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Legislative Bargaining with Heterogeneous Disagreement Values: Theory and Experiments

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Abstract

We study a legislative bargaining game in which failure to agree in a given round may result in a breakdown of negotiations. In that case, each player receives an exogenous 'disagreement value'. We characterize the set of stationary subgame perfect equilibria under all q-majority rules. Under unanimity rule, equilibrium payoffs are strictly increasing in disagreement values. Under all less-than-unanimity rules, expected payoffs are either decreasing or first increasing and then decreasing in disagreement values. We conduct experiments involving three players using majority and unanimity rule, finding support for these predictions.

Keywords: legislative bargaining; majority rule; unanimity rule; risk of breakdown; experiments.
JEL Classification: C78, C92, D71, D72.

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1 Introduction

One of the central questions in political economy concerns the optimal rule to be used for collective decision making in a group. Although real-world institutions are far more complex, a number of important insights can be gained from considering the choice among alternative *q-majority* rules: How many members of a group should be required to consent before the group undertakes some collective action such as passing a new law or engaging in a joint project?

In a seminal analysis of this problem, Buchanan and Tullock (1962) identified a fundamental tradeoff between what they called 'external costs' and 'decision costs'. According to the authors, more inclusive rules may help to prevent that decisions taken will inflict ('external') harm on non-consenting parties. On the other hand, such rules (especially unanimity rule) may increase the costs associated with decision making itself. This may be true not just for purely logistical or statistical reasons, but also because such rules create incentives for individual participants to adopt a tougher bargaining stance. In the case of unanimity rule, each member may be tempted to *hold out* and refuse agreement in an effort to elicit compensation in exchange for her support.

In prior research, Miller and Vanberg (2013, and 2015) have experimentally investigated the effects of alternative decision rules in the context of a simple multilateral bargaining game first proposed by Baron and Ferejohn (1989) (henceforth BF). In the BF game, a group of individuals has the opportunity to divide a fixed amount of some resource (e.g. money). Bargaining proceeds over a potentially infinite number of rounds. Within each round, one member is randomly chosen to propose a division, which is immediately voted on. If the proposal passes (according to the decision rule being employed), the game ends and players receive their respective shares. If it fails, bargaining proceeds to a new round. This process continues until a proposal passes. Payoffs are discounted, such that delay before reaching agreement is costly and inefficient. (Results from these experiments will be described briefly in the next section.)

Although it represents a purely distributive decision problem, the BF game is an attractive "workhorse" model of group decision making more generally. It can be interpreted to reflect a situation in which a group is deciding whether to undertake a collective action or "project". Under this interpretation, the resource being divided represents a "surplus" that would result if the project is undertaken. This interpretation is appropriate if (a) the action being considered is efficient (in the sense that it generates a positive surplus), (b) transfers are possible (such that the surplus can be reallocated), and (c) group members are identical, as reflected by the symmetry of the game.[1]

If we want to interpret the BF game in this way, however, it would seem to miss important features of real-world collective choice situations. In most contexts, it is important to distinguish between the *substantive proposal* under consideration (e.g. to build a bridge), and the *transfers* that might be attached to such a proposal in an attempt to secure agreement. This distinction is not (explicitly) made in the BF model, since preferences are about transfers only. Thus an implicit assumption is that all participants have identical preferences with respect to the substantive proposal under consideration. In most interesting contexts, however, individuals have *heterogeneous preferences* with respect to the substantive proposal. A more realistic model should therefore allow that players differ in the value they attach to agreement *per se*, in addition to transfers.

We introduce heterogeneity in preferences for agreement by modifying the BF model in two ways. First, we assume that whenever a proposal fails, bargaining will continue with probability δ, and otherwise "break down" (end without agreement). Second, we assume that in case of failure, each member receives an exogenously given non-negative payoff (*disagreement value*).[2] Most importantly, these values are assumed to *differ* between members.

These modifications are interesting for at least two reasons. First, the introduction of heterogeneous disagreement values is an easy way to exogenously manipulate the 'toughness' of the bargaining stances. By analyzing the relationship between disagreement values and expected payoffs, we can investigate the incentives to adopt a 'tough' bargaining stance under differ-

[1] Another implicit assumption is that transfers are constrained such that it is not possible to make any player *worse* off.

[2] The disagreement value can be interpreted in several ways. One is that it represents the value that a player attaches to the *status quo*, which will prevail if agreement is not reached. Another is that it represents a reward directly associated with a failure to agree. For example, a 'tough' bargainer can be thought of as an individual who anticipates an extrinsic (social) or intrinsic (emotional) reward in case negotiations end with disagreement. Under each interpretation, a large disagreement value is likely to be associated with a reluctance to agree. An alternative approach would be to endow each member with an *agreement value* which is received only in case agreement is reached. For technical reasons, the approach we are taking is easier.

ent decision rules. Second, the introduction of asymmetry is likely to make agreement among experimental subjects more difficult, increasing observable delays and creating more room for *variation* in delay under different treatment conditions (decision rules).

Our main theoretical result is that a player's expected equilibrium payoff is increasing in his disagreement value under unanimity rule, but not so under any less-than-unanimity rule. Under less inclusive rules, payoffs are either decreasing in disagreement values (such that the player with the smallest disagreement value achieves the highest expected payoff), or first increasing and then decreasing (such that some other player, but never the one with the highest disagreement value, achieves the greatest payoff). Substantively, this means that a 'tough' bargaining stance (as modeled by a large disagreement payoff) is beneficial under unanimity rule but possibly harmful under majority rule.

Our experimental results provide partial support for these theoretical predictions. Consistent with our model, we find that the player with the largest disagreement value indeed achieves the largest payoff under unanimity rule. Under majority rule, however, that player is included in others' coalitions significantly less often. None the less, we do not find that this results in consistently lower average payoffs. A statistically significant disadvantage in terms of expected payoffs is found only if the highest disagreement value is very large compared to others.

2 Related Literature

Despite its simplicity, the BF game is rich in strategic possibilities and admits multiple subgame perfect equilibria. The theoretical literature has focused on symmetric stationary equilibria, which are (essentially) unique (Eraslan, 2002, and Norman, 2002).These equilibria are characterized by three empirically testable features. First, proposers form "minimal winning coalitions", allocating positive payoffs only to the number of players required to agree. Second, the distribution of payoffs within the coalition is unequal, favoring the proposer. Third, the first proposal passes immediately, so there is no delay. All three of these properties are independent of the decision rule being used (majority, qualified majority, or unanimity rule).

These (symmetric stationary subgame perfect) equilibrium properties of the BF model have been experimentally investigated in a number of papers

(McKelvey, 1991, Fréchette et al, 2003, 2005a, 2005b, Diermeier and Morton 2005, Agranov and Tergiman, 2014a, and Bradfield and Kagel, 2015). All of these studies investigated the simple majority rule version of the game. The central findings include: (1) Subjects do indeed form minimal winning coalitions, allocating positive shares to a bare minimum of players. (2) The distribution of shares within the majority coalition is generally more equal than theory predicts, and (3) The vast majority of games end in immediate agreement to the first proposal made.

Building on this literature, Miller and Vanberg (2013, and 2015) investigated the effects of different decision rules (majority and unanimity rule) within experimental BF games. Inspired by Buchanan and Tullock (1962), their main hypotheses were (a) that unanimity rule would be associated with greater costly delay in reaching agreement, and (b) that this effect would be driven in part by "tougher" bargaining at the individual level. The main finding from these studies was that unanimity rule is indeed associated with significantly greater delay.[3] Both studies also find some support for the notion that individuals adopt a 'tougher' stance under unanimity rule, more often voting 'no' on a given proposal and making less generous offers when proposing. Most importantly, and consistent with prior experiments, both studies find relatively little delay overall, as well as a tendency to agree on *symmetric* distributions - most commonly an equal split within a minimal winning coalition.

The effect of heterogeneous disagreement values in the event of a breakdown of negotiations has been studied theoretically for the case of two players (see Binmore et al., 1986); these results can be easily extended to n players and unanimity rule[4]. To the best of our knowledge, the case of general q-majority rules has not been studied. The closest paper we are aware of is Banks and Duggan (2006). In their model, players receive a flow payoff every bargaining period; for every period in which agreement has not been

[3]Agranov and Tergiman (2014b) have replicated this result using larger groups. Their study focuses on the effects of (verbal) communication. Their main result is that communication significantly increases proposer power under majority rule, causing outcomes to be more consistent with theoretical predictions.

[4]Binmore et al. (1986) use the alternating-offer procedure of Rubinstein (1982) rather than the random proposer procedure of Binmore (1987), Baron and Ferejohn (1989) and Okada (1996). This makes little difference to the results under unanimity rule, namely that a player benefits from having a higher disagreement value, and that disagreement values remain relevant even as the breakdown probability goes to zero.

reached, they receive a status quo payoff. They prove existence of SSPE for very general policy spaces and voting rules. In one of the special cases they consider (model 5), the policy space is the unit simplex and the decision rule is simple majority; this is similar to our model with the additional restriction $q = \frac{n+1}{2}$ (n odd). They discuss only equilibria in which all players have the same continuation value, noting that the equilibrium may or may not be of this type depending on the parameters. We provide a full characterization for arbitrary values of the parameters and of q.

A small number of experimental studies have introduced asymmetries into the BF framework. Diermeier and Morton (2005) and Fréchette et al. (2005a) consider the case in which some players are more likely to be selected as proposers. Other papers consider asymmetric voting rules in which some players are favored, either by being given veto power (Kagel et al., 2010) or by having more votes than other players (Fréchette et al., 2005c, and Drouvelis et al., 2010). In some cases, having more votes than other players does not theoretically confer any objective advantage, and subject behavior broadly confirms this (Diermeier and Morton, 2005, Fréchette et al., 2005b, and Drouvelis et al., 2010). Diermeier and Gailmard (2006) study a 1-round version of the game in which failure to agree results in exogenously given payoffs that differ between players. This is a particular case of our model presented below with $\delta = 0$ (i.e. certain breakdown). Some of their findings are similar to ours, in particular, the player with the highest disagreement value is often excluded.

3 Model

Let $N = \{1, 2, ..., n\}$ be the set of players and q be the number of votes needed to pass a proposal, where $q \leq n$. Within each round, Nature selects a proposer randomly, with each player having a $\frac{1}{n}$ probability of being selected. The proposer can propose any vector $x = (x_1, ..., x_n)$, provided that $x_i \geq 0$ for all i and $\sum_{i \in N} x_i \leq 1$. All players vote on the proposal.[5] If there are at least q votes in favor, the proposal passes. Otherwise, there is a probability $0 \leq \delta < 1$ that bargaining moves to the next round, and a probability $(1 - \delta)$ that breakdown occurs, in which case payoffs are given by a vector $r = (r_1, ..., r_n)$ of disagreement values. Players are labeled so

[5] We assume that all players vote as if they are pivotal. This assumption may be replaced by the assumption that voters vote sequentially.

that $0 \leq r_1 \leq r_2 \leq ... \leq r_n$. The game continues until either a proposal is accepted, or breakdown occurs. Note that under all decision rules, agreement is efficient if and only if $\sum_{i \in N} r_i \leq 1$.

3.1 Equilibrium concept

A strategy in this game specifies a) what proposal a player makes when recognized as proposer, and b) how he would vote on any proposal made by other players. In principle, these actions could depend upon the history of play. For example, the proposal that a player makes in a given round may depend on prior proposals or voting decisions. Following the prior literature on legislative bargaining, we will exclude such behavior and focus on equilibria in which players use stationary strategies. Such strategies require that players make the same (possibly random) proposal in each round, and vote the same way on others' proposals. A subgame perfect equilibrium in which players use stationary strategies is called a stationary subgame perfect equilibrium (SSPE).

Following common practice, we will refer to a player's expected utility given that a proposal has (just) been rejected as the player's *continuation value*. For *any* profile of stationary strategies (equilibrium or not), there is an associated vector of continuation values, which are the same in all rounds. Also, for any profile of stationary strategies, there is an associated vector of *expected payoffs* computed at the beginning of the game, before Nature selects a proposer. Given a profile of stationary strategies σ, we will denote player i's continuation value as $z_i(\sigma)$ and player i's expected payoff as $y_i(\sigma)$; we will drop σ from the notation if no confusion arises.[6]

In a SSPE, continuation values act as prices: it is optimal for player i to vote in favor of any proposal with $x_i \geq z_i$, and to vote against otherwise. As a proposer, player i looks for the $q-1$ players with the lowest z_j and sets $x_j = z_j$ for these players, keeping the remainder. If the remainder is below z_i, it is better for player i to make a proposal that will be rejected and get z_i.[7] We can then distinguish between equilibria with *no delay* or

[6]A player's continuation value differs from his expected payoff for two reasons. First, future payoffs may be discounted. Second, breakdown may occur before another round of bargaining begins.

[7]In case of indifference, we assume that players break ties in favor of agreement, that is, responders vote in favor of proposals with $x_i = z_i$ and proposers offer $q-1$ players their continuation value if the remaining payoff $x_i \geq z_i$. This assumption simplifies the

immediate agreement (in which all players prefer to make a proposal that will be accepted) and equilibria with *delay* (in which at least one player prefers to make a proposal that will be rejected, and hence disagreement occurs with positive probability).

3.2 Example

Before presenting a general analysis, we will illustrate our main results using a simple example. Assume that the group consists of $n = 3$ players, with disagreement values $r = (0, 0, \frac{1}{2})$. Player 3, having the largest disagreement value, can be thought of as 'tougher' than the other players. Let the continuation probability be given by $\delta = 2/3$. Thus, in case a proposal fails, the game will end without agreement with probability $\frac{1}{3}$.

Suppose first that the group makes decisions using *unanimity rule.* Then, there exists a stationary subgame perfect equilibrium in which the continuation values are $z = \left(\frac{2}{18}, \frac{2}{18}, \frac{11}{18}\right)$. The strategies are as follows. If, say, player 1 is selected as a proposer, he offers $\frac{2}{18}$ to player 2 and $\frac{11}{18}$ to player 3, keeping $1 - \frac{2}{18} - \frac{11}{18} = \frac{5}{18}$. Similarly, player 3 would offer $\frac{2}{18}$ to each of the other two players and keep $\frac{14}{18}$. Given these strategies, expected payoffs are $y_1 = y_2 = \frac{1}{3}\frac{5}{18} + \frac{2}{3}\frac{2}{18} = \frac{3}{18}$ and $y_3 = \frac{1}{3}\frac{14}{18} + \frac{2}{3}\frac{11}{18} = \frac{12}{18}$. We can check that the z values we have provided are indeed the continuation values for the players. Given the strategies, $z_1 = z_2 = \frac{2}{3}\frac{3}{18} + \frac{1}{3}0 = \frac{2}{18}$ and $z_3 = \frac{2}{3}\frac{12}{18} + \frac{1}{3}\frac{1}{2} = \frac{11}{18}$.

Under unanimity rule, we see that the players' expected payoffs are increasing in disagreement values. Mr. 3, being the 'toughest' player, is more expensive to 'buy' into a coalition. Since all players have to be 'bought', he is paid more for his vote than the other two players. Under unanimity rule, it is good to be the 'toughest' player (to have a large disagreement value).

Next, suppose that the group decides using *majority rule.* In this case, there exists a stationary subgame perfect equilibrium in which the continuation values are $z = (\frac{1}{4}, \frac{1}{4}, \frac{1}{3})$. As in the case of unanimity rule, these continuation values are increasing in the disagreement values. Again, Mr. 3's vote is 'more expensive' than that of the other two players. However, in the case of majority rule, this causes the other players to exclude Mr. 3 from the coalition when they propose. If player 1 or 2 proposes, 3 is excluded and the other player is offered $\frac{1}{4}$. If player 3 proposes, he offers one of the other players $\frac{1}{4}$ (with equal probability). Expected payoffs are then

analysis and makes little difference to equilibria; see Appendix A.1.

$y_1 = y_2 = \frac{1}{3}[1 - \frac{1}{4}] + \frac{1}{3}\frac{1}{4} + \frac{1}{3}\frac{1}{2}\frac{1}{4} = \frac{3}{8}$; $y_3 = \frac{1}{3}\left[1 - \frac{1}{4}\right] = \frac{1}{4}$. Again, it can be easily checked that these expected payoffs are consistent with the conjectured continuation values.

Under majority rule, the players' expected payoffs are not increasing in their disagreement values. Player 3's continuation value is larger, making his vote more expensive. This is why he is excluded whenever others propose. His expected payoff under majority rule is smaller than that of the other players. Under majority rule, being 'tough' (having a large disagreement value) can be a disadvantage.

To summarize, the example demonstrates a set of patterns that turn out to be more general. Under all decision rules, *continuation values* z are increasing in disagreement values. Under unanimity rule, the same is true for *expected payoffs* y. Under all less-than-unanimity rules, by contrast, expected payoffs are either decreasing or non-monotone in disagreement values, and the player with the largest disagreement value never achieves the greatest payoff.[8] The following subsection establishes that all equilibria without delay satisfy these properties. Equilibria with delay are discussed in Appendix A.6.

3.3 Equilibria with no delay

Recall that a no-delay equilibrium is one in which all players make proposals that pass. In order to characterize the properties of such equilibria, as well as the conditions under which they occur, it will be useful to consider how the payoffs y_i and continuation values z_i are related to one another.

When a proposal passes, we refer to the players who vote in favor as the *coalition* that forms, and to players in the coalition other than the proposer as the *coalition partners*. In any no-delay equilibrium, coalition partners receive z_i, and the proposer receives $1 - \sum_{j \in T \setminus \{i\}} z_j$, where T is (one of) the 'cheapest' coalition(s) that includes i.[9] All other players receive 0.

As we show in Appendix A.2, continuation values are ordered like the disagreement values, i.e.,

$$z_1 \leq z_2 \leq \ldots \leq z_n.$$

[8] Another general pattern is that a player's payoff is larger if he proposes than when he is a coalition partner.

[9] If the cheapest coalition is not unique, player i can mix. By definition, the total 'price' of all cheapest coalitions is the same.

It follows that (one of) the cheapest coalition(s) a player can buy consist of himself plus the 'first' $(q-1)$ other players (those with the smallest continuation values z_j). Let $Z_q = \sum_{j=1}^{q} z_q$. This would be the minimum 'price' to pay for the first q votes. However, player i must buy only $(q-1)$ votes. If his own continuation value is no larger than z_q, he must pay $Z_q - z_i$ for his cheapest coalition(s) (he need not pay himself). If his own continuation value is strictly larger than z_q, he must pay $Z_q - z_q$ (he need not buy player q). It follows that player i's payoff as a proposer is $(1 - Z_q + \min\{z_i, z_q\})$.

We denote the ex ante probability of player i being in the coalition that forms by μ_i. Naturally, player i will be in the coalition whenever he is the proposer; this occurs with probability $\frac{1}{n}$. Then the equilibrium probability of i being a coalition *partner* is $\mu_i - \frac{1}{n}$. As a coalition partner, player i's payoff is given by z_i.

From what we have said so far, it follows that the expected equilibrium payoff for player i, denoted y_i, is related to the continuation values and inclusion probabilities as follows.

$$y_i = \frac{1}{n}\left(1 - Z_q + \min\{z_i, z_q\}\right) + \left(\mu_i - \frac{1}{n}\right) z_i$$

In case a proposal fails, bargaining continues with probability δ, in which case players' expected payoffs are again given by y_i. Bargaining will break down with probability $1-\delta$, in which case players receive their disagreement value r_i. Therefore, player i's continuation value is given by

$$z_i = \delta y_i + (1-\delta) r_i.$$

Combining these equations yields n equations relating the vector of continuation values $z = (z_1, ..., z_n)$ to the vector of inclusion probabilities $\mu = (\mu_1, ..., \mu_n)$. Specifically,

$$z_i = \begin{cases} \frac{1}{1-\delta\mu_i}\left((1-\delta)r_i + \frac{\delta}{n}(1 - Z_q)\right) & z_i \le z_q \\ \frac{1}{1-\delta\left(\mu_i - \frac{1}{n}\right)}\left((1-\delta)r_i + \frac{\delta}{n}(1 - Z_q + z_q)\right) & z_i > z_q \end{cases} \quad i = 1, ..., n$$

In equilibrium, the inclusion probabilities μ must reflect the fact that proposers will buy the cheapest available coalition given z. An equilibrium can therefore be constructed as follows. (Details are presented in Appendix A.3.)

Begin by conjecturing that $L \in \{0, 1, ..., q-1\}$ players are strictly cheaper than player q. For these players, we must have $\mu_i = 1$ as they are included in all coalitions. Then for these players

$$z_i = \frac{1}{1-\delta}\left((1-\delta)r_i + \frac{\delta}{n}(1-Z_q)\right) \quad i = 1, ..., L$$

Also assume that $H \in \{0, 1, ..., n-q\}$ players are strictly more expensive than player q. For these players, we must have $\mu_i = \frac{1}{n}$ as they are included only in their own coalitions. Therefore

$$z_i = (1-\delta)r_i + \frac{\delta}{n}(1-Z_q+z_q) \quad i = n-H+1, ..., n$$

The remaining $M = n-L-H \geq 1$ players are exactly as expensive as player q and so we have

$$z_q = \frac{1}{(1-\delta\mu_i)}\left((1-\delta)r_i + \frac{\delta}{n}(1-Z_q)\right) \quad i = L+1, ..., n-H$$

Although the individual inclusion probabilities for these players are not immediately determined, it's clear that the *average* inclusion probability for these players must be exactly large enough to achieve an expected coalition size of q. Specifically, it must equal $\bar{\mu}_M = \frac{q-L-\frac{H}{n}}{n-H-L}$. Therefore the corresponding equations can be combined to yield

$$z_q = \frac{1}{(1-\delta\bar{\mu}_M)}\left((1-\delta)\bar{r}_M + \frac{\delta}{n}(1-Z_q)\right),$$

where $\bar{r}_M = \frac{1}{n-H-L}\sum_{i=L+1}^{n-H} r_i$ is the average disagreement value among those players for whom $z_i = z_q$.

Suppose for example that $M = n$. Then, $\bar{\mu}_M = \frac{q}{n}$ and $Z_q = qz_q$. The last equation then reduces to $z_q = (1-\delta)\bar{r}_N + \frac{\delta}{n}$. This makes sense, since assuming immediate agreement, $\sum_{i \in N} z_i = \delta + (1-\delta)\sum_{i \in N} r_i$. If $M = n$, all players have the same continuation value, which must then equal the previous expression divided by n.[10]

In similar fashion, the equations derived above can be solved explicitly for any conjecture regarding the numbers L, M, and H. Subsequently, the

[10]As mentioned in Section 2, this is the special case of our model considered by Banks and Duggan (2006). We provide a more complete characterization.

resulting vector of continuation values can be inspected to verify that the first L players are strictly cheaper than player q, etc. If so, the combination of continuation values and implied inclusion probabilities constitute a SSPE of the model. We show in the Appendix that each conjecture regarding L, M, and H indeed uniquely determines the continuation values and inclusion probabilities (proposition 3). Furthermore, there is only one combination of L, M, and H that leads to an equilibrium (proposition 5). Hence, all no-delay SSPE have the same payoffs. Based on our analysis, we are able to construct equilibria for all constellations of the parameters r_i and δ. In what follows, we concentrate on stating our main results.

3.4 Unanimity rule ($q = n$)

Under unanimity rule, immediate agreement occurs in equilibrium if and only if it is efficient, and both continuation values and expected payoffs are increasing in disagreement values.

Proposition 1. *For $q = n$, immediate agreement occurs if and only if $\sum_{i \in N} r_i \leq 1$. In this case, the SSPE is unique.*[11] *Expected equilibrium payoffs are given by*

$$y_i = \frac{1}{n}\left(1 - \sum_{i \in N} r_i\right) + r_i,$$

and continuation values are given by

$$z_i = \frac{\delta}{n}\left(1 - \sum_{i \in N} r_i\right) + r_i.$$

If $\sum_{i \in N} r_i > 1$, disagreement occurs with probability 1 and $z_i = y_i = r_i$.

Proof. See Appendix A.4. □

Note that the expected payoffs are independent of δ, such that the profile of disagreement payoffs $(r_i)_{i \in N}$ remains relevant even if the continuation probability becomes arbitrarily close to 1. For all δ, players effectively share the surplus from agreement equally. This result is analogous to the results for two-player bargaining with breakdown probability. Most importantly, note that expected equilibrium payoffs are increasing in disagreement values.

[11] For the case $\sum_{i \in N} r_i = 1$, the tie-breaking rule is used to select a unique equilibrium.

Although expected payoffs do not depend of δ, the outcome of the game, conditional on the identity of the proposer, does. This can be seen by comparing the expressions for y_i and z_i. Since $z_i < y_i$, it follows that the payoff conditional on being a responder is lower than that conditional on being proposer. This difference, which indicates a proposer advantage, is decreasing in δ and vanishes in the limit when δ approaches 1.

3.5 Less-than-unanimity rules ($q < n$)

For less-than-unanimity rules, we can establish a sufficient condition for immediate agreement. Under unanimity rule, immediate agreement occurred if it was potentially Pareto improving. Under majority rule, an analogous condition holds that takes into account that only q players need to agree in order for a proposal to be implemented: if $\sum_{i=1}^{q-1} r_i + r_n < 1$, each player can find a coalition of q players to which they belong for which $\sum_{i \in S} r_j < 1$, hence each proposer can find a coalition for which agreement is potentially Pareto improving, and all equilibria involve immediate agreement in this case.

We also show that, even though SSPE with immediate agreement may differ in the strategies that are played, they all lead to the same expected payoffs and continuation values for the players.

Finally, we establish a crucial difference between unanimity and less-than-unanimity decision rules: even though continuation values are ranked in the same way as disagreement values, expected equilibrium payoffs are not. Expected equilibrium payoffs follow one of two basic patterns: they are either decreasing in disagreement values, or first increasing and then decreasing in disagreement values (in both cases, there may be an additional flat region after the decreasing part). The player with the greatest disagreement value never gets the highest expected payoff.

Proposition 2. *Let $q < n$.*

(i). If $\sum_{i=1}^{q-1} r_i + r_n < 1$, then all SSPE exhibit immediate agreement.

(ii). Continuation values and expected equilibrium payoffs in a no-delay equilibrium are uniquely determined.

(iii). Continuation values are weakly increasing in the r-values.

(iv). Inclusion probabilities are weakly decreasing in the r-values.

(v). The player with the greatest disagreement value never gets the highest expected payoff.

Proof. See Appendix.

It follows from propositions 1 and 2 that efficiency of agreement (i.e., $\sum_{j\in N} r_j < 1$) is a sufficient condition for immediate agreement under all decision rules. However, under less-than-unanimity rules, agreement may occur even though it is inefficient.

4 Experimental Design

4.1 Experimental Procedures

We conducted several games involving 3 players with a divisible amount of 100 'tokens'. The continuation probability was $\delta = \frac{2}{3}$. This was implemented using a virtual die roll, with the game continuing until the number rolled exceeded four. Prior to a game, each subject was randomly assigned a disagreement value from the set $\{0, 20, 40, 60\}$. Instructions referred to "default tokens" which the subject would receive unless the group agreed.

Within each game, the sequence of events was as follows. First, each subject was randomly assigned a letter i.d. ('A','B','C') which remained fixed throughout the game. Next, disagreement values were assigned. Each subject was informed about the disagreement values for all members of the three-person group. These also remained fixed throughout the game. At the beginning of a given round of the game, one subject was randomly chosen to make a proposal. All subjects were immediately informed of the proposer's i.d., and the chosen subject was prompted to enter a proposal, consisting of three positive integers which sum to at most 100 tokens. After the proposer clicked 'ok', this proposal was displayed to all members of the group, who were then prompted to vote either 'yes' or 'no'. Following this, detailed results of the vote were displayed to all subjects. This included the proposal made, the i.d. of the proposer, and the individual votes cast by each player ('A','B', 'C'). If the proposal failed, the results screen also informed subjects of the outcome of a die roll to determine whether the game would continue. If so, a new round began. In games that continued beyond round 1, a detailed history table showed the proposer i.d., the proposal made, as well as the individual votes, for all prior rounds of the game.

Each participant played either 10 or 15 games, with random rematching of subjects between games.[12] At the end of the experiment, one game was

[12]Following these games, subjects played an additional 10 or 15 games in which disagreement values were private information. Results from these games will be discussed in

randomly chosen to be paid. The exchange rate was 1 token = 0.25 EUR. Subjects also received a 3 EUR show-up fee. The experiment was conducted at the University of the Basque Country in Bilbao. Participants were undergraduate students of economics and business.

We conducted four sessions, two involving majority and the other two unanimity rule.[13] Sessions involved between 18 and 36 subjects divided into matching groups of size 6. We have a total of 19 matching groups, 10 in the unanimity treatment and 9 under majority. It follows that the raw numbers of observations from our experiment break down as shown in Table 1.

TABLE 1. TREATMENTS, SUBJECTS AND OBSERVATIONS

	Short sessions (10 games)	Long sessions (15 games)
majority rule	6 matching groups 36 subjects 120 proposals 360 voting decisions	3 matching groups 18 subjects 90 proposals 270 voting decisions
unanimity rule	6 matching groups 36 subjects 120 proposals 360 voting decisions	4 matching groups 24 subjects 120 proposals 360 voting decisions

4.2 Hypotheses

As indicated, our experimental games involve $n = 3$ subjects and two possible values for q: $q = 3$ (unanimity rule) and $q = 2$ (majority rule). The randomly assigned disagreement values took on four possible values, $r_i \in \{0, 0.2, 0.4, 0.6\}$. Finally, the continuation probability was $\delta = \frac{2}{3}$ in all cases. Not all possible combinations of disagreement values occurred in our experiment. For all combinations that did occur, Table A1 in the Appendix presents the equilibrium values for z_i (player i's continuation value and the 'price' for his vote), y_i (player i's expected equilibrium payoff) and μ_i (the probability of being in the final coalition *as a proposer or as a coalition partner*).

Our aim in the experiment will not be to test the precise point predictions presented in Table A1. Instead we will be interested in the following quali-

a follow-up paper.

[13]In each treatment condition, one session consisted of 10 games and one session consisted of 15 games

tative predictions concerning the relationship between disagreement values, probabilities of being in the coalition, offers, and expected payoffs. Moreover, we will be especially interested in comparing these relationships under majority vs. unanimity rule.

For all hypotheses, it should be noted that the predictions concerning expected equilibrium payoffs are based on the assumption that all player types are chosen as proposers exactly $\frac{1}{3}$ of the time. The appropriate way to test these hypotheses is therefore to first calculate average payoffs each type achieves *within* each role (proposer or responder), and then to calculate the 'expected' payoffs by calculating the weighted sum of those values. (Here, 'within each role' might best be operationalized as 'given the role the player is assigned at the beginning of the game', i.e., if I am proposer in round 1, the theory says I should get a certain amount. If my game actually goes on to round 3 where I am responding, it is still valid to say that for this game the relevant prediction was what I would get as proposer. An alternative way of testing would be looking at the role at the point in time when agreement is reached.)

Hypothesis 1. When all players have the same disagreement value, offers made to coalition partners do not differ between majority and unanimity rule.[14]

Hypothesis 2. Under unanimity rule: When disagreement values differ, the player with the largest disagreement value (a) exhibits a larger acceptance threshold, (b) is offered more, and (c) achieves a larger average payoff than the other player types *both as proposer and as coalition partner*.

Hypothesis 3. Under majority rule: When disagreement values differ, the player with the largest disagreement value (a) exhibits a larger acceptance threshold, (b) is less often included in others' coalitions, (b') is not offered more than others when included, and (c) achieves a smaller average payoff when he is responder than do the other player types. As proposer, he achieves at least as much as other types.

Hypotheses 1-3 are valid for all situations in which the sum of the disagreement values is less than 100 tokens, such that agreement is efficient. Our final hypothesis concerns the only situation that occurred in our experiment in which agreement is actually inefficient.

[14] In particular, agreement is reached immediately under both rules and therefore the average payoff is $\frac{100}{3}$ tokens. If all disagreement values are zero, coalition partners must be offered $\frac{2}{3}$ of this value, i.e. $\frac{200}{9}$. Since only integer amounts can be offered, proposers would have to offer others 23 tokens.

Hypothesis 4. When agreement is *inefficient*, no agreement occurs under unanimity rule. Under majority rule, agreement always occurs if $r_1 + r_3 < 100$.

5 Results

5.1 Proposals

Table 2 summarizes the average offers made under unanimity rule, for each of the constellations of r_i values that occurred in our experiment. Looking first at the symmetric situations, we find no difference in the amounts offered when $r = (0, 0, 0)$ vs. $r = (20, 20, 20)$. Contrary to the theoretical prediction, offers are not larger in the latter case.

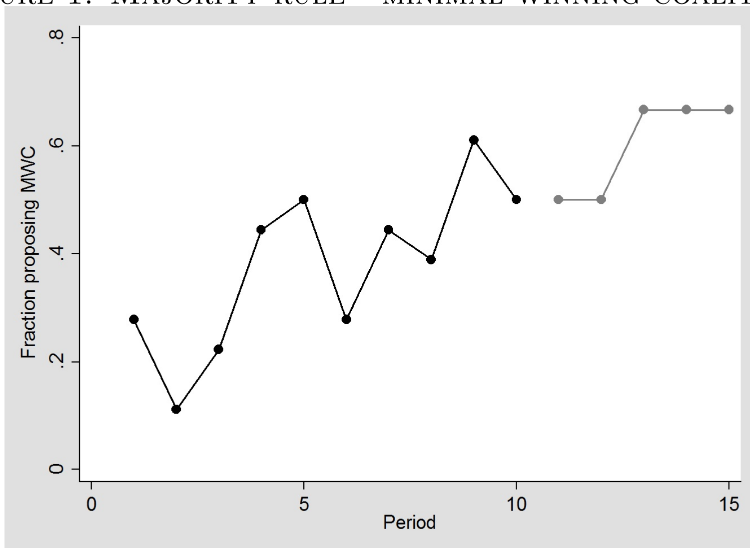
Turning to the asymmetric situations, we find that the player with the largest disagreement value is consistently offered more, consistent with our main theoretical prediction.[15] However, contrary to our prediction, we find that players with $r_i > 20$ are offered *less* than r_i on average. Given that all players must agree for a proposal to pass, this is perhaps surprising and lets us anticipate that many of these proposals are likely to fail. Generally, actual offers are less sensitive to disagreement values than predicted offers.[16]

Result 1: *Under unanimity rule, the player with the largest disagreement value is consistently offered more.*

[15]We use the Mann-Whitney U-test to establish that the amount offered to the player with the largest disagreement value is statistically different from the amount offered to the other players. To control for the dependency of observations, we use matching group level data. Focusing on average positive offers: $Z = -2.065$, $p = 0.039$, for $r = (0, 0, 20)$; $Z = -3.628$, $p < 0.001$, for $r = (0, 0, 40)$; $Z = -3.682$, $p < 0.001$, for $r = (0, 0, 60)$; $Z = -3.780$, $p < 0.001$, for $r = (0, 20, 60)$; $Z = -2.309$, $p = 0.021$, for $r = (20, 20, 40)$; $Z = -2.309$, $p < 0.001$, for $r = (20, 40, 60)$.

[16]This echoes the results of Anbarci and Feltovich (2013) for two-player bargaining with a deadline.

Figure 1. Majority rule - minimal winning coalitions



Note: For the first ten periods, we pool the data from the two majority sessions (black line). For periods 11-15, data come from a single session (grey line).

Table 2. Average offers - Unanimity

Disagreement values (r_1, r_2, r_3)	Predicted offers (z_1, z_2, z_3)	Average offers $(\widehat{z}_1, \widehat{z}_2, \widehat{z}_3)$
$(0, 0, 0)$	$(23, 23, 23)$	$(30, 30, 30)$
$(20, 20, 20)$	$(29, 29, 29)$	$(31, 31, 31)$
$(0, 0, 20)$	$(18, 18, 38)$	$(28, 28, 32)$
$(0, 0, 40)$	$(13, 13, 53)$	$(25, 25, 37)$
$(0, 0, 60)$	$(9, 9, 69)$	$(21, 21, 49)$
$(0, 20, 60)$	$(4, 24, 64)$	$(17, 26, 44)$
$(20, 20, 40)$	$(24, 24, 44)$	$(25, 25, 38)$
$(20, 40, 60)$	$(20, 40, 60)$	$(22, 28, 37)$

Next, consider proposals being made under majority rule. Theoretically, we predict that proposers build minimal winning coalitions. Figure 1 displays the fraction of proposals that allocate zero to one of the responders. This proportion increases over time.

Table 3 summarizes information on proposals conditional on the responder types. Minimal winning coalitions are more frequently proposed when the responders have different disagreement values. In these cases, the player with the largest disagreement value is less often included. However, this result is only statistically significant when $r_i = 60$.[17] If we focus on inclusion frequencies in MWCs, we find that players with the largest disagreement value are included in less that one third of the coalitions when $r_i = 40$ and almost never included when $r_i = 60$. When included, players with high r_i are not necessarily offered more.[18]

Result 2: *Under majority rule, the player with the largest disagreement value is not offered more as a coalition partner.*

Table 3. Proposals given responders' r values (majority rule)

$\{r_1, r_2\}$	Frequency of MWC	Inclusion frequencies p_1	p_{1MWC}	p_2	p_{2MWC}	Average *positive* offers Offer 1	Offer 2
{0,0}	0.28	0.87	0.5	0.87	0.5	26	26
{20,20}	0.39	0.81	0.5	0.81	0.5	30	30
{0,20}	0.58	0.58	0.36	0.74	0.64	27	33
{0,40}	0.48	0.83	0.64	0.69	0.36	32	32
{0,60}	0.46	0.91	0.87	0.57	0.13	33	26
{20,40}	0.67	0.83	0.75	0.50	0.25	32	43
{20,60}	0.39	0.94	0.86	0.67	0.14	35	29

Similar information is summarized in Table 4, where we compare predicted and observed offers in the various situations. In symmetric situations, there is no difference between $r = (0, 0, 0)$ and $r = (20, 20, 20)$, contrary to the theoretical prediction. For all asymmetric and efficient situations taken together, we find that Player 3 (with the largest disagreement value) (a) is

[17]Mann-Whitney U-test, using matching group level inclusion frequencies: $Z = -0.442$, $p = 0.658$, for $\{r_1, r_2\} = \{0, 20\}$; $Z = -0.728$, $p = 0.467$, for $\{r_1, r_2\} = \{0, 40\}$; $Z = 2.743$, $p = 0.006$, for $\{r_1, r_2\} = \{0, 60\}$; $Z = -0.943$, $p = 0.346$, for $\{r_1, r_2\} = \{20, 40\}$; $Z = 1.626$, $p = 0.104$, for $\{r_1, r_2\} = \{20, 60\}$.

[18]Mann-Whitney U-test, using matching group level average positive offers: $Z = -0.960$, $p = 0.337$, for $\{r_1, r_2\} = \{0, 20\}$; $Z = -0.354$, $p = 0.724$, for $\{r_1, r_2\} = \{0, 40\}$; $Z = 1.815$, $p = 0.069$, for $\{r_1, r_2\} = \{0, 60\}$; $Z = -1.155$, $p = 0.248$, for $\{r_1, r_2\} = \{20, 40\}$; $Z = 1.309$, $p = 0.191$, for $\{r_1, r_2\} = \{20, 60\}$.

not less often included ($Z = 1.281$, $p = 0.200$).[19] However, when $r_i > 20$, Player 3 is marginally less often included ($Z = 1.643$, $p = 0.100$), and when $r_i = 60$, Player 3 is less often included ($Z = 2.219$, $p = 0.026$). This lends partial support to our predictions. We also find that Player 3 (b) is not offered more than others when included ($Z = -0.132$, $p = 0.895$) and (c) is offered less than he is under unanimity rule ($Z = -2.776$, $p = 0.005$).

We now compare average offers under unanimity and majority rule. Consistent with hypothesis 1, when all players have the same disagreement values, offers made to coalition partners do not differ between unanimity and majority rule ($Z = -1.022$, $p = 0.307$). Looking at all asymmetric and efficient situations together, we find that players 1 and 2 (who have smaller r_i) are offered more, on average, than they are under unanimity rule ($Z = 1.796$, $p = 0.072$). These results are consistent with the theoretical predictions. The only deviation from the theory is that the player with the largest disagreement value is included too often when his disagreement value is small ($r_i = 20$).

Table 4. Average positive offers and inclusion frequencies - Majority

Disagreement values (r_1, r_2, r_3)	Predicted offers	Average positive offers received	Predicted inclusion frequencies	Inclusion frequencies
$(0, 0, 0)$	$(23, 23, 23)$	$(28, 28, 28)$	(50,50,50)	$(87, 87, 87)$
$(20, 20, 20)$	$(29, 29, 29)$	$(30, 30, 30)$	(50,50,50)	$(81, 81, 81)$
$(0, 0, 20)$	$(24, 24, 24)$	$(28, 28, 34)$	(70,70,09)	$(67, 67, 78)$
$(0, 0, 40)$	$(25, 25, -)$	$(26, 26, 32)$	(75,75,0)	$(85, 85, 69)$
$(0, 0, 60)$	$(25, 25, -)$	$(35, 35, 28)$	(75,75,0)	$(89, 89, 48)$
$(0, 20, 60)$	$(28, 29, -)$	$(25, 34, 27)$	(100,50,0)	$(75, 86, 69)$
$(20, 20, 40)$	$(31, 31, 31)$	$(32, 32, 43)$	(66,66,18)	$(83, 83, 50)$
$(20, 40, 60)$	$(36, 36, 36)$	$(26, 29, 36)$	(94,52,4)	$(44, 92, 71)$

Result 3: *Under majority rule, the player with the largest disagreement value is less often included in others' coalitions when r_i is greater than the equal split.*

[19] This result is driven by the fact that the player with the largest disagreement value is as likely to be included as the other players when $r_i = 20$ ($Z = -0.816$, $p = 0.414$). Interestingly, this is the only case where this player disagreement value is below the equal split.

5.2 Individual voting and aggregate rate of passage

Regarding voting behavior, recall that the predicted acceptance thresholds for each combination of values that occurred in our experiment can be found in Table A.1 in the Appendix. It follows from this table that players with the largest disagreement value should be more 'demanding' under unanimity rule than under majority rule, meaning that they must be offered more in order to vote yes. The reverse should be true for players with smaller r_i. To test these predictions, we would ideally want to observe players' *acceptance thresholds*. Unfortunately, these are not observed in our experiment.[20] As an alternative approach, we estimate regression models in which the dependent variable is the decision to vote yes. Results are reported in Table 5.

Controlling for the kind of offer being considered, we find that the player with the largest disagreement value is less likely to vote yes under unanimity rule than under majority rule. This lends support to the idea that he is more 'demanding' under unanimity rule. However, we do not find the opposite effect for those with smaller disagreement values - contrary to our theoretical prediction.[21]

Result 4: *The player with the largest disagreement value is less likely to vote yes under unanimity rule than under majority rule.*

[20] An alternative design would have asked subjects to state an acceptance threshold. A disadvantage of this approach is that we would have *forced* subjects to use a cutoff strategy that depends only on their own payoff.

[21] We may add the following observations regarding voting behavior. First, players more often vote yes the more they are offered, and the less the proposer keeps for himself. Second, the probability of voting yes is decreasing in r_i. Finally, having the largest r_i value does not in itself affect the probability of voting yes, except under unanimity rule.

Table 5. Probability of voting yes

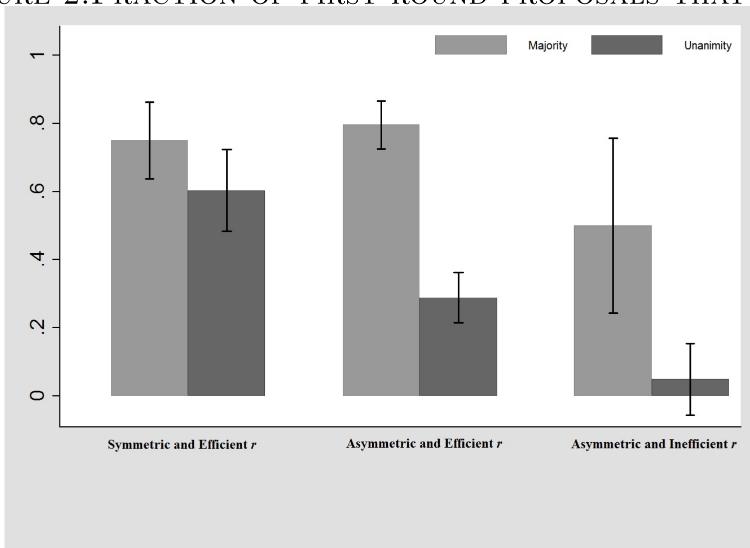
Random-effects probit regression - marginal effects					
Dependent variable: "Voting yes" (asymmetric and efficient situations)					
	All r_i	All r_i	All r_i	Low & Medium r_i	Highest r_i
Unanimity	-0.237*	-0.338**	-0.304**	0.0231	-0.871***
	(0.130)	(0.151)	(0.142)	(0.219)	(0.336)
Own share	0.028***	0.051***	0.046***	0.080***	0.048***
	(0.005)	(0.007)	(0.007)	(0.013)	(0.014)
Proposer's share	-0.006	-0.011***	-0.011***	-0.012**	-0.013
	(0.004)	(0.004)	(0.004)	(0.005)	(0.012)
$r_i = 20$		-0.378**		-0.620**	1.407***
		(0.183)		(0.242)	(0.524)
$r_i = 40$		-0.952***			0.856***
		(0.211)			(0.303)
$r_i = 60$		-1.726***			
		(0.222)			
Highest			-1.138***		
			(0.159)		
Period	-0.015	-0.017	-0.020	0.005	-0.041
	(0.015)	(0.017)	(0.016)	(0.023)	(0.034)
Observations	498	498	498	339	159
Number of id	113	113	113	109	94

Note: The unit of analysis is individual acceptance behavior; marginal effects from random-effect probit regressions presented; standard errors in parentheses.
***, **, and * indicate statistical significance at the 1%, 5%, and 10% level.

Next consider the rate of passage. Focusing on the very first round of bargaining, Figure 2 shows the fraction of proposals that pass immediately. We observe a significantly lower rate of passage under unanimity rule ($Z = 1.726$, $p = 0.084$, for symmetric r; $Z = 3.681$, $p < 0.001$, for asymmetric and efficient r; $Z = 2.291$, $p = 0.004$, for inefficient r).

When agreement is inefficient, all first-round proposals should be rejected under unanimity rule; we observed only one agreement (out of 20 groups) in this case. There are many inefficient agreements under majority rule, though their frequency is far from the theoretically predicted 100%.

FIGURE 2. FRACTION OF FIRST ROUND PROPOSALS THAT PASS



5.3 Realized payoffs

Finally, let us look at the payoffs realized within the experiment. We present predicted and realized payoffs separately for proposer and responder roles in symmetric (Table 6) and asymmetric (Table 7) situations. For instance, in row 1 of table 6, the 11.5 comes from the fact that, conditional on being a responder, a subject is included in a MWC with probability 1/2, in which case he is paid 23. So a responder's expected payoff is 11.5.

For symmetric situations and in line with our theoretical predictions, we find that the average realized payoff is smaller under majority rule than under unanimity rule for responders ($Z = -2.346$, $p = 0.019$) and larger for proposers ($Z = 4.086$, $p < 0.001$). We find no difference in payoffs between the two symmetric situations. This means that breakdown was rare in the experiment, as otherwise the payoffs in $(0, 0, 0)$ would have been significantly lower than in $(20, 20, 20)$.

Table 6. Average payoffs earned in the experiment - Symmetric situations

			Predicted			Realized		
RULE	ROLE	(r_1, r_2, r_3)	$predict_1$	$predict_2$	$predict_3$	u_1	u_2	u_3
majority	responder	(0,0,0)	11.5	11.5	11.5	23	23	23
		(20,20,20)	14.5	14.5	14.5	25	25	25
	proposer	(0,0,0)	77	77	77	44	44	44
		(20,20,20)	76	76	76	46	46	46
unanimity	responder	(0,0,0)	23	23	23	26	26	26
		(20,20,20)	29	29	29	31	31	31
	proposer	(0,0,0)	54	54	54	31	31	31
		(20,20,20)	42	42	42	32	32	32

For asymmetric situations, our main theoretical prediction is that the player with the largest disagreement value should achieve the largest payoff under unanimity rule (both as a proposer and as a responder), but the lowest payoff under majority rule (as a responder). The data are summarized in Table 7. Under unanimity rule, we find that the player with largest r_i earns more than the other players ($Z = -6.728$, $p < 0.001$, for responders; $Z = -5.689$, $p < 0.001$, for proposers). However, under majority rule, he does not consistently earn less ($Z = 0.827$, $p = 0.408$, for responders; $Z = -0.770$, $p = 0.441$, for proposers).

Result 5: *In asymmetric situations and under unanimity rule, the player with the largest disagreement value earns more than the other players.*

Result 6: *In asymmetric situations and under majority rule, the player with the largest disagreement value does not achieve a lower average payoff. A statistically significant disadvantage in terms of expected payoffs is found only if the highest disagreement value is very large compared to others.*

Table 7. Average payoffs earned in
the experiment - Asymmetric situations

			Predicted			Realized		
RULE	ROLE	(r_1, r_2, r_3)	$predict_1$	$predict_2$	$predict_3$	u_1	u_2	u_3
unanimity	responder	(0,0,20)	18	18	38	21	21	28
		(0,0,40)	13	13	53	20	20	37
		(0,0,60)	9	9	69	8	8	51
		(0,20,60)	4	24	64	7	24	56
		(20,20,40)	24	24	44	22	22	40
		(20,40,60)	20	40	60	20	40	58
	proposer	(0,0,20)	44	44	64	23	23	37
		(0,0,40)	34	34	74	25	25	47
		(0,0,60)	22	22	82	13	13	54
		(0,20,60)	12	32	62	10	21	56
		(20,20,40)	32	32	52	20	20	45
		(20,40,60)	20	40	60	20	40	60
majority	responder	(0,0,20)	17	17	2.2	18	18	30
		(0,0,40)	18.6	18.6	0	20	20	25
		(0,0,60)	18.6	18.6	0	34	34	16
		(0,20,60)	28	14.5	0	18	31	19
		(20,20,40)	20.5	20.5	5.5	27	27	22
		(20,40,60)	20.5	20.5	5.5	19	35	31
	proposer	(0,0,20)	76	76	76	51	51	47
		(0,0,40)	75	75	75	38	38	37
		(0,0,60)	75	75	75	45	45	20
		(0,20,60)	71	72	72	46	47	61
		(20,20,40)	69	69	69	52	52	-
		(20,40,60)	64	64	64	43	50	60

These patterns appear to be at most roughly consistent with our main theoretical prediction. A possible reason for the discrepancy is the fact that the theory predicts immediate agreement. In the experiment, a non-trivial number of groups failed to agree before negotiations broke down. When this occurs, it is clear that a larger disagreement payoff is an advantage. Substantively, however, this is not an advantage *within the negotiation.* Therefore it is perhaps interesting to separately consider the payoffs achieved only in those cases where agreement was actually reached. When we focus on those cases only, we find that the player with the largest payoff does consistently achieve the lowest payoff as a responder under majority rule ($p = 0.004$).

6 Conclusion

We study a legislative bargaining game in which failure to agree in a given round may result in a breakdown of negotiations. In that case, each player receives an exogenous 'disagreement value'. We characterize the set of stationary subgame perfect equilibria under all q-majority rules. Under unanimity rule, equilibrium payoffs are strictly increasing in disagreement values. Under all less-than-unanimity rules, expected payoffs are either decreasing or first increasing and then decreasing in disagreement values.

A different way to model a situation where some players have less to lose than others if agreement is delayed is to introduce heterogeneous discount factors. Intuitively, a player with a greater discount factor has less to lose from delay since the value of the pie is less heavily discounted; similarly, a player with a greater disagreement value has less to lose from delay since he receives a greater payoff in the event of a breakdown of negotiations. An important difference is that in the case of heterogeneous discount factors immediate agreement is always efficient, and no player can be harmed by an agreement compared to the situation of perpetual disagreement.

The two-player game with asymmetric discount factors (and unanimity) was solved by Rubinstein (1982) for the case of alternating offers; the random proposers case is very similar (Binmore, 1987). The Baron-Ferejohn model with possibly asymmetric discount factors and general q-voting rules has been studied in Eraslan (2002) and Kalandrakis (2015); both papers establish uniqueness of equilibrium payoffs using different methods. Continuation values are nondecreasing in discount factors (Eraslan, 2002); similarly, we find that continuation values are nondecreasing in disagreement values. For the case of unanimity rule, expected equilibrium payoffs are increasing in the discount factors (Eraslan, 2002). However, under majority rule, expected equilibrium payoffs are decreasing in discount factors provided that the discount factors are sufficiently high (Kawamori, 2005); this result is analogous to our result that, when the continuation probability is sufficiently close to 1, expected equilibrium payoffs are decreasing in disagreement values (Appendix A.6). We analyze expected equilibrium payoffs more fully in our setting, since we do not restrict ourselves to the limit case in which the continuation probability is sufficiently close to 1, and we are able to show that there are only two possibilities for expected equilibrium payoffs: they are either decreasing in disagreement values, or they are first increasing and then decreasing in disagreement values.

We conducted an experiment designed to investigate games involving 3 players, comparing majority and unanimity rule treatments. Our results lend qualitative support for several of our main predictions. Specifically, we find that the player with the largest disagreement value indeed achieves the largest payoff under unanimity rule. Under majority rule, however, that player is included in others' coalitions significantly less often. None the less, we do not find that this results in consistently lower average payoffs. A statistically significant disadvantage in terms of expected payoffs is found only if the highest disagreement value is very large compared to others.

Substantively, our results support the notion that 'being tough' (having a large disagreement value) may be advantageous under unanimity rule, but bad under majority rule. This, in turn, suggests that more inclusive decision rules may create incentives for players to *'act'* tough under unanimity rule. In ongoing theoretical work, we are studying a version of the game in which disagreement values are privately known. Our main conjecture is that, in such a game, players may attempt to *'signal'* a higher disagreement value by acting *'tough'* under unanimity rule, but not under majority rule.

A Appendix

A.1 Characterization of equilibrium

For reasons that will become clear, it is possible and convenient to formulate a set of necessary and sufficient conditions for an SSPE in terms of the continuation values associated with the equilibrium strategies.

Lemma 1. *Let $\sigma = (\sigma_1, ..., \sigma_n)$ be a combination of stationary strategies and $(z_1, ..., z_n)$ its associated vector of continuation values. The strategy combination σ is an SSPE if and only if the following conditions are satisfied:*

1. As a responder, player i votes 'yes' on any proposal with $x_i > z_i$ and 'no' on any proposal with $x_i < z_i$.

2. If $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j < 1$, the only proposals that player i makes with a positive probability as a proposer are such that $x_j = z_j$ for all $j \in T\backslash\{i\}$, $x_i = 1 - \sum_{j\in T\backslash\{i\}} z_j$ and $x_j = 0$ for all $j \in N\backslash T$, where $T \in \arg\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j$. These proposals are always accepted.

3. If $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j > 1$, player i always makes a proposal that would be rejected.

Proof. 1. This follows from subgame perfection and our assumption that players always vote as if they are pivotal.

2. Suppose $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j < 1$ and let $T \in \arg\min_{S\ni i,|S|=q} \sum_{j\in S} z_j$. Player i can propose $x_j = z_j + \epsilon$ for $j \in T\backslash\{i\}$, $x_j = 0$ for $j \in N\backslash T$ and $x_i = 1 - \sum_{j\in T\backslash\{i\}} z_j - (q-1)\epsilon$ for a sufficiently small $\epsilon > 0$. This proposal would be accepted and gives player i a payoff above z_i, which would be the payoff from making a proposal that would be rejected. Hence, player i will never make a proposal that would be rejected since there is a more favorable proposal that would be accepted. Take any of the proposals that player i makes with positive probability in equilibrium, and let Q be the set that votes in favor of this proposal. It must be the case that $x_j = z_j$ for all $j \in Q\backslash\{i\}$, since $x_j < z_j$ would lead to a rejection and $x_j > z_j$ could not be optimal since player i could always do better by reducing x_j while keeping the inequality $x_j > z_j$. Also, $Q \in \arg\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j$, since otherwise player i could do better by proposing coalition T and offering $z_j + \epsilon$ to each player in $T\backslash\{i\}$ for a sufficiently small ϵ.

3. If $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j > 1$, it is not possible to find a proposal that would give player i and $q-1$ other players at least their continuation value, hence it is optimal for player i to make a proposal that will be rejected. □

Note that the concept of SSPE imposes no restrictions on behavior in the knife-edge case $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j = 1$. Player i may or may not offer their continuation value to players in some $T \in \arg\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j$ and, even if player i does, the remaining players in T may or may not accept the proposal. However, such a situation however can only arise if disagreement values are very high. As we will see in section A.3 (lemma 3 and corollary 1), $\sum_{j=1}^{q-1} r_j + r_n < 1$ implies that $\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j < 1$ for all i.

Note also that the lemma does not constrain players to vote yes to *all* proposals with $x_i = z_i$. In principle they could vote no, but only in the aforementioned case $\arg\min_{S:S\ni i,|S|=q} \sum_{j\in S} z_j = 1$ or off the equilibrium path, i.e, as a response to a proposal that is never actually made.

In our discussion henceforth we will assume that both proposers and responders break ties in favor of agreement.[22]

A.2 Ranking of continuation values

The characterization of equilibrium above depends on the continuation values z_i; these values are endogenous. It will be useful to have results in terms of the exogenous disagreement values r_i. The following lemma shows that the z_i values are ranked in the same order as the r_i values, though as we will see some strict inequalities may become weak inequalities.

Lemma 2. *Let $r_i \leq r_j$. Then $z_i \leq z_j$ in any SSPE.*

Proof. Suppose $r_i \geq r_j$ but $z_j > z_i$.

If $\min_{S:S\ni j,|S|=q} \sum_{k\in S} z_k > 1$, player j never makes acceptable proposals. In this case nobody would make acceptable proposals involving player j and it is clear that $z_j \leq z_i$, since i gets at least the same as j in the event of disagreement and may get something in the event of agreement (while j is sure to get nothing).

Now consider the case $\min_{S:S\ni j,|S|=q} \sum_{k\in S} z_k \leq 1$, so that player j finds it profitable to make acceptable proposals (and so does player i). Delay is not completely ruled out in this case, since there may be players other than i and j who cannot find a profitable coalition. Denote by α the probability

[22] Eraslan and McLennan (2013, appendix A) formally show that there is little loss of generality in requiring responders to accept proposals when indifferent. Banks and Duggan (2006) incorporate this tie-breaking rule into their definition of equilibrium.

that one of those players is selected to make a proposal; $\alpha \geq 0$.[23]

Denoting player k's payoff conditional on being proposer as π_k, continuation values are given by

$$z_i = \frac{\delta}{n}\pi_i + \delta\left[\mu_i - \frac{1}{n} + \alpha\right] z_i + (1-\delta)r_i \quad (1)$$

$$z_j = \frac{\delta}{n}\pi_j + \delta\left[\mu_j - \frac{1}{n} + \alpha\right] z_j + (1-\delta)r_j \quad (2)$$

The inequality $z_j > z_i$ implies $\pi_i \leq \pi_j$, but the difference $\pi_j - \pi_i$ cannot exceed $z_j - z_i$. We also know that $\mu_i \geq \mu_j$. This is because players other than i and j would never include j in the coalition and exclude i. As for i and j themselves, suppose player j does not propose to player i with certainty. Then there must be a coalition T such that $j \in T$, $i \notin T$ that is optimal for player j, that is, $\sum_{k\in T} z_k \leq \sum_{k\in S} z_k$ for all $S \supseteq \{i,j\}$. But then $\sum_{k\in T\backslash\{j\}\cup\{i\}} z_k < \sum_{k\in S} z_k$ for all $S \supseteq \{i,j\}$, that is, player i would never involve player j in the coalition as $T\backslash\{i\}\cup\{j\}$ would be strictly cheaper than any coalition involving j.

$$\begin{aligned} z_i &= \frac{\delta}{n}\pi_i + \delta\left[\mu_j + (\mu_i - \mu_j) - \frac{1}{n} + \alpha\right] z_i + (1-\delta)r_i \\ z_j &\leq \frac{\delta}{n}\left(\pi_i + z_j - z_i\right) + \delta\left[\mu_j - \frac{1}{n} + \alpha\right] z_j + (1-\delta)r_j \end{aligned}$$

Subtracting and collecting terms

$$\begin{aligned} z_j - z_i &\leq \frac{\delta}{n}(z_j - z_i) + \delta\left[\mu_j - \frac{1}{n} + \alpha\right](z_j - z_i) - (\mu_i - \mu_j)z_i - (1-\delta)(r_i - r_j) \\ (z_j - z_i)\left(1 - \delta\mu_j - \delta\alpha\right) &\leq -(\mu_i - \mu_j)z_i - (1-\delta)(r_i - r_j) \end{aligned}$$

Since the LHS is strictly positive and the RHS is nonpositive, we have a contradiction. □

[23]This paragraph assumes that ties are always broken in favor of agreement. If we do not impose this, player j could make proposals that are rejected with positive probability if $\min_{S:S\ni j,|S|=q} \sum_{k\in S} z_k = 1$. This does not affect the discussion on π_i and π_j. We would then have two separate values, α_i and α_j, where α_k would be the probability that a proposal is made by a player other than k and rejected. We would then have $\alpha_i \geq \alpha_j$, which goes in the same direction as $\mu_i \geq \mu_j$, and the proof can be easily adapted.

A.3 Equilibria with immediate agreement

It will be useful to distinguish between equilibria in which all players make acceptable proposals and equilibria in which some players do not. We will refer to the former as equilibria with *no delay* or *immediate agreement*, and to the latter as equilibria involving *delay*. We begin by presenting sufficient conditions for immediate agreement.

Lemma 3. *If there exists a coalition S with $|S| = q$ and $\sum_{j \in S} r_j < 1$, then all players in S make acceptable proposals in any SSPE.*

Proof. We start by establishing that, for any $S \subseteq N$, $\sum_{j \in S} r_j < 1$ implies $\sum_{j \in S} z_j < 1$. Since total payoffs for S add up to less than 1 in the event of disagreement and can add up to at most 1 in the event of agreement, the maximum possible value of $\sum_{j \in S} z_j$ is $\delta + (1 - \delta) \sum_{j \in S} r_j < 1$. The result then follows from part 2 of lemma 1. □

Corollary 1. *If $\sum_{j=1}^{q-1} r_j + r_n < 1$, any SSPE exhibits immediate agreement.*

If agreement is efficient, corollary 1 implies that immediate agreement will occur in any SSPE for any decision rule. When $q < n$, corollary 1 also implies that immediate agreement may occur even if it is not efficient. In fact, the following example shows that immediate agreement may occur even in the extreme case where $\sum_{j \in S} r_j > 1$ for all S such that $|S| = q$.

Let $n = 3$, $q = 2$, $r_i = \frac{9}{15}$ and $\delta = 0.5$. There is an equilibrium in which proposers offer $\frac{7}{15}$ to a randomly selected partner, and responders accept all proposals that give them at least $\frac{7}{15}$. (There is also an equilibrium in which agreement is never reached).

To see that this is an equilibrium, note that the continuation values that follow from the strategies are determined as follows. When proposing, a player receives $1 - \frac{7}{15} = \frac{8}{15}$. If not proposing, they are included in the coalition with probability $\frac{1}{2}$; in this case they earn $\frac{7}{15}$. Thus, a player's expected payoff is $\frac{1}{3}\frac{8}{15} + \frac{2}{3}\frac{1}{2}\frac{7}{15} = \frac{5}{15}$. Therefore, the continuation value is $\delta \frac{5}{15} + (1 - \delta)\frac{9}{15}$; since $\delta = 0.5$ this is $\frac{7}{15}$.

As the above example shows, there is a very strong pressure for immediate agreement under less-than-unanimity rule. We now characterize the properties of equilibria which exhibit immediate agreement. It will be useful to define the following sets:

$$\begin{aligned}\mathcal{L} &= \{i \in N : z_i < z_q\} \\ \mathcal{M} &= \{i \in N : z_i = z_q\} \\ \mathcal{H} &= \{i \in N : z_i > z_q\}\end{aligned}$$

That is, the set $\mathcal{L}$ is the set of players whose votes are "cheaper" than that of player q, and the set $\mathcal{M}$ contains all players who are as expensive as player q, while the set $\mathcal{H}$ contains those that are strictly more expensive. Therefore, when any player proposes, an optimal strategy involves buying all players in L and as many players in $\mathcal{M}$ as are necessary to build a coalition of size q. Clearly the set $\mathcal{M}$ is always nonempty, though one or both of the sets $\mathcal{L}$ and $\mathcal{H}$ may be empty.

We will denote the cardinalities of those sets by L, M and H respectively. By lemma 2, any SSPE must have $z_1 \leq z_2 \leq ... \leq z_n$. Hence, in order to know the partition into the three sets it is sufficient to know the *cardinalities* L, M and H.

We begin by showing that a no-delay equilibrium is characterized by L, M and H in the sense that the continuation values z_i, expected payoffs y_i and inclusion probabilities μ_i are uniquely determined by L, M and H, though there may be several strategy combinations that lead to the same values of z_i, y_i and μ_i.[24,25]

Proposition 3. *Given an SSPE with immediate agreement, the partition of the player set into the sets $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{H}$ uniquely determines the equilibrium values of y, z and μ.*

Proof. In a no-delay equilibrium, the proposer offers z_i to $q-1$ other players (the ones with the $q-1$ lowest values of z_i) and 0 to the remaining players.[26]

[24] We are not (yet) claiming that the no-delay equilibrium is unique, just that other no-delay equilibria would lead to different values of L, M and H.

[25] It is a known feature of legislative bargaining models that a given vector of equilibrium expected payoffs may be supported by several strategy combinations (see e.g. Eraslan and McLennan, 2013).

[26] Note that it cannot be optimal for the proposer to offer their continuation value to more than $q-1$ players because he would be better-off by excluding the coalition partner with the highest z_i. The only possible exception would be if $z_i = 0$ for more than $q-1$ players, but this is not possible since each of those players could form a coalition with the rest and get a positive payoff as a proposer, which would contradict $z_i = 0$.

Define $Z_q := \sum_{i \leq q} z_i$ and $Z_L := \sum_{i \in L} z_i$. By definition, $Z_q = Z_L + (q - L)z_q$. Continuation values are given by the following equations:

$$\begin{aligned}
\mathcal{L} &: \quad z_i = \delta\left[\frac{1}{n}(1 - Z_q + z_i) + \left(\mu_i - \frac{1}{n}\right) z_i\right] + (1-\delta)r_i. \\
\mathcal{M} &: \quad z_i = \delta\left[\frac{1}{n}(1 - Z_q + z_i) + \left(\mu_i - \frac{1}{n}\right) z_i\right] + (1-\delta)r_i. \\
\mathcal{H} &: \quad z_i = \delta\left[\frac{1}{n}(1 - Z_q + z_q) + \left(\mu_i - \frac{1}{n}\right) z_i\right] + (1-\delta)r_i.
\end{aligned}$$

As a proposer, a player buys the votes from the cheapest $q-1$ *other* players. Recall that the total continuation value of the cheapest q players is Z_q. Players in $\mathcal{L}$ are themselves among the cheapest q, so they pay $Z_q - z_i$. Players in $\mathcal{H}$ are not among the cheapest q, so they can buy the cheapest $q-1$ votes and pay $Z_q - z_q$. Players in $\mathcal{M}$ can be thought of as paying $Z_q - z_i$ or $Z_q - z_q$, since $z_i = z_q$. Expected equilibrium payoffs are thus $y_i = \frac{1}{n}(1 - Z_q + \min(z_i, z_q)) + \left(\mu_i - \frac{1}{n}\right) z_i$. Continuation values are given by $z_i = \delta y_i + (1-\delta)r_i$.

Consider players in $\mathcal{M}$. Collecting terms and taking into account that by definition $z_i = z_q$ in this set we find

$$\mathcal{M} : z_q = \delta\left[\frac{1}{n}(1 - Z_q) + \mu_i z_q\right] + (1-\delta)r_i$$

This has a very clear interpretation: if bargaining goes on after a rejection, player i gets his continuation value whenever he is part of a coalition, and on top of that he gets the proposer surplus with probability $\frac{1}{n}$ (the proposer surplus is the difference between i's payoff as a proposer and i's payoff as a coalition partner; its value for players in $\mathcal{L}$ or $\mathcal{M}$ is $1 - Z_q$).

Collecting terms again

$$\mathcal{M} : (1 - \delta\mu_i)\, z_q = \frac{\delta}{n}(1 - Z_q) + (1-\delta)r_i \tag{3}$$

We also know that $\sum_{i \in N} \mu_i = q$, that is, the coalition that forms always contains exactly q players. Moreover, $\mu_i = 1$ for all i in $\mathcal{L}$ and $\mu_i = \frac{1}{n}$ for all i in $\mathcal{H}$. That is, players in $\mathcal{L}$ are always included in the coalition, and players in $\mathcal{H}$ are only included when they are proposers. Hence, $\sum_{i \in M} \mu_i = q - L - \frac{H}{n}$. Note that $\sum_{i \in M} \mu_i > \frac{M}{n}$, which, since each player in $\mathcal{M}$ is the proposer with probability $\frac{1}{n}$, implies that, collectively, players in $\mathcal{M}$ have a positive probability of being coalition partners.[27]

If we add up all the equations (3),

$$Mz_q - \delta\left(q - L - \frac{H}{n}\right)z_q = M\frac{\delta}{n}(1 - Z_q) + (1-\delta)\sum_{i\in M} r_i$$

Dividing everything by M,

$$z_q - \delta\frac{\left(q - L - \frac{H}{n}\right)}{M}z_q = \frac{\delta}{n}(1 - Z_q) + (1-\delta)\overline{r}_M$$

where $\overline{r}_M$ is the average value of r in the set $\mathcal{M}$, $\frac{\sum_{i\in M} r_i}{M}$.

We also know $Z_q = Z_L + (q - L)z_q$, so we can get an equation with two unknowns, Z_L and z_q.

$$z_q - \delta\frac{\left(q - L - \frac{H}{n}\right)}{M}z_q = \frac{\delta}{n}(1 - Z_L - (q-L)z_q) + (1-\delta)\overline{r}_M$$

Collecting terms

$$\left[1 - \delta(q - L)\left(\frac{1}{M} - \frac{1}{n}\right) + \delta\frac{H}{Mn}\right]z_q = \frac{\delta}{n}(1 - Z_L) + (1-\delta)\overline{r}_M \qquad (4)$$

This gives us an equation where, given H, M and L, the only unknowns are z_q and Z_L.

If all players are in $\mathcal{M}$, the equation simplifies to $z_q = \frac{\delta}{n} + (1-\delta)\overline{r}_N$, which is clear since, assuming immediate agreement, $\sum_{i\in N} z_i = \delta + (1-\delta)\sum_{i\in N} r_i$. If all players are in $\mathcal{M}$ they all have the same continuation value, which must then equal the previous expression divided by n.

For players in $\mathcal{L}$, since $\mu_i = 1$, we find

$$\mathcal{L} : z_i = \delta\left[\frac{1}{n}(1 - Z_q + z_i) + \frac{n-1}{n}z_i\right] + (1-\delta)r_i$$

$$z_i = \delta\left[\frac{1}{n}(1 - Z_q) + z_i\right] + (1-\delta)r_i \qquad (5)$$

[27] If $L = 0$ we have $\sum_{i\in M}\mu_i = q - \frac{H}{n}$, which, since $q > 1$ and $H < n$ implies $\sum_{i\in M}\mu_i > 1$. If $L > 0$, by definition we have $q - L \geq 1$ and $n - H > M$, hence $\sum_{i\in M}\mu_i \geq 1 - \frac{H}{n} = \frac{n-H}{n} > \frac{M}{n}$.

$$z_i - r_i = \frac{\delta}{1-\delta}\frac{1}{n}(1 - Z_q) \tag{6}$$

We then see that $z_i - r_i$ is a constant, that is, all players in $\mathcal{L}$ get the same surplus above r_i.

Replacing Z_q by its value we find

$$z_i - r_i = \frac{\delta}{1-\delta}\frac{1}{n}(1 - Z_L - (q-L)z_q) \tag{7}$$

If we instead add up the equations (5) for players in $\mathcal{L}$, we find

$$Z_L = \delta\left[\frac{L}{n}(1 - Z_q) + Z_L\right] + (1-\delta)L\overline{r}_L$$

Replacing Z_q by its value in terms of z_q and Z_L and collecting terms:

$$\left(1 - \delta + \delta\frac{L}{n}\right) Z_L = \delta\frac{L}{n}(1 - (q-L)z_q) + (1-\delta)L\overline{r}_L \tag{8}$$

The set $\mathcal{H}$ is residual. For players in this set,

$$\mathcal{H} : z_i = \frac{\delta}{n}(1 - Z_q + z_q) + (1-\delta)r_i \tag{9}$$

Clearly, z_i is increasing in r_i for players in the set $\mathcal{H}$.

The values Z_L and z_q can be found from the system of two linear equations and two unknowns (4) and (8). Once these two values have been found, z_i values for players in $\mathcal{L}$ can be found from (7) and z_i values for players in $\mathcal{H}$ can be found from (9). The probabilities of inclusion in the coalition for players in $\mathcal{H}$ and $\mathcal{L}$ are known; the probabilities of inclusion μ_i for players in $\mathcal{M}$ can be found from (3). Expected payoffs $(y)_{i\in N}$ are found from the equation $z_i = \delta y_i + (1-\delta)r_i$.

□

Proposition 4. *For $q < n$, expected equilibrium payoffs in any no-delay SSPE are strictly increasing in r_i within the set $\mathcal{L}$, strictly decreasing in r_i within the set $\mathcal{M}$, and constant within the set $\mathcal{H}$. Furthermore, expected payoffs for all players in $\mathcal{M}$ are at least as high as those for players in $\mathcal{H}$.*

Proof. Since all players in $\mathcal{L}$ are in the final coalition with probability 1, expected payoffs y_i and continuation values z_i are ranked in the same way within this set. Expected payoffs for a player in L are found from the equation $y_i = \frac{1}{n}(1 - Z_q + z_i) + \frac{n-1}{n} z_i$. Collecting terms, $y_i = \frac{1}{n}(1 - Z_q) + z_i$.

Players in $\mathcal{M}$ all get $1 - Z_q + z_q$ as proposers and z_q as responders, thus any difference in expected payoffs must be due to differences in μ_i. Since μ_i is decreasing in r_i by (3), y_i is decreasing in r_i as well.

Finally, since no player in $\mathcal{H}$ receives proposals from other players, all players in $\mathcal{H}$ have $y_i = \frac{1}{n}(1 - Z_q + z_q)$, which does not depend on r_i.

Since players in $\mathcal{M}$ and $\mathcal{H}$ get the same payoff as proposers and players in $\mathcal{H}$ never get any proposals from other players, it is clear that expected payoffs for a player in $\mathcal{M}$ cannot be lower than those of a player in $\mathcal{H}$. Indeed, they will typically be strictly higher, except for non-generic cases in which player $L+M$ (the most expensive player in $\mathcal{M}$) does not get any proposals. We know that collectively the players in $\mathcal{M}$ have a positive probability of being coalition partners, hence player $L+1$ must have a strictly higher payoff than players in $\mathcal{H}$. □

The sets $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{H}$ are endogenous. Putting proposition 4 and lemma 2 together allows us to state a result in terms of the exogenous values r_i. Under all less-than-unanimity rules, expected equilibrium payoffs are either decreasing or first increasing and then decreasing in r_i. The player with the highest expected equilibrium payoff is either player L or player $L + 1$.

Corollary 2. *The highest expected equilibrium payoff is achieved by (one or more) player(s) for whom $r_i \leq r_q$.*

There may be multiple no-delay SSPE. However, all no-delay SSPE are equivalent in the sense that they lead to the same expected equilibrium payoffs, continuation values and probabilities of being included in the final coalition, though they may differ in the exact mixed strategies used, as in the original model (see Eraslan and McLennan (2013)).

Proposition 5. *Let σ and σ' be two no-delay SSPE. Then $y = y'$, $z = z'$ and $\mu = \mu'$.*

Proof. By contradiction, suppose σ and σ' are two SSPE that induce different partitions of the set N. We will denote by $\mathcal{L}$, $\mathcal{M}$ and $\mathcal{H}$ the sets associated to σ (the sets associated to σ' will be denoted by $\mathcal{L}'$, $\mathcal{M}'$ and $\mathcal{H}'$). Recall that continuation values must respect the order of the disagreement values, hence if $\mathcal{L}$ and $\mathcal{L}'$ have the same cardinality the actual sets must also coincide. Similarly, if $\mathcal{L} \neq \mathcal{L}'$, one of the two sets must be a strict subset of the other.

Case 1. Suppose $\mathcal{L} = \mathcal{L}'$. Then $\mathcal{M} \neq \mathcal{M}'$ (otherwise the partitions would be identical).

1. a) Suppose $Z_q = Z'_q$. Recall that the equation for the continuation value of a player in set $\mathcal{L}$ is

$$z_i = r_i + \frac{\delta}{1-\delta}\frac{1}{n}(1 - Z_q). \tag{10}$$

The same equation holds in equilibrium σ', with z_i being replaced by z'_i and Z_q being replaced by Z'_q.

Since by assumption $Z_q = Z'_q$, it follows that $z_i = z'_i$ for all players in $\mathcal{L} \cap \mathcal{L}'$. Since, also by assumption, $\mathcal{L} = \mathcal{L}'$, we have $Z_L = Z'_{L'}$, and (given that $Z_q = Z_L + (q - L)z_q$ and $Z'_q = Z'_{L'} + (q - L')z'_q$) $z_q = z'_q$.

Since the partitions are different, $\mathcal{M} \neq \mathcal{M}'$. Given our result on how the z_i values are ranked in the same way as the r_i values, and given that $\mathcal{L} = \mathcal{L}'$ by assumption, one of the two sets must be a subset of the other. Without loss of generality, let $\mathcal{M} \subsetneq \mathcal{M}'$. Let $j \in \mathcal{M}' \backslash \mathcal{M}$ (hence $j \in \mathcal{H}$). Since player j is one of the expensive players in equilibrium σ and one of the medium players in equilibrium σ', and, as we have shown, $z_q = z'_q$, it must be the case that $z_j > z'_j$. We now show that this is not possible.

The following equation holds for a player in $\mathcal{M} \cup \mathcal{H}$ (and hence for player j, since $j \in \mathcal{H}$):

$$z_j = \frac{\delta}{n}[1 - Z_q + z_q] + \delta\left(\mu_j - \frac{1}{n}\right) z_j + (1-\delta)r_j. \tag{11}$$

Collecting terms in j, we find

$$\left(1 + \frac{\delta}{n} - \delta\mu_j\right) z_j = \frac{\delta}{n}[1 - Z_q + z_q] + (1-\delta)r_j \tag{12}$$

An analogous equation holds for a player in $\mathcal{M}' \cup \mathcal{H}'$ (and hence for player j, since $j \in \mathcal{M}'$):

$$\left(1 + \frac{\delta}{n} - \delta\mu'_j\right) z'_j = \frac{\delta}{n}[1 - Z'_q + z'_q] + (1-\delta)r_j. \tag{13}$$

By assumption, $Z'_q = Z_q$ and, as we have shown, this implies $z'_q = z_q$. Hence, player j gets the same payoff as proposer in both equilibria. The only way in which $z_j > z'_j$ is if $\mu_j > \mu'_j$. However, since $j \in \mathcal{H}$, j is never a responder in equilibrium σ but may be a responder in equilibrium σ', implying $z_j \leq z'_j$, a contradiction.

1. b) Suppose $Z_q \neq Z'_q$. Without loss of generality, let $Z_q < Z'_q$.

Coming back to equation (10), this implies $z_i > z'_i$ for all $i \in \mathcal{L} \cap \mathcal{L}'$. Given that $\mathcal{L}$ and $\mathcal{L}'$ coincide, it follows that $Z_L > Z'_{L'}$, which together with $Z_q < Z'_q$ implies $z_q < z'_q$.

Consider the set $\mathcal{M} \cap \mathcal{M}'$ (clearly, this set is nonempty since $q \in \mathcal{M} \cap \mathcal{M}'$). For any $i \in \mathcal{M}$, $z_i = z_q$ and $z'_i = z'_q$, so (replacing $z_i = z_q$ in (12) and collecting terms) we can write i's equilibrium comntinuation value as

$$(1 - \delta\mu_i)z_q = \frac{\delta}{n}(1 - Z_q) + (1 - \delta)r_i. \tag{14}$$

Analogously, for any $i \in \mathcal{M}'$,

$$(1 - \delta\mu'_i)z'_q = \frac{\delta}{n}(1 - Z'_q) + (1 - \delta)r_i. \tag{15}$$

Given that $Z'_q > Z_q$ but $z_q < z'_q$, it must be the case that $\mu'_i > \mu_i$. Now consider the total probability of being involved in a coalition in equilibrium. The total probability is $\sum_{i \in N} \mu_i = \sum_{i \in N} \mu'_i = q$. Further, each player must be included if he is selected to be the proposer, hence $\mu_i \geq \frac{1}{n}$ for all i. There is then a total probability of being coalition partner of $q-1$, which may be distributed differently in the two equilibria. Given that players in $\mathcal{H}$ (respectively $\mathcal{H}'$) never get proposals, we have a total probability of $q-1$ to be distributed between players in $\mathcal{L} \cup \mathcal{M}$ in equilibrium σ, and between players in $\mathcal{L}' \cup \mathcal{M}'$ in equilibrium σ'. Further, note that all players in $\mathcal{L}$ have $\mu_i = 1$, and all players in $\mathcal{L}'$ have $\mu'_i = 1$. Since by assumption $\mathcal{L} = \mathcal{L}'$, we have $\sum_{i \in \mathcal{M}}(\mu_i - \frac{1}{n}) = \sum_{i \in \mathcal{M}'}(\mu'_i - \frac{1}{n})$, and the only way in which $\mu'_i > \mu_i$ for all players in $\mathcal{M} \cap \mathcal{M}'$ is if $\mathcal{M}' \subsetneq \mathcal{M}$ (otherwise we would "run out of probability").

Let j be a player in $\mathcal{M} \backslash \mathcal{M}'$. If we consider equilibrium σ, using

(11) we have (since the worst-case scenario is $\mu_j = \frac{1}{n}$):

$$\begin{aligned} z_j &= z_q \geq \frac{\delta}{n}(1 - Z_q + z_q) + (1-\delta)r_i \\ (1 - \frac{\delta}{n})z_q &\geq \frac{\delta}{n}(1 - Z_q) + (1-\delta)r_i \end{aligned}$$

If we consider equilibrium σ', where $j \in \mathcal{H}'$, we have

$$\begin{aligned} z'_q &< z'_j = \frac{\delta}{n}(1 - Z'_q + z'_q) + (1-\delta)r_i \\ (1 - \frac{\delta}{n})z'_q &< \frac{\delta}{n}(1 - Z'_q) + (1-\delta)r_i \end{aligned}$$

Putting the two expressions together, since $Z'_q > Z_q$, we find

$$(1-\frac{\delta}{n})z'_q < \frac{\delta}{n}(1-Z'_q)+(1-\delta)r_i < \frac{\delta}{n}(1-Z_q)+(1-\delta)r_i \leq (1-\frac{\delta}{n})z_q$$

which implies $z'_q < z_q$, a contradiction.

Case 2. Suppose $\mathcal{L} \neq \mathcal{L}'$. Without loss of generality, let $\mathcal{L} \subsetneq \mathcal{L}'$. There are two possible cases, depending on how Z_q compares with Z'_q.

2. a) Suppose $Z_q \leq Z'_q$. We can write $Z_q \leq Z'_q$ as

$$\sum_{i \in \mathcal{L}} z_i + (q-L)z_q \leq \sum_{i \in \mathcal{L}} z'_i + \sum_{i \in \mathcal{L}' \setminus \mathcal{L}} z'_i + (q - L')z'_q$$

By equation (10), $Z_q \leq Z'_q$ implies $z_i \geq z'_i$ for all $i \in \mathcal{L} \cap \mathcal{L}'$, hence

$$(q-L)z_q \leq \sum_{i \in \mathcal{L}' \setminus \mathcal{L}} z'_i + (q - L')z'_q$$

By definition, $z'_i < z'_q$ for all $i \in \mathcal{L}'$. Hence, the equation above indicates that z_q is at most as large as a weighted average of several values, the largest of which is z'_q. Thus, $z_q < z'_q$.

If we now look at players $j \in \mathcal{M} \cap \mathcal{M}'$ (a set that includes player q), comparing equations (14) and (15) we see that, given that

$Z_q \leq Z'_q$, the only way in which $z_q < z'_q$ is if $\mu_i < \mu'_i$ for all these players.

Recall that the probability of being a coalition partner is $\mu_i - \frac{1}{n}$ ($\mu'_i - \frac{1}{n}$ in equilibrium σ'). It holds that $\sum_{i \in \mathcal{L} \cup \mathcal{M}} \left(\mu_i - \frac{1}{n}\right) = \sum_{i \in \mathcal{L}' \cup \mathcal{M}'} \left(\mu'_i - \frac{1}{n}\right) = q - 1$. Consider the allocation of this probability, starting by player 1 onwards. Players in $\mathcal{L} \cap \mathcal{L}'$ (i.e., players in $\mathcal{L}$) have $\mu_i = \mu'_i = 1$. Players in $\mathcal{M} \cap \mathcal{L}'$ have $\mu_i \leq \mu'_i = 1$. Players in $\mathcal{M} \cap \mathcal{M}'$ have $\mu_i < \mu'_i$. It then follows that $\mathcal{L}' \cup \mathcal{M}' \subsetneq \mathcal{L} \cup \mathcal{M}$, that is, the total probability $q - 1$ must be exhausted earlier in the equilibrium σ'. Hence, the set $\mathcal{M} \backslash (\mathcal{L}' \cap \mathcal{M}')$ is nonempty. Let j be a player in this set. Player j is in set $\mathcal{M}$ in the equilibrium σ, but is in set $\mathcal{H}'$ in equilibrium σ'. Since $z_q < z'_q$, this implies $z_j = z_q < z'_q < z'_j$. We can then find a contradiction by the same reasoning as in case 1b).

2. b) Let $Z_q > Z'_q$. Then $z_i < z'_i$ for all $i \in \mathcal{L}$. We then have

$$Z_q = \sum_{i \in \mathcal{L}} z_i + (q - L) z_q > \sum_{i \in \mathcal{L}} z'_i + \sum_{i \in \mathcal{L}' \backslash \mathcal{L}} z'_i + (q - L') z'_q = Z'_q.$$

Hence, $(q - L) z_q > \sum_{i \in \mathcal{L}' \backslash \mathcal{L}} z'_i + (q - L') z'_q$. This does not seem to give us a clear relationship between z_q and z'_q, though it tells us that $z_q > z'_i$, where $i \in \mathcal{L}' \backslash \mathcal{L}$. In other words, players in $\mathcal{M} \cap \mathcal{L}'$ have $z_i > z'_i$. Consider the equations for z_i and z'_i, $i \in \mathcal{M} \cap \mathcal{L}'$. Since $i \in \mathcal{M}$, we have

$$(1 - \delta \mu_i) z_i = \frac{\delta}{n}(1 - Z_q) + (1 - \delta) r_i.$$

On the other hand, since $i \in \mathcal{L}'$, we have

$$(1 - \delta) z'_i = \frac{\delta}{n}(1 - Z'_q) + (1 - \delta) r_i.$$

Since $\mu_i \leq 1$ and $Z_q > Z'_q$, it follows that $z'_i > z_i$, a contradiction.

□

Hence, SSPE payoffs are unique if the sufficient conditions for immediate agreement are satisfied; if not, there may be multiple equilibria.

A.4 Proof of propositions 1 and 2

Propositions 1 and 2 easily follow from the results of the previous section.

Proof of proposition 1. If $\sum_{i\in N} r_i < 1$, all SSPE must have immediate agreement by corollary 1. If $\sum_{i\in N} r_i > 1$, each player can ensure disagreement by rejecting all proposals and proposing $x_i = 1$, hence $\sum_{i\in N} z_i > 1$ and no agreement can occur. If $\sum_{i\in N} r_i = 1$, it is an equilibrium for all players to propose $x = r$ and accept any proposal with $x_i \geq r_i$, and this is the equilibrium selected by our tie-breaking rule.

In a no-delay equilibrium under unanimity rule, each player offers the other $n-1$ players their continuation value and $\mu_i = 1$ for all players, hence

$$y_i = \frac{1}{n}\left[1 - \sum_{j\in N\setminus\{i\}} z_j\right] + \frac{n-1}{n} z_i = \frac{1}{n}\left[1 - \sum_{j\in N} z_j\right] + z_i \tag{16}$$

Continuation values are related to expected payoffs by the equation

$$z_j = \delta y_j + (1-\delta) r_j \tag{17}$$

If we add up equations (17) and take into account that $\sum_{j\in N} y_j = 1$ in a no-delay equilibrium,

$$\sum_{j\in N} z_j = \delta + (1-\delta)\sum_{j\in N} r_j \tag{18}$$

If we take equation (16) and replace z_i by its value from (17) and $\sum_{j\in N} z_j$ by its value from (18), we obtain an equation with y_i as the only unknown. Solving this equation, we get $y_i = \frac{1}{n}[1 - \sum_{j\in N} r_j] + r_i$, and, using (17), $z_i = \frac{\delta}{n}[1 - \sum_{j\in N} r_j] + r_i$.

Proof of Proposition 2. Parts (i)-(iii) follow directly from corollary 1, proposition 5 and lemma 2 respectively.

(iv) In a no-delay equilibrium, players in $\mathcal{L}$ are included in the final coalition with probability 1 (the maximum possible), and players in $\mathcal{H}$ are included in the final coalition with probability $\frac{1}{n}$ (the minimum possible, since proposers always include themselves). Within the set $\mathcal{M}$, the probability of inclusion is decreasing in r_i as can be seen from equation (3).

(v) This is a consequence of proposition 4. Since expected equilibrium payoffs are increasing within $\mathcal{L}$, decreasing within $\mathcal{M}$ and constant (but not higher than those in $\mathcal{M}$) within $\mathcal{L}$, there are four possible cases:

If $\mathcal{N} = \mathcal{M}$, expected payoffs are decreasing in the r-values.

If only $\mathcal{M}$ and $\mathcal{H}$ are nonempty, expected payoffs are first decreasing and then constant in the r-values.

If only $\mathcal{L}$ and $\mathcal{M}$ are nonempty, expected payoffs are first increasing and then decreasing in the r-values.

If all three sets are nonempty, expected payoffs are first increasing, then decreasing and then constant in the r-values.

A.5 Limit results as $\delta \to 1$

Proposition 6. *If $q < n$, there is a value $\overline{\delta} < 1$ such that the unique no-delay SSPE of the game $G(n, q, \delta, r)$ has $N = \mathcal{M}$ for $\overline{\delta} < \delta < 1$.*

Proof. $\mathcal{L}$ and $\mathcal{H}$ must both be empty when δ is sufficiently close to 1. This can be proved by contradiction.

Suppose $\mathcal{L} \neq \varnothing$ in equilibrium for $\delta \to 1$. If we look at equation (6) for z_i in $\mathcal{L}$, we see that $\frac{1-Z_q}{1-\delta}$ becomes unbounded when $q < n$ as $\delta \to 1$, a contradiction. This is because in a no-delay equilibrium we have $\sum_{i \in N} z_i = \delta + (1-\delta)\sum_{i \in N} r_i$, therefore the largest continuation value must be at least $\frac{\delta}{n} + (1-\delta)\overline{r}_N$. Since $q < n$, $Z_q \leq \sum_{i \in N} z_i - z_n \leq \delta\frac{n-1}{n} + (n-1)(1-\delta)\overline{r}_N$, and $1 - Z_q \geq 1 - \delta\frac{n-1}{n} - (n-1)(1-\delta)\overline{r}_N$. When $\delta \to 1$, the lower bound for $1 - Z_q$ approaches $\frac{1}{n} > 0$. More generally, $\mu_i < 1$ for all i for δ sufficiently large (also for players in $\mathcal{M}$), as otherwise we could rewrite (3) and find that z_q becomes unbounded in the same way.

Similarly, suppose $\mathcal{H} \neq \varnothing$ and take player $n \in \mathcal{H}$. Let m be the player with the smallest r_i in $\mathcal{M}$ (in general the identity of player m may depend on δ). We now show that $z_m - z_n = \delta\left(\mu_m - \frac{1}{n}\right) z_q - (1-\delta)(r_n - r_m) > 0$ for δ close enough to 1, a contradiction. To see this, note that expression $(1-\delta)(r_n - r_m)$ clearly converges to 0, since $r_n - r_m \leq r_n - r_1$ and $\delta \to 1$. It remains to be shown that $\left(\mu_m - \frac{1}{n}\right) z_q$ remains strictly positive as $\delta \to 1$. First we show that $\mu_m > \frac{1}{n}$ for all values of δ. Given that $\mathcal{L} = \varnothing$ for a sufficiently large value of δ and $q \geq 2$, $\sum_{i \in \mathcal{M}} \mu_i = q - \frac{H}{n} = q - 1 + \frac{M}{n} \geq 1 + \frac{M}{n}$. Since m is the player with the smallest value of r_i, he is also the player with the highest μ_i in $\mathcal{M}$ according to equation (3), hence $\mu_m \geq \frac{1}{M} + \frac{1}{n} \geq \frac{2}{n}$. It remains to be shown that $z_q > 0$ for $\delta \to 1$. This is clear since, using (3), $z_q \geq \frac{\delta}{n}(1 - Z_q) + (1-\delta)r_m$, and we have established that $1 - Z_q$ remains positive as $\delta \to 1$. Hence the set $\mathcal{H}$ is empty for δ close enough to 1. □

Corollary 3. *In the limit when $\delta \to 1$, $y_i = \frac{1}{n}$ for all $i \in N$.*[28]

Hence, there is a sharp contrast between the results for $q = n$ and for $q < n$ when $\delta \to 1$. Under unanimity rule, the r-values remain relevant even if $\delta \to 1$. Under any majoritarian rule, continuation values are less sensitive to the r-values, which is consistent with players being unable to unilaterally secure these values, and these values become irrelevant in the limit as $\delta \to 1$.

A.6 Equilibria with delay

Consider a SSPE that involves delay, i.e. at least one player, when proposing, makes a proposal that is not accepted. Call him player i. Then it follows from Lemma 1 and our tie-breaking rule that the cheapest q-coalition including Mr. i has $\sum_S z_j > 1$. This in turn implies that no other player will build a coalition that includes Mr. i. Thus, the only way that Mr. i influences the game is that he will 'stall' when it is his turn to propose, thereby increasing the chance that breakdown will occur before an 'active' player is chosen.

Suppose there are k such 'stallers' in equilibrium. The remaining players make proposals that are accepted, and so they are essentially playing an equilibrium with immediate agreement, however in a 'transformed' game in which the probability of breakdown is increased. What is the new probability of breakdown?

After a proposal fails, breakdown will occur *immediately* with probability $(1 - \delta)$. If not, then with probability $\frac{k}{n}$, a staller will propose next, in which case failure will again occur with probability $(1 - \delta)$, etc. So the probability that breakdown will occur is

$$(1 - \tilde{\delta}) = (1 - \delta) + \frac{\delta k}{n}\left[(1 - \delta) + \frac{\delta k}{n}\left(\cdots\right)\right] = 1 - \frac{\delta(n - k)}{n - \delta k}$$

and so the modified continuation probability is

$$\tilde{\delta} = \frac{\delta(n - k)}{n - \delta k}$$

which is equal to δ when $k = 0$ and equal to zero when $k = n$.

[28] When all players are in $\mathcal{M}$ we have $z_q = \frac{\delta}{n} + (1 - \delta)\overline{r}_N$. Putting this together with the equation $z_q = \delta y_i + (1 - \delta) r_i$, we see that both z_i and y_i converge to $\frac{1}{n}$ as $\delta \to 1$.

The expected payoffs and continuation values of the remaining $n-k$ players are determined as in equilibria with immediate agreement. For the 'stallers', expected payoffs are simply

$$y_j = \rho r_j$$

where ρ is the probability that breakdown will occur before an active player is chosen to propose.

$$\rho = \frac{k}{n}(1-\tilde{\delta}) = \frac{(1-\delta)k}{n-\delta k}$$

Two cases are possible: (1) If $k \leq n-q$, agreement will occur if one of the non-stallers is chosen to propose. The payoffs and continuation values of these players satisfy exactly the conditions we have previously derived. And the continuation values of the stallers must be strictly greater than the others. (2) $k = n$, i.e. no agreement occurs.

A.7 The three-player case

The three-player case can be solved by enumeration. With three players there are only four possible types of no-delay equilibria: $L = 1$ and $M = 2$; $L = 1, M = 1$ and $H = 1$; $M = 3$; $M = 2$ and $H = 1$. For each of these four possible types we can find conditions on the parameters r and δ in order for this type of equilibrium to exist. The no-delay equilibrium is unique as we know from proposition 5, hence each parameter combination is compatible with only one type of equilibrium.

Proposition 7. *Let $r_1 + r_3 < 1$. There is a unique no-delay SSPE for each combination of r and δ. The no-delay SSPE is of one of four possible types:*
a) L=1,M=1,H=1 occurs if $r_1 < r_2 < r_3$ and $\delta < \underline{\delta}$.
b) L=1, M=2 occurs if $r_2 > \frac{r_1+r_3}{2}$ and $\underline{\delta} \leq \delta < \overline{\delta}$.
c) M=2, H=1 occurs if $r_2 < \frac{r_1+r_3}{2}$ and $\underline{\delta} \leq \delta < \overline{\delta}$.
d) M=3 occurs if $\delta \geq \overline{\delta}$.

The idea of the proof is as follows. We first conjecture a particular value for L, M and H (for example, $L = 1$ and $M = 2$). This conjecture leads to a system of equations that can be solved for z_i and μ_i (see section A.3). In order for the solution to be an equilibrium, the found values of z_i must be consistent with our initial conjecture (in the example, the found value of z_1

must be below the found value of $z_2 = z_3$), and any mixed strategies that are played must involve probabilities between 0 and 1 (in the example, player 1 is mixing between proposing to player 2 and proposing to player 3). These conditions leads to inequalities involving the parameters r_1, r_2, r_3 and δ.

Even though the proof is quite lengthy, the intuition behind the result is clear.

First, there are some conditions on the r_i values. In particular, case $L = 1$, $M = 1$, $H = 1$ is only possible if all values are different. $L = 1$, $M = 2$ is only possible if $r_2 > \frac{r_1+r_3}{2}$, so that r_2 and r_3 are close together relative to r_1 and players 2 and 3 can be grouped together in the same class. $M = 2$, $H = 1$ is only possible if $r_2 < \frac{r_1+r_3}{2}$ so that players 1 and 2 can be grouped together in the same class.

Second, there are some conditions on δ. $L = 1$, $M = 1$, $H = 1$ occurs when $r_1 < r_2 < r_3$ and $\delta = 0$ since $z_i = r_i$ in this case. The inequality $z_1 < z_2 < z_3$ can be sustained as long as δ is sufficiently low, so that the difference in r_i overrules the fact that players with a lower r_i get more proposals. $L = 1$, $M = 2$ occurs when $z_1 < z_2 = z_3$, which can be sustained for $r_2 > \frac{r_1+r_3}{2}$ given an intermediate value of δ. On the one hand, δ needs to be sufficiently high so that player 1's strategy can compensate the difference between r_2 and r_3 by proposing to player 2 more often (if $r_2 = r_3$ there is no difference to compensate, so $\underline{\delta} = 0$), but sufficiently low to keep z_1 below z_2 despite player 1 getting more proposals. Likewise, $M = 2$, $H = 1$ occurs when $z_1 = z_2 < z_3$, which can be sustained for $r_2 < \frac{r_1+r_3}{2}$ given an intermediate value of δ. The continuation probability δ needs to be sufficiently high so that player 3's strategy can compensate the difference between r_1 and r_2, but sufficiently low to keep z_1 and z_2 below z_3, despite player 3 getting no proposals (if $r_1 = r_2$, $\underline{\delta} = 0$). Finally, $M = 3$ needs a sufficiently high value of δ so that $z_1 = z_2 = z_3$ despite the possible differences between r_1, r_2 and r_3. The thresholds $\underline{\delta}$ and $\overline{\delta}$ have a different expression in terms of the r-values depending on whether $r_2 < \frac{r_1+r_3}{2}$ or $r_2 > \frac{r_1+r_3}{2}$ (both formulas are equivalent when $r_2 = \frac{r_1+r_3}{2}$). The reason for this is that different inequalities are binding depending on the parameters.

A.8 Equilibria for situations in the experiment

The following table presents the equilibrium predictions for those situations that actually occurred within the experiment. Our main hypotheses are based on these values.

Table A1. Equilibrium predictions

Disagreement values	Majority rule			Unanimity rule	
(r_1, r_2, r_3)	(z_1, z_2, z_3)	(μ_1, μ_2, μ_3)	(y_1, y_2, y_3)	(z_1, z_2, z_3)	(y_1, y_2, y_3)
(0,0,0)	$(23, 23, 23)$	$(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$	$(33, 33, 33)$	$(23, 23, 23)$	$(33, 33, 33)$
(0,0,20)	$(24, 24, 24)$	$(.8, .8, .39)$	$(37, 37, 27)$	$(18, 18, 38)$	$(27, 27, 47)$
(0,0,40)	$(25, 25, 30)$	$(\frac{5}{6}, \frac{5}{6}, \frac{1}{3})$	$(38, 38, 25)$	$(13, 13, 53)$	$(20, 20, 60)$
(0,0,60)	$(25, 25, 37)$	$(\frac{5}{6}, \frac{5}{6}, \frac{1}{3})$	$(38, 38, 25)$	$(9, 9, 69)$	$(13, 13, 73)$
(0,20,60)	$(28, 29, 36)$	$(1, \frac{2}{3}, \frac{1}{3})$	$(43, 34, 24)$	$(4, 24, 64)$	$(7, 27, 67)$
(20,20,20)	$(29, 29, 29)$	$(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$	$(33, 33, 33)$	$(29, 29, 29)$	$(33, 33, 33)$
(20,20,40)	$(31, 31, 31)$	$(.77, .77, .45)$	$(37, 37, 27)$	$(24, 24, 44)$	$(27, 27, 47)$
(20,40,60)	$(36, 36, 36)$	$(.95, .67, .39)$	$(43, 33, 23)$	$(20, 40, 60)$	$(20, 40, 60)$

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