Optimal Transshipments and Orders:
A Tale of Two Competing and Cooperating Retailers

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joint work with
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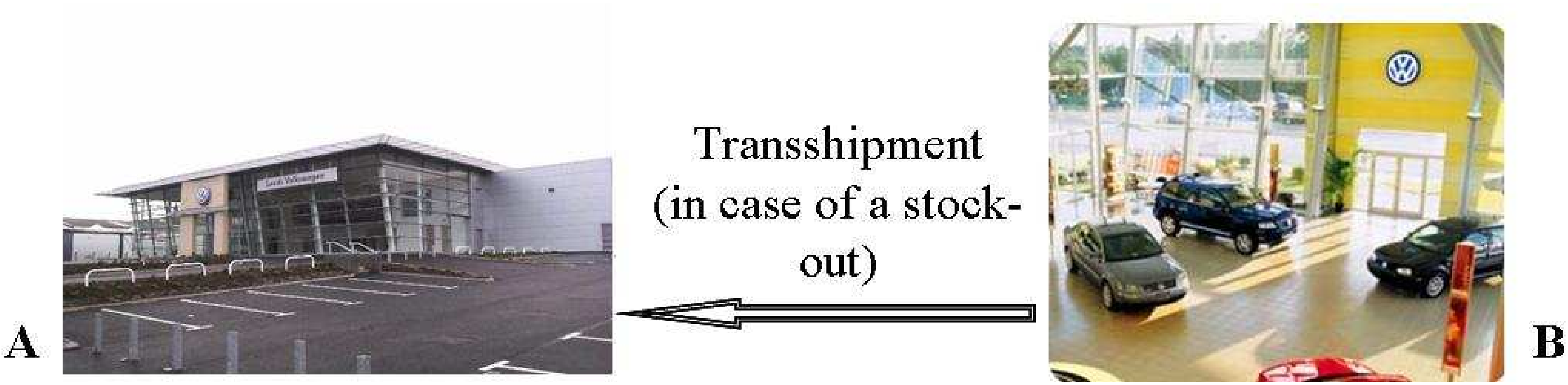

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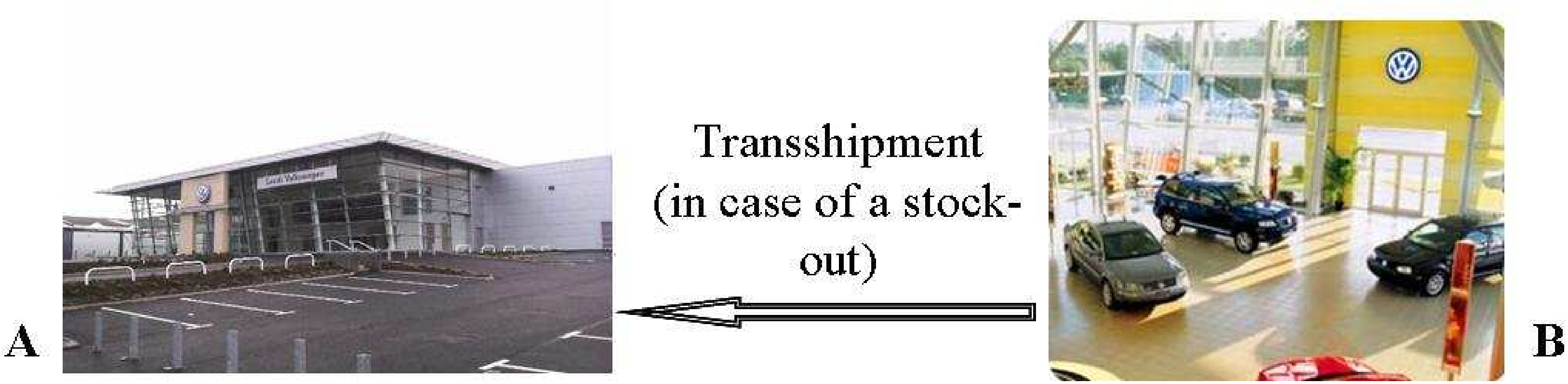

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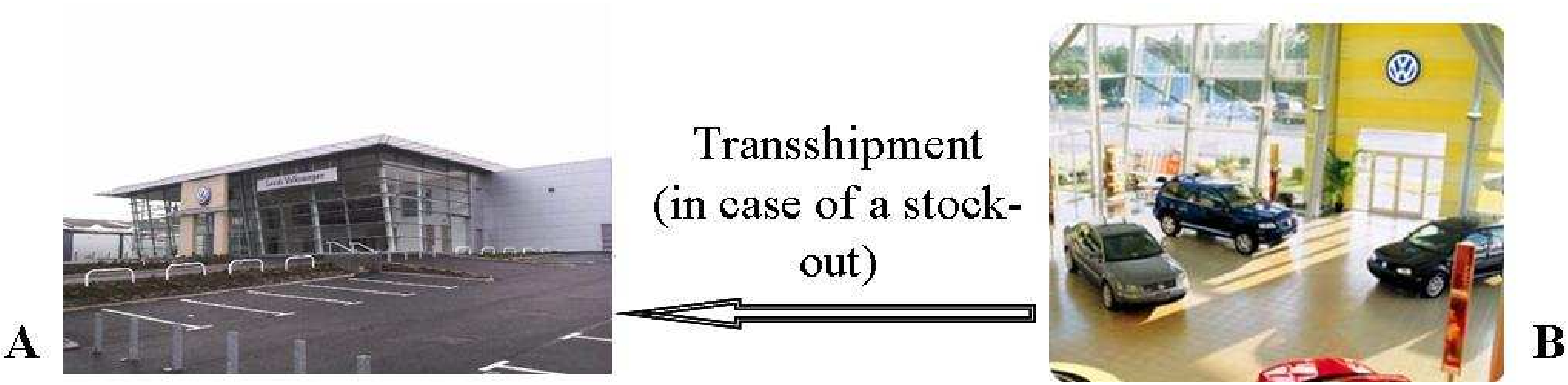
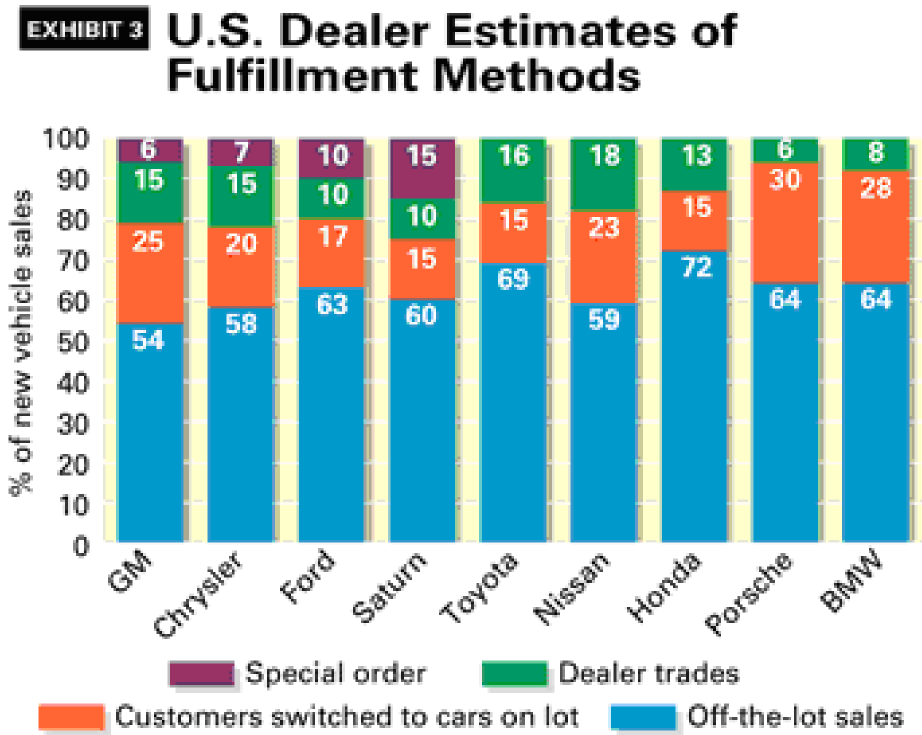


- Decentralized systems such as, Honda, and General Motors have intranet systems for their independent retailers.
- Observed in a variety of industries such as apparel, toys, furniture, IT products, aircraft and auto spare parts, etc.

Cooperation



*Supply Chain Management Review, September 1, 1997

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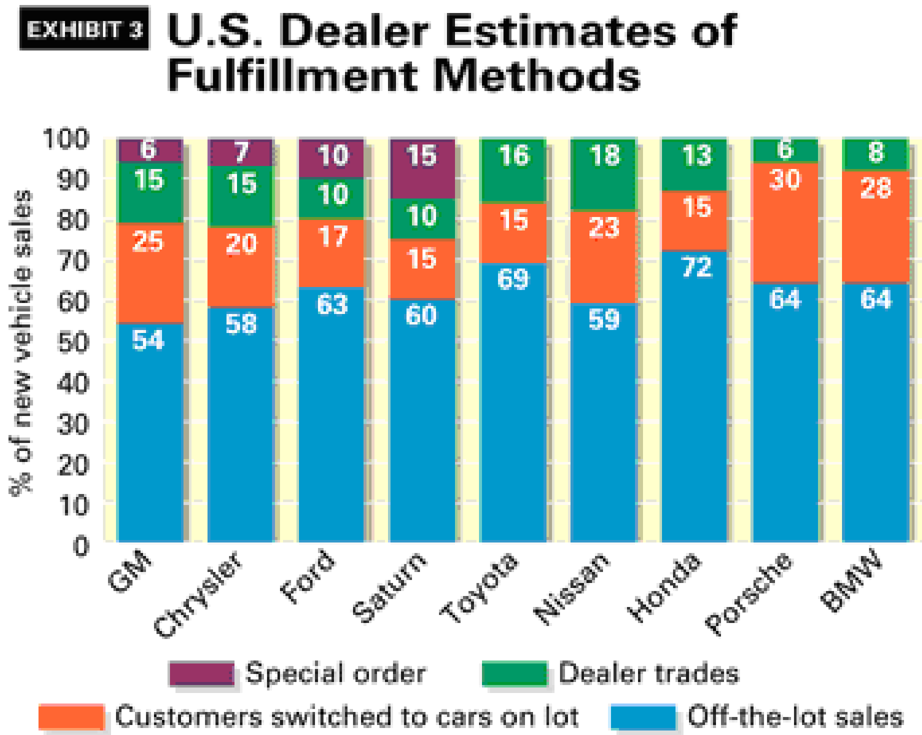

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- "Life without dealer trades would be a whole lot of special orders", WardsAuto.com, Dec 1, 2006.

Cooperation is increasing, because...

Item	Early 70s	Late 90s
Vehicle Models	140	260
Amusement Parks	362	1174
Prescription Drugs	6,131	7,563
OTC Pain Relievers	17	141
McDonald's Menu Items	13	43
Frito-Lay Chip Varieties	10	78
Levi's Jean Styles	41	70
Running Shoe Styles	5	285
Bicycle Types	8	31
Soft Drinks	26	252
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- With developed information technology, easy information exchange
- Cheaper 3PL services

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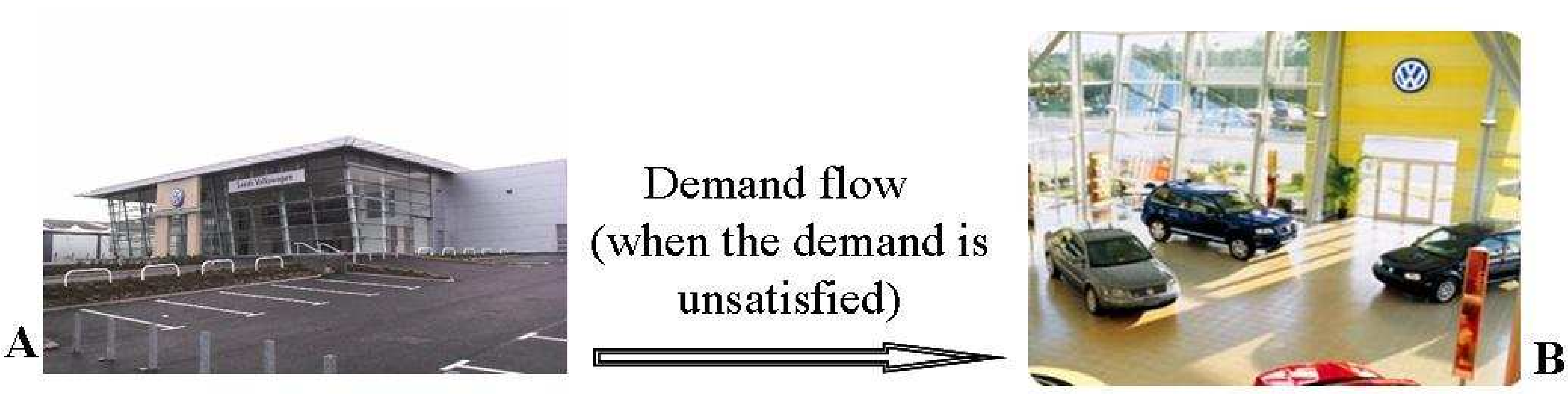

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- Demand flow is effected by brand loyalty and communication between retailers.
- Therefore, a retailer with inventory may (may not) send a transshipment to satisfy a retailer (flowed customer) demand.

Scenario

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- During the cycle, in case of a stock-out, the stocked-out retailer can make a transshipment request to the other retailer.
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  - or, reject the request expecting the current demand to flow to own store.

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- Each retailer has to answer following questions to maximize his/her payoffs:
  - How much should a retailer order?
  - How to accept/reject opponent's transshipment requests?

Agenda

- Literature and contribution
- Development of expected profit functions
- Optimal transshipment policies
- Analysis of the ordering game
- Sensitivity and performance analysis
- Summary and conclusion

Literature and Contribution

Paper	Objective		Pooling Policy			Demand Flow
	Central.	Decentral.	Only Complete	Partial		
				Stat.	Non-Stat.	
Krishnan and Rao (1965)	√		√			
Comez *et al.* (2006)	√				√	
Anupindi *et al.*(1999)		√	√			√
Rudi *et al.* (2001)		√	√			
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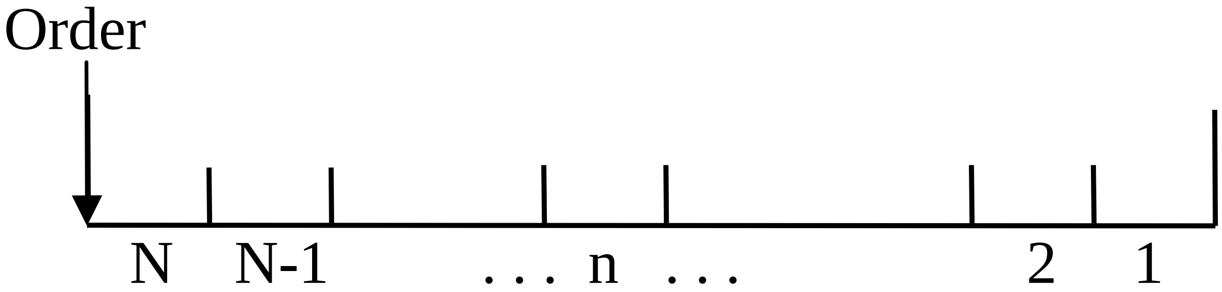
We fill the gap in the literature for

- Optimal and dynamic transshipment policies in a finite decentralized system
- Demand flow in a partial pooling system

Problem Setting



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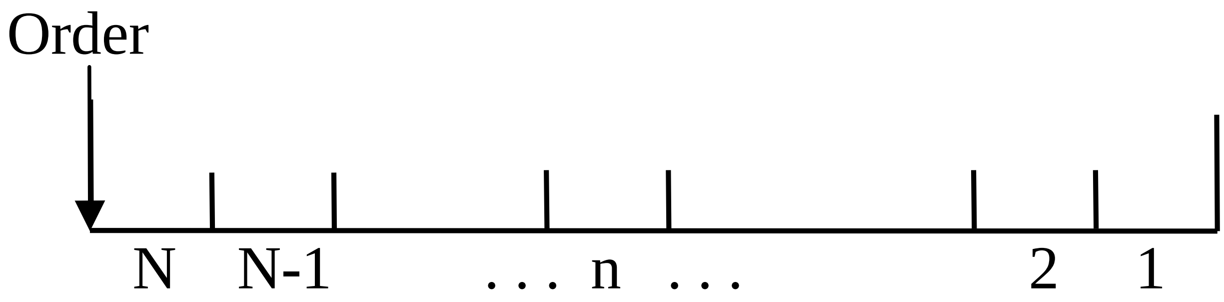

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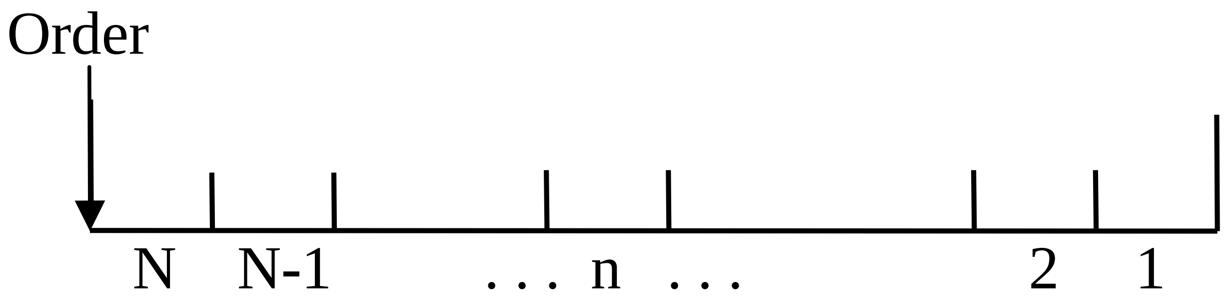

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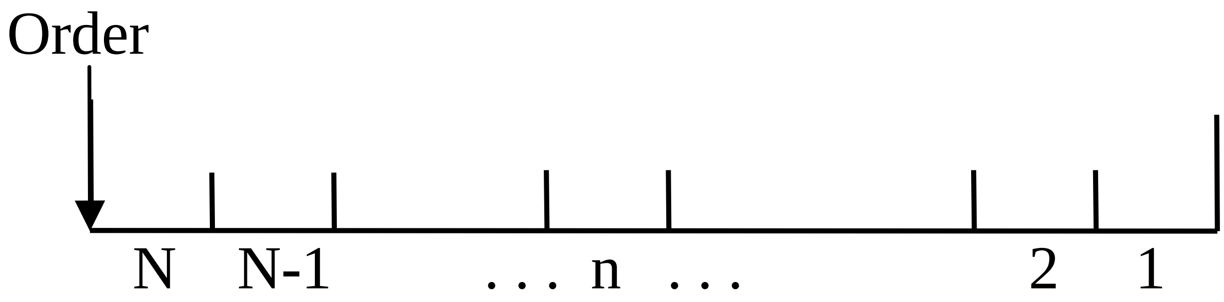

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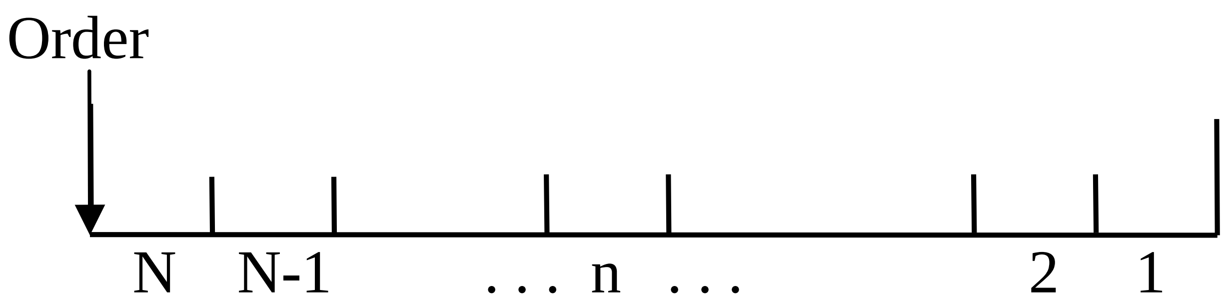

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- Each unit is sold to the customer for a revenue of r.

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At the end of the cycle, each remaining unit is salvaged at s_1, s_2,

$c \geq s_1, s_2$.

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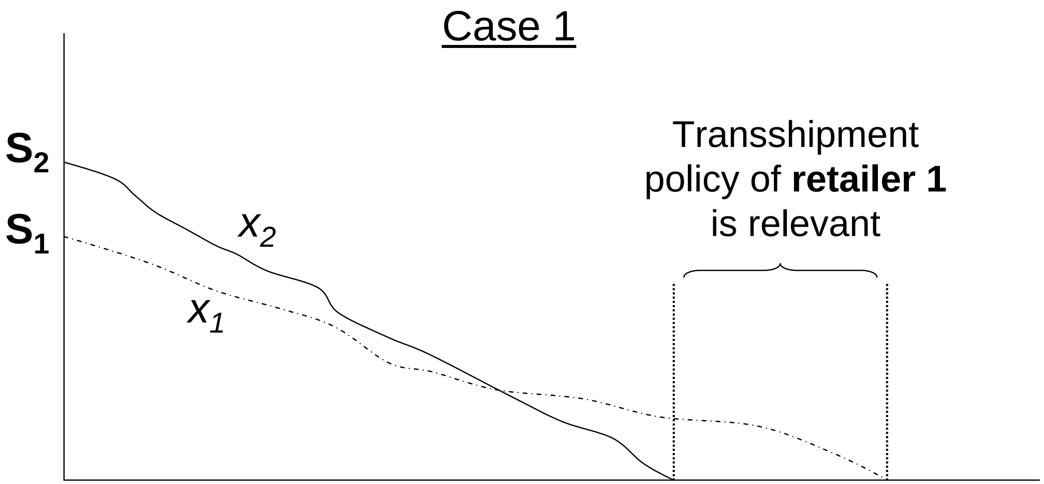
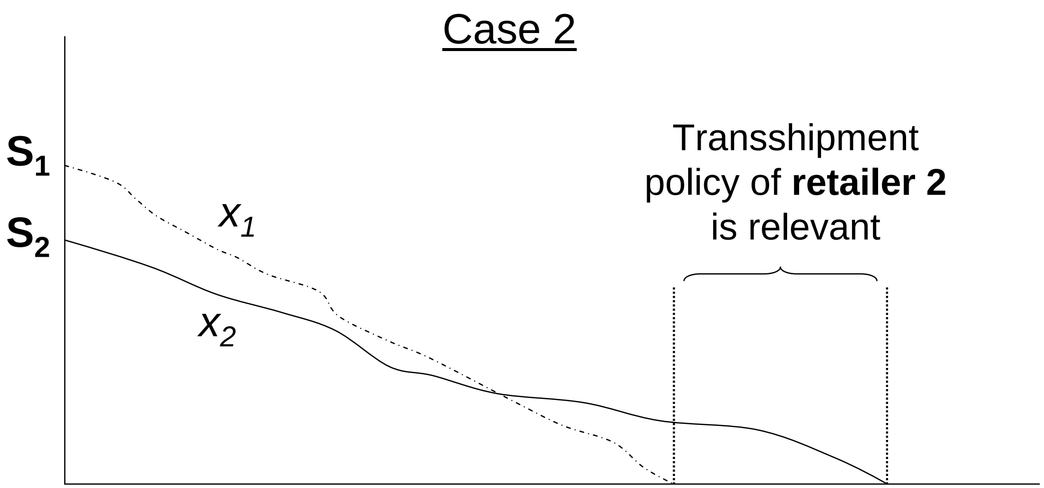
Remark: $\pi_n^i(x_1, x_2)$ is obtained by making optimal transshipment decisions.

Only One Case May Happen In a Cycle





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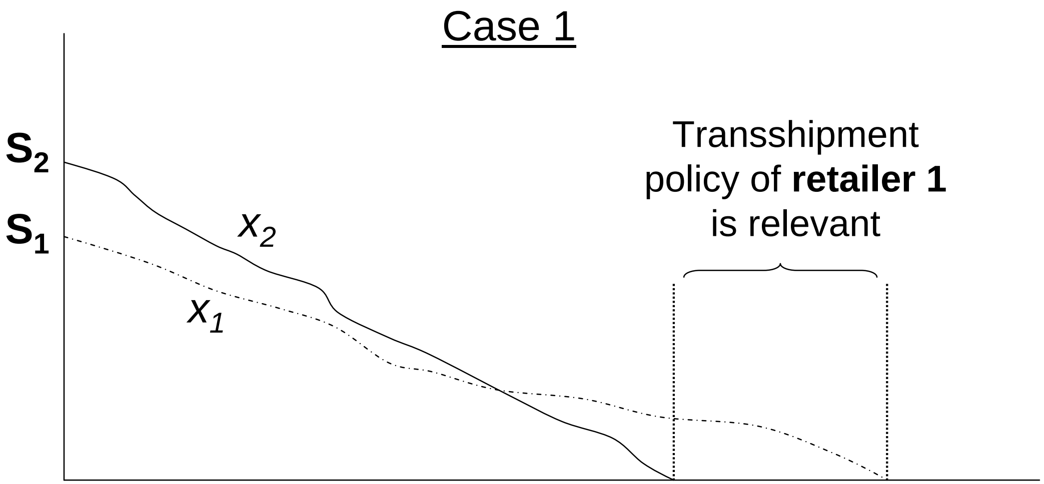

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$$+ (1 - p_1 - p_2)\pi_{n-1}^1(x_1, x_2), \qquad x_1, x_2 \in \mathcal{N}.$$

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- When retailer 2 stocks-out before retailer 1:

$$\pi_n^1(x_1,0) = p_1[r+\pi_{n-1}^1(x_1-1,0)] + (1-p_1-p_2)\pi_{n-1}^1(x_1,0)$$
$$+p_2\max\{t+\pi_{n-1}^1(x_1-1,0), \theta(r+\pi_{n-1}^1(x_1-1,0)) + (1-\theta)\pi_{n-1}^1(x_1,0)\}$$

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This completes the construction of $\pi_n^1(x_1, x_2)$ under Case 1, i.e., retailer 2 stocks-out before retailer 1.

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$$\underbrace{\theta(r + \pi_{n-1}^1(x_1 - 1, 0))}_{\textit{Demand flows}} + \underbrace{(1-\theta)\pi_{n-1}^1(x_1, 0)}_{\textit{Demand is lost}} \leq \underbrace{t + \pi_{n-1}^1(x_1 - 1, 0)}_{\textit{Exp. profit from accept}}$$

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(iii) The marginal benefit of keeping extra inventory is increasing in n*:*

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- Hold-back levels are increasing (decreasing) in n (time): $\tilde{x}_1^i \leq \tilde{x}_2^i \leq \ldots \leq \tilde{x}_n^i \ldots$

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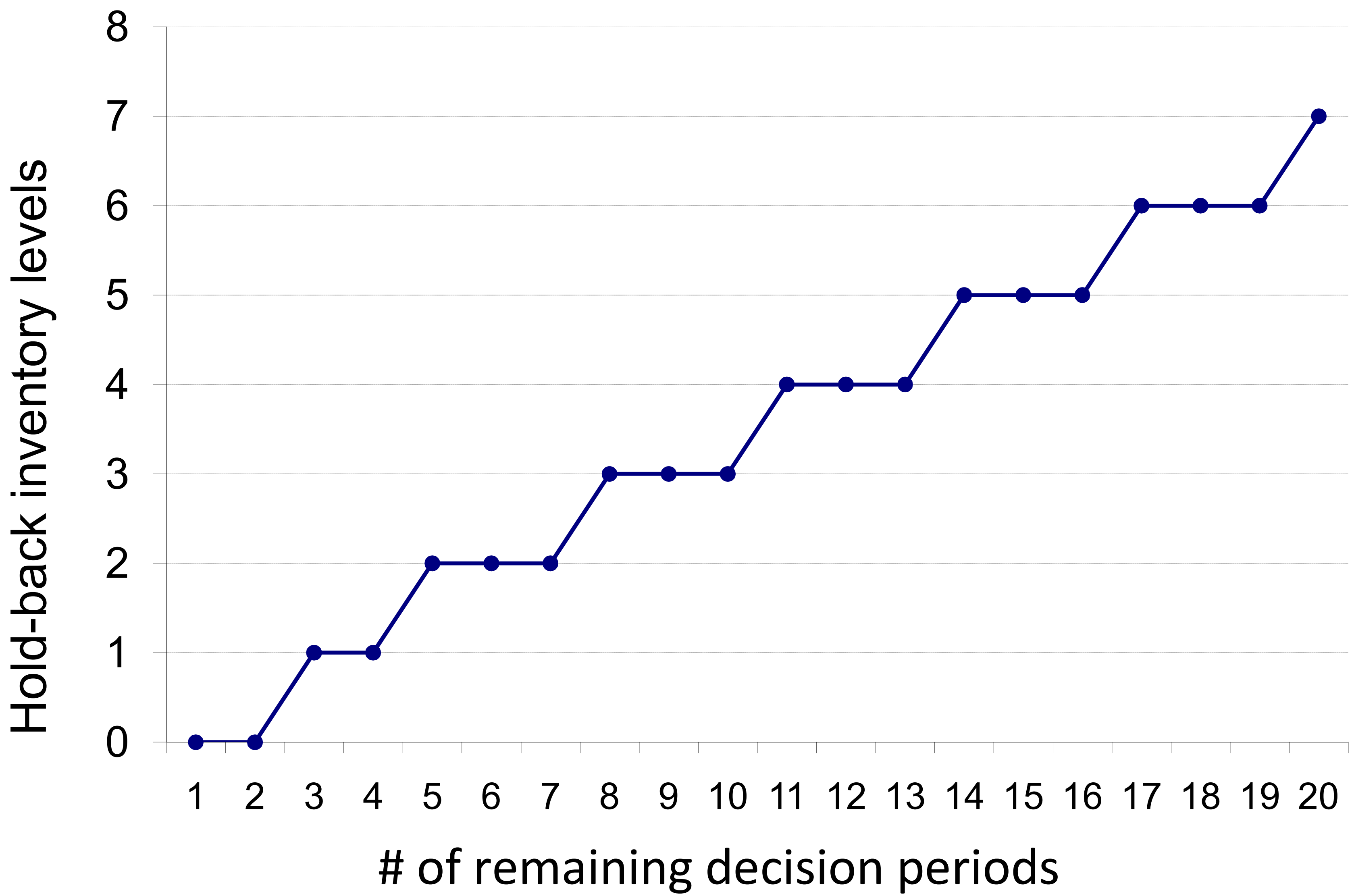
  - hold-back level decreases by at most 1 in time, $\tilde{x}_{n+1}^i - \tilde{x}_n^i \leq 1$.

An Example Transshipment Policy of Retailer 1



$$p_1 = p_2 = 0.2, r = 10, t = 6, \theta = 0.3, s_1 = 1$$

Analysis of Ordering Game

- The optimal ordering level of a retailer is a best response function:

$$S_1^*(S_2) = \arg\max_{S_1} J^1(S_1, S_2)$$

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- Ordering game has a mixed strategy equilibrium.

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- Pure strategy Nash equilibrium exists in two-player submodular games.

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- there exists a pure-strategy Nash equilibrium in inventory levels (S_1^*, S_2^*).

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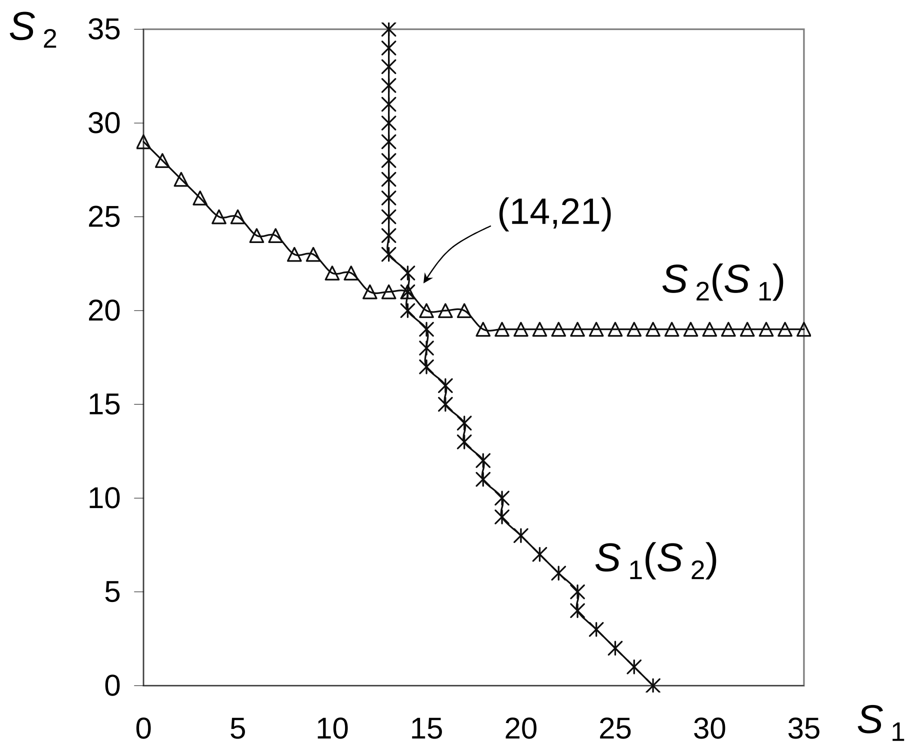
- Numerical study shows submodularity is valid over most of the strategy space.
- Nash equilibrium exists for all numerical studies.

Ordering game where $t < r - \tau$

Best response functions for a sample problem

$N = 60$ and $p_1 = 0.2, p_2 = 0.3, r = \$13, t = \$6, c = \$4, \tau = \$1, \theta = 0.2, s_1 = s_2 = \$2$



$$(S_1^*, S_2^*) = (14, 21)$$

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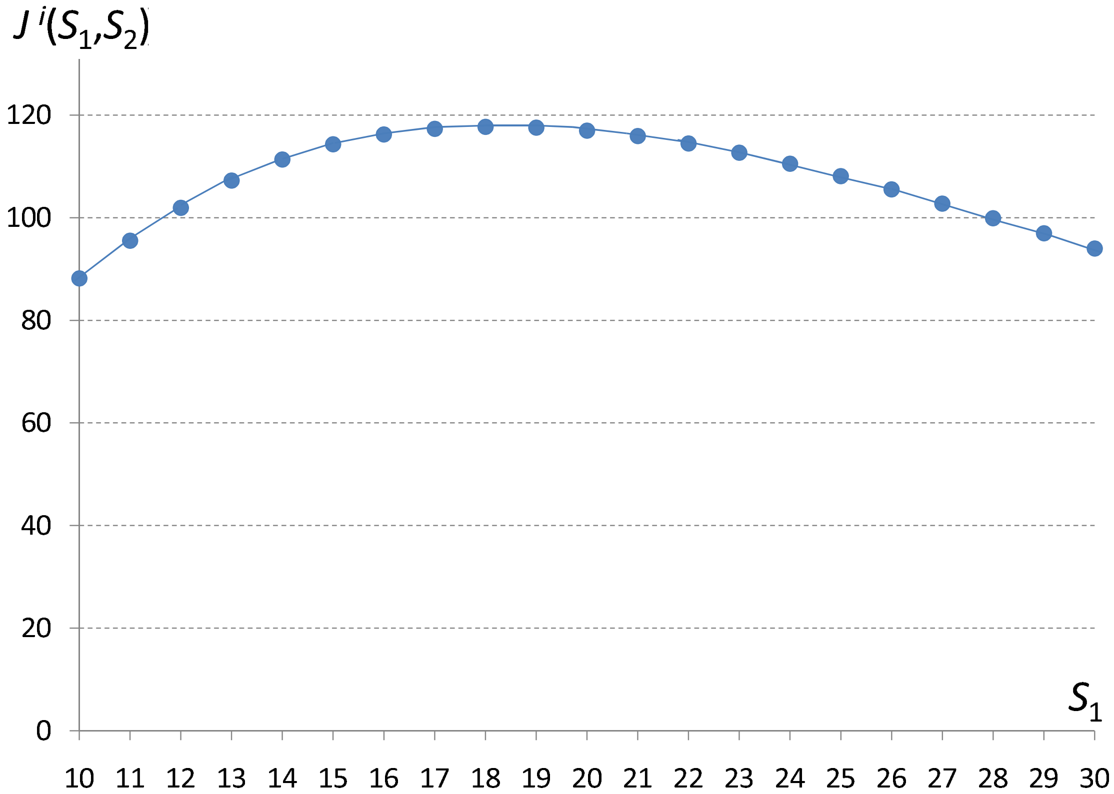

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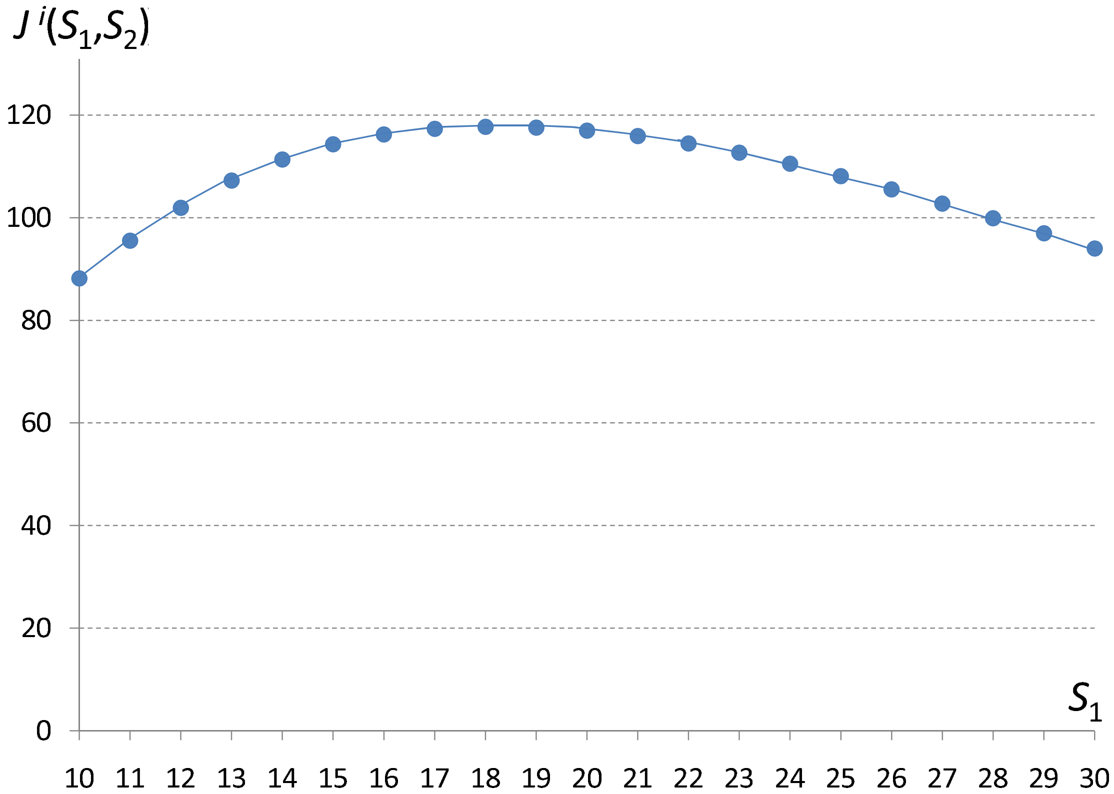

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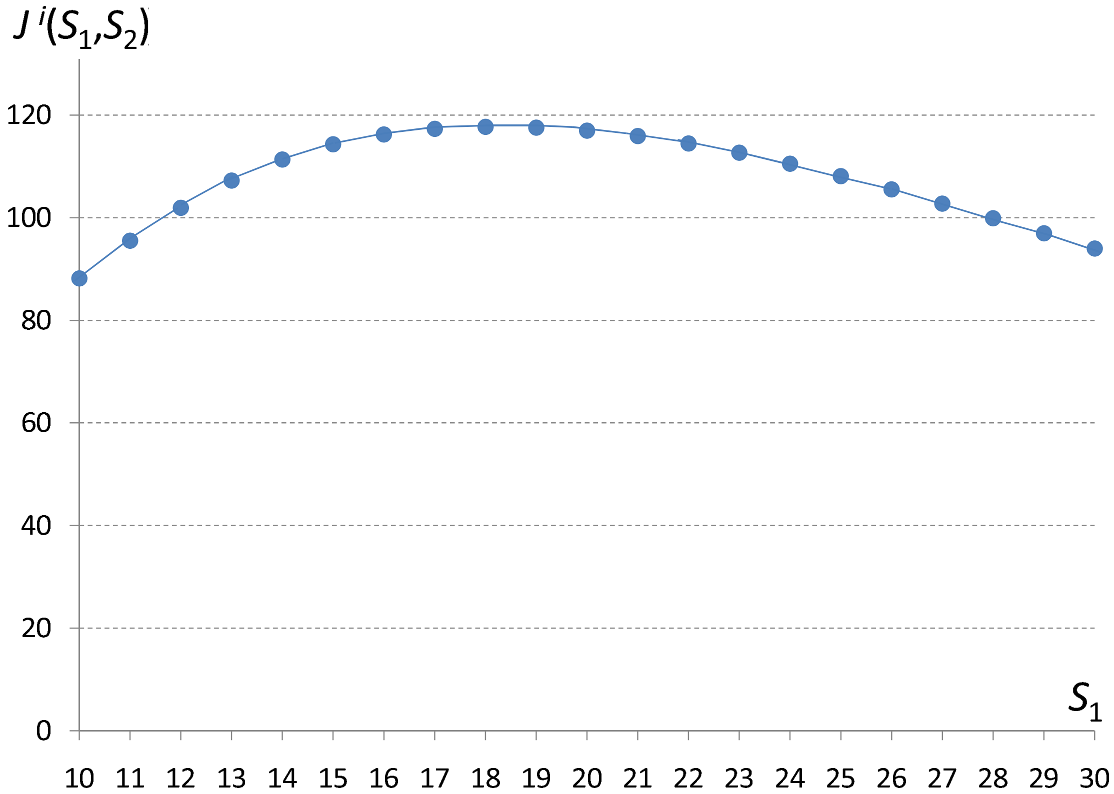

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- The ordering game with extended payoff functions has a pure strategy Nash equilibrium.

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- Increase in expected demand or demand flow leads to more competition, so less cooperation.

Numerical Analysis of Retailers' Benefit

- 3000 problems are analyzed by generating random parameters with uniform distributions

$p_1 \sim U(0.1, 0.25)$	$p_2 \sim U(0.1, 0.25)$	$c \sim U(3, 5)$
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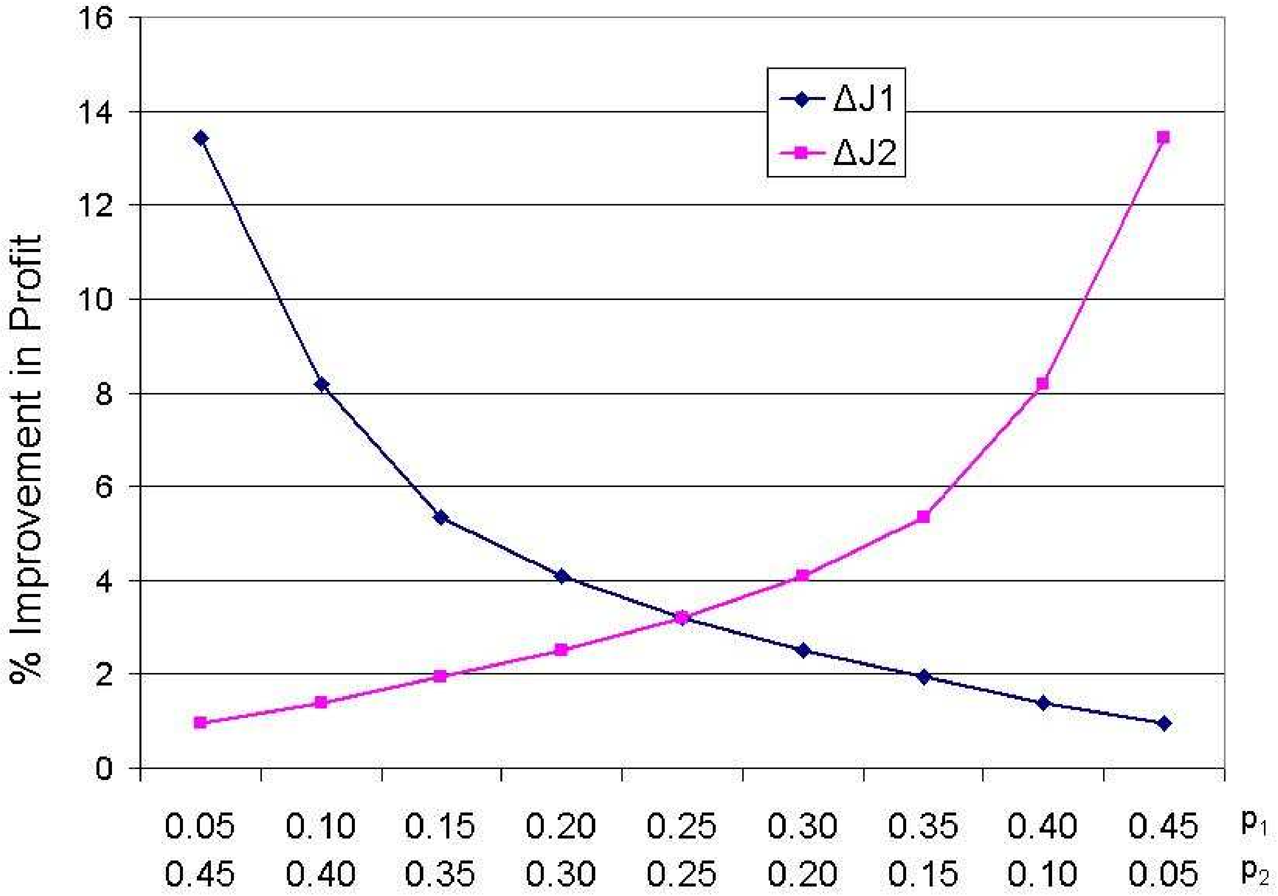
- Average increase in the profit of a retailer wrt no pooling is 3.3%, with a maximum of 9.6%.

Numerical Analysis of Retailers' Benefit

- The retailer with relatively low expected demand benefits from the transshipment more.



$$\tau = 1, t = 6, r = 10, s_1 = s_2 = 0, c = 2.5$$

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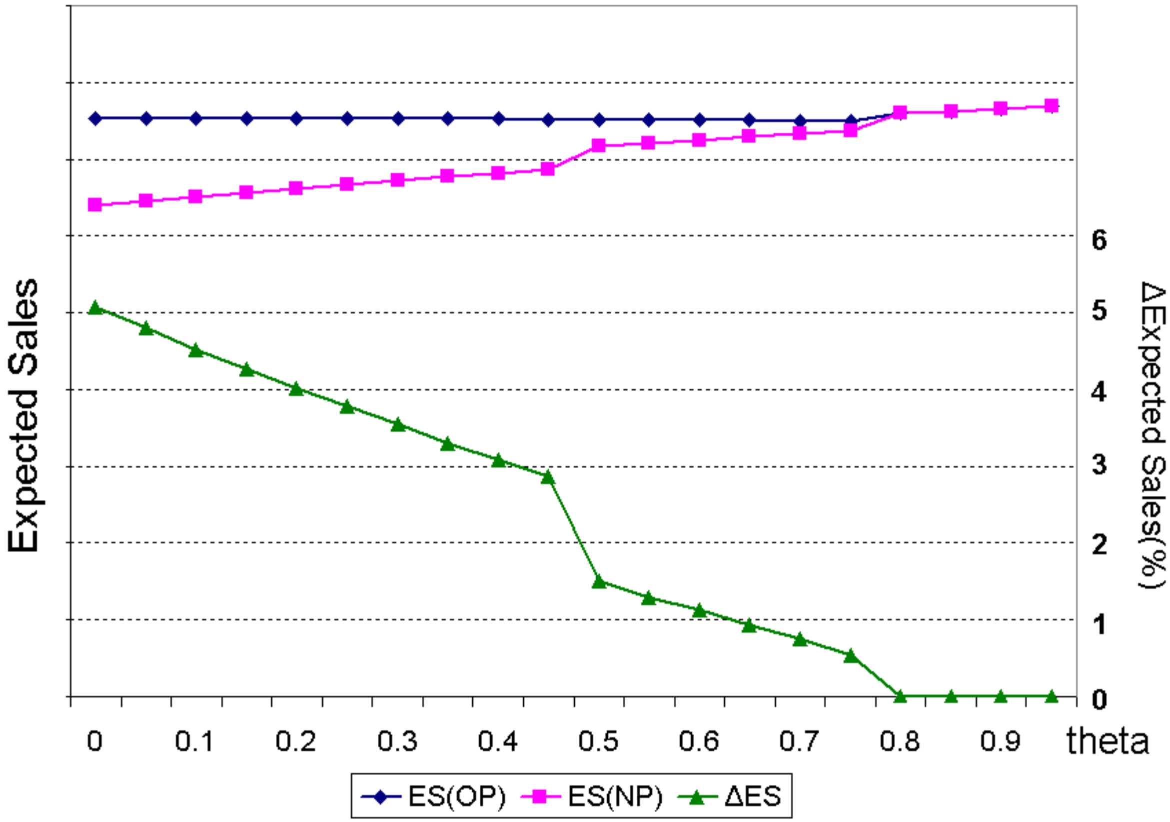

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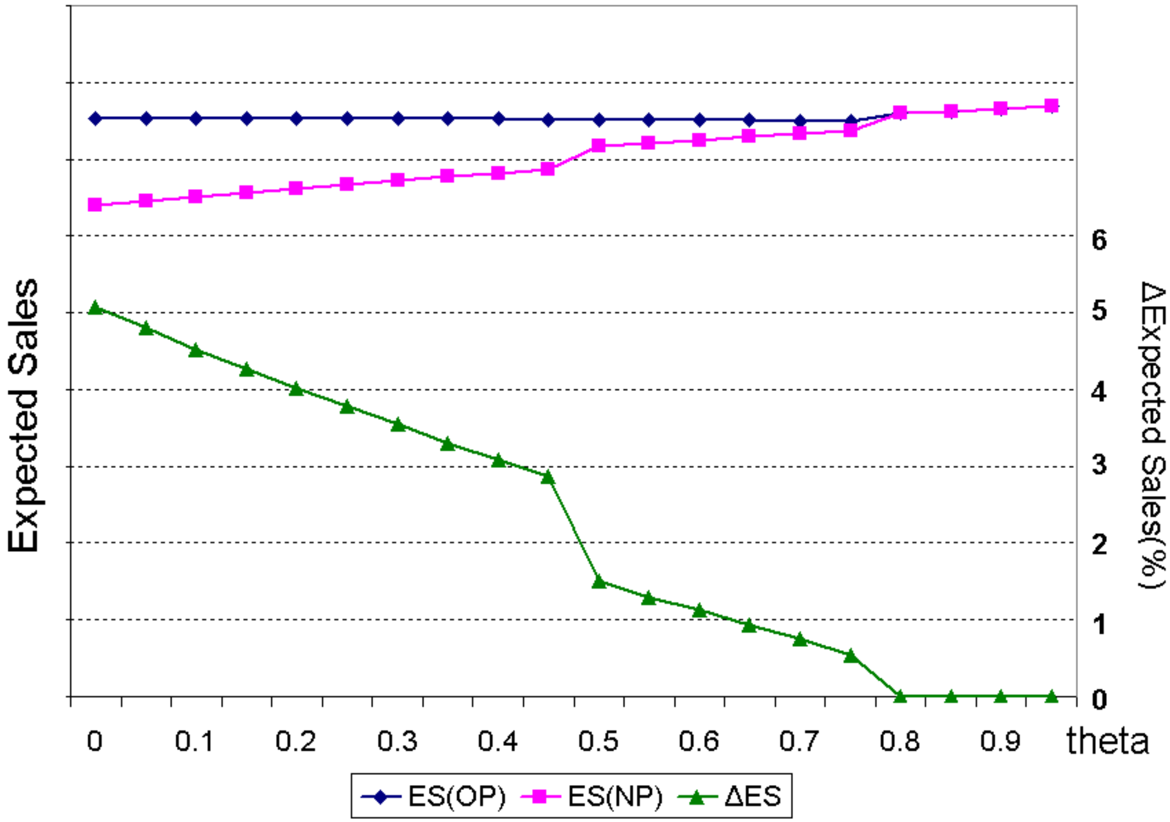

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- For 3000 problems, average improvement in total expected sales by optimal pooling wrt no pooling is 2.1%(max 7.8%).

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- There exists Nash equilibrium for retailers ordering decisions (i) for omnipotent requested retailer, (ii) in general for extended payoffs.
- The level of competition effects the willingness to cooperate.
- Both retailers and the manufacturer benefit from the optimal transshipment.

Thank you