

# A Story of Maxwell's Displacement Current

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# Agenda

- Brief story of the development of Maxwell's displacement current
- To reflect on
  - Elements of creativity
  - Qualities necessary for nourishing it
- Hence to grasp some of the attributes desired for employability:
  - Openness, life-long learning, creativity

# Reference

- This presentation material is prepared essentially from the following paper:
  - K.T. Selvan, “A revisit of scientific and philosophical perspectives on Maxwell’s displacement current,” *IEEE Antennas and Propagation Magazine*, vol. 51, no. 3, June 2009, pp. 36-46.

# What is displacement current?

- Ampere's law:

$$\nabla \times \mathbf{H} = \mathbf{J}_C \quad (1)$$

- This law does not hold in regions of open circuits where conduction current ( $\mathbf{J}_C$ ) is not present. To cure this defect, Maxwell modified (1) thus:

$$\nabla \times \mathbf{H} = \mathbf{J}_C + \frac{\partial \mathbf{D}}{\partial t} \quad (2)$$

- The term  $\partial \mathbf{D} / \partial t$  is displacement current
- Maxwell treated this as a source term, just like the conduction current

# Highlights

- The discovery of displacement current represents a major, theoretically motivated innovation
- Arrived at based on mechanical considerations
- A very good illustration of the way of innovators
- Much admired
- Also criticized!

# Fundamental questions

- Is the displacement current equivalent to an electric current?
- Does the displacement current produce a magnetic field?

# The context: A medium

- 17<sup>th</sup> century philosophers: understanding of nature calls for mechanical explanations
- Maxwell shared this belief
- He therefore rejected Weber's action-at-a-distance theory, admitting though that the two approaches were mathematically equivalent
- He believed that 'it is a good thing to have two ways of looking at a subject, and to **admit** that there are two ways of looking at it'

# Maxwell's changing ideas

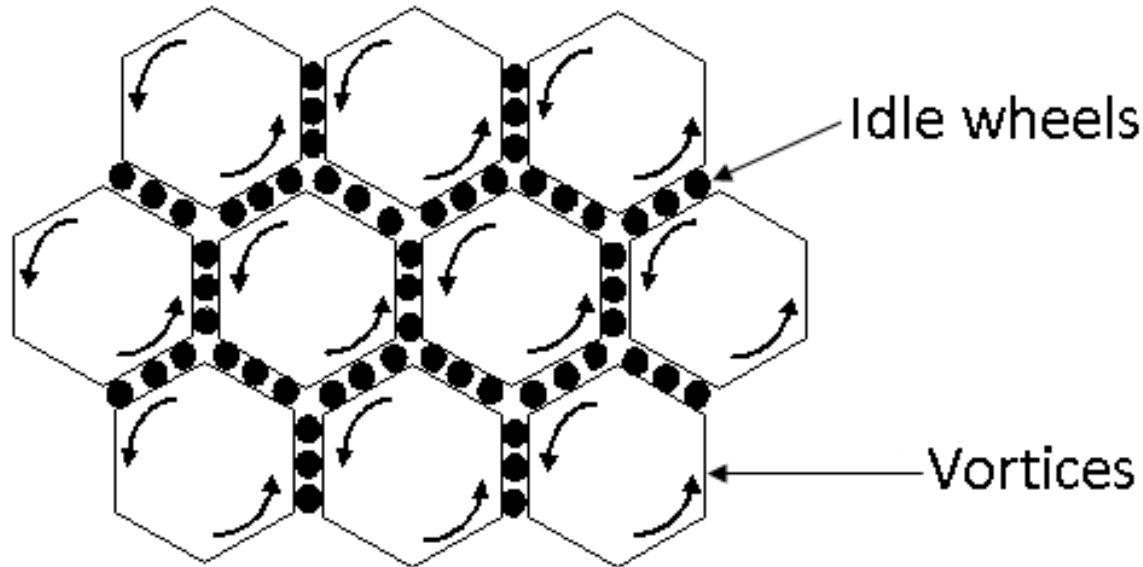
- Though committed to mechanical approach in his 'period of intensive innovation,' Maxwell displayed changing attitude
- Initial belief: Physical analogies
  - Purely mathematical theory would result in losing 'sight of the phenomena to be explained'
  - Physical hypothesis could just be a 'partial explanation' and thus be a threat to furthering investigations
  - Employed in 'Faraday's lines of force,' 1855

- Middle period: Need for medium
  - Clue from William Thomson's work on molecular vortices
  - Rejected the possibility of mathematical theory without mechanical representation
  - Initial formulation of major innovations
  - 'Treatise,' 1864
- Later period: Skeptical towards mechanical representation

# What they said on physical models

- Neils Bohr:
  - ‘A mere formal mathematical structure would obscure the physical core of the problem, and hence a complete physical explanation should absolutely precede the mathematical formulation’
- Einstein
  - ‘The external conditions do not permit the scientist to let himself be too much restricted in the construction of his conceptual world. He, therefore, must appear as a type of unscrupulous opportunist’
  - ‘Only an incomplete picture (model) of the physical universe is obtainable in the pursuit of science’

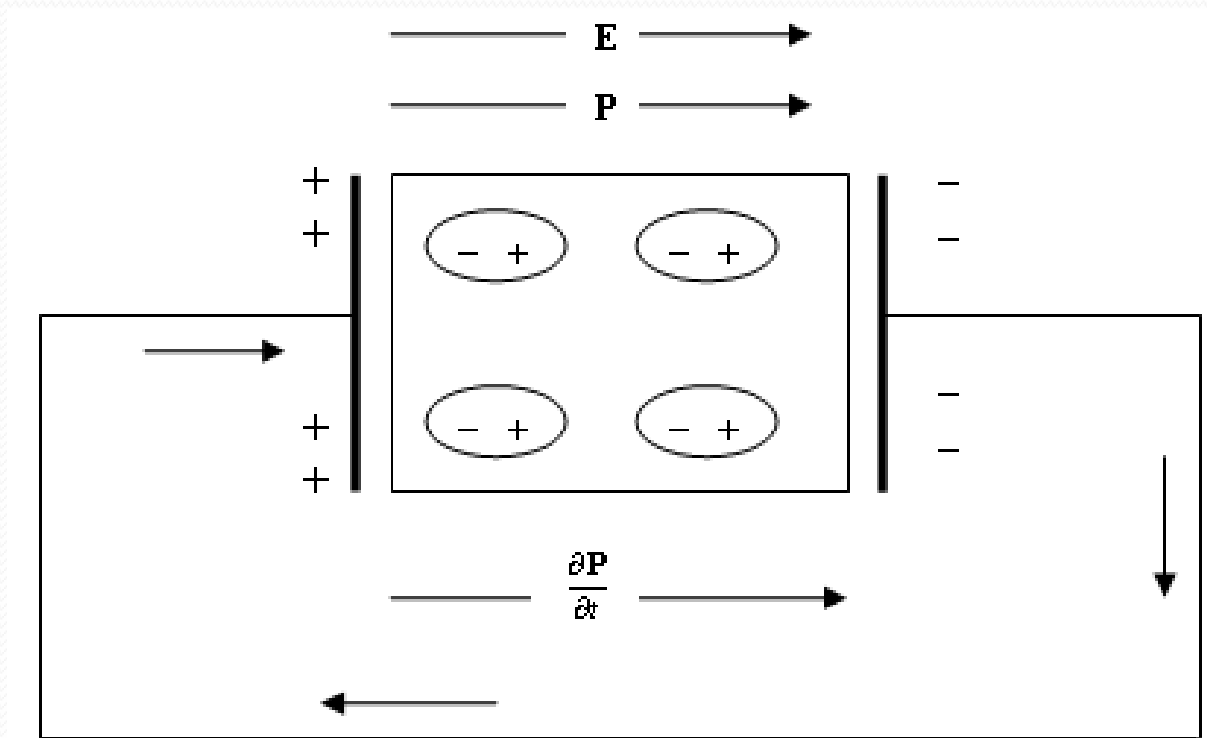
# Maxwell's model: Molecular vortices



- Originally proposed by Thomson
- Idle wheels can move from vortex to vortex in conductors
- Can't leave their place in insulators, causing distortion
- This difference used to explain accumulation of charges at insulator-conductor boundary

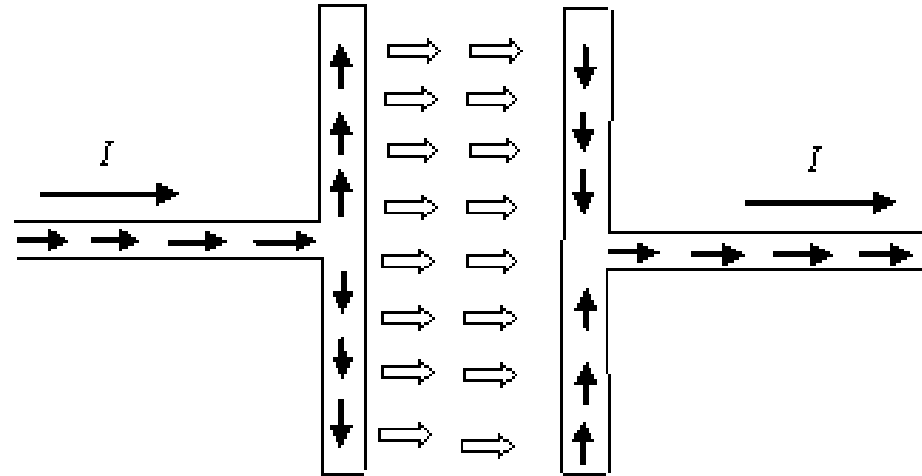
# Introduction of displacement current

- Done to make Ampere's circuital law applicable to open circuits
- Maxwell explained that existence of *open* electric currents had no experimental evidence and thus needed an explanation
  - Thus not quite convinced about its reality himself!



Charging capacitor circuit

- Polarization due to acting EMF tipped to be the physical cause of displacement current



- With displacement current introduced, every electric current – conduction and displacement – could be considered to form a closed circuit

# Displacement current as source term

- Using scalar potential  $\Phi$  and vector potential  $\mathbf{A}$ :

$$\mathbf{E} = -\nabla\Phi - \frac{\partial\mathbf{A}}{\partial t} \qquad \mathbf{B} = \nabla \times \mathbf{A} \qquad (3)$$

- Inhomogeneous Maxwell's equations:

$$-\nabla^2\Phi - \frac{\partial}{\partial t}(\nabla \cdot \mathbf{A}) = \frac{\rho}{\epsilon_0} \qquad (4)$$

$$-\nabla^2\mathbf{A} + \nabla(\nabla \cdot \mathbf{A}) = \mu_0 \left[ \mathbf{J}_C + \frac{\partial\mathbf{D}}{\partial t} \right] \qquad (5)$$

- Coulomb gauge:

$$\nabla \cdot \mathbf{A} = 0 \quad (6)$$

- Lorentz gauge:

$$\nabla \cdot \mathbf{A} = -\frac{1}{c^2} \frac{\partial \Phi}{\partial t} \quad (7)$$

- The potentials using Coulomb gauge are:

$$-\nabla^2 \Phi = \frac{\rho}{\epsilon_0} \quad -\nabla^2 \mathbf{A} = \mu_0 \left[ \mathbf{J}_C + \frac{\partial \mathbf{D}}{\partial t} \right] \quad (8)$$

- Maxwell treating displacement current as source term motivated by the use of Coulomb gauge [Jackson]

# Does displacement current cause magnetic field?

- Purcell:

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} = \epsilon \frac{\partial \mathbf{E}}{\partial t} \quad (9)$$

$$\nabla \times \mathbf{J}_d = \nabla \times \epsilon \frac{\partial \mathbf{E}}{\partial t} = \epsilon \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) = -\epsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} \quad (10)$$

- For slowly varying fields,  $\nabla \times \mathbf{J}_d = 0$ . Hence  $\nabla \cdot \mathbf{J}_d$  should be non-zero
- Magnetic field of any radial, symmetrical current distribution must be zero by symmetry

- Bartlett:

- Magnetic field can be estimated without including displacement current when the Biot-Savart law is employed:

$$c\mathbf{B}(r) = -\int \nabla \times \mathbf{J}_T \frac{d\tau'}{|\mathbf{r} - \mathbf{r}'|} \quad (11)$$

- Also attempted measurements, but with no success

- Rosser:

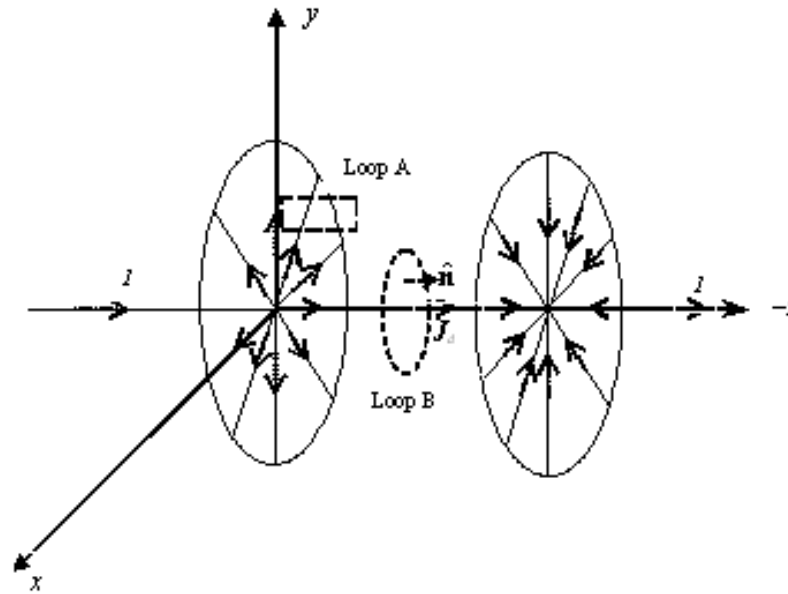
- By using  $\partial E / \partial t$  found from (3) in (8):

$$\begin{aligned}\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mu_o \mathbf{J}_C + \mu_o \epsilon_o \nabla \frac{\partial \phi}{\partial t} \\ &= -\mu_o \left( \mathbf{J}_C - \epsilon_o \nabla \frac{\partial \phi}{\partial t} \right) = -\mu_o \mathbf{J}_t \quad (12)\end{aligned}$$

- In obtaining (12), Coulomb gauge is used
- $\mathbf{J}_t$  is transverse current density, calculable using conduction current and scalar potential
- Thus no part of the displacement current gives rise to a magnetic field!

# The displacement current as an alternative approach

- Jackson:
  - Capacitor magnetic field estimation by using Ampere's law requires
    - the inclusion of displacement current to obtain correct result for arbitrary choices of contours
    - only the conduction currents for certain contours



- Application on loop A needs only the conduction current
- Application on loop B requires the inclusion of the displacement current

- Zapolsky:
  - Rosser's argument is semantic. By appropriate substitutions, the wave equation can be written as

$$\nabla^2 \mathbf{A} = -\mu_o \left( \mathbf{J}_C + \frac{\partial \mathbf{D}}{\partial t} \right) \quad (13)$$

- The displacement current is thus hidden in Rosser's equation (12)!

- It is interesting to note what Maxwell had said:
  - *‘These equations may be regarded as the principal relations among the quantities ... They may be combined so as to eliminate some of these quantities, but our object ... is to express every relation of which we have any knowledge. To eliminate a quantity which expresses a useful idea would be rather a loss than a gain...’*

# Scholarship on electromagnetic equivalence of $\partial\mathbf{D}/\partial t$ to current

- Roche:
  - The displacement current of a rapidly changing induced electric field generates a significant magnetic field
  - Hertz's demonstration
  - In the Coulomb gauge, the displacement current can be considered to be equivalent to an electric current
  - But the Coulomb gauge is not a physical gauge and the equivalence of  $\partial\mathbf{D}/\partial t$  to current is always purely fictional
  - Term very important, but the name should not be mentioned to students, as it is in no sense an electric current!

- Jackson:
  - The Coulomb gauge is no more or less physical than any other
- Yaghjian:
  - There is mathematically nothing wrong with the Coulomb gauge
- Bartlett:
  - Why labor the Coulomb gauge when the Lorentz gauge is generally much easier to use?

- Jackson:
  - Maxwell is wrong (only) if he asserts that the displacement current is a real external current density on a par with the conduction current density, but he is right if he says that it is electromagnetically equivalent to current
- Warbarton:
  - The name has lost its relevance
  - It might mislead students
- Rosser:
  - To avoid confusion, name to be changed

# The lessons

- Variation in perspectives are widely (and inevitably) prevalent and are desirable in research
- Innovation often demands adaptability, rather than rigidity, of ideas, to evolving research paradigms
- Science does not develop in simplistic way
- The image of the certainty of scientific knowledge is not to be taken for granted

# In summary...

- The concept of displacement current is ‘closest to...a genuine, useful, profound theory...built purely speculatively’ [Einstein]
- Without the displacement current, the treatment of electromagnetic waves would have been absurdly complicated [French and Tesson]
- Its development surrounded by a variety of positive and negative discussion
- But then that is the way of science and human thought – and hence of innovation!

# Conclusion

- It pays to discuss the term displacement current and its development!
- The story helps us contemplate about
  - Being open and receptive
  - Being reflective
  - Being a life-long learner!
- Employability in modern life demands all these qualities!



Thank You!