2) the average appeal of rival (domestically-based) firms' products in the foreign market (y). To do the first, one can incorporate signals from domestic sales, $s_{jt}^d = \beta_d z_j + \epsilon_t^x$, where $t = 1, ...T$ indexes the periods that the firm has been in operation. To do the second one can incorporate signals from the foreign sales of rival firms in the same industry $s_{kt}^y = \beta_y z_j + \epsilon_{jkt}^y$, where $k = 1, ...n_{yt}$ indexes the number of such rival matches that the firm has observed as of time t. These modifications lead to straightforward generalizations of (6) and (7). Among other things, they generate learning spillovers that accelerate aggregate export responses to positive early experiences in new foreign markets.

3.3 Search intensity

As information accrues to a seller about foreign buyers' demand for its product, it adjusts the intensity with which it searches for new buyers. Let the firm experience new matches with hazard λ when it spends $c(\lambda)$ on search activities, where $c(\cdot)$ is increasing and convex.[7]

[7] Following Arkolakis (2008), if we think that the market has M potential buyers and sampling occurs without replacement we can generalize the hazard rate to be $\widetilde{\lambda} = \lambda \cdot h(n)$ where $h(n)$ is decreasing in n, bounded on [0,1], and $h(M) = 0$. For example, if the probability of a match is proportional to the pool of potential buyers who have not yet been visited, this function might take the form: $h(n) = \frac{M-n}{M}$. Working

Then if the firm has received an average signal of $\overline{s}_n$ after n encounters, the value of continued searching in the foreign market is $V(\widehat{z}_j^n, n, X_t, \varphi_{jt})$, where:

$$\begin{aligned} & V(\widehat{z}_j^n, n, X_t, \varphi_{jt}) \qquad (8) \\ = \; & \max_{\lambda} \left\{ -c(\lambda) + \lambda \int_z \widetilde{\pi}(X_t, z, \varphi_{jt}) dF(z|\widehat{z}_n, n) \right. \\ & \left. + \frac{1}{1+r} \int_{X'.} \int_{\varphi'} \left[(1-\lambda) V(\widehat{z}_j^n, n, X', \varphi') + \lambda \int_{\widehat{z}'} V(\widehat{z}', n+1, X', \varphi') d\Phi(\widehat{z}'|\widehat{z}_j^n) \right] dG(X', \varphi'|X_t, \varphi_{jt}) \right\}. \end{aligned}$$

Here r is the discount rate, $F(z|\widehat{z}_n, n)$ is the posterior distribution for z after the n^{th} match, and $\Phi(\widehat{z}'|\widehat{z}_j^n) = N(\widehat{z}_j^n, \sigma_{n+1})$ is the posterior distribution for z that the firm expects to prevail after the $n + 1^{st}$ match, given $\widehat{z}_n$.

Three margins of firm-level export response are characterized by this value function. First, average sales per foreign transaction at time t ($\overline{X}_{jt}$) are determined by product appeal (z_j), productivity (φ_{jt}), and macro conditions (X_t). Second, buyers per firm are governed by the search intensity, λ, which responds to sales history (X_{ijt-n}). (The case of zero buyers corresponds to non-participation in export markets, of course, but it does not imply $\lambda = 0$.).

3.4 Stationary State

We consider an environment where firms are buffeted by shocks to their macroeconomic environment and to their own productivity, and in which they start out ignorant of their product appeal abroad, but learn about it over time. Hence some key variables in our model are highly

against this effect is the possibility that as matches accumulate, a firm's reputation grows, and it becomes *less* costly to reach new custumers. Hence a general expression for $h(n)$ that does not impose a sign on its derivative may be the most appropriate formulation. If this function is identified, it provides a test of Arkolakis (2008).

nonstationary, and it is necessary to use numerical techniques to characterize its transition dynamics. Nevertheless it is useful to consider what happens in a stable environment in which all learning has taken place.

We thus ask what happens if $(X_t, \varphi_{jt}) = (X', \varphi')$ and $n \to \infty$ so that $\overline{s}_n \to z$ and new matches convey no further information. Asymptotically, the distinction between $V(\widetilde{z}_n^j, n, X', \varphi')$ and $V(\widetilde{z}', n{+}1, X', \varphi')$ disappears, and the problem becomes $rV(z) = \max_\lambda \left\{-c(\lambda) + \lambda \widetilde{\pi}(X, z, \varphi_j)\right\}$. The solution is:

$$V(z) = \frac{-c(\lambda^*) + \lambda^* \widetilde{\pi}(X, z, \varphi_j)}{r},$$

where λ^* solves $c'(\lambda^*) = \widetilde{\pi}(X, z_j, \varphi_j)$. So, not surprisingly, steady state search efforts and the present value of participating in foreign markets are monotonically increasing in the payoff to a successful match. As in Arkolakis (2008), more efficient firms (with higher φ_j) undertake more search effort and encounter more buyers. However, firms learn about their product appeal as acquire buyers in our model, they adjust their search intensity accordingly, and they lose buyers over time as matches go sour. In a stationary equilibrium–with no macro or idiosyncratic shocks, and after all learning has taken place— firms settle into constant search intensities. If firm j chooses match hazard λ_j^* in this stationary equilibrium, it sells to an average number of buyers $n(j)$ that satisfies $\delta n(j) = \lambda_j^*$.

3.5 Specification for Numerical Solution

To solve the problem numerically we parameterize the cost of matching as:

$$c(\lambda) = b\left(\frac{\lambda}{1-\lambda}\right)^2 + f \cdot 1\left[\lambda > 0\right], \quad \lambda \epsilon [0, 1) \tag{9}$$

where f is the fixed cost of maintaining positive levels of search. Finally, we (arbitrarily) choose parameter values to be as indicated in Table 1. We treat shocks to efficiency and macroeconomic shocks as following independent first-order autoregressive processes, so that:

$$\begin{aligned} \varphi_{jt+1} &= \psi\varphi_{jt} + \epsilon_t^{\varphi} \\ X_{t+1} &= \phi X_t + \epsilon_t^X \end{aligned} \tag{10}$$

where:

$$\begin{aligned} &\ln \epsilon_t^{\varphi} \,\tilde{}\, N(0, \sigma_{\varphi}^2) \\ &\ln \epsilon_t^{X} \,\tilde{}\, N(0, \sigma_X^2). \end{aligned} \tag{11}$$

The model thus has four sources of variance: each firm's underlying true product appeal z, which is drawn from $N(0, \sigma_z^2)$, the noise around true product appeal associated with each match, which is drawn from $N(0, \sigma_\epsilon^2)$, the shocks to productivity, ϵ_t^{φ}, and the macroeconomic shock, ϵ_t^X, both given in (11).

The model is fully described by the expression for profit (3), from which we can calculate, using (10), the expected value of a relationship (**??**), the equation for updating beliefs about product appeal (6), the value function (8) and the cost function (9). The complete set of parameters of the model are given in the following Table:

Table: Parameters for Simulations			
Parameter		*base value*	*alternative calibration*
rate of time preference	r	0.05	0.05
rate of separation	δ	0.25	0.20
search cost function scale parameter	b	0.20	0.20
profit function scale parameter	c	0.00001	0.05
fixed cost of searching	f	0.01	0.02
standard deviation of noise in signal	σ_ϵ^2	1.67	0.30
standard deviation of product appeal	σ_z^2	1.76	0.30
root of efficiency process	ϕ	0.90	0.90
root of macro process	φ_X	0.80	0.80
standard deviation of efficiency innovation	σ_ϕ^2	0.10	0.10
standard deviation of macro innovation	σ_X^2	0.40	0.16

where we indicate the values we place on them in our baseline simulation.

3.6 Calibration

To calibrate our simulations, we draw parameter values from a variety of sources. The separation rate δ is based on match-specific survival rates among all Colombian exporters to the U.S. market. The root φ_X and innovation variance σ_X^2 for the market-wide shock are based on a simple AR1 fit to the Colombian real exchange rate. The root ϕ and variance σ_ϕ^2 are based on plant-level panel-based estimates of productivity processes among Colombian firms. The cross-buyer variance in product appeal for a given seller, σ_ϵ^2, is estimated using the subsample of matches corresponding to sellers with more than one buyer, and is calculated as the residual variation in match value after controlling for buyer fixed effects. The variation in product appeal effects, σ_z^2, is calculated as the variation in match value (using the full sample of matches) less σ_ϵ^2. The profit function scaling paramter is chosen to equate average match value to the average match value in our sample. Finally, for this draft, the cost function parameters b and f are chosen to generate dynamics that qualitatively replicate those reviewed in section 2 above.

Clearly, this calibration is very crude and preliminary. Among other problems, the estimates of σ_z^2 and σ_ϵ^2 are subject to selection bias, and they presume that firms do not distinguish sub-markets for particular types of products from each other. For example, an exporter of men's shirts is presumed to face markets with the same characteristics as an exporter of flat-rolled steel. We therefore experient with an alternative calibration in which we cut both variances by roughly a factor of 5. (This adjustment required an associated adjustment in the profit function scaler, c.) In future work, the model will be econometrically estimated.

3.7 Numerical Procedure

Our numerical solution proceeds in four steps.

3.7.1 Markov transition matrices for signals

With our parameterization learning occurs very fast. After encountering twenty buyers the seller's uncertainty about its product's true appeal has dwindled, according to (7), to having a standard deviation of .07. Hence we limit the number or encounters that convey information about product appeal to twenty, and treat the seller as fully informed about its own product appeal thereafter. For transitions across the first twenty periods we discretize the space of noisy signals into 50 signals evenly spaced across 3 standard deviations, and calculate the Markov transition matrices across each possible pair, using (7).

3.7.2 Markov transition matrices for macroeconomic and efficiency processes

We follow a similar procedure of discretizing efficiency levels into 20 possible values and the macroeconomic states into 8 possible values. We then calculate the Markov transition probabilities across them. For purposes of our simulations we assume that all macro uncertainty comes from the exchange rate, and we fit an AR1 process to the dollar-peso exchange rate to impute transition probabilities.

3.7.3 Discretization of Effort

We allow for 50 possible effort levels across $[0, 1]$.

3.7.4 Value Function Iteration

We solve for the value function V and associated policy function for λ that solves (8) for different numbers of meetings n, profit signals i, efficiencies k, and macroeconomic shocks m.

3.8 Policy functions

The first panel of figure 1 above shows the value of access to foreign buyers that firms perceive after one signal, as a function of the signal they have received. Not surprisingly, there is a positive relationship, and firms that receive better signals choose to search more intensively. The second panel of this figure shows how values and search intensities have changed after 5 signals have accrued. (The horizontal axis is the posterior mean after 5 signals, $\widehat{z}^5$.) Note that the value of search has fallen relative to its value after one signal for those firms with low average signals because these signals become increasing precise as experience accumulates. (When five buyers tell you they don't care for your product, there is a good chance that your product has poor market potential.) The last two panels of figure 1 translate search values into match probabilities, and tell the same qualitative story. Below some threshold signal, the return to search is less than the associated fixed cost (f), and so no search takes place. If f were to increase, this cutoff would shift to the right (not pictured).

Figure 2 shows how the policy function characterized in figure 1 translates into behavior for a simulated set of 1,000 firms. Here the horizonal axis is true z value rather than signal. The first panel describes match hazards for a new cohort of firms, none of which has received any signals yet. Since all firms share the same priors at this point there is no relationship between z values and search intensity. However, some firms don't search at all because their current productivity level is low. After 5 periods, a relationship between z and search intensity emerges, but heterogeneity in behavior remains, given z. (Refer to the bottom panel.) This reflects both productivity differences and differences in the idiosyncratic features of the buyers (ϵ's) that the exporters have randomly matched with. It also reflects the magnitude of fixed search costs.

Figure 3 depicts match probabilities as functions of exporters' productivity levels and the macro state. Not surprisingly, improvements in either encourage search. The shape of the function changes over time, however, as seen in figures 1 and 2. In particular, after 5 matches have accrued, those firms with relatively low productivity have been convinced to stop looking for buyers, and the truncaction point is particularly high when the macro state (i.e., the real exchange rate) is poor. Thus macro conditions may explain the cross-cohort variation in export market participation seen in Table 1 above.

3.9 Export Trajectories

To link this model back to export flows, we need simply keep track of match patterns, random productivity shocks, and random separation patterns for a simulated set of firms. For these simulations we use the actual real dollar-peso exchange rate, but we retain the assumption that firms base their beliefs about future exchange rate realizations on the AR1 process we have estimated. (That is, firms are assumed to have rational expectations, not perfect foresight.)

Figure 4 shows clients per exporter and total exports for 1,000 firms exploring a new market that appears in year 0. (Imagine a Caribbean island dismantling prohibitive trade barriers, and the population of Colombian producers commencing to learn about the islanders' demand for their products.) Note that, although the number of exporters falls over time (not pictured), clients per surviving exporter and total exporters both rise. They climb particularly rapidly in the early years because the firms which have received positive signals tend to (1) be exporting relatively more, and (2) intensify their search as these signals accrue. Eventually firms learn their true z values and settle into a stable search intensity. This translates into a stable number of clients pe firm.

Figure 5 shows the five year transition densities for numbers of clients, providing a simulated analog to Table 3 above. The top panel describes transitions between year 1 and year 5, when firms are in their early learning stages. Because they have not yet built their clientele, most of the action involves movement between 0, 1 and 2 clients. After 5 periods have elaspsed, many firms that have low product appeal have dropped out, and the remaining firms have built larger client bases (bottom panel).

The export aggregates associated with this new market exploration are depicted in the panels of figure 6. Each of these graphs also includes the simulated exhange rate series, which appears as the lower line.[8] The first panel indicates that during the early years, exchange rate effects are dominated by learning effects, but eventually, exports start to track the exchange rate–both because shipment values depend upon the exchange rate and because search intensities increase when the Colombian peso depreciates (second panel and fourth panel). The number of exporters is not very sensitive to the exchange rate because matches last for multiple periods, and exporters have no incentive to terminate clients when appreciation occurs. These features of the model induce a kind of hysteresis in trade flows. However, instead of attributing irreversibilities to one-time market entry costs, as in the existing literature, we attribute irreversibiliteis to the fact that devaluation induces learning but appreciation does not induce forgetting.

Table 1 above defined the year t cohort to consist of all firms who exported in period t but did not export in period t-1. Applying this definition to our simulated data, we obtain figure 7. The four panels correspond to the cohorts that enter in years 2, 3, 4 and 5. Consistent with table 1, our model shows that membership in each cohort drops off rapidly after its first year. (Refer to the lower line each graph.) However, total exports go through a growth period as those cohort members who survive add to their client base, and thus exports per suriviving cohort member climb rapidly in the early years of each cohort's existence. This, too, matches up well to the actual data.

4 Econometric estimation (to come)

[8]Note that the units differ across graphs. The exchange rate process used in this experiment is obtained by fitting a simple AR1 to real Colombian peso-dollar rate (1982-2007).

5 Conclusions (to come)

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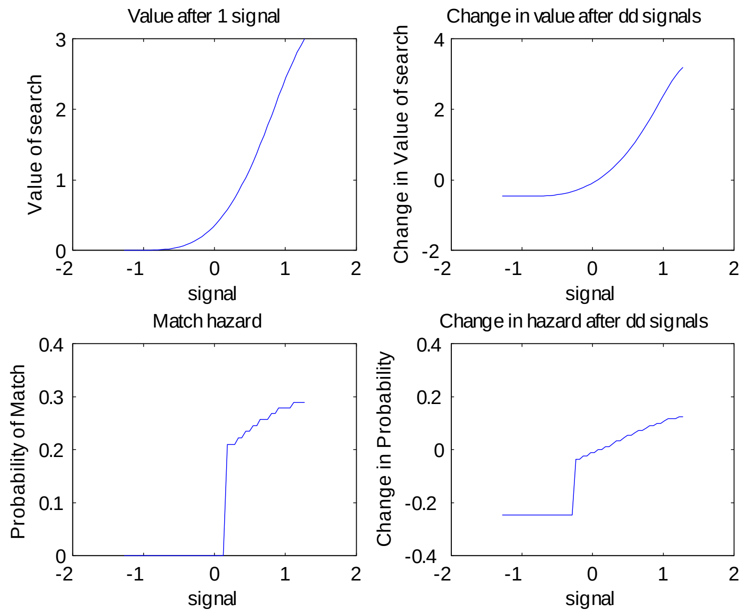

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Figure 1:

Signal, value, match hazard, and learning

2

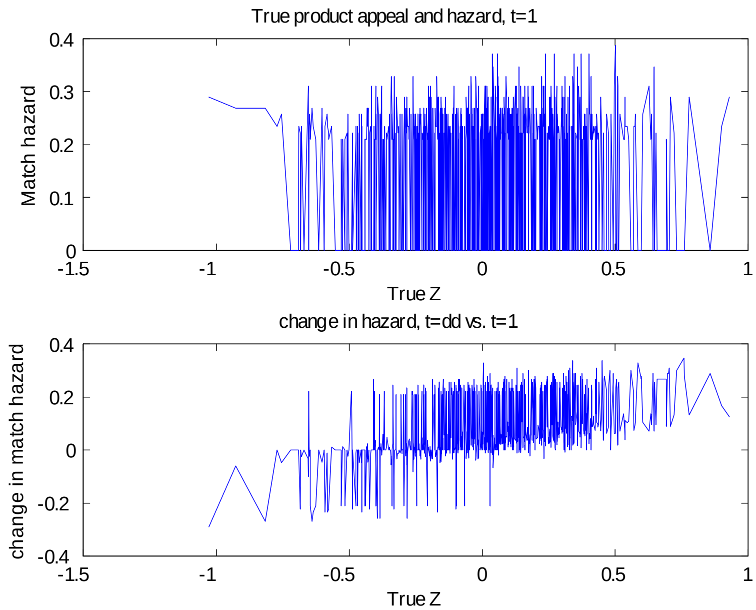

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Figure 2:

True product appeal and match hazard: initial and change after 5 signals

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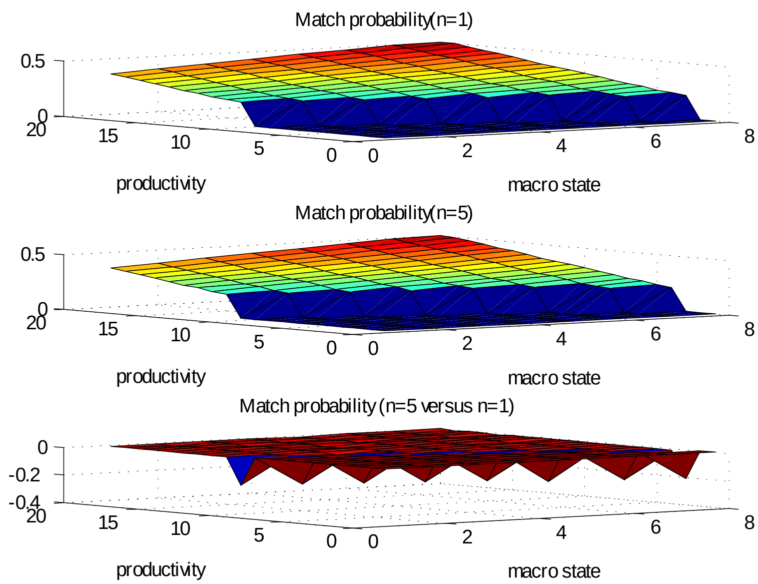

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Figure 3:

Match Probabilities, firm productivity, and macro state

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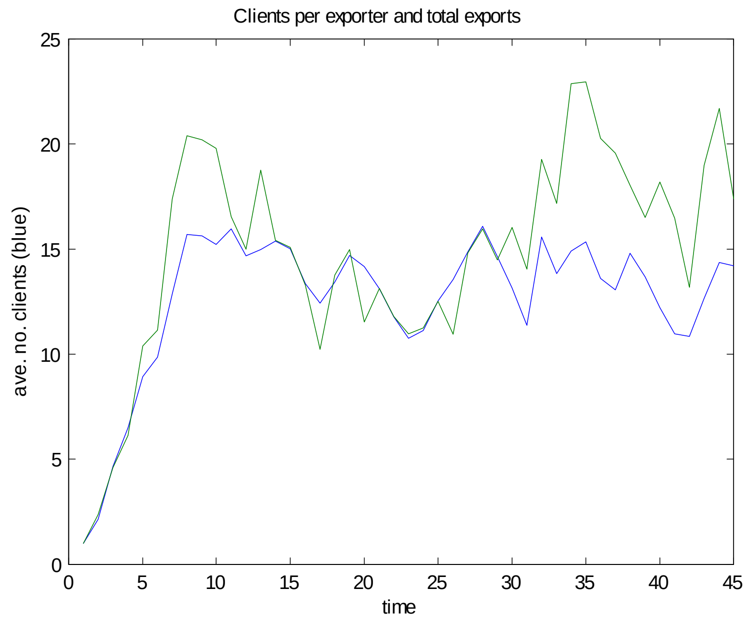

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Figure 4:

Clients per exporter and total exports: new cohort through time

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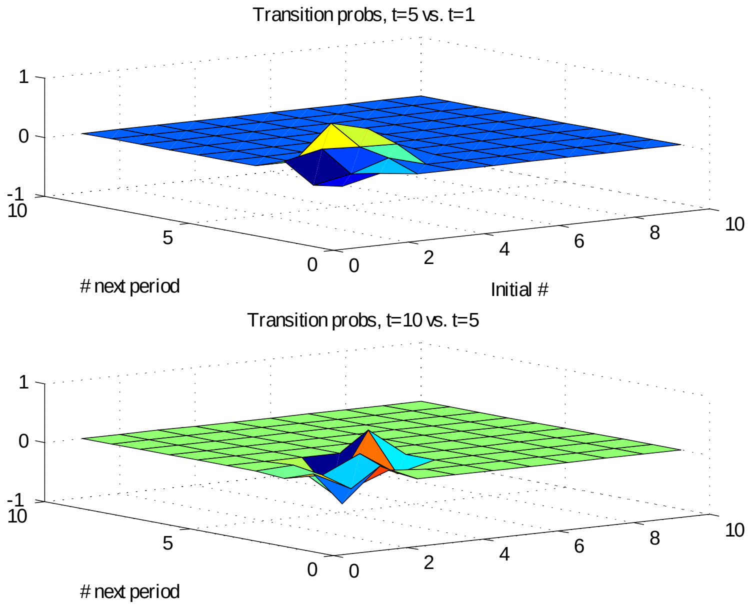

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Figure 5:

5-period transition probabilities (conditional densities)

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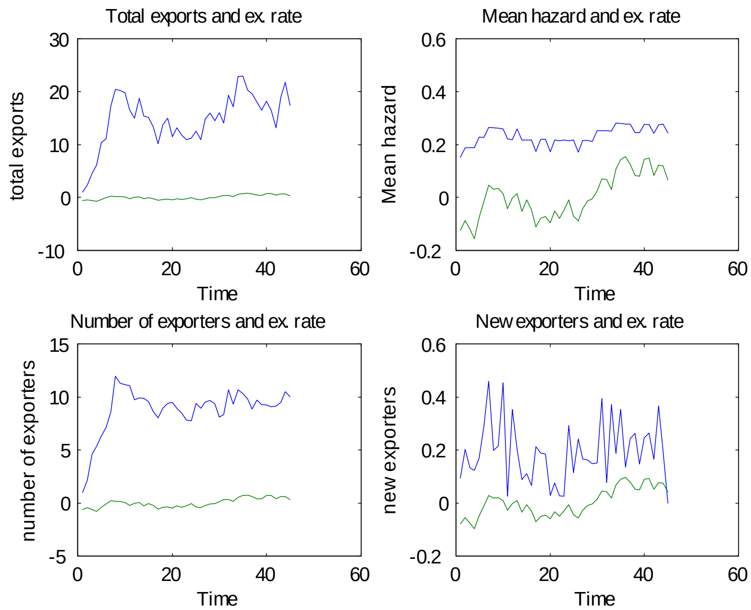

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Figure 6:

Export aggregates and the real exchange rate (new cohort)

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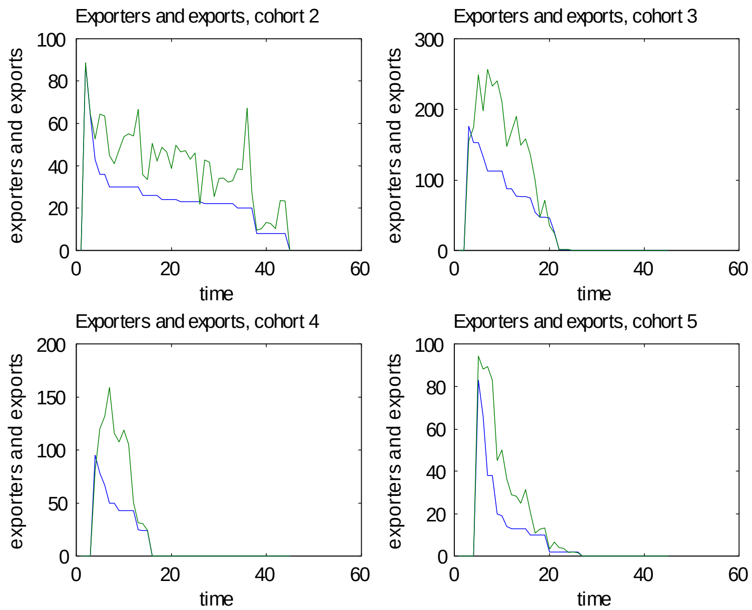

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Figure 7: Cohort-specific exports and exporters