INVESTMENT, OUTPUT AND INTEREST RATE POLICY WHEN CAPITAL IS MOBILE*

David Fielding and Paul Mizen

The determination of long-run aggregate supply is a feature which short-run models of macroeconomic policy coordination tend to neglect but which has implications for the setting of interest rates. In this paper we develop an open-economy model of output maximisation, where investment and capital flows are crucial determinants of output and are in turn influenced by interest rate policy and credible control of inflation. The model represents an extension to short-run models of government policy-making, and emphasises that coordination of relative interest rates is as important as control of inflation.

Long-run issues relating to investment and output are often neglected in the literature on macroeconomic policy co-ordination.1 Short-term models of economic coordination typically consider a world in which the aggregate supply curve is vertical and the capital stock is fixed. Investment, if it is modelled at all, appears only as a component of aggregate demand, and does not influence the supply side. In a model with a longer time horizon, however, in which the capital stock in the production function is not fixed, investment does influence the supply side. The aggregate supply curve under these conditions is no longer vertical but upward-sloping because the full-employment level of output rises as the capital stock grows. The purpose of this paper is to construct such a model, which motivates long-run macroeconomic policy coordination for small inter-connected open economies.2

There are several distinctive features of our extended model. First, our model deals with open economies between which there are substantial potential capital flows in either direction. The factors which affect inflows and outflows of capital take a central role in the analysis, and we identify relative interest rates as the principal determinant of these flows.3 Given that governments control their domestic interest rates, the coordination of interest rates between governments requires game theoretic analysis. Secondly, investment is a function of the real interest rate; our model shows that credible control of inflation, a main focus of short-run stabilisation models, is still likely to be an important consideration. When long-run considerations about investment and

---

* We wish to thank Robin Cubitt, David Greenaway, Chris Milner, seminar participants at Princeton University, and two anonymous referees for helpful comments and suggestions on the paper, but acknowledge that remaining errors and omissions are our own responsibility.


2 Currie et al. (1988) deal with the nature of long-run versus short-run considerations in their introduction to an edited book arising from a CEPR conference and they usefully identify the point that issues of growth and trade are long-term concerns, whilst balance of payments and open economy dimensions to macroeconomics are shorter-term considerations. They use these considerations to motivate North–South discussions in macroeconomic modelling; and we also use them, but to extend a short-term model to allow for investment and growth considerations. There is a more recent paper by Sinclair (1989) which also deals with international policy coordination but not with respect to the question of investment and interest policy.

3 We acknowledge that taxes and subsidies can also be used to influence the flow of capital and the Appendix explains how the model can be amended to deal with these.
output are introduced, coordination over interest rates and the level of inflation are equally important and credibility remains an important issue. All governments are likely to be concerned with generating favourable conditions for long-term economic growth and inward foreign investment.

The next Section develops the model in which long-run income considerations are made explicit, and uses a simple model of coordination amongst small, open economies whose governments have as an objective the maximisation of equilibrium income. Section II sets out the implications and conclusions of the paper.

I. INTEREST RATE POLICY, INVESTMENT, AND INCOME DETERMINATION

We assume that there are two countries whose governments can control three macroeconomic policy tools (income taxes, the nominal interest rate, and the nominal money supply) and are attempting to maximise aggregate income. We begin by characterising the Nash equilibrium policy for each government, in which the other government’s policy is taken as given. We assume for simplicity that the population is constant, and that government policy is consistent with an equilibrium in which income and the capital stock are constant. Since in equilibrium income is constant, then with the assumption of a Quantity Theory equation the rate of growth of the nominal money supply will equal the rate of inflation ($\pi$). As a result the government can be modelled as having direct control over $\pi$ (and therefore the real interest rate, $R$) in the long run. The possibility of welfare-improving policy coordination arises because capital is mobile between countries, and its location is affected by international interest rate differentials. The inflow of capital might take the form of loanable funds from which domestic investors borrow, or direct foreign investment by multinational investors.

We begin by describing the constraints which a government faces from the behaviour of firms and consumers, the external balance and its own budget constraint, and the national accounting identity.

Private Firms

Private firms are profit maximising and their output is a function which depends positively on the stock of public and private capital and negatively on inflation:

$$Y = \xi K_p \alpha \xi \theta \pi$$

Equation (1) is an aggregate Cobb–Douglas production function: for a given level of inflation, output depends on the stock of publicly owned capital ($K_G$) and privately owned capital ($K_P$). $K_G$ can be thought of as the level of infrastructure, although it might also reflect direct investment by the

---

4 The assumption of a constant population keeps the model as simple as possible but means that equilibrium is defined in terms of the level rather than the rate of growth of income. It is quite straightforward to extend the model to the case of a constant positive rate of population growth with an equilibrium ratio of output and investment to employment and a positive equilibrium income growth rate.
government in private firms. Higher inflation \((\pi)\) has a detrimental impact on output because it introduces distortions to the allocation of resources. Firms maximise profits, so there is an equality between the marginal product of capital and the real interest rate:

\[
R = \alpha \xi K_p^{\alpha - 1} K_G^\beta (\sigma - \theta \pi) Y = \alpha Y K_p \quad 1 \geq \alpha, \beta, \gamma \geq 0; \xi, \sigma, \theta \geq 0. \tag{2}
\]

\(R\) is defined as the difference between the nominal interest rate \((r)\) and inflation \((\pi)\):

\[
R = r - \pi. \tag{3}
\]

\textit{Consumers}

Private consumption depends just on disposable income \((Y - T)\) and inflation:

\[
C = \chi_0 (Y - T) - \chi_1 \pi \quad 1 > \chi_0, \chi_1 > 0. \tag{4}
\]

\(T\) is total income tax revenue. Comparison of consumption with income gives the level of saving, but the location of saving – at home or abroad – depends on domestic and foreign interest rates. So we now turn to the external balance and in particular to the capital account.

\textit{External Balance}

The external position is determined by a balance of payments (BOP) equilibrium condition which ensures that the current account plus the capital account must equal zero. Since domestic and foreign assets are imperfect substitutes, the capital account will depend on the difference between the domestic nominal interest rate, \(r\) and the foreign nominal interest rate, \(r^*\) adjusted for currency appreciation. The capital account is based on a portfolio balance model in which financial capital is mobile hence, \textit{ceteris paribus}, capital is attracted to countries offering higher relative real interest rates. However, differences in asset characteristics with regard to maturity and risk mean that persistent interest rate differentials are consistent with a capital account equilibrium. Linearising this relationship, the BOP condition becomes:

\[
V + \eta_0 + \eta_1 (r - r^* - \dot{e}) I_p = 0 \quad \eta_0, \eta_1 > 0, \tag{5}
\]

where nominal interest rates are \(r\) and \(r^*\), and the rate of appreciation of the domestic price of foreign currency is given by \(\dot{e}\). The current account surplus is given by the first term, \(V\), and the capital account by the second and third terms where \(\eta_0\) represents an exogenous net inflow or outflow of capital. For a given level of domestic private investment demand \((I_p)\), the net flow of capital depends on the interest rate differential. A higher level of domestic investment will also attract international capital \textit{ceteris paribus}, however, since for a given relative interest rate a more buoyant domestic economy will attract more international funds.\(^5\)

\(^5\) Note that ours is a long-run model in which a country must have a sustainable balance of payments position: the current account and capital account must offset each other in equilibrium.

© Royal Economic Society 1997
By invoking relative purchasing power parity, which can also be justified both empirically and intuitively over the long run, we have:

\[ \dot{\epsilon} = \pi - \pi^*, \]  

where \( \pi^* \) is the foreign inflation rate. Then by substitution of (3) and (6) into (5) the BOP condition becomes:

\[ V + \eta_0 + \eta_1 (R - R^*) I_P = 0 \quad \eta_0, \eta_1 > 0. \]  

(7)

Not only does the government have to attract funds from abroad into the country, but it also has to prevent the flow of assets abroad by offering a sufficiently high nominal rate of interest, especially if inflation is high. The problem for the government is to achieve a balance between public and private investment. If it sets the interest rate too high then funds may flow in from abroad to finance more aggregate investment, but firms will face such high capital costs that private investment will be very low. On the other hand, if the rate is too low the sources of funds may dry up, since the savings of domestic residents flow abroad and the capital from abroad does not flow in. This constraint on total available loanable funds means that whilst private investment is high there are no funds left over for any public investment.

Model Closure

Income, investment, and consumption are linked by the national accounting identity:

\[ Y = C + I_P + I_G + V, \]  

where \( I_G \) is public investment. The equilibrium capital stock conditions for public and private investment are given by:

\[ I_P = \frac{K_P}{\delta}, \quad I_G = \frac{K_G}{\delta}, \]  

(9)

where \( \delta \) is the reciprocal of the rate of capital depreciation. Public investment is constrained by a financing condition:

\[ I_G = T. \]  

(10)

In the long run, the budget must balance, and net government borrowing is

---

\(^6\) Regarding the non-equalisation of real interest rates in our model, a long-run capital account deficit (which is caused by a long run \( R < R^* \)) is sustainable because the current account \( (V) \) can be permanently in surplus. For any interest rate vector \( (R, R^*) \) resulting from the two governments’ policy choices the nominal exchange rate adjusts to a level which delivers the appropriate current account balance. Any balance of payments deficit leads to excess supply of domestic currency and hence currency depreciation, which leads to an improvement in the current account. *Mutatis mutandis*, a capital account surplus can be accommodated by a current account deficit. Note also that since the purchasing power parity condition is specified as a relative condition, and inflation rates do not necessarily have to be equal, the exchange rate can follow a long-term trend of depreciation or appreciation, so even with real interest equality nominal rates might differ.

© Royal Economic Society 1997
zero.\(^7\) Government capital expenditure must equal tax revenue. For simplicity, other forms of expenditure are set at zero.

**Derivation of the Long-Run Equilibrium**

The Nash solution to the model is obtained by deriving each government’s optimal real interest rate policy, given the behaviour of the other. A key feature of the analysis will be the shape of the reaction function and the welfare consequences that follow from it. We will express the solution in terms of real interest rates so that the importance of both nominal interest rate policy and credible inflation policy is apparent.

The derivation of the government’s optimal interest rate policy comes from its choices of \(T\), \(R\) and \(\pi\) to maximise \(Y\). Its choice of \(T\) amounts to a choice of \(I_G\) and hence in equilibrium of \(K_G\). Its choice of \(R\), given its control of \(K_G\) and \(\pi\), amounts to a choice of \(K_P\) (see (2)). Therefore, we can model the government as choosing \(K_G\), \(K_P\), and \(\pi\) so as to maximise \(Y\), treating the variables \(T\), \(I_G\), \(I_P\), \(R\), \(C\), and \(V\) as endogenous. Using (1)–(2), (4), and (7)–(10), the maximisation problem collapses to:

\[
\begin{align*}
\text{maximise} & \quad Y = \xi K_P^\alpha K_G^\beta (\sigma - \theta \pi)^\gamma \\
\text{subject to} & \quad \xi K_P^\alpha K_G^\beta (\sigma - \theta \pi)^\gamma \left(1 - \chi_0 + \frac{\eta_1 \alpha}{\delta}\right) = \frac{[K_P + K_G(1 - \chi_0)]}{\delta} \\
& \quad -\chi_1 \pi - \eta_0 + \frac{\eta_1 R^* K_P}{\delta}.
\end{align*}
\]

Solving the problem as a Lagrangian and defining \(\lambda\) as the multiplier corresponding to the constraint, the first order conditions imply:

\[
\begin{align*}
\left[\frac{1}{\lambda} + (1 - \chi_0) + \frac{\eta_1 \alpha}{\delta}\right] \alpha \xi K_P^{\alpha - 1} K_G^\beta (\sigma - \theta \pi)^\gamma - \frac{1}{\delta} \frac{\eta_1 R^*}{\delta} &= 0 \quad (12a) \\
\left[\frac{1}{\lambda} + (1 - \chi_0) + \frac{\eta_1 \alpha}{\delta}\right] \beta \xi K_P^\alpha K_G^{\beta - 1} (\sigma - \theta \pi)^\gamma - \frac{(1 - \chi_0)}{\delta} &= 0 \quad (12b) \\
-\left[\frac{1}{\lambda} + (1 - \chi_0) + \frac{\eta_1 \alpha}{\delta}\right] \gamma \theta \xi K_P^\alpha K_G^{\beta - 1} (\sigma - \theta \pi)^{\gamma - 1} + \chi_1 &= 0. \quad (12c)
\end{align*}
\]

These conditions imply that:

\[
K_G = \frac{\beta}{\alpha} \left(\frac{1 + \eta_1 R^*}{1 - \chi_0}\right) K_P
\]

and

\[
\pi = -\frac{\alpha}{\delta \chi_1} \left(\frac{1 + \eta_1 R^*}{\delta \chi_1}\right) K_P + \frac{\sigma}{\beta}. \quad (14)
\]

\(^7\) Strictly, there is another component of revenue, inflation seigniorage; however, the real money stock is typically so small that this component is tiny compared to borrowing and fiscal revenue.

© Royal Economic Society 1997
Equations (13) and (14) are substituted into the constraint, which, by assuming that \(\alpha + \beta + \gamma = 1\), yields a tractable equation for \(K_P\). The level of the private capital stock resulting from the government’s choice of \(T\), \(R\), and \(\pi\) will be:

\[
K_P = \frac{\delta (\eta_0 + \chi_1 \sigma)}{\left\{1 + \eta_1 R^* - x\left(\frac{\beta}{\alpha(1 - \chi_0)}\right)^\theta \left(\frac{\theta \gamma}{\alpha \chi_1}\right)^\gamma (1 + \eta_1 R^*)^{(\beta + \gamma)} / (\delta (1 - \chi_0) + \eta_1 \alpha)\right\}}.
\]  

(15)

It can be seen that \(K_P\), and therefore \(R\), since the two variables are related by (2), depend on \(R^*\). \(\partial K_P / \partial R^*\) has a negative component and a component of positive sign, corresponding to the two expressions in the denominator of the RHS of (15). An increase in \(R^*\) entails a worsening in the capital account; this will entail currency depreciation and a current account improvement in order to maintain BOP equilibrium. This increase in a component of aggregate demand means that the government must engineer a decrease in another component, i.e. private investment, in order to maintain macroeconomic equilibrium. In the long run, lower private investment means a lower level of the private capital stock. This effect tends to make \(\partial K_P / \partial R^*\) negative. However, an increase in \(R^*\) also raises the government’s chosen ratio of \(K_G\) and \(\sigma - \theta \pi\) to \(K_P\) (with less financial capital around for private investment, it is easier to achieve a certain level of output by lowering inflation or increasing taxes and public capital expenditure). This leads to an overall increase in aggregate supply, requiring a corresponding increase in aggregate demand which the government achieves by stimulating higher private investment. However, it is sensible to assume that the denominator of equation (15) is positive (otherwise the equilibrium capital stock will be negative), and hence \(\partial K_P / \partial R^* < 0\).

In order to explore the characteristics of equilibrium in the model, now allowing for an endogenous foreign interest rate, we need to determine the shape of the reaction function \(R(R^*)\). Assuming that the foreign interest rate is determined by a process analogous to the one determining the domestic interest rate, equilibrium will be defined by the interest rate vector \((R, R^*)\) which satisfies both the domestic and foreign reaction functions. The domestic reaction function \(R(R^*)\) can be derived by substitution of (13)–(14) into the marginal efficiency of capital schedule, (2). Assuming that \(\alpha + \beta + \gamma = 1\), we have:

\[
R = \alpha \xi \left(\frac{\beta}{\alpha(1 - \chi_0)}\right)^\gamma \left(\frac{\theta \gamma}{\alpha \delta \chi_1}\right)^\gamma (1 + \eta_1 R^*)^{(\beta + \gamma)}.
\]  

(16)

The reaction function has a positive slope and \((\beta + \gamma < 1)\) is concave. However, we do not know whether the slope of the reaction function is greater or less than unity.

Assuming that the interest rate in the foreign country is determined by a process similar to that in the home country, the two reaction functions, \(R(R^*)\)
and \( R^*(R) \), will both have slopes either greater or less than unity in the region of equilibrium. We consider both cases.

If the reaction functions have slopes less than unity, then we will be in a world depicted by Fig. 1. The figure shows the two reaction functions and the isoquants corresponding to the equilibrium at point \( Q \). Isoquants closer to the origin represent higher levels of output in each respective country, since when the other country’s interest rate is lower the capital stock and so output in each country are higher. Output in the two countries is \((Y_0, Y_*^0)\). There is a core of interest rate combinations, \( \Gamma \), which deliver higher output in both countries, so \( Q \) is not Pareto optimal. It would be better for both countries for the domestic government to have a lower reaction function (moving down \( R^*(R) \) into the core), and Pareto optimal outcomes (where the isoquants are tangential) are likely to be possible only if both governments coordinate to lower their reaction functions.

Operating with a lower reaction function means maximising some function other than \( \bar{Y} = \xi K^\alpha P^\beta G^\gamma \). There are many alternatives to achieve this end, the most relevant of which, to the present political climate, is to ‘under’ or ‘over’-value inflation. Equation (16) tells us that if the government maximised a function with a lower value of \( \gamma \) and \( \theta \) (as it were, undervaluing low inflation) then its reaction function would be lower for each level of \( R^* \). In such a case, it pays for the government to be ‘soft’ about inflation.

If, on the other hand, the slope of the reaction functions is greater than unity, then we are in a world depicted by Fig. 2. Here, points in the core can be achieved by upward shifts in the reaction function. This is the case where the domestic government should be excessively ‘hard’ about inflation, behaving as if \( \gamma \) and \( \theta \) were higher than is actually the case. In this scenario, there is a role for an independent central banker who has an excessive taste for low inflation.

---

* For the sake of visual clarity, the reaction functions in the figures are drawn as linear functions and the isoquants as strictly convex functions.

© Royal Economic Society 1997
Ultimately, optimal government policy depends crucially on the slope of the two reaction functions and particularly on whether they are likely to have a slope greater than unity. Hence, policy prescriptions regarding inflation targets require some knowledge of the parameters, which for individual countries would be a matter of empirical investigation. In all the cases we have considered the Nash equilibrium is not Pareto efficient and the feasibility of Pareto improvements will depend on the political economy of policy-making. The government will have to commit itself credibly to goals which are not Nash strategies. In some scenarios (e.g. when a ‘tough’ inflation policy is welfare improving) a way of achieving the desired credibility may be by delegation of power to an independent central bank, with stronger preferences for low inflation.

II. CONCLUSIONS

The paper constructs a model of coordination amongst small, open economies and analyses capital flows, long-run growth and investment. This contrasts with models which focus on short-run stabilisation and allows us to consider the long-run policy objectives for interconnected small, open economies. In the context of global financial markets, the question is how to attract an inward flow of capital and to stimulate long-term economic growth. The critical factor in this model is the real interest rate differential, and hence, both nominal interest rates and inflation are important in determining the flow of capital into or out of the country.

9 Inspection of equation (16) indicates that higher values of $\xi$ (the intercept of the log-linear production function), $\alpha$ and $\beta$ (which captures the productivity of private and public investment), $\eta_1$ (the sensitivity of the capital account to interest rate differentials) and $\chi_0$ (the average propensity to consume) will tend to make the reaction function steeper. Higher values of $\delta$ (the reciprocal of the depreciation rate) and $\chi_1$ (the sensitivity of aggregate consumption to inflation) tend to make the reaction function flatter.

© Royal Economic Society 1997
Setting the correct real rate of interest so as to attract capital is important; yet the achievement of a Pareto optimum requires the coordination of interest rate policies. Pareto optimal inflation and interest rate policies involve commitment to targets which are different from those of the Nash equilibrium. If governments simply set interest rates to maximise output taking foreign rates as given, then growth rates will be universally lower than if interest rates are set in consultation with foreign governments. This result follows intuitively from the fact that economic growth is affected by domestic investment through the level of the real interest rate and the marginal product of capital, but governments face the temptation to increase their own interest rate, relative to the foreign rate, to attract financial capital from abroad. Since the relative rate is compatible with any level of the real interest rate governments have an incentive to coordinate interest rates at a level which jointly maximises growth and delivers Pareto improvements to welfare relative to the Nash equilibrium.

Since capital flows are dependent on real interest rates, the Pareto optimum requires credible commitment to inflation targets, and, whilst the paper shows that the 'hardness' or 'softness' towards these targets depends on the particular parameter values, credibility and coordination over inflation are required too.

The conclusion we draw is that in a long-term model, with growth and capital mobility, a coordinated approach to interest rate and inflation policy is necessary for welfare improvements. Unilateral pursuit of interest rate and inflation policies will be sub-optimal. Governments facing global markets for capital and investment funds must coordinate both nominal and interest rates and inflation.

University of Nottingham

References


Appendix

The model in the paper can be amended to include the role of investment subsidies (or taxes). Constraints on space prevent the elaboration of details here, but the model can be extended in the following way. We modify the aggregate production function to allow for the resource costs of administering the subsidy: this is captured by the term $(\nu-\zeta_t)\kappa$ in (A.3), where $t$ is the rate of subsidy. (We assume that the private sector bears the administration costs.) Also, we modify the government budget constraint, so that

© Royal Economic Society 1997
public investment equals income tax revenue ($T$) less the cost to the government of financing the investment subsidy ($tI_p$):

$$I_o = T - tI_p.$$  \hspace{1cm} (A 1)

The private marginal cost of capital is now $R - t$, so the profit maximisation condition is:

$$R - t = \frac{\alpha Y}{K_p}.$$  \hspace{1cm} (A 2)

The maximisation problem collapses to:

$$\begin{align*}
\text{maximise} \quad & \quad Y = \xi K_p^\alpha K_G^\beta (\sigma - \theta \pi)\gamma (v - \zeta_t)^\varepsilon \\
\text{subject to} \quad & \quad \xi K_p^\alpha K_G^\beta (\sigma - \theta \pi)\gamma (v - \zeta_t)^\varepsilon \\
 & \quad \left(1 - \chi_0 + \frac{\eta_1 \alpha}{\delta}\right)
\end{align*}$$

Solving the problem as a Lagrangian as before, the first-order conditions yield:

$$\begin{align*}
K_o &= \beta \left[1 + \eta_1 R^* (\eta_1 + \chi_0)\right] K_p \\
\pi &= -\frac{\gamma}{\alpha} \left[1 + \eta_1 R^* (\eta_1 + \chi_0)\right] K_p + \frac{\sigma}{\delta} \\
t &= -\frac{\kappa (1 + \eta_1 R^*) - \frac{\alpha Y}{\xi}}{(\kappa - \alpha)}. \hspace{1cm} (A 6)
\end{align*}$$

Note if $\kappa = 0$ then $t = \nu/\zeta$, but if this is the case then taxes do not impinge on the maximand. Even when $\kappa > 0$, $t$ is still pinned down straight away and the solution follows through after substitution for $t$ into (A 4) and (A 5). It is clear that there are game theoretic aspects to the choice of $t$ but we ignore those aspects here due to space constraints.