DEFORMATION BEHAVIOUR OF GRANULAR MATERIALS UNDER REPEATED DYNAMIC LOAD

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1. INTRODUCTION

Pavement design is a process intended to find the most economical combination of layer thickness and material type for the pavement, taking into account the properties of the soil foundation and the traffic to be carried during the service life of the road. Traditional design methods are more or less empirical in Germany but there is, worldwide, an increasing desire to develop analytical approaches. A prerequisite of a successful analytical method is the experimental measurement and appropriate mathematical characterisation of the permanent deformation behaviour of unbound granular materials (UGM). Triaxial tests for the investigation of permanent deformation behaviour are the basis of these studies.

2. THE SHAKEDOWN CONCEPT

For a successful pavement design, the pavement must be able to resist the accumulation of permanent deformation. The permanent deformation of Unbound Granular Material (UGM) and other materials leads to irreversible deformations at the pavement surface. Thus, in practice, a pavement construction should be designed in such a way that no, or only small, permanent deformations appear in each layer. For design purposes, this implies that the maximum load level which is associated with a solely resilient response must be known and then not exceeded, if uncontrolled permanent deformations are to be prevented. This has raised the possibility of the existence of a critical stress level between stable and unstable conditions in a pavement. According to the "shakedown concept", this is termed the "shakedown limit". The shakedown concept has been used to describe the behaviour of conventional engineering structures under repeated cyclic loading. In summary, the concept maintains that there are four categories of material response under repeated loading: purely elastic, elastic shakedown, plastic shakedown and incremental collapse or ratcheting [3].

Fig. 1 Elastic/plastic behaviour under repeated cyclic pressure and tensile load [1].
3. RESEARCH PROJECT AND TESTING PROCEDURE

This paper reports on one aspect of an ongoing research project at the Dresden University of Technology, Germany, which is aimed at developing a fundamental model for the calculation of permanent deformation behaviour. Part of the project’s goal is to find the critical stress condition that defines the boundary between stable (non-rutting) and unstable (rutting) conditions in a pavement.

Triaxial tests were carried out on UGM as part of a collaboration with the University of Nottingham, England. A crushed Granodiorite aggregate was tested. For these tests the constant confining pressure was set at levels of 70, 140, 210 and 280 kPa. After the confining pressure was reached, additional dynamic (frequency = 5 Hz) vertical stress (deviator stress) pulses were applied.

4. TEST RESULTS

There are two different approaches to describe the deformation behaviour, the macromechanical and the micromechanical method. The micromechanical method synthesises the deformation behaviour from a study of localised material characteristics like friction between the grains, particle shape, density, grain abrasion, grain crushing etc.

With the Distinct-Element-Method originally developed by Cundall [9] it is possible to model materials that consist of individual grains or particles where the grains roll or slide on other grains. However, the modelling of pavement constructions with the Distinct-Element-Method for a large number of load repetitions is not possible at this time because of the restricted computing capacity. Furthermore, the relevant micromechanical characteristics and processes are not yet fully understood.

For the macromechanical modelling, several methods are available. One method is to develop the macromechanical model in combination with the characterisation of the micromechanical processes in the grain assembly. Another way is to determine the model only in an empirical way. However, the former method is much to be preferred as it allows prediction beyond the limits of experience and allows the user to understand its limitations.

It is characteristic for the deformation behaviour of the grain assembly under cyclic loads that consolidation, distortion processes and the deformation of single grains occur. The deformation behaviour can be separated into different ranges (comprising stable and unstable behaviour). Even at small stresses resilient and permanent deformations occur. The stress-strain relationship for granular materials is given by a non-linear curve, which is not retraced on the removal of stresses but forms a hysteresis loop. The form of the hysteresis loop will produce the values for the permanent and resilient deformations per load cycle (Fig. 2).

\[
W = 0.5 \left( \int s \cdot \text{de} - \int e \cdot \text{ds} \right)
\]

where \( \sigma \) = stress and \( \varepsilon \) = strain. The greatest part (≈ 95%) of this work is transformed into heat energy. Only a small part of this work will be accumulated [11].

In the triaxial testing performed, three types of permanent strain accumulation were observed. None of the test results showed the first type of behaviour – “purely elastic” – as suggested by Fig. 1. Probably this type of response does not occur in the UGMs in

\[
\text{Stress} \quad \text{Strain} \quad \text{permanent strain} \quad \text{resilient strain}
\]

Fig. 2 Hysteresis loop for viscous-elastic plastic behaviour.
pavements, as evidenced by the occurrence of post-compaction strain [4]. However three distinct types of behaviour were observed (which we name A, B and C) and each is now described.

4.1 Range A – Plastic Shakedown Range

Fig. 3 shows the resilient and permanent strains versus number of stress cycles for a Range A material. Here the response is plastic for a finite number of load applications, but after completion of the post-compaction period the response becomes entirely resilient and no further permanent strain occurs. The Shakedown state is reached. For this range the level of accumulated strain depends on the stress level. Also, inspection of the individual test results shows that the number of cycles required, before plastic strain ceases, increases with applied stress level increase.

A pavement with material in this condition would come to a stable equilibrium behaviour in response to the loading [7, 14]. It ‘shakes-down’. Range A behaviour is, therefore, permitted in the pavement, provided the total accumulated strain is sufficiently small.

![Fig. 3 Resilient and permanent strains versus load cycles, \(\sigma_D = 105 \text{ kPa}, \sigma_\text{3} = 210 \text{ kPa}, \text{Range A}\)](image)

4.2 Range C – incremental collapse

Lines like those on Fig. 4 (stress below static failure stress, decreasing resilient strains) and 5 (stress above static failure stress, increasing resilient strains) indicating continuing incremental plastic deformation with each stress cycle. Thus, in both cases, the response is always plastic and each stress application results in a progressive increment of the permanent strain [2]. The initial behaviour as shown in Fig. 5 is probably the same as in Fig. 4, but compressed into a fewer number of stress applications due to the greater cyclic stress level applied.

![Fig. 4 Resilient and permanent strains versus load cycles, \(\sigma_\text{D} = 840 \text{ kPa}, \sigma_\text{3} = 210 \text{ kPa}, \text{Range C}\)](image)

Range C behaviour in a pavement would result in the failure of the pavement by shear deformation in the UGM layer or/and probably by overstress of the bound layers, being experienced as rutting at the pavement surface. This range should not appear in a well-
designed pavement. It is also possible that plastic behaviour will only appear locally in the UGM layer. If so, no deformation in the UGM layer will occur, because the bound layers will bridge these zones while they are less than a certain size.

Fig. 5 Resilient and permanent strains versus load cycles, $\sigma_0 = 840$ kPa, $\sigma_3 = 140$ kPa, Range C

4.3 Range B – intermediate response – plastic creep

Fig. 6 shows an intermediate response. During the first stress cycles the high level of plastic strain rate decreases for the time being to a low, nearly constant level. The number of stress cycles for reaching this constant level of strain rate depends on the material and the stress level. This number of stress cycles may mark the end of post-compaction. Because of the almost constant level of strain rate, a near-linear rise of permanent strain is observed for tests with 100,000 stress cycles. A test with 700,000 stress cycles showed that a further increase of permanent deformation occurs with a growing number of stress cycles. A slow increase of the permanent strain rate occurred after 380,000 stress cycles. At 700,000 stress cycles it comes, like Range C, to an incremental collapse. Kolisoja [10] conducted triaxial tests on a crushed rock and found similar results. He observed that although the behaviour of an aggregate during the first 100,000 looks like stabilizing it can turn to failure if cycling at the same stress level is continued long enough.

Fig. 6 Resilient and permanent strains versus number of load cycles, $\sigma_0 = 280$ kPa, $\sigma_3 = 140$ kPa

Range B

4.4 Resilient strain

It can be observed that both Range A and Range B specimens exhibit a constant level of resilient strain during these tests and that the level of resilient strain depends on the stress level. If the material comes to an incremental collapse, as Range B, the resilient strains will
slowly increase. However, a significant decrease of resilient deformation is initially observed with an increasing number of stress cycles in Range C. If the cyclic stress level exceeds the static failure line, an increase of resilient deformation with increasing number of stress cycles will occur (Fig. 5) after the initial decrease.

Ranges A and B can also be separated on the basis of resilient deformation behaviour. Within Range A the resilient deformations increase with rising stress ratios a stiffening non-linear response to deviator stress level. With a further increase in stress ratio a rapid increase in resilient deformation is observed on the transition to Range B (Fig. 7). Because of this, a two stage resilient response is observed indicating that the deformation behaviour in Range A and Range B is different in type. The transition from Range A to Range B occurs at nearly the same level of resilient deformations. The observed boundaries between Range A and Range B are similar to the ones obtained from the permanent deformation behaviour.

5. MICROMECHANICAL PROCESSES

In the triaxial test the deformation behaviours just described can be observed, but the test does not immediately provide an explanation for the behaviour. For this reason it is desirable to describe the large scale observations of deformation behaviour of the granular material with a consideration of the manner in which the inter-particle response within the granular matrix may be responsible for that macro behaviour (as observed in Ranges A, B and C).

At low levels of stress (i.e. Range A macro behaviour) the initial, post compaction, plastic strain is most probably due to limited particle re-orientation and breakage. There may be a little inter-particle attrition, but this is expected to be insignificant. However, immediately after compaction strains, a wider resilient hysteresis loop (indicating greater energy loss per cycle of loading) is observed than after 270,000 stress cycles. Changes in the shape of the loop give information about the different deformation mechanisms. The shape of the hysteresis loop at 270,000 stress cycles can be explained by the Hertz contact theory [6].

Certainly, as the plastic strain rate per cycle continues to decrease and a purely resilient state is reached, there can be no ongoing damage. Once the point of almost pure resilience has been reached, the cyclic strain behaviour must be due only to the deformation of the single grains and to very limited recoverable particle rotations. Some minor frictional losses must still occur as the macro-scale resilient stress-strain response in Range A retains some very small hysteresis (Fig.8), which means that energy still dissipates. This frictional loss is associated with only a very small continuing accumulating of permanent strain. So these repeated applications of stress must be, in essence, non-damaging. In particular, on the basis of the foregoing, particle crushing/breakage probably does not occur to any significant amount [5].
At greater imposed stress levels a similar micro-scale behaviour is proposed. However, the residual, constant rate of accumulation of plastic strain experienced in Range B, together with a wider resilient hysteresis loop (indicating greater energy loss per cycle of loading) suggests that, after post-compaction rearrangement, particle breakage and inter-particle slip, continued frictional energy loss is now indicative of ongoing damage. Given the slow rate of plastic strain accumulation which is observed and, hence, the slow rate of damage which is inferred, it seems probable that this damage is more likely to take the form of particle contact attrition rather than particle breakage. The resilient strain behaviour must be due to the deformation of the single grains, to recoverable particle rotations and to additional recoverable slip between particles.

During Range A and Range B behaviours an increasing stiffness with increasing stress can be observed. This is conventionally explained in terms of Hertz contact theory [6]. With an increasing force to be carried by the inter-particle contacts, the centres of individual aggregate particles are forced closer together. Thus size of the inter-particle contacts must increase due to the compression of those contacts. Thus the contact stresses do not increase as fast as the externally applied stresses and a convex stress strain curve will result as seen in Fig. 7. Taking the above, micro-mechanical, explanation concerning the difference between Range A and Range B behaviours, then it will be evident that the response to each cycle of stress in Range B must include the effects of recoverable particle rotations and of additional recoverable slip between particles - effects not experienced in Range A. Thus we expect to see some discontinuity in the stiffness-stress curve on transition from Range A level stresses to Range B level stresses. And this is, indeed, seen in Fig. 7. Once within Range B we would expect to, and do, see the Hertzian stiffening response once again.

In both Ranges A and B, once a constant level of resilient strain is reached then constant volume permanent deformation is observed. In Range A this would be a natural consequence of the cessation of all particle damage. In Range B the ongoing, low-level and constant rate of damage would be expected to be linked with a constant resilient behaviour only if the condition of the particle contacts at the end of a cycle is the same as it was at the beginning. For this to be the case it is necessary to postulate that the small amount of damage debris generated has no significant effect on the resilient deformation behaviour. This may be the case if new fines can be accommodated within the void space. If this was the case then a constant volume (or even slightly compressive) plastic strain response might be anticipated in such circumstances.

In those cases in Range B where plastic strain rate begins to increase again after many cycles in which the plastic strain rate had been small (e.g. as in Fig. 6), it seems likely that the grain attrition effects the collapse. According to this understanding, the resistance to the friction between the grains and also the angle of internal friction are decreasing (probably due to a polishing effect at the grain contact points). The above explanations can also help to provide an explanation of the change in stiffness with number of stress cycles, as seen in Figure 6.
At even higher externally imposed stress levels (Range C) a different mechanism must be in play. The plastic strain rate never drops to a low level such that it is impossible to define the end of post-compaction behaviour. The hysteresis loops are always large, indicating significant energy loss per cycle (Figs. 9 and 10). Thus a greater degree of damage must occur almost from the outset of repeated load application than in Ranges A and B. Particle re-orientation and slip between particles would provide such an explanation. Particle breakage is expected to occur if the applied stress causes the strength of the grains to be exceeded. This particle breakage would then allow relatively large-scale particle re-orientation and a non-stable aggregate skeleton such that the observed large plastic strain rates become credible.

In Range C there are two different resilient processes (decreasing and increasing resilient strains) to explain, depending on the applied stress level. The dissimilar shapes of the hysteresis loops indicate that different micromechanical processes must be in play. If damage has allowed a denser structure to develop, the number of grain contacts will increase. This helps to explain why, with respect to resilient strains, Range C behaviour can be associated with a stiffening response. The incremental collapse will then occur because the friction between the grains is not sufficient anymore, to resist the external stress.

If the resilient strains are increasing and rapid plastic collapse occurs (e.g. Fig. 5), then it appears that particle damage now produces a weaker material skeleton and/or poorer inter-particle contacts. A simple explanation of this would be:

a) the removal of high quality contacts because they break,
b) the generation of excess fines leading to a lower mass strength for the granular material, and
c) shear dilation causing a lower density and, thus, fewer particle contacts.
6. SUMMARY

By considering the observed behaviours just described, it can be seen that Fig. 1 is not a good representation of them. Instead, Fig. 11 is introduced as a summary of the observed types. They are:

A Plastic shakedown
B Intermediate response
C Incremental Collapse

\[ \sigma_1 = \alpha \cdot \sigma_3 \cdot \beta \]

where \( \sigma_1 \) = total axial stress, \( \sigma_3 \) = cell pressure and \( \alpha, \beta = \) material constants [7], [8]. The analysis of the test results revealed an exponential relation (Equation 2) between the applied stresses (\( \sigma_1/\sigma_3 \)) and the different deformation behaviours, Ranges A, B and C.

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8. BIBLIOGRAPHICAL REFERENCES


