Double shearography for engineering metrology: optical and digital approach

V.M. Murukeshan *, Ng Chee Keong, Krishnakumar V, Ong Lin Seng, A. Asundi

School of Mechanical and Production Engineering, Nanyang Technological University, North Spine (N3), Level 2, Nanyang Avenue, 639798, Singapore

Received 6 December 2000; received in revised form 8 February 2001; accepted 26 February 2001

Abstract

This paper presents a double shearographic configuration based on the optical method by using two Michelson interferometers in tandem. The problems associated with the extraction of second order derivatives by optical means and a comparison with the proposed novel approach by digital means are discussed in this paper. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Shearography; Curvature; ESPI; Image processing; Metrology

1. Introduction

For plate bending problems and flexural analysis, the second order derivatives of the displacements are required for calculating the stress components. Hence, it is necessary to differentiate the slope data to obtain curvature information. The methods reported in literature have some limitations in regard to the experimental set-up, illumination or filtering process. Sharma et al. [1] used a multiple aperture interferometer with two wedges that can measure slope and curvature simultaneously. Similar approach was adopted by Tay et al. [2] and Ganesan et al. [3] for determining curvatures. In these methods the shear is fixed depending on the wedge angle and can not be varied simultaneously. Further, the use of apertures in front of the imaging lens results in loss of light at the image plane, thus necessitating the use of a high power laser especially for large objects. Also the shear can be introduced only along a single direction for a given set-up. Subramaniam et al. [4] and Li et al. [5] reported a method for obtaining pure curvature fringes based on photographic recording. While many studies have been reported based on photographic recording (double-exposure method), few have been reported so far for recording the curvature fringes electronically. The methods based on multi-aperture technique cannot be implemented in real-time techniques like electronic speckle pattern interferometry (ESPI) or electronic shearography where the recording is by digital means. Rastogi [7] has proposed a technique for curvature and twist measurement of a deformed object using 3-beam illumination and by image processing techniques. The method previously reported by our group has a limitation of poor curvature fringe quality [8]. In this paper we present a comparative study of the double shearographic technique by an optical and a digital approach with a novel proposal in order to obtain curvature fringes.

2. Double shearography: Michelson interferometers in tandem

Fig. 1 shows the optical double shearographic configuration of the experimental set-up. It is configured by aligning two Michelson interferometers in tandem. As one of the procedures, the mirrors M1 through M4 are adjusted such that there is no shear along the y-direction and two of the four images sheared along the x-direction coincide. This results in three images as shown in Fig. 2b. Since the middle image is formed by the superposition of two images, the amplitude of light forming that image will be double. The complex amplitudes of light reaching the point \( P(x_i, y_i) \) on the image plane, from the three adjacent object points [say, \( o_1(x, y) \), \( o_2(x - \delta x, y) \) and \( o_3(x + \delta x, y) \)] will be \( 2a \exp[i\phi_1(x, y)] \), \( a \exp[i\phi_2(x, y)] \) and \( a \exp[i\phi_3(x + \delta x, y)] \), respectively.
The intensity at point \( P \) is given by
\[
I = 2\alpha^2 \left[ 3 + 2 \cos(\phi_{21}) + 2 \cos(\phi_{21} + \phi_{32}') + \cos(\phi_{32}) \right],
\] (1)
where
\[
\phi_{21} = \phi_2 - \phi_1, \quad (2a)
\]
\[
\phi_{32} = \phi_3 - \phi_2, \quad (2b)
\]
\[
\phi_{32}' = (\phi_3 - \phi_1) - (\phi_2 - \phi_1). \quad (2c)
\]
Here \( \phi_1, \phi_2, \phi_3 \) are the phases and \( \delta \phi \) is the amplitude. \( \delta x \) is the shear in the object plane given by
\[
\delta x = M \delta x' \quad \text{where} \quad \delta x' \quad \text{is the image plane shear and} \quad M \quad \text{is the magnification.}
\]
For the intensity calculation, we follow the approach of Tay et al. [2], Murukeshan et al. [6] and which, briefly stated, is as follows:

The brightness at any point \( P \) in a speckle correlogram produced at the CCD plane can be written as [3,6].
\[
I(x, y) \propto 1 + V \cos(\delta \phi),
\] (2)
where \( V \) is the fringe visibility.

The term \( \delta \phi \) is given as
\[
\delta \phi \approx \frac{4\pi}{\lambda} \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \right),
\] (3)
for small amounts of shear. These fringes represent contours of derivatives of displacement in the shear direction. When two such slope fringes are subtracted following an additional shear, the resulting moiré [3,6] fringe pattern is represented by
\[
\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \delta x^2 = \frac{(2m + 1)\lambda}{4}, \quad m = 0, \pm 1, \pm 2, \ldots, \quad (4)
\]
where \( \delta x \) is the shear along the \( x \)-direction (\( x \)-shear) and \( "w" \) is the out of plane displacement. This combined fringe pattern depicts first order derivative (slope) and the second order derivative (curvature) of displacement.

But the curvature fringes are hardly visible when we follow this approach. From Fig. 2b, it can be inferred that coinciding the two sheared images (Fig. 2a) in opposite directions forms three images, the central image having intensity double that of the other two images. Similar results can be obtained by configuring different shear elements either in tandem or in combination.

Single optical shearing followed by digital shearing can also achieve this. Single shearing is given by blocking one of the mirrors in the Michelson interferometers in order to act as a single optical shearing configuration. The fiber-optic shearographic configuration presented in our earlier work can also be used for single shearing [9]. The limitations of the double shear configuration shown in Fig. 1 and their modifications are explained in the following sections.

3. Results and discussion

3.1. Double shear configuration: optical method

The test object is a circular diaphragm (\( \phi = 40 \text{ mm} \)) made of polymethyl methacrylate (PMMA) sheet. It is clamped along its edges and a central displacement of 10 \( \mu \text{m} \) is applied. The shear is set to 7 mm. Using Fig. 1, the curvature fringes are hardly visible presumably due to (a) the low spatial resolution normally associated with the CCD camera and (b) the doubly intense central image when compared to the other two images.

In order to improve the contrast and to observe the curvature fringes the following is carried out. The digitally processed slope fringe pattern was copied and stored into the memory buffer and a digital shear was applied. The two slope fringes were then subtracted from each other to obtain curvature fringes. The fringe pattern shows a combination of perturbed slope and curvature fringes [Fig. 3].
Fig. 3. Combined (and disturbed) fringe pattern of slope and curvature using Fig. 1.

Fig. 4. Phase modulo $2\pi$ obtained after incorporating phase shifting and single optical shear.

3.2. Double shear configuration: incorporation of phase shifting technique

The same Fig. 1 is used as well as the test specimen. Single shearing is given by blocking one of the mirrors in the Michelson interferometers in order to act as a single optical shearing configuration. Instead of giving a small shear, a large shear is given. This is mainly due to the decorrelation of many displacement fringes on deformation with small shear [10]. The object before deformation is stored in the frame grabber board. When the object is given the central load, correlation fringes are seen on the TV monitor in real-time [11]. Thus as a first step, the slope fringes are obtained in real-time on the TV Monitor. A phase modulator (Piezo-driven mirror [9,11]) is attached to one of the mirrors of the Michelson interferometers (in Fig. 1.) for incorporating phase shifting. Application of a voltage will induce a phase difference. By using the PZT attached mirror phase modulator, phase shifts of 0, $\pi/2$, $\pi$, $3\pi/2$ are given and four phase shifted fringe patterns are stored in the frame grabber board of the associated image processing card. The developed software is used to calculate the $2\pi$ phase (Fig. 4). Phase modulo $2\pi$ is then digitally sheared for the same shear amount. Using the developed software and the image processing card, the panned modulo $2\pi$ phase is subtracted from the initial stored modulo $2\pi$ phase. This results in the moiré phase pattern corresponding to the curvature or the second derivative of the out-of plane displacement of the deformed object (Fig. 5.). The shear can be varied continously and adjusted for the required magnitude and orientation by adjusting the mirrors. Though we can see the curvature contours by this method, the quality is poor due to the superposed slope fringes. Hence, the following digital scheme is developed.

4. Curvature fringes by the developed digital method

A novel method is proposed in order to obtain good quality curvature fringes. In this method, the shear fringe is subtracted from its shifted version to obtain curvature fringes. Shifting and subtraction of the slope fringes is performed digitally. Let $f(x, y)$ be the original image and let the shear constrained in one dimension say the x direction is $\delta$ such that the shifted image is $f(x + \delta, y)$. The resultant fringe is,

$$g(x, y) = f(x, y) - f(x + \delta, y)$$

(4)

The curvature fringes are seen in the image $g(x, y)$. This is similar to the double shear interferometer in which a single optical shearing and a single digital shearing are given. The curvature fringes are seen as moiré. These fringes have a lower spatial frequency. Hence to improve the contrast the following is done:

We low-pass Fourier filter $g(x, y)$ such that

$$g_{fl}(x, y) = g(x, y) * * h(x', y'),$$

(5)

where "**", denotes convolution and $h(x, y)$ is the low-pass filtering function. We selectively low-pass filter $g(x, y)$ with a high frequency components and is filtered less along the $x$ direction than that along the $y$ direction. $h(x', y')$ is an asymmetric Gaussian in this case. The width of the Gaussian is made different between the $x$ and $y$ axis such that $h(x' = x/u, y' = y/v), u < v$, or if $H(x', y')$ is the Fourier transform of $h(x, y)$ then $f^{x'} = fx/c, f^{y'} = fy/d, c > d$. 

$$f^{x'} = \frac{fx}{c}, f^{y'} = \frac{fy}{d}$$
Fig. 6. Curvature fringe pattern after the incorporation of the proposed quasi-real time method.

Fig. 7. Second order derivatives of displacement: Comparison between theoretical and experimental values.

Low-pass filtering gives the overall curvature fringes improving the fringe quality (Fig. 6). Fig. 7 shows a comparison between the theoretical and experimental data for the second-order derivative along the specimen center.

5. Conclusion

In this paper we have shown that the digital approach described here can effectively be applied to obtain curvature.

The desired magnitude and direction of shear can be selected by this configuration. The approach presented in this paper can be used to obtain the second derivative of displacements, which is a directly proportionate quantity to bending moments. A stress analyst can use this as a direct tool to estimate strain, stress and bending moments of an object under test.

Acknowledgements

The authors gratefully acknowledge School of Mechanical and Production Engineering, NTU, Singapore for the support.

References