The Sustainability of Economic Unions
in the Third-Market Model

by

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Abstract
This paper considers the formation of economic unions as an alternative to an export subsidies trade war situation in the context of the third-market model of Brander and Spencer (1985). We consider the possibility of an economic union of two producing countries plus a third importing country, with a common economic authority choosing an optimal common output subsidy in the union. We show that even although an economic union formed by all the countries may Pareto dominate the non-union situation, if we take account of all possibilities of coalition formation in hand of the countries, the coalition formed by all the countries is not stable in the single-shot game. Specifically, the only stable coalition is the one formed by the more competitive country and the consuming country when the more efficient firm is sufficiently more competitive than the less competitive firm. Finally, it is shown in an infinitely repeated version of the game that the economic union formed by the three countries is sustainable as a perfect equilibrium if the two producing countries are sufficiently similar and the discount factor is sufficiently large.

Outline
1. Introduction
2. The Model
3. The Stability of the Economic Unions in the Basic Single-Shot Game
4. The Sustainability of the Economic Union formed by the Three Countries
5. Final Remarks
Non-Technical Summary

In this paper we focus on the incentives for countries to form an economic union as an alternative to a trade war where producing countries use export subsidies in order to gain an advantage. We consider a ‘third market model’ in which there are two producing countries and one consuming country, with the structure of the model as follows: in each of the producing countries a firm is located that produces a homogeneous good with constant returns to scale; the firms compete on a Cournot basis; outside an economic union, a country can use subsidies to improve the competitive position of the domestic firm; there are asymmetric production costs between the firms.

In our analysis, we depart from the traditional literature by considering the possibility of the three countries forming an economic union, with an economic authority choosing a common level of subsidy per unit of output. The objective of the authority is to maximize the joint welfare of the three countries net of subsidies. The total amount of subsidies in the union has to be paid by the three countries. Given the joint financial responsibility by the countries in the union, the relevant question is how to share the subsidies received by the firms among the three countries, in such a way that the three countries have an incentive to form an economic union.

We find that the optimal subsidy, output and profit of firms within the economic union formed by the three countries are larger than the equilibrium values in the non-cooperative export subsidy game. We show that if the difference in competitiveness between the producing countries is low enough, the total social welfare under the economic union formed by the three countries is greater than the total social welfare under the non-union situation. Therefore, there exists in an economic union an equilibrium that Pareto dominates the non-union situation.

The decision of the countries to form an economic union produces two effects. On the one hand, the so-called technological effect, implies that increases in subsidies in an economic union cause the less efficient firm increase its market share. Therefore, the economy as a whole is less efficient. On the other hand, the subsidy effect, implies that as total output increases prices are driven downwards and hence consumer surplus increases. When the competitive differences are sufficiently low, both firms will increase their outputs and the second effect will dominate the first, and vice versa. In the second case, total welfare is lower under an economic union and no division of revenues can make all countries better off.

In what follows, we assume that the difference in competitiveness is such that an economic union Pareto dominates the non-union situation. We study the stability of the economic union in the basic single-shot game by considering four possible coalitions: a three country union; a union formed by one producing
country and the consuming country; and the non-union situation. Each country can talk to another and decide whether to form an economic union and how to share the cost of the subsidy to be paid to the firm(s) inside the economic union. We find that (i) the unique stable coalition is the one formed by the more efficient producing country plus the consuming country, as long as the efficient firm has a sufficient competitive advantage and (ii) no situation is stable if the two firms are sufficiently similar.

Finally, it is shown in a dynamic version of the game, that the economic union formed by the three countries can be sustainable as a perfect equilibrium, if the two producing countries are sufficiently similar and the discount factor is sufficiently large. In this case, the economic union is sustainable as a perfect equilibrium by the threat of deviations by the producing countries. However, the greater the difference in competitiveness between the producing countries, the larger the discount factor required to sustain the economic union. Thus, it is more likely that an economic union formed by the three countries can be sustained the greater the size of the demand, the lower the cost differences between firms and the more efficient the economy is as a whole.
1 Introduction

In this paper we focus on the incentives of countries to form an economic union as an alternative to a trade war among producing countries giving a production subsidy to their firms. Balassa (1961) distinguishes several stages or levels of economic integration among countries, such as free trade areas, customs unions, common markets and monetary unions\(^1\). In this paper, we consider an economic union as an area in which there exists a common economic authority that choose the same level of output subsidies for all the firms in an oligopolistic industry\(^2\).

We consider a simple model with two producing countries and one consuming country, the so called third-market model introduced by Brander and Spencer (1985) to provide an illustration of strategic trade policy\(^3\). Brander and Spencer were the first to show that, in a Cournot duopoly setting, countries have incentives to give an output subsidy to domestic firms because it raises firms' market share and payoffs at the expense of its foreign competitors. Further analysis of firm asymmetries in the strategies subsidies model can be found in De Meza (1986) and Neary (1994). Collie (1993) uses a linear version of the model in order to study the sustainability of free trade in an infinitely repeated game. For an excellent survey about strategic trade policy and the different aspects and extensions of the third-market model see Brander (1995).

The structure of the model is as follows. In each of the producing countries it is located a firm. Both firms produce an homogeneous good with constant return to scale and compete à la Cournot. Outside the economic union, each country may use subsidies to improve the relative position of the domestic firm in a noncooperative competition with the foreign firm. We consider asymmetry between production costs between the firms. We assume that countries' decision on subsidies is irreversible and prior to decision of

\(^1\)See Hine (1994) for a survey on the international economic integration literature.

\(^2\)The reason because we study uniform output subsidies is that it is difficult, both politically and administratively, to implement differential subsidies. In particular, the European Union has attempted to harmonize all aid being given by the governments of member states to the different industrial sectors. This was done to eliminate or reduce distortion of competition within the EU.

\(^3\)Brander (1995) define the third-market model as one in which firms from a domestic country and firms from a foreign country compete only in a third market.
firms on output. This situation is model as a two-stage game. In stage 1, countries simultaneously choose a subsidy level per unit of output. In stage 2, firms simultaneously choose output levels for the third market. Brander and Spencer (1985) showed that noncooperative behavior yields suboptimal subsidies policies compared to the situation in which the producing countries are taken together.

In our analysis, we depart from the traditional literature by considering the possibility of coalitions among the three countries (i.e. the two producing countries and the consuming country) forming an economic union, in which there is an economic authority choosing a common level of subsidy per unit of output. The objective is to maximize the joint welfare of the three countries net of subsidies. The total amount of subsidies in the union have to be paid by the three countries. Given the joint financial responsibility by the countries in the union, the relevant question is how to share the subsidies received by the firms among the three countries, in such a way that the three countries have incentives to form an economic union. It is showed that if the difference in competitiveness among the firms are low enough, it is profitable for the three countries to engage in an economic union. However, if we consider other possibilities of coalition formation (i.e. one of the producing countries plus the consuming country), we find that (i) the unique stable coalition is the one formed by the more efficient producing country plus the consuming country if the more efficient firm is sufficiently more competitive that the other firm and that (ii) there is no situation that can be regarded as stable if the two firms are sufficiently similar. Finally, it is shown in a dynamic version of the game that the economic union formed by the three countries can be sustainable as a perfect equilibrium if the two producing countries are sufficiently similar and the discount factor is sufficiently large.

The paper is organized as follows. Section 2 presents the model and compares the economic union formed by the three countries with the non-union situation. The stability of the economic unions in the basic single-shot game is studied in Section 3. Section 4 looks at the infinitely repeated version of the game presented in Section 2. Finally, Section 5 concludes.
2 The Model

Following Brander and Spencer (1985), we use a simplified version of the third-market model of international trade under oligopolistic competition. The description of the model is as follows. There are two domestic firms, each of them located in a different country. Firms are quantity setters, produce an homogeneous good and have no objectives other than profits. Without loss of generality, we will assume that country 1’s firm is more efficient than country 2’s firm. Both firms have constant return to scale cost functions denoted by $C_1(x_1) = cx_1$ and $C_2(x_2) = \beta cx_2$, where $c > 0$ and $\beta > 1$. Let $x = x_1 + x_2$ be total output and $p(x) = a - x$ be the inverse demand function, with $a > 0$. Let $s_i$ be the subsidy per unit of output received by firm $i$ and $M_i$ be the total amount of money to be paid by country $i$. Firms’ profit functions are given by

$$\Pi_1(x_1, x, s_1) = (a - c - x)x_1 + s_1x_1$$

$$\Pi_2(x_2, x, s_2) = (a - \beta c - x)x_2 + s_2x_2$$

and given that there is no consumption in the producing countries the social welfare of country $i$, for $i = 1, 2$ depends only on firm’s profits and the value of subsidy payments, and it is defined as:

$$U_i(x_i, x, s_i) = \Pi_i(x_i, x, s_i) - M_i$$

whereas the social welfare of country 3 is defined as:

$$U_3(x) = V_3(x) - M_3$$

where $V_3(x) = \frac{a^2}{2}$ is the consumer surplus. Let $A = a - \beta c$ and $tA = a - c$, where $t = \frac{a - c}{a - \beta c}$ can be interpreted a measure of the difference in competitiveness between the firms and will turn out to be very useful in simplifying notation. Note that the difference in competitiveness is decreasing in the size of the demand ($a$), and increasing in the cost differences among the firms ($\beta$).
and in the efficiency of the economy (c). To ensure that there is always an interior solution in the case in which the countries compete in subsidies, where both firms export positive quantities to the third country, it will be assumed that \( A > 0 \) and \( t < \frac{3}{2} \). Let the total social welfare be obtained as the sum of the welfare of the three countries.

We now study the relationship between a situation in which countries compete in subsidies with a situation in which the three countries decide to form an economic union with a common economic authority that sets a common level of subsidies for both firms in order to maximize the joint social welfare of the three countries\(^4\).

2.1 Non-Cooperative Subsidies

In the situation in which countries compete in subsidies, we assume that the decision on them is irreversible and prior to decision of firms on output. We will refer to this situation as the non-union situation or the trade war situation. We denote the Nash equilibrium variables in this situation with the superscript \( nu \) (non-union). The Nash equilibrium subsidies, output and profits subsidies for each firm are

\[
\begin{align*}
\sigma_1^{nu} &= \frac{A(3t - 2)}{5}, \quad \sigma_2^{nu} = \frac{A(3 - 2t)}{5} \\
x_1^{nu} &= \frac{2A(3t - 2)}{5}, \quad x_2^{nu} = \frac{2A(3 - 2t)}{5} \\
\Pi_1^{nu} &= \frac{4}{25} [A(3t - 2)]^2, \quad \Pi_2^{nu} = \frac{4}{25} [A(3 - 2t)]^2
\end{align*}
\]

(4)

The noncooperative Nash subsidy equilibrium is characterized by positive production subsidies in both producing countries. However, the more cost competitive firm gets the higher level of subsidy and the subsidy differential equals the production cost differential (see Neary, 1994).

\(^4\)Note that the free trade situation can be understood as an economic union with a common level of subsidies equal to zero.
In this situation each of the producing countries pays the total amount of subsidies to his firm and country 3 pays nothing. Therefore, $M_i^{wu} = s_i^{wu} x_i^{wu}$ for $i = 1, 2$ and $M_3^{wu} = 0$. Operating, the social welfare at the Nash equilibrium for each of the three countries can be defined as

$$U_1^{wu} = \frac{2}{25} [A (3t - 2)]^2,$$

$$U_2^{wu} = \frac{2}{25} [A (3 - 2t)]^2$$

(5)

$$U_3^{wu} = \frac{2}{25} [A (t + 1)]^2$$

As it is shown by Collie (1993) the social welfare of the producing countries will usually be lower at the Nash equilibrium in production subsidies than under free trade. Therefore, both countries will usually lose if there is a trade war as Bhagwati (1988) claimed. Only with significant costs differences it is possible that the more efficient country will be better off at the Nash equilibrium in export subsidies than under free trade. Finally, country 3 increases the social welfare given that the subsidies decrease the price and increase total output\(^5\).

### 2.2 Economic Union formed by the Three Countries

In the situation in which the three countries decide to form an economic union we assume the existence of a common economic authority. This common authority decides a common level of subsidies for both firms such that the social welfare of the economic union is maximized. As before, we assume that the decision on the level of subsidy is irreversible and prior to decision of firms on output. We denote the equilibrium variables in this situation with the superscript 123 (the three countries in the economic union). We assume that there is a joint financial responsibility by the countries in the union. The social welfare function in this case is defined as:

\(^5\)As Brander and Spencer (1985) say, if countries 1 and 2 wish to subsidize country 3 to consume the goods they produce, so much the better for country 3.
\[ U^{123}(s) = \Pi_1(x_1, x, s) + \Pi_2(x_2, x, s) + V_3(x, s) - M \quad (6) \]

where \( M = \sum_{j=1}^{3} M_j \). The relevant question is how to share the total amount of money received by the firms among the three countries, in such a way that the three countries have incentives to form an economic union. In this situation, the total amount of money to be paid to the firms is given by \( s^{123}x^{123} \), and \( M_j^{123} = \alpha_j^{123} s^{123} x^{123} \), where \( \alpha_j^{123} \in (0, 1) \) is the proportion of the subsidy paid by country \( j \), and \( \sum_{j=1}^{3} \alpha_j^{123} = 1 \). The Nash equilibrium common subsidy, output, and profits of each firm in the economic union of the three countries are:

\[ s^{123} = \frac{A(t + 1)}{4} \]

\[ x_1^{123} = \frac{A(3t - 1)}{4}, \quad x_2^{123} = \frac{A(3 - t)}{4} \quad (7) \]

\[ \Pi_1^{123} = \frac{1}{16} [A(3t - 1)]^2, \quad \Pi_2^{123} = \frac{1}{16} [A(3 - t)]^2 \]

As we can observe, the optimal subsidy, output and profits of both firms in the economic union of the three countries are larger than the equilibrium values in the non-cooperative export subsidy game. Operating, the Nash equilibrium social welfare of each country is

\[ U_1^{123} = \frac{1}{16} [A(3t - 1)]^2 - \alpha_1^{123} \frac{1}{8} [A(t + 1)]^2 \]

\[ U_2^{123} = \frac{1}{16} [A(3 - t)]^2 - \alpha_2^{123} \frac{1}{8} [A(t + 1)]^2 \quad (8) \]

\[ U_3^{123} = \frac{1}{8} [A(t + 1)]^2 (\alpha_1^{123} + \alpha_2^{123}) \]
In Proposition 1 we show that if the difference in competitiveness between the producing countries is low enough, the total social welfare under the economic union formed by the three countries is greater than the total social welfare under the non-union situation. Therefore, there exist proportions of the monetary cost of the union to be paid by the countries such that the economic union formed by the three countries Pareto dominates the non-union situation.

**Proposition 1** For all \( t \leq \frac{11}{9} \) the economic union formed by the three countries Pareto dominates the non-union situation, each of the producing countries pays a greater amount of money and the firms have greater payoffs.

**Proof: See Appendix.**

The decision of the countries to form an economic union produces two effects. On the one hand, the so-called *technological effect*, implying that the increases in the level of subsidies per unit of output in the economic union compared to the non-union situation \( (s^1 > s^3 > s^2_{nu}) \) makes the less efficient firm to increase its market share. Therefore, the economy as a whole produces less efficiently. On the other hand, the *subsidy effect*, implying that total output increases, prices are driven downwards and therefore the consumer surplus increases. In this sense, when the difference in competitiveness is sufficiently low, \( t \leq \frac{11}{9} \), both firms increase their outputs and their payoffs and the second effect prevail over the first one. However, when the difference in competitiveness is sufficiently high, \( t > \frac{11}{9} \), the more efficient firm contracts his output and payoff and the first effect prevail over the second one. As a result, the total social welfare under the economic union is lower than under the non-union situation, and therefore there are no proportions of the money to be paid to the firms by each country such that all the countries be better off\(^6\).

From Proposition 1 it can be concluded that not only the country 3 pays a positive proportion of the amount of money to be paid to the firms (up to 36 percent), but also the producing countries pay more money than in the non-union situation.

In what follows, we will assume that \( t \leq \frac{11}{9} \) to ensure that the situation in which the three countries form an economic union is Pareto efficient compared

\(^6\)Note that when \( t = 1 \), both firms are identical, the technological effect disappears and the economic union is always welfare improving with respect to the non-union situation.
to the non-union situation. At the light of Proposition 1, one would be
tempted to say that the three countries are willing to form an economic union.
However, by doing so we would not be taking account of all possibilities of
coaition formation among the countries. In the next section, we study the
stability of the economic union formed by the three countries and of any
other coalition with the restriction that in any situation firms’ payoff must
be greater than in the non-union situation.

3 The Stability of the Economic Unions in
the Basic Single-Shot Game

In order to study the stability of the economic union formed for the three
countries in the basic single-shot game, we consider the following four possible
coalitions: the three countries forming the economic union, the economic
unions formed by one producing country and the consuming country, (i.e.
countries 1 and 3 or countries 2 and 3) and the non-union situation. The
idea is that countries can talk each other and decide whether to form an
economic union and how to share the cost of the subsidy to be paid to the
firm(s) inside the economic union.\footnote{As Brander and Spencer (1985) showed, when there is no consumption of the good
by the two producing countries, the jointly optimal policy of the economic union formed
by the producing countries would be to tax production. We do not consider the coalition
formed by the producing countries by two reasons: (a) it is unlikely that the producing
countries be able to make binding agreements of this sort and (b) the payoff of the more
efficient firm is lower under the economic union of the two producing countries than under
the non-union situation.}

3.1 Economic Union formed by the More Competitive
Country and the Consuming Country

We first analyze the situation in which countries 1 (the most competitive
country) and 3 form an economic union with a common authority that decide
the level of subsidy to be paid to firm 1. Country 2 (the less competitive
country) is out the economic union. We denote the equilibrium variables in
this situation with the superscript 13. The social welfare functions for the
economic union and for the country out the economic union are:
$$U^{13}(s) = \Pi^{13}_1(x_1, x, s) + V^{13}_3(x, s) - M^{13}_1 - M^{13}_3$$

$$U^{13}_2(s) = \Pi^{13}_2(x_2, x, s) - M^{13}_2$$

Of course, the best response of country 2 to this situation is to set an unilateral subsidy, with \( M^{13}_2 = s^{13}_2 x^{13}_2 \). Inside the economic union, the total amount of money to be paid to firm 1 is given by \( s^{13}_1 x^{13}_1 \), and \( M^{13}_1 = \alpha^{13}_1 s^{13}_1 x^{13}_1 \) and \( M^{13}_3 = (1 - \alpha^{13}_1) s^{13}_1 x^{13}_1 \), where \( \alpha^{13}_1 \in (0, 1) \) is the proportion of the subsidy paid by country 1, and \( 1 - \alpha^{13}_1 \) by country 3. The Nash equilibrium subsidies, output and profits of each firm are

\[
\begin{align*}
  s^{13}_1 &= tA, \quad s^{13}_2 = 0 \\
  x^{13}_1 &= tA, \quad x^{13}_2 = 0
\end{align*}
\]

(10)

\[
\begin{align*}
  \Pi^{13}_1 &= (tA)^2, \quad \Pi^{13}_2 = 0
\end{align*}
\]

where the equilibrium subsidy in the union (received by firm 1) is independent of the level of subsidy chosen by country 2. However, as we can observe, the output of the firm in country 2 is zero in this situation. From that, the equilibrium social welfare for each country is

\[
\begin{align*}
  U^{13}_1 &= (tA)^2 \left( 1 - \alpha^{13}_1 \right) \\
  U^{13}_2 &= 0 \\
  U^{13}_3 &= (tA)^2 \left( \alpha^{13}_1 - \frac{1}{2} \right)
\end{align*}
\]

9
3.2 Economic Union formed by the Less Competitive Country and the Consuming Country

We second analyze the situation in which countries 2 (the one with the higher cost firm) and 3 form an economic union with a common authority that decide the level of subsidies to be paid to firm 2, and country 1 is out of the economic union. We denote the equilibrium variables in this situation with the superscript 23. The social welfare functions for the economic union and for the country out the economic union are:

\[ U^{23}(s) = \Pi_2^{23}(x_2, x, s) + V_3^{23}(x, s) - M_2^{23} - M_3^{23} \]

\[ U_1^{23}(s) = \Pi_1^{23}(x_1, x, s) - M_1^{23} \quad (12) \]

As above, the best response of country 1 to this situation is to set an unilateral subsidy, with \( M_1^{23} = s_1^{23} x_1^{23} \). Inside the economic union, the total amount of money to be paid to firm 2 is given by \( s_2^{23} x_2^{23} \), and \( M_2^{23} = \alpha_2^{23} s_2^{23} x_2^{23} \) and \( M_3^{23} = (1 - \alpha_2^{23}) s_2^{23} x_2^{23} \), where \( \alpha_2^{23} \in (0, 1) \) is the proportion of the subsidy paid by country 2, and \( 1 - \alpha_2^{23} \) by country 3. Operating, the Nash equilibrium subsidies, outputs and profits of each firm are:

\[ s_1^{23} = \frac{A(t - 1)}{2}, \quad s_2^{23} = A \]

\[ x_1^{23} = A(t - 1), \quad x_2^{23} = \frac{A(3 - t)}{2} \quad (13) \]

\[ \Pi_1^{23} = (c_2 - c_1)^2, \quad \Pi_2^{23} = \left( \frac{2a - 3c_2 + c_1}{2} \right)^2 \]

where the equilibrium subsidy in the union (received by firm 2) is independent of the level of subsidy chosen by country 1 and even although \( t > 1 \), we have that \( s_2^{23} > s_1^{23} \) contrary to the traditional result that the low cost firm gets
a higher level of subsidy. From that, the Nash equilibrium social welfare for each country is:

\[ U_{1}^{23} = \frac{[A(t - 1)]^2}{2} \]

\[ U_{2}^{23} = \frac{[A(3 - t)]^2}{4} - \alpha_{2}^{23} \frac{(A)^2 (3 - t)}{2} \]  
(14)

\[ U_{3}^{23} = \frac{[A(t + 1)]^2}{8} - (1 - \alpha_{2}^{23}) \frac{(A)^2 (3 - t)}{2} \]

3.3 Economic Union of Two Countries versus Non-Cooperative Export Subsidies

The following step is to compare all possible situations. We compare the above two situations (economic union of one producing country and the consuming country) with the non-union situation.

In Lemma 2 and 3 we show that under the economic union formed by one of the producing countries and the consuming country the sum of the social welfare of these countries is greater than under the non-union situation.

**Lemma 2** Under the economic union formed by the more competitive country and the consuming country, the social welfare of these countries and the total social welfare are greater than under the non-union situation\(^8\).

**Proof:** See Appendix.

\(^8\)In the case in which \( t = 1 \), the economic union formed by countries 1 and 3 is Pareto efficient compared to the no-union situation if and only if \( \frac{82}{70} < \alpha_{1}^{13} < \frac{97}{70} \) or equivalently, country 3 is willing to pay a proportion of the total subsidy between 8 and 18 percent. Given symmetry, the situation of the union formed by countries 2 and 3 is analogous.
Lemma 3 Under the economic union formed by the less competitive country and the consuming country, the social welfare of these countries and the total social welfare is greater than under the non-union situation.

Proof: See Appendix.

Note that from the non-union situation, there always exists a profitable deviation of any coalition formed either by one producing country and the consuming country or by the three countries. Moreover, all economic unions share the same features, (a) they are welfare improving for the countries forming the economic union with respect to the non-union situation, (b) the countries in the economic union pay more money and (c) firms inside the economic unions have greater payoffs.

3.4 Two Countries Union versus Three Countries Union

We now compare the economic union formed by the three countries with the economic union of one producing country and the consuming country.

In Lemma 4 we show that the total social welfare under the economic union formed by the producing country with the more efficient firm and the consuming country is greater than the total social welfare under the economic union formed by the three countries. Since the economic union formed by the more competitive country and the consuming country can be regarded as the optimal situation in which the output is the one that maximize social welfare and the less efficient firm does not produce, this result is not very surprising.

Lemma 4 Under the economic union formed by the more competitive country and the consuming country, the welfare of these countries and the total social welfare is greater than under the economic union formed by the three countries.

Proof: See Appendix.

The intuition of the above result is that the technological and the subsidy effects go in the same direction. The increases in the level of subsidies per unit of output in the economic union formed by the more competitive country
and the consuming country, compared to economic union formed by the three
countries \((s_1^{13} > s_1^{23} > s_2^{13} = 0)\) makes the less efficient firm not to produce
and the total output to increase. Therefore, the economy as a whole produces
more efficiently, the consumer surplus increases and the more efficient firm
payoffs are greater.

In Lemma 5 we show that the total social welfare in the economic union
formed by the three countries is greater than the total social welfare in the
economic union formed by the producing country with the less efficient firm
and the consuming country.

**Lemma 5** The economic union formed by the three countries Pareto domi-

nates the economic union formed by the less competitive country and the

***Proof:*** See Appendix.

The intuition of the above result is that in this case the subsidy effect is

cancel out by the technological effect. Total output remain unchanged in the

pass from the economic union formed by the three countries to the economic

union formed by the less competitive country and the consuming country,

but the more efficient firm contracts his output. Therefore, the economy as

a whole produces less efficiently, and there exist proportions of the money
to be paid to the firms such that all the countries are better off under the

economic union formed by the three countries.

Finally, in Theorem 1, we show that the economic union formed by the

more competitive country and the consuming country is stable if the differ-

cence in competitiveness between the firms is large enough.

**Theorem 6** There exists a cut-off level of the difference in competitiveness,

\(t^* \in (1, \frac{11}{8})^a\), such that for all \(t < t^* + \) there is no economic union that can

be regarded as stable and for all \(t \geq t^* + \) the unique stable coalition is the one

formed by the more competitive country plus the consuming country.

***Proof:*** See Appendix.

\[-t^* = \frac{171}{108} - \frac{20}{108} \sqrt{2}\]
The economic intuition behind Theorem 6 is the following. The economic union formed by the more competitive country and the consuming country maximizes the total social welfare. However, under this situation the social welfare of the less competitive country is zero. Therefore, this country is willing to pay almost all the monetary cost of the economic union with the consuming country. The greater are the difference in competitiveness between both firms, more inefficiently produces the economic union formed by the less competitive country and the consuming country and more difficult is that the less competitive country may guarantee to the consuming country the social welfare that this country gets under the economic union with the more competitive country.

4 The Sustainability of the Economic Union formed by the Three Countries

In this section, we study whether the economic union formed by the three countries can be supported as a perfect equilibrium of an infinitely repeated version of the game in Section 3, using trigger strategies as in Friedman (1971). Collie (1993) considers a dynamic version of Brander and Spencer (1985) export subsidies model and finds that free trade can be supported if the countries are sufficiently similar and the discount factor is sufficiently low.

Let $\delta$ be the discount factor which is between zero and one and will be assumed to be the same for the three countries. The strategy of each country is to play the economic union formed by the three countries until the other countries deviate by forming other coalition. The strategy for any of the producing countries is thereafter (i) to offer an economic union to the consuming country if in the last period there was an economic union between any of the producing countries and the consuming country and (ii) to go to the non-union situation if the consuming country offer it an economic union or if in the last period there was a non-union situation. The strategy for the consuming country is (i) to accept the most profitable economic union in each period if its social welfare is greater than the welfare under the non-union situation and to go the non-union situation otherwise and (ii) to remain in the non-union situation if in the last period there was a non-union situation. We will assume that the non-union situation is the default situation when there
is no agreement between the countries (i.e. starting from the economic union formed by the three countries, if one of the producing countries proposes an economic union to the consuming country and it rejects the offer, then the countries would be under the non-union situation).

From Proposition 1 we know that none of the countries has incentive to break the economic union formed by the three countries and go to the non-union situation. We have also the following previous result regarding the social welfare of the more competitive country.

**Lemma 7** The more competitive country always have incentive to avoid the deviation from the economic union formed by itself and the consuming country by the coalition formed by the less competitive country and the consuming country.

**Proof:** See Appendix.

**The Consuming Country** In the infinitely repeated game, assuming the other two countries use the trigger strategies, if the consuming country deviates from the economic union formed by the three countries by offering an economic union to one of the producing countries its social welfare when it deviates will be $U_3^{nu}$ thereafter. From Proposition 1, we know that country 3 will not have incentive to deviate. Obviously, to accept the most profitable economic union is an optimal response for the consuming country to the producing countries whenever its social welfare is greater than under the non-union situation.

**The Less Competitive Country** In the infinitely repeated game, assuming the other two countries use the trigger strategies, from Proposition 1 and Lemma 5, it is clear that the less competitive country has no incentive to deviate. If in the last period there was an economic union between the other two countries, since country 2 gets zero social welfare anyway, an optimal response is to offer an economic union to the consuming country yielding it zero social welfare. Finally, we have the following result.
Lemma 8  For the less competitive country, to reject the offer of the consuming country to form an economic union among them is an optimal response if its discount factor is sufficiently large.

Proof: See Appendix.

The More Competitive Country  In the infinitely repeated game, assuming the other two countries use the trigger strategies, if the less competitive country offer an economic union to the consuming country and this country accepts, it is clear from Lemma 7 that the best response by the more competitive country is to offer an economic union to the consuming country in the next period. Finally, we have the following two results.

Lemma 9  For the more competitive country, to reject the offer of the consuming country to form an economic union among them is an optimal response if its discount factor is sufficiently large and the producing countries are sufficiently similar.

Proof: See Appendix.

Theorem 10  The economic union formed by the three countries is sustainable as a perfect Nash equilibrium using trigger strategies if the two producing countries are sufficiently similar, $t < t^*$, and the discount factor is sufficiently large.

Proof. The economic union formed by the three countries is sustainable as a perfect Nash equilibrium using trigger strategies if $0 \leq \delta \leq 1$ and $\delta \geq \delta^*_1$. First, we know that the consuming country has no incentive to deviate. Second, from Lemma 8 we have that if $\delta \geq \delta^*_2$, by deviating the less competitive country will have a social welfare lower than under the non-union situation that it is also lower than the social welfare of this country under the economic union formed by the three countries by Proposition 1, $(U^{13}_2 < U^{3u}_2 < U^{12}_2)$. Third, from Lemma 9 we have that if the producing countries are sufficiently similar and $\delta \geq \delta^*_1$, by deviating the more competitive country will have a social welfare lower than under the non-union situation that it is also lower than the social welfare of this country under the economic union.
union formed by the three countries by Proposition 1, \((U_{i}^{12} < U_{i}^{ua} < U_{i}^{23})\). Therefore, if the producing countries are sufficiently similar and by taking \(\delta \geq \delta_{i}^{*}\) (where \(\delta_{i}^{*} \geq \delta_{j}^{*}\)), no country will deviate and the economic union formed by the three countries is sustainable. ■

The economic union formed by the three countries is sustained as a perfect equilibrium by the threat of deviations by the producing countries. However, the interesting point to note is that the larger is the difference in competitiveness between the producing countries, the larger is the discount factor required to sustain the economic union formed by the three countries. Thus, it is more likely that an economic union formed by the three countries can be sustained the greater is the size of the demand, the lower are the cost differences among the firms and the more efficient is the economy taken as a whole.

5 Final Remarks

In this paper, we have presented a simple model of economic unions as an alternative to an export subsidy trade war in the context of the third-market model. We consider the possibility of an economic union of two producing countries plus a third importing country, with a common economic authority that choose the optimal subsidy in the union. We show that even although an economic union formed by all the countries may Pareto dominate the non-union situation, if we take account of all possibilities of coalition formation in hand of the countries, the coalition formed by all the countries is no stable in the single-shot game. Specifically, there is a profitable deviation by the more competitive country and the consuming country from the economic union formed by all the countries, that is stable if the more efficient firm is sufficiently more competitive than the less competitive firm.

Finally, it is shown in a repeated version of the game that the economic union formed by the three countries is sustainable as a perfect equilibrium if the two countries are sufficiently similar and the discount factor is sufficiently large. The economic union formed by the three countries is sustained by the threat of sequence of continuous deviations if any country deviate by leaving the economic union with the three countries and forming an economic union with the consuming country. The economic union formed by the three countries is more difficult to sustain as the difference in competitiveness between
the two producing countries increases and therefore it is more likely that an economic union be stable the greater are the size of the demand, and the lower are the cost differences among the firms and the inefficiency of the economy.

The assumption of linear demand has been made to get explicit solutions, and without this assumption it would not be possible to compare the social welfare in the different policy regimes. The assumption of constant return to scale is a standard assumption in the strategic trade policy literature. However, we left as an open question how the results obtained in this paper might change if we assume technologies with non-decreasing return to scale.
References


Appendix

Proof of Proposition 1: We make the proof in three steps.

Step 1. The welfare of all the countries is greater under the economic union than in the non-union situation. In the case of country 3, the union will be preferred if

\[
\frac{[A(t + 1)]^2}{8} (\alpha_1^{123} + \alpha_2^{123}) = U_3^{123} > U_3^{nu} = \frac{2}{25} [A(t + 1)]^2
\]  

(15)

Rearranging terms we get that

\[
\alpha_1^{123} + \alpha_2^{123} > \frac{64}{100}
\]

(16)

or equivalently \( \alpha_3^{123} < \frac{26}{100} \).

For country 1, the union will be preferred if

\[
\frac{[A(3t - 1)]^2}{16} - \alpha_1^{123} \frac{[A(t + 1)]^2}{8} = U_1^{123} > U_1^{nu} = \frac{2}{25} [A(3t - 2)]^2
\]

(17)

Rearranging terms we get that

\[
\alpha_1^{123} < \frac{234t - 103 - 63t^2}{50(t + 1)^2}
\]

(18)

Finally, for country 2, the union will be preferred if

\[
\frac{[A(3 - t)]^2}{16} - \alpha_2^{123} \frac{[A(t + 1)]^2}{8} = U_2^{123} > U_2^{nu} = \frac{2}{25} [A(3 - 2t)]^2
\]

(19)

Rearranging terms we get that

\[
\alpha_2^{123} < \frac{234t - 63 - 103t^2}{50(t + 1)^2}
\]

(20)

Expressions (16), (18) and (20) are compatible if and only if
that holds all \( t \in \left(1, \frac{11}{9}\right)\).

Step 2. The amount of money paid by each of the producing countries is greater under the economic union than in the non-union situation. In the case of country 1, it must hold that \( 1 > \frac{M_{1u}}{M_1} \), or equivalently

\[
1 > \frac{16}{25} \frac{1}{\alpha_1^{13}} \left( \frac{3t - 2}{t + 1} \right)^2
\]

(22)

Since \( \alpha_1^{13} < \frac{234 - 103 - 63t^2}{50(t+1)^2} \), we have that expression (22) is equivalent to

\[
351t^2 - 618t + 231 < 0
\]

(23)

In the case of country 2, it must hold that \( 1 > \frac{M_{2u}}{M_2} \), or equivalently

\[
1 > \frac{16}{25} \frac{1}{\alpha_2^{13}} \left( \frac{3 - 2t}{t + 1} \right)^2
\]

(24)

Since \( \alpha_2^{13} < \frac{234 - 103 - 63t^2}{50(t+1)^2} \), we have that expression (24) is equivalent to

\[
231t^2 - 618t + 351 < 0
\]

(25)

Finally expressions (23) and (25) holds for all \( t \leq \frac{11}{9} \).

Step 3. The firms have greater payoffs under the economic union than in the non-union situation. It follows by direct comparison of firms’ payoffs under the economic union and the non-union situation.

**Proof of Lemma 2:** In the case of country 1, the union with 3 will be preferred to the non-union situation if

\[
(tA)^2 \left(1 - \alpha_1^{13}\right) = U_1^{13} > U_1^{uu} = \frac{2}{25} [A (3t - 2)]^2
\]

(26)
or equivalently

\[ \alpha_{13}^{13} < \frac{7t^2 + 24t - 8}{25t^2} \]  

(27)

and for country 3, the union with 1 will be preferred if

\[(tA)^2 \left( \alpha_{13}^{13} - \frac{1}{2} \right) = U_{3}^{13} > U_{3}^{uu} = \frac{2}{25} [A (t + 1)]^2 \]  

(28)

or equivalently

\[ \alpha_{13}^{13} > \frac{29t^2 + 8t + 4}{50t^2} \]  

(29)

Note that for (27) and (29) to be compatible we have

\[ 3t^2 - 8t + 4 < 0 \]  

(30)

that holds for all \( t \leq \frac{11}{7} \). Finally the total social welfare under the economic union formed by countries 1 and 3 is greater than in the non-union situation if \( 3t^2 - 88t + 56 < 0 \) that holds for all \( t \leq \frac{11}{7} \).

**Proof of Lemma 3:** In the case of country 2, the union with 3 will be preferred if:

\[ \frac{[A (3 - t)]^2}{4} - \alpha_{23}^{23} A \left( \frac{A (3 - t)}{2} \right) = U_{2}^{23} > U_{2}^{uu} = \frac{2}{25} [A (3 - 2t)]^2 \]  

(31)

or equivalently

\[ \alpha_{23}^{23} < \frac{153 - 7t^2 - 54t}{50 (3 - t)} \]  

(32)

and for country 3, the union with 2 will be preferred if:
\[ \frac{[A(t+1)]^2}{8} - (1 - \alpha_2^{23}) A \frac{A(3-t)}{2} = U_3^{23} > U_3^{1u} = \frac{2}{25} [A(t+1)]^2 \] (33)

or equivalently

\[ \alpha_2^{23} > 1 - \frac{9 (t+1)^2}{100 (3-t)} \] (34)

Note that for expressions (32) and (34) be compatible we have

\[ t^2 - 2t - 3 < 0 \] (35)

that holds for all \( t \leq \frac{11}{9} \). Finally the total social welfare under the economic union formed by countries 2 and 3 is greater than in the non-union situation if \( 124t^2 - 152t - 276 < 0 \) that holds for all \( t \leq \frac{11}{9} \). □

**Proof of Lemma 4:** Since the welfare of the less competitive country is zero under the economic union of the most competitive country and the consuming country, we have only to show that the total social welfare under the economic union formed by the three countries is lower than the total social welfare of the three countries under the economic union formed by countries 1 and 3

\[ \frac{(A)^2 (5t^2 - 6t + 5)}{8} = U_1^{123} + U_2^{123} + U_3^{123} < U_1^{13} + U_2^{13} + U_3^{13} = \frac{(tA)^2}{2} \] (36)

or equivalently

\[ t^2 - 6t + 5 < 0 \] (37)

that holds for all \( t \leq \frac{11}{9} \). Finally, for the consuming country to prefer the economic union with the most competitive country to the economic union formed by the three countries, it must hold that \( U_3^{13} > U_3^{123} \), or equivalently

\[ \alpha_1^{13} > \frac{(t+1)^2 (\alpha_1^{23} + \alpha_2^{23})}{8t^2} + \frac{1}{2} \] (38)
Proof of Lemma 5: First we show that the total social welfare under the economic union formed by the three countries is greater than the sum of the welfare of the three countries under the economic union formed by countries 2 and 3

\[
\frac{(A^2(\nu^2-6t+5))}{8} = U_1^{123} + U_2^{123} + U_3^{123} > U_1^{23} + U_2^{23} + U_3^{23} = \frac{(A^2(7t^2-14t+11))}{8}
\]

or equivalently

\[t^2 - 4t + 3 < 0\]

that holds for all \(t \leq \frac{11}{9}\). Second we show that

\[\frac{(A^2[3-t]^2 + 2(t+1)^2 t \alpha_1^{23})}{16} = U_2^{123} + U_3^{123} > U_2^{23} + U_3^{23} = \frac{(A^2[2;3-t]^2 + (t+1)^2 - 4(3-t))}{8}\]

or equivalently

\[\alpha_1^{123} > \frac{3t^2 + 6t - 13}{2(t+1)^2}\]

that holds for all \(t \leq \frac{11}{9}\). Finally, for the consuming country to prefer the economic union with the less competitive country to the economic union formed by the three countries, it must hold that \(U_3^{23} > U_3^{123}\), or equivalently

\[\alpha_1^{23} > 1 - \frac{(t + 1)^2 (1 - \alpha_1^{123} - \alpha_2^{23})}{4(3 - t)}\]

\[\blacksquare\]
Proof of Theorem 6: Note that the only situation that can be stable is the economic union formed by countries 1 and 3. This situation will be stable if there is no coalition and no profitable deviation by this coalition. Therefore, we have to check the coalition formed by the countries 2 and 3, the coalition formed by the three countries yielding to the non-union situation and the coalition formed by the three countries yielding the economic union formed by the three countries.

From Lemma 4, we know that the economic union formed by the three countries is not Pareto efficient compared to the economic union formed by countries 1 and 3.

From Lemma 2, we know that countries 1 and 3 prefer the economic union formed by both of them to the non-union situation if

\[
\frac{29t^2 + 8t + 4}{50t^2} \leq \alpha_{13}^1 \leq \frac{7t^2 + 24t - 8}{25t^2}
\] (44)

Finally, note that the coalition formed by countries 2 and 3 would not deviate if

\[
(tA)^2 \left(\alpha_{13}^1 - \frac{1}{2}\right) = U_2^{13} + U_3^{13} \geq U_2^{23} + U_3^{23} = \frac{(A)^2 (7 - 6t + 3t^2)}{8}
\] (45)

or equivalently if

\[
\alpha_{13}^1 \geq \frac{1}{2} + \frac{7 - 6t + 3t^2}{8t^2}
\] (46)

Note that for expressions (44) and (46) be compatible it must hold that

\[
342t - 119t^2 - 239 \geq 0
\] (47)

Note that (47) holds for all \( t \in \left[\frac{171}{119} - \frac{20}{119} \sqrt{2}, \frac{11}{17} \right] \) but it does not hold for all \( t < \frac{171}{119} - \frac{20}{119} \sqrt{2} \).

Proof of Lemma 7. On the one hand, the more competitive country avoid the deviation by the coalition formed by the less competitive country
and the consuming country if \( \alpha_{13}^{13} \geq \frac{1}{r} + \frac{7-\alpha_{2}^{2}+3\alpha^2}{8\sigma^2} \). On the other hand, the social welfare of the more competitive country is greater under the economic union formed by itself and the consuming country than under the economic union formed by the less competitive country and the consuming country if

\[
\alpha_{13}^{13} < \frac{t^2 + 2t - 1}{2t^2}
\]

(48)

Note that for expressions (46) and (48) to be compatible we have that

\[
3t^2 - 14t + 11 < 0
\]

(49)

that holds for all \( t \leq \frac{11}{9} \).

Proof of Lemma 8. If the less competitive country rejects the offer of the consuming country, thereafter the three countries will be under the non-union situation. If the less competitive country accepts the offer of the consuming country, in the first period it will get a social welfare of \( U_{2}^{23} = \frac{A^2}{2} \left( \frac{3-t}{2} - \alpha_{23}^{23} \right) \), but in the following periods it will get a social welfare of zero. Therefore, for the less competitive country to reject the offer of the consuming country will be an optimal response if

\[
\frac{1}{1 - \delta_2} \frac{2}{25} A^2 (3 - 2t)^2 > \frac{A^2}{2} \left( \frac{3-t}{2} - \alpha_{23}^{23} \right)
\]

(50)

or equivalently if

\[
\alpha_{23}^{23} > \frac{(3-t)}{2} - \frac{4 (3-2t)^2}{25 (3-t) (1-\delta_2)}
\]

(51)

However, the right hand side of expression (51) is lower or equal to zero whenever

\[
\delta_2^{*} \geq 1 - \frac{8 (3-2t)^2}{25 (3-t)^2}
\]

(52)
Therefore, we have that for all $t < \frac{3}{7}$, there exists values of the discount factor such that to reject the offer of the consuming country is an optimal response for the less competitive country. ■

**Proof of Lemma 9.** If the more competitive country rejects the offer of the consuming country, hereafter the three countries will be under the non-union situation. If the more competitive country accepts the offer of the consuming country, in the first period it will get a social welfare of $U_1^{13} = t^2 A^2 (1 - \alpha_1^{13})$, but in the following periods it will get a social welfare of $U_1^{13} = \frac{A^2 (t^2 + 6t - 7)}{8}$ given the strategy of the less competitive country. Therefore, for the more competitive country to reject the offer of the consuming country will be an optimal response if

\[
\frac{1}{1 - \delta_1} \frac{2}{25} A^2 (3t - 2)^2 > t^2 A^2 (1 - \alpha_1^{13}) + \frac{A^2 (t^2 + 6t - 7)}{8} \frac{\delta_1}{1 - \delta_1},
\]

or equivalently

\[
\alpha_1^{13} > 1 - \frac{16 (3t - 2)^2 - 25 \delta_1 (t^2 + 6t - 7)}{200t^2 (1 - \delta_1)}
\]

(54)

Where the right hand side of expression (54) is lower or equal to zero whenever

\[
\delta_1^* \geq \frac{200t^2 - 16 (3t - 2)^2}{175t^2 - 150t + 175}
\]

(55)

Therefore, we have that for all $t < \frac{171}{119} - \frac{20}{119} \sqrt{2}$, there exists a discount factor such that to reject the offer of the consuming country is an optimal response for the more competitive country. ■