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On a Mechanism that Improves Efficiency and Reduces Inequality in Voluntary Contribution Games

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Abstract

We consider the class of linear voluntary contribution games under the general assumption of heterogeneous endowments. In this context, we assess the performance of the Galbraith Mechanism (GM) relative to a fixed equal sharing allocation in both theory and experiments. Three main empirical results emerge. First, the GM raises average contributions significantly above those under an equal-shares allocation. Second, the GM simultaneously reduces income inequality as it improves efficiency. Third, a player's contribution and allocation behaviour is sensitive to her position in the endowment distribution. In all their decision-making, agents consistently place greater emphasis on contribution levels when they are rich, and on contribution ratios (contributions relative to endowments) when they are poor.

1. Introduction

The challenge of finding a mechanism whereby agents can be encouraged to contribute in public goods games, where full contributions is the social optimum but zero contributions is in their private interests, has attracted significant research interest. Recognizing that agents may differ in their endowments has two implications for this search. Firstly, the mechanism must be able to encourage differently endowed agents to contribute differently. Specifically, the rich must be induced to contribute more than the poor, which would seem likely to require that they rich receive the larger share of the output. Secondly, heterogeneous endowments add a distributive concern to the standard objective of improved efficiency. Will increased contributions require an allocation of the public output so biased in favor of the rich that inequality is increased? Or can we find a mechanism that avoids such an equity-efficiency tradeoff?

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In this paper we demonstrate that the Galbraith Mechanism (GM), introduced in Dong et al. (2019) (DFL), is such a mechanism. The public output game can be viewed as having two stages; contributions are made in the first stage and then the output is allocated in the second. Under the GM, having observed the contributions in the first stage, each player is asked to allocate a share of the public output to each of the other players in the second stage. The final distribution of the public output is determined by these allocations. In a controlled laboratory experiment, where subjects had equal endowments, DFL found the subjects' contributions improved substantially after the implementation of the GM, averaging 92% of endowments in the final round, where 83% of subjects contributed fully.

They also found that, in the allocation stage, a vast majority of players tended to allocate to others in proportion to their relative contributions, behavior consistent with the notion of 'fairness' that agents should be rewarded in accordance with their contributions. When we introduce heterogeneous endowments, as in the new experiments reported in the current paper, we are faced with two alternative allocations that agents might view as fair. The first is proportional allocations based on absolute contributions (PC), where agents essentially ignore differences in endowments. The second is proportional allocations based on the ratio of contributions to endowments (PCR), where agents take full account of the differences in the ability to contribute. We use both allocation behaviors to identify a set of parameter values that allow for a range of possible outcomes in our experiments and find that both have a role to play in our results.

Our first empirical result establishes that, under certain conditions and consistent with our expectations based on theory, the GM significantly improves efficiency relative to the standard model where the public output is distributed evenly. Since endowments are heterogeneous, we can also use their Gini coefficients to compare inequality in the final income distribution under the GM with inequality in both the initial endowment distribution and the distribution which results if we use the alternative equal-sharing mechanism. Our second result is that the GM seems able to overcome potential clashes between equity and efficiency. In the cases where the GM improves efficiency it also reduces inequality, relative to both the endowment distribution and the equal-sharing alternative.

When we consider individual subjects' contribution and allocation decisions, we find that these appear to be sensitive to the subjects' endowment levels. In all treatments, subjects when poor contribute a higher share of their endowments than they do when rich. Conversely subjects when rich contribute a larger amount than they do when poor. Further, when it comes to allocating under the GM, agents allocate to each other according to a weighted average of PC and PCR; putting a higher weight on the PCR when poor than when rich. This gives us our third result. In their decision making, agents consistently place greater emphasis on contribution levels when they are rich, and on contribution ratios when they are poor.

Finally, we can interpret the subjects' allocation behaviors in terms of their willingness to redistribute. As noted, if subjects focus on the PC in making their allocation decisions, they are effectively ignoring the endowment distribution. While if subjects focus on the PCR then they are taking the endowment distribution fully into account when making their allocations. In fact, it appears that subjects allocate using a weighted average of both - but the weights depend on their current endowment in a consistent way. When they are rich agents put a higher weight on PC than when poor. This gives a corollary to our third result. An agent is more willing to redistribute on the basis of endowment differences when poor than when rich.

We see the main contribution of this paper as establishing the conditions under which the GM can, when agents have unequal endowments, simultaneously improve efficiency and reduce inequality, a pair of accomplishments which are conventionally regarded as being mutually exclusive in many economic settings (e.g. Okun, 2015). In doing this we generalize the DFL analysis beyond the common endowments case and illustrate how agents' contribution and allocation behaviours appear to be sensitive to their position in the endowment distribution. We also examine the characteristics of distribution neutral contributions under an equal-sharing allocation rule and draw the relevant conclusions for the redistributive effects of the contributions that we observe.

In outline, the remainder of the paper is as follows. The next section sets the scene and reviews the relevant experimental literature. Section 3 provides the theoretical background to our experiments. Section 4 sets up the experiments and section 5 presents the results. This followed by a summary and conclusions in Section 6.

2. Context and Literature

In many circumstances, economic agents can collectively take advantage of opportunities not available to them individually to create a collective good that can be shared. Such circumstances are broadly defined in economic theory as public goods production problems. One obvious example would be team production by agents under the broad supervision of a principal. The principal organises the team, the team members contribute to the production; the principal collects the output, rewards the team members and retains the residual (profit) for himself. The nature of the reward structure is important to the outcome because it is often difficult or costly for the principal to observe the agents' individual contributions. Since effort is costly to the agent, a fixed wage or salary structure, which does not provide a link between agent effort and agent reward, can lead to shirking and a loss of income for the principal. Profit sharing has been suggested as a response, but under equal shares, which is the natural allocation for a principal to impose when she cannot observe individual agents' behaviour, a free-rider problem arises since each agent bears the full cost of their effort but only reaps $1/N$ th of the benefit in an N -agent team. Agents are unlikely to contribute the socially optimal

(full) effort under an equal-sharing allocation.

That said, experiments using the voluntary contribution mechanism (VCM) tend to find that contributions significantly exceed the Nash-equilibrium prediction (typically zero contributions); but also fall significantly short of the social optimum (full contributions). When participants are equally endowed, initial contributions typically lie between 40 and 60 percent of the endowment, decay over time but remain at 15–25% of the endowment by the final round, although by then about 70 percent of subjects are contributing zero (Ledyard, 1995; Ostrom, 2000). Agents appear to make tentative steps towards cooperating in the direction of the social optimum, before disillusion eventually sets in and contributions decline in later rounds.

In practice, teams are unlikely to be composed of identically endowed individuals. Inequality in endowments does not change the essential nature of the problem – with equal shares, zero contributions is still the Nash Equilibrium, full contributions is still the social optimum. In his early review of public goods experiments, Ledyard (1995) conjectured that heterogeneity in endowments would tend to decrease total contributions compared to homogeneity, but the subsequent evidence has been mixed. Where there is significant inequality in endowments, most studies find a significant reduction in total contributions (see Cherry et al. 2005, Buckley and Croson, 2006, Reuben and Riedl, 2013, and Hargreaves Heap et al., 2016, 2017)¹, though there are exceptions that find significantly increased total contributions (Visser and Burns, 2006; Prediger, 2011; Chan et al. 1996); or no significant change in contributions at all (Hofmeyr et al. 2007).²

Interestingly, any reduction in total contributions does not appear to be reflected in proportionate reductions at all endowment levels. Hargreaves Heap et al. (2016) test how inequality in endowments affects contributions to a public good, while controlling for possible endowment size effects. They consider groups of three differently-endowed agents and find that the key adverse effect of inequality on contributions arises because the rich reduce their contribution ratios (contribution as a share of endowment), while the poor and middle maintain their contribution ratios, in comparison with the corresponding uniform endowment games. They emphasize that the fall in the contribution ratio of the rich relative to the poor is a robust pattern and that this difference in behavior drives the fall in overall contributions under inequality.³ Cherry et al. (2005) also find that subjects with high endowments contributed significantly less when they

¹Other experimental studies indicate that heterogeneous valuations of the public good lead less frequently to the efficient outcome (see e.g. Nikiforakis et al. 2012) and to lower (unconditional) contributions (Fischbacher, et al. 2014)

²Some part of these differences may be attributable to differences in other characteristics of the experiments – single-shot versus repeated games; fixed groups versus random reassignments etc.

³Hargreaves Heap et al. (2017) note that an effect of the introduction of team competition with unequal teams is that the contribution ratio of the rich jumps back up to that of the other income groups. Empirical evidence from non-linear public good games also suggests that the poor tend to over-contribute and the rich to under-contribute (Chan et al., 1996).

were in heterogeneous groups.

These observations have prompted research to explain this behaviour.⁴ Some explanations involve non-traditional preferences. If agents care not only about their own consumption but also about the consumption of others, then voluntarily contributing to provide public goods can be equilibrium behavior. Other explanations involve notions of reciprocity (Sugden, 1984). The principle of reciprocity requires that if members of the group are voluntarily contributing to a public good from which an agent derives benefits, then that agent is ‘morally bound’ to reciprocate and contribute to the good, even though self-interest would suggest otherwise. Agents following this principle will act as conditional co-operators, contributing when others contribute, but also defecting if they defect. The evidence suggests that players begin cooperatively but cooperation cannot be maintained (Keser and van Winden, 2000; Fischbacher et al., 2001).

If contribution behavior is being driven by notions of fairness and reciprocity, then we should be able to extract information on the underlying notions of fairness from the observed contributions. There are two obvious candidates for fair contributions - equal absolute contributions or equal contribution ratios. While these are the same when endowments are identical, Sugden (1984) suggested that, where endowments are heterogeneous, reciprocating agents will contribute their ‘fair share’ to the provision of a public good, provided that others are also contributing - fair share in this context being the same contribution ratios. There is some experimental evidence that agents cooperate in this way (Hofmeyr et al., 2007; Keser et al., 2014; Keser and Schmidt, 2014). But, this fair-share rule has its limitations. When the asymmetry in the endowments becomes so large that one of the agents loses interest in the group optimum, the norm appears to shift from equal contribution ratios to equal contribution levels and the group contribution level declines significantly (Keser et al., 2014). Recalling that it is the rich who appear to renege on the equal ratios, it seems likely that an agent’s notion of what is a ‘fair’ contribution may very well depend on her (relative) endowment.

The typically observed patterns of behaviour in the experimental literature have also prompted research suggesting various mechanisms through which contributions can be improved. A number of studies have demonstrated that players exhibit social preferences to ‘punish’ those who free-ride on the group production (Fehr & Gächter, 2000) and to ‘reward’ those who contribute more than the group average (Sefton et al., 2007; Nosenzo

⁴Models of non-traditional preferences have included altruism (Becker, 1974), where an agent’s welfare function includes the consumption of others; warm-glow altruism (Andreoni, 1990), where altruism is strengthened by a warm glow from giving; and inequality aversion (Bolton and Ockenfels, 2000; Fehr and Schmidt, 1999), where agents dislike unequal net incomes or consumption. While these models have been shown to successfully predict contribution behavior in linear public goods experiments with identical players, Buckley & Croson (2006) observe that their prediction that the poor should contribute either less absolutely or a lower proportion of their endowments than the rich when endowments are heterogeneous are opposite to the experimental outcomes.

& Sefton, 2012). The option to punish and reward has, therefore, proved effective in moving the outcome towards social efficiency, but only if the right conditions are in place (e.g. a cost-effective punishment structure and the absence of antisocial punishment). The practicality of implementing costly punishment within organisations remains an issue (Nikiforakis, 2008).⁵

Other mechanisms move away from the fixed share allocation.⁶ In the principal-agent context, fixed (and equal) shares seems the logical allocation structure, given the principal’s ignorance of the individual agents’ endowments and contributions.⁷ But while the principal may be unable to observe agents’ efforts, there will be occasions where the agents themselves are in a position to observe others’ abilities and actions.⁸ The challenge then for the principal is to design a mechanism that elicits and uses this information to induce the appropriate levels of effort from the agents. One approach has been to introduce an allocator, who can be from inside (a ‘stakeholder’) or outside (a ‘spectator’) the team. In Van der Heijden et al. (2009) and Drouvelis et al. (2017) a stakeholder is assigned the role of team leader, tasked with determining the allocation of output to all members, including herself since she is also a contributor. While production significantly increases relative to an equal-sharing allocation, the self-interest of the leader can prevent the attainment of full efficiency. In Stoddard, Cox & Walker (2019) the output is distributed to members by a third-party spectator whose own reward is determined by the total contributions of the group. In general, spectators allocate shares to members based on the rank order of their contributions. In our context, it is as if the uninformed principal appointed an outsider as ‘manager’ whose role is to observe the efforts of the agents and to allocate from the pool. Since the manager’s reward is also related to the pool size, she is motivated to allocate so as to encourage full contributions.⁹ These monitoring costs would then be subtracted from the total output to determine the return to the principal.

Two further mechanisms have been found to be quite successful when endowments are homogeneous. Baranski (2016) allows the players’ shares of the output to be determined through a Baron & Ferejohn (1989) multilateral bargaining process in which

⁵See Chaudhuri (2010) for a review.

⁶Other adjustments involve adding further dimensions to the overall game. For example, if teams are put in competition (via a standard VCM stage for each team individually that leads into a Tullock game between teams) this tends to overcome the free rider problem (Hargreaves Heap, Ramalingam & Stoddard, 2017).

⁷In fact equal shares is the best fixed-share option even if the principal knows the agents endowments. See Appendix A

⁸Freeman (2008) reports survey results showing: that most workers believe they are able to detect shirking by co-workers; that those participating in a profit-sharing scheme are more likely to act against shirking; and that such anti-shirking behaviour tends to reduce shirking.

⁹Although this does not always happen. Stoddard et al. (2019), find instances of ‘rogue’ spectators who do not reward greater contributions with larger shares.

each player’s probability of being the proposer can be either fixed or increasing in their contributions. In each period, after a division is proposed, the remaining players can vote to agree or disagree with the proposal. The bargaining process ends if a majority (specified by some rule) agree with the proposer and the fund is divided as per the proposal. In Baranski’s experiments, subjects steadily raise their contributions averaging 85.4% of endowment in the last 5 periods. By the last period of play, 86 out of 120 (72%) subjects contribute all their endowment. Potential distortions from allocating to self and to obtain votes remain, but the former is moderated by the bargaining structure which inhibits proposers from being too greedy.

An alternative allocation mechanism, involving only the agents, is the GM noted earlier.¹⁰ In the context of an uninformed principal and informed agents, the GM is a method of determining an informed allocation which requires only that the principal collate the allocation shares proposed by the agents and distribute accordingly. No informed spectator or bargaining process is required. Since the GM is the allocation mechanism that we employ below, we discuss it in some detail here. The GM takes the form of a two-stage game. In the first (VCM) stage, each agent chooses a contribution level and in the second stage, after having observed others’ contributions, each agent proposes a share of the public output to be received by each of the remaining agents. An agent’s final share depends on the other agents’ allocations toward her. The crucial feature of the GM is that how an agent allocates shares in the second stage does not affect her own payoff. Therefore, agents are free to punish, to reward, to allocate equally or even randomly to the other agents, while no costs are incurred by any agents in the allocation exercise. Provided they allocate in a way that encourages agents to contribute in the first stage a more efficient outcome will be achieved. DFL report that under the GM, contributions averaged 80% of endowment overall and averaged 91.6% in the final round, where 82.8% of players contribute fully. Most allocations under the GM are related to players’ contributions and the overall outcomes are consistent with players following a proportional rule.¹¹

When endowments are heterogeneous, subjects’ contribution behavior may be sensitive to two matters - where they are in the hierarchy (i.e. rich, middle or poor), and how they perceive the source of the inequality. Starmans et al. (2017) argue that it is not inequality per se that bothers people, but economic unfairness, and there is evidence to support this view (e.g., Bortolotti et al., 2017; Breza, Kaur, and Shamdasani, 2018;

¹⁰The “Galbraith Mechanism” label arises as the idea was inspired by John Kenneth Galbraith who, in an aside in *The Great Crash 1929*, described a bonus sharing scheme used by the National City Bank in the U.S. in the 1920s. Under this scheme each officer would sign a ballot giving an estimated share of the bonus pool towards each of the other eligible officers, himself excluded. The average of these shares would then guide the final allocation of the bonus to each of the officers (Galbraith, 1963, p.171).

¹¹This is consistent with the principle of ‘distributive justice’ that a player’s entitlement towards some group outcome should be proportional to her contribution to that outcome (Konow, 2000).

Fehr, 2018). Prominent normative theories of justice assume that inequalities arising from factors under an agent’s control should not be eliminated (e.g., Roemer, 1998; Konow, 2003). These theories receive experimental support, which suggests that most agents do not eliminate inequalities that are due to merit or for which agents can be held responsible (Konow, 2000; Cappelen et al., 2007, 2013; Mollerstrom et al., 2015). The broad conclusion seems to be that if subjects had equal opportunities and the income differences are purely attributable to subjects’ actions, then agents are disinclined to redistribute; while if income differences are purely attributable to luck, then agents are more inclined to redistribute. But importantly, this statement reflects the aggregation of quite different individual preferences, as emphasised by Cappelen et al., (2013).¹² Given that the differences in endowments imply different abilities to contribute, agents may differ in their views on what constitutes a ‘fair’ pattern of contributions, and these views may depend on the agent’s position in the endowment distribution and extent of the inequality.¹³ Baranski (2016) notes that, at the bargaining stage in his model, subjects seem to abide by the most convenient norm of fairness when they propose allocations. In his case (with common endowments) low contributors propose allocations that on average yield lower outcome inequality, while high contributors are more likely to allocate shares proportional to agent’s contributions.

Finally, the hypothesis that subjects may restrict their contributions if they view others as receiving a disproportionate share of the benefits is supported by the experiments of Kuy & Salmon (2013). They measured subjects’ willingness to approve Pareto improvements when the improvements mainly favored the already rich. They found that the poor did not choose the contribution which would maximize their own income (as well as achieve social efficiency). While we expect such behavior to be moderated as the opportunity cost (in terms of sacrificed income gains) increases (e.g. Charness and Rabin, 2002; Engelmann and Strobel, 2004), it may explain the situations that arise in some of the treatments we consider below.

¹²Cappelen et al., (2013) construct an experiment that involves a risk-taking phase followed by a redistribution phase. They consider three ‘fairness views’. ‘Ex ante fairness’ is the view that if initial opportunities are equal there is no argument for redistribution of the gains and losses from risk-taking; ‘ex post fairness’ favours redistribution however the inequality came about; and, a middle ground that they refer to as ‘choice egalitarianism’, which holds people responsible for their choices but not for their luck. Such a view would endorse ex post redistribution between lucky and unlucky risk-takers but not between risk-takers and participants who choose the safe alternative. They find that the ex ante fairness view has the largest share of support (40%); but most subjects favour some ex post redistribution, even when people had the same ex ante opportunities. Among those favouring redistribution, only a minority (30%) endorses equalization of all ex post inequalities, and most favour equalizing ex post inequalities resulting from differences in luck among risk-takers. But the authors emphasize that their evidence reveals considerable disagreement on how to allocate fairly the gains and losses from risk taking.

¹³Experimental evidence suggests that the origin of endowments does not matter for contributions. Subjects contributed about the same level regardless of whether their endowments are earned or windfall (see Clark 2002, and Cherry, T., S. Kroll, & J. F. Shogren 2005).

In the next section we set out the VCM model more formally and then illustrate the constraints which must be imposed on contributions to avoid income redistribution under an equal sharing allocation rule. This is followed by a formal exploration of our ‘fair’ allocations (PC and PCR) under the GM. The section concludes with the details of our experimental treatments derived from this analysis and the statement of our hypotheses derived from the experimental literature just reviewed.

3. Theory and Hypotheses

We consider the DFL model with heterogeneous endowments. There are three agents (players) and one principal, in the context of a two-stage game. Each player, indexed $i \in N = \{1, 2, 3\}$, has an initial endowment of $e_i > 0$ and takes an action $c_i \in E_i = \{0, 1, \dots, e_i\}$ in the first stage.¹⁴ The agents do so simultaneously. The players’ actions determine a joint monetary outcome $\Pi = \beta \sum_{i=1}^3 c_i$ which must be allocated among the players, where $\beta > 1$ is a parameter that represents the scale of returns of the production function. The payoff function of player i is given by

$$\pi_i = e_i - c_i + q_i \Pi, \quad (3.1)$$

where q_i is the share of the joint monetary outcome received by player i .

Under the standard assumption that the players receive equal shares of the output (ES), that is $q_i = \frac{1}{3}$ for each i , the payoff to player i is given by

$$\begin{aligned} \pi_i &= e_i - c_i + \frac{1}{3} \beta \sum_{i=1}^3 c_i, \\ &= e_i + \frac{1}{3} \beta \sum_{j \in N \setminus \{i\}} c_j + \left(\frac{1}{3} \beta - 1 \right) c_i. \end{aligned} \quad (3.2)$$

For any given contributions by the other players, the best response of player i is full contributions ($c_i = e_i$) if and only if $\beta \geq 3$ and zero contribution otherwise.¹⁵

3.1. Redistribution and Distribution Neutral Contributions (DNC)

In experiments, subjects do make positive contributions even when $\beta < 3$, and these contributions differ in a systematic way depending on endowments.¹⁶ Some perspective

¹⁴Our notations slightly differ from DFL as we use c_i in the place of e_i for contribution levels and e_i instead of \bar{e}_i for player i ’s endowment.

¹⁵Since $\sum_{i=1}^3 \pi_i = \sum_{i=1}^3 e_i + (\beta - 1) \sum_{i=1}^3 c_i$, it is socially optimal for players to contribute $\sum_{i=1}^3 e_i$ (0) if and only if $\beta > (<) 1$.

¹⁶Since low-income subjects can realize higher net benefits from mutual cooperation, they have stronger incentives to cooperate, at least initially, in the hope that high-income subjects will reciprocate.

on this contribution behaviour can be gained if we consider explicitly the income distribution implications of the way in which the joint output is allocated. The equal-shares allocation commonly assumed will involve redistribution, unless subjects contribute in a particular way, and this could be a considerable redistribution if contributions are high.¹⁷

What then is the distribution of the contributions that leaves the income distribution unaffected once the output has been distributed equally? We explore this in detail in Appendix A1 and simply report the results here. If E denotes the total endowment given to the subjects, and C the total contributions that these players make, we find that the contribution share of player i ($\varphi_i \equiv \frac{c_i}{C}$) must satisfy the following equation if i 's income share is to remain equal to her endowment share ($\theta_i \equiv \frac{e_i}{E}$).

$$\varphi_i = \theta_i + \beta \left(\frac{1}{3} - \theta_i \right) \quad (3.3)$$

If a subject's endowment share is equal to the mean of the distribution ($\frac{1}{3}$), then she should contribute in proportion to her endowment share. Otherwise, a subject with an endowment share smaller (larger) than the mean should have a higher (lower) contribution share than her endowment share, with the magnitude of these differences increasing in the scale parameter (β). This implies that the poor must contribute a larger share of their endowment than the rich if the income distribution is to be maintained. Further, if we convert the above equation to contribution levels, we find that DNC requires that the contribution level of the rich is lower than that of the poor.¹⁸ Seen in this light the apparently modest contribution of the rich observed in public goods experiments may reflect a greater underlying willingness to redistribute than appears at first glance.

3.2. The Galbraith Mechanism (GM)

Under the GM the allocation takes place in the second stage as follows. Each player i observes all actions taken in the first stage and proposes share a_{ij} of the outcome to each player $j \neq i$ such that

$$a_{ij} \in [0, 1] \text{ and } a_{ij} + a_{ik} = 1, \text{ where } k \neq j \text{ and } k \neq i \quad (3.4)$$

¹⁷Indeed, if all subjects were to contribute fully they would simultaneously achieve the socially optimal output and a completely even distribution of income. Even if all subjects contribute equal amounts, the fact that they all receive the same payment from the pool (an amount higher than their contributions) will mean inequality (as measured by the Gini coefficient) is reduced because there is a disproportionate increase in the income of the poorer subjects.

¹⁸Illustrative numbers are given in Table 1 below.

In other words, each player proposes a fraction of Π to be received by each of the other players. They do so simultaneously. We let q_i denote player i 's final share of the outcome Π and we assume that it is determined by: $q_i = \frac{a_{ji} + a_{ki}}{3}$. While the GM has freed each player from the constraint of having to protect her own interests at the second stage, it can be easily verified that any allocation constitutes a second stage Nash Equilibrium. Hence, at this point we have no unique theoretical prediction on how a player might allocate between the other two players. One method of removing the resulting arbitrariness is by explicitly incorporating a behavioural component into the payoff function, which could be seen as reflecting the player's subjective notion of a "fair" allocation. But rather than imposing a solution in this way, we follow DFL and leave the question of how the players actually allocate to be uncovered in the experiments that follow.

In their investigation of equilibria in this contribution game with common endowments, DFL found a link between efficiency and "fairness". Distributive justice is often defined by the principle that a player's entitlement towards some group outcome should be proportional to her contribution to that outcome. DFL established that a necessary and sufficient condition for PC allocation behaviour under the GM to support efficiency (full contributions) as a SPNE of this contribution game, was that $\beta \geq \frac{6}{5}$. However, they also found that $\beta \leq \frac{3}{2}$ was necessary and sufficient to obtain zero contributions as a SPNE of this game. In these circumstances it is reasonable to consider $\frac{3}{2}$ as the lowest lower bound to support full contributions as a SPNE as only then does the GM give clear theoretical predictions. When we relax the assumption of common endowments, we find that the lower bounds on β for which full contributions are attainable as part of a SPNE now depend on the distribution of endowments as well as players' allocation behaviors. We illustrate this in the next subsection.

As in DFL, we represent the GM as an extensive game, with simultaneous moves at each stage, using Moore and Repullo's (1988) formulation. Under their formulation, at each node, all players know the entire history of the moves preceding it, and they can therefore use history-dependent strategies. Following Moore and Repullo, we define a strategy of player i by the pair $s_i \equiv (c_i, a_i(c))$, where c_i is a contribution level of the player at his first information set and $a_i(c) \equiv (a_{ij}(c), a_{ik}(c))$ is an allocation function that depends on the first stage contribution tuple c and that satisfies the definition of a_{ij} . As result, $q_i = \frac{a_{ji} + a_{ki}}{3}$ must also depend on tuple c and we can therefore denote q_i by function $q_i(c)$. The allocation function $a_i(c)$ prescribes an action to player i for each of her remaining information sets. We then refer to triple $s = (s_1, s_2, s_3)$ as a strategy profile of the game. We use the same definitions of Nash Equilibrium (NE) and Subgame Perfect Nash Equilibrium (SPNE) as in Moore and Repullo.

It can easily be verified that every function $a(c) = (a_1(c), a_2(c), a_3(c))$ prescribes a Nash equilibrium in each stage two subgame. Then, by backward induction, it suffices to find the Nash Equilibrium of the resultant first stage game, by taking function $a(c)$

as given. Then, $c^* \equiv (c_i^*, c_{-i}^*)$ is a Nash Equilibrium of the resultant first stage game if and only if for all i and for all c_i , we have

$$\pi_i((c_i^*, c_{-i}^*), (a(c_i^*, c_{-i}^*))) \geq \pi_i((c_i, c_{-i}^*), (a(c_i, c_{-i}^*))) \quad (3.5)$$

It should be clear that the satisfaction of the above inequality, together with the fact that every allocation function $a(c)$ prescribes a Nash equilibrium in each second stage subgame, imply that s^* is a SPNE of the game, where

$$s^* = ((c_1^*, a_1(c^*)), (c_2^*, a_2(c^*)), (c_3^*, a_3(c^*))) \quad (3.6)$$

We are interested in allocation functions that support full contribution tuple $e = (e_1, e_2, e_3)$ as part of a SPNE. We call e the full contribution equilibrium.

Remark 1 Under the GM, $\beta \leq \frac{3}{2}$ is necessary and sufficient for $(0, 0, 0)$ to be part of a SPNE for all possible $q_i(c)$.

Proof: See the appendix A2.

In the paper, we often refer to $(0, 0, 0)$ as the zero contribution equilibrium. Since we are interested in instances in which our proposed mechanism rules out the zero contribution equilibrium, we only assess the effectiveness of the mechanism in supporting the full contribution equilibrium for all $\beta > \frac{3}{2}$.¹⁹

3.3. Specific Allocations under the GM

We now consider two specific allocation behaviours. To simplify the discussion, where necessary we assume a particular form of the endowment distribution (which we will use in our experiments) that depends explicitly on a single dispersion parameter δ . In particular, we assume the following.

$$\begin{aligned} e_1 &= (1 + \delta) \epsilon \\ e_2 &= \epsilon \\ e_3 &= (1 - \delta) \epsilon \\ \text{where } \delta &\in (0, 1) \end{aligned} \quad (3.7)$$

We let player 1 (R) represent the ('rich') player with the largest endowment, player 2 (M) denote the ('middle') player with the mean endowment, and player 3 (P) denote the ('poor') player with the smallest endowment. We can assume, without loss of generality $\epsilon = 1$.²⁰

¹⁹It is possible that both a full contribution equilibrium and a low contribution equilibrium exist when $\beta \leq \frac{3}{2}$ for some allocations in the GM.

²⁰In our experiments $\epsilon = 8$.

3.3.1. Allocation Shares Proportional to Contributions (PC)

If players ignore endowment differences and simply allocate shares based on relative contributions, then player i allocation to player j will be given by

$$a_{ij}(c) = \begin{cases} \frac{c_j}{c_j + c_k} & \text{if } c_j + c_k > 0 \\ \frac{1}{2} & \text{otherwise.} \end{cases} \quad (3.8)$$

While the full details of derivations are provided in Appendix A2, we can use the particular form of the endowment distribution and the above equation to determine the share of each player in the full contributions equilibrium under the PC allocation as

$$q_1(e) = \frac{1 + \delta}{3} \left(\frac{4 + \delta}{2(2 + \delta)} \right) < \frac{1 + \delta}{3} \quad (3.9)$$

$$q_2(e) = \frac{4}{3} \left(\frac{1}{4 - \delta^2} \right) > \frac{1}{3} \quad (3.10)$$

$$q_3(e) = \frac{1 - \delta}{3} \left(\frac{4 - \delta}{2(2 - \delta)} \right) > \frac{1 - \delta}{3} \quad (3.11)$$

If full contributions are achieved under the PC allocation, then in the final income distribution the rich player has an income share lower than his endowment share, while the income shares of the other two players have increased. Furthermore, this redistribution is increasing in δ .

Proposition 1 Suppose that $\beta > \frac{3}{2}$, then $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the PC allocation.

Proof: See the appendix A2.

Proposition 1 shows that (i) under the proportional allocation behavior, for all $\beta > 3/2$, there exist a SPNE in full contribution. Moreover, the equilibrium is independent of the endowment inequality parameter. Taking account of our comment above, we see that, under the proportional allocation, allowing for heterogeneous endowments does not change the lower bound for full contributions to be part of a SPNE and for zero contributions to be ruled out as a SPNE. The proportional allocation is robust to unequal endowments.

3.3.2. Allocation Shares Proportional to Contribution Ratios (PCR)

In this case players' views of fairness take full account of endowment differences and they allocate according to contribution ratios. Player i allocation to player j will be given by

$$a_{ij}(c) = \begin{cases} \frac{\frac{c_j}{e_j}}{\frac{c_j}{e_j} + \frac{c_k}{e_k}} & \text{if } \frac{c_j}{e_j} + \frac{c_k}{e_k} > 0 \\ \frac{1}{2} & \text{otherwise.} \end{cases} \quad (3.12)$$

The full details of derivations are provided in Appendix A3, but we can use the particular form of the endowment distribution and the above equation to again determine the share of each player in the full contributions equilibrium under the PCR allocation as

$$q_1(e) = \frac{1}{3} < \frac{1 + \delta}{3} \quad (3.13)$$

$$q_2(e) = \frac{1}{3} \quad (3.14)$$

$$q_3(e) = \frac{1}{3} > \frac{1 - \delta}{3} \quad (3.15)$$

Proposition 2 Suppose that

$$\beta \geq \begin{cases} \frac{3}{2} & \text{if } \delta \leq \frac{1}{2} \\ 1 + \delta & \text{otherwise} \end{cases}.$$

Then, $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the PCR allocation.

Proof: See the appendix A3.

Proposition 2 shows that, under the PCR allocation, the lower bound for full contributions to be part of a SPNE and for zero contributions to be ruled out as a SPNE is increasing in the dispersion parameter δ . If full contributions are achieved under the PCR allocation, then the final income distribution has equal shares for all players. Compared with the endowment distribution, the rich player has a lower income share, the middle player's income share is unchanged and the income share of the poor player has increased. If the scale of returns is sufficiently high that full contributions are attained, then there is no trade-off of equity for efficiency under the PCR allocation. Quite the opposite, retaining some of the initial inequality also requires a sacrifice in efficiency.

Of course, the same claim can be made for the equal shares allocation, where $\beta \geq 3$, is necessary and sufficient for full contributions to be a SPNE under an equal shares allocation. Thus, full contributions and an equal income distribution could be achieved simultaneously. So, for the PCR allocation to have been preferred by the principal, it would have had to be capable of achieving a full contributions equilibrium at some $\beta < 3$.

3.3.3. Combined Allocations MPCR

Define the allocation share of player j to player i , a_{ji} as follows

$$a_{ji} = \gamma a_{ji}^{PC} + (1 - \gamma) a_{ji}^{PCR} \quad (3.16)$$

where $\gamma \in [0, 1]$. We called the above allocation the mixed PC and PCR allocation (MPCR).

In Appendix A4 we investigate the bounds on β for allocations that are a linear combination of the PC and PCR allocations to support the full contribution equilibrium while ruling out the zero contribution equilibrium. This information will be useful because our empirical results suggest that the allocations of a representative subject can be approximated by a linear combination of these forms. We show that the lower bound on β in these cases is the correspondingly weighted average of the bounds on PC and PCR.

Proposition 3 Suppose that

$$\beta \geq \left\{ \begin{array}{ll} \frac{3}{2} & \text{if } \delta \leq \frac{1}{2} \\ \frac{1}{\left(\frac{2\gamma}{3} + \frac{1-\gamma}{1+\delta}\right)} & \text{if } \delta > \frac{1}{2} \end{array} \right\}.$$

Then, $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the MPCR allocation. Moreover, the above lower bound is decreasing in γ .

Proof: See the appendix A4.

3.4. Summary and Hypotheses

Based on the modelling above we construct Table 1 which summarises the relevant information for the experiments described in the next section. Note that we will test the GM under two different levels of inequality, namely $\delta = 1/2$ and $\delta = 3/4$. The table presents, for each level of inequality, the minimum β required to support full contributions as part of a SPNE, depending on players' allocation behavior, while ruling out the zero contribution equilibrium. We will also include two control treatments, one for each of level of inequality, in which the public output is allocated in equal shares to each of the players.

Table 1.1: Summary of Endowment Data and β Thresholds

$$\text{Low Inequality } \delta = \frac{1}{2}$$

	Rich	Middle	Poor
e_i	12	8	4
$\frac{e_i}{E}$	0.5	0.33	0.17
β threshold (PC)	1.5	1.5	1.5
q_i (PC)	0.45	0.36	0.19
β threshold (PCR)	1.5	1.5	1.5
q_i (PCR)	0.33	0.33	0.33

High Inequality $\delta = \frac{3}{4}$

	Rich	Middle	Poor
e_i	14	8	2
$\frac{e_i}{E}$	0.58	0.33	0.08
β threshold (PC)	1.5	1.5	1.5
q_i (PC)	0.5	0.39	0.11
β threshold (PCR)	1.75	1.75	1.75
q_i (PCR)	0.33	0.33	0.33

Notes: e_i is the endowment of player i , E is the total endowment and q_i is the output share received by player i .

Table 1.2: Control Rounds 1-5 $\delta = \frac{5}{8}$, $\beta = 1.7$

e_i	13	8	3
$\pi_i(e_1, e_2, e_3)$ (ES)	13.6	13.6	13.6
$\pi_i(3, 3, 3)$ (ES)	15.1	10.1	5.1
$\frac{c_i}{C}$ (DNC)	0.19	0.33	0.48

Notes: c_i is the contribution of player i and C is the total contribution. The β threshold is for the rich player to contribute fully assuming that the others contribute fully. This threshold also works for the other two players as it is the maximum of all such β thresholds across the players.

A comparison of output shares at full contributions with endowment shares reveals that all the allocations considered involve redistribution and this redistribution reduces inequality. In the case of ES and PCR, inequality is eliminated at full contributions. The reduction in inequality under PC is much more muted. Away from full contributions the change in the income distribution will depend on the actual contributions of the players, but a presumption remains that a significant increase in average contributions will be accompanied by a reduction in inequality. This is because, beyond some point

the increase in average contributions can only happen if the contributions of the rich increase disproportionately.

In the first 5 (control) rounds of our experiment, we will not implement the GM, but instead employ the equal sharing mechanism standard in the public goods literature. The lower part of Table 1 relates to these rounds, in which δ is set at $5/8$ and β at 1.7. If all subjects contribute fully, then they all gain. But these gains differ quite widely across the subjects; the rich gain only 0.6 while the poor gain 10.6. If all subjects contribute the same amount (equal to the lowest endowment, 3), again they all gain, but this time the gains are equal (at 2.1) and the effect on income inequality is less pronounced.²¹ If contributions are to be such that the income distribution is unchanged, then the poor must put up 48% of the total contributions, compared with only 19% for the rich. We suggest that very few subjects are likely to view such a pattern of contributions as ‘fair’, and that contributions under equal shares are likely to result in reduced inequality.

Our choices of values for the scale of returns parameter (β) are guided by the numbers reported in Table 1. We know that full contributions is an SPNE under ES if $\beta > 3$, so our principal-agent problem only exists for $\beta < 3$.²² We choose 4 different values for β . The objective is to include some values under which full contributions is a SPNE for both allocation types and inequality levels, and others where full contributions is not supported as an SPNE in at least one case. Thus when $\beta = 2.2$, both allocations can achieve an SPNE, and the same is true of $\beta = 1.8$, but this value is close to the threshold of 1.75 for the PCR in the high inequality case. Only PC has full contributions as a SPNE at $\beta = 1.6$ for all inequality levels. While if $\beta = 1.2$ zero contributions are an SPNE of the game, lending some ambiguity to the possible outcomes.

Based on all we have considered, what do we expect our experiments to show? From DFL we expect subjects to allocate in accordance with some notion of fairness in the second stage and this to quickly impact on contributions in the first stage. Since fair shares are positively related to contributions (directly in the case of PC and more indirectly in the case of PCR) we expect average contributions to increase (relative to the equal shares control). Further, as fair allocations also involve inequality reducing redistribution, we expect the increase in average contributions to be accompanied by a reduction in inequality (which we will measure by the Gini Index). Thus, we expect to find that the GM both improves efficiency and reduces inequality.

Whether the GM will achieve full contributions and full equality is more problematic. Our choice of parameters allows for such a possibility, but only if the subjects make the appropriate allocation choices. The endowment differences are based entirely on luck and experimental subjects have been shown to be tolerant of significant redistribution in such circumstances. That said, experimental subjects also have been found to have

²¹ The highest income is now 3 times the lowest compared with 4 times at the endowment stage.

²² Although our results below strongly suggest that β might need to be significantly greater than 3 to actually achieve full contributions when endowments are heterogeneous.

quite divergent views on how much redistribution is ‘fair’. Achieving full efficiency may involve a wide disparity in the gains from contributing, and subjects may be willing to restrict contributions that are pareto improving, if they feel others are getting a disproportionate share of the benefits.

Using the results of this section and drawing on our brief review of the existing experimental literature we put forward the following hypotheses. First, regarding the control treatments in which we impose equal sharing of the public output, we have

ES1 Average contributions decline as the rounds progress and

ES2 Increased endowment inequality will lead to lower average contributions.

Hypothesis ES1 is a standard result, independent of endowment heterogeneity. ES2 may or may not be confirmed – the experimental literature is ambiguous on this. The next hypotheses are alternative hypotheses relating contributions to endowments. The positive contributions that we expect to observe in the control cases (where self-interest would imply zero contributions) reflect reciprocity based on some underlying notion of fairness. The two alternative notions of fairness that arise in our case relate to equal contribution ratios and equal contribution levels. Thus, we have for any given level of endowment inequality:

ES3 (a) The contribution ratio of the rich is not significantly different from that of the poor; or (b) the contribution level of the rich is not significantly different from that of the poor.

Second, for the GM treatments we have several hypotheses derived from theory, the performance of the GM under homogeneous endowments and the behaviour of subjects under equal shares when endowments are heterogeneous. These are:

GM1 For a given the level of endowment inequality, average contributions will (a) be higher than under the equal sharing mechanism; (b) increase or remain steady as the rounds progress; and (c) be positively related to the scale of returns (β).

GM2 For a given scale of returns, average contributions will be negatively related to the level of endowment inequality.

GM3 For given levels of endowment inequality and scale of returns, (a) the contribution ratio of the rich is not significantly different from that of the poor; or (b) the contribution level of the rich is not significantly different from that of the poor.

We expect that the allocation behavior of subjects might reflect a mix of proportional allocations based on contributions (PC) and proportional allocations based on contribution ratios (PCR), and that this mix may vary depending on the scale of returns and the level of endowment inequality. But we are unable to draw any hypotheses on these variations based on the literature. Since both allocations tend to reduce inequality, however, we expect that:

GM4 For a given level of endowment inequality, the final income distribution is less unequal than both (a) the endowment distribution; and (b) that in the corresponding control treatment.

These are the hypotheses that we take to the data.

4. Experimental Design and Procedure

As explained in the previous section, we focus on two levels of endowment inequality, under each of which we ran four treatments with different β , providing a total of eight GM treatments. As in DFL, we also required a baseline against which to compare the effects of the GM. We therefore implemented Control treatments for both Low Inequality (LI) and High Inequality (HI), which used an equal sharing mechanism. In each Control treatment, we set β at 2.2, equal to its highest value in any GM treatment. Thus, we compare the effectiveness of the GM against our baseline under circumstances where the scale of returns on production should either make a successful outcome under the GM as difficult as in the baseline (when $\beta = 2.2$), or even more difficult (when $\beta < 2.2$). In short, we set tough tests for the GM to pass.

All treatments consisted of 15 rounds of the game. In all cases, subjects first played five rounds with the equal sharing mechanism. In the GM treatments, the GM was introduced in the sixth round and then remained in force until the end of the experiment. In the Control treatments, the equal sharing mechanism instead continued throughout. In the first five rounds, we set $\beta = 1.7$ and $\delta = 5/8$ in all treatments.²³ Our interest is in studying behaviour under the GM in rounds 6-15, and comparing this against behaviour during the same rounds in the Control treatments. Based on the public goods game literature, we expected contributions to decline across the first five rounds; by introducing the GM after this point, our aim was to test the impact of a potentially efficiency-enhancing mechanism from a starting point of low efficiency (as was done by DFL and as has been done in tests of other efficiency-improving mechanisms (e.g. Fehr and Gächter, 2000)).

The treatment names and design can be found in Table 2. Subjects initially received instructions for the first five rounds and were provided no information about rounds 6-15 until after the end of the fifth round. All treatments used a stranger matching design, with subjects randomly re-allocated to new trios after each round, and unable to track the identity of other players across rounds. In each round, it was randomly determined which subjects within each trio would be rich, middle or poor. Each session consisted of nine subjects.

We employed a between-subjects design with each subject only participating in one treatment. 36 sessions were run, with a total of 324 subjects. Most treatments had four sessions each, while there were three sessions each for the Low Inequality (LI) GM treatments with $\beta = 1.2$ and 1.6 and the High Inequality (HI) GM treatments

²³Therefore, not only did we hold constant across treatments the conditions under which the first five rounds were played, we also held constant across treatments the fact that both β and δ changed when subjects entered the sixth round.

with $\beta = 1.8$ and 2.2 .²⁴ We ran 20 sessions at Zhejiang University of Finance and Economics (ZUFE), and 16 sessions at the University of Nottingham Ningbo China (UNNC); at ZUFE subjects were recruited using the lab’s WeChat-based participant management system, while at UNNC they were recruited using the ORSEE platform (Greiner, 2015). All subjects were Chinese undergraduate or postgraduate students.²⁵ The language of instruction was Chinese.²⁶ The instructions were distributed in written form and also read aloud by the experimenter; subjects were required to correctly answer comprehension test questions before rounds 1 and 6. Copies of the instructions (English and Chinese) are available in Online Appendix B.

Neutral language was used. Subjects’ initial endowments were referred to as tokens held in their Individual Fund. In the first stage, they were asked to choose a number of tokens – which could be any integer between 0 and their full endowment – to move into a Group Fund. It was explained that any token kept in an Individual Fund would be worth 1 experimental currency unit (ECU) to that subject, while any token moved to the Group Fund would become worth β ECU and would then be allocated between group members depending on the sharing mechanism being used in the given round. In rounds with the GM, subjects were told that after the first stage of the game they would receive feedback on each member’s contribution (all interactions were anonymous, but group members were identified as either player 1, 2 or 3, so each member’s initial endowment would also be common knowledge). Each would then be assigned one-third of the ECUs generated in the Group Fund and tasked with allocating this between the other two players. Under the GM, each player’s earnings were given by the number of tokens kept in their Individual Fund, added to the ECUs allocated to them by each of the other two players.²⁷ Under the equal sharing mechanism, earnings were instead given by tokens kept in the Individual Fund, added to one-third of the ECUs created in the Group Fund. At the end of each round, subjects received a summary of the starting endowments, first stage contributions, second stage allocation decisions (in rounds with the GM) and eventual earnings of all players in their trio. The experiment was computerized, using Z-Tree (Fischbacher, 2007).

²⁴We were unable to fill fourth sessions for these treatments, because campus access restrictions during the Covid-19 pandemic prevented the laboratories at which we collected data from recruiting participants from neighbouring universities, as they would otherwise have been able to do to increase sample sizes

²⁵The average age was 20. 61% of subjects were female. Subjects were students of a wide range of disciplines, the most common being economics (44%) and business (24%).

²⁶Following best practice, the instructions – initially written in English – were translated by one person into Chinese, and independently back-translated into English by another; all discrepancies between the original and the back-translation were resolved through consultation with a bilingual research assistant, who also checked in full the Chinese version for accuracy.

²⁷Allocations were not required to be integers (in many cases this would be impossible) but instead could be made with a resolution of 0.1.

The ECU subjects accumulated from all rounds would be exchanged into Chinese yuan at the end of the experiment, at an exchange rate of 2.5 ECU = 1 yuan. Sessions lasted about one hour on average, and mean earnings were 64.65 yuan.²⁸ The experiments were conducted between April and July 2021, during which time this was worth approximately 10 USD.

Table 2: Treatment Design (treatment names and mechanisms for rounds 6-15)

		β	δ	Sessions	Subjects
LI control	ES	2.2	0.5	UNNC 2 + ZUFE 2	36
LI GM2.2	GM	2.2	0.5	UNNC 2 + ZUFE 2	36
LI GM1.8	GM	1.8	0.5	UNNC 2 + ZUFE 2	36
LI GM1.6	GM	1.6	0.5	UNNC 1 + ZUFE 2	27
LI GM1.2	GM	1.2	0.5	UNNC 1 + ZUFE 2	27
HI control	ES	2.2	0.75	UNNC 2 + ZUFE 2	36
HI GM2.2	GM	2.2	0.75	UNNC 1 + ZUFE 2	27
HI GM1.8	GM	1.8	0.75	UNNC 1 + ZUFE 2	27
HI GM1.6	GM	1.6	0.75	UNNC 2 + ZUFE 2	36
HI GM1.2	GM	1.2	0.75	UNNC 2 + ZUFE 2	36
Total				UNNC 16 + ZUFE 20	324

5. Results

5.1. Average Contributions

We focus first on average contribution levels, aggregating across all players in each trio. Figure 1 plots, by treatment, how these evolve across all rounds. Low Inequality treatments are displayed in the top panel, while High Inequality treatments are shown in the bottom panel. Note that 8 is the maximum possible average contribution, as this is equal to the average endowment of tokens within each trio.

In all treatments, average contributions declined during the control period (rounds 1 to 5).²⁹ In every case, even in the Control treatments, we then observe a jump in

²⁸Note that this represented a high level of incentivization for a lab experiment in China, and much more than most students would be able to earn from available sources of employment. In 2021, legal minimum wages in Zhejiang Province, where both universities are located, were in the vicinity of 20 RMB per hour.

²⁹We find no significant differences in contributions between different treatments in rounds 1-5, as established by regressions reported in Table C1, Online Appendix C. This is important as it suggests that the treatment effects we will report for rounds 6-15 are due to the treatment variables themselves, rather than the result of random sampling errors which would be likely to also manifest in observable differences in behaviour during the first five rounds.

contribution levels in round 6.³⁰ After round 6, average contributions again drifted downwards in both Control treatments, confirming Hypothesis ES1.³¹ This mirrors the common pattern of low cooperation observed in lab experiments using equal sharing mechanisms. However, Hypothesis ES2, that average contributions are significantly lower with higher inequality, is not supported other than in Round 6, as demonstrated by regression output reported in Table C2, Online Appendix C.³²

The evolution of average contributions differs markedly between treatments once the GM is introduced in round 6. For both degrees of inequality, contributions in rounds 6-15 under the GM exceeded those under the ES allocation, regardless of the level of β the GM operated under, consistent with Hypothesis GM1(a). In the Low Inequality GM1.2, High Inequality GM1.2 and High Inequality GM1.6 treatments, contributions remained roughly steady throughout these rounds, indicating that the effectiveness of the GM was modest in these cases.³³ In all other treatments, average contributions rose, consistent with Hypothesis GM1(b).³⁴ We note that the cases where average contributions remained steady are those we flagged where PCR does not lead to full contributions as part of SPNE for the contribution game and PC does not lead to full contributions as a unique SPNE in two of the cases.

Under Low Inequality GM1.6 and High Inequality GM1.8 subjects were contributing a majority of their tokens by the final round (5.48 and 5.33 respectively), while in the remaining three treatments average contributions were above 7 by the penultimate

³⁰This may be because in the Control treatments β increases in round 6, or may simply be a ‘restart effect’ (e.g. Andreoni, 1988; Chaudhuri, 2018).

³¹In every session of both Control treatments, average contributions are lower in round 15 than in round 6. Pooling across both Control treatments, a Wilcoxon signed-rank test finds this difference in within-session averages between rounds 6 and 15 to be significant ($p = 0.01$). A simple OLS model run on all data from rounds 6-15 of the Control treatments, in which the dependent variable is a subject’s contribution and the sole independent variable is the round number, finds a significant negative relationship between the dependent and independent variables ($p = 0.003$). See Regression (2) in Table C2, Online Appendix C.

³²Mann-Whitney tests, comparing session averages in contributions between the High and Low Inequality Control treatments produce results echoing those of the regressions: the High Inequality sessions have significantly higher contributions specifically in round 6 ($p = 0$)

³³DFL also found that the positive impact of the GM was relatively modest for $\beta = 1.2$ with no inequality in endowments. However, in their case they did find contributions continued to increase somewhat over the 10 rounds the GM was used.

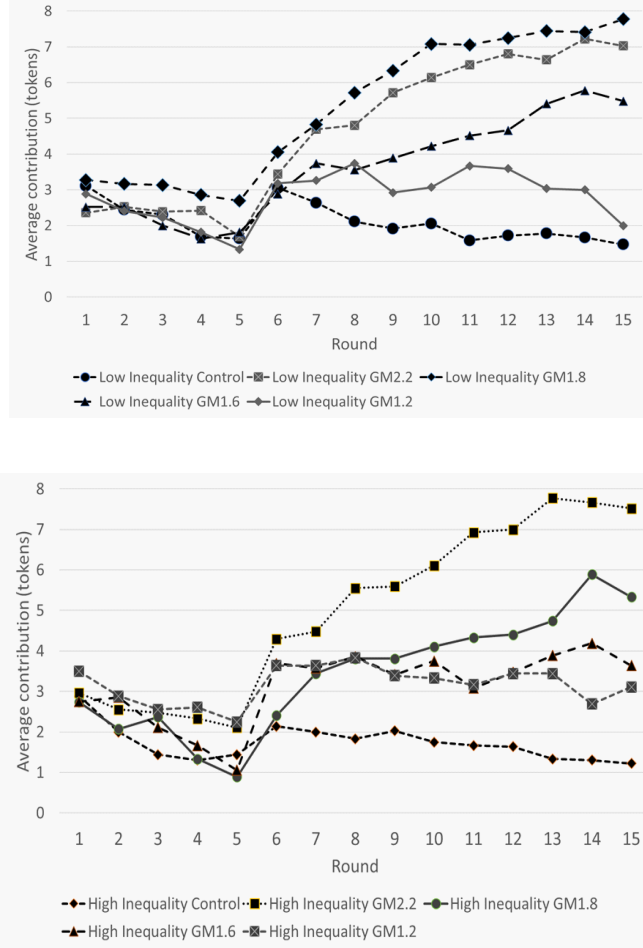
³⁴To formally verify this, we ran simple OLS models – reported in Table C3, Online Appendix C – which regressed amount contributed against round number, separately on data from rounds 6-15 of each of the eight GM treatments. The relationship is positive and significant at the 5% level for all Low Inequality treatments with $\beta \geq 1.6$ and for the High Inequality GM2.2 treatment, and statistically insignificant for other treatments. The insignificant result for High Inequality GM1.8, despite the visible upward trend, may be related to only having three sessions in this treatment (the standard errors are clustered by session). We are unable to address Hypothesis GM1(b) using non-parametric tests on session averages due to an insufficient number of sessions in each treatment.

round. This suggests that the GM can (almost) support full contributions under $\delta = 1/2$ if β is at least 1.8, and under $\delta = 3/4$ if β is at least 2.2. In two out of the four Low Inequality GM1.8 sessions, one out of the three High Inequality GM1.8 sessions, and one out of the four Low Inequality GM2.2 sessions, all nine subjects contributed fully in the final round.

To address the significance of treatment differences, in Table 3 we present OLS regressions in which the dependent variable is an individual's contribution level in a given round. Our primary analysis comes from Regression (1), which uses data from across rounds 6-15 and from all treatments. We include dummy variables for each level of β under which the GM operated; the coefficients on these variables are interpreted as the increase in contributions under the GM at each β relative to the Control treatments, thereby providing a general test of the effectiveness of the GM under each β , pooling across the High and Low Inequality conditions. Also included is a dummy for High Inequality, and controls for subjects' age, gender and degree (a dummy for those majoring in economics) as well as the location of the experiment.³⁵ Standard errors are adjusted for heteroskedasticity by treating each session as providing one cluster of observations.

³⁵We note that the results in this table and in subsequent tables do not meaningfully differ between UNNC and ZUFE despite these being two culturally quite different universities. Adding interaction terms between university and treatment variables in the regressions does not yield significant results.

Figure 1: Average Contributions



The coefficients on the β -dummies in Regression (1) are positive and significant, indicating that even the GM with β as low as 1.2 significantly outperformed the baseline equal sharing mechanism (where, recall, β was much higher).³⁶ This confirms Hypothesis GM1(a). At the bottom of Table 3 we report p-values on linear restriction tests, comparing contribution levels between the different GM treatments. All pairwise comparisons are significant except for GM2.2 – GM1.8 and GM1.6 – GM1.2; this implies that in general increasing β does increase contributions, but not substantially when β rises above 1.8, perhaps because contributions are already close to a maximum.

³⁶ Similar results are obtained by Mann-Whitney tests: session averages in amount contributed across rounds 6–15 are significantly higher in all four β versions of the GM treatments than in the Control treatments (all $p < 0.05$).

The other regressions in Table 3 disaggregate the analysis. Regressions (2) and (3) include data from only the Low and High Inequality treatments respectively. These results also show that the GM always outperformed the baseline, although the effect is only (weakly) significant for $\beta = 1.2$ under High Inequality. The linear restriction tests show GM2.2 significantly outperforms GM1.8 under High Inequality, but the two do not significantly differ under Low Inequality. Regressions (4) and (5) include all treatments but data only from rounds 6 and 15 respectively. In round 6, the Control treatments are only significantly outperformed by the GM with $\beta = 2.2$ and $\beta = 1.6$ (and none of the GM treatments significantly differ from one another), while in round 15 each GM treatment yields significantly higher contributions than the Control. This confirms that the positive effects of the GM take time to emerge most strongly.

Overall, these results establish that the efficiency-generating power of the GM is robust to different combinations of β and δ . Hypothesis GM1(c), that average contributions are increasing in β for a given δ , is broadly supported. However, there is only partial support for Hypothesis GM2, that average contributions are decreasing in δ for a given β . Figure 1 demonstrates some tendency for contributions to be greater under the lower δ , especially for $\beta = 1.8$ and $\beta = 1.6$. Regression (1) also finds this, although the negative coefficient on High Inequality is significant only at the 10% level. Regressions run specifically on data from the GM treatments with a given β (reported in Table C4, Online Appendix C) find the difference between High and Low inequality treatments is significant for $\beta = 1.8$ but not other levels of β .³⁷

³⁷Under PCR, increasing δ from 0.5 to 0.75 results in the minimum β to support full contributions increasing from 1.5 to 1.75. So, we would expect the level of inequality to most strongly affect contributions when β is in or close to this range. PC supports full contributions for both values of β and δ .

Table 3: OLS regressions on tokens contributed (rounds 6-15)

	(1) Full Sample	(2) HI	(3) LI	(4) r6	(5) r15
GM2.2	4.087*** (0.355)	4.463*** (0.320)	3.915*** (0.510)	1.092** (0.391)	5.790*** (0.467)
GM1.8	3.595*** (0.528)	2.302** (0.684)	4.566*** (0.338)	0.678 (0.566)	5.357*** (0.654)
GM1.6	2.160*** (0.243)	1.926*** (0.289)	2.546*** (0.256)	0.710* (0.328)	3.188*** (0.439)
GM1.2	1.393* (0.580)	1.473† (0.829)	1.322 (0.782)	0.722 (0.614)	1.406* (0.566)
HI	-0.596† (0.342)			-0.142 (0.355)	-0.652 (0.416)
ZUFE	-0.187 (0.319)	0.494 (0.429)	-0.648† (0.318)	-0.398 (0.413)	-0.416 (0.398)
Age	0.0366 (0.0563)	0.127 (0.0785)	-0.0391 (0.0550)	0.0602 (0.0834)	0.0981 (0.110)
Female	-0.445* (0.177)	-0.674* (0.281)	-0.298 (0.203)	-0.578* (0.268)	0.0120 (0.301)
Economics	0.227 (0.203)	-0.282 (0.277)	0.598* (0.220)	-0.104 (0.329)	0.558 (0.372)
Constant	1.730 (1.128)	-0.481 (1.618)	3.043* (1.115)	2.148 (1.579)	-0.345 (2.133)
Observations	3240	1620	1620	324	324
R^2	0.224	0.190	0.310	0.044	0.378

Linear Restriction Tests (p-values)

GM2.2 vs GM1.8	0.419	0.005	0.248	0.515	0.567
GM2.2 vs GM1.6	<0.001	<0.001	0.015	0.347	<0.001
GM2.2 vs GM1.2	<0.001	0.003	0.011	0.575	<0.001
GM1.8 vs GM1.6	0.012	0.577	<0.001	0.954	0.005
GM1.8 vs GM1.2	0.007	0.458	<0.001	0.955	<0.001
GM1.6 vs GM1.2	0.213	0.586	0.144	0.985	0.007

Notes: *r6* and *r15* refer to data only from rounds 6 and 15 respectively. The dependent variable is the number of tokens contributed. Only data from rounds 6-15 are included. The omitted treatment category is Control. Standard errors, in parentheses, are clustered by session (36 clusters in Regressions (1), (4) and (5); 18 in Regressions (2) and (3)). † $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

5.2. Gini Coefficients

We illustrate the average Gini coefficients of the income distribution after each round for each treatment in Figure 2. The results resemble a mirror image of Figure 1, and in combination the two figures clearly illustrate that inequality is high when efficiency is low and vice versa. The Gini measures for the two endowment distributions (0.222 and 0.333 for the low and for the high inequality, respectively) are also indicated by the points LI and HI on the vertical axes in Figure 2. The control and all the GM treatments result in reduced inequality relative to the endowment distribution, with the exception of the control under low inequality. This confirms Hypothesis GM4(a).³⁸ Table 4 reports the equivalent regressions to those in Table 3 with the Gini as the dependent variable. The significance of coefficients almost exactly reflects that in Table 3. GM1.2 is never significant, but all the other scale of return dummies are, except in regression (4) which is for round 6 only, where none of these dummies are significant.³⁹ When it comes to reducing inequality, the GM outperforms the control treatment, except for $\beta = 1.2$ when there is no significant difference between them. The linear restriction tests are mostly significant, which shows that inequality generally decreases as the scale of returns increases. Hypothesis GM4(b), that for a given level of endowment inequality, the final income distribution is less unequal than that of the corresponding control, is strongly supported.

³⁸We ran Wilcoxon signed rank tests, separately for the GM treatments at the four different levels of β , testing whether the average Gini coefficient in a given session, based on post-round income and pooling all observations from rounds 6-15, is significantly different from that session's endowment Gini level. All test results are significant at the 5% level. We lack a sufficient number of sessions to conduct such tests separately for all eight treatments.

³⁹The significance of the dummy denoting the high inequality treatment in Table 6 indicates that, while the GM has been successful in reducing inequality in both the high and low inequality treatments, inequality still remains higher in the former.

Figure 2: Gini Coefficients

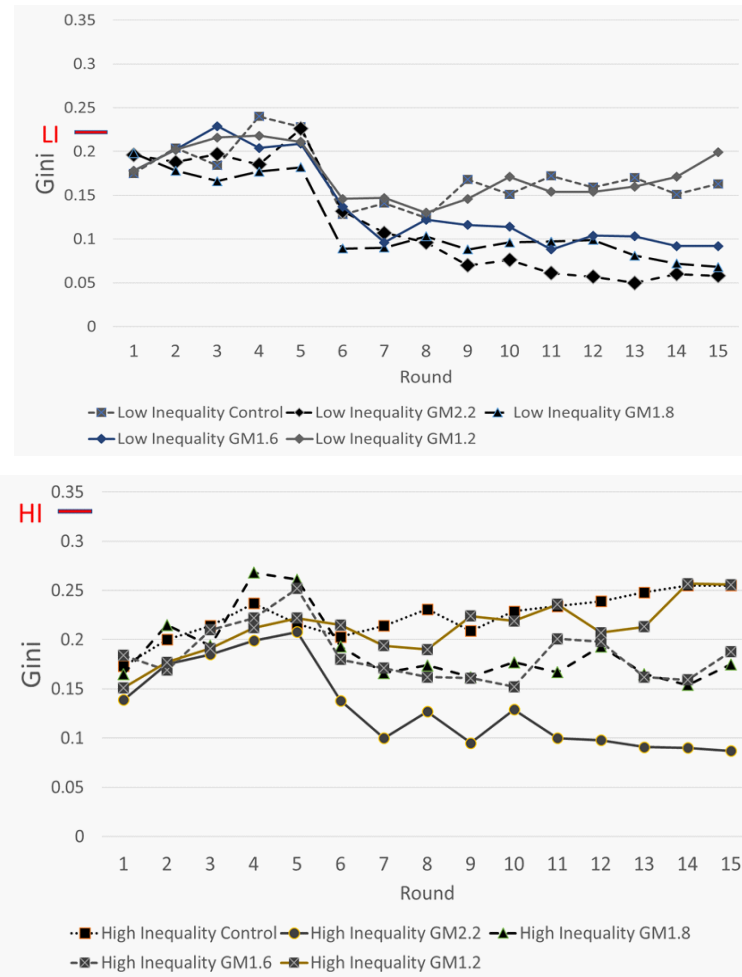


Table 4: OLS regressions on Gini (rounds 6-15)

	(1) Full Sample	(2) HI	(3) LI	(4) r6	(5) r15
GM2.2	-0.0990*** (0.0139)	-0.127*** (0.0224)	-0.0779*** (0.0121)	-0.0299 (0.0281)	-0.129*** (0.0193)
GM1.8	-0.0632*** (0.0134)	-0.0581* (0.0216)	-0.0661*** (0.0130)	-0.0275 (0.0232)	-0.0877*** (0.0230)
GM1.6	-0.0531*** (0.0123)	-0.0582** (0.0189)	-0.0490*** (0.0112)	-0.0107 (0.0163)	-0.0692*** (0.0182)
GM1.2	-0.00522 (0.0209)	-0.0112 (0.0345)	-0.00132 (0.0198)	0.0141 (0.0229)	0.0170 (0.0218)
HI	0.0657*** (0.0100)			0.0583*** (0.0154)	0.0774*** (0.0123)
Constant	0.179*** (0.0308)	0.266*** (0.0470)	0.177*** (0.0325)	0.219*** (0.0500)	0.183*** (0.0394)
Observations	1080	540	540	108	108
R^2	0.400	0.280	0.292	0.245	0.629

Linear Restriction Tests (p-values)

GM2.2 vs GM1.8	0.010	0.002	0.374	0.935	0.046
GM2.2 vs GM1.6	<0.001	<0.001	0.014	0.433	<0.001
GM2.2 vs GM1.2	<0.001	0.004	0.001	0.145	<0.001
GM1.8 vs GM1.6	0.377	0.996	0.158	0.391	0.345
GM1.8 vs GM1.2	0.007	0.184	0.004	0.110	<0.001
GM1.6 vs GM1.2	0.022	0.160	0.025	0.219	<0.001

Notes: *r6* and *r15* refer to data only from rounds 6 and 15 respectively. The dependent variable is the group-level Gini. Only data from rounds 6-15 are included. There is one observation per each group of three players. The omitted treatment category is Control. Standard errors, in parentheses, are clustered by session (36 clusters in Regressions (1), (4) and (5); 18 in Regressions (2) and (3)). Control variables for Age, Location, Female and Major were included, but found to be insignificant except for two isolated cases, both negative – age in regression (4) and major in regression (3). † $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

5.3. Contributions by Player Types

To better understand what drives the average contribution results, we next consider how much an agent contributes under different circumstances. Table 5 presents the average

contribution ratios by subjects in the role of rich, middle and poor in each treatment, both combined across rounds 6-15 and in round 15 only. In every treatment, averaging across rounds 6-15, the poor contributed the highest proportion of their endowments, followed by middle and then rich.

Regression results (in Table C5, Online Appendix C) for the control treatments show that Hypothesis ES3(a) ‘that the contribution ratio of the rich is not significantly different from that of the poor’ is rejected when the data is combined across rounds 6-15 and both control treatments; it is also weakly rejected for the treatment with high inequality; but cannot be rejected for the treatment with low inequality. In conjunction with Table 5 these results indicate that the contribution ratio of the poor is significantly higher than that of the rich when inequality is high, but is not significantly different if inequality is low. The alternate Hypothesis ES3(b) ‘that the contribution level of the rich is not significantly different from that of the poor’ is also rejected therefore. Here we find that the contribution levels of the rich are significantly higher than those of the poor. This reflects a pattern of behavior that persists throughout the remainder of our experimental results. In terms of contributions to public output, subjects make the highest absolute contributions when rich, and contribute the highest income share when poor.

Table 5: Average proportion of endowment contributed by player type

	Poor	Middle	Rich
	r6-15	r6-15	r6-15
LI Control	0.288	0.278	0.219
HI Control	0.333	0.233	0.182
LI GM2.2	0.869	0.759	0.679
HI GM2.2	0.961	0.8	0.754
LI GM1.8	0.89	0.811	0.787
HI GM1.8	0.861	0.586	0.448
LI GM1.6	0.733	0.561	0.485
HI GM1.6	0.867	0.488	0.382
LI GM1.2	0.667	0.41	0.292
HI GM1.2	0.875	0.48	0.323
	r15	r15	r15
LI Control	0.188	0.219	0.16
HI Control	0.208	0.177	0.131
LI GM2.2	0.958	0.885	0.847
HI GM2.2	1	0.986	0.905
LI GM1.8	1	0.969	0.965
HI GM1.8	1	0.778	0.556
LI GM1.6	0.833	0.708	0.62
HI GM1.6	0.958	0.448	0.387
LI GM1.2	0.611	0.361	0.056
HI GM1.2	0.958	0.5	0.244

Notes: r6 - 15 refers to data from round 6 to round 15. r15 refers to data only from round 15.

The differences in contribution ratios between types are starkest in the GM treatments with lower β . High contribution ratios by the poor are uniform across almost all GM treatments; averaging over all rounds they always contributed the majority of their endowments, and by the final round their average contributions had increased to 100% or close in all treatments except for Low Inequality GM1.2. The contribution ratios of the middle were more variable, reaching close to full contributions in the final rounds of the Low Inequality GM2.2, Low Inequality GM1.8 and High Inequality GM2.2 treatments, but falling below 50% in the treatments with $\beta = 1.2$. The rich also raised their contributions to close to 100% in the Low Inequality GM2.2, Low Inequality GM1.8 and High Inequality GM2.2 treatments, but their contributions were even lower than those of the middle in the other treatments and fell close to zero by the last rounds of the GM1.2 treatments. We discuss these differences in Section 6.

The regression results (in Table C6, Online Appendix C) indicate that Hypothesis GM3(a) ‘that for given levels of endowment inequality and scale of returns, the contribution ratio of the rich is not significantly different from that of the poor’; can be rejected in favor of the poor contributing a higher proportion than the rich in all cases. Similarly (see Table C7, Online Appendix C), Hypothesis GM3(b) ‘that for given levels of endowment inequality and scale of returns, the contribution level of the rich is not significantly different from that of the poor, can be rejected in favor of the rich having higher contribution levels than the poor when $\beta \geq 1.6$ (except for when $\beta = 1.8$ and inequality is high). There is no significant difference in the contribution levels when $\beta = 1.2$. While both contributions and contribution ratios are generally higher, the pattern of the poor having the higher contribution ratio and the rich the higher contribution level found in the control treatments persists under the GM.

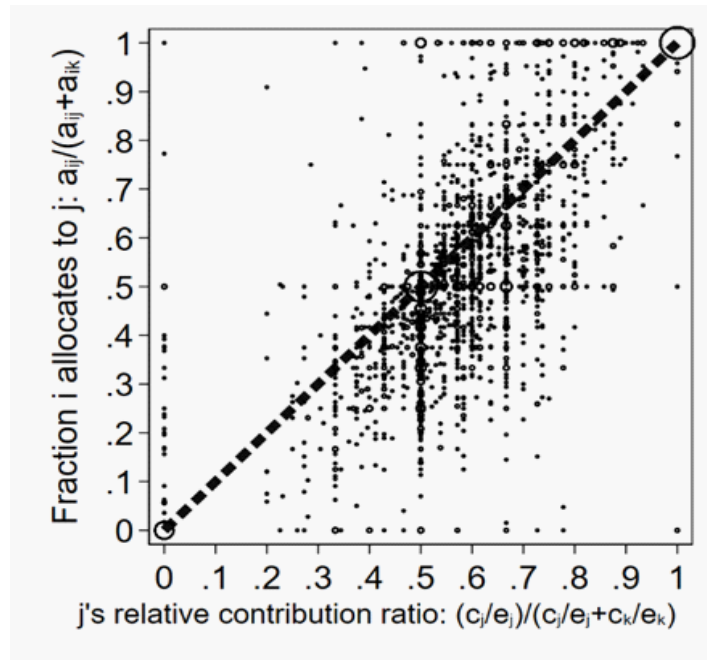
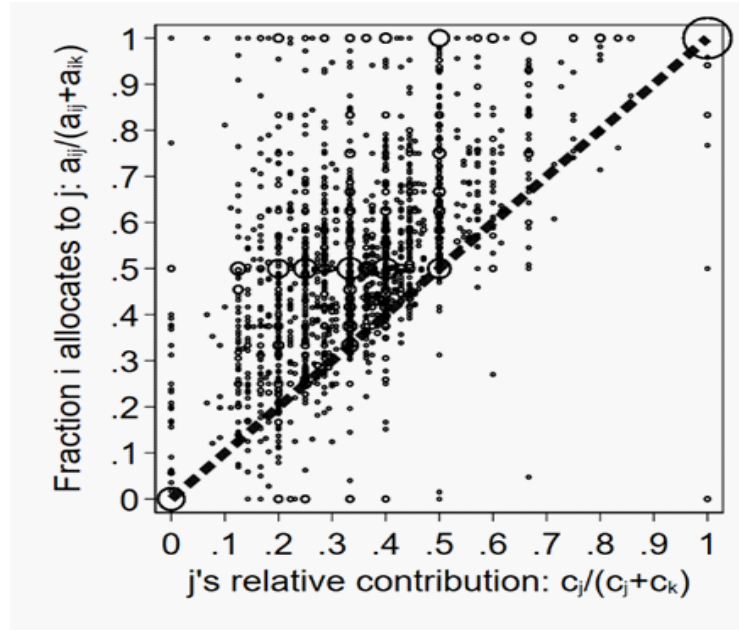
5.4. Allocations by player type

We turn next to the question of how players allocate the first stage output in the game’s second stage. Recall that under the GM, each subject must allocate one-third of the ECU in the Group Fund between the two other players. The following analysis considers only each player i ’s allocation towards player j , where player j is defined as the remaining player with the lower initial endowment. Player i ’s allocation towards the other player (player k) can be deduced trivially as the remaining part of the one-third of the Group Fund that player i did not allocate to player j .

The horizontal axis in the upper panel of Figure 3 represents the absolute amount contributed in the first stage by player j relative to that of player k – that is, $\frac{c_j}{c_j+c_k}$. The vertical axis represents the fraction of player i ’s combined allocation given to player j – that is, $\frac{a_{ij}}{a_{ij}+a_{ik}}$. Circle sizes represent the relative frequency of particular observations. Data is pooled from all GM treatments, rounds 6-15. Allocations along the 45-degree line would be consistent with player i following PC behavior. In contrast to DFL, who found that with equal endowments about half of the allocations in the experiment were located on this 45-degree line, in our case we observe that only 14.7% are.⁴⁰ However, deviations from the line tend to be systematic: 76.9% of the observations lie above it. This reveals that allocators tend to reward a given contribution more if it comes from the player with the lower endowment.

⁴⁰DFL note that in some cases it is mathematically impossible for the allocator to set $\frac{a_{ij}}{a_{ij}+a_{ik}}$ exactly equal to $\frac{c_j}{c_j+c_k}$. They therefore define allocations as proportionalist if the difference between the two is no more than 0.05. Under this criterion, 28.8% of the allocations in our study are consistent with PC, compared to 70.6% in DFL.

Figure 3: Allocation Decisions



As few subjects appear to be perfect PC allocators, this raises the possibility that many are perfect PCR allocators, i.e. the fraction they allocate as player i to player j is given by the proportion of player j 's endowment that j contributed relative to the proportion of player k 's endowment that k contributed. In such cases $\frac{a_{ij}}{a_{ij}+a_{ik}} = \frac{c_j/e_j}{c/e_j+c_k/e_k}$. The lower panel of Figure 3 explores this possibility. Here, the horizontal axis represents $\frac{c_j/e_j}{c/e_j+c_k/e_k}$, while the vertical axis still represents $\frac{a_{ij}}{a_{ij}+a_{ik}}$, and an observation along the 45-degree line would be consistent with player i being a perfect PCR allocator.⁴¹ We see that allocations are generally targeted more closely to the 45-degree line than they are in the upper panel, with 19.9% of observations lying precisely on the line and 40.6% at most 0.05 units above or below it, suggesting that PCR allocation behavior is empirically more relevant than PC allocation behavior. However, there is in this case a slight tendency for observations to lie below the 45-degree line (53.7% do) rather than above it (only 26.4%). This reveals that subjects who contributed a given proportion of their endowment – relative to that contributed by their team-mate – received a larger allocation if they were contributing out of a larger endowment. In other words, the allocation a player received was positively influenced by this player's absolute contribution, not just by the proportion of their endowment they contributed.

The insights above are confirmed by the OLS regressions in Table 6. The dependent variable is the fraction each player i allocates to each player j . Player j 's bilateral relative contribution RC $\frac{c_j}{c_j+c_k}$ and relative contribution ratio RCR $\frac{c_j/e_j}{c/e_j+c_k/e_k}$ are both included as independent variables, along with dummy variables for treatment (level of β and δ) and controls for location and demographics. In regression (1), which contains the full sample, both the RC and the RCR terms have strongly significant positive coefficients, which shows that allocators do indeed respond to both. However, the coefficient on the RCR is much larger, from which we can discern that allocators primarily reward contributors on the basis of their contribution ratios. This regression indicates that aggregate allocations can be characterized as reflecting a mix of PC and PCR allocation behaviors.⁴²

Intriguingly, however, the weights on the PC and PCR allocations seem to depend on the allocator's endowment, as revealed by regressions (2), (3) and (4) which separately analyze the allocation choices by player type. The rich are slightly biased towards RC

⁴¹In both panels of Figure 3, for observations where the values of the numerator and denominator of the term represented on the horizontal axis are both equal to zero, we set the value of the variable equal to 0.5.

⁴²Any equal shares allocations would appear on a horizontal line through 0.5 on the vertical axis on each panel of Figure 3, and there are slight clusters along these lines. 13.0% of observations are at exactly 0.5 and 27.1% are between 0.45 and 0.55. We cannot strictly attribute all of these observations to a pure equal sharing allocation strategy, however, because they are also consistent with the other types of allocation strategies, including but not restricted to cases where both players made equal contributions or had equal contribution ratios.

over RCR, which results in them rewarding the middle handsomely for contributing a mediocre share of their endowment.⁴³ The poor, on the other hand, base their allocations strongly on RCR, to the extent that RC does not figure as a significant motivator; which tends to result in the rich receiving little payback for contributions that are large in absolute terms but stingy in relation to their endowment. The rich favour the (relatively) rich, while the poor do the opposite. What is particularly noteworthy here is that the allocation behavior is so strongly influenced by a subject's status within the trio, even though this status can change every round and despite the fact that a subject's allocation decision has no effect on her own earnings in the round in question.^{44,45}

Table 6: OLS regressions on allocation to player j

	(1) Full Sample	(2) P	(3) M	(4) R
RC	0.335*** (0.0383)	0.111 (0.112)	0.319*** (0.0719)	0.500*** (0.0559)
RCR	0.615*** (0.0546)	0.903*** (0.123)	0.622*** (0.106)	0.407*** (0.0622)
GM1.8	-0.0223 (0.0158)	-0.0190 (0.0122)	-0.0135 (0.0236)	-0.0333 (0.0199)
GM1.6	-0.0116 (0.0151)	0.0106 (0.0150)	-0.0131 (0.0236)	-0.0311 (0.0187)
GM1.2	-0.00791 (0.0130)	0.00381 (0.0188)	-0.0166 (0.0189)	-0.0141 (0.0159)
HI	0.0134 (0.00958)	-0.00623 (0.00960)	0.00954 (0.0172)	0.0510** (0.0151)
Constant	-0.0368 (0.0734)	0.121† (0.0606)	-0.0976 (0.103)	-0.0770 (0.0950)
Observations	2520	840	840	840
R^2	0.621	0.718	0.618	0.509

⁴³ A Chow Test on regression (1) with added dummy variables for Middle and Rich and interactions between those variables and RC and RCR finds that RC weakly differs between poor and rich ($p=0.099$) and RCR differs between poor and rich ($p=0.014$). Neither significantly differ between Poor and Middle.

⁴⁴ While the weights estimated on PC and PCR in these models differ depending on the allocator's status, they do not substantively differ depending on the levels of β and δ . See the regressions in Table C8, Online Appendix C, which run separate regressions for different treatments.

⁴⁵ In our data, females are more pro-redistribution than males. The significant coefficient on Female in Equation (1) indicates that, for any given set of contributions by Players j and k , a female allocator tends to give more to Player j than a male would. This is particularly the case when the allocator is rich (regression (4)). Note that we also detected a gender difference in Table 3, with females contributing significantly less than males in stage one of the game.

Notes: *The dependent variable is the share of player i 's combined allocation going to player j (the disadvantaged player). Data from all GM treatments, rounds 6-15, are included. The omitted treatment category is GM2.2. Standard errors, in parentheses, are clustered by session (28 clusters in each regression). Controls for location and demographics were also included and were insignificant, except that older subjects tended to allocate less to the poor and more to the middle, while female subjects allocated more to the rich. $\dagger p < 0.1$, $* p < 0.05$, $** p < 0.01$, $*** p < 0.001$.*

5.5. Relationship between contributions and earnings

We have seen how contribution levels vary across treatments and player types, and we have studied the patterns of allocation behavior. We now ask how useful the latter are in explaining the former. One question to consider is under which circumstances it is, in reality, profitable for subjects to contribute fully under the GM. To address this, we plot the average net return subjects receive on each available contribution level, where net return is defined as the ECUs a player receives through allocations in stage two minus the ECUs the player sacrificed through contributions in stage one. Figure 4 presents these plots separately for each type of player in each treatment. The contribution level for which average net returns are highest represents a player's expected ex-post payoff-maximizing strategy.

The following observations stand out from Figure 4. First, contributing fully always maximizes expected payoffs for the poor; in every treatment, the highest returns for the player with the lowest endowment came from contributing the whole endowment. Secondly, the middle maximizes expected payoffs by fully contributing when $\beta = 2.2, 1.8$ or 1.6 .⁴⁶ When $\beta = 1.2$, there is no clear relationship between the middle's contribution and the payoff earned. Thirdly, the rich maximizes expected payoffs by fully contributing only under $\beta = 2.2$ or 1.8 .⁴⁷ When $\beta = 1.6$, there is an unclear relationship between the contributions and payoffs for the rich, while for $\beta = 1.2$ they would generally maximize payoffs by contributing zero (as their received allocations tend to be less than the amount they contribute for any positive contribution). These results are consistent with our expectations based on the theory discussed above in the sense that when $\beta \geq 1.8$ full contributions is an SPNE under both the PC and PCR allocations, while when $1.8 \geq \beta \geq 1.6$ full contributions is only an SPNE for the PC allocation. In those

⁴⁶There is a caveat. The highest average net returns for the Middle in the High Inequality GM2.2 and Low Inequality GM1.6 treatments actually came from contributing 2 and 7 tokens respectively. However, these behaviours were only observed on three and one instances respectively, so these high returns are likely to be outliers. In both treatments, there is a clear upward trend in the net returns as the Middle's contributions increase towards the maximum.

⁴⁷A similar caveat applies as in the previous footnote. The expected payoff maximizing contribution levels are 10 in High Inequality GM2.2 and 11 in Low Inequality GM1.8, but this is based on only 16 and 3 observations respectively.

cases when full contributions is not a SPNE, it is especially the rich that don't find it in their interests to contribute fully. Note that this is partly the consequence of the different allocation strategies being followed by different player types; the poor have a particularly strong tendency to follow PCR allocations, which result in lower benefits to the rich than PC allocations would.

These findings help to explain the contributions proffered by each player type, as reported in Table 5. Specifically, the low contributions of the Rich in the $\beta = 1.2$ sessions and mediocre contributions in the $\beta = 1.6$ sessions, the mediocre contributions of the Middle under $\beta = 1.2$, and the high contributions of the Poor in all cases, are consistent with subjects following rational income maximizing strategies. Relative to expected ex post payoff maximization, we do see some tendency for under-contribution. For instance, the Middle did not reach anywhere close to full contributions under $\beta = 1.6$, even though doing so would have ex post maximized their expected payoffs. This may be explained by risk aversion, or even a variant of betrayal aversion (e.g. Bohnet et al., 2008). It is noteworthy, however, that we do not observe any obvious influence of social preferences, which in many other economic games with voluntary contributions have been found to push contributions well above the level predicted by standard economic theory (see e.g. Chaudhuri, 2011).

Overall, it is clear that the GM with heterogeneous endowment is able to work to some extent even with low returns to production, because players with low endowments can be easily motivated to contribute. However, the GM is most effective when $\beta = 1.8$ or even higher, because only then might all player types be incentivized to contribute fully.

Figure 4: Net Returns on Contributions



Note: Data from rounds 6-15 is included.

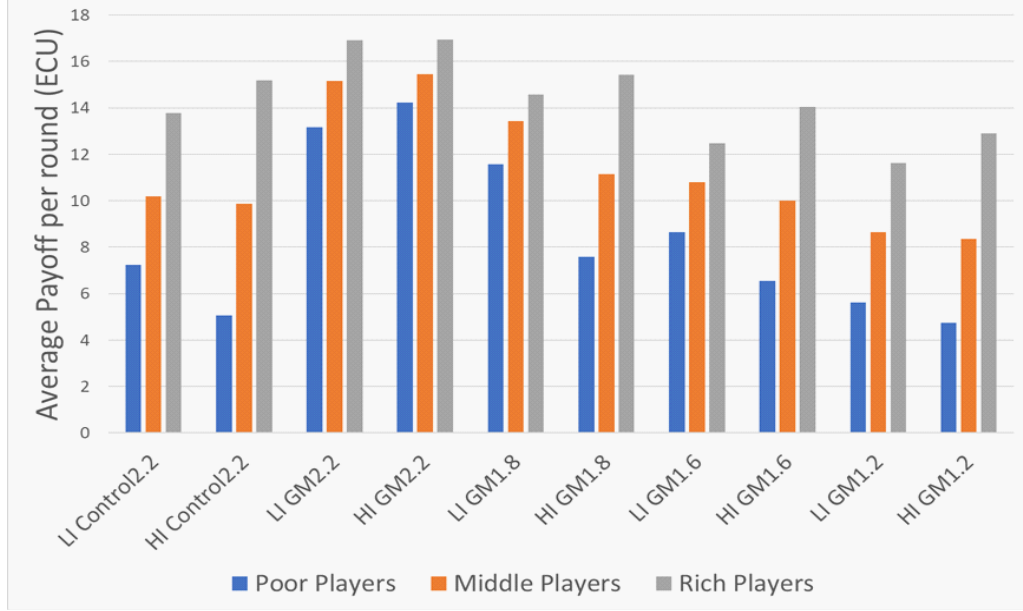
5.6. Payoffs

Although allocation strategies based on contribution ratios – rather than contribution levels – disincentivize contributions from well-endowed players, they do have the effect, likely to be considered beneficial, of strongly redistributing from rich to poor, thereby mitigating the inequality built into the start of each round.

To explore this effect, Figure 5 presents average payoffs per round for the different player types in each treatment during rounds 6-15. In the treatments where the GM is successful in achieving high contributions, the inequality in payoffs between types is quite small. For instance, in High Inequality GM2.2, where the starting endowments are 2, 8 and 14, respectively, the corresponding average payoffs per round using the GM mechanism are 14.2, 15.5 and 16.9. In contrast, treatments in which the GM is less successful in boosting contributions leave large amounts of inequality between types – in High Inequality GM1.2, for example, the average payoffs per round for the three player types are 4.8, 8.4 and 12.9. This illustrates an intriguing feature of the GM: inequality is minimized in cases where efficiency is maximized, contrary to what is conventionally expected in other economic contexts (e.g. Okun, 2015).

Figure 5 serves to illustrate two effects in particular. First, in mitigating inequality, the GM does a better job than the equal sharing mechanism. For instance, the High Inequality Control treatment results in average payoffs per round of 4.8, 8.4 and 12.9 for the three player types – much more dispersed than are the average payoffs in the GM treatment with equivalent β and δ . Second, the differences in earnings between treatments are small for rich players but dramatic for poor players. When the GM works well, it's the less well-endowed who primarily benefit from it.

Figure 5: Average Payoffs



6. Summary and Conclusions

As noted in the Introduction, our experiments generate results relating to efficiency, inequality and agent contribution and allocation behavior. We discuss each in turn.

6.1. Efficiency

The GM improves efficiency. Average contributions in the GM treatments exceeded those in the equal-shares control treatments for both levels of inequality and all levels of the scale of returns considered. As expected, this performance was sensitive to the level of endowment inequality and the scale of returns. By the final round a group's total contributions in GM2.2 and LI GM1.8 averaged 93% of the group's combined endowments, with 82% of subjects contributing fully, numbers comparable to the overall averages found in DFL (92% and 83%, respectively). The GM was less successful at lower levels of the scale of returns. The corresponding numbers for HI GM1.8 and GM1.6 were 61% and 43%; and for GM1.2 32% and 28%. All of these are significantly higher than those for the control treatments (17% and 0%).

While the GM has been shown to work well even when subjects have heterogeneous endowments, there is clearly scope for further refinements. Our experiments have identified the endowment types that are 'under-contributing' (see the discussion below) which

is where these refinements might best be aimed. But in their absence, our results suggest that if choosing between two teams with the same average endowments, the principal might find it more profitable to employ that which has the more uniform endowment.

6.2. Inequality

At the same time as the GM improves efficiency it reduces inequality. The ‘fair’ allocations that the subjects employ induce higher contributions and the two in combination involve redistribution in favor of the subject with the lower endowment. But there is a limit. Unless the scale of returns is sufficiently high ($\beta \geq 1.8$), the rich agent may be unwilling to make a full contribution. In general there is no tradeoff between efficiency and equity for the agents under the GM. The only trade-off involves the principal offering a higher β (thereby sacrificing profits) to extract the last few tokens from the rich agent.

We have used the principal-agent problem as the context for our discussion of the GM, and in that context income inequality among the agents may be of no concern to the principal. However, this does not mean it is entirely irrelevant. It is surely comforting for the principal to know that any concern that the agents have about income inequality when making their allocations does not seem likely to hamper the principal’s objective of raising contributions.

Our results indicate that the principal will need to have some information on the general distribution of the agents’ endowments in order to set the appropriate scale of returns to attract full contributions. But, as long as the agents themselves are aware of each other’s precise endowments and contributions before the allocation stage, the GM can deliver full contributions without the principal having this specific knowledge.

6.3. Agent Behavior

As noted earlier, individual subjects’ contribution and allocation decisions seem to be sensitive to their endowment levels in a consistent way. As the experimental literature shows, under the equal-share allocation, subjects when poor contribute a higher share of their endowments than they do when rich. Moreover, subjects when rich contribute a larger amount than they do when poor. Our experiments show that this contribution behavior continues to hold under the GM, even as the GM raises average contributions. Further, when it comes to allocations under the GM, agents allocate to each other according to a weighted average of PC and PCR; putting a higher weight on the PCR when poor than when rich. Thus, in both their contribution and allocation decision-making, agents consistently place greater emphasis on contribution levels when they are rich, and on contribution ratios when they are poor.

A corollary of this is that we can use the subjects' allocation behavior to infer something about their willingness to redistribute. If subjects focus on the PC in making their allocation decisions, they are effectively ignoring the endowment distribution. While if they focus on the PCR then they are taking the endowment distribution fully into account when making their allocations. In our experiments subjects appear to be using a weighted average of both; however, the weights depend on their current endowment in a consistent way. When they are rich agents put a higher weight on PC than when they are poor, implying that an agent is more willing to redistribute on the basis of income differences when poor than when rich.

Our experimental results showed that where the GM failed to support full contributions (in treatments with $\beta = 1.8$ or lower) the main barrier was the reluctance of subjects to contribute when they have the largest endowment. In sub-section 5.5 above we found evidence that these subjects under-contribute relative to their payoff maximization contribution. This cannot be because they are concerned that larger contributions might lead to greater inequality (i.e. inequality aversion), because we have shown that the opposite is the case. Could it be that, beyond some point, efficiency-improving contributions are held back by the rich precisely because others receive a disproportionate share of the benefits, and the rich regard this as an 'unfair' erosion of their status associated with higher relative income? ⁴⁸

More generally, it was clear from our review of the experimental literature on the VCM that subjects take more than narrow self-interest (i.e. own income) into account in making their decisions. Early experiments with common endowments and equal-shares allocations found positive contributions where self-interest suggested otherwise. A range of answers (e.g. altruism, inequality aversion, cooperation) were then offered to the question 'why do subjects contribute so much?' When heterogeneous endowments were investigated, the question became 'why do some agents contribute (disproportionately) more than others?' and some of the previous answers (e.g. altruism and inequality aversion) were found wanting. Now the GM poses the opposing quandary. 'Why do (some) agents hold back their contributions when narrow self-interest suggests the contrary?' Risk and betrayal aversion, along with potential erosion of status, are possible solutions, and finding a set of preferences to answer all these questions is a challenge for future research.

6.4. Concluding Remarks

Overall, the GM is showing itself to be a promising mechanism. DFL demonstrated it to be efficiency-enhancing and here we have shown that its efficiency-enhancing properties are robust to a more realistic environment involving heterogeneous endowments. Fur-

⁴⁸A parallel result to that of Kuy & Salmon (2013) who found that the poor were reluctant to contribute fully if the rich received a disproportionate share of the benefits.

thermore, we have shown it simultaneously reduces inequality. In the experiments above we excluded the possibility of discriminatory preferences or collusion by subgroups of players, by maintaining players' anonymity. We also rotated players across endowment levels and teams between rounds. In practice an individual player's endowment (ability) doesn't vary this much in a short space of time; team membership is likely to remain fairly constant across several rounds; and team members are unlikely to be anonymous, opening the possibility that allocations may be influenced by interpersonal relations as well as contributions.⁴⁹ It is time to test the GM in settings even closer to reality.

7. Appendix

7.1. Appendix A1

7.1.1. Fixed allocation share

In a fixed share allocation, a player's allocation share is unaffected by his contributions – i.e.

$$q_i(c) = \bar{q}_i \text{ for all } c.$$

Then the condition for player i to prefer full contributions e_i to any other contribution level c_i , given the other players make full contributions is

$$\begin{aligned} \bar{q}_i \beta E &\geq \bar{q}_i \beta [c_i + E_{-i}] + e_i - c_i \text{ or} \\ \beta &\geq \frac{1}{\bar{q}_i} \end{aligned}$$

Only when β exceeds the inverse of the smallest allocation share are full contributions a part of an equilibrium of the game. The smallest allocation share is maximised when all shares are equal, in which case we require $\beta \geq 3$. If the principal were to allocate fixed shares and wished to minimise β , then equal shares cannot be improved upon.

7.1.2. Distribution Neutral Contributions

Player j 's initial income is given by e_j , and the initial aggregate income is E . We let y_j denote j 's income and Y denote the aggregate income, after contributions have been made and the output has been allocated. Then DNC will need to satisfy the condition that all subjects' income shares are the same before and after the output has been allocated. That is,

$$\frac{y_j}{Y} = \frac{e_j}{E} \equiv \theta_j \text{ for all } j, \tag{7.1}$$

⁴⁹Further discussion of this possibility can be found in Dong (2017).

where θ_j is player j 's endowment share. Equation (7.1) can be rewritten as

$$\frac{y_j}{e_j} = \frac{Y}{E} \equiv \rho \text{ for all } j, \quad (7.2)$$

Player j 's contribution is c_j , and we denote total contributions by C . Under an equal shares allocation

$$y_j = e_j - c_j + \frac{\beta}{3}C \quad (7.3)$$

Substituting in condition (7.2) gives

$$\frac{y_j}{e_j} = 1 - \frac{c_j}{e_j} + \frac{\beta}{3} \frac{C}{e_j} \quad (7.4)$$

$$= 1 + \frac{\frac{\beta}{3}C - c_j}{e_j} \quad (7.5)$$

Hence,

$$\begin{aligned} \rho - 1 &= \frac{C \left(\frac{\beta}{3} - \frac{c_j}{C} \right)}{e_j} \\ &= \frac{\frac{C}{E} \left(\frac{\beta}{3} - \varphi_j \right)}{\theta_j} \text{ for all } j \end{aligned} \quad (7.6)$$

where $\varphi_j \equiv \frac{c_j}{C}$ is player j 's contribution share. Solving (7.6) for φ_j we find

$$\varphi_j = \frac{\beta}{3} - \theta_j [\rho - 1] \frac{E}{C} \quad (7.7)$$

Summing across the agents gives

$$\sum_{j=1}^3 \varphi_j = \beta - [\rho - 1] \frac{E}{C} \sum_{j=1}^3 \theta_j. \quad (7.8)$$

Since $\sum_{j=1}^3 \varphi_j = \sum_{j=1}^3 \theta_j = 1$, we can solve for $\rho - 1 = [\beta - 1] \frac{C}{E}$ and hence substitute back into (7.7), obtaining

$$\varphi_j = \theta_j + \beta \left[\frac{1}{3} - \theta_j \right] \quad (7.9)$$

The above equation gives the contribution shares required to maintain the income distribution under an equal-shares allocation. If a subject's endowment share is

equal to the mean of the distribution ($1/3$), then she should contribute in proportion to her endowment share. Otherwise, a subject with an endowment share smaller (larger) than the mean should have a higher (lower) contribution share than her endowment share, with the magnitude of these differences increasing in the scale parameter (β). This implies that the poor must contribute a larger share of their endowment than the rich if the income distribution is to be maintained. Converting the equation to contribution levels we find:

$$\begin{aligned} c_j - c_k &= [\theta_j - \theta_k] [1 - \beta] C \\ \text{for all } k &\neq j \end{aligned} \quad (7.10)$$

Since $\beta > 1$, (7.10) implies that the contribution of the subject with the larger endowment share will be lower than that of the subject with the lower endowment share.

7.1.3. Distribution Neutral Allocation (DNA)

A DNA will need to satisfy the condition that all players' income shares are the same before and after the contributions and allocations have occurred (i.e. after the game has been played) that is, equations (7.1) and (7.2) above are satisfied.

Consider an allocation to player j (A_j) such that

$$A_j = c_j + \theta_j (\beta - 1) C \quad (7.11)$$

That is, the allocation to player j is equal to her contribution plus a share of the surplus equal to her initial income share.

Then

$$\begin{aligned} y_j &= e_j - c_j + A_j \\ &= e_j + \theta_j (\beta - 1) C \\ &= e_j + \frac{e_j}{E} (\beta - 1) C \\ &= e_j \left(1 + (\beta - 1) \frac{C}{E} \right) \end{aligned}$$

Hence (7.11) is a DNA allocation. Note that (7.11) gives the amount allocated to player j . If we convert this to j 's allocated share of the pool (a_j), we obtain

$$a_j = \frac{A_j}{\beta C} = \frac{c_j}{\beta C} + \frac{\beta - 1}{\beta} \theta_j \quad (7.12)$$

$$\begin{aligned}
&= \frac{1}{\beta} \varphi_j + \left[1 - \frac{1}{\beta}\right] \theta_j \\
&= \theta_j + \frac{1}{\beta} [\varphi_j - \theta_j]
\end{aligned} \tag{7.13}$$

Thus j 's allocation share under a distribution neutral allocation is:

(i) A positively weighted average of j 's endowment and contribution shares, with the weight on the endowment share increasing in β .

(ii) Equal to its endowment share, plus an adjustment that depends on the difference between j 's contribution and endowment shares. If j 's contribution share exceeds her endowment share, she receives an allocation share larger than her endowment share to maintain the income distribution.

If all players contribute fully, then $\varphi_j = \theta_j$ for all j , and j 's allocation share is equal to her endowment share.

7.2. Appendix A2

7.2.1. Proof of Remark 1

Setting $c^* = (0, 0, 0)$, inequality (3.5) becomes

$$q_i(c^*)\beta [0] + e_i \geq q_i(c_i, c_{-i}^*)\beta [c_i] + e_i - c_i, \text{ where } c_i \neq 0.$$

The above can be simplified to

$$q_i(c_i, c_{-i}^*)\beta \leq 1.$$

If $q_i(c_i, c_{-i}^*) = 0$, then the above inequality is satisfied for all β . If $q_i(c_i, c_{-i}^*) \neq 0$, then the above inequality is satisfied for all β satisfying

$$\beta \leq \frac{1}{q_i(c_i, c_{-i}^*)}.$$

Since $q_i(c_i, c_{-i}^*)$ can take a maximum value of $\frac{2}{3}$ under the GM, we obtain the desired inequality. \square

7.2.2. Proof of Proposition 1 (PC)

Let e denote tuple (e_1, e_2, e_3) . Our aim is to show that $\beta > \frac{3}{2}$ implies that $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the PC allocation. We can therefore set $c^* = e$ in inequality (3.5) so that (3.5) becomes

$$\pi_i((e_i, e_{-i}), (a(e_i, e_{-i}))) \geq \pi_i((c_i, e_{-i}), (a(c_i, e_{-i}))) \tag{7.14}$$

for each i and for all c_i . Using the formula for π_i , the above can be reduced to

$$q_i(e)\beta [e_i + e_j + e_k] \geq q_i(c_i, e_{-i})\beta [c_i + e_j + e_k] + e_i - c_i. \quad (7.15)$$

Note that under the GM, under the PC allocation, for a given arbitrary contribution tuple c , $q_i(c) = \frac{1}{3} \left[\left(\frac{c_i}{c_i + c_j} \right) + \left(\frac{c_i}{c_i + c_k} \right) \right]$. To prove proposition 1, it suffices to show that the above inequality holds for all $\beta > \frac{3}{2}$ for each i and for all c_i . Since players are heterogeneous, we will show that the inequality is satisfied for each player. Indeed, we shall consider a pair of inequalities for each player, one in which $c_i > 0$ and another in which $c_i = 0$.

We start with player 2. First, suppose that $c_2 > 0$. Using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, and after simplifying, inequality (7.15) reduces to the following.

$$3\beta q_2(e) - 1 \geq \frac{\beta c_2}{3} \left(\frac{[2c_2 + 2][c_2 + 2]}{c_2^2 + 2c_2 + 1 - \delta^2} \right) - c_2 \quad (7.16)$$

We next consider the case where $c_2 = 0$. Then, the inequality becomes

$$q_2(e)\beta [e_1 + e_2 + e_3] \geq e_2 \text{ or}$$

$$3\beta q_2(e) \geq 1$$

This leads to the following lower bound on β for the case where $c_2 = 0$

$$\beta \geq \frac{1}{3q_2(e)}$$

Now, using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, after simplifying, it can be shown that $q_2(e) = \frac{4}{3} \left(\frac{1}{4 - \delta^2} \right)$. Thus, the lower bound on β for the case where $c_2 = 0$ becomes

$$\beta \geq \frac{4 - \delta^2}{4}. \quad (7.17)$$

As result, (7.16) and (7.17) give the inequalities that need to be satisfied so that player 2 does not deviate from the equilibrium.

We next consider player 1. First suppose that $c_1 > 0$. Using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, and after simplifying, inequality (7.15) reduces to the following.

$$3\beta q_1(e) - (1 + \delta) \geq \frac{\beta c_1}{3} \left(\frac{[2c_1 + 2 - \delta][c_1 + 2 - \delta]}{c_1^2 + (2 - \delta)c_1 + 1 - \delta} \right) - c_1 \quad (7.18)$$

We next suppose that $c_1 = 0$. Then, the inequality becomes

$$q_1(e)\beta [e_1 + e_2 + e_3] \geq e_1 \text{ or}$$

$$\beta \geq \frac{1 + \delta}{3q_1(e)}$$

Now, using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, after simplifying, it can be shown that $q_1(e) = \frac{1+\delta}{3} \left(\frac{4+\delta}{2(2+\delta)} \right)$. Thus, the lower bound on β becomes

$$\beta \geq \frac{2(2 + \delta)}{4 + \delta} \quad (7.19)$$

As result, (7.18) and (7.19) give the inequalities that need to be satisfied so that player 1 does not deviate from the equilibrium.

We finally consider player 3. First, suppose that $c_3 > 0$. Using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, and after simplifying, inequality (7.15) reduces to the following.

$$3\beta q_3(e) - (1 - \delta) \geq \frac{\beta c_3}{3} \left(\frac{[2c_3 + 2 + \delta][c_3 + 2 + \delta]}{c_3^2 + (2 + \delta)c_3 + 1 + \delta} \right) - c_3 \quad (7.20)$$

We next suppose that $c_3 = 0$. Then the inequality becomes

$$q_3(e)\beta [e_1 + e_2 + e_3] \geq e_3 \text{ or}$$

$$\beta \geq \frac{1 - \delta}{3q_3(e)} = \frac{(1 - \delta)}{3} \cdot \frac{1}{q_3(e)}$$

Now, using the $q_i(c)$ formula and expressing the endowments in terms of the dispersion parameter δ viz. (3.7), with the assumption that $\epsilon = 1$, after simplifying, it can be shown that $q_3(e) = \frac{1-\delta}{3} \left(\frac{4-\delta}{2(2-\delta)} \right)$. Thus, the lower bound on β becomes

$$\beta \geq \frac{2(2 - \delta)}{4 - \delta} \quad (7.21)$$

As result, (7.20) and (7.21) give the inequalities that need to be satisfied so that player 3 does not deviate from the equilibrium.

Next, for the case where $c_i > 0$, for all $\beta > \frac{3}{2}$, we establish a useful monotonicity property of the RHS of inequalities (7.16), (7.18) and (7.20) for players 2, 1 and 3 respectively. The result is given in terms of the following lemma.

Lemma 1: Suppose that $\beta > \frac{3}{2}$, then the RHS of inequalities (7.16), (7.18) and (7.20) for players 2, 1 and 3 respectively are strictly monotonically increasing in c_i for all i and $c_i > 0$.

Proof : Since $c_i \in E_i \subseteq [0, e_i]$, it suffices to show that for each i , the partial derivative of the RHS of inequalities (7.16), (7.18) and (7.20) are strictly positive when $\beta > \frac{3}{2}$, for all $c_i \in (0, e_i]$. We start with player 2. The derivative of the RHS of the inequality (7.16) is given by

$$\frac{\beta}{3} \left\{ \frac{(c_2^2 + 2c_2 + 1 - \delta^2)(6c_2^2 + 12c_2 + 4)}{(c_2^2 + 2c_2 + 1 - \delta^2)^2} - \frac{((2c_2^3 + 6c_2^2 + 4c_2))(2c_2 + 2)}{(c_2^2 + 2c_2 + 1 - \delta^2)^2} \right\} - 1$$

It can be shown that the above expression is strictly positive if and only if the following inequality is satisfied.

$$(c_2^2 + 2c_2 + 1 - \delta^2) \left([2\beta - 1] c_2^2 + [4\beta - 2] c_2 + \frac{4}{3}\beta - (1 - \delta^2) \right) - \frac{\beta}{3} [2c_2^3 + 6c_2^2 + 4c_2] [2c_2 + 2] > 0 \quad (7.22)$$

For player 1, it can be shown that the derivative of the RHS of the inequality (7.18) is given by

$$\frac{\beta}{3} \left\{ \frac{(c_1^2 + (2 - \delta)c_1 + 1 - \delta)(6c_1^2 + 6(2 - \delta)c_1 + (2 - \delta)^2)}{(c_1^2 + (2 - \delta)c_1 + 1 - \delta)^2} - \frac{(2c_1^3 + 3(2 - \delta)c_1^2 + (2 - \delta)^2 c_1)(2c_1 + 2 - \delta)}{(c_1^2 + (2 - \delta)c_1 + 1 - \delta)^2} \right\} - 1$$

It can be shown that the above expression is strictly positive if and only if the following inequality is satisfied.

$$(c_1^2 + (2 - \delta)c_1 + 1 - \delta) \left([2\beta - 1] c_1^2 + [2\beta(2 - \delta) - (2 - \delta)] c_1 + \frac{\beta}{3}(2 - \delta)^2 - (1 - \delta) \right) - \frac{\beta}{3} [2c_1^3 + 3(2 - \delta)c_1^2 + (2 - \delta)^2 c_1] [2c_1 + (2 - \delta)] > 0 \quad (7.23)$$

For player 3, it can be shown that the derivative of the RHS of the inequality (7.20) is given by

$$\frac{\beta}{3} \left\{ \frac{(c_3^2 + (2 + \delta)c_3 + 1 + \delta)(6c_3^2 + 6(2 + \delta)c_3 + (2 + \delta)^2)}{(c_3^2 + (2 + \delta)c_3 + 1 + \delta)^2} - \frac{((2c_3^3 + 3(2 + \delta)c_3^2 + (2 + \delta)^2 c_3))(2c_3 + 2 + \delta)}{(c_3^2 + (2 + \delta)c_3 + 1 + \delta)^2} \right\}$$

It can be shown that the above expression is strictly positive if and only if the following inequality is satisfied.

$$(c_3^2 + (2 + \delta)c_3 + 1 + \delta) \left([2\beta - 1]c_3^2 + [2\beta(2 + \delta) - (2 + \delta)]c_3 + \frac{\beta}{3}(2 + \delta)^2 - (1 + \delta) \right) - \frac{\beta}{3} [2c_3^3 + 3(2 + \delta)c_3^2 + (2 + \delta)^2c_3] [2c_3 + (2 + \delta)] > 0 \quad (7.24)$$

Since $c_i \in (0, e_i]$ and $\beta > \frac{3}{2}$, it can be shown that the coefficients of the terms in c_2^n , c_1^n and c_3^n (where $n = 0, 1, 2, 3, 4$) on the LHS of inequalities (7.22), (7.23) and (7.24) respectively, which are polynomials of order 4, are strictly positive. Therefore, the RHS of (7.16), (7.18) and (7.20) are strictly increasing in c_i . \square

Proof of Proposition 1 : From Lemma 1, we know that the RHS of (7.16), (7.18) and (7.20) are strictly increasing in c_i for players 2, 1, and 3 respectively, and therefore, fixing c_{-i} at e_{-i} , no $c_i \in (0, e_i)$ can be part of a SPNE. Now, $\beta > \frac{3}{2}$ also ensures that inequalities (7.17), (7.19) and (7.21) are satisfied and, therefore, deviations by choosing $c_i = 0$ are not profitable for all i . Moreover, at $c_i = e_i$, (7.16), (7.18) and (7.20) are satisfied with equality. Hence, $c_i = e_i$ is part of the proposed SPNE of the game under the PC allocation. \square

7.3. Appendix A3

7.3.1. Proof of Proposition 2 (PCR)

Our aim is to show that $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the PCR allocation if

$$\beta \geq \left\{ \begin{array}{ll} \frac{3}{2} & \text{if } \delta \leq \frac{1}{2} \\ 1 + \delta & \text{otherwise} \end{array} \right\}.$$

As in the proof of proposition 1, we divide inequality (7.15) into two cases; (i) $c_i = 0$ and (ii) $c_i > 0$. Then, (7.15) can be rewritten as

$$q_i(e)\beta[e_i + e_j + e_k] \geq q_i(0, e_{-i})\beta[e_j + e_k] + e_i \quad \text{if } c_i = 0, \text{ and} \quad (7.25)$$

$$\left[\beta \frac{q_i(e)}{c_i(e)} - 1 \right] e_i \geq \left[\beta \frac{q_i(c_i, e_{-i})}{c_i(c_i, e_{-i})} - 1 \right] c_i \quad \text{if } c_i > 0. \quad (7.26)$$

In the latter case, for endowment tuple e , we denote $\frac{q_i(e)}{c_i(e)}$ by $R(e)$ and $\frac{q_i(c_i, e_{-i})}{c_i(c_i, e_{-i})}$ by $R_i(c_i, e_{-i})$.⁵⁰

Note that under the GM, under the PCR allocation, for a given arbitrary contribution tuple c , $q_i(c) = \frac{1}{3} \left[\frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + \frac{e_j}{e_j}} + \frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + \frac{e_k}{e_k}} \right]$. Thus, for each i , we have $q_i(e) = \frac{1}{3}$ and

$$\begin{aligned} q_i(c_i, e_{-i}) &= \frac{1}{3} \left[\frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + 1} + \frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + 1} \right] \\ &= \frac{2}{3} \left[\frac{c_i}{c_i + e_i} \right] \end{aligned}$$

It can be shown that the $q_i(c_i, e_{-i})$ function above under the PCR satisfies the following.⁵¹

(I) Strict Monotonicity over all pairs (c_i, e_{-i}) : $q_i(e) > q_i(c_i, e_{-i})$ for all i and (c_i, e_{-i}) such that $c_i \neq e_i$.

(II) Maximum punishment for zero effort: $q_i(0, e_{-i}) = 0$ for all i .

We first consider inequality (7.26), which can be rewritten as follows

$$\begin{aligned} [\beta R_i(e) - 1] e_i &\geq [\beta R_i(c_i, e_{-i}) - 1] c_i \text{ or} \\ \beta (R_i(e) e_i - R_i(c_i, e_{-i}) c_i) &\geq e_i - c_i \end{aligned} \tag{7.27}$$

Now, assumption (I) ensures that $(R_i(e) e_i - R_i(c_i, e_{-i}) c_i) > 0$. *This is because*

$$(R_i(e) e_i - R_i(c_i, e_{-i}) c_i) > 0 \text{ iff}$$

$$\frac{R_i(e)}{R_i(c_i, e_{-i})} > \frac{c_i}{e_i} \text{ iff}$$

$$q_i(e) [e_i + e_j + e_k] > q_i(c_i, e_{-i}) [c_i + e_j + e_k] \text{ (which holds by (I))}$$

Inequality (7.27) implies a lower bound on β for the case where $c_i > 0$, as follows.

$$\beta \geq \frac{e_i - c_i}{R_i(e) e_i - R_i(c_i, e_{-i}) c_i} \tag{7.28}$$

The above inequality will be used in proof of proposition 2 for the case where $c_i > 0$.

⁵⁰Note that a similar approach was employed in DFL.

⁵¹Incidentally, the allocation under PC also satisfies these properties.

Proof of Proposition 2 We first consider the case where $c_i > 0$. Thus, we can use inequality(7.28) directly. In order to do this, we first compute the following expressions.

$$R_i(e) = \frac{\bar{s}}{3e_i}$$

$$\text{where } \bar{s} = e_i + e_j + e_k$$

$$\begin{aligned} R_i(c_i, e_{-i}) &= \frac{\frac{2}{3} \left[\frac{c_i}{c_i + e_i} \right]}{\frac{c_i}{c_i + \bar{s} - e_i}} \\ &= \frac{\frac{2}{3} \left[c_i + \bar{s} - e_i \right]}{c_i + e_i} \end{aligned}$$

Hence,

$$R_i(e) e_i = \frac{\bar{s}}{3}$$

and

$$R_i(c_i, e_{-i}) c_i = \frac{2}{3} c_i \left[\frac{c_i + \bar{s} - e_i}{c_i + e_i} \right]$$

Therefore, (7.28) becomes

$$\beta \geq \frac{3(e_i^2 - c_i^2)}{-2c_i^2 - c_i \bar{s} + \bar{s} e_i + 2c_i e_i}$$

Using (3.7) and letting $\epsilon = 1$, we have the following bounds for for case where $c_i > 0$, for each player.

For player 2,

$$\begin{aligned} \beta &\geq \frac{3(1 - c_2^2)}{-2c_2^2 - 3c_2 + 3 + 2c_2} = \\ &\frac{3(1 + c_2)}{2c_2 + 3} \end{aligned}$$

As the RHS of the above inequality is increasing in c_2 , we set $c_2 = e_2$ to obtain

$$\beta \geq \frac{6}{5} \tag{7.29}$$

For player 3,

$$\beta \geq \frac{3 \left((1 - \delta)^2 - c_3^2 \right)}{-2c_3^2 - 3c_3 + 3(1 - \delta) + 2c_3(1 - \delta)} = \frac{3(1 - \delta + c_3)}{2c_3 + 3}$$

As the RHS of the above inequality is increasing in c_3 , we set $c_3 = e_3$ to obtain

$$\beta \geq \frac{6(1 - \delta)}{5 - 2\delta} \quad (7.30)$$

The RHS of the above is decreasing in δ .

For player 1,

$$\beta \geq \frac{3 \left((1 + \delta)^2 - c_1^2 \right)}{-2c_1^2 - 3c_1 + 3(1 + \delta) + 2c_1(1 + \delta)} = \frac{3(1 + \delta + c_1)}{2c_1 + 3}$$

Suppose $\delta = \frac{1}{2}$, then we obtain

$$\beta \geq \frac{3}{2}$$

Suppose $\delta < \frac{1}{2}$. Then, the RHS of the inequality is increasing in c_1 , we set $c_1 = e_1$ to obtain

$$\beta \geq \frac{6(1 + \delta)}{2(1 + \delta) + 3}$$

The RHS of the above is increasing in δ .

Suppose $\delta > \frac{1}{2}$. Then, the RHS of the inequality is decreasing in c_1 , we set $c_1 \rightarrow 0$ to obtain

$$\beta \geq \lim_{c_1 \rightarrow 0} \frac{3(1 + \delta + c_1)}{2c_1 + 3} = \frac{3(1 + \delta)}{3} = 1 + \delta$$

The RHS of the above is increasing in δ . We therefore conclude that the lower bound for player 1 of

$$\beta \geq \left\{ \begin{array}{ll} \frac{3}{2} & \text{if } \delta \leq \frac{1}{2} \\ 1 + \delta & \text{otherwise} \end{array} \right\} \quad (7.31)$$

will ensure that the LHS is greater or equal to the RHS for all values of δ . (as noted in Remark 1 after equation (3.6), we do consider bounds below $\frac{3}{2}$ in all of our results as for these values, a zero effort SPNE always exists). We also observe from inequalities (7.29), (7.30) and (7.31) that the maximum lower bounds of all the three players is

given by inequality (7.31), that is, only the bound of player 1 (the rich player). Hence, in order to ensure that no player deviates at the proposed SPNE, we need to impose inequality (7.31) for the case where $c_i > 0$.

For the case where $c_i = 0$, using inequality (7.25), it can be shown that $\beta \geq 1$ for player 3, $\beta \geq 1 - \delta$ for player 2 and $\beta \geq 1 + \delta$ for player 1 are sufficient for the inequality to hold for the respective players. But the latter conditions are implied by the bounds given in (7.31). This completes the proof. \square

7.4. Appendix A4

7.4.1. Proof of Proposition 3 (MPCR)

Our aim is to show that $s = ((e_1, a_1(e)), (e_2, a_2(e)), (e_3, a_3(e)))$ is a SPNE of the game, where function $a(e) = (a_1(e), a_2(e), a_3(e))$ satisfies the MPCR allocation if

$$\beta \geq \left\{ \begin{array}{ll} \frac{3}{2} & \text{if } \delta \leq \frac{1}{2} \\ \frac{1}{\left(\frac{2\gamma}{3} + \frac{1-\gamma}{1+\delta}\right)} & \text{if } \delta > \frac{1}{2} \end{array} \right\}$$

For a given arbitrary contribution tuple c , the allocation share of player j to player i , a_{ji} as follows

$$\begin{aligned} a_{ji} &= \gamma a_{ji}^{PC} + (1 - \gamma) a_{ji}^{PCR} \\ &= \gamma \left(\frac{c_i}{c_i + c_k} \right) + (1 - \gamma) \frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + \frac{c_k}{e_k}} \\ &\text{if both } c_i + c_k > 0 \text{ and } \frac{c_i}{e_i} + \frac{c_k}{e_k} > 0 \text{ hold.} \end{aligned}$$

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It can be shown that the above is a well-defined allocation in the GM. It can also be shown that for a given arbitrary contribution tuple c , the resultant share of player i will be given by

$$\begin{aligned} q_i(c) &= \gamma q_i^{PC} + (1 - \gamma) q_i^{PCR} \\ &= \frac{\gamma}{3} \left[\left(\frac{c_i}{c_i + c_j} \right) + \left(\frac{c_i}{c_i + c_k} \right) \right] + \\ &\quad \frac{(1 - \gamma)}{3} \left[\frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + \frac{c_j}{e_j}} + \frac{\frac{c_i}{e_i}}{\frac{c_i}{e_i} + \frac{c_k}{e_k}} \right] \end{aligned}$$

⁵²As before, we assume that each of the two terms in the expression takes value $\frac{1}{2}$ if its denominator is equal to zero.

It suffices to show that inequality (7.15) holds for all c_i . The following definitions will help in establishing the proof of the proposition.

Let $\beta [e_i + e_j + e_k] \equiv U_i(e)$ and $\beta [c_i + e_j + e_k] \equiv U_i(c_i, e_{-i})$. Moreover, let $e_i - c_i \equiv \Delta c_i$, $q_i(e) - q_i(c_i, e_{-i}) \equiv \Delta q(c_i)$ and $U_i(e) - U_i(c_i, e_{-i}) \equiv \Delta U(c_i)$. Then, inequality (7.15) becomes

$$(q_i(c_i, e_{-i}) + \Delta q(c_i))(U_i(c_i, e_{-i}) + \Delta U(c_i)) \geq q_i(c_i, e_{-i})U_i(c_i, e_{-i}) + \Delta c_i$$

which reduces to

$$\beta \Delta c_i q_i(c_i, e_{-i}) + \Delta q(c_i) U_i(c_i, e_{-i}) + \beta \Delta q(c_i) \Delta c_i \geq \Delta c_i \quad (7.32)$$

We will show the inequality holds for player 1 (the rich player).⁵⁴

We first establish the lower bounds on β for the case where $\gamma = 0$.

Using (3.7) and with the assumption that $\epsilon = 1$, it can be shown that for player 1

$$\Delta q_1^{PCR} = \frac{1}{3} \left(\frac{\Delta c_1}{c_1 + 1 + \delta} \right)$$

Then, the above inequality becomes

$$\frac{2\beta}{3} \left(\frac{c_1 \Delta c_1}{c_1 + 1 + \delta} \right) + \frac{\beta}{3} \left(\frac{\Delta c_1 [c_1 + 2 - \delta]}{c_1 + 1 + \delta} \right) + \frac{\beta}{3} \left(\frac{\Delta c_1 \cdot \Delta c_1}{c_1 + 1 + \delta} \right) \geq \Delta c_1$$

which reduces to

$$\begin{aligned} \frac{\beta}{3} \left[\left(\frac{2c_1}{c_1 + 1 + \delta} \right) + \left(\frac{[c_1 + 2 - \delta]}{c_1 + 1 + \delta} \right) + \left(\frac{\Delta c_1}{c_1 + 1 + \delta} \right) \right] &\geq 1 \\ \frac{\beta}{3} &\geq \frac{c_1 + 1 + \delta}{2 + 3c_1 - \delta + 1 + \delta - c_1} \end{aligned}$$

Simplifying, we have

$$\frac{\beta}{3} \geq \frac{c_1 + 1 + \delta}{3 + 2c_1} \quad (7.33)$$

This leads to the exact same inequality as what was found in the proof of proposition 2, that is, (7.31) and hence, we have the following bound.

$$\beta \geq \left\{ \begin{array}{l} \frac{3}{2} \text{ if } \delta \leq \frac{1}{2} \\ 1 + \delta \text{ otherwise} \end{array} \right\}$$

⁵³ As before, we obtain that each of the two terms in the expression takes value $\frac{1}{2}$ if its denominator is equal to zero.

⁵⁴ The same can be established for players 2 and 3 using similar steps.

We next consider the case where $\gamma = 1$ for which we have

$$q_1(e) = \frac{1}{3} \left(\frac{1+\delta}{2+\delta} + \frac{1+\delta}{2} \right)$$

and

$$\Delta q_1^{PC} = \frac{1}{3} \left(\frac{1+\delta}{2+\delta} - \frac{c_1}{(c_1+1)} + \frac{1+\delta}{2} - \frac{c_1}{(c_1+1-\delta)} \right)$$

Simplifying, we have

$$\Delta q_1^{PC} = \frac{1}{3} \left(\frac{\Delta c_1}{(c_1+1)(2+\delta)} + \frac{(1-\delta)\Delta c_1}{2(c_1+1-\delta)} \right)$$

Inequality (7.32) becomes

$$\frac{\beta}{3} \left[\left(\frac{c_1}{c_1+1} + \frac{c_1}{c_1+1-\delta} \right) + (c_1+2-\delta) \left(\frac{1}{(c_1+1)(2+\delta)} + \frac{(1-\delta)}{2(c_1+1-\delta)} \right) + \Delta c_1 \left(\frac{1}{(c_1+1)(2+\delta)} + \frac{(1-\delta)}{2(c_1+1-\delta)} \right) \right] \geq 1,$$

which reduces to

$$\frac{\beta}{3} \left[\frac{c_1 + \frac{3}{2+\delta}}{c_1+1} + \frac{c_1 + \frac{3(1-\delta)}{2}}{c_1+1-\delta} \right] \geq 1$$

Let $K \equiv \frac{3}{2+\delta}$ and $L \equiv \frac{3(1-\delta)}{2}$. Then, after simplifying, the above inequality becomes

$$\frac{\beta}{3} \geq \frac{c_1^2 + (2-\delta)c_1 + 1 - \delta}{2c_1^2 + (2-\delta + K + L)c_1 + L + K(1-\delta)} \quad (7.34)$$

Multiplying by the denominator on both sides, we have quadratic expressions on both sides on the inequality. We can therefore make the RHS 0. Then, we claim that $\beta > \frac{3}{2}$ is a sufficient condition for the coefficient terms in c_1^n (where $n = 0, 1, 2$) of the reduced LHS of the inequality to be positive. For the term in c_1^2 , this conclusion is straightforward. For the constant term, we need to show that

$$\frac{\beta}{3} (L + K(1-\delta)) \geq 1 - \delta$$

The above simplifies to

$$\frac{\beta}{2} \geq \frac{2+\delta}{4+\delta}$$

which is satisfied by $\beta > \frac{3}{2}$. We next turn to the term in c_1 which leads to the following inequality.

$$\frac{\beta}{3} (2-\delta + K + L) \geq (2-\delta)$$

The above can be simplified to the following

$$\frac{\beta}{3} \geq \frac{8 - 2\delta^2}{-5\delta^2 - 3\delta + 20}$$

which is satisfied by $\beta > \frac{3}{2}$.

Now that we have inequalities (7.33) and (7.34) for the extreme cases where $\gamma = 0$ and $\gamma = 1$, we can establish the bounds for the mixed proportional allocation, that is, when $q_i = \gamma q_i^{PC} + (1 - \gamma) q_i^{PCR}$. Let $A = \frac{3(c_1+1+\delta)}{3+2c_1}$ and $B = \frac{3(c_1^2+(2-\delta)c_1+1-\delta)}{2c_1^2+(2-\delta+K+L)c_1+L+K(1-\delta)}$ from inequalities (7.33) and (7.34) respectively. Then, by the previous argument for $\gamma = 0$, we know that A is bounded above by $\frac{3}{2}$ if $\delta \leq \frac{1}{2}$ and by $1 + \delta$ otherwise. We also know by the inequalities for the case where $\gamma = 1$, that B is bounded above by $\frac{3}{2}$.⁵⁵

Then, on the one hand, from the above arguments, we have the following two inequalities.

$$\gamma \frac{1}{B} \beta \geq \gamma$$

and

$$(1 - \gamma) \frac{1}{A} \beta \geq 1 - \gamma$$

which can be combined to the following inequality.

$$\gamma \frac{1}{B} \beta + (1 - \gamma) \frac{1}{A} \beta \geq 1 \tag{7.35}$$

On the other hand, inequality (7.32) becomes

$$\begin{aligned} & \gamma (\beta \Delta c_i q_i^{PC}(c_i, e_{-i}) + \Delta q^{PC}(c_i) U_i(c_i, e_{-i}) + \beta \Delta q^{PC}(c_i) \Delta c_i) \\ & + (1 - \gamma) (\beta \Delta c_i q_i^{PCR}(c_i, e_{-i}) + \Delta q^{PCR}(c_i) U_i(c_i, e_{-i}) + \beta \Delta q^{PCR}(c_i) \Delta c_i) \geq \Delta c_i \end{aligned}$$

The above simplifies to

$$\gamma \frac{1}{B} \beta + (1 - \gamma) \frac{1}{A} \beta \geq 1,$$

which is nothing but inequality (7.35).

Hence, the bound for the mixed allocation becomes

$$\begin{aligned} \beta & \geq \frac{1}{\inf_{c_1} \left(\frac{\gamma}{B} + \frac{1-\gamma}{A} \right)} \\ & \geq \frac{1}{\left(\inf_{c_1} \frac{\gamma}{B} + \inf_{c_1} \frac{1-\gamma}{A} \right)} \end{aligned}$$

⁵⁵In fact these bounds are the respective supremums of A and B over e_1 .

$$= \frac{1}{\left(\frac{2\gamma}{3} + \frac{1-\gamma}{1+\delta}\right)} \text{ if } \delta > \frac{1}{2} \quad (7.36)$$

If $\delta \leq \frac{1}{2}$, the bound is $\frac{3}{2}$. Now taking the derivative of the RHS of (7.36) with respect to γ , we have

$$\frac{dRHS}{d\gamma} = - \left(\frac{1}{\left(\frac{2\gamma}{3} + \frac{1-\gamma}{1+\delta}\right)} \right)^2 \left(\frac{2}{3} - \frac{1}{1+\delta} \right) \text{ if } \delta > \frac{1}{2}$$

Since $\frac{2}{3} - \frac{1}{1+\delta} \geq 0$, the sign of the above is negative and hence, we get a negative relationship between γ (the weight assigned to the PC) and the lowest beta needed to support full effort as SPNE. This completes the proof. \square

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Online Appendix C

Table C1: OLS regressions on tokens contributed (rounds 1-5)

	(1) Full Sample	(2) High Inequality	(3) Low Inequality
GM2.2	0.261 (0.322)	0.645 (0.503)	0.00706 (0.373)
GM1.8	0.495 (0.482)	-0.0382 (0.509)	0.810 (0.686)
GM1.6	0.102 (0.245)	0.314 (0.307)	-0.0744 (0.382)
GM1.2	0.507 (0.600)	0.810 (0.847)	0.0692 (0.841)
High Inequality	-0.184 (0.303)		
ZUFE	-0.499 [†] (0.287)	-0.446 (0.376)	-0.472 (0.421)
Age	0.128 (0.0772)	0.181 [†] (0.0970)	0.0968 (0.118)
Female	-0.00683 (0.189)	-0.400 (0.248)	0.357 (0.248)
Economics	0.195 (0.175)	0.214 (0.212)	0.129 (0.257)
Constant	-0.255 (1.543)	-1.411 (1.847)	0.232 (2.316)
Observations	1620	810	810
R ²	0.021	0.044	0.037
Linear Restriction Tests (p-values)			
GM2.2 vs GM1.8	0.654	0.230	0.301
GM2.2 vs GM1.6	0.605	0.477	0.833
GM2.2 vs GM1.2	0.706	0.866	0.942
GM1.8 vs GM1.6	0.403	0.439	0.237
GM1.8 vs GM1.2	0.987	0.392	0.482
GM1.6 vs GM1.2	0.502	0.543	0.871

Notes: The dependent variable is number of tokens contributed. Only data from rounds 1-5 are included. The omitted treatment category is *Control*. Standard errors, in parentheses, are clustered by session (36 clusters in Regression (1); 18 in Regressions (2) and (3)).

[†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C2: OLS regressions on tokens contributed in Control treatments (rounds 6-15)

	(1) All	(2) All	(3) Round 6	(4) Round 15
High Inequality	-0.346 (0.284)		-0.976* (0.403)	-0.258 (0.458)
ZUFE	0.351 (0.323)		-0.0849 (0.496)	0.202 (0.421)
Age	0.0871 (0.0851)		0.103 (0.156)	0.0848 (0.113)
Female	-0.149 (0.427)		-0.249 (0.702)	0.348 (0.449)
Economics	-0.132 (0.434)		0.136 (0.804)	0.308 (0.305)
Period		-0.123** (0.028)		
Constant	0.290 (1.605)	3.139*** (0.319)	1.202 (2.991)	-0.713 (2.167)
Observations	720	720	72	72
R ²	0.022	0.036	0.049	0.037

Notes: The dependent variable is number of tokens contributed. Only data from Control treatments and rounds 6-15 are included. Standard errors, in parentheses, are clustered by session (8 clusters).

[†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C3: OLS regressions on tokens contributed in GM treatments (rounds 6-15)

	(1) $\beta = 2.2,$ $\delta = 0.75$	(2) $\beta = 1.8,$ $\delta = 0.75$	(3) $\beta = 1.6,$ $\delta = 0.75$	(4) $\beta = 1.2,$ $\delta = 0.75$	(5) $\beta = 2.2,$ $\delta = 0.5$	(6) $\beta = 1.8,$ $\delta = 0.5$	(7) $\beta = 1.6,$ $\delta = 0.5$	(8) $\beta = 1.2,$ $\delta = 0.5$
Period	0.409* (0.087)	0.303 (0.126)	0.021 (0.064)	-0.081 (0.070)	0.380** (0.063)	0.381* (0.096)	0.300* (0.032)	-0.081 (0.029)
Constant	2.001 (0.996)	1.043 (0.905)	3.441* (0.663)	4.216 [†] (1.451)	1.909 [†] (0.620)	2.493 (1.402)	1.266 [†] (0.401)	4.001 [†] (1.110)
Observations	270	270	360	360	360	360	270	270
R ²	0.075	0.055	0.001	0.005	0.133	0.113	0.128	0.010

Notes: The dependent variable is the number of tokens contributed. Only data from the GM treatments and rounds 6-15 are included. Each model includes data only from one treatment, as indicated at the top of the table. Standard errors, in parentheses, are clustered by session (3 clusters in Regressions (1), (2), (7) and (8); 4 in Regressions (3), (4), (5) and (6)). [†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C4: OLS regressions on tokens contributed (rounds 6-15)

	(1) GM2.2	(2) GM1.8	(3) GM1.6	(4) GM1.2
High Inequality	0.465 (0.676)	-2.475* (0.742)	-0.693 (0.377)	0.277 (1.266)
ZUFE	-0.290 (0.682)	-0.707 (0.486)	-0.662 [†] (0.282)	0.415 (1.025)
Age	-0.0400 (0.0814)	0.0427 (0.148)	0.234 (0.142)	0.0380 (0.195)
Female	-0.382 (0.328)	-0.763 (0.726)	-1.209** (0.317)	-0.173 (0.294)
Economics	-0.181 (0.524)	0.646 (0.536)	0.821* (0.240)	-0.106 (0.426)
Constant	7.167** (1.615)	6.313 (3.394)	0.509 (2.879)	2.244 (3.534)
Observations	630	630	630	630
R ²	0.008	0.111	0.093	0.009

Notes: The dependent variable is number of tokens contributed. Only data from GM treatments and rounds 6-15 are included. Regression (1) includes data only from sessions with $\beta=2.2$; Regression (2) includes data only from sessions with $\beta=1.8$; Regression (3) includes data only from sessions with $\beta=1.6$; Regression (4) includes data only from sessions with $\beta=1.2$. Standard errors, in parentheses, are clustered by session (7 clusters in each model).

[†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C5: OLS regressions on Contribution Ratios and Contributions in Control Treatments - analysis of player type

	Contribution Ratios			Contribution Levels			
	(1) All Control	(2) HI	(3) LI	(1) All Control	(2) HI	(3) LI	
Middle	-0.0547 (0.034)	-0.1000 (0.0634)	-0.00937 (0.0216)	1.137*** (0.155)	1.200* (0.284)	1.075** (0.171)	
Rich	-0.110* (0.0352)	-0.152 [†] (0.0511)	-0.0687 (0.0450)	1.675** (0.333)	1.875* (0.512)	1.475 [†] (0.479)	
Constant	0.310*** (0.0291)	0.333* (0.0583)	0.287*** (0.0143)	0.908*** (0.109)	0.667* (0.117)	1.150*** (0.0571)	
Obs	720	360	360	720	360	360	
R ²	0.031	0.059	0.014	0.138	0.201	0.096	

Notes: Only data from the Control treatments and rounds 6-15 are included. The omitted player type is Poor. Standard errors, in parentheses, are clustered by session (8 clusters in Regression (1); 4 in Regressions (2) and (3)). [†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C6: OLS regressions on contribution ratios in GM treatments – analysis of player type

	(1) $\beta = 2.2,$ $\delta = 0.75$	(2) $\beta = 1.8,$ $\delta = 0.75$	(3) $\beta = 1.6,$ $\delta = 0.75$	(4) $\beta = 1.2,$ $\delta = 0.75$	(5) $\beta = 2.2,$ $\delta = 0.5$	(6) $\beta = 1.8,$ $\delta = 0.5$	(7) $\beta = 1.6,$ $\delta = 0.5$	(8) $\beta = 1.2,$ $\delta = 0.5$
Middle	-0.161 (0.0611)	-0.275* (0.0361)	-0.379** (0.0323)	-0.395* (0.113)	-0.109 [†] (0.0397)	-0.0781* (0.0243)	-0.172** (0.0164)	-0.257* (0.0507)
Rich	-0.207* (0.0217)	-0.413* (0.0566)	-0.485*** (0.0360)	-0.552** (0.0794)	-0.190 [†] (0.0666)	-0.103 [†] (0.0392)	-0.248* (0.0463)	-0.375* (0.0725)
Constant	0.961*** (0.0223)	0.861** (0.0712)	0.867*** (0.0426)	0.875*** (0.0663)	0.869*** (0.0384)	0.890*** (0.0456)	0.733** (0.0653)	0.667* (0.144)
Observations	270	270	360	360	360	360	270	270
R ²	0.129	0.232	0.420	0.344	0.084	0.033	0.148	0.235

Notes: The dependent variable is the contribution ratio. Only data from the GM treatments and rounds 6-15 are included. Each model includes data only from one treatment, as indicated at the top of the table. The omitted player type is *Poor*. Standard errors, in parentheses, are clustered by session (3 clusters in Regressions (1), (2), (7) and (8); 4 in Regressions (3), (4), (5) and (6)). We note that in the regressions with only 3 clusters, an F-statistic cannot be estimated. [†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C7: OLS regressions on tokens contributed in GM treatments – analysis of player type

	(1) $\beta = 2.2,$ $\delta = 0.75$	(2) $\beta = 1.8,$ $\delta = 0.75$	(3) $\beta = 1.6,$ $\delta = 0.75$	(4) $\beta = 1.2,$ $\delta = 0.75$	(5) $\beta = 2.2,$ $\delta = 0.5$	(6) $\beta = 1.8,$ $\delta = 0.5$	(7) $\beta = 1.6,$ $\delta = 0.5$	(8) $\beta = 1.2,$ $\delta = 0.5$
Middle	4.478** (0.359)	2.967 [†] (0.704)	2.167*** (0.0874)	2.092 (1.117)	2.600** (0.426)	2.933*** (0.171)	1.556** (0.137)	0.611 (0.359)
Rich	8.633** (0.476)	4.556 (1.612)	3.608* (0.678)	2.767 (1.354)	4.675* (0.987)	5.883** (0.642)	2.889** (0.195)	0.833 (0.770)
Constant	1.922*** (0.0446)	1.722** (0.142)	1.733*** (0.0852)	1.750*** (0.133)	3.475*** (0.153)	3.558*** (0.182)	2.933** (0.261)	2.667* (0.577)
Observations	270	270	360	360	360	360	270	270
R ²	0.676	0.258	0.289	0.129	0.409	0.541	0.240	0.022

Notes: The dependent variable is tokens contributed. Only data from the GM treatments and rounds 6-15 are included. Each model includes data only from one treatment, as indicated at the top of the table. The omitted player type is *Poor*. Standard errors, in parentheses, are clustered by session (3 clusters in Regressions (1), (2), (7) and (8); 4 in Regressions (3), (4), (5) and (6)). We note that in the regressions with only 3 clusters, an F-statistic cannot be estimated. [†] $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table C8: OLS regressions on allocation to player j

	(1) High Inequality	(2) Low Inequality	(3) GM2.2	(4) GM1.8	(5) GM1.6	(6) GM1.2
$e_j/(e_j + e_k)$	0.358*** (0.0470)	0.304*** (0.0415)	0.265** (0.0536)	0.321* (0.100)	0.331*** (0.0407)	0.414** (0.0904)
$(e_j/f_j)/(e_j/f_j + e_k/f_k)$	0.574*** (0.0710)	0.698*** (0.0539)	0.668*** (0.0798)	0.679** (0.136)	0.661*** (0.0530)	0.506** (0.123)
GM1.8	-0.0588** (0.0178)	0.00503 (0.0165)				
GM1.6	-0.0491* (0.0183)	0.0215 (0.0133)				
GM1.2	-0.0309† (0.0163)	0.00471 (0.0149)				
ZUFE	- 0.000962 (0.0114)	-0.0181 (0.0130)	0.0165 (0.0159)	-0.0440† (0.0215)	-0.0206 (0.0215)	- 0.000131 (0.0134)
Age	0.00838† (0.00455)	- 0.000963 (0.00283)	- 0.00675† (0.00338)	0.00936 (0.00551)	0.0104 (0.00850)	0.00710 (0.00436)
Female	0.0397* (0.0134)	0.00167 (0.0108)	0.0153 (0.0187)	0.0263† (0.0128)	0.0165 (0.0196)	0.0255 (0.0251)
Economics	-0.0173 (0.0129)	0.0220 (0.0141)	0.00341 (0.0151)	0.0116 (0.0131)	0.00861 (0.0299)	-0.0218 (0.0199)
High Inequality			0.0372 (0.0204)	-0.0107 (0.0198)	-0.0143 (0.0100)	0.0283† (0.0141)
Constant	-0.0693 (0.0952)	0.0283 (0.0539)	0.163† (0.0705)	-0.170 (0.112)	-0.174 (0.166)	-0.0760 (0.0728)
Observations	1260	1260	630	630	630	630
R ²	0.618	0.637	0.441	0.586	0.624	0.704

Notes: The dependent variable is the share of player i's combined allocation going to player j (the disadvantaged player). Data from GM treatments, rounds 6-15, are included; Regression (1) contains only data from *High Inequality* sessions; Regression (2) contains only data from *Low Inequality* sessions; Regression (3) contains only data from sessions with $\beta=2.2$; Regression (4) contains only data from sessions with $\beta=1.8$; Model (5) contains only data from sessions with $\beta=1.6$; Regression (6) contains only data from sessions with $\beta=1.2$. The omitted treatment category is *GM2.2*. Standard errors, in parentheses, are clustered by session (14 clusters in regressions (1) and (2); 7 clusters in other regressions).

† $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$