Large firms and heterogeneity: the structure of trade and industry under oligopoly *

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ABSTRACT: We develop a model of trade with endogeneity in key features of industry structure linked to heterogeneous cost structures under Cournot competition. As such we address firm heterogeneity in international trade in a setting with strategic interaction between firms as opposed to existing models where firms are atomistic. Entry/exit dynamics are therefore more complex but also richer. The equilibrium value of the main variables like market price and total sales wander around as firms enter and exit. The model nests two workhorse trade models, the Brander & Krugman reciprocal dumping model and the Ricardian technology-based trade model, as special cases. We examine both free entry and limited entry (free exit) cases. The model generates testable predictions on the probability of zero trade flows and the pattern of export prices and generates endogenous moves from zero to non-zero trade flows due to entry and exit. As two countries move from autarky to free trade, market prices fall, but along the path from autarky to free trade, market prices might increase at certain points. Also with lower trade costs the least productive firms get squeezed out of the market, exporting firms gain market share, and more firms become trade oriented.

Keywords: Firm heterogeneity, Oligopoly, Composition effects of trade liberalization

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Heterogeneous Firms, the Structure of Industry, & Trade under Oligopoly

ABSTRACT: We develop a model of trade with endogeneity in key features of industry structure linked to heterogeneous cost structures under Cournot competition. As such we address firm heterogeneity in international trade in a setting with strategic interaction between firms as opposed to existing models where firms are atomistic. Entry/exit dynamics are therefore more complex but also richer. The equilibrium value of the main variables like market price and total sales wander around as firms enter and exit. The model nests two workhorse trade models, the Brander & Krugman reciprocal dumping model and the Ricardian technology-based trade model, as special cases. We examine both free entry and limited entry (free exit) cases. The model generates testable predictions on the probability of zero trade flows and the pattern of export prices and generates endogenous moves from zero to non-zero trade flows due to entry and exit. As two countries move from autarky to free trade, market prices fall, but along the path from autarky to free trade, market prices might increase at certain points. Also with lower trade costs the least productive firms get squeezed out of the market, exporting firms gain market share, and more firms become trade oriented.

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1 Introduction

Research on the impact of globalization on firms has shown that the reallocation effects of trade are an important mechanism linking openness to productivity. Bernard and Jensen (2004) find that almost half of the rise of manufacturing total factor productivity in the USA between 1983 and 1992 is linked to a reallocation effect of resources towards more productive and trade oriented firms. Episodes of liberalization in developing countries also show the importance of changes in firm composition – composition effects (Tybout 2001).

A number of models of heterogeneous productivity have been put forward in the recent theoretical trade literature to explain composition effects of trade. A standard result is that processes of firm formation involving heterogeneous cost structures across firms imply beneficial reallocation effects from trade liberalization linked to rationalization of the population of firms. Less efficient firms producing for the domestic market are squeezed out by more efficient trading firms. For example, Melitz (2003) introduces heterogeneous productivity in a monopolistic competition framework with CES-preferences, while Bernard, et al (2003) include heterogeneous productivity in a model with Bertrand competition. Working with monopolistic competition models, Baldwin and Robert-Nicoud (2005) examine linkages between trade and growth given firm heterogeneity, while Ghironi and Melitz (2004) examine macroeconomic dynamics.

An important feature of the monopolistic competition models of Melitz (2003) and Melitz and Ottaviano (2008) is that there is no strategic interaction between firms. Firms are atomistic in the market, which enables the definition of a smooth entry and exit process. When firms exit due to an exogenous shock and are replaced by new entering firms, nothing changes in the market equilibrium. In the real world, trade seems to be dominated by a small number of very large firms that interact strategically. Also, the replacement of dying firms by new ones is likely to affect the market equilibrium. To account for this fact, we explore heterogeneous productivity in a model with oligopoly characterized by Cournot competition. Basic aspects of market structure – markups, industrial concentration, relative firm positions, and prices for domestic and export markets – are endogenous. They depend on the interaction between the technology set, market size, and trade openness. We do not assume a specific distribution of initial productivities, as is the case with Bernard, et al (2003) and Melitz and Ottaviano (2008), to generate our results. Preferences are assumed to be CES across different sectors. For entry and exit dynamics we expose the model in two different ways. We start with firms drawing a new productivity parameter each period upon paying a sunk entry cost. This can be interpreted as firms making annual production plans. Later on we model entry/exit like in Melitz (2003) with firms having the same productivity over their whole life time and exiting with a fixed death probability δ. We show that all the results for the easier entry/exit process carry through for the more complicated entry/exit process. As mentioned, in Melitz (2003) and Melitz and Ottaviano (2008) this leads to a steady state of entry and exit where specific draws of entering firms or specific firms dying does not affect the equilibrium value of variables like market price and cutoff cost level, because firms are atomistic. In our model this is different: the market price wanders around from period to period.

The model we develop is a two-country, multi-sector model of trade under Cournot competition.[1] The value added of this approach is threefold. First, the model accounts for strategic interaction between firms in a firm heterogeneity setting with endogenous dynamics in equilibrium values due to entry and exit, while generating a rich set of results linked to composition effects. In particular, we find that a larger market size generates a lower market price by inducing more entry. Interestingly, as two countries integrate their markets and move from autarky to free trade, the market price does not go down monotonically and can jump up at certain levels

[1] Van Long et al. (2007) also address firm heterogeneity in an oligopoly model. Their paper is focused on a different set of issues however, the interaction of trade and R&D.

of trade costs. This result is due to possible exit induced by the fact that lower trade costs drive down profits for domestic sales, an effect that possibly outweighs the increased profits from exporting sales. Another important result is that we can generate endogenous variation in the probability of zero trade flows and the fob export price due to entry and exit of firms and the variation in productivities drawn.

Second, the model nests two workhorse trade models, the Brander and Krugman (1983) reciprocal dumping model and the Ricardian model, as special cases. Third, the model generates testable predictions on the probability of zero trade flows and export prices. The pattern of zeros and unit values has emerged as a particularly important issue in the recent empirical trade literature. (See Baldwin and Harrigan 2007, Baldwin and Taglioni 2006). In the model developed here, the effect of a larger distance between countries on the probability of zero trade flows and fob export prices is ambiguous. When the number of entrants does not change, the probability of zero trade flows declines with a lower distance, when the number of entrants induced by a smaller distance declines, the probability of zero trade flows rises and when the number of entrants due to a msaller distance rises, the effect is ambiguous. A larger market size of the importer country increases the probability of zero trade flows and increases the fob export price.

The model in this paper is related to two strands of literature in international trade. The first one is the firm heterogeneity literature, which is already discussed above. The paper differentiates itself from that literature by focusing on oligopoly and thereby allowing for strategic interaction, which generates entry and exit dynamics not present in the existing models. The second related strand of literature is the one on oligopoly in international trade. This literature can be characterized by the following three dimensions. First, whether firms are homogeneous or heteregeneous, second whether there is free entry or not and third whether the model is embedded in a general equilibrium framework. Brander and Krugman (1983) explore a model with homogeneous firms in a partial equilibrium setting both with and without free entry. The main innovation of their paper is to show that reciprocal duming can emerge in an oligopolistic market.

Van Long and Soubeyran (1997) study a model with heterogeneous firms in a partial equilibrium setting without free entry. Their main innovation is to show that the Herfindahl index of concentration is related to the variance of marginal costs of the different firms with implications for optimal strategic trade policy. A third paper in the literature on trade and oligopoly is Neary

(2009), featuring homogenous firms in a general equilibrium setting without free entry imposed. His main innovation is to show how oligopoly can be modeled in a general equilibrium setting by assuming a continuum of sectors. The current paper is different from the other papers in the oligopoly and trade literature, as it combines firm heterogeneity with free entry in a general equilibrium setting. This allows the study of reallocation effects of trade in a market setting with strategic interaction.

The paper is organized as follows. In Section 2 we lay out the basic model. Section 3 introduces international trade and contains four subsections. The first subsection abstracts from free entry, the second subsection adds a free entry condition, the third subsection addresses the effect of distance and market size on zeros and fob prices and the fourth subsection points out how the Brander&Krugman model and the Ricardian model can be seen as special cases of our model. Section 4 concludes.

2 The Basic Model

This section lays out the basics of the model without trade (or identically for an integrated or single global economy without trade costs). Industrial concentration emerges endogenously as a function of the degree of firm heterogeneity and market size, while the relationship of concentration to price depends on the cost structure.

We start by assuming that there are $Q + 1$ sectors in the economy, Q oligopolistic sectors producing q_j and 1 sector producing z under conditions of perfect competition. In the first sections it is assumed that the Cournot sectors are symmetric. Later on this assumption is relaxed when asymmetries in national technology sets, country size, and policy are explored. Throughout it is assumed that there are sufficient sectors in the economy so that the effect of a price change on demand through the price index is negligible for firms. (There is no numeraire problem). There are L equal agents each supplying 1 unit of labor. All profit income from the Cournot sectors goes to the economic agents. The utility function of each agent is CES. The optimization problem of the consumer generates the following market demand functions in the Cournot sectors q_j and in the perfect competition sector z:

$$q_j = \frac{IP_U^{\sigma-1}}{p_j^{\sigma}} \tag{1}$$

$$z = IP_U^{\sigma-1} \tag{2}$$

The price of good z is normalized at 1 and I is the endogenous income of all agents, the sum of labor and profit income. P_U is the consumer price index, corresponding to one unit of utility:

$$P_U = \left[\sum_{j=1}^{Q} p_j^{1-\sigma} + 1 \right]^{\frac{1}{1-\sigma}} = \left[Q p^{1-\sigma} + 1 \right]^{\frac{1}{1-\sigma}} \tag{3}$$

Until we relax out symmetry assumptions, we will focus on one representative Cournot sector. (We will warn the reader when we drop these assumptions.) This means we can drop the sector index j for now. Labor is the only factor of production and there is a labor force of size L. One unit of labor is needed to produce one unit of the perfect competition good y. This means the wage is equal to 1. In the q sectors productivity is heterogeneous. One unit of labor can be transformed into $1/c_i$ units of q for the i-th firm which has marginal cost of production c_i. There are no fixed costs of production. Therefore the cost function of firm i is given by

$$C_i (q_i) = c_i q_i \tag{4}$$

There is Cournot competition between the different firms in the q-sectors. So, firms maximize profits towards quantity supplied, taking the quantity supplied by other firms as given. Profit of firm i is given by:

$$\pi_i = p q_i - c_i q_i \tag{5}$$

The first order condition is defined as:

$$\frac{\partial \pi_i}{\partial q_i} = p \left[1 - \frac{1}{\sigma} \frac{q_i}{q} \right] - c_i = 0 \tag{6}$$

With $q = \sum_{i=1}^{n} q_i$. n is the number of firms in the market. Using the first order condition, the second order condition can be written as follows (derivation in appendix A):

$$-\frac{1}{\sigma} \frac{p}{q} \left[\frac{(\sigma + 1) c_i - (\sigma - 1) p}{p} \right] < 0 \tag{7}$$

Using the definition for market share, $\theta_i = \frac{q_i}{q}$, the first order condition can be rewritten as:

$$p \left(1 - \frac{\theta_i}{\sigma} \right) = c_i \tag{8}$$

$$\theta_i = \sigma \frac{p - c_i}{p} \tag{9}$$

The marginal revenues on the LHS of equation (9) should be at least as large as the marginal costs on the RHS. The larger is market share θ_i, the lower is marginal revenue. So, for positive sales ($\theta_i \geq 0$) which are implicitly imposed, a firm can satisfy the FOC by just reducing its market share as long as its marginal cost is smaller than the market price. The cutoff cost level c^* is defined as the cost level with which a firm would just stay in the market. This cutoff cost level c^* is equal to the market price p. The highest cost firm staying in the market has a cost level equal or just below the cutoff cost level and selling an amount just above zero. In the actual market equilibrium, the highest cost firm that can stay in the market can have a cost lower than c^*. This depends upon the specific sample of firms drawn from an initial cost distribution at a specific point in time. Hence, the market price can vary over time with firms entering and exiting. Still, the expression for the market price p and total sales q can be determined as a function of average costs $\bar{c}$ and the number of firms n at an arbitrary point in time. And also the free entry condition can be determined, although it depends upon variables that vary over time. First, let us focus on the equilibrium expressions without imposing a free entry condition.

Appendix A shows that the reaction functions are stable when $\sigma > 3/2$. Although it is possible that a very efficient firm dominating the market with a very large market share has a reaction function with positive slope, the condition $\sigma > 3/2$ guarantees that the equilibrium is stable, see for discussion the appendix.

The equilibrium price and quantities sold can be found for a given number of firms. Suppose for now there are n firms, endogenizing this later on with a free entry condition. Combining the demand equation in (1) with n first order conditions in equation (6) and with the equation for the sum of market shares, one can find the following solutions for the market price p, total sector sales q and sales of an individual firm q_i:

$$p = \frac{\sigma}{\sigma n - 1} \sum_{i=1}^{n} c_i = \frac{\sigma n}{\sigma n - 1} \bar{c} \tag{10}$$

$$q = \frac{I P_U^{\sigma-1}}{\bar{c}^{\sigma}} \left(\frac{\sigma n - 1}{\sigma n} \right)^{\sigma} \tag{11}$$

$$q_i = \sigma I P_U^{\sigma-1} \frac{\frac{\sigma n}{\sigma n - 1} \bar{c} - c_i}{\bar{c}^{\sigma+1}} \left(\frac{\sigma n - 1}{\sigma n} \right)^{\sigma+1} \tag{12}$$

with $\bar{c}$ the unweighted average cost of firms, $\bar{c} = \frac{1}{n} \sum_{i=1}^{n} c_i$.

Using the fact that the price is equal to the cutoff cost level, the price equation (10) can be rewritten to solve for the number of firms as a function of the cutoff cost level and average cost:

$$n = \frac{1}{\sigma} \frac{c^*}{c^* - \bar{c}} = \frac{1}{\sigma} \frac{\overline{m}}{\overline{m} - 1} \tag{13}$$

With $\overline{m}$ an average markup measure defined as price divided by unweighted average cost, $\overline{m} = p/\bar{c}$. On the basis of equation (13) we make the following Proposition.

Proposition 1 *The market structures and average markups of industries are related to the degree of heterogeneity. In particular, the less the degree of cost heterogeneity, the more competitive the structure of the industry and the smaller the average markup.*

Equation (13) shows that an increase in the number of firms implies that the firm with the highest cost needs to have a cost parameter ever closer to average cost. Therefore, the cost levels of firms become ever closer to each other with more firms in the market. Proposition 1 highlights how the market structure and the cost structure in the model are interrelated. When we observe more competitive industries with more firms in equilibrium, this means the cost levels of firms should be closer to each other.[2]

Next, we add free entry to the model. This will endogenise the number of entrants n^e. Firms make production plans for each period. Basically, we interpret the entry and exit mechanism for firms each period as reflecting a mechanism where firms make new production plans each year. This interpretation implies that firms also have to pay sunk entry costs and draw a new cost parameter each period.

Operationally, we assume that firms pay a sunk entry cost f_e each period to draw a cost parameter c randomly from a set of initial costs C with a certain discrete distribution of costs $F(c)$ with lower bound $\underline{c}$ and upper bound $\bar{c}$. Hence, uncertainty about productivity is a barrier to entry for firms. They start to produce when they can make positive operating profits. Production plans are good for only one period. Every period all producing firms have to pay again sunk entry costs to draw again from the cost distribution for plans for the next period.

We start with a model where firms have to draw a new productivity parameter each period

[2] Working with a more restricted model without entry, Van Long and Soubeyran (1997) find similar results. They show that the variance of the cost distribution and the Herfindahl index of industry concentration are positively related in a model with Cournot competition: a larger variance leads to more industry concentration. From equation (13) above, it is clear that this result is more general, and holds with entry and exit as well.

to keep the exposition of the model straightforward and simple. After this exposition we will set up the model where firms live for more than one period, until they die according to a fixed death probability, the entry/exit process of Melitz (2003), and show that an equilibrium exists.

The sunk entry costs use labor. As free entry leads to almost zero expected profits, all profit income on average is used to pay labor in the entry sector.[3] Therefore, total income in the economy is fixed and equal to the amount of labor (with wages normalized at 1).

The major change in this model relative to models without strategic interaction is that there is no steady state of entry and exit. The market price will typically be different in each period, dependent upon the cost draws of the entering firms. Still we can characterize the equilibrium based upon a zero cutoff profit condition (ZCP) and a free entry condition (FE). Also we will be able to determine the sign of the effect of a larger market and (later in the open economy) of lower trade costs upon the expected market price.

We proceed in two steps. First we show that a unique Nash equilibrium emerges for a given number of entrants n^e and a given cost draw for the entering firms. Second, we show that for each initial state in the previous period there is a number of entrants n^e that leads to satisfaction of the free entry condition.

The ZCP follows from the fact that without fixed costs zero profit implies that price should be equal or just above marginal cost for the cutoff firm. The FOC in equation (6) shows that this firm will reduce market share to (just above) zero, to satisfy the first order condition and make non-negative profit. As the distribution of initial costs is discrete, the cutoff cost level c^* is equal or smaller than the market price p:

$$c^* \leq p \tag{14}$$

For a specific cost draw of the n^e entrants, a unique market price will emerge as stated in the following proposition:

Proposition 2 *For a given number of entrants n^e and a given set of cost draws $\{c_1, c_2, ..., c_{n^e}\}$, a unique Nash equilibrium of number of producing firms, n, sales $\{q_1, q_2, ..., q_n\}$ and market price p will emerge.*

The proof of proposition 2 proceeds as follows. We can rank the marginal cost draws from

[3]Profit can be slightly positive, when the free entry condition is not exactly satisfied, as we work with a discrete number of entrants.

small to large as $c_1 \leq c_2 \leq ... \leq c_{n^e}$. The number of producing firms n is then determined by the following conditions:

$$p_n \geq c_n; \; p_{n+1} < c_{n+1} \tag{15}$$

p_n is the market price with the n most efficient cost drawing firms producing. Hence, with n firms the highest cost firm can still produce profitably and with $n+1$ firms the highest cost firm could not survive. Inequalities (15) imply that $p_{n-1} \geq c_{n-1}$ and $p_{n+2} > c_{n+2}$ and hence imply a unique n that satisfies these inequalities, because p_n decreases in n as we show now. We start from equation (10) and add Δn firms with average cost $\overline{c}_j$. This generates a change in price of:

$$\begin{aligned} \Delta p &= \frac{\sigma}{\sigma (n + \Delta n) - 1} \left(\sum c_i + \overline{c}_j\right) - p \\ &= \frac{\sigma \Delta n}{\sigma (n + \Delta n) - 1} \left(\overline{c}_j - p\right) \end{aligned}$$

$\Delta p < 0$ when $\overline{c}_j < p$ and $\Delta p = 0$ when $\overline{c}_j = p$, i.e. when all entering firms would have a cost equal to the ruling market price. Entry of firms with a cost higher than the ruling market price, i.e. $\overline{c}_j > p$, would lead to an increase of the market price, but is not possible as these entering firms would not survive, as we have that $\Delta p < \overline{c}_j - p$, hence the new market price p' would be below the cost of the entered firms $\overline{c}_j$. Or in other words, if a firm with marginal cost higher than the ruling market price comes in, the market price increases insufficiently for this firm to make positive profit and hence it will not enter.

Hence, for a given set of cost draws there is a unique market price p, a unique number of producing firms n and the sales of each firm q_i follows from equation (12).

To determine the number of entrants n^e, we add a free entry condition. We start with a model where firms make annual production plans and draw a new marginal cost parameter each period, that is independent upon cost draws in other periods. We show now that the FE will lead to a unique number of entrants n^e in this case. The FE is given by equalizing the ex ante expected profits from entry with the sunk entry cost. The FE expression is more complicated than in models without strategic interaction, like Melitz (2003) and Melitz and Ottaviano (2008). First, as the market price p depends upon the marginal costs drawn, the expected profit is written as an expected value over the market price p given the cost parameter drawn and the number of entrants, $j\left(p \mid c, n^e\right)$ and the cost distribution, $f\left(c\right)$. The distribution of p is conditional upon the number of entering firms n^e, as the number of entering firms affects probability distribution of the market price. Second, the free entry condition will in general not hold exactly and is therefore written in inequality terms:

$$\overline{\pi}\left(n^e+1\right) \leq f_e \leq \overline{\pi}\left(n^e\right)$$

$\overline{\pi}$ is the profit unconditional upon entry. We can elaborate upon this FE as follows:

$$\sum_{c\in C}\sum_{p\in M(c,n^e+1)} \pi\left(p,c\right) I\left[p\geq c\right] j\left(p\mid c,n^e+1\right) f\left(c\right) \leq f_e \leq \sum_{c\in C}\sum_{p\in M(c,n^e)} \pi\left(p,c\right) I\left[p\geq c\right] j\left(p\mid c,n^e\right) f\left(c\right) \quad (16)$$

To generate expected profit we sum over all possible cost distributions and conditional upon cost c for one firm and number of entrants n^e as well over all possible prices, expressed by the conditional price density $j\left(p \mid c, n^e\right)$. Note that density of costs $f\left(c\right)$ is independent of the number of entrants: the probability to draw a certain cost for an entrant is independent of the number of other entrants. The set $M\left(c,y\right)$ is the set of all possible market prices when 1 firm has drawn cost c and the number of entrants is y. $\pi\left(p,c\right)$ is the profit of a firm facing market price p and having marginal cost c:

$$\pi\left(p,c\right) = \sigma I P_u^{\sigma-1}\frac{\left(p-c\right)^2}{p^{\sigma+1}} I\left[p\geq c\right] \quad (17)$$

$I\left[p\geq c\right]$ is the indicator function for price larger than marginal cost. It features in the FE, as we sum over all possible cost draws. Therefore, we have to add that only firms with costs lower than the market price make positive profits.

The following proposition claims that the free entry condition generates a unique equilibrium for the number of entering firms n^e, i.e. a unique long run equilibrium.

Proposition 3 *There is a unique number of entering firms n^e that leads to satisfaction of the free entry condition in equation (16).*

The proof of proposition 3 proceeds in three steps. We start by showing that the probability density function $J\left(p \mid c, n^e\right)$ first order stochastically dominates (FOSD) $J\left(p \mid c, n^e+1\right)$ for any n^e and for any c. From the proof of proposition 2 it follows that an increase in the number of producing firms has a non-increasing effect on the market price p. Using this result we know that the entry of one additional firm leads to an upward shift of the support of probability function $j\left(p \mid c, n^e\right)$. All points in the support of the probability function either do not change when the additional entering firm draws a cost higher than the ruling market price without this firm or the points in the support shift to the left when the entering firm draws a cost lower than the ruling market price. In formal terms, for any market price p given n^e entrants, we have that the market price z given $n^e + 1$ is $z = p - x$ with x having a distribution function with $I_x(0) = 0$. Define the distribution function of the market price with $n^e + 1$ entrants as $J(p \mid c, n^e + 1)$. Then we have that for any non-decreasing function $v : \mathbb{R} \to \mathbb{R}$,

$$\sum v(z)\, j(z \mid c, n^e + 1) = \sum \left(\sum v(p - x)\, i(x)\right) j(p \mid c, n^e) \leq \sum v(p)\, j(p \mid c, n^e)$$

Hence, $J(p \mid c, n^e)$ FOSD $J(p \mid c, n^e + 1)$ for any level of n^e and c. Second, we need to show that profit, $\pi(p, c)$, rises in p. This is done in Appendix A. FOSD of $J(p \mid c, n^e)$ over $J(p \mid c, n^e + 1)$ implies that the expected value of any function rising in p taken over the distribution function $J(p \mid c, n^e)$ is larger than when it is taken over $J(p \mid c, n^e + 1)$. Third, we note that the density function of costs conditional upon the number of entrants n^e is independent of the number of entrants n^e. This implies that the expected profit expression in equation (16) declines in n^e and hence there is a unique n^e that satisfies the FE.

Now we generalize the entry/exit dynamics and assume that firms stay in the market until they leave with a fixed death probability δ. As there are no fixed costs, firms will always stay in the market until they die, either producing when their marginal costs c is smaller than market price p or not producing (lingering) in a period where $p < c$, waiting for the price to go up. The implication is that the market price is stochastic and varies over time. When firms die this has a non-negligible impact on profits of the remaining firms. The stochastic market price implies that possible entrants have to form expectations about possible profits when they enter conditional upon the state of the economy before entry. This implies that there will be a FE condition with corresponding number of entrants $n^e(s)$ for each initial state s.

To express the free entry condition, we have to define the Markov process describing the dynamics of the different variables. A state s_t in period t is characterized by a tuple $d(s_t) = \left\{p(s_t), n(s_t), n^L(s_t)\right\}$. p is the market price, n the number of producing firms, n^L is the number of lingering firms. Based upon these variables an entering firm has sufficient information to calculate expected profits and the dynamics are described by a stationary Markov process. The state space is given by the set S.

The sequence of events is as follows. A firm considering entrance in period t observes s_{t-1}

and calculates the expected profit from entry which depends upon the profits expected in period $t+1, t+2, \ldots$. As we will show the expected profit declines in the number of entrants, hence given initial state s_{t-1} a certain number of firms pays the sunk entry costs and decides to enter. After entrance firms (both existing firms and just entered ones) die according to a fixed death probability δ.[4] Next, the firms that can make positive profits start to produce, the others linger and wait for later periods to start to produce. As firms do not have to incur fixed costs, firms will never exit definitively from the market unless they die, i.e. there is no optimal stopping rule. As a consequence, the absence of fixed costs makes the model substantially less complicated. An implication of the discussed sequence of events is that there is a positive probability that there is no supply in a certain period, as all firms could have died.

For each initial state we can write a free entry condition that determines the number of entrants for that initial state. As a result, the state variables in period $t+1$ are a function of the state variables in period t and the number of entrants in period $t+1$ which are solved from the FE conditions. A complication is that we have to define the state in period $t+1$ conditional upon one firm having marginal cost c. This is the firm whose expected profits are under examination in the FE.[5] Defining states in this way allows us to sum the expected profit of the firm under consideration over all possible cost draws and all periods (with for each period a fixed death probability). If we would sum first over possible states unconditional of the cost level and then for each state over possible cost draws, we do not get the correct expected profit as firms draw costs only once and forever.[6]

Hence, we have transition probabilities $\Pr\left(u_t, n_{t+1}^e(u_t), c \mid s_{t-1}, n_t^e(s_{t-1}), c\right)$. An equilibrium is defined as a rule characterizing for each initial state $s \in S$ the number of entrants $n^e(s)$ leading to satisfaction of all the free entry conditions.

As implied by the previous paragraph on information we assume that firms do observe the market price in the previous period and the number of producing and lingering firms, but they

[4]This timing seems weird, but is necessary to guarantee that the Markov process is stationary. The transition probabilities should not depend upon whether the firm under consideration for the free entry condition has just entered or is already producing for some periods. Also, Hopenhayn (1992) assumes that firms face a shock to productivity immediately after entrance and before production starts. In the Melitz entry/exit process that we use and that builds on Hopenhayn (1992), the shock facing firms is dying. Hence, the timing is consistent with Hopenhayn (1992).

[5]The firm we are considering has either entered in period $t+1$ or in a previous period. Hence, the market price in period $t+1$ is conditional upon one firm having cost c, be it an entrant or an already producing firm. This is necessary for the Markov process to be stationary and not dependent upon the one firm having just entered or being in the market already.

[6]To phrase it differently, we cannot reverse the order of conditioning and summation over states and marginal costs.

do not know the marginal costs drawn by the other firms. This information is sufficient to calculate transition probabilities and thus expected profit.

Firms calculate transition probabilities, conditional upon the observed state in the last period to determine whether it pays off to enter or not. As firms live for more than one period, we also have to take into account expected profits in future periods, which depend upon transition probabilities towards states in future periods. As such we can write down the free entry condition in period t for initial state s_{t-1} as follows:

$$\begin{aligned}
&\sum_{i=0}^{\infty}(1-\delta)^{i+1}\sum_{c\in C}\Big\{\sum_{u\neq s}\pi\left(p\left(u_{t+i}\right),c\right)\Pr\left(u_{t+i},n^{e}\left(u_{t+i}\right),c\mid s_{t-1},n^{e}\left(s_{t-1}\right)+1,c\right) \\
&\qquad +\pi\left(p\left(s_{t+i}\right),c\right)\Pr\left(s_{t+i},n^{e}\left(s_{t+i}\right)+1,c\mid s_{t-1},n^{e}\left(s_{t-1}\right)+1,c\right)\Big\}f\left(c\right) \\
&\leq f_{e}\leq\sum_{c\in C}\left(\sum_{u\in S}\pi\left(p\left(u_{t+i}\right),c\right)\Pr\left(u_{t+i},n^{e}\left(u_{t+i}\right),c\mid s_{t-1},n^{e}\left(s_{t-1}\right),c\right)\right)f\left(c\right)
\end{aligned} \tag{18}$$

As mentioned before, as the number of firms is discrete we have to write the FE in terms of inequalities. The FE for state s_{t-1} is written such that with $n^{e}\left(s_{t+i}\right)+1$ entrants in state s_{t+i}, profits are lower than the sunk entry costs and with $n^{e}\left(s_{t+i}\right)$ entrants, profits are larger than sunk entry costs. Hence, the number of entrants on which we condition on the LHS is $n^{e}\left(s_{t-1}\right)+1$ for state s in period $t-1$, but also in all other periods.[7]

The number of free entry conditions is equal to the number of states, by slight abuse of notation also defined as S. An equilibrium of the model solves the number of entrants corresponding to each initial state using all free entry conditions. Hence, there are S inequalities in S unknowns. We can rewrite the conditional probability in terms of one period transition probabilities as follows using the Markovproperty:

$$\begin{aligned}
&\Pr\left(u_{t+i},n^{e}\left(u_{t+i}\right),c\mid s_{t-1},n^{e}\left(s_{t-1}\right),c\right) \\
=\ &\sum_{v_{t+i-1}\in S}...\sum_{v_{t-1}\in S}\Pr\left(u_{t+i},n^{e}\left(u_{t+i}\right),c\mid v_{t+i-1},n^{e}\left(v_{t+i-1}\right),c\right) \\
&...\Pr\left(v_{t},n^{e}\left(v_{t}\right),c\mid s_{t-1},n^{e}\left(s_{t-1}\right),c\right)
\end{aligned}$$

[7]A technical detail is that formally we should have expressed expected profit with the probability that a firm dies within the summation terms, as states are defined unconditional upon one firm not dying. That means that the profit expression for each state should have been multiplied by the probability that one firm dies given the state. But it is easily shown (derivation available upon request) that the probability that one firm dies is independent of the state, i.e. of market price and number of firms. Therefore, we can take this probability out of the summation term.

We omit the expression for $\Pr\left(u_{t+i}, n^e\left(u_{t+i}\right) \mid s_{t-1}, n^e\left(s_{t-1}\right)+1\right)$, it is straightforward, but very cumbersome. We therefore also display only the greater than part of the FE in the remainder, as it is sufficient for our purposes. We have S FEs defined for each initial state s_{t-1}:

$$f_e \leq \sum_{i=0}^{\infty}(1-\delta)^i \sum_{c\in K}\left\{\sum_{u_{t+i}\in S}...\sum_{u_t\in S}\pi\left(p\left(u_{t+i}\right),c\right)\Pr\left(u_{t+i}, n^e\left(u_{t+i}\right) \mid u_{t+i-1}, n^e\left(u_{t+i-1}\right)\right)\right.$$
$$\left. ...\Pr\left(u_t, n^e\left(u_t\right) \mid s_{t-1}, n^e\left(s_{t-1}\right)\right)\right\} f\left(c\right) \tag{19}$$

We notice that the S FEs are stationary, i.e. it does not matter for which period they are defined. This implies that the optimal number of entrants are stationary, $n^e\left(s_t\right)=n^e\left(s\right)$. We define an equilibrium of the free entry model as follows:

- **Definition** An equilibrium of the free entry model consists of a mapping $n^e\left(s\right): S \rightarrow \mathbb{N}$ such that for all $s \in S$

  (i) If $n^e\left(s\right) \geq 0$, then inequality (19) is satisfied

  (ii) If $n^e\left(s\right) = 0$, then the LHS of inequality (19) is smaller than 0

Hence, definition 2 states that a solution of the model requires a mapping from the state space S to a natural number n^e solving the FE-inequality for each inequality for which n^e is larger than 0 and having an expected profit smaller than 0 in state s when the number of entrants for that state, $n^e\left(s\right)$ is 0. We now state the following proposition:

Proposition 4 *There exists an equilibrium mapping $n^e\left(s\right)$ from initial state s to number of entrants n^e leading to an equilibrium as defined in definition 2.*

To prove existence of equilibrium, we will prove, using first order stochastic dominance (FOSD), that expected profit for each FE given initial state s is rising in the number of entrants for each initial state u, $n^e\left(u\right)$ with u equal or unequal to s, for given number of entrants for other initial states, v, $n^e\left(v\right)$, $v \neq s$. We will show that this is sufficient for the existence of equilibrium.

As we have a Markov process specifying transition probabilities over a set, we define FOSD as follows in this case, following the definition proposed by Green, McKelvey and Packel (1983).

Consider a state space X with transition probabilities from state i to j defined by the function $g\left(j \mid i\right)$. Define a function $L: X \rightarrow \mathbb{R}^+$. Define for any $c \in \mathbb{R}$ the subset S_c as a subset of the state space X that satisfies:

$$S_c = \{x \in X \mid L(x) \leq c\} \tag{20}$$

$g_y = g(\cdot \mid y) : X \to \mathbb{R}$ is the measure defining the probability to go from state y to some specific state in X. We say that the Markov process g first order stochastically dominates (FOSD) h with respect to L iff for all $c \in \mathbb{R}$ and for all $y \in X$:

$$h(S_c \mid y) \geq g(S_c \mid y) \tag{21}$$

Hence, stochastic dominance in this case says that the probability to go to any subset S_c of X from any initial value y is larger for the process described by h than for the process described by g.

The following definition of FOSD is equivalent. The Markov process g FOSD h with respect to L iff for any non-decreasing function $w : \mathbb{R} \to \mathbb{R}$ and for any initial state y:

$$\sum_{x \in X} w(L(x))\, g(x \mid y) \geq \sum_{x \in X} w(L(x))\, h(x \mid y) \tag{22}$$

We will prove now that the Markov process $\Pr(u, n^e, c \mid s, n^e, c)$ FOSD $\Pr(u, n^e, c \mid s, n^e + 1, c)$ with respect to the function $L(S) = p(S)$ with the second Markov process defined such that the number of entrants n^e is larger for the second Markov process for one or more initial states s. We proceed as follows. We note from the proof of proposition 2 that an increase in the number of producing firms has a non-increasing effect on the market price p. Using this result we know that the entry of one additional firm leads to a leftward shift of the support of probability function $\Pr(u, n^e, c \mid s, n^e, c)$ for any initial state s. All points in the support of the probability function either do not change when the additional entering firm draws a cost higher than the ruling market price without this firm or the points in the support shift to the left when the entering firm draws a cost lower than the ruling market price. In formal terms for any market price p given n^e entrants, we have that the market price z given $n^e + 1$ is $z = p - q$ with q having a distribution function with $I_q(0) = 0$, a state space Q and a probability function i_q.

Hence, we have that for any non-decreasing function $w : \mathbb{R} \rightarrow \mathbb{R}$ and and any initial state s,

$$\begin{aligned} & \sum_{u \in S} w\left(z\left(u\right)\right) \Pr\left(u, n^e, c \mid s, n^e + 1, c\right) \\ = & \sum_{u \in S} \left(\sum_{q \in Q} w\left(p\left(u\right) - q\right) i_q\left(q\right) \right) \Pr\left(u, n^e, c \mid s, n^e, c\right) \\ \leq & \sum_{u \in S} w\left(p\right) \Pr\left(u, n^e, c \mid s, n^e, c\right) \end{aligned} \tag{23}$$

Hence, $\Pr\left(u, n^e, c \mid s, n^e, c\right)$ FOSD $\Pr\left(u, n^e, c \mid s, n^e + 1, c\right)$. In Appendix A it is shown that profit decreases in market price p. Hence, when the number of entrants in period t increases for any initial state s_{t-1}, expected profit in period t will decline. Still, we also need to know the impact of more entrants in period t on profits in future periods. To show that profits in other periods (for other states) also decrease, we use that the Markov process $\Pr\left(u, n^e, c \mid s, n^e, c\right)$ is stochastically increasing with respect to $L\left(S\right) = p\left(S\right)$ for given number of entrants in other states.

Applying the definition for X and S_c (equation (20)) above, we say that a Markov process g is stochastically increasing with respect to L if for all y, x

$$L\left(y\right) \geq L\left(x\right) \Rightarrow g\left(S_c \mid y\right) \leq g\left(S_c \mid x\right)$$

In our application this definition says that starting from a higher price, we will go to a higher expected price in the next period. To prove that our Markov process is stochastically increasing for given number of entrants, we can use the same line of reasoning as in the proof of FOSD above. Suppose $p\left(s_t\right) = p\left(u_t\right) + b$, $b > 0$. It is easy to show that market price given s_t, $z_{t+1}\left(s_t\right)$ is $z_{t+1} = p_{t+1}\left(u_t\right) + q$ with q having a a distribution function with $I_q\left(0\right) = 0$, a state space Q and a probability function i_q. The proof is now analogue to the proof of FOSD above. As an increase in $n^e\left(s\right)$ leads to a lower price in the next period and by the property of stochastic increase also in future periods, we know that an increase in $n^e\left(s\right)$ generates also a lower expected profit given initial state $u \neq s$.

Hence, we have shown that in the FEs defined in equation (19) for each given initial state s, the expected profit declines in the number of entrants $n^e\left(u\right)$ with $u = s$ or $u \neq s$ for given number of entrants for other initial states $n^e\left(v\right)$, $v \neq s$. So, we have S inequality constraints in S variables with each function defining the inequality constraint declining in all variables.

The proof of existence of equilibrium entry levels requires showing the existence of equilibrium of the following system of S equations and inequalities in S unknowns:

$$\begin{aligned} f_s\left(x\left(s\right), x\left(\_s\right)\right) &= 0, \forall s \text{ with } x\left(s\right) > 0 \\ f_s\left(x\left(s\right), x\left(\_s\right)\right) &< 0, \forall s \text{ with } x\left(s\right) = 0 \end{aligned}$$

In the present context this is relatively straightforward. Expected profit is downward sloping in the number of new entrants, while we assume there are some states where expected profits are non-negative with entry. As such, we can focus on the expected profit of a representative entrant, which is declining in the number of entrants, and so solve for the maximum number of entrants such that the expected profit of each entrant is non-negative. Existence follows from the Arrow-Enthoven theorem.

3 International Trade

International trade is introduced in a setup with two countries, not necessarily equal. We make a distinction between a situation where a free entry condition does not apply and the situation where a free entry condition does apply. We start with the first approach, that can be interpreted in two ways. First, we can consider it as a short-run approach, where new firms do not come in yet to drive expected profits to zero. Second, it can be interpreted as an analysis of the effect of changes in variables like market size and trade costs, when these changes are sufficiently small that they do not induce any change in the number of entering firms for the different initial states. As we work with a discrete number of firms, free entry conditions are expressed in terms of inequalities and sufficiently small changes in exogenous variables will possibly not change the number of entering firms.

After the no free entry analysis, we introduce a free entry condition and analyse the effects of unilateral and bilateral liberalization. In a third subsection we do additional comparative statics analyses on the effect of distance and market size on the probability of zero trade flows and on fob prices. In the last subsection it is shown that the Brander&Krugman model and the Ricardian comparative advantage model are nested cases of our model.

3.1 International Trade without Free Entry

Before introducing trade, we explore the typical experiment in the trade literature of doubling the market size to mimic for the effects of a move from autarky to free trade. In the no-free-entry case the effect of an increase in market size is straightforward as pointed out in the following proposition:

Proposition 5 *An increase in market size increases the sales of all firms proportionally in the no-free-entry case without a change in market price.*

Proposition 5 can be proved as follows. Rewrite the market share equation (9) as follows:

$$\begin{aligned} q_i &= \sigma \frac{p - c_i}{p} q \\ &= \sigma \frac{p - c_i}{p} \frac{(L + \Pi) P_u^{\sigma - 1}}{p^\sigma} \end{aligned}$$

Π is total profit income in the economy. It is easy to show (see the derivation of equation (B.7) in the appendix) that $I = L + \Pi$ is given by the following expression:

$$L + \Pi = \frac{L}{1 - \frac{Q P_U^{\sigma-1}(p - \tilde{c})}{p^\sigma}} \tag{24}$$

$\tilde{c} = \sum_{i=1}^{n} c_i \theta_i$ is the market share weighted average cost. From equation (9) and (10) it follows that market price p and market share θ_i do not depend upon L. Hence, an increase in L leads to a proportional increase in total income $L + \Pi$, a proportional increase in demand in one Cournot sector q and also a proportional increase in production of each producer, q_i.

In the next subsection we will see that with a free entry condition, the market price does change. Now we discuss the finding that lower trade costs do affect the market price, also without imposing a free entry condition. The reason is that the market price depends upon the costs of the firms in the market and they change when trade costs change.

We introduce international trade between two countries $a, b = H, F$ with markets effectively segmented by trading costs. In particular we now introduce iceberg trade costs τ_{ba} in the Cournot sectors, meaning that marginal cost for delivery into country a is increased at the rate τ relative to production and delivery for the domestic market. There are no fixed or beachhead trade costs, and the trading costs preclude return exports. We focus on the impact of increased globalization (i.e. falling trade costs).

Under our assumptions about trade costs, the equilibrium market price in the representative Cournot sector becomes:

$$p_a = \frac{\sigma n_a}{\sigma n_a - 1} \bar{c}_a \tag{25}$$

with $\bar{c}_a = \frac{1}{n_a}\left[\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau_{ba} c_{ixb}\right]$ and $n_a = n_{da} + n_{xb}$. In equation (25), there is a direct effect of lower trade costs from country b to country a, τ_{ba}, on the market price: exporting firms have lower costs and therefore average costs decline. And there is an indirect effect, because firms producing for the domestic market can disappear and exporting firms can appear on the market. It can be shown that this indirect effect is 0 at the margin (see appendix B). Therefore, the relative change in the market price is equal to:

$$\widehat{P}_a = \frac{\sum_{i=1}^{n_{xb}} \tau_{ba} c_{ixb}}{\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau_{ba} c_{ixb}} \widehat{\tau}_{ba} \tag{26}$$

The elasticity of the market price with respect to trade costs, $\varepsilon_{p_a,\tau_{ba}}$, is between 0 and 1:

$$\varepsilon_{p_a,\tau_{ba}} = \frac{\sum_{i=1}^{n_{xb}} \tau_{ba} c_{ixb}}{\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau_{ba} c_{ixb}} \tag{27}$$

From equations (26) and (27) we make the following proposition:

Proposition 6 *Without imposing a free entry condition, a decline in trade cost τ_{ba} into country a leads to a lower market price p_a in country a. The elasticity of the market price p_a with respect to trade costs τ_{ba} is between 0 and 1.*

Equation (26) shows that a decline of trade costs τ drives down the market price. The domestic cutoff marginal cost is equal to the market price, so it also declines. Proposition 6 applies both to unilateral and bilateral liberalization.

Several other Propositions can be made on the effect of trade liberalization in the short run.

Proposition 7 *Some of the least productive firms are squeezed out of the market with a decline in trade cost τ.*

How many firms are squeezed out of the market depends on the price distribution of the firms, i.e. it depends on how far the highest cost firms are from the old market price.

Proposition 8 *More of the remaining firms export with a decline in trade cost τ.*

More firms can enter the export market, as the exporting cutoff marginal cost rises when τ declines:

$$c_{xb}^{*} = \frac{P_a}{\tau_{ab}} \tag{28}$$

$$\widehat{c}_{xb}^{*} = \widehat{P}_a - \widehat{\tau}_{ba} = -\left(1 - \varepsilon_{p_a,\tau_{ba}}\right)\widehat{\tau}_{ba} \tag{29}$$

Proposition 9 *For all firms in the market, markups from domestic sales decline and markups from exporting sales rise with a decline in trade cost τ.*

Markups of all domestic sales decline, as the costs of the firms remain equal, whereas the market price declines. Markups of the exporting firms rise with trade liberalization, as the effect of the declining trade costs dominates the effect of the decrease in market price in the exporting market. Using the letter m to indicate markup, the following can be derived:

$$m_{ixa} = \frac{P_b}{\tau_{ab} c_{ixa}} \tag{30}$$

$$\hat{m}_{ixa} = \widehat{p}_b - \widehat{\tau}_{ab} = \left(\varepsilon_{p_b,\tau_{ab}} - 1\right)\widehat{\tau}_{ab} \tag{31}$$

As in almost any model of international trade (for example Armington) firms increase their market share on the exporting market and their market share is reduced in domestic markets. But the relative gain and loss of exporters and domestic producers displays an interesting pattern:

Proposition 10 *Large low cost firms lose less market share on the domestic market than small high cost firms and small high cost exporting firms gain more market share on the export market than large low cost firms*

Proposition 10 follows from totally differentiating the expressions for market shares:

$$d\theta_{ida} = \sigma \frac{c_{ida}}{p_a} \varepsilon_{p_a,\tau_{ba}} \widehat{\tau}_{ba} \tag{32}$$

$$d\theta_{ixa} = \sigma \frac{c_{ixb}}{p_b} \left(\varepsilon_{p_b,\tau_{ab}} - 1\right)\widehat{\tau}_{ab} \tag{33}$$

Therefore small firms lose relatively more market share on the domestic market and small firms gain relatively more market share on the exporting market than large firms. So, more efficient big firms do not gain more from improved market access abroad than less efficient small firms.

Essentially, big firms already have a strong position in an exporting market, so they cannot grow as much as a result of trade liberalization as small firms.[8]

Consider next the welfare effect of trade liberalization assuming equal countries. This is complicated by the fact that income is endogenous as it depends on profit income in the imperfect competition sector. With free entry profit income is driven to zero, but in the no free entry case profit income is non-zero and varies.

Welfare per worker in country a is equal to utility per worker in that country:

$$W_a = U_a = \frac{I_a}{P_{Ua}} = \frac{L_a + \Pi_a}{L_a P_{Ua}} \tag{34}$$

Π_a is total profit income in the economy. Elaborating upon this equation (see appendix B) assuming that both countries are equal, one arrives at the following expression:

$$W = \frac{(Qp + p^{\sigma})}{p^{\sigma} + Q\tilde{c}} \frac{1}{P_U} \tag{35}$$

$\tilde{c}$ are the market share weighted average costs, $\tilde{c} = \sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix}$. Log-linearizing welfare towards trade costs τ from equation (35) and treating the price and the market share weighted average costs as endogenous, one finds (derivation in appendix B):

$$\hat{W} = \left[\frac{\sigma p^{\sigma}}{Qp + p^{\sigma}} - \frac{\sigma p^{\sigma}}{p^{\sigma} + Q\tilde{c}} \right] \hat{p} - \frac{Q}{p^{\sigma} + Q\tilde{c}} d\tilde{c} \tag{36}$$

The first term in (36) is the welfare gain through a decline in price. As expected the gain for the consumer from lower prices outweighs the loss of a lower profit income with lower prices. The second term measures the possible gain from trade liberalization of lower costs leading to a higher profit income. Elaborating on the cost effect, $d\tilde{c}$, one gets:

$$\begin{aligned} \hat{W} \quad &= \quad - \left[\frac{\sigma p^{\sigma}}{Qp + p^{\sigma}} - \frac{\sigma p^{\sigma}}{p^{\sigma} + Q\tilde{c}} \right] \hat{p} \\ &\quad - \frac{Q}{p^{\sigma} + Q\tilde{c}} \left[\sum_{i=1}^{n_d} c_i \theta_{id} \hat{\theta}_{id} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \hat{\theta}_{ix} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \hat{\tau} \right] \end{aligned} \tag{37}$$

[8]This set of results, related in particular to Proposition 10, has interesting political economy implications beyond the scope of this paper. As trade liberalization progresses, the dominant domestic firms gain relative domestic position (known as "standing" in the antidumping and trade safeguards literature). Assuming that lobbying efficiency is a function of industry concentration, increased concentration of firms with standing (i.e. the domestic industry) may increase their ability to organize and seek protection or relief against further drops in trade costs and foreign competition.

$$
\begin{aligned}
\hat{W} &= -\left[\frac{\sigma p^{\sigma}}{Qp+p^{\sigma}} - \frac{\sigma p^{\sigma}}{p^{\sigma}+Q\tilde{c}}\right]\varepsilon_{p,\tau}\hat{\tau} \\
&\quad -\frac{Q}{p^{\sigma}+Q\tilde{c}}\left[\sum_{i=1}^{n_d}\sigma\frac{c_i^2}{p}\varepsilon_{p,\tau,SR} - \sum_{i=1}^{n_x}\sigma\frac{\tau^2 c_i^2}{p}\left(1-\varepsilon_{p,\tau,SR}\right) + \sum_{i=1}^{n_x}\tau c_i\theta_{ix}\right]\hat{\tau} \qquad (38)
\end{aligned}
$$

Equation (37) and (38) can be interpreted as follows. In both equations is the first term on the RHS again the welfare gain from a lower market price. The second term on the RHS measures the effect on profit income through changed costs. In both (37) and (38) the first term between the second brackets measures the gain from the declining market share of domestic producing firms. The second term between the second brackets measures the loss from the rising market share of exporting firms. The third term measures the welfare gain from lower trade costs with trade liberalization.

Proposition 11 *Like in Brander and Krugman (1983) the welfare effect of trade liberalization can be negative at first when the tariff is reduced from a prohibitive level, due to the increased costs of cross-hauling associated with the first units traded. However, unlike Brander and Krugman (1983), the welfare effect can also be positive when the tariff is reduced from a prohibitive level.*

Unlike in the model of Brander and Krugman (1983) the welfare effect of trade liberalization when the tariff is reduced from a prohibitive level is ambiguous. It depends on the cost structure of firms whether the welfare effect is positive or negative. It can be shown under what condition the welfare effect is negative in general, but this condition is cumbersome and does not lend itself to any interpretation. (See footnote 9 below for proof by example).[9] The ambiguity vanishes for low trade costs.

Proposition 12 *The welfare effect of trade liberalization is unambiguously positive when the tariff is negligible or small, like in Brander and Krugman (1983)*

[9] As proof of ambiguity, we can offer two examples to show that the welfare effect can go both ways. First an example of a negative welfare effect from trade liberalization. Suppose there are two identical countries with each three firms. They have marginal costs of 1, 1 and 2. The autarky market price will be 2. The iceberg trade costs are equal to 2. This implies that 2 firms can export, but with a market share of 0. Substitution elasticity σ is equal to 1. Equation (38) can be applied to show that a marginal reduction of the tariff decreases welfare with $\frac{1}{2}\frac{Q}{1+Q}$. An example where the welfare effect is positive is the following. Again there are two identical countries with each three firms. Marginal costs are 1, 2 and 3. The autarky market price is 3. Iceberg trade costs are 3. So, only one firm can export. Furthermore, the substitution elasticity σ is 1, so utility is Cobb-Douglas. There are two sectors in the economy and the Cournot sector has CES-weight (Cobb-Douglas parameter) α. When the tariff is reduced from the prohibitive level, the welfare effect from equation (38) is equal to $(1-\alpha)\frac{5}{9}-\frac{4}{9}$ So, when the Cournot sector is small enough ($\alpha < 1/5$), the welfare effect of trade liberalization is positive.

Proposition 12 follows immediately from equation (38). When the tariff is equal to 1, the first two terms between brackets in equation (38) are equal. So, only negative terms are left and therefore the welfare effect from trade liberalization is positive. Brander and Krugman (1983) only show that the welfare effect is positive when the tariff is negligible. In the present heterogeneous productivity model one can say more on when the welfare effect is positive. Elaborating upon equation (38), the following expression can be derived for the welfare effect of trade liberalization (see appendix B):

$$\begin{aligned} \hat{W} &= -\left[\frac{\sigma p^{\sigma}}{Qp + p^{\sigma}} - \frac{\sigma p^{\sigma}}{p^{\sigma} + Q\tilde{c}}\right]\hat{p} \\ &\quad -\frac{Q}{p^{\sigma} + Q\tilde{c}}\frac{\sigma n}{p^{2}(\sigma n - 1)}\sum_{i=1}^{n_x}\tau c_i\left[n\mu_c\left(\mu_c + p - 2\tau c_i\right) + (n-1)\,Var\left(c_i\right)\right]\hat{\tau} \end{aligned} \tag{39}$$

In equation (39) μ_c and $Var\left(c_i\right)$ are respectively the mean and variance of the marginal costs of domestic and exporting firms,

$$\mu_c = \frac{1}{n}\left[\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x}\tau c_i\right] \tag{40}$$

$$Var\left(c_i\right) = \frac{1}{n-1}\left(\sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x}\tau^2 c_i^2 - n\mu_c^2\right). \tag{41}$$

Note that the summation in equation (39) is over all the terms between brackets. It can also be shown that the welfare effect is positive when the following condition is satisfied:

$$\frac{Var\left(c_i\right)}{\mu_c^2} \geq \frac{n}{(\sigma n - 1)(n-1)} \tag{42}$$

From equation (39) and (42) the following statements can be made:

Proposition 13 *The welfare effect of trade liberalization is positive when the exporting firms are efficient relative to average market costs. In particular, the welfare effect is unambiguously positive when all exporting firms have marginal costs inclusive of trade costs lower than the average of market price and average costs.*

Proposition 14 *The welfare effect of trade liberalization is positive when the coefficient of variation of the cost distribution is larger than the square root of* $\frac{n}{(\sigma n-1)(n-1)}$.

Proposition 13 follows from equation (39). When $\mu_c + p$ is larger than $2\tau c_i$ all terms in equation (39) will be negative and hence the welfare effect of trade liberalization will be positive. Intuitively, when the exporting firms are productive, their gain in market share at the expense of domestic producing firms represents a welfare gain. More productive firms replace less productive firms. But when the exporting firms' marginal costs inclusive of trade costs are larger than the marginal costs of the domestic producing firms, the shift in market share towards exporting firms can represent a loss. In some cases this loss can be larger than the welfare gain due to lower prices and lower trade costs, as shown by the example above.

Proposition 14 follows from (42). It can be interpreted as follows. When the variance of firms' costs is large relative to average firms' costs, the fraction of relatively inefficient exporting firms will be small. So, the welfare loss from an increasing market share of relatively inefficient exporting firms will be smaller than the welfare gain from a decreasing market share of domestic producing inefficient firms. The next section shows that the welfare effect from trade liberalization is unambiguously positive with free entry.

3.2 International Trade with Entry

Like in the previous subsection, we start by pointing out the effect of an increase in market size L for both entry and exit processes, as an indicator for the effect of a move from autarky to free trade. We start with the annual production plans entry/exit process. We have the following result based upon the closed economy free entry condition as displayed in equation (16):

Proposition 15 *A larger market size L leads to a lower expected market price Ep.*

To prove proposition 15 we start by noting that a larger market size L raises expected profit $\pi(p, c)$ for each market price p. This can be seen easily from equations (16) and (17). This implies that an increase in L leads to an increase in the number of entrants n^e to restore the free entry condition. Now we can define the expected market price as an expectation over the price distribution:

$$Ep = \sum_{c \in C} \sum_{p \in M(c, n^e)} pj(p \mid c, n^e) f(c) \tag{43}$$

By the fact that the number of entrants n^e rises with a larger L and the fact that $j(p \mid c, n^e)$ FOSD $j(p \mid c, n^e + 1)$ for all n^e, we have the result in proposition 15.

To do comparative statics for the Melitz entry/exit process, we sum expected profit in the FEs defined in (19) over the initial states S in period $t-1$ generating the following expression:

$$f_e \leq \sum_{i=0}^{\infty} (1-\delta)^{i+1} \sum_{c\in K} \left(\sum_{s_{t+i}\in S} \pi\left(p\left(s_{t+i}\right), c\right) \Pr\left(s_{t+i}, n^e\left(s_{t+i}\right), c\right) \right) f\left(c\right) \tag{44}$$

The probabilities are the limiting probabilities, that follow from the transition probabilities.[10] We can simplify expression (44) recognizing that the state space is stationary and so omitting time indexes. This generates:

$$\frac{f_e\left(1-\delta\right)}{\delta} \leq \sum_{c\in K} \left(\sum_{s\in S} \pi\left(p\left(s\right), c\right) \Pr\left(s, n^e\left(s\right), c\right) \right) f\left(c\right) \tag{45}$$

Inequality (45) can be used for comparative statics analysis as it is a necessary condition for equilibrium. In particular, we address the effect of a larger market size L, generating the following result:

Proposition 16 *A larger market size leads to a lower average price Ep when the free entry condition applies, with Ep defined as the average price using limiting probabilities.*

To prove proposition 16, we proceed in three steps. First, we note that profit in (45) rises in L. This follows from the expression for profit in equation (17) and equation (24) for total income I as a function of L.

Second, we use a result in Green, McKelvey and Packel (1983) that FOSD of Markov process g over h combined with g stochastically increasing (and a technical condition on continuity), both with respect to the function $L : S \rightarrow p$ implies FOSD of the stationary distribution corresponding to g over the one corresponding to h. We have shown both FOSD of $\Pr\left(u, n^e, c \mid s, n^e, c\right)$ over $\Pr\left(u, n^e, c \mid s, n^e+1, c\right)$ and $\Pr\left(u, n^e, c \mid s, n^e, c\right)$ being stochastically increasing in section 2. This implies FOSD of the stationary distribution $\Pr\left(s, n^e, c\right)$ over $\Pr\left(s, n^e+1, c\right)$, with the number of entrants in the second distribution being larger for one or more initial states. If the increase in L is large enough that the FE in (45) got violated, the number of entrants has to increase for one or more states s to still satisfy the FE in equation (45).

[10]Strictly, as certain states are not aperiodic, we cannot define the probabilities as probabilities, but we should see them as the long-run proportion of time that the process is in a certain state. For our purposes of comparative statics using stochastic dominance this does not pose problems.

Third, define the average price as follows:

$$Ep = \sum_{c \in K} \left(\sum_{s \in S} p \Pr\left(s, n^e\left(s\right), c\right) \right) f\left(c\right) \tag{46}$$

From the FOSD of $\Pr(s, n^e, c) f(c)$ over $\Pr(s, n^e + 1, c) f(c)$ and the fact that for at least one state the number of entrants has to increase, we know that $E(p \mid c)$ has to decrease for each cost c and hence also Ep has to decrease.

An alternative way to prove that a larger market size L leads to a lower average price starts also from the observation that expected profit in equation (45) rises in L. Then, we observe that average price is calculated from the same limiting distribution of states as expected profit and that profit rises when price rises. This implies that the change in the number of entrants, irrespective of whether they rise or fall, that leads to a fall in expected profit and thereby a restoration of the free entry condition will also lead to a fall in average price. This proves that an increase in L leads to a decrease in average price.

Next, we define the equilibrium in a model with trade between two countries with index a and b starting again with annual production plans model. The equilibrium is based on the combined ZCP and FE conditions. We only display the conditions for country a, the expressions for the other country are similar.

There are two ZCPs for domestic and exporting sales of firms in country a:

$$c^*_{da} \leq p_a \tag{47}$$

$$c^*_{xa} \leq \frac{p_b}{\tau} \tag{48}$$

For the annual production plans entry/exit process we have the following FE for country a:

$$\overline{\pi}_{ad}\left(n^e_a + 1, n^e_b\right) + \overline{\pi}_{ax}\left(n^e_a + 1, n^e_b\right) \leq f_e \leq \overline{\pi}_{ad}\left(n^e_a, n^e_b\right) + \overline{\pi}_{ax}\left(n^e_a, n^e_b\right) \tag{49}$$

With the profit functions $\overline{\pi}_{ad}\left(n^e_a, n^e_b\right)$ and $\overline{\pi}_{ax}\left(n^e_a, n^e_b\right)$ defined as follows:

$$\overline{\pi}_{ad}\left(n^e_a, n^e_b\right) = \sum_{c \in C_a} \sum_{p_a \in M\left(c, n^e_a, n^e_b\right)} \pi\left(p_a, c\right) I\left[p_a \geq c\right] j\left(p_a \mid c, n^e_a, n^e_b\right) f\left(c\right)$$

$$\overline{\pi}_{ax}\left(n^e_a, n^e_b\right) = \sum_{c \in C_a} \sum_{p_b \in M\left(c, n^e_a, n^e_b\right)} \pi\left(p_b, \tau c\right) I\left[p_b \geq \tau c\right] j\left(p_b \mid \tau c, n^e_a, n^e_b\right) f\left(c\right)$$

The different sets and functions are defined similarly as in the model with one country, but we have to condition now on the number of entrants in both countries. It is important to recognize that there might be multiple equilibria for the number of entrants in the open economy, as there can be different combinations of the number of entrants in both countries that lead to satisfaction of the free entry conditions in the two countries.

We can make the following statement on the effect of bilateral trade liberalization

Proposition 17 *Lower trade costs have an ambiguous effect on the market price. Specifically, (i) as trade costs decline from a prohibitive level to a negligible level, the market price decreases; (ii) when the number of entrants does not change, lower trade costs lead to lower market prices; (iii) when a change in the number of entrants is induced, the market price might either go up or down.*

From this proposition the following picture emerges. As two countries move from autarky to free trade the market price goes down. Along the path of lower trade costs, market prices go down smoothly when no change in the number of entrants is induced. But at certain spikes where violation of the free entry condition causes either more or less entrants, the market price moves with a shock. When more entry is induced the price moves down and when the number of entrants declines the market price jumps up. The latter is possible with lower trade costs when the higher profits from exporting are dominated by lower profits from domestic sales caused by a lower market price.

Formally, claim (i) of proposition 17 follows from proposition 15. Claim (ii) is implied by proposition 6. Claim (iii) is proved by example. First we note that exporting profits increase with lower trade costs for each level of trade costs and each market price (see proof in Appendix C). Domestic profits decline, as the price distribution shifts to the left for each price. The decrease in domestic profit might dominate the increase in exporting profit and this could induce a lower number of entrants and as such a higher price. We consider an example to show this possibility. In Appendix C we study trade between two economies where the number of entrants into the market is two each period. Firms can either draw a marginal cost of 1 or a marginal cost of 2. When trade costs fall from the prohibitive level of 8/3, market prices fall and domestic profits go down more than exporting profits go up. Profits keep on falling also for lower trade costs. If the sunk entry costs are such that the free entry condition was just satisfied, the number of entrants will fall to one in one of the countries. This will cause a higher expected price. At the same time, there are other levels of trade costs where exporting profits go up more than domestic profits go down, which can induce entry and as such a jump down in the expected market price.[11]

For the Melitz entry/exit process, the FE is more complicated, but results still carry through. Again we have in each country an FE for each initial state. The state space is now defined as a function of variables in both countries, i.e. $s = \left\{p_a(s), n_a(s), n_a^L(s), p_b(s), n_b(s), n_b^L(s)\right\}$. The FE for country a for initial state s_{t-1} can be written as follows:

$$f_e \leq \sum_{i=0}^{\infty}(1-\delta)^i \sum_{c\in K}\left\{\sum_{u_{t+i}\in S}...\sum_{u_t\in S}\pi_a\left(p_a\left(u_{t+i}\right), p_b\left(u_{t+i}\right), \tau, c\right)\Pr\left(u_{t+i}, n_a^e\left(u_{t+i}\right) \mid u_{t+i-1}, n_a^e\left(u_{t+i-1}\right)\right)\right.$$
$$\left. ...\Pr\left(v_t, n_a^e\left(v_t\right) \mid s_{t-1}, n_a^e\left(s_{t-1}\right)\right)\right\} f(c) \quad (50)$$

With profit $\pi_a\left(p_a(s), p_b(s), \tau, c\right)$ the sum of domestic and exporting profit of a firm conditional upon state s, trade costs τ and marginal cost c, defined as:

$$\pi_a\left(p_a(s), p_b(s), \tau, c\right) = \pi_{ad}\left(p_a(s), c\right) I\left[p_a(s) \geq c\right] + \pi_{ax}\left(p_b(s), \tau c\right) I\left[p_b(s) \geq \tau c\right]$$

Hence, there are a total of $2S$ free entry conditions that solve for $2S$ entrants for each state in both countries. The definition of an equilibrium and the proof of existence of equilibrium is a straightforward extension of the proof in the closed economy, as expected profit for each initial state will still increase in the number of entrants for each state.

We can make the following statement on the effect of bilateral trade liberalization

Proposition 18 *Lower trade costs have an ambiguous effect on the market price. Specifically, (i) as trade costs decline from a prohibitive level to a negligible level, the market price decreases; (ii) when the number of entrants does not change, lower trade costs lead to lower market prices; (iii) when a change in the number of entrants is induced, the market price might either go up or down.*

Formally, claim (i) of proposition 17 follows from proposition 16. Claim (ii) is implied by proposition 6. Claim (iii) is proved as follows. First we note that exporting profits increase with lower trade costs for each level of trade costs and each market price (see proof in Appendix C).

[11] The development of prices as trade costs shift down in a move from autarky to free trade reflects the ambiguity in the response of the number of firms to a larger market. If the number of firms goes up less than proportionally with an increase in market size, the move from autarky to free trade has to be accompanied by a decrease in the number of firms in each market. This could be driven by less producing firms for a given number of entrants as the cutoff cost level shifts down or it could be caused by a lower number of entrants in each market.

Domestic profits decline, as the price distribution shifts to the left for each price. The decrease in domestic profit might dominate the increase in exporting profit and this could induce a lower number of entrants and as such a higher price. In particular, we are sure this happens at a level of trade costs with no trade for each state, i.e. at the prohibitive tariff level for each state.[12] The tariff reduction is such that there will be trade for one or more states, but but the change in exporting profit is zero as firms entering the export market have a market share of zero in the export market, whereas the declining market price causes lower profits of firms already producing for the domestic market.

Empirical work by Besedes and Prusa (2006) and Nitsch (2007) shows that there is considerable variability in the probability of positive trade flows in detailed product categories. The current model provides an explanation for this phenomenon from the simple fact that entry from and exit into the market causes significant variability in the price and thus in the possible profitability of exporting. Dying firms are replaced by new ones and this has a non-negligible and possibly large effect on the market equilibrium and so might induce moves from zero to one in the trade matrix and vice versa. Other firm heterogeneity models can also explain the moves in and out of exporting, but only as a result of shocks to exogenous variables as the steady state of entry and exit is a smooth process in these models that does not cause changes in the endogenous variables. In our model variability of prices and hence moves from one to zero in the trade matrix emerges endegenously from the entry process of firms.

3.3 Zeros and Fob Prices

In the recent literature there is interest in the question why many entries in the trade matrix are zero (Baldwin and Harrigan (2007), Helpman, Melitz and Rubinstein (2008)). In this section we explore the effect of distance and importing country size on the probability of zero trade and on fob prices in our model. We start by noticing that the probability of non-zero trade flows and fob export prices move always in the same direction. The probability of a zero export flow in a certain sector from country a to b is determined by the exporting cutoff cost level, $c^*_{ax} = \frac{p_b}{\tau_{ab}}$. When one firm has a cost smaller than this cutoff level, there are non-zero exports. The cutoff level is equal to the fob export price, as all firms charge the same price

[12] The development of prices as trade costs shift down in a move from autarky to free trade reflects the ambiguity in the response of the number of firms to a larger market. If the number of firms goes up less than proportionally with an increase in market size, the move from autarky to free trade has to be accompanied by a decrease in the number of firms in each market. This could be driven by less producing firms for a given number of entrants as the cutoff cost level shifts down or it could be caused by a lower number of entrants in each market.

under oligopoly. Hence, when the cutoff cost level moves up, the fob price goes up and the probability of a non-zero trade flow increases. To be breve we therefore only report the effect on the probability of non-zero trade flows.

We start with the effect of distance, mimicked by a decrease in iceberg trade costs, where we have the following result:

Proposition 19 *A lower distance between trading partners as mimicked by lower iceberg trade costs leads to (i) a smaller probability of zero trade flows when the number of entrants in both countries does not change; (ii) a smaller probability of zero trade flows, when the number of entrants in the importing country decreases due to lower trade costs; (iii) an indeterminate effect on the probability of zero trade flows, when the number of entrants in the importing country rises because of lower trade costs.*

The first part of proposition 19 follows directly from equation (27). The elasticity of the domestic market price with respect to trade costs is between 0 and 1. This implies that $c^*_{ax} = \frac{p_b}{\tau_{ab}}$ will rise with lower trade costs, as the denominator τ_{ab} declines at a larger rate than the numerator p_b. Hence, it becomes easier to export. The second claim follows from the fact that a decline in the number of entrants raises the market price p_b. Therefore, in $c^*_{ax} = \frac{p_b}{\tau_{ab}}$ the numerator rises whereas the denominator declines, implying the cutoff cost level has to rise making it easier to export. Finally, when the number of entrants increases, the market price declines and therefore both numerator and denominator in $c^*_{ax} = \frac{p_b}{\tau_{ab}}$ decline. As we cannot determine the relative size of the effects, it is not clear what happens to the exporting cutoff and thus with the probability of zero trade flows.

Consider next the effect of importing country size on the probability of zero trade flows and export price, as summarized in the following proposition:

Proposition 20 *A larger market size has (i) no effect on the probability of zero trade flows when the number of entrants in the market does not change; (ii) a positive effect on the probability of zero trade flows when entry in the importing country or the exporting country is induced due to a larger market size.*

The first part of this proposition is implied by the fact that market size does not affect the market price when the number of entrants does not change, as pointed out in proposition 5. For the second claim we notice that a larger market induces more entry and lower prices as pointed out in proposition 16. With two countries, this can either happen in one of the two countries, depending upon how close the expected profit was to the sunk entry costs before market extension. However, in both cases the effect is a lower market price and therefore a larger probability of zero trade flows.

Baldwin and Harrigan (2007) compare different models of international trade on their predictions of the effect of distance and importing country size on the probability of zero trade flows and fob prices. From table 1 in their paper it is clear that the model in this paper generates mostly the same predictions as the Melitz and Ottaviano (2008) model in this regard. The predictions are different from the model proposed by Baldwin and Harrigan (2007), which seems to align with the empirical findings presented in their paper. However, whereas the model of Baldwin and Harrigan (2007) contains product differentiation and quality differences, the oligopoly model in this paper describes a setting with homogeneous products. Therefore, the predictions from this model should be tested with data from homogeneous goods sectors and not with a dataset of all sectors as Baldwin and Harrigan (2007) do. Intuitively, the different predictions can be clearly explained from the different modeling setups. Baldwin and Harrigan (2007) adapt the Melitz firm heterogeneity model to allow for quality differences. More productive firms charge higher instead of lower prices, because they sell higher quality products involving also higher marginal costs. The probability of zero trade flows rises with distance in our model (in the no-free-entry case) and in Baldwin and Harrigan (2007). A larger distance makes it in both models more likely that trade costs are too high and that no firm is productive enough to sell profitably in the export market. The probability of zero trade flows rises in importing country size in our model and declines in importing country size in Baldwin and Harrigan (2007). The intuition in our model is that a larger market leads to tougher competition, more entry of firms and lower prices. Henceforth, it becomes harder to export to that market. The model of Baldwin and Harrigan (2007) features fixed export costs. In a larger market it is easier to earn these fixed costs back and therefore also the less productive firms with lower quality and lower price can sell in the market profitably.[13]

A larger distance leads to higher fob export prices in Baldwin and Harrigan (2007) and lower export prices in our model (in the no-free-entry case). In both models a larger distance

[13]A larger market also implies a lower price index and therefore less sales for an individual firm, making it more difficult to sell profitably in the export market. Apparently the direct effect of market size dominates. An effect of market size on profit margins is absent in the model of Baldwin and Harrigan (2007) , because they work with CES and thus fixed markups.

makes it harder to export and therefore only more productive firms can export. In our model with homogeneous goods more productive firms charge lower prices, whereas in Baldwin and Harrigan (2007) they charge higher prices, because the quality of the good is larger. Finally, the export price declines in both models in the importing country size. The reason is different, however. In our model prices are lower in a larger market due to intenser competition and for given trade costs this leads to lower export prices as well. In Baldwin and Harrigan (2007) it is easier to earn back the fixed export costs in a larger market. Therefore, also lower quality, lower price exporters can sell profitably and the average export price will be lower. It could be an interesting exercise to see if the predictions of Baldwin and Harrigan (2007) on the probability of trade zeros and export zeros carry through in a sample of sectors with homogeneous goods or if our model of oligopoly predicts better for these type of sectors.

3.4 Technology Asymmetries

With technological asymmetries, the Ricardian comparative advantage can be treated as a nested case of the present model. Comparative advantage is introduced in this case as follows. There are two types of sectors, country a has a comparative advantage in the A sectors and country b has a comparative advantage in the B sectors. Comparative advantage is modeled by the lower and upper bound of the initial distribution of productivities. As only the lower bound $\underline{c}$ appears in the relevant ZCP and FE equations (when the market price is smaller than the upper bound, which is assumed), attention can be restricted to these. The following assumptions are made to define comparative advantage:

$$\underline{c}_{aA} < \underline{c}_{bA} \tag{51}$$

$$\underline{c}_{aB} > \underline{c}_{bB} \tag{52}$$

$\underline{c}_{aA}$ is the lower integration frontier in country a in the A sectors, i.e. in the sectors in which country a has a comparative advantage. To show that Ricardian comparative advantage is a nested case of the model, the distribution of productivities within a country is squeezed, i.e. the heterogeneity of firms is reduced. The productivity differences between countries remain. When the within country distribution of productivities collapses to a single point, the model converges either to a Ricardian model with perfect competition or a Brander and Krugman (1983) Cournot model with specialization, depending on whether the sunk entry costs disappear or not.

Before the distribution of productivities is narrowed, the following relations between the lower bounds, market prices and trade costs apply:

$$\underline{c}_{aA} < \underline{c}_{bA} < p_{bA}/\tau < p_{bA} \tag{53}$$

$$\underline{c}_{aA} < \underline{c}_{bA} < p_{aA}/\tau < p_{aA} \tag{54}$$

The focus in the discussion is on sector A, because sector B is just its mirror image with a comparative advantage for country b. Equation (53) ensures that at least some firms in country a can export in their comparative advantage sector A and that at least some firms in country b can produce for the domestic market. Equation (54) guarantees that some firms in country r can also export in their comparative disadvantage market A and that firms in country a can sell in their domestic market in their comparative advantage sector A. Hence, there is two-way trade in sector A.

Next, suppose that the distribution of productivities becomes more homogenous. This can be seen as a narrowing of the distribution of productivities. The lower bound moves up and the upper bound moves down. However, only the lower bound appears in the combined ZCP/FE condition, so mathematically a more homogenous productivity distribution comes down to an increase in the lowest cost.

Uncertainty about productivity is a barrier to entry for firms. The sunk entry costs are dependent on uncertainty about the prospective productivity. Firms have to incur research costs to get rid of the uncertainty about their productivity. This interpretation of the sunk entry costs implies that a squeezing of the productivity distribution should decrease the sunk entry costs. The effect of squeezing the distribution of productivities on market prices depends on the size of the change in the sunk entry cost f_e. When this change is small, the market prices will have to rise to keep on satisfying the free entry condition.

Suppose that the distribution of productivities becomes concentrated in one point. Then two questions remain. First, does the model converge to a Ricardian comparative advantage model with perfect competition or a Brander and Krugman Cournot model? Second, will there be full specialization across countries? To address the first question, where the model converges to depends on what happens with sunk entry costs. When some sunk entry costs remain, because uncertainty about productivity is not the only source of the sunk costs, the model remains Cournot. The market price becomes higher than marginal costs to cover the sunk entry costs and the number of firms is limited. When uncertainty is the only source of sunk costs and so when there are no sunk costs left when the distribution of productivities collapses to a single point, the model converges to a perfect competition Ricardian model. Marginal cost will be equal to the market price and the number of firms becomes infinite as is clear from equation (13).[14]

Proposition 21 *When the distribution of productivities becomes concentrated in one point the model either converges to a Brander&Krugman Cournot model or a Ricardian perfect competition model depending on the presence of sunk (or fixed) costs. Two-way trade emerges either from cost heterogeneity or the presence of sunk (or fixed) entry costs.*

Whether there will be full specialization depends on the relation between market prices and marginal cost levels that emerges. There will be full specialization when:

$$\tau \underline{c}_{aA} > p_{aA} > \underline{c}_{aA} \tag{55}$$

$$\underline{c}_{bA} > p_{bA} > \tau \underline{c}_{aA} \tag{56}$$

Equation (55) implies that no firm from country b can profitably export to country a. Equation (56) means that no firm from country b can profitably sell in its home market, whereas the lowest cost producer from country a can. The model converges either to a Cournot model or a Ricardian perfect competition model depending on the presence of sunk costs. There is no strict link between the appearance of full specialization and the type of market competition that emerges. There can be full specialization with Cournot competition when productivity differences are large enough. Also, the Ricardian model does not imply full specialization. A country could still produce for its own market in the Ricardian model in its comparative disadvantage sector when trade costs are large enough. But two way trade is only possible with Cournot competition. Moreover, full specialization is more likely in the Ricardian model without fixed costs, because market prices become equal to marginal costs (inclusive of trade costs) in that case.

Proposition 22 *When the distribution of productivities collapses to a single point, full specialization is more likely with lower trade costs, a larger cost difference between countries and the*

[14] It should be noted that there are no wage differences in the present model. Modeling wage differences, possibly along the line of the Dornbusch-Fischer-Samuelson model of Ricardian trade with a continuum of goods and technology asymmetries, constitutes a possible extension of the present model

absence of sunk costs.

4 Summary and Conclusions

We have developed a model of firm heterogeneity and trade in an oligopoly setting featuring strategic interaction between firms. This approach leads to a set of results familiar from the recent Bertrand and Chamberlinian monopolistic competition literature with cost heterogeneity. Market prices decline, the least productive firms get squeezed out of the market and exporting firms gain market share when trade costs fall. These results hold in cases with and without free entry. The move from autarky to free trade with free entry displays an interesting pattern: the market price goes down but in a non-smooth way. There can be points where the market price jumps up as the number of entrants falls. We also generated predictions on zero trade flows and export prices as a function of distance and importing market size and showed that the probability of zero trade flows varies endogenously over time due to entry and exit of firms. With asymmetric countries, the Brander & Krugman's (1983) reciprocal dumping model and the Ricardian comparative advantage model can be nested as special cases. Possible extensions of the model are the introduction of wage differences between the two countries, political economy applications (as domestic industry concentration is endogenous to the evolution of trade policy), and specifying a distribution of costs enabling simulations with the model with more countries and more sectors.

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Appendix A Basic Model

The appendices show how to derive equations from the main text.

Equation 7: SOC

Differentiating the FOC in equation 6 with respect to firm sales q_i leads to:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2p}{\sigma q} + \frac{(\sigma+1)\,p}{\sigma^2 q}\theta_i \tag{A.1}$$

Substituting the first order condition, $\theta_i = \sigma \frac{p-c_i}{p}$ into (A.1), generates equation (7) in the main text:

$$\begin{aligned}\frac{\partial^2 \pi_i}{\partial q_i^2} &= -\frac{1}{\sigma}\frac{p}{q}\left[2 - \frac{\sigma+1}{\sigma}\theta_i\right] = -\frac{1}{\sigma}\frac{p}{q}\left[2 - \frac{\sigma+1}{\sigma}\sigma\frac{p-c_i}{p}\right] \\ &= -\frac{1}{\sigma}\frac{p}{q}\frac{2p-(\sigma+1)\,p+(\sigma+1)\,c_i}{p} = -\frac{1}{\sigma}\frac{p}{q}\frac{(\sigma+1)\,c_i-(\sigma-1)\,p}{p}\end{aligned}$$

From the first expression it is clear that the SOC is satisfied whenever $\sigma \geq 1$ and $\theta_i < 1$.

Condition on σ and market shares θ for reaction functions to generate stable equilibrium

We can totally differentiate the FOC in equation (6) with respect to q_i and q_j, where q_j are the sales of one other firm, some other firms or all other firms. This gives:

$$\begin{aligned}-\frac{1}{\sigma}\frac{p}{q}\left(1-\frac{1}{\sigma}\frac{q_i}{q}\right)(dq_i+dq_j) - \frac{1}{\sigma}\frac{p}{q}dq_i + \frac{1}{\sigma}p\frac{q_i}{q^2}(dq_i+dq_j) &= 0 \\ -\left(1-\frac{1}{\sigma}\frac{q_i}{q}\right)(dq_i+dq_j) - dq_i + \frac{q_i}{q}(dq_i+dq_j) &= 0\end{aligned}$$

From this equation we get the reaction function of firm i in response to a change in the behavior of firm j:

$$\frac{dq_i\,(q_j)}{dq_j} = -\frac{1-\frac{\sigma+1}{\sigma}\frac{q_i}{q}}{2-\frac{\sigma+1}{\sigma}\frac{q_i}{q}} = -\frac{1-\frac{\sigma+1}{\sigma}\theta_i}{1+\left(1-\frac{\sigma+1}{\sigma}\theta_i\right)} \tag{A.2}$$

This shows that the reaction function has a positive slope if $\frac{\sigma+1}{\sigma}\theta_i > 1$, hence for firms with a large market share. Still, we can show that the model generates a stable equilibrium as long as $\sigma > 3/2$.

To show this, we start from the observation that the model generates a stable equilibrium when the product of a firm's own reaction function and the reaction function of the other firm is smaller than 1, also if one of the reaction functions is positive. A firm with a positive reaction function does increase its sales in reaction to an increase in sales of the other firm. But the reaction of the other firm will be to reduce its sales. This leads to lower sales of the firm with a positively sloped reaction function. The dynamics set in motion do lead back to the equilibrium when the product of the slopes of the reaction functions is smaller than 1 in absolute terms.

Hence, we have the following condition for a stable equilibrium:

$$\begin{aligned} \frac{dq_i}{dq_i} &= \left| \frac{dq_i\left(q_j\right)}{dq_j} \frac{dq_j\left(q_i\right)}{dq_i} \right| < 1 \\ &= \left| \frac{1-\frac{\sigma+1}{\sigma}\theta_i}{1+\left(1-\frac{\sigma+1}{\sigma}\theta_i\right)} \frac{1-\frac{\sigma+1}{\sigma}\theta_j}{1+\left(1-\frac{\sigma+1}{\sigma}\theta_j\right)} \right| < 1 \end{aligned} \tag{A.3}$$

One can show from equation (A.2) that the slope of the reaction function increases monotonically in θ_i and is thus at its maximum when θ goes to 1. Calculations show that the product of the reaction functions is also at its maximum when one of the θ's goes to 1 and the other to 0. One can see from equation (A.3) that this product is smaller than 1, when $\sigma > 3/2$, which is hence a sufficient condition for stability of the reaction functions.

Profit rising in market price

Differentiating profit as defined in equation (17) with respect to the market price gives[15]:

$$\begin{aligned} \frac{\partial \pi}{\partial p} &= \sigma L P_u^{\sigma-1} \left(\frac{2\left(p-c\right)}{p^{\sigma+1}} - \left(\sigma+1\right) \frac{\left(p-c\right)^2}{p^{\sigma+2}} \right) I\left[c \leq p\right] + \pi\left(p,c\right) \frac{\partial I\left[c \leq p\right]}{\partial p} \\ &= \frac{\sigma L P_u^{\sigma-1}}{p^{\sigma}} \frac{p-c}{p} \left(2 - \left(\sigma+1\right) \frac{p-c}{p}\right) + \pi\left(p,c\right) \frac{\partial I\left[c \leq p\right]}{\partial p} \\ &= \frac{L P_u^{\sigma-1}}{p^{\sigma}} \theta\left(c\right) \left(\frac{\left(\sigma+1\right)c - \left(\sigma-1\right)p}{p} \right) + \pi\left(p,c\right) \frac{\partial I\left[c \leq p\right]}{\partial p} \end{aligned} \tag{A.4}$$

The first term in (A.4) is positive by the SOC in equation (7). This reflects two opposite forces: firstly, a decline in the market price leads to larger market sales in the entire industry and thus a larger profit conditional upon entry. Secondly, a decline in market price decreases the profit margin (weighted by the market share θ). This is due to a decline in the profit margin $p-c$ and to the declining market share. The second effect dominates the first effect. The second term is also nondecreasing in p, as the probability to draw a c smaller than p rises with p. Hence profit rises in the market price p for each value of p and c.

[15]As there is a continuum of sectors, the impact of market price on individual profit through aggregate profit in the whole economy is negligible.

Appendix B The No-Free-Entry Case

Equation 26: Direct and indirect effect of trade liberalization in short-run free exit case

The market price is defined in equation (25)

$$p_a = \frac{\sigma}{\sigma\left(n_{da} + n_{xb}\right) - 1}\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb}\right) \tag{B.1}$$

Totally differentiating equation (B.1) with respect to p and τ, one finds:

$$\begin{aligned} dp_a &= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau + \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb} + \sum_{j=1}^{dn_{da}} c_{jda} + \sum_{j=1}^{dn_{xb}} \tau c_{jxb}\right)}{\sigma\left(n_a + dn_{da} + dn_{xb}\right) - 1} \\ &\quad - \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb}\right)}{\sigma n_a - 1} \end{aligned} \tag{B.2}$$

dn_{xb} is the change in the number of exporting firms, i.e the exporting firms that are entering the market because of the change in tariffs and dn_{ds} is the change in the number of domestic firms, i.e. the domestic firms that have to leave the market. These firms that are entering the export market and leaving the domestic market all have marginal costs (inclusive of trade costs for the exporters) equal to the market price. Therefore, equation (B.2) can be written as:

$$\begin{aligned} dp_a &= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau + \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb} + \left(dn_{xb} + dn_{da}\right) p\right)}{\sigma\left(n_a + dn_{xb} + dn_{da}\right) - 1} - \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb}\right)}{\sigma n_a - 1} \\ &= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau \\ &\quad + \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb} + \left(dn_{xb} + dn_{da}\right) p\right)\left(\sigma n_s - 1\right)}{\left(\sigma\left(n_a + dn_{xb} + dn_{da}\right) - 1\right)\left(\sigma n_a - 1\right)} \\ &\quad - \frac{\sigma\left(\sum_{i=1}^{n_{da}} c_{ida} + \sum_{i=1}^{n_{xb}} \tau c_{ixb}\right)\left(\sigma\left(n_a + dn_{xb} + dn_{da}\right) - 1\right)}{\left(\sigma\left(n_a + dn_{xb} + dn_{da}\right) - 1\right)\left(\sigma n_a - 1\right)} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau \\
&+ \frac{\sigma\left(dn_{xb}+dn_{da}\right)p_a\left(\sigma n_a-1\right)-\sigma\left(\sum_{i=1}^{n_{da}} c_{ida}+\sum_{i=1}^{n_{xb}} \tau c_{ixb}\right)\sigma\left(dn_{xb}+dn_{da}\right)}{\left(n_a-1+dn_{xb}+dn_{da}\right)\left(n_a-1\right)} \\
&= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau + \frac{\sigma\left(dn_{xb}+dn_{da}\right)\left(p\left(\sigma n_a-1\right)-\sigma n_a E c\right)}{\left(n_a-1+dn_{xb}+dn_{da}\right)\left(n_a-1\right)} \\
&= \sum_{i=1}^{n_{xb}} c_{ixb} d\tau
\end{aligned}
$$

So, the effect through a change in the number of firms is zero. The direct effect remains which is positive. Using relative changes, one arrives at equation (26) in the main text.

Equation 35: Welfare in free exit case

Welfare per worker is defined in equation (34) of the main text as:

$$
U_a = \frac{I_a}{P_{Ua}} = \frac{L_a + \Pi_a}{L_a P_{Ua}} \tag{B.3}
$$

Labor income is fixed. All Cournot-sectors are equal. Therefore total profit Π is equal to:

$$
\Pi_a = Q\pi_a = Q\left(p_a q_{da} + p_b q_{xa} - \sum c_{ia} q_{ida} - \sum \tau c_{ia} q_{ixa}\right) \tag{B.4}
$$

π_s is profit income in one Cournot-sector. To proceed one needs to assume that the two countries are equal. This implies that (B.4) can be rewritten as:

$$
\begin{aligned}
\frac{\Pi}{Q} &= p\left(q_d + q_x\right) - \sum_{i=1}^{n_d} c_i q_{id} - \sum_{i=1}^{n_x} c_i \tau q_{ix} \\
\frac{\Pi}{Q} &= pq - q\sum_{i=1}^{n_d} c_i \theta_{id} - q\sum_{i=1}^{n_x} c_i \tau \theta_{ix} \\
\frac{\Pi}{Q} &= \frac{I P_U^{\sigma-1}}{p^{\sigma-1}} \left(\frac{p - \sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} c_i \tau \theta_{ix}}{p}\right) \qquad \text{(B.5)} \\
\frac{\Pi}{Q} &= \left(L + \Pi\right) \frac{P_U^{\sigma-1}}{p^{\sigma}} \left(p - \tilde{c}\right) \qquad \text{(B.6)}
\end{aligned}
$$

The term $\sum_{i=1}^{n_d} c_i \theta_{id} + \sum_{i=1}^{n_x} c_i \tau \theta_{ix}$ in (B.5) represents the market share weighted average of costs, $\tilde{c}$. Therefore, $\left(p - \sum_{i=1}^{n_d} c_i \theta_{id} - \sum_{i=1}^{n_x} c_i \tau \theta_{ix}\right)$ is defined as the market share weighted average profit per unit of sales, $\tilde{\pi}$. Solving for $L+\Pi$ from (B.6) yields:

$$\begin{aligned}
\frac{\Pi}{Q} - \Pi\frac{P_U^{\sigma-1}}{p^\sigma}(p-\tilde{c}) &= L\frac{P_U^{\sigma-1}}{p^\sigma}(p-\tilde{c}) \\
\Pi\left(1-\frac{QP_U^{\sigma-1}}{p^\sigma}(p-\tilde{c})\right) &= QL\frac{P_U^{\sigma-1}}{p^\sigma}(p-\tilde{c}) \\
\Pi &= \frac{QL\frac{P_U^{\sigma-1}}{p^\sigma}(p-\tilde{c})}{1-\frac{QP_U^{\sigma-1}}{p^\sigma}(p-\tilde{c})} \\
L+\Pi &= \frac{L}{1-\frac{QP_U^{\sigma-1}(p-\tilde{c})}{p^\sigma}} \qquad \text{(B.7)}
\end{aligned}$$

Substituting equation (B.7) into equation (B.3), one finds the following expression for welfare per worker, equation (35) in the main text:

$$\begin{aligned}
W &= \frac{L}{1-\frac{QP_U^{\sigma-1}(p-\tilde{c})}{p^\sigma}}\frac{1}{LP_U} = \frac{1}{1-\frac{Q(p-\tilde{c})}{(Qp^{1-\sigma}+1)p^\sigma}}\frac{1}{P_U} = \frac{1}{1-\frac{Q(p-\tilde{c})}{Qp+p^\sigma}}\frac{1}{P_U} \\
W &= \frac{(Qp+p^\sigma)}{p^\sigma+Q\tilde{c}}\frac{1}{P_U} \qquad \text{(B.8)}
\end{aligned}$$

Equation 36: Relative Welfare Change in free exit case

Log-differentiating equation (B.8) with respect to trade costs τ, treating the market price p, the price index P_U and average costs $\tilde{c}$ as endogenous generates equation (36) in the main text:

$$\begin{aligned}
\hat{W} &= \frac{Qp+\sigma p^\sigma}{Qp+p^\sigma}\hat{p} - \frac{\sigma p^\sigma\hat{p}+Qd\tilde{c}}{p^\sigma+Q\tilde{c}} - \hat{P}_U \\
&= \left(\frac{Qp+\sigma p^\sigma}{Qp+p^\sigma} - \frac{\sigma p^\sigma}{p^\sigma+Q\tilde{c}}\right)\hat{p} - \frac{Q}{p^\sigma+Q\tilde{c}}d\tilde{c} - \frac{Qp^{1-\sigma}}{Qp^{1-\sigma}+1}\hat{p} \\
\hat{W} &= \left(\frac{\sigma p^\sigma}{p^\sigma+Qp} - \frac{\sigma p^\sigma}{p^\sigma+Q\tilde{c}}\right)\hat{p} - \frac{Q}{p^\sigma+Q\tilde{c}}d\tilde{c} \qquad \text{(B.9)}
\end{aligned}$$

Equation 39 and 42: Conditions for positive welfare effect of trade liberalization

Starting from equation (B.9), one can elaborate on the term $d\tilde{c}$. Using equations (9), (32), (33), $d\tilde{c}$ can be rewritten as follows:

$$d\tilde{c} = \sum_{i=1}^{n_d} c_i\frac{p-c_i}{p}\frac{c_i}{p-c_i}\varepsilon_{p,\tau}\hat{\tau} + \sum_{i=1}^{n_x}\tau c_i\frac{p-\tau c_i}{p}\frac{\tau c_i}{p-\tau c_i}(\varepsilon_{p,\tau}-1)\hat{\tau} + \sum_{i=1}^{n_x}\tau c_i\frac{p-\tau c_i}{p}\hat{\tau} \qquad \text{(B.10)}$$

Using equation (27) one can substitute for the price elasticity in equation (B.10) to get:

$$
\begin{aligned}
d\tilde{c} &= \sum_{i=1}^{n_d}\sigma\frac{c_i^2}{p}\frac{\sum\limits_{i=1}^{n_x}\tau c_i}{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}\hat{\tau}+\sum_{i=1}^{n_x}\sigma\frac{\tau^2 c_i^2}{p}\frac{\sum\limits_{i=1}^{n_x}\tau c_i}{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}\hat{\tau}-2\sum_{i=1}^{n_x}\sigma\frac{\tau^2 c_i^2}{p}\hat{\tau}+\sum_{i=1}^{n_x}\sigma\frac{p\tau c_i}{p}\hat{\tau}d\tilde{c} \\
& \sigma\sum_{i=1}^{n_d}\frac{c_i^2}{p}\frac{\sum\limits_{i=1}^{n_x}\tau c_i}{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}\hat{\tau}+\sigma\sum_{i=1}^{n_x}\frac{\tau^2 c_i^2}{p}\frac{\sum\limits_{i=1}^{n_x}\tau c_i}{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}\hat{\tau} \\
& -\sigma\left(2\sum_{i=1}^{n_x}\frac{\tau^2 c_i^2}{p}-\sum_{i=1}^{n_x}\frac{p\tau c_i}{p}\right)\frac{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}{\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i}\hat{\tau} \\
d\tilde{c} &= \frac{1}{p\left(\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i\right)}*
\end{aligned}
$$

$$
\left[\sigma\sum_{i=1}^{n_d}c_i^2\sum_{i=1}^{n_x}\tau c_i\hat{\tau}+\sigma\sum_{i=1}^{n_x}\tau^2 c_i^2\sum_{i=1}^{n_x}\tau c_i\hat{\tau}-\sigma\left(2\sum_{i=1}^{n_x}\tau^2 c_i^2-p\sum_{i=1}^{n_x}\tau c_i\right)\left(\sum_{i=1}^{n_d}c_i+\sum_{i=1}^{n_x}\tau c_i\right)\hat{\tau}\right] \tag{B.11}
$$

Next, $p=\frac{\sigma}{\sigma n-1}\left(\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i\right)$, $\mu_c=\frac{1}{n}\left(\sum\limits_{i=1}^{n_d}c_i+\sum\limits_{i=1}^{n_x}\tau c_i\right)$, also implying that, $p=\frac{\sigma n}{\sigma n-1}\mu_c$ can be used to rewrite (B.11) as:

$$
d\tilde{c}=\frac{\sigma n}{p^2(\sigma n-1)}\left[\left(\sum_{i=1}^{n_d}c_i^2+\sum_{i=1}^{n_x}\tau^2 c_i^2\right)\sum_{i=1}^{n_x}\tau c_i\hat{\tau}-\left(2\sum_{i=1}^{n_x}\tau^2 c_i^2-p\sum_{i=1}^{n_x}\tau c_i\right)n\mu_c\hat{\tau}\right] \tag{B.12}
$$

The following expression on the variance of costs is used:

$$
Var(c_i)=\frac{1}{n-1}\left(\sum_{i=1}^{n_d}c_i^2+\sum_{i=1}^{n_x}\tau^2 c_i^2-\frac{1}{n}\left(\sum_{i=1}^{n_d}c_i+\sum_{i=1}^{n_x}\tau c_i\right)^2\right)=\frac{1}{n-1}\left(\sum_{i=1}^{n_d}c_i^2+\sum_{i=1}^{n_x}\tau^2 c_i^2-n\mu_c^2\right) \tag{B.13}
$$

Substituting equation (B.13) into equation (B.12) leads to the following expression:

$$
d\tilde{c}=\frac{\sigma^2 n}{p^2(\sigma n-1)}\left[\left((n-1)Var(c_i)+n\mu_c^2\right)\sum_{i=1}^{n_x}\tau c_i\hat{\tau}-\left(2\sum_{i=1}^{n_x}\tau^2 c_i^2-p\sum_{i=1}^{n_x}\tau c_i\right)n\mu_c\hat{\tau}\right] \tag{B.14}
$$

Bringing the summation of $\sum\limits_{i=1}^{n_x}\tau c_i$ outside the brackets in equation (B.14) gives the final expression for $d\tilde{c}$ in equation (39) in the main text:

$$
d\tilde{c}=\frac{\sigma n}{p^2(\sigma n-1)}\sum_{i=1}^{n_x}\tau c_i\left[n\mu_c(\mu_c+p-2\tau c_i)+(n-1)Var(c_i)\right]\hat{\tau}
$$

Inequality (42) can be derived as follows. The $d\tilde{c}$ part of the welfare change in equation (B.9) can be written as:

$$
\begin{aligned}
d\tilde{c} &= \sum_{i=1}^{n_d} c_i \theta_{id} \hat{\theta}_{id} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \hat{\theta}_{ix} + \sum_{i=1}^{n_x} \tau c_i \theta_{ix} \hat{\tau} \\
d\tilde{c} &= \sum_{i=1}^{n_d} \sigma \frac{c_i^2}{p} \hat{p} + \sum_{i=1}^{n_x} \sigma \frac{\tau^2 c_i^2}{p} \left(\hat{p} - \hat{\tau}\right) + \sum_{i=1}^{n_x} \sigma \tau c_i \frac{p - \tau c_i}{p} \hat{\tau} \\
d\tilde{c} &= \frac{1}{p} \sum_{i=1}^{n_d} \sigma c_i^2 \frac{\sum_{i=1}^{n_x} \tau c_i}{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i} \hat{\tau} - \sum_{i=1}^{n_x} \sigma \tau^2 c_i^2 \frac{\sum_{i=1}^{n_d} c_i}{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i} \hat{\tau} + \sum_{i=1}^{n_x} \sigma \tau c_i \left(p - \tau c_i\right) \frac{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i}{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i} \hat{\tau} \\
d\tilde{c} &= \frac{1}{p} \frac{\sigma}{\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i} \left(\sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_d} c_i \hat{\tau} + \sum_{i=1}^{n_x} \tau c_i \left(p - \tau c_i\right) \left(\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \hat{\tau} \right)
\end{aligned}
$$

$$
d\tilde{c} = \frac{\sigma}{p^2 \left(n-1\right)} \left(\sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i \hat{\tau} - \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_d} c_i \hat{\tau} + \left(p \sum_{i=1}^{n_x} \tau c_i - \sum_{i=1}^{n_x} \tau^2 c_i^2 \right) \left(\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \hat{\tau} \right) \tag{B.15}
$$

The third term between brackets in equation (B.15), the gain through lower trade costs, should be positive as $p \geq \tau c_i \; \forall i$. This generates the following condition:

$$
\sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_d} c_i \leq p \sum_{i=1}^{n_x} \tau c_i \left(\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) - \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_x} \tau c_i \tag{B.16}
$$

Substituting the condition in (B.16) into the first two terms of $d\tilde{c}$ in equation (B.15) one can proceed as follows:

$$
\begin{aligned}
d\tilde{c} &\geq \left[\sum_{i=1}^{n_d} c_i^2 \sum_{i=1}^{n_x} \tau c_i - p \sum_{i=1}^{n_x} \tau c_i \left(\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) + \sum_{i=1}^{n_x} \tau^2 c_i^2 \sum_{i=1}^{n_x} \tau c_i \right] \hat{\tau} \\
d\tilde{c} &\geq \sum_{i=1}^{n_x} \tau c_i \left[\left(\sum_{i=1}^{n_d} c_i^2 + \sum_{i=1}^{n_x} \tau^2 c_i^2 \right) - p \left(\sum_{i=1}^{n_d} c_i + \sum_{i=1}^{n_x} \tau c_i \right) \right] \hat{\tau} \\
d\tilde{c} &\geq \sum_{i=1}^{n_x} \tau c_i \left[\left(n-1\right) Var\left(c_i\right) + n \mu_c^2 - pn\mu_c \right] \hat{\tau} \\
d\tilde{c} &\geq \sum_{i=1}^{n_x} \tau c_i \left[\left(n-1\right) Var\left(c_i\right) + n \mu_c \left(\mu_c - p\right) \right] \hat{\tau} \\
d\tilde{c} &\geq \sum_{i=1}^{n_x} \tau c_i \left[\left(n-1\right) Var\left(c_i\right) + n \mu_c \left(\mu_c - \frac{\sigma n}{\sigma n - 1} \mu_c \right) \right] \hat{\tau} \\
d\tilde{c} &\geq \sum_{i=1}^{n_x} \tau c_i \left[\left(n-1\right) Var\left(c_i\right) + \frac{\sigma n}{\sigma n - 1} \mu_c^2 \right] \hat{\tau}
\end{aligned} \tag{B.17}
$$

Table 1: Cost draws, resulting market prices and probabilities in example with 2 entrants and 2 possible cost draws

$\mathbf{c}_s$	c_r	$\mathbf{p}_s$ with $\tau = 8/3$	p_s with $\tau = 2$	Probability
$(1,1)$	$(1,1)$	$p_a(1,1) = 4/3$	$p_a(1,1) = 4/3$	1/16
$(1,1)$	$(1,2)$	$p_a(1,1) = 4/3$	$p_a(1,1) = 4/3$	1/8
$(1,1)$	$(2,2)$	$p_a(1,1) = 4/3$	$p_a(1,1) = 4/3$	1/16
$(1,2)$	$(1,1)$	$p_a(1,2) = 2$	$p_a(1,2,2,2) = 2$	1/8
$(1,2)$	$(1,2)$	$p_a(1,2) = 2$	$p_a(1,2,2) = 2$	1/4
$(1,2)$	$(2,2)$	$p_a(1,2) = 2$	$p_a(1,2) = 2$	1/8
$(2,2)$	$(1,1)$	$p_a(2,2) = 8/3$	$p_a(2,2,2,2) = 16/7$	1/16
$(2,2)$	$(1,2)$	$p_a(1,1) = 8/3$	$p_a(2,2,2) = 12/5$	1/8
$(2,2)$	$(2,2)$	$p_a(1,1) = 8/3$	$p_a(2,2) = 8/3$	1/16

So, from the inequality in (B.17) $d\tilde{c}$ is positive whenever $\frac{Var(c_i)}{\mu_c^2} \geq \frac{\sigma n}{(n-1)(\sigma n-1)}$, condition (42) in the main text.

Appendix C Free Entry Case

Exporting profit declining in trade costs

Exporting profit of a firm with cost c in country a is defined as:

$$\pi_{xa}(p_b, \tau_{ab}c) = \sigma I_a P_{ua}^{\sigma-1} \frac{(p_b - \tau_{ab}c)^2}{p_b^{\sigma+1}} \tag{C.1}$$

Log differentiating exporting profit in equation (C.1) with respect to τ_{ab}, we find:

$$\begin{aligned} \frac{\partial \ln \pi_{xa}}{\partial \ln \tau_{ab}} &= \frac{2\left(p_b \varepsilon_{p_b,\tau_{ab}} - \tau_{ab}c\right)}{p_b - \tau_{ab}c} - (\sigma+1)\,\varepsilon_{p_b,\tau_{ab}} \\ &= -(\sigma-1)\,\varepsilon_{p_b,\tau_{ab}} - 2\frac{\tau_{ab}c}{p_b - \tau_{ab}c}\left(1 - \varepsilon_{p_b,\tau_{ab}}\right) \end{aligned}$$

As $0 < \varepsilon_{p_b,\tau_{ab}} \leq 1$ from equation (27), we have shown that exporting profit rises with lower τ.

Example to show that number of entrants can decline and market price can increase in response to lower trade costs

There are two countries indicated by subscript a and b. We assume that the sunk entry costs are such that there are two entrants in both countries, $n_a^e = n_b^e = 2$. There are two marginal costs in the initial cost distribution, $c_1 = 1$, $c_2 = 2$, both with probability 1/2. We assume $\tau = 8/3$. This implies that we have the following nine possible cost draws, with corresponding prices (as a function of the cost draws of producing firms) and probabilities.

Hence, the prohibitive tariff level is $8/3$. When τ is reduced from $8/3$ it is obvious that the market price decreases for the last three possible cost draws. As a result π_{da} decreases in these three cases, whereas π_{sx} is not affected, as exporting profits start at a zero level. Hence, expected profit declines. When the free entry condition was just satisfied, this implies that the number of entrants drops to one in one of the countries. This leads to an increase in the expected price, from 2 to $3,64$. It seems paradoxical that the decrease in the domestic profit due to a lower price caused exit and thereby largely raises the price in the end. This is due to the fact that the old equilibrium with two entrants in both countries is not an equilibrium anymore, as expected profit would be lower than sunk entry costs.

Calculations (available upon request) show that also when trade costs start from a lower than the prohibitive level the expected profit still declines when trade costs fall.

If we set trade costs at a level slightly larger than $\tau = 2$, expected profit rises with lower trade costs, thus possibly inducing entry.