On the Evolution of U.S. Temperature Volatility

Francis X. Diebold
University of Pennsylvania

Glenn D. Rudebusch
Federal Reserve Bank of San Francisco

June 20, 2019

Climate Change...

... is a multidimensional shift in the joint conditional probability distribution describing the state of the atmosphere, oceans, and fresh water (including ice).

Many aspects of climate change

One of the most important is temperature

Many aspects of temperature

– Cold spell durations; frost days; growing season length; ice days; summer days; tropical nights; warm spell durations; ...

– Average Temperature (AVG = .5 MAX + .5 MIN)

– Diurnal Temperature Range (DTR = MAX - MIN)

DTR is an efficient estimator of underlying daily quadratic variation

DTR

– Overall environmental and human health impacts

– Broad industries like agriculture
$(Output = f(K, L, AVG, DTR, ...))$

– Specific parts of agriculture, like wine production:

> *Diurnal temperature variation is of particular importance in viticulture. Wine regions situated in areas of high altitude experience the most dramatic swing in temperature variation during the course of a day. In grapes, this variation has the effect of producing high acid and high sugar content as the grapes' exposure to sunlight increases the ripening qualities while the sudden drop in temperature at night preserves the balance of natural acids in the grape.*

Data

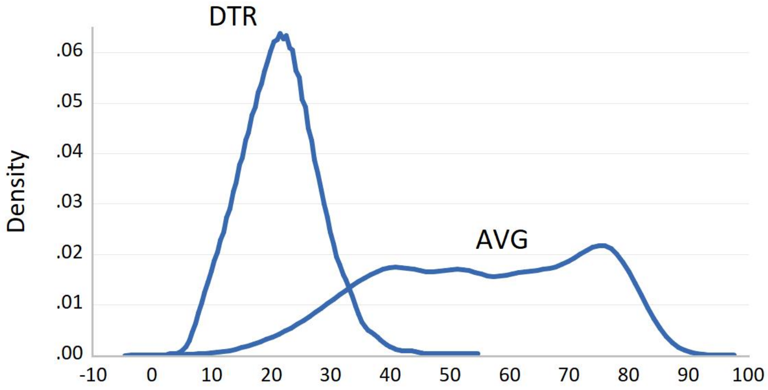
Daily MAX and MIN measured in degrees Fahrenheit, obtained from the U.S. National Ocean and Atmospheric Administration (NOAA) Global Historical Climate Network database (GHCN-daily)

`https://www.ncdc.noaa.gov/ghcn-daily-description`

Sample period is 01/01/1960 – 12/31/2017

Penn
University of Pennsylvania

Estimated Densities, AVG and DTR, Philadelphia



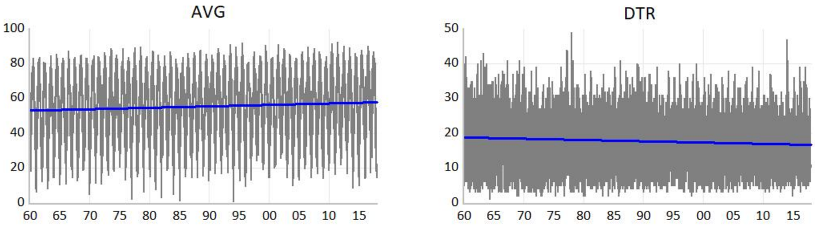
Linear Trend

$$Y \to c, TIME,$$

where $TIME_t = t$.

We use Newey-West HAC s.e.'s here and throughout.

Data and Estimated Trends, AVG and DTR, Philadelphia

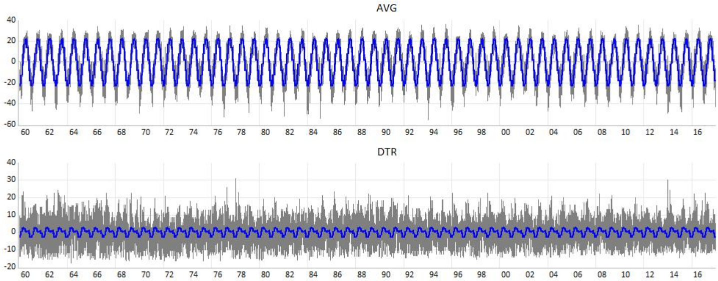


Fixed Seasonality

$$\widetilde{Y} \;\rightarrow\; D_1, ..., D_{12},$$

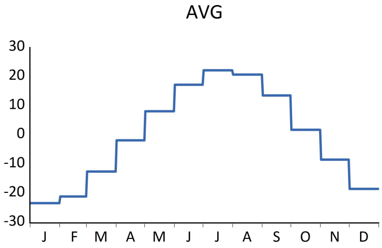
where $\widetilde{Y}$ is de-trended AVG or DTR, $D_{it} = 1$ if day t is in month i and 0 otherwise.

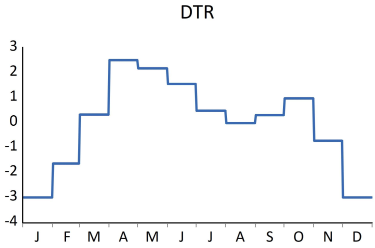
De-Trended Data and Estimated Fixed Seasonal, AVG and DTR, Philadelphia



Penn
University of Pennsylvania

Estimated Fixed Twelve-Month Seasonal Pattern, AVG and DTR, Philadelphia



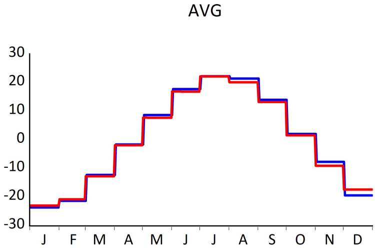


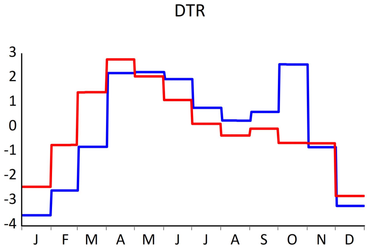
Evolving Seasonality

$$\widetilde{Y} \;\rightarrow\; (D_1, ..., D_{12}), \;\; (D_1 \cdot TIME, ..., D_{12} \cdot TIME),$$

where $\widetilde{Y}$ is de-trended AVG or DTR, $D_{it} = 1$ if day t is in month i and 0 otherwise, and $TIME_t = t$.

Estimated Evolving Twelve-Month Seasonal Patterns, AVG and DTR, Philadelphia, 1960 vs. 2017





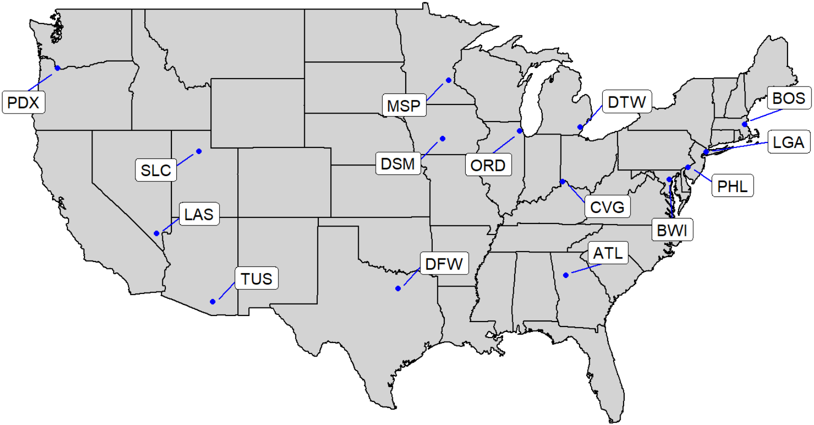
Blue is 1960 and red is 2017.

A Joint Regression

$$
\begin{aligned}
Y \;\rightarrow\; & c,\; TIME, Y(-1), \\
& D_1,\; \ldots,\; D_6,\; D_8,\; \ldots,\; D_{12}, \\
& D_1 \cdot TIME,\; \ldots,\; D_6 \cdot TIME,\; D_8 \cdot TIME,\; \ldots,\; D_{12} \cdot TIME,
\end{aligned}
$$

where Y is AVG or DTR, $TIME_t = t$, $Y(-1)$ denotes a 1-day lag, and $D_{it} = 1$ if day t is in month i and 0 otherwise.

Fifteen Measurement Stations



"The CME Fifteen"

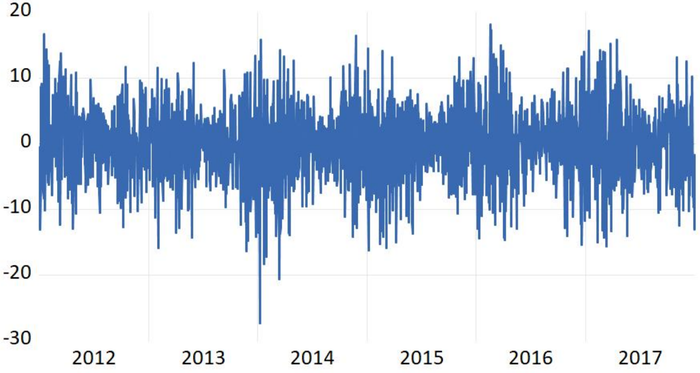
AVG Regression Ten Cities

(1) *station*	(2) $\Delta trend$	(3) $p(nt)$	(4) $p(ns)$	(5) $p(nts)$	(6) ρ	(7) R^2
ATL	4.09*	0.00	0.00	0.99	0.76*	0.90
CVG	2.23*	0.00	0.00	0.94	0.74*	0.89
DFW	3.18*	0.00	0.00	0.55	0.72*	0.89
DSM	3.57*	0.00	0.00	0.17	0.76*	0.91
LGA	2.52*	0.00	0.00	0.96	0.69*	0.91
LAS	5.77*	0.00	0.00	0.41	0.82*	0.96
ORD	2.52*	0.00	0.00	0.78	0.74*	0.90
PDX	2.37*	0.00	0.00	0.26	0.76*	0.90
PHL	4.42*	0.00	0.00	0.95	0.72*	0.91
TUS	4.66*	0.00	0.00	0.31	0.79*	0.93
Median	3.38	0.00	0.00	0.67	0.75	0.91

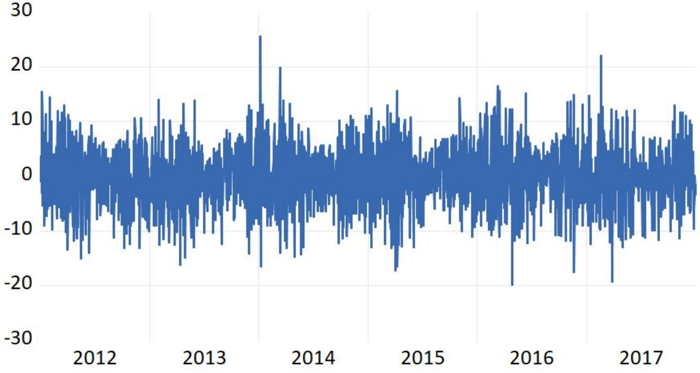
DTR Regression, Ten Cities

(1) station	(2) $\Delta trend$	(3) $p(nt)$	(4) $p(ns)$	(5) $p(nts)$	(6) ρ	(7) R^2
ATL	-1.61*	0.00	0.00	0.14	0.38*	0.18
CVG	-1.29*	0.00	0.00	0.04	0.32*	0.17
DFW	-1.29*	0.00	0.00	0.64	0.40*	0.17
DSM	-0.50	0.06	0.00	0.03	0.32*	0.15
LGA	-0.17	0.00	0.00	0.00	0.26*	0.11
LAS	-6.91*	0.00	0.00	0.13	0.46*	0.37
ORD	-2.01*	0.00	0.00	0.00	0.30*	0.20
PDX	-1.66*	0.00	0.00	0.63	0.50*	0.45
PHL	-2.08*	0.00	0.00	0.00	0.34*	0.19
TUS	0.52	0.06	0.00	0.04	0.51*	0.35
Median	-1.45	0.00	0.00	0.04	0.36	0.19

Residual Heteroskedasticity, AVG, PHL
Zoom in to any Five-Year Span, e.g., 2012-2017:



Residual Heteroskedasticity, DTR, PHL
Zoom in to any Five-Year Span, e.g., 2012-2017:

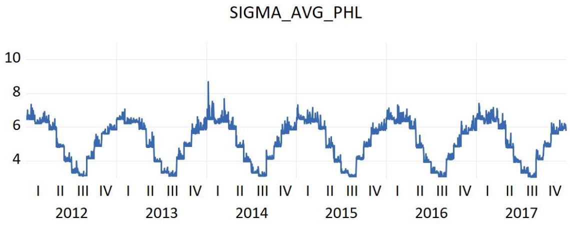


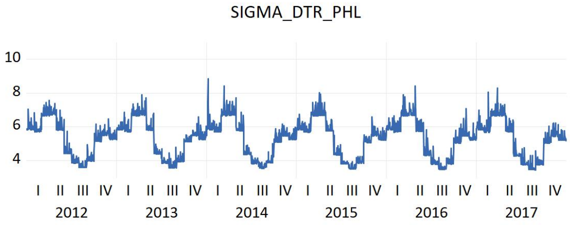
Conditional Variance Regression

$$e^2 \rightarrow c, \; TIME, e^2(-1),$$
$$D_1, \; ..., \; D_6, \; D_8, \; ..., \; D_{12},$$
$$D_1 \cdot TIME, \; ..., \; D_6 \cdot TIME, \; D_8 \cdot TIME, \; ..., \; D_{12} \cdot TIME,$$

where e^2 is the squared residual from the earlier conditional mean regression (for AVG or DTR)

Estimated Conditional Standard Deviations, PHL Zoom in to a Five-Year Span, e.g., 2012-2017:





AVG, Conditional Variance Dynamics, Ten Cities

(1)	(2)	(3)	(4)	(5)	(6)	(7)
station	$\Delta trend$	$p(nt)$	$p(ns)$	$p(nts)$	ρ	R^2
ATL	-0.29	0.23	0.00	0.34	0.07*	0.11
CVG	-0.43*	0.00	0.00	0.52	0.04*	0.10
DFW	-0.11	0.82	0.00	0.77	0.09*	0.11
DSM	-0.34*	0.00	0.00	0.25	0.05*	0.08
LGA	-0.17*	0.00	0.00	0.02	0.04*	0.05
LAS	-0.06	0.46	0.00	0.39	0.08*	0.03
ORD	-0.79*	0.00	0.00	0.40	0.04*	0.05
PDX	-0.03	0.38	0.00	0.30	0.10*	0.02
PHL	-0.25*	0.00	0.00	0.29	0.05*	0.06
TUS	-0.02	0.03	0.00	0.03	0.03*	0.04
Median	-0.21	0.02	0.00	0.32	0.05	0.06

DTR, Conditional Variance Dynamics, Ten Cities

(1)	(2)	(3)	(4)	(5)	(6)	(7)
station	$\Delta trend$	$p(nt)$	$p(ns)$	$p(nts)$	ρ	R^2
ATL	-0.86*	0.00	0.00	0.00	0.01	0.10
CVG	-0.64*	0.00	0.00	0.72	0.03*	0.05
DFW	-0.44	0.12	0.00	0.91	0.03*	0.11
DSM	-0.50*	0.01	0.00	0.87	0.01	0.06
LGA	-0.24*	0.01	0.00	0.10	0.04*	0.02
LAS	-1.23*	0.00	0.00	0.00	0.04*	0.04
ORD	-1.05*	0.00	0.00	0.02	0.04*	0.03
PDX	-0.79*	0.00	0.00	0.01	-0.01	0.05
PHL	-0.89*	0.00	0.00	0.02	0.07*	0.05
TUS	0.21	0.15	0.00	0.63	0.01	0.04
Median	-0.72	0.00	0.00	0.06	0.03	0.05

A Stochastic Model

$$Y_t = c + \beta \, TIME_t + \sum \delta_i \, D_{it} + \sum \gamma_i \, D_{it} \, TIME_t + Y_{t-1} + \varepsilon_t$$

$$\varepsilon_t = \sigma_t \, v_t$$

$$\sigma_t^2 = c' + \beta' \, TIME_t + \sum \delta_i' \, D_{it} + \sum \gamma_i' \, D_{it} \, TIME_t + \alpha \, \varepsilon_{t-1}^2$$

$$v_t \sim iid \, f(0, 1)$$

Standardization

Since

$$\varepsilon_t = \sigma_t \, v_t$$

$$v_t \sim iid \; f(0,1),$$

we have

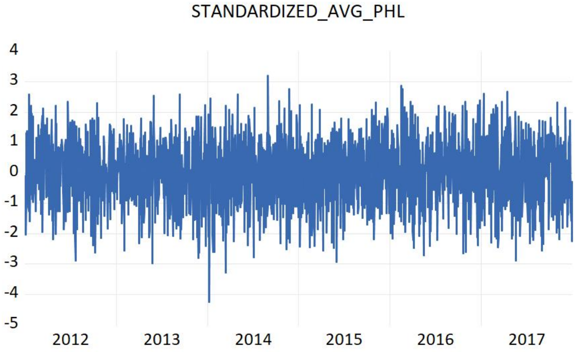
$$\frac{\varepsilon_t}{\sigma_t} \sim iid \; f(0,1)$$

Empirically, does standardization remove heteroskedasticity?

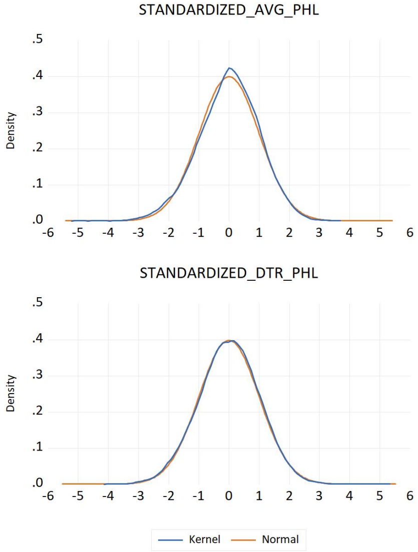
Empirically, what is the shape of the conditional density, f?

Penn
University of Pennsylvania

Standardization Removes Heteroskedasticity



Standardization Produces Normality



Residual Skewness and Kurtosis, Ten Cities

(1)	(2)	(3)	(4)	(5)
	AVG:		DTR:	
station	*skew*	*kurt*	*skew*	*kurt*
ATL	-0.74	3.49	-0.16	3.80
CVG	-0.41	3.39	0.02	2.74
DFW	-0.84	4.22	0.05	2.86
DSM	-0.21	3.16	0.14	2.78
LGA	0.16	3.19	0.30	3.14
LAS	-1.09	5.22	-0.50	3.03
ORD	-0.07	2.96	0.19	3.11
PDX	-0.18	3.26	0.01	3.16
PHL	-0.02	2.90	-0.20	3.15
TUS	-0.88	4.12	-0.66	3.52
Median	-0.20	3.33	0.17	3.13

Penn
University of Pennsylvania

Conclusions and Future Directions

Our time series representation of DTR and AVG shows:

– Strong and different DTR and AVG cond mean trends

– Strong and different DTR and AVG cond mean seasonality

– Evolving DTR cond mean seasonality

– Strong DTR and AVG cond *variance* seasonality

– Gaussian innovations to both DTR and AVG

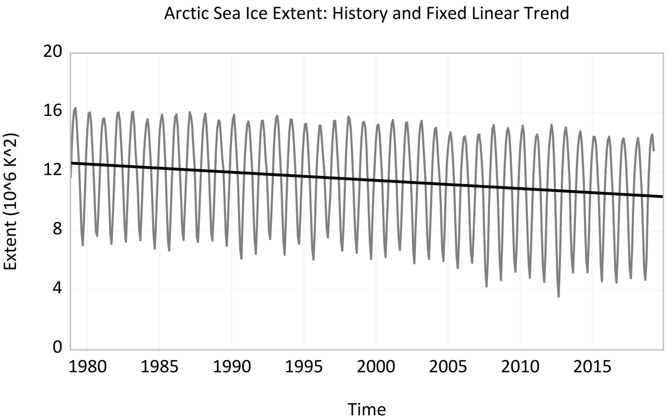
Possible applications of such analysis:

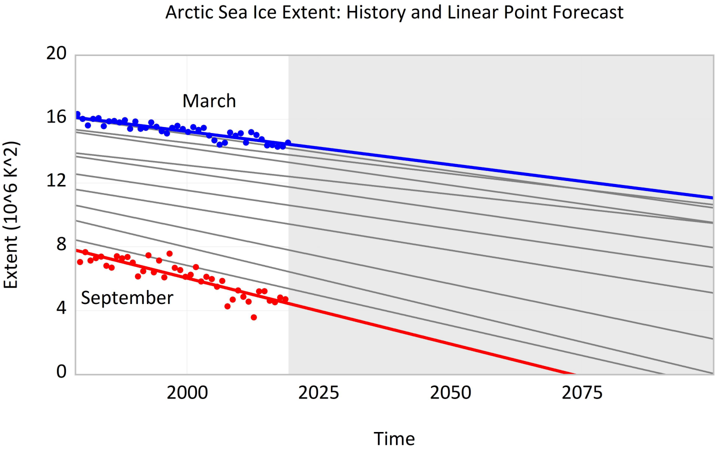
– Guiding decisions

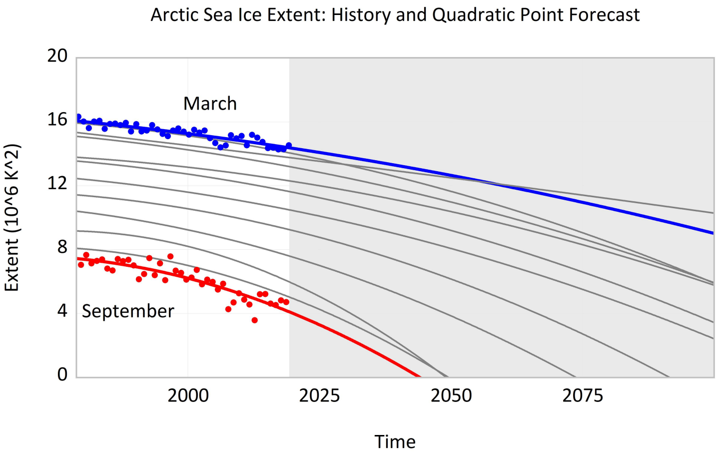
– Assessing and refining structural climate models

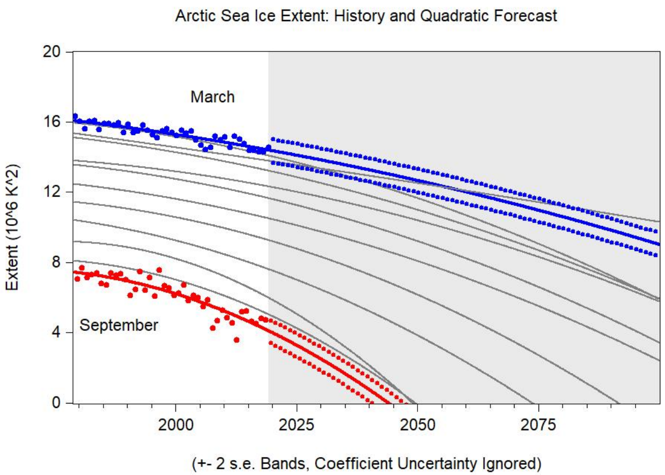
– Pricing weather derivatives

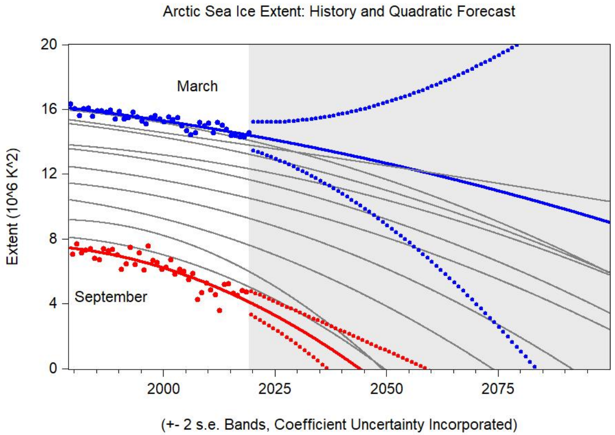
– Guiding related modeling efforts; e.g., arctic sea ice

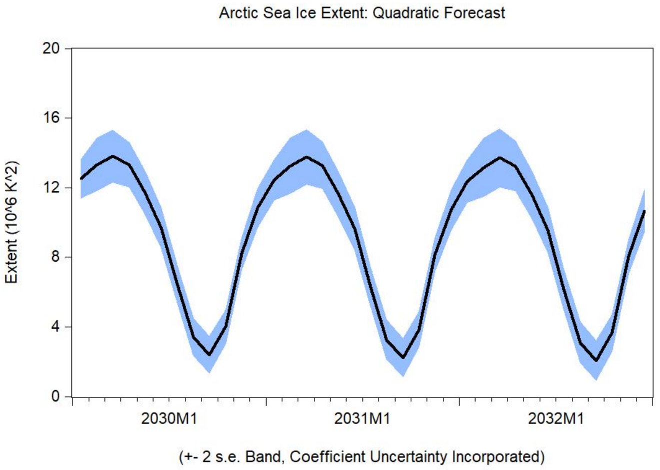

Arctic Sea Ice Extent: History and Fixed Linear Trend
Extent (10^6 K^2)
20
16
12
8
4
0
1980
1985
1990
1995
2000
2005
2010
2015
Time

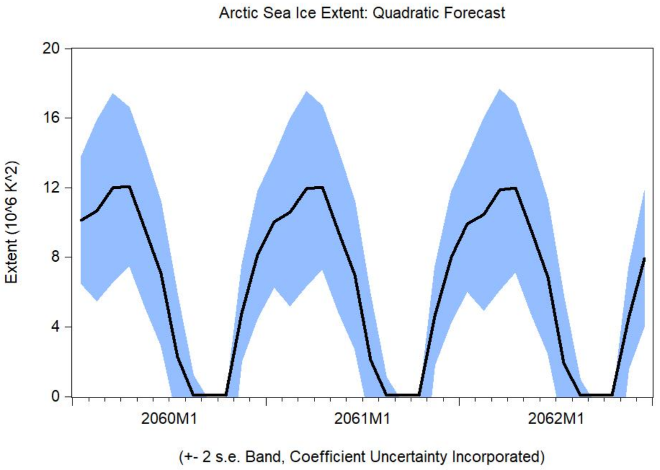

Arctic Sea Ice Extent: History and Linear Point Forecast
20
16
12
8
4
0
March
September
Extent (10^6 K^2)
2000
2025
2050
2075
Time


Arctic Sea Ice Extent: History and Quadratic Point Forecast
20
16
12
8
4
0
March
September
2000
2025
2050
2075
Extent (10^6 K^2)
Time


Arctic Sea Ice Extent: History and Quadratic Forecast
20
16
12
8
4
0
March
September
Extent (10^6 K^2)
2000
2025
2050
2075
(+- 2 s.e. Bands, Coefficient Uncertainty Ignored)


Arctic Sea Ice Extent: History and Quadratic Forecast
March
September
Extent (10^6 K^2)
20
16
12
8
4
0
2000
2025
2050
2075
(+- 2 s.e. Bands, Coefficient Uncertainty Incorporated)


Arctic Sea Ice Extent: Quadratic Forecast
20
16
12
8
4
0
Extent (10^6 K^2)
2030M1
2031M1
2032M1
(+- 2 s.e. Band, Coefficient Uncertainty Incorporated)


Arctic Sea Ice Extent: Quadratic Forecast
20
16
12
8
4
0
Extent (10^6 K^2)
2060M1
2061M1
2062M1
(+- 2 s.e. Band, Coefficient Uncertainty Incorporated)