Worked solution to Q6(a) F31ST1 paper '06-'07

Show that $\int_0^\infty f(v)dv = 1$

That is, show that:

$$4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 exp\left(\frac{-mv^2}{2kT}\right)dv = 1 \tag{1}$$

We therefore need to evaluate $\int_0^\infty v^2 exp(\frac{-mv^2}{2kT})dv$

Compare to standard integral given in formula sheet: $\int_0^\infty x^2 exp(-x^2)dx = \sqrt{\pi}/4$

Let

$$x^2 = \frac{mv^2}{2kT} \Rightarrow v^2 = \frac{2kT}{m}x^2 \tag{2}$$

i.e.

$$dv = \sqrt{\frac{2kT}{m}}dx \tag{3}$$

$$\Rightarrow \int_0^\infty v^2 exp\left(\frac{-mv^2}{2kT}\right)dv = \int_0^\infty x^2\frac{2kT}{m}exp(-x^2)\sqrt{\frac{2kT}{m}}dx \tag{4}$$

Substituting RHS of equation [4] into [1] and rearranging,

$$\Rightarrow 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^{3/2}\int_0^\infty x^2 exp(-x^2)dx = 1 \tag{5}$$

But we know that $\int_0^\infty x^2 exp(-x^2)dx = \sqrt{\pi}/4$ from the formula sheet so substitute $\sqrt{\pi}/4$ for integral. You will find that the identity in [5] holds.