A Theoretical and Empirical Comparison of Systemic Risk Measures

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Abstract

We propose a theoretical and empirical comparison of the most popular systemic risk measures. To do so, we derive the systemic risk measures in a common framework and show that they can be expressed as linear transformations of firms' market risk (e.g., beta). We also derive conditions under which the different measures lead similar rankings of systemically important financial institutions (SIFIs). In an empirical analysis of US financial institutions, we show that (1) different systemic risk measures identify different SIFIs and that (2) firm rankings based on systemic risk estimates mirror rankings obtained by sorting firms on market risk or liabilities. One-factor linear models explain most of the variability of the systemic risk estimates, which indicates that standard systemic risk measures fall short in capturing the multiple facets of systemic risk.

Keywords: Banking Regulation, Systemically Important Financial Firms, Marginal Expected Shortfall, SRISK, CoVaR, Systemic vs. Systematic Risk.

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1 Introduction

The recent financial crisis has fostered extensive research on systemic risk, either on its definition, measurement, or regulation.[1] Of particular interest is the identification of the financial institutions that contribute the most to the overall risk of the financial system – the so-called Systemically Important Financial Institutions (SIFIs). The Financial Stability Board (2011) defines SIFIs as "financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause significant disruption to the wider financial system and economic activity". As they pose a major threat to the system, regulators and policy makers from around the world have called for tighter supervision, extra capital requirements, and liquidity buffers for SIFIs (Financial Stability Board, 2011).[2]

In practice, there are two ways of measuring the contribution of a given firm to the overall risk of the system. A first approach, called the Supervisory Approach, relies on firm-specific information on size, leverage, liquidity, interconnectedness, complexity, and substitutability. This approach uses data on positions provided by the financial institutions to the regulator (FSB-IMF-BIS, 2009, Basel Committee on Banking Supervision, 2011, Financial Stability Oversight Council, 2012, Gourieroux, Heam and Monfort, 2012, and Greenwood, Landier and Thesmar, 2012). A second approach relies on publicly available market data, such as stock returns, option prices, or CDS spreads, as they are believed to reflect all information about publicly traded firms. Three prominent examples of such measures are the *Marginal Expected Shortfall* (MES) of Acharya et al. (2010), the *Systemic Risk Measure* (SRISK) of Acharya, Engle and Richardson (2012) and Brownlees and Engle (2012), and the *Delta Conditional Value-at-Risk* (ΔCoVaR) of Adrian and Brunnermeier (2011).[3] Very few crisis-related papers made a higher impact both in the academia and on the regulatory debate than this series of papers. Over the past four years, dozens of research papers have discussed, implemented, and sometimes generalized, these systemic risk measures.[4] Furthermore, in discussions with central bankers and regulators, we learned that these measures are monitored by the Federal Reserve, along with the CDS-based distress insurance premium of Huang, Zhou and Zhu (2012).

[1] Bisias et al. (2012) survey 31 measures of systemic risk in the economic and finance literature.

[2] For some banks, the benefits of being designated a SIFI outweigh the costs. As put by Douglas Flint, the chairman of HSBC (http://www.guardian.co.uk/business/2011/nov/06/banks-disappointed-not-on-g-sifi-list): "I see it as a label that would attract customers, because such banks would be forced to hold more capital and be subject to more intense regulation".

[3] Other related papers include Elsinger, Lehar and Summer (2006), Gauthier, Lehar and Souissi (2009), Huang, Zhou and Zhu (2009, 2012), Drehmann and Tarashev (2011), Gray and Jobst (2011), Kritzman et al. (2011), Acharya and Stefen (2012), Billio et al. (2012), Giglio (2012), Gourieroux and Monfort (2012), and White, Kim and Manganelli (2012).

[4] See for instance Adams et al. (2010), Fong and Wong (2010), Danielson et al. (2011, 2012), Colletaz, Pérignon and Hurlin (2012), Engle, Jondeau and Rockinger (2012), Idier, Lamé and Mésonnier (2012), Lopez-Espinosa et al. (2012a,b), Cao (2013), and Ergun and Girardi (2013). For recent media coverage, see Bloomberg Businessweek (2011), The Economist (2011), and Rob Engle's interview on CNBC (2011). For online computation of systemic risk measures, see the Stern-NYU's V-Lab initiative at http://vlab.stern.nyu.edu/welcome/risk/.

The goal of this paper is to propose a comprehensive comparison of the major systemic risk measures (MES, SRISK, and ΔCoVaR). The three systemic risk measures we consider in this paper have nice economic interpretations. First, the MES corresponds to a firm's expected equity loss when market falls below a certain threshold over a given horizon, namely a 2% market drop over one day for the short-run MES, and a 40% market drop over six months for the long-run MES (LRMES). The basic idea is that the banks with the highest MES contribute the most to market declines; thus, these banks are the greatest drivers of systemic risk. Second, the SRISK measures the expected capital shortfall of an institution conditional on a crisis occurring. The intuition is that the firm with the largest capital shortfall that occurs precisely during the system crisis, should be considered as the most systemically risky. Third, the CoVaR corresponds to the Value-at-Risk (VaR) of the financial system conditionally on a specific event affecting a given firm. The contribution of a firm to systemic risk (ΔCoVaR) is the difference between its CoVaR when the firm is, or is not, in financial distress. As an illustration, we display in Figure 1, the evolution of the three risk measures for Lehman Brothers between 2000 and 2008. We see that all three risk measures raise around 2006 and that SRISK increases much more, in relative terms, than the other two measures.

[Insert Figure 1]

There are two main parts in our analysis. First, we propose a theoretical comparison of these measures by deriving them in a common framework. We show that (1) MES corresponds to the product of the market's expected shortfall (market tail risk) and the firm beta (firm systematic risk) and that (2) ΔCoVaR corresponds to the product of the firm VaR (firm tail risk) and the linear projection coefficient of the market return on the firm return. Furthermore, (3) we derive conditions under which the different measures lead to similar rankings of SIFIs. Second, we propose an empirical comparison of the systemic risk measures by considering a sample of top US financial institutions over the period 2000 - 2010. This comparison aims to answer the following key questions: Do the different risk measures identify the same SIFIs? And if not, what are the reasons? Our empirical analysis delivers some key insights on systemic risk. First, we show that different risk measures lead to identifying different SIFIs. On most days, there is not a single institution simultaneously identified as a top-10 SIFI by the three measures. Second, there is a strong positive relationship between MES and firm beta, which implies that systemic risk rankings of financial institutions based on MES mirror rankings obtained by sorting firms on betas. Third, we reach a similar conclusion for SRISK and liabilities. Fourth, as the empirical ΔCoVaR of a firm is strongly correlated with its VaR, ΔCoVaR brings limited added value over and above

VaR to forecast systemic risk. In a linear regression analysis, we show that a one-factor model explains between 83% and 100% of the variability of the systemic risk estimates, which indicates that standard systemic risk measures fall short in capturing the multiple facets of systemic risk.

Our paper makes several contributions to the academic literature on systemic risk. To the best of our knowledge, this is the first attempt to derive the major systemic risk measures within a common framework. Our analytical expressions allow us to uncover the theoretical link between systemic risk and standard financial risks (systematic risk, tail risk, correlation, and beta), as well as firm characteristics such as leverage and market capitalization. Unlike purely empirical horse races, our theoretical comparison is not plagued by estimation risk or concerns about sample composition and sample periods. Another reason for us to not running an empirical horse race is that it is impossible to measure ex post the contribution of a given firm to the risk of the system. As a result, there is no benchmark and we cannot assess the validity of a given measure by analysing its forecasting errors.[5] One could argue that we instead could use as a benchmark the actual list of the Global SIFIs published by the Financial Stability Board (2012), and see which measure can best reproduce it. However, in such an analysis we first must assume the truthfulness of the list and moreover we could always imagine a parametric systemic risk measure sufficiently flexible to reproduce any particular ranking on a given date.

The rest of the paper is organized as follows. Section 2 provides the general definitions of the three considered systemic risk measures and presents the common framework used for the comparison. Section 3 proposes a theoretical analysis of the MES, SRISK, and ΔCoVaR measures. In Section 4, we describe the data and present the main empirical findings. Section 5 summarizes and concludes.

2 Methodology

2.1 Definitions

In this section, we provide a formal definition for the considered systemic risk measures. We consider N firms and denote r_{it} the return of firm i at time t. Similarly, the market return is the value-weighted average of all firm returns, $r_{mt} = \sum_{i=1}^{N} w_{it} r_{it}$, where w_{it} denotes the relative market capitalization of firm i.

[5] Differently, Sedunov (2012) tests whether measures of systemic risk exposures can forecast financial institutions' returns during systemic crisis periods in 1998 and 2008. Giglio, Kelly and Qiao (2012) evaluate the empirical success of systemic risk measures, based on their predictive ability for low quantiles of the conditional distribution of macroeconomic outcomes.

MES

First, the MES is the marginal contribution of an institution i to systemic risk, as measured by the Expected Shortfall (ES) of the system. Originally proposed by Acharya et al. (2010), the MES was recently extended to a conditional version by Brownlees and Engle (2012). By definition, the ES at the $\alpha\%$ level is the expected return in the worst $\alpha\%$ of the cases, but it can be extended to the general case, in which the returns exceed a given threshold C. Formally, the conditional ES of the system is defined as:[6]

$$ES_{mt}(C) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C) = \sum_{i=1}^{N} w_{it}\mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C). \tag{1}$$

Then, the MES corresponds to the partial derivative of the system ES with respect to the weight of firm i in the economy (Scaillet, 2004).[7]

$$MES_{it}(C) = \frac{\partial ES_{mt}(C)}{\partial w_{it}} = \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C). \tag{2}$$

The MES can be viewed as a natural extension of the concept of marginal VaR proposed by Jorion (2007) to the ES, which is a coherent risk measure (see Artzner et al., 1999). It measures the increase in the risk of the system (measured by the ES) induced by a marginal increase in the weight of firm i in the system. The higher the firm MES, the higher the individual contribution of the firm to the risk of the financial system.

SRISK

Second, the SRISK measure proposed by Acharya, Engle and Richardson (2012) and Brownlees and Engle (2012) extends the MES in order to take into account both the liabilities and the size of the financial institution. The SRISK corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system. In this perspective, the firms with the largest capital shortfall are assumed to be the greatest contributors to the crisis and are the institutions considered as most systemically risky. We follow Acharya, Engle and Richardson (2012) and define the SRISK as:

$$\begin{aligned} SRISK_{it} &= \max\left[0\ ;\ \overbrace{k\ (D_{it} + (1 - LRMES_{it})\, W_{it})}^{Required\ Capital} - \overbrace{(1 - LRMES_{it}) W_{it}}^{Available\ Capital}\right] && (3) \\ &= \max\left[0\ ;\ k\ D_{it} - (1-k)\ \ W_{it}\ \ (1 - LRMES_{it})\right] && (4) \end{aligned}$$

[6] We follow the original notations of the different authors: ES, MES, VaR, CoVaR and ΔCoVaR are typically negative. The only exception is the SRISK which is typically a positive number when a firm suffers from a capital shortage.

[7] To simplify the notation, we use MES_{it} (respectively ES_{it}) instead of $MES_{i,t|t-1}$ (respectively $ES_{i,t|t-1}$), but it should be understood as the conditional MES (respectively ES) computed at time t given the information available at time $t-1$.

where k is the prudential capital ratio, D_{it} is the book value of total liabilities, and W_{it} is the market capitalization or market value of equity. Note that the SRISK, which is positive by convention, is an increasing function of the liabilities and a decreasing function of the market capitalization. Then, the SRISK can be viewed as an implicit increasing function of the quasi-leverage (leverage thereafter) defined by one plus the ratio of the book value of total liabilities to the market capitalization.

The SRISK also considers the interconnection of a firm with the rest of the system through the long-run marginal expected shortfall (LRMES). The latter corresponds to the expected drop in equity value the firm would experiment should the market falls by more than a given threshold within the next six months. Acharya, Engle and Richardson (2012) propose to approximate it using the daily MES (defined for a threshold C equal to 2%) as $LRMES_{it} \simeq 1 - \exp\left(18 \times MES_{it}\right)$. This approximation represents the firm expected loss over a six-month horizon, obtained conditionally on the market falling by more than 40% within the next six months (for more details, see Acharya, Engle and Richardson, 2012).

ΔCoVaR

The third systemic risk measure is the ΔCoVaR of Adrian and Brunnermeier (2011). This measure is based on the concept of Value-at-Risk, denoted VaR(α), which is the maximum loss within the α%-confidence interval (see Jorion, 2007). Then, the CoVaR corresponds to the VaR of the market return obtained conditionally on some event $\mathbb{C}\left(r_{it}\right)$ observed for firm i:[8]

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|\mathbb{C}(r_{it})} \;\middle|\; \mathbb{C}\left(r_{it}\right)\right) = \alpha. \tag{5}$$

The ΔCoVaR of firm i is then defined as the difference between the VaR of the financial system conditional on this particular firm being in financial distress and the VaR of the financial system conditional on firm i being in its median state. To define the distress of a financial institution, various definitions of $\mathbb{C}\left(r_{it}\right)$ can be considered. Because they use a quantile regression approach, Adrian and Brunnermeier (2011) consider a situation in which the loss is precisely equal to its VaR:

$$\Delta CoVaR_{it}\left(\alpha\right) = CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}. \tag{6}$$

A more general approach would consist in defining the financial distress of firm i as a situation in which the losses exceed its VaR (see Ergun and Girardi, 2012):

$$\Delta CoVaR_{it}\left(\alpha\right) = CoVaR_t^{m|r_{it}\leq VaR_{it}(\alpha)} - CoVaR_t^{m|r_{it}=Median(r_{it})}. \tag{7}$$

[8] To simplify the notations, we neglect the conditioning with respect to past information, but the CoVaR is a conditional VaR with respect to both $\mathbb{C}\left(r_{it}\right)$ observed for firm i and the past returns $r_{m,t-k}$.

2.2 A Common Framework

The different systemic risk measures analyzed in this paper have been developed within very different frameworks. For instance, Adrian and Brunnermeier (2011) allow for tail dependences and use a quantile regression approach to estimate the ΔCoVaR. Differently, Brownlees and Engle (2012) model time-varying linear dependencies and use a multivariate GARCH-DCC model to compute the MES. Hence, their direct comparison is not straightforward since some empirical differences may be due to the estimation strategies. Differently, we derive all these risk measures within a unified theoretical framework to provide a level playing field. Following Brownlees and Engle (2012), we consider a bivariate GARCH process for the demeaned returns:

$$r_t = H_t^{1/2} \; \nu_t \tag{8}$$

where $r_t' = (r_{mt} \; r_{it})$ denotes the vector of market and firm returns and where the random vector $\nu_t' = (\varepsilon_{mt} \; \xi_{it})$ is *i.i.d.* and has the following first moments: $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t \nu_t') = I_2$, a two-by-two identity matrix. The H_t matrix denotes the conditional variance-covariance matrix:

$$H_t = \begin{pmatrix} \sigma_{mt}^2 & \sigma_{it} \; \sigma_{mt} \; \rho_{it} \\ \sigma_{it} \; \sigma_{mt} \; \rho_{it} & \sigma_{it}^2 \end{pmatrix} \tag{9}$$

where σ_{it} and σ_{mt} denote the conditional standard deviations and ρ_{it} the conditional correlation. No particular assumptions are made about the bivariate distribution of the standardized innovations v_t, which is assumed to be unknown. We only assume that the time-varying conditional correlations ρ_{it} fully captures the dependence between firm and market returns.[9] Formally, this assumption implies that the standardized innovations ε_{mt} and ξ_{it} are independently distributed at time t.

3 A Theoretical Comparison of Systemic Risk Measures

3.1 MES

Given Equations (8) and (9), the MES can be expressed as a function of the firm return volatility, its correlation with the market return, and the comovement of the tail of the distribution (See Appendix A):

$$\begin{aligned} MES_{it}(C) \;\; &= \;\; \sigma_{it} \; \rho_{it} \; \mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \\ &\quad + \sigma_{it} \; \sqrt{1-\rho_{it}^2} \; \mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right). \end{aligned} \tag{10}$$

The MES is expressed as a weighted function of the tail expectation of the standardized market residual and the tail expectation of the standardized idiosyncratic firm residual. As the dependence between market and firm returns is completely captured by their correlation, the conditional

[9] We will relax this assumption in the empirical analysis in Section 4.

expectation $\mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right)$ is null. In order to facilitate the comparison with the ΔCoVaR, we consider a threshold C equal to the conditional VaR of the market return, which is defined as $\Pr\left[r_{mt} < VaR_{mt}(\alpha) \middle| \mathcal{F}_t\right] = \alpha$ where $\mathcal{F}_t$ denotes the information set available at time t.

Proposition 1 *The MES of a given financial institution i is proportional to its systematic risk, as measured by its time-varying beta. The proportionality coefficient is the expected shortfall of the market:*

$$MES_{it}(\alpha) = \beta_{it}\ ES_{mt}(\alpha) \tag{11}$$

where $\beta_{it} = cov(r_{it}, r_{mt})/var(r_{mt}) = \rho_{it}\sigma_{it}/\sigma_{mt}$ denotes the time-varying beta of firm i and $ES_{mt}(\alpha) = \mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < VaR_{mt}(\alpha))$ is the expected shortfall of the market.

The proof of Proposition 1 is in Appendix A.[10] This proposition has two main implications. First, on a given date, the systemic risk ranking of financial institutions based on MES (in absolute value) is strictly equivalent to the ranking that would be produced by sorting firms according to their betas. Indeed, since the system ES is not firm-specific, the greater the sensitivity of the return of a firm with respect to the market return, the more systemically-risky the firm is. Consequently, under our assumptions, identifying SIFIs using MES is equivalent to consider the financial institutions with the highest betas. Second, for a given financial institution, the time profile of its systemic risk measured by its MES may be different from the evolution of its systematic risk measured by its conditional beta. Since the market ES may not be constant over time, forecasting the systematic risk of firm i may not be sufficient to forecast the future evolution of its contribution to systemic risk.

Note that Proposition 1 is robust with respect to the choice of the threshold C that determines the system crisis. For any threshold $C \in \mathbb{R}$, the MES is still proportional to the time-varying beta (see proof in Appendix A). The only difference is that the proportionality coefficient, $\mathbb{E}_{t-1}(r_{mt} \mid r_{mt} < C)$, is different from the system ES if $C \neq VaR_{mt}(\alpha)$. However, this coefficient remains common to all firms.

3.2 SRISK

We show in Section 2 that SRISK is a function of the MES. As a result, a corollary of Proposition 1 is that SRISK can be expressed as a function of the beta, liabilities, and market capitalization:

$$SRISK_{it} \simeq \max\left[0\ ;\ k\ D_{it} - (1-k)\ W_{it}\ \exp\left(18 \times \beta_{it} \times ES_{mt}(\alpha)\right)\right]. \tag{12}$$

[10] For some particular distributions, both the ES and the MES of the market returns can be expressed in closed form. For instance, if ε_{mt} follows a standard normal distribution, then $VaR_{mt}(\alpha) = \sigma_{mt}\Phi^{-1}(\alpha)$ and $ES_{mt}(\alpha) = -\sigma_{mt}\phi\left(\Phi^{-1}(\alpha)\right)/\alpha$, where $\phi(.)$ and $\Phi(.)$, respectively, denote the standard normal probability distribution function and cumulative distribution function. Therefore, $MES_{it}(\alpha) = -\beta_{it}\sigma_{mt}\lambda\left(\Phi^{-1}(\alpha)\right)$, where $\lambda(z) = \phi(z)/\Phi(z)$ denotes the Mills ratio.

SRISK is an increasing function of the systematic risk, as measured by the conditional beta. However, unlike with MES, systemic-risk rankings based on SRISK are not equivalent to rankings based on betas. SRISK-based rankings also depend on the liabilities and on the market capitalization of the financial institution. Recall that the system ES is typically a negative number, $ES_{mt}(\alpha) < 0$. Consequently, when $\beta_{it} > 0$, SRISK is a decreasing function of the market capitalization. Then, if we consider a given level of liabilities, SRISK increases with leverage.

Accounting for market capitalization and liabilities in the definition of the systemic risk measure tends to increase the systemic risk score of large firms. This result is in line with the too-big-to-fail paradigm, whereas the MES tends to be naturally attracted by interconnected institutions (through the beta), which is more in line with the too-interconnected-to-fail paradigm (Markose et al., 2010). In that sense, the SRISK can be viewed as a compromise between both paradigms.

3.3 ΔCoVaR

In our theoretical framework, it is also possible to express ΔCoVaR*,* defined for a conditioning event $\mathbb{C}(r_{it}) : r_{it} = VaR_{it}(\alpha)$, as a function of the conditional correlations, volatilities, and VaR. Given Equations (8) and (9), we obtain the following result:

Proposition 2 *The $\Delta CoVaR$ of a given financial institution i is proportional to its tail risk, as measured by its VaR. The proportionality coefficient corresponds to the linear projection coefficient of the market return on the firm return.*

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it} \; [VaR_{it}(\alpha) - VaR_{it}(0.5)] \tag{13}$$

where $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$. If the marginal distribution of the returns is symmetric around zero, $\Delta CoVaR$ is strictly proportional to VaR:

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it} \; VaR_{it}(\alpha) . \tag{14}$$

The proof of Proposition 2 is in Appendix B.[11] The fact that the proportionality coefficient between ΔCoVaR and VaR is firm-specific has some strong implications. Let us, for instance, consider two financial institutions i and j, with $VaR_{it} < VaR_{jt}$. Given the relative correlations between the returns of firms i and j with the market return (respectively ρ_{it} and ρ_{jt}), and the volatilities σ_{it} and σ_{jt}, we could observe $\Delta CoVaR_{it} < \Delta CoVaR_{jt}$ or $\Delta CoVaR_{it} > \Delta CoVaR_{jt}$. This means that the most risky institution in terms of VaR is not necessarily the most systemically risky institution. In other words, on a given date, the systemic risk ranking over N financial institutions based on

[11] Adrian and Brunnermeier (2011) derive the CoVaR and the ΔCoVaR under the normality assumption. They show that $\Delta CoVaR_{it}(\alpha) = \rho_{it}\sigma_{mt}\Phi^{-1}(\alpha)$ or equivalently $\gamma_{it}\sigma_{it}\Phi^{-1}(\alpha)$, where $\sigma_{it}\Phi^{-1}(\alpha)$ denotes the VaR(α) of the firm.

ΔCoVaR is not equivalent to a VaR-based ranking. In that sense, ΔCoVaR is not equivalent to VaR as already pointed out by Adrian and Brunnermeier (2011) in their Figure 1. Indeed, they report a weak relationship between an institution's risk in isolation, measured by its VaR, and its contribution to system risk, measured by its ΔCoVaR. However, for a given institution, ΔCoVaR is proportional to VaR. Consequently, forecasting the future evolution of the contribution of firm i to systemic risk is equivalent to forecast its risk in isolation.

Note that the proportionality coefficient in Equation (14), γ_{it}, is not always time-varying. For instance, when the variance-covariance matrix is constant, the proportionality coefficient is constant. Similarly, the framework of Adrian and Brunnermeier (2011), in which ΔCoVaR is estimated through quantile regression in order to capture the potential nonlinear dependencies between returns, also leads to a constant proportionality coefficient.

Proposition 2 highlights two important differences between the MES, SRISK, and ΔCoVaR measures. First, MES is a marginal risk measure (defined by the first derivative of the system ES) whereas ΔCoVaR is an incremental risk measure (defined by the difference between two conditional VaRs). SRISK is a compromise between both approaches in the sense that it is based on a marginal measure of the return interconnectedness, through the MES, but also on nominal values such as liabilities and market capitalization. Second, MES depends on the linear projection coefficient of the firm return onto the market return, i.e., the beta, whereas ΔCoVaR is based on the linear projection coefficient of the market return on the firm return. This difference in the definition of the conditioning event is quite important. Indeed, MES and SRISK are fundamentally linked to the sensitivity of the firm return to the market return. In contrast, ΔCoVaR captures the sensitivity of the market return with respect to the firm return.

3.4 Comparing Systemic-Risk Rankings

The main objective of any systemic risk analysis is to rank firms according to their systemic risk contribution and, in turn, identify the SIFIs. The key question is then to determine whether the different systemic risk measures lead to the same conclusion. A natural way to answer this question is to analyze their ratio.

Proposition 3 *For a given financial institution i at time t, the ratio between its $\Delta CoVaR$ and its MES is:*

$$\frac{\Delta CoVaR_{it}\left(\alpha\right)}{MES_{it}\left(\alpha\right)} = f_{it} \times g_{mt}. \tag{15}$$

If the marginal distribution of the firm return is symmetric, $f_{it} = VaR_{it}\left(\alpha\right)/\sigma_{it}^2$ and $g_{mt} = \sigma_{mt}^2/ES_{mt}\left(\alpha\right)$. If the distribution is not symmetric, $VaR_{it}\left(\alpha\right)$ is replaced by $VaR_{it}\left(\alpha\right) - VaR_{it}\left(0.5\right)$.

The ΔCoVaR/MES ratio is the product of two terms. The first term is firm-specific (f_{it}), whereas the second is common to all firms (g_{mt}).[12] The fact that this ratio is firm-specific implies that the systemic risk rankings based on the two measures may not be the same. Consider two different financial institutions i and j such that i is more systemically risky than j according to ΔCoVaR, $\Delta CoVaR_{it} < \Delta CoVaR_{jt}$. It is possible to observe a situation where i is less risky than j according to the MES measure, $MES_{it} > MES_{jt}$. In other words, the SIFIs identified by the MES and by the ΔCoVaR may not be the same. Note that this result can be extended to the SRISK since the latter depends on MES.

Our theoretical framework also permits to derive conditions under which both rankings are convergent, respectively divergent.

Proposition 4 *A financial institution i is more systemically risky that an institution j according to the MES and the $\Delta CoVaR$ measures, $MES_{it}(\alpha) \leq MES_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if:*

$$\rho_{it} \geq \max\left(\rho_{jt}, \frac{\rho_{jt}\ \sigma_{jt}}{\sigma_{it}}\right) \tag{16}$$

and if the conditional distributions of the two standardized returns r_{it}/σ_{it} and r_{jt}/σ_{jt} are identical and location-scale.

The proof of Proposition 4 is in Appendix C.[13] The interepretation of this result works as follows. If $\sigma_{it} \geqslant \sigma_{jt}$, inequality (16) becomes $\rho_{it} \geq \rho_{jt}$. In the other case, if $\sigma_{it} < \sigma_{jt}$, the inequality becomes $\rho_{it} \geq \rho_{jt}\sigma_{jt}/\sigma_{it}$. In both cases, the interpretation is the same: the higher the correlation between the returns of the SIFIs and the market, the more likely it is that MES and $\Delta CoVaR$ will lead to a convergent diagnostic. This result comes from the fact that correlation captures both the sensitivity of the system return with respect to the firm return (ΔCoVaR dimension) and the sensitivity of the firm return with respect to the system return (MES dimension).

The systemic risk rankings based on SRISK and ΔCoVaR can also be compared. In this case, the comparison depends on the liabilities and market capitalizations of the two firms. For simplicity, let us consider two financial institutions with the same level of liabilities.

Proposition 5 *A financial institution i is more systemically risky than a financial institution j (with the same level of liabilities) according to the SRISK and the $\Delta CoVaR$ measures, $SRISK_{it}(\alpha) \geq$*

[12] If we assume normality for the marginal distributions of ε_{mt} and ξ_{it}, this ratio has a closed form:

$$\frac{\Delta CoVaR_{it}(\alpha)}{MES_{it}(\alpha)} = -\left(\frac{\sigma_{mt}}{\sigma_{it}}\right)\frac{\Phi^{-1}(\alpha)}{\lambda\left(\Phi^{-1}(\alpha)\right)}.$$

[13] If the conditional distributions are not identical and/or not location-scale, the corresponding condition has the same form and implies that the correlation ρ_{it} exceeds a given threshold (see Appendix C).

$SRISK_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$, if

$$\rho_{it} \geq \rho_{jt} \quad and \quad W_{it} \leq W_{jt} \times \exp\left[18 \times ES_{mt}(\alpha) \times (\beta_{jt} - \beta_{it})\right] \tag{17}$$

where W_{it} and W_{jt} denote the market capitalizations of both firms.

The proof of Proposition 5 is in Appendix D. ΔCoVaR and SRISK provide a similar systemic risk ranking if and only if (*i*) the correlation of the riskier firm with the system is higher than the correlation of the less risky institution and (*ii*) if the riskier firm has the lower market capitalization. Since both firms are assumed to have the same level of liabilities, this condition means that the ranking are similar if the riskier financial institution has the higher leverage. In other words, if the SIFIs have a high leverage and are very correlated with the system, ΔCoVaR and SRISK will lead to the same conclusion. As soon as one of these conditions is violated, the ranking of the financial institutions will be divergent.

4 An Empirical Comparison of Systemic Risk Measures

We have shown in our theoretical analysis that systemic risk measures (1) can be expressed as linear transformations of market risk measures (ES, VaR, beta) and (2) lead similar rankings under rather restrictive conditions. These results have been derived within the common framework presented in Section 2.2. However, in practice, the dependence between financial asset returns may be richer (i.e., not linear) than in Section 2.2 and thus our results may not hold in real financial markets.

For this reason, in this section, we relax the assumptions made in Equations (8) and (9) for asset returns. In our empirical analysis, we implement the same estimation methods as in the original papers presenting the MES, SRISK, and CoVaR, and we use the same sample as in Acharya et al. (2010) and Brownlees and Engle (2012). This sample contains all U.S. financial firms with a market capitalization greater than \$5 billion as of end of June 2007 (See Appendix E for a list of the 94 sample firms). For our sample period, January 3, 2000 - December 31, 2010, we extract daily firm stock returns, value-weighted market index returns, number of shares outstanding, and daily closing prices from CRSP. Quarterly book values of total liabilities are from COMPUSTAT. Following Brownless and Engle (2011), we estimate the MES and SRISK using a GARCH-DCC model. The coverage rate is fixed at 5%, and the threshold C is fixed to the unconditional market daily VaR at 5%, which is equal to 2% in our sample. The ΔCoVaR is estimated with a quantile regression as proposed by Adrian and Brunnermeier (2011). We discuss in detail the estimation techniques of all systemic risk measures in Appendix F.

4.1 Rankings: SIFI or not SIFI?

In practice, systemic risk measures are used to classify firms between SIFIs and non-SIFIs. The formers are more closely scrutinized by regulators and are subject to additional capital requirements and/or liquidity buffers. Within a given bucket of SIFIs, the level of extra capital requirement is the same regardless of the exact ranking of the firm within the bucket. The goal is then to identify the top tier banks in terms of contribution to the risk of the system. Of lesser importance is the exact value of the systemic risk measures or the exact ranking of the bank. In order to compare the SIFIs identified by several systemic risk measures, we need to set the size of the SIFI group. In the rest of the analysis, we use the top 10 financial institutions, which corresponds to approximately 10% of our sample. It is also close to the actual number of US SIFIs (namely 8) identified by the Financial Stability Board (2012) in its list of global systemically important banks. As a robustness check, we also provide results based on the top 20 financial institutions.

The main finding from this preliminary analysis is that the *different* risk measures identify *different* SIFIs. For instance, Table 1 displays the tickers of the top 10 financial institutions according to their systemic risk contribution measured by the MES, SRISK, and ΔCoVaR, respectively, for the last day of our sample period (December 31, 2010). On that day, there is not a single institution simultaneously identified as a SIFI by the three measures. Only two financial institutions (Bank of America and American International Group) are simultaneously identified by MES and SRISK, whereas ΔCoVaR identifies only three financial institutions (H&R Block, Marshall & Ilsley, and Janus Capital) in common with MES but none with SRISK. Furthermore, the SRISK-based top 10 list is clearly tilted towards the largest financial institutions (Bank of America, Citigroup, JP Morgan, etc.), whereas it is not necessarily the case for MES and ΔCoVaR. Indeed, these measures do not take into account the market capitalization and level of liabilities of the financial institutions. Note that we reach a similar conclusion when we consider the top 20 financial institutions, with only three firms being simultaneously identified by the three risk measures (See Appendix G).

[Insert Table 1]

The findings about diverging rankings is not specific to any particular date. Indeed, out of 2,767 days in our sample, there are 1,263 days (45.7%) during which none of the 94 financial institutions is jointly included in the top 10 ranking of the three risk measures. Figure 2 shows the daily percentage of concordant pairs between the top 10 SIFIs identified by the different risk measures. On average, the percentage of concordant pairs between MES and SRISK is 18.9%, which means that, on average, only two SIFIs out of ten are common to both measures. Over our 11-year sample, this percentage has ranged between 0% and 60%; the latter percentage corresponding to the peak of the crisis in October 2008. During a crisis, the MES tends to rise because asset correlation goes to one and both beta and ES increase. Similarly, the SRISK is rising because both liabilities and correlation increase and market capitalization drops (see Equation 12). The figures are much lower for SRISK and ΔCoVaR, with on average 9.9% of concordant pairs. The highest level of similarity is obtained for MES and ΔCoVaR, with an average percentage of concordant pairs of 43%. We see in Appendix G that the conclusion remains the same when we focus on the top 20 firms.

[**Insert Figure 2**]

Even if these systemic risk measures are divergent, they deliver a consistent ranking for a given institution. Indeed, for each measure, we compute the Kendall rank-order correlation coefficient between the systemic risk ranking obtained at time t and the one obtained at time $t-1$. The average correlations are 91.3% for MES, 97.7% for SRISK, and 93.4% for ΔCoVaR, and are always statistically significant. This result indicates that the rankings produced by these measures are stable through time. This is a nice property to have since it would make little sense for a measure to regularly classify a bank as SIFI on one day, and as non-SIFI on the following day. Therefore, the divergence of the systemic risk rankings is not due to the instability of a particular measure but instead to their fundamental differences.

4.2 Main Forces Driving Systemic Risk Rankings

After having shown that rankings vary across systemic risk measures, we investigate the reasons for these variations. We display in Table 2 the top 10 SIFIs, as of December 31, 2010, according to the three systemic risk measures, as well as the top 10 firms based on market capitalization, liabilities, leverage, beta, and VaR.[14] There are three striking results in this table. First, MES and beta tend to identify the same SIFIs. On that day, seven out of the ten highest beta firms are also identified among the top 10 SIFIs according to their MES. Even if the rankings provided by the two measures are not exactly the same, the 70% match between the MES and beta provide empirical support to Proposition 1. Indeed, the ranking based on MES is, in practice, mainly driven by systematic risk. Second, the SRISK-based ranking is mainly sensitive to the liabilities/leverage of the firms. We have shown in the previous section, that the SRISK can be considered as a compromise between the too-big-to-fail paradigm (through the liabilities/leverage) and the too-interconnected-to-fail paradigm (through the beta). However, in practice the SRISK-based ranking seems to be largely determined by the indebtedness of the firms. On that day, eight out of the top 10 SIFIs identified

[14] See Appendix F for a discussion of the estimation of the firms' beta and VaR.

by the SRISK, are also the financial institutions with the highest level of liabilities and seven have the highest leverage. On the contrary, only two are in the high-beta list. Third, the ΔCoVaR ranking is not determined by the VaR, since only three out of the top 10 SIFIs are also in the high-VaR list. These results are by no means specific to this date as shown in Figure 3 and remain pervasive during the entire sample period. Furthermore, they also hold valid when we consider the top 20 firms.

[Insert Table 2 and Figure 3]

We investigate further the relationship between MES and beta in Figure 4. This scatter plot compares the average MES, $\overline{MES}_i(\alpha) = T^{-1}\sum_{t=1}^{T}|MES_{it}(\alpha)|$, to the average beta, $\overline{\beta}_i = T^{-1}\sum_{t=1}^{T}\beta_{it}$, for the 61 firms that have been continuously traded during our sample period.[15] This plot confirms the strong relationship between MES (y-axis) and firm beta (x-axis). In line with Proposition 1, the OLS estimated slope coefficient (0.0248) is extremely close to the unconditional ES of the market at 5%, 0.0252 or 2.52% (see Equation 11).[16] The main implication of this result is that systemic risk rankings of financial institutions based on their MES tend to mirror rankings obtained by sorting firms on betas.

[Insert Figure 4]

Should we worry about the fact that MES and beta give similar rankings? We think that this is a serious concern for the following reasons. First, if beta is believed to be a good proxy for systemic risk, why not ranking firms on betas in the first place? Second, this leads to confusion between *systemic risk* and *systematic risk* (market risk). The latter being already accounted for in the banking regulation since the 1996 Amendment of the Basel Accord as regulatory capital depends on the banks' market risk VaR. Third, betas tend to increase during economic downturns, which makes MES procyclical.

Although the SRISK is by construction a function of the MES, it is much less sensitive to beta. Unlike for MES-beta (top panel in Figure 3, 85.1% match), the matching is far from being perfect for SRISK-beta, with an average percentage of concordant pairs of 23.3%. SRISK rankings is more closely related to leverage (71.4% match on average), especially during relatively calm periods. Until the beginning of 2007, the percentage of concordant pairs was about 100%: the ranking

[15] The data requirement allows us to estimate the average ES of the market return over the same period for all firms.

[16] Similar results (not reported) are obtained when we consider unconditional (constant) betas rather than conditional betas, or when we consider the firm MES and beta at a given point in time rather than averages.

produced by the SRISK was exactly the same as the leverage-based ranking for the top 10 SIFIs. However, this perfect concordance disappears during the crisis and the percentage of concordant pairs between SRISK and leverage falls to 20% in 2008. This difference can be explained by the increase in correlations, and consequently in the MES, observed during the crisis. Such an increase implies a modification of the weight given in the SRISK to the interconnectedness measure compared to the size of the firm. As a consequence, during the crisis, the percentage of concordance between the SRISK and beta rankings increases to reach 60% in October 2008 (second panel in Figure 3). On the contrary, the matching between the SRISK and the liabilities-based rankings has been close to 100% since the 2008 crisis. Consequently, the SRISK tend to identify the same SIFIs as the leverage in quiet periods and the same SIFIs as the liabilities during crisis periods.

As for the ΔCoVaR, we see that the ranking is pretty much orthogonal to other rankings. Of particular interest is the little overlap between the ΔCoVaR ranking and the VaR ranking (bottom panel in Figure 3). As already pointed out by Adrian and Brunnermeier (2011) in their Figure 1, ΔCoVaR is not equivalent to VaR. In Figure 5, we replicate their Figure 1 by comparing the averages $\Delta\overline{CoVaR}_i = T^{-1}\sum_{t=1}^{T}\Delta CoVaR_{it}(\alpha)$ and $\overline{VaR}_i = T^{-1}\sum_{t=1}^{T} VaR_{it}(\alpha)$ for the 94 sample firms. We also report a weak relationship between an institution's risk in isolation, measured by its VaR, and its contribution to system risk, measured by its ΔCoVaR. In that sense, ΔCoVaR is definitely not VaR.

[Insert Figure 5]

However, the latter conclusion is more questionable for a given institution. Figure 6 compares the dynamics of the ΔCoVaR and VaR of Bank of America over the entire sample period. We see that the two lines match almost perfectly and there is a theoretical reason for this. Indeed, with quantile regression, ΔCoVaR is strictly proportional to the VaR (see Appendix F). Hence, for a given financial institution, ΔCoVaR is nothing else but VaR. This result is robust to the estimation method used. Indeed, the correlation is still equal to one if we include state variables in the quantile regression. When the ΔCoVaR is estimated with a DCC model (not reported), the correlation is not one anymore but remains very high. This strong relationship between ΔCoVaR and VaR in the time series domain has some important implications. Consider a given bank that wants to lower its systemic risk score. Given the fact that the key driver of the bank's ΔCoVaR is the VaR of its stock return, the bank has to make its stock return distribution less leptokurtic and/or skewed.

[Insert Figure 6]

The main forces driving these three systemic risk measures can be summarized in a simple regression. We consider for each systemic risk measure a single-factor model in which the measure is successively explained by the market capitalization, liabilities, leverage, beta, and VaR. We consider two types of regressions: cross-sectional regressions for each of the 757 days in the sample and time-series regressions for each of the 94 sample firms. In Table 3, we report the average, minimum, maximum and standard deviations of the R^2 associated to the 757 or 94 regressions, respectively. The sample period covers 2008-2010.

[Insert Table 3]

In the cross-sectional dimension, 95% of the variance of the MES of the firms is explained by the beta. This result confirms our previous findings about the similarities in the rankings produced by the two measures. However, we can also observe that in the time series dimension, 95% of the variance of the MES is explained by the VaR. The results for the SRISK confirm that it is much highly correlated to the leverage and liabilities rather than to the beta of the firm. The average R^2 of the cross-section regressions with the liabilities is equal to 83%, whereas it is only equal to 11% for beta. As for ΔCoVaR, we get a perfect correlation in time series with the VaR of the firms, for the above-mentioned reasons. In cross-section, the average R^2 of the five models for the ΔCoVaR is relatively low (the maximum average R^2 is 32% for beta). Overall our regression results clearly indicate that each considered systemic risk measure captures one dimension only of systemic risk, and this dimension corresponds to either the market risk (VaR or beta) or the leverage of the firm.

One could argue that the large R^2 reported in Table 3 (time series panel) may be the sign of a spurious regression. It is indeed well known that time series regressions of non-stationary and non-cointegrated series can lead to artificially inflated R^2. To rule out this explanation, we run all the time series regressions taking the variables in first differences and the average R^2 remain high for all three measures (average R^2 (all) is 0.9061 for MES, 0.6522 for SRISK, and 1 for ΔCoVaR). Note that the perfect correlation between VaR and ΔCoVaR is a direct consequence of the quantile regression method used to generate the ΔCoVaR (see Equation (F10) in Appendix F).

5 Conclusion

Systemic risk is one of the most elusive concepts in finance. In practice, a good risk measure for systemic risk should capture many different facets that describe the importance of a given financial institution in the financial system. For instance, the Financial Stability Board states that systemic risk score should reflect size, leverage, liquidity, interconnectedness, complexity, and substitutability. In this paper, we have studied several popular systemic risk measures that are currently used by central banks and banking regulatory agencies. Our findings indicate that these measures fall short in capturing the multifaceted nature of systemic risk. We have shown, both theoretically and empirically, that most of the variability of these three systemic measures can be captured by one market risk measure or firm characteristics.

The quest for a proper systemic risk measures is still ongoing but we have reasons to remain optimistic as more data become available, with better quality, higher frequency, and wider scope (see G20 Data Gaps Initiative, Cerutti, Claessens and McGuire, 2012). Given the very nature of systemic risk, future risk measures should combine various sources of information, including balance-sheet data and proprietary data on positions (e.g., common risk exposures à la Greenwood, Thesmar and Landier, 2012) and market data (e.g., CDS à la Giglio, 2012). Future research on systemic risk should also consider the definition of the perimeter of the financial system.

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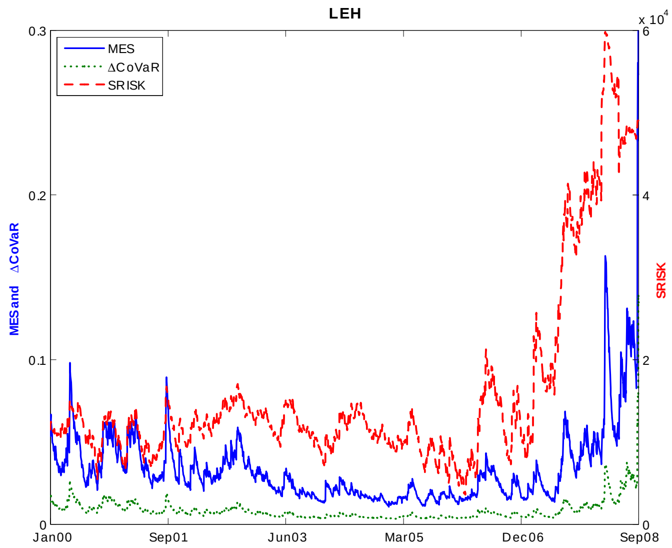


Figure 1: **Time Series Evolution of Systemic Risk Measures:** The figure displays the MES (solid line, left axis), the ΔCoVaR (dotted line, left axis) and the SRISK (dashed line, right axis) of Lehman Brothers (LEH).

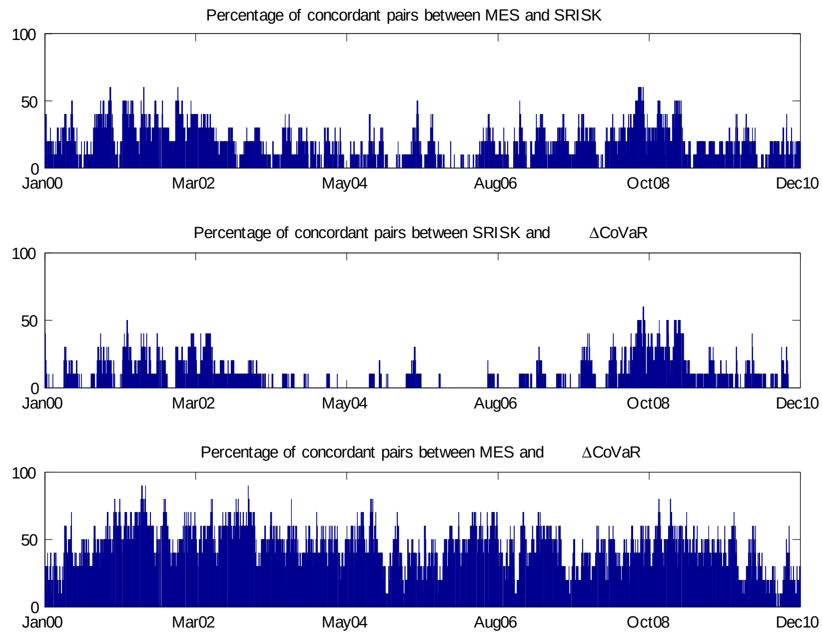


Figure 2: **Different Risk Measures, Different SIFIs:** These figures show the daily percentage of concordant pairs between the top ten financial institutions based one MES and SRISK (top panel), the top 10 financial institutions based on SRISK and ΔCoVaR (middle panel), and the top 10 financial institutions based on ΔCoVaR and MES (bottom panel).

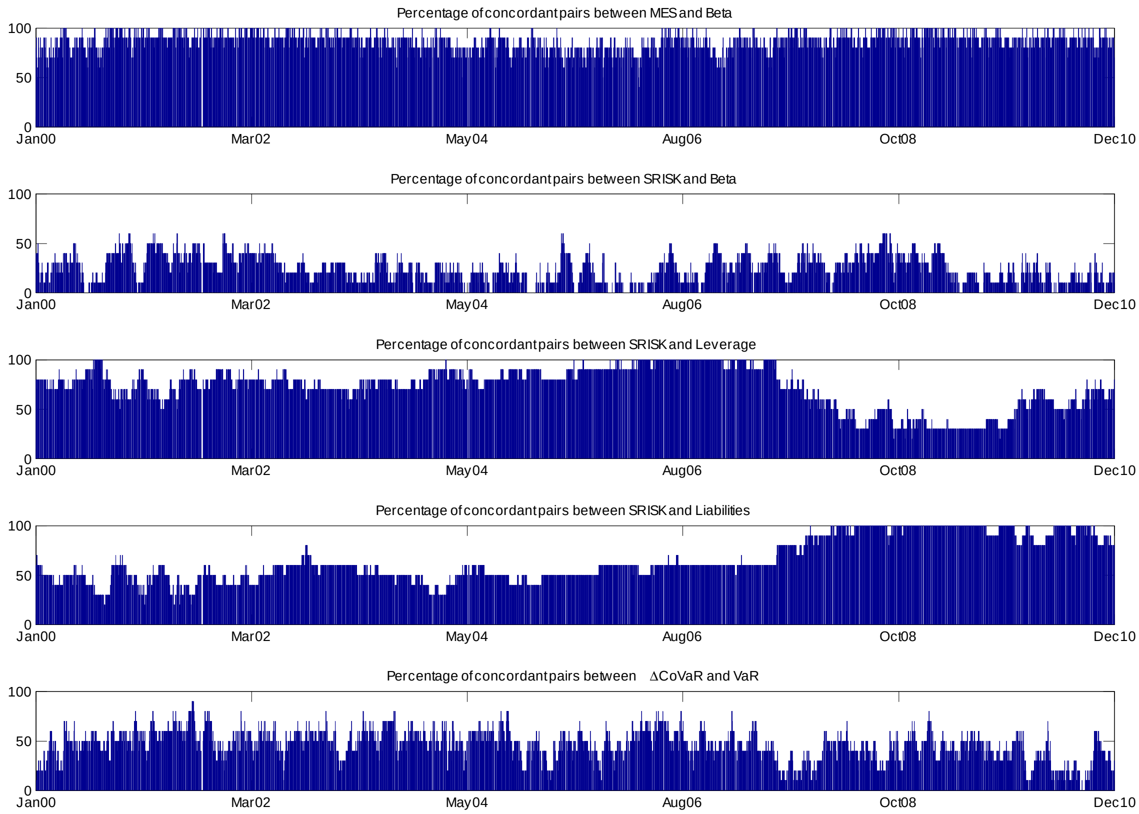


Figure 3: **Driving Forces of Systemic Risk Rankings:** The top figure shows the daily percentage of concordance between the top 10 financial institutions given the MES and the top 10 financial institutions given the beta. The next three figures show the daily percentage of concordance between the first 10 financial institutions given the SRISK and the first 10 financial institutions given the beta, leverage or liabilities. The bottom figure shows the daily percentage of concordance between the top 10 financial institutions given the ΔCoVaR and the top 10 financial institutions given the VaR.

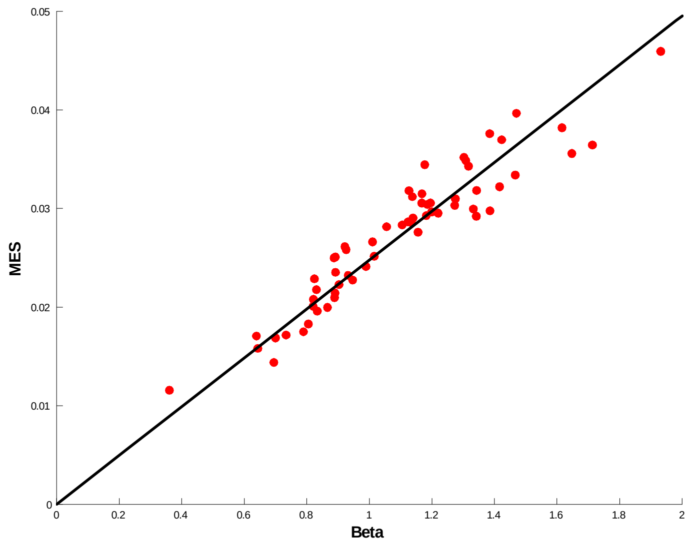


Figure 4: **Systemic Risk or Systematic Risk?** The scatter plot shows the strong cross-sectional link between the time-series average of the MES at 5% estimated for each institution (y-axis) and its beta (x-axis). The beta corresponds to the average of the time-varying beta β_{it}. Each point represents a financial institution and the solid line is the OLS regression line with no constant. The estimation period is from 01/03/2000 to 12/31/2010.

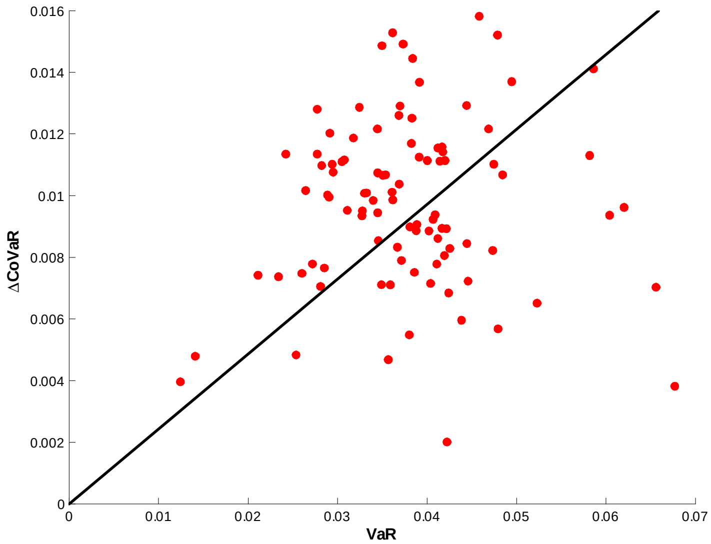


Figure 5: **ΔCoVaR is not Equivalent to VaR in the Cross-Section:** The scatter plot shows the cross-sectional link between the time-series average of the ΔCoVaR estimated for each institution (y-axis) and its VaR at 5% (x-axis). Each point represents an institution and the solid line is the OLS regression line with no constant. The estimation period is from 01/03/2000 to 12/31/2010.

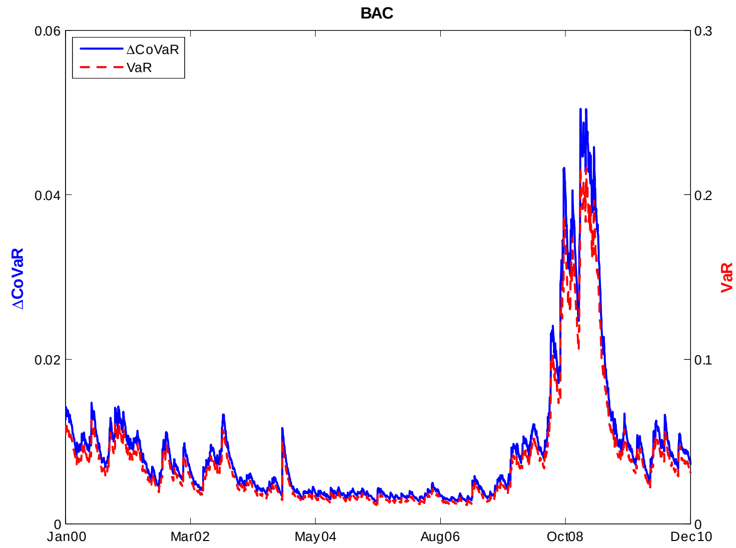


Figure 6: **ΔCoVaR is Equivalent to VaR in Time Series:** The figure displays the ΔCoVaR (solid line, left y-axis) and the 5%-VaR (dashed line, right y-axis) of Bank of America (BAC).

Table 1: Systemic Risk Rankings

Rank	MES	SRISK	ΔCoVaR
1	MBI	BAC	HRB
2	AIG	C	MI
3	MI	JPM	BEN
4	CBG	MS	CIT
5	RF	AIG	WU
6	LM	MET	AIZ
7	JNS	PRU	AXP
8	HRB	HIG	JNS
9	BAC	SLM	NYB
10	UNM	LNC	MTB

Notes: The column labeled MES displays the ranking of the top 10 financial institutions based on MES, ranked from most to least risky. The following two columns display the top 10 financial institutions based on SRISK and ΔCoVaR, respectively. The ranking is for December 31, 2010. See Appendix E for the list of firm names and tickers.

Table 2: Systemic Risk Rankings and Firm Characteristics

Rank	MES	SRISK	ΔCoVaR	MV	LTQ	LVG	β	VaR
1	MBI	BAC	HRB	JPM	BAC	SLM	MBI	MBI
2	AIG	C	MI	WFC	JPM	HIG	LM	MI
3	MI	JPM	BEN	C	C	LNC	JNS	AIG
4	CBG	MS	CIT	BAC	WFC	MS	MI	RF
5	RF	AIG	WU	GS	GS	PRU	CBG	HRB
6	LM	MET	AIZ	BRK	MS	MET	AIG	SNV
7	JNS	PRU	AXP	USB	MET	GNW	ACAS	HBAN
8	HRB	HIG	JNS	AXP	AIG	BAC	AMTD	BAC
9	BAC	SLM	NYB	MET	PRU	AIG	BAC	FITB
10	UNM	LNC	MTB	MS	HIG	RF	ETFC	JNS

Pairs	MES	SRISK	ΔCoVaR	MV	LTQ	LVG	β	VaR
SRISK	2	–						
ΔCoVaR	3	0	–					
MV	1	5	1	–				
LTQ	2	8	0	7	–			
LVG	3	8	0	3	6	–		
β	7	2	2	1	2	2	–	
VaR	7	2	3	1	2	3	5	–

Notes: In the upper panel, the column labeled MES displays the ranking of the top 10 financial institutions based on MES, listed from most to least risky. The following seven columns display the top 10 financial institutions based on SRISK, ΔCoVaR, market value of equity (MV), liabilities (LTQ), leverage (LVG), conditional beta (β), and VaR, respectively. In the lower panel, we report the number of concordant pairs between two risk measures or firm characteristics. The ranking is for December 31, 2010. See Appendix E for the list of firm names and tickers.

Table 3: Explaining Systemic Risk Measures by Market Risk and Firm Characteristics

	Time series						Cross-section					
MES	MV	LTQ	LVG	beta	VaR	all	MV	LTQ	LVG	beta	VaR	all
average R^2	0.3210	0.1742	0.3661	0.2820	0.9510	0.9687	0.0071	0.0403	0.2591	**0.9571**	0.7968	0.9837
min R^2	0.0002	0.0000	0.0003	0.0000	0.5498	0.7610	0.0000	0.0000	0.0137	**0.7198**	0.3972	0.9433
max R^2	0.8360	0.7991	0.8305	0.9758	0.9990	0.9992	0.0452	0.1852	0.7883	**0.9946**	0.9785	0.9986
std R^2	0.2272	0.1736	0.2232	0.2410	0.0727	0.0436	0.0086	0.0416	0.1477	**0.0319**	0.1100	0.0105
	Time series						Cross-section					
SRISK	MV	LTQ	LVG	beta	VaR	all	MV	LTQ	LVG	beta	VaR	all
average R^2	0.6117	0.2533	0.4888	0.2405	0.6064	0.9373	0.3197	**0.8341**	0.1840	0.1173	0.0592	0.9932
min R^2	0.0022	0.0000	0.0001	0.0003	0.0004	0.7750	0.0085	**0.2569**	0.0110	0.0034	0.0022	0.9807
max R^2	0.9635	0.9603	0.9551	0.8215	0.9086	0.9930	0.5759	**0.9952**	0.4103	0.3331	0.2269	0.9995
std R^2	0.2428	0.2441	0.2295	0.1990	0.2189	0.0391	0.1073	**0.1279**	0.0757	0.0661	0.0445	0.0036
	Time series						Cross-section					
ΔCoVaR	MV	LTQ	LVG	beta	VaR	all	MV	LTQ	LVG	beta	VaR	all
average R^2	0.3235	0.1870	0.3642	0.2645	**1.0000**	1.0000	0.0092	0.0269	0.0725	0.3297	0.2510	0.4413
min R^2	0.0022	0.0000	0.0001	0.0000	**1.0000**	1.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.1470
max R^2	0.8478	0.7876	0.7453	0.9799	**1.0000**	1.0000	0.0594	0.2486	0.8027	0.8577	0.8763	0.9149
std R^2	0.2244	0.1766	0.2178	0.2339	**0.0000**	0.0000	0.0117	0.0430	0.1261	0.1957	0.1781	0.1760

Notes: This table presents some R^2 statistics (average, minimum, maximum, and standard deviation) obtained by regressing a systemic risk measure (respectively, MES in the upper panel, SRISK in the middle panel, and ΔCoVaR in the lower panel) on one or five (all) market risk measures or firm characteristics: market value of equity (MV), liabilities (LTQ), leverage (LVG), beta, and VaR. We consider two types of regressions: time series regressions for each of the 94 sample firms (left column) and cross-sectional regressions for each of the 757 days in the sample period (right column). Each regression is run with a constant term over an estimation period covering January 2, 2008 - December 31, 2010. Bold figures indicate the explanatory variable that leads to the highest average R^2.

Appendix A: Proof of Proposition 1 (MES)

Proof. Let us consider the Cholesky decomposition of the variance-covariance matrix H_t:

$$H_t^{1/2} = \begin{pmatrix} \sigma_{mt} & 0 \\ \sigma_{it}\ \rho_{it} & \sigma_{it}\ \sqrt{1-\rho_{it}^2} \end{pmatrix} \tag{A1}$$

Given Equation (8), the market and firm returns can be expressed as:

$$r_{mt} = \sigma_{mt}\ \varepsilon_{mt} \tag{A2}$$

$$r_{it} = \sigma_{it}\ \rho_{it}\ \varepsilon_{mt}\ +\ \sigma_{it}\ \sqrt{1-\rho_{it}^2}\ \xi_{it}. \tag{A3}$$

For any conditioning event C:

$$\begin{aligned} MES_{it}(C) &= \mathbb{E}_{t-1}(r_{it} \mid r_{mt} < C) \\ &= \sigma_{it}\ \rho_{it}\ \mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \qquad \text{(A4)} \\ &\quad + \sigma_{it}\ \sqrt{1-\rho_{it}^2}\ \mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right). \qquad \text{(A5)} \end{aligned}$$

If we assume that ξ_{it} and ε_{mt} are independent, we have:

$$MES_{it}(C) = \sigma_{it}\ \rho_{it}\ \mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\right) \tag{A6}$$

or equivalently:

$$MES_{it}(C) = \sigma_{it}\ \rho_{it}\ \mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid r_{mt} < C\right). \tag{A7}$$

Let $\beta_{it} = cov\,(r_{it}, r_{mt})\,/var\,(r_{mt}) = \rho_{it}\sigma_{it}/\sigma_{mt}$ denotes the time-varying beta of firm i. Combining β_{it} with Equation (A7), we obtain:

$$\begin{aligned} MES_{it}(C) &= \beta_{it}\ \sigma_{mt}\ \mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid r_{mt} < C\right) \\ &= \beta_{it}\ \mathbb{E}_{t-1}\left(r_{mt} \mid r_{mt} < C\right). \qquad \text{(A8)} \end{aligned}$$

The MES is expressed as the product between the time-varying beta and the truncated expectation of the market return for any given threshold C. By definition, the expected shortfall of the market return $ES_{mt}(\alpha)$ corresponds to the truncated expectation of the market return for a given threshold equal to the conditional VaR (Jorion, 2007), $C = VaR_{mt}(\alpha)$:

$$ES_{mt}(\alpha) = \mathbb{E}_{t-1}\left(r_{mt} \mid r_{mt} < VaR_{mt}(\alpha)\right). \tag{A9}$$

Then, the MES defined for the specific event $C = VaR_{mt}(\alpha)$, denoted $MES_{it}(\alpha)$, is simply expressed as the product of time-varying firm beta and expected shortfall of the market return:

$$MES_{it}(\alpha) = \beta_{it}\ ES_{mt}(\alpha). \tag{A10}$$

■

Appendix B: Proof of Proposition 2 (ΔCoVaR)

Proof. We consider two cases: a general case with $\rho_{it} \neq 0$ and a special case with $\rho_{it} = 0$. Given Equations (8) and (9), if $\rho_{it} \neq 0$ then the market return can be expressed as:

$$r_{mt} = \frac{\sigma_{mt}}{\sigma_{it}\rho_{it}}\ r_{it} - \frac{\sigma_{mt}\sqrt{1-\rho_{it}^2}}{\rho_{it}}\ \xi_{it}. \tag{B1}$$

For each conditioning event form $\mathbb{C}(r_{it}) : r_{it} = C$, CoVaR is defined as follows:

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|r_{it}=C}\ \middle|\ r_{it} = C\right) = \alpha \tag{B2}$$

or equivalently:

$$\Pr\left(\xi_{it} \leq \frac{\rho_{it}}{\sigma_{mt}\sqrt{1-\rho_{it}^2}}\left(\frac{\sigma_{mt}}{\sigma_{it}\rho_{it}}C - CoVaR_t^{m|r_{it}=C}\right)\ \middle|\ r_{it} = C\right) = 1-\alpha. \tag{B3}$$

In the special case where the conditional mean function of ξ_{it} is linear in r_{it}, the first two conditional moments of ξ_{it} given $r_{it} = C$ can be expressed as:

$$\begin{aligned}
\mathbb{E}\left(\xi_{it}\ |\ r_{it} = C\right) &= \frac{cov\left(\xi_{it}, r_{it}\right)}{\sigma_{it}^2} \times C \\
&= \frac{\sigma_{it}\sqrt{1-\rho_{it}^2}}{\sigma_{it}^2} \times C \\
&= \frac{\sqrt{1-\rho_{it}^2}}{\sigma_{it}} \times C
\end{aligned} \tag{B4}$$

$$\begin{aligned}
\mathbb{V}\left(\xi_{it}\ |\ r_{it}\right) &= \mathbb{V}\left(\xi_{it}\right) - \mathbb{V}_{r_{it}}\left[\mathbb{E}\left(\xi_{it}\ |\ r_{it}\right)\right] \\
&= \mathbb{V}\left(\xi_{it}\right) \times \left[1 - \left(\frac{cov\left(\xi_{it}, r_{it}\right)}{\sigma_{it}^2}\right)^2 \sigma_{it}^2\right] \\
&= 1 - \left(\frac{\sigma_{it}\sqrt{1-\rho_{it}^2}}{\sigma_{it}^2}\right)^2 \sigma_{it}^2 \\
&= \rho_{it}^2.
\end{aligned} \tag{B5}$$

Consider $G(.)$ the conditional (location-scale) demeaned and standardized cdf of ξ_{it} such that:

$$\mathbb{E}\left[\frac{1}{\rho_{it}}\left(\xi_{it} - \frac{\sqrt{1-\rho_{it}^2}}{\sigma_{it}} \times C\right)\ \middle|\ r_{it} = C\right] = 0 \tag{B6}$$

$$\mathbb{V}\left[\frac{1}{\rho_{it}}\left(\xi_{it} - \frac{\sqrt{1-\rho_{it}^2}}{\sigma_{it}} \times C\right)\ \middle|\ r_{it} = C\right] = 1. \tag{B7}$$

Thus, Equation (B3) is expressed as:

$$\frac{1}{\rho_{it}}\left[\frac{\rho_{it}}{\sigma_{mt}\sqrt{1-\rho_{it}^2}}\left(\frac{\sigma_{mt}}{\sigma_{it}\rho_{it}}\ C - CoVaR_t^{m|r_{it}=C}\right) - \frac{\sqrt{1-\rho_{it}^2}}{\sigma_{it}} \times C\right] = G^{-1}\left(1-\alpha\right).$$

By rearranging these terms, we write the general expression of the CoVaR:

$$CoVaR_t^{m|r_{it}=C} = -\sigma_{mt}\ \sqrt{1-\rho_{it}^2}\ G^{-1}\left(1-\alpha\right) + \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}\ C. \tag{B8}$$

The CoVaR defined for the conditioning event $\mathbb{C}(r_{it}) : r_{it} = Median(r_{it})$, has a similar expression:

$$CoVaR_t^{m|r_{it}=Median(r_i)} = -\sigma_{mt}\sqrt{1-\rho_{it}^2}\; G^{-1}(1-\alpha) + \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}}\; F^{-1}(0.5). \tag{B9}$$

where $F(.)$ denotes the marginal cdf of the firm return. Then, for each conditioning event form $\mathbb{C}(r_{it}) : r_{it} = C$, the ΔCoVaR is defined as:

$$\begin{aligned}\Delta CoVaR_{it}(C) &= CoVaR_t^{m|r_{it}=C} - CoVaR_t^{m|r_{it}=Median(r_{it})} \\ &= \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}} \times [C - Median(r_{it})] \qquad \text{(B10)} \\ &= \gamma_{it} \times [C - Median(r_{it})] \qquad \text{(B11)}\end{aligned}$$

where $\gamma_{it} = \rho_{it}\sigma_{mt}/\sigma_{it}$ denotes the time-varying linear projection coefficient of the market return on the firm return. If the marginal distribution of r_{it} is symmetric around zero, then $F^{-1}(0.5) = 0$, and we have:

$$\Delta CoVaR_{it}(C) = \frac{\rho_{it}\sigma_{mt}}{\sigma_{it}} \times C = \gamma_{it} \times C. \tag{B12}$$

As in Adrian and Brunnermeier (2011), ΔCoVaR denoted $\Delta CoVaR_{it}(\alpha)$ and defined for a conditioning event $\mathbb{C}(r_{it}) : r_{it} = VaR_{it}(\alpha)$ is:

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it} \times [VaR_{it}(\alpha) - VaR_{it}(0.5)] \tag{B13}$$

or

$$\Delta CoVaR_{it}(\alpha) = \gamma_{it} \times VaR_{it}(\alpha) \tag{B14}$$

if the marginal distribution of the firm return is symmetric around zero.

We now consider the case where $\rho_{it} = 0$ and the bivariate process becomes:

$$r_{mt} = \sigma_{mt}\;\varepsilon_{mt} \tag{B15}$$

$$r_{it} = \sigma_{it}\;\xi_{it} \tag{B16}$$

$$(\varepsilon_{mt},\; \xi_{it}) \sim D \tag{B17}$$

where $\nu_t = (\varepsilon_{mt}, \xi_{it})'$ satisfies $\mathbb{E}(\nu_t) = 0$ and $\mathbb{E}(\nu_t\nu_t') = I_2$, and D denotes the bivariate distribution of the standardized innovations. It is straightforward to show that:

$$\Pr\left(r_{mt} \leq CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)} \;\middle|\; r_{it} = VaR_{it}(\alpha)\right) = \Pr\left(r_{mt} \leq CoVaR_t^{m|r_{it}=VaR_{it}(\alpha)}\right) = \alpha.$$

Hence, we have $CoVaR_{it}(\alpha) = \sigma_{mt}F_m^{-1}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) = 0$, where $F_m(.)$ denotes the cdf of the marginal distribution of the standardized market return. $\blacksquare$

Appendix C: Proof of Proposition 4 (Rankings MES-ΔCoVaR)

Proof. First, given Equation (13), the inequality $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$ is then equivalent to:

$$\frac{\rho_{it}}{\sigma_{it}} \times [VaR_{it}(\alpha) - VaR_{it}(0.5)] \leq \frac{\rho_{jt}}{\sigma_{jt}} \times [VaR_{jt}(\alpha) - VaR_{jt}(0.5)]. \tag{C1}$$

If we assume that the conditional distribution of the firm return is a location scale distribution, then $VaR_{it}(\alpha) = \sigma_{it} F_i^{-1}(\alpha)$ where $F_i^{-1}(\alpha)$ denotes the conditional α-quantile of the standardized return r_{it}/σ_{it}. The inequality becomes:

$$\rho_{it} \times \left[F_i^{-1}(\alpha) - F_i^{-1}(0.5)\right] \geq \rho_{jt} \times \left[F_j^{-1}(\alpha) - F_j^{-1}(0.5)\right]. \tag{C2}$$

For simplicity, we assume that the two conditional distributions for firms i and j are identical, i.e., $F_i^{-1}(.) = F_j^{-1}(.) = F^{-1}(.)$. The difference $F^{-1}(\alpha) - F^{-1}(0.5)$ is typically a negative number, so the inequality $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$ can be reduced to the simple condition $\rho_{it} \geq \rho_{jt}$.

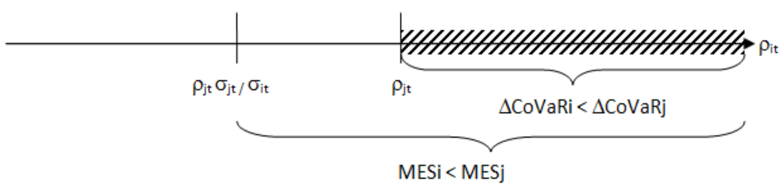
$$\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha) \Longleftrightarrow \rho_{it} \geq \rho_{jt}. \tag{C3}$$

Second, the inequality $MES_{it}(\alpha) \leq MES_{jt}(\alpha)$ means that $\beta_{it} \geq \beta_{jt}$ since the system ES is negative, $ES_{mt} < 0$. Given the definition of conditional beta, this inequality is equivalent to the condition $\sigma_{it}\rho_{it} \geq \sigma_{jt}\rho_{jt}$:

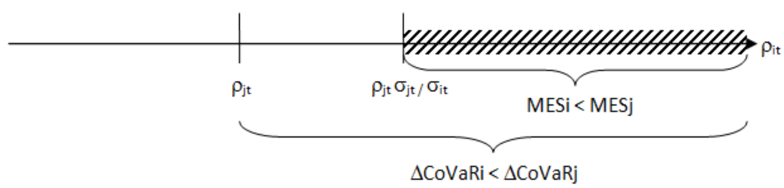
$$MES_{it}(\alpha) \leq MES_{jt}(\alpha) \Longleftrightarrow \sigma_{it}\rho_{it} \geq \sigma_{jt}\rho_{jt}. \tag{C4}$$

We have simultaneously $MES_{it}(\alpha) \leq MES_{jt}(\alpha)$ and $\Delta CoVaR_{it}(\alpha) \leq \Delta CoVaR_{jt}(\alpha)$ when conditions (C3) and (C4) are satisfied. Given the relative values of the volatilities, two cases can be studied separately.

Case a: $\sigma_{it} \geq \sigma_{jt}$. Conditions (C3) and (C4) are satisfied if $\rho_{it} \geq \rho_{jt}$.



Case b: $\sigma_{it} < \sigma_{jt}$. Conditions (C3) and (C4) are satisfied if $\rho_{it} \geq \rho_{jt}\sigma_{jt}/\sigma_{it}$.



Then, the systemic risk rankings (MES and ΔCoVaR) of both financial institutions are identical when we have:

$$\rho_{it} \geq \max\left(\rho_{jt}, \frac{\rho_{jt}\sigma_{jt}}{\sigma_{it}}\right). \tag{C5}$$

If the two conditional distributions $F_i(.)$ and $F_j(.)$ are different, but location-scale, this condition becomes:

$$\rho_{it} \geq \max\left(\rho_{jt}, \rho_{jt}\frac{\left[F_j^{-1}(\alpha) - F_j^{-1}(0.5)\right]}{\left[F_i^{-1}(\alpha) - F_i^{-1}(0.5)\right]}\right) \tag{C6}$$

and if they are not location-scales it is:

$$\rho_{it} \geq \max\left(\rho_{jt}, \rho_{jt}\frac{\sigma_{it}\left[VaR_{jt}\left(\alpha\right) - VaR_{jt}\left(0.5\right)\right]}{\sigma_{jt}\left[VaR_{it}\left(\alpha\right) - VaR_{it}\left(0.5\right)\right]}\right). \tag{C7}$$

■

Appendix D: Proof of Proposition 5 (Rankings SRISK-ΔCoVaR)

Proof. Given Equation (12) of the SRISK, firm i is more risky than firm j if:

$$k\ D_{it} - (1-k)\ W_{it}\ \exp\left(18 \times \beta_{it} \times ES_{mt}\left(\alpha\right)\right) \geq k\ D_{jt} - (1-k)\ W_{jt}\ \exp\left(18 \times \beta_{jt} \times ES_{mt}\left(\alpha\right)\right).$$

For simplicity, we consider two firms with the same level of liabilities, $D_{it} = D_{jt}$. Then, the inequality $SRISK_{it}\left(\alpha\right) \geq SRISK_{jt}\left(\alpha\right)$ is equivalent to:

$$W_{it} \exp\left(18 \times \beta_{it} \times ES_{mt}\left(\alpha\right)\right) \leq W_{jt} \exp\left(18 \times \beta_{jt} \times ES_{mt}\left(\alpha\right)\right). \tag{D1}$$

As shown in Appendix C, under some mild assumptions, we have:

$$\Delta CoVaR_{it}\left(\alpha\right) \leq \Delta CoVaR_{jt}\left(\alpha\right) \Longleftrightarrow \rho_{it} \geq \rho_{jt}. \tag{D2}$$

The systemic risk ranking given by the SRISK and the ΔCoVaR are convergent when conditions (D1) and (D2) are satisfied. These conditions can be expressed as constraints on both the correlation and the market value of the riskiest firm i:

$$\rho_{it} \geq \rho_{jt} \quad \text{and} \quad W_{it} \leq W_{jt} \exp\left[18 \times ES_{mt}\left(\alpha\right) \times \left(\beta_{jt} - \beta_{it}\right)\right]. \tag{D3}$$

■

Appendix E: Dataset

Tickers and Company Names by Industry Groups

Depositories (29)		Insurance (32)	
BAC	Bank of America	ABK	Ambac Financial Group
BBT	BB&T	AET	Aetna
BK	Bank of New York Mellon	AFL	AFLAC
C	Citigroup	AIG	American International Group
CBH	Commerce Bancorp	AIZ	Assurant
CMA	Comerica Inc.	ALL	Allstate Corp.
HBAN	Huntington Bancshares	AOC	Aon Corp.
HCBK	Hudson City Bancorp	WRB	W.R. Berkley Corp.
JPM	JP Morgan Chase	BRK	Berkshire Hathaway
KEY	Keycorp	CB	Chubb Corp.
MI	Marshall & Ilsley	CFC	Countrywide Financial
MTB	M&T Bank Corp.	CI	CIGNA Corp.
NCC	National City Corp.	CINF	Cincinnati Financial Corp.
NTRS	Northern Trust	CNA	CNA Financial Corp.
NYB	New York Community Bancorp	CVH	Coventry Health Care
PBCT	Peoples United Financial	FNF	Fidelity National Financial
PNC	PNC Financial Services	GNW	Genworth Financial
RF	Regions Financial	HIG	Hartford Financial Group
SNV	Synovus Financial	HNT	Health Net
SOV	Sovereign Bancorp	HUM	Humana
STI	Suntrust Banks	LNC	Lincoln National
STT	State Street	MBI	MBIA
UB	Unionbancal Corp.	MET	MetLife
USB	US Bancorp	MMC	Marsh & McLennan
WB	Wachovia	PFG	Principal Financial Group
WFC	Wells Fargo & Co	PGR	Progressive
WM	Washington Mutual	PRU	Prudential Financial
WU	Western Union	SAF	Safeco
ZION	Zions	TMK	Torchmark
		TRV	Travelers
		UNH	UnitedHealth Group
		UNM	Unum Group
Broker-Dealers (10)		**Others (23)**	
AGE	A.G. Edwards	ACAS	American Capital
BSC	Bear Stearns	AMP	Ameriprise Financial
ETFC	E*Trade Financial	AMTD	TD Ameritrade
GS	Goldman Sachs	AXP	American Express
LEH	Lehman Brothers	BEN	Franklin Resources
MER	Merill Lynch	BLK	BlackRock
MS	Morgan Stanley	BOT	CBOT Holdings
NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
SCHW	Schwab Charles	CBSS	Compass Bancshares
TROW	T. Rowe Price	CIT	CIT Group
		CME	CME Group
		COF	Capital One Financial
		EV	Eaton Vance
		FITB	Fifth Third Bancorp
		FNM	Fannie Mae
		FRE	Freddie Mac
		HRB	H&R Block
		ICE	Intercontinental Exchange
		JNS	Janus Capital
		LM	Legg Mason
		NYX	NYSE Euronext
		SEIC	SEI Investment Company
		SLM	SLM Corp.

Appendix F: Estimation Methods

In order to compute the MES, the SRISK and the beta for each financial institution, we implement the estimation method of Brownlees and Engle (2012) and use the model defined in Equations (8) and (9). The conditional variances σ_{it}^2 and σ_{mt}^2 are modeled according to a TGARCH specification (Rabemananjara and Zakoïan, 1993). The time-varying correlations ρ_{it} are modeled with a symmetric DCC model. We estimate the model in two steps, using Quasi Maximum Likelihood (QML). Given the estimated correlations and variances, $\widehat{\rho}_{it}$, $\widehat{\sigma}_{it}^2$ and $\widehat{\sigma}_{mt}^2$, we estimate the beta, MES and SRISK as follows:

Beta: Given the market model defined in Equations (8) and (9), the estimated time-varying beta of the firm i is:

$$\widehat{\beta}_{it} = \frac{\widehat{\rho}_{it}\ \widehat{\sigma}_{it}}{\widehat{\sigma}_{mt}}. \tag{F1}$$

In order to asses the robustness of our results we also consider a constant beta estimated by OLS with a linear market model $r_{it} = \alpha_i + \beta_i\ r_{mt} + \varepsilon_t$.

MES and SRISK: When we allow for nonlinear dependencies between the firm and market returns, the MES can no longer be expressed as the product of the market ES and the time-varying beta of this firm. Indeed, the conditional tail expectation $\mathbb{E}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < C/\sigma_{mt}\right)$ in the expression of the MES (Equation 10) can differ from zero. This term captures the tail-spillover effects from the financial system to the financial institution that are not captured by the correlation. Additionally, if both marginal distributions of the standardized returns are unknown, then the conditional expectation $\mathbb{E}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < C/\sigma_{mt}\right)$ is also unknown. Consequently, both tail expectations must be estimated. To do so, we follow Brownlees and Engle (2012) and use a nonparametric kernel estimation method (Scaillet, 2005). We consider an unconditional threshold C equal to the unconditional VaR of the system.[17] Then, if the standardized innovations ε_{mt} and ξ_{it} are *i.i.d.*, the nonparametric estimates of these tail expectations are given by:

$$\widehat{\mathbb{E}}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \kappa\right) = \frac{\sum\limits_{t=1}^{T} K\left(\frac{\kappa-\varepsilon_{mt}}{h}\right)\ \varepsilon_{mt}}{\sum\limits_{t=1}^{T} K\left(\frac{\kappa-\varepsilon_{mt}}{h}\right)} \tag{F2}$$

$$\widehat{\mathbb{E}}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \kappa\right) = \frac{\sum\limits_{t=1}^{T} K\left(\frac{\kappa-\varepsilon_{mt}}{h}\right)\ \xi_{it}}{\sum\limits_{t=1}^{T} K\left(\frac{\kappa-\varepsilon_{mt}}{h}\right)} \tag{F3}$$

where $\kappa = VaR_m\left(\alpha\right)/\sigma_{mt}$, $K(x) = \int_{-\infty}^{x/h} k(u)\ du$, $k(u)$ is a kernel function, and h is a positive bandwidth parameter. Following Scaillet (2005), we fix the bandwidth at $T^{-1/5}$ and choose the standard normal probability distribution function as a kernel function, i.e., $k\left(u\right) = \phi\left(u\right)$. The final elements needed to compute the MES are the conditional variance and correlation estimated with a GARCH-DCC model. Then, the MES is defined as:

$$\begin{aligned} \widehat{MES}_{it}\left(VaR_m\left(\alpha\right)\right) \quad &= \quad \widehat{\sigma}_{it}\ \widehat{\rho}_{it}\ \widehat{\mathbb{E}}_{t-1}\left(\varepsilon_{mt} \mid \varepsilon_{mt} < \kappa\right) \\ &\quad + \widehat{\sigma}_{it}\ \sqrt{1-\widehat{\rho}_{it}^2}\ \widehat{\mathbb{E}}_{t-1}\left(\xi_{it} \mid \varepsilon_{mt} < \kappa\right). \end{aligned} \tag{F4}$$

[17] Results obtained with $C = VaR_{mt}\left(\alpha\right)$, where $VaR_{mt}\left(\alpha\right)$ denotes the conditional VaR, are similar and available upon request.

The LRMES is deduced from the MES by using to the approximation proposed by Acharya, Engle and Richardson (2012), $LRMES_{it} \simeq 1 - \exp\left(18 \times MES_{it}\right)$. This approximation represents the firm expected loss over a six-month horizon, obtained conditionally on the market falling by more than 40% within the next six months (for more details, see Acharya, Engle and Richardson, 2012). Finally, the SRISK is deduced from the LRMES according to Equation (4):

$$\widehat{SRISK}_{it} = \max\left[0\ ;\ k\ D_{it} - (1-k)\ W_{it}\ \left(1 - \widehat{LRMES}_{it}\right)\right] \tag{F5}$$

where D_{it} is the quarterly book value of total liabilities, k is the prudential capital ratio (set to 8%), and W_{it} is the daily market value of equity.

VaR: The unconditional VaR of the system return, used to define the conditioning event in the MES, is simply estimated by the empirical quantile of the past returns:

$$\widehat{VaR}_m\left(\alpha\right) = percentile\left(\left\{r_{mt}\right\}_{t=1}^{T}, \alpha\right). \tag{F6}$$

The conditional VaR of firm i, used in the ΔCoVaR definition, is computed from the QML estimated conditional variances issued from the TGARCH model. If we assume that the marginal distribution of the standardized firm returns is a location-scale distribution, the conditional VaR satisfies $\widehat{VaR}_{it}\left(\alpha\right) = F_i^{-1}\left(\alpha\right)\ \widehat{\sigma}_{it}$, where $F_i\left(.\right)$ denotes the true distribution of the standardized returns r_{it}/σ_{it} and $\widehat{\sigma}_{it}^2$ is the estimated conditional variance. Because the quantile $F_i^{-1}\left(\alpha\right)$ is unknown, we estimate it by its empirical counterpart.

ΔCoVaR: For any conditioning event $\mathbb{C}\left(r_{it}\right) : r_{it} = C_t$, $\forall C_t \in \mathbb{R}$, the CoVaR satisfies:

$$\int_{-\infty}^{CoVaR_t^{m|C_t}} f_{r_i,r_m}\left(x, C_t\right) dx = \alpha \int_{-\infty}^{\infty} f_{r_i,r_m}\left(x, C_t\right) dx \tag{F7}$$

where $f_{r_i,r_m}\left(x,y\right)$ denotes the joint distribution of $\left(r_{it}, r_{mt}\right)$. There is no closed form for the CoVaR, but it can be estimated in various ways including a copula function, a time-varying second-order moments model, or by bootstrapping past returns. Adrian and Brunnermeier (2011) suggest to use a standard quantile regression (Koenker and Bassett, 1978) of the market return on a particular firm return for the α-quantile:

$$r_{mt} = \mu_\alpha^i + \gamma_\alpha^i r_{it}. \tag{F8}$$

For a conditioning event $\mathbb{C}\left(r_{it}\right) : r_{it} = VaR_{it}\left(\alpha\right)$, where $VaR_{it}\left(\alpha\right)$ denotes the conditional VaR of the i^{th} financial institution, the CoVaR defined by:

$$\Pr\left(r_{mt} \leq CoVaR^{m|VaR_{it}(\alpha)}\ \middle|\ r_{it} = VaR_{it}\left(\alpha\right)\right) = \alpha \tag{F9}$$

is estimated by $\widehat{CoVaR}^{m|VaR_{it}(\alpha)} = \widehat{\mu}_\alpha^i + \widehat{\gamma}_\alpha^i \widehat{VaR}_{it}\left(\alpha\right)$, where $\widehat{\mu}_\alpha^i$ and $\widehat{\gamma}_\alpha^i$ denote the estimated parameters of the quantile regression. A similar result is obtained for the CoVaR defined for the median state of the institution, $\widehat{CoVaR}_t^{m|Median(r_i)} = \widehat{\mu}_\alpha^i + \widehat{\gamma}_\alpha^i \widehat{VaR}_{it}\left(0.5\right)$. Then, by definition, the ΔCoVaR is equal to:

$$\Delta\widehat{CoVaR}_{it}\left(\alpha\right) = \widehat{\gamma}_\alpha^i\left[\widehat{VaR}_{it}\left(\alpha\right) - \widehat{VaR}_{it}\left(0.5\right)\right]. \tag{F10}$$

In order to assess the robustness of our results, we consider two alternative estimators of the CoVaR (not presented). The first one is based on an augmented quantile regression:

$$r_{mt} = \mu_\alpha^i + \gamma_\alpha^i r_{it} + \psi_\alpha^i M_{t-1} \tag{F11}$$

where M_{t-1} denotes a vector of lagged state variables as in Adrian and Brunnermeier (2011). The second estimator is based on a GARCH-DCC model: the ΔCoVaR is obtained from the estimated time-varying second-order moments. Given Equations (8) and (9), the estimated DCC-ΔCoVaR is defined as:

$$\Delta\widehat{CoVa}R_{it}\left(\alpha\right) = \widehat{\gamma}_{it}\left[\widehat{VaR}_{it}\left(\alpha\right) - \widehat{VaR}_{it}\left(0.5\right)\right] \tag{F12}$$

where $\widehat{\gamma}_{it} = \widehat{\rho}_{it}\widehat{\sigma}_{mt}/\widehat{\sigma}_{it}$.

Appendix G: Robustness Check

Table G1: Systemic Risk Rankings (Top 20 Firms)

Rank	MES	SRISK	ΔCoVaR
1	MBI	BAC	HRB
2	AIG	C	MI
3	MI	JPM	BEN
4	CBG	MS	CIT
5	RF	AIG	WU
6	LM	MET	AIZ
7	JNS	PRU	AXP
8	HRB	HIG	JNS
9	BAC	SLM	NYB
10	UNM	LNC	MTB
11	ACAS	GS	EV
12	STI	RF	PGR
13	ETFC	PFG	HCBK
14	AMTD	GNW	LM
15	HBAN	STI	MBI
16	SNV	MI	TROW
17	LNC	MBI	GS
18	FITB	ETFC	MMC
19	HIG	COF	BLK
20	CIT	SNV	RF

Notes: The column labeled MES displays the ranking of the top 20 financial institutions based on MES, ranked from most to least risky. The following two columns display the top 20 financial institutions based on SRISK and ΔCoVaR, respectively. The ranking is for December 31, 2010. See Appendix E for the list of firm names and tickers.

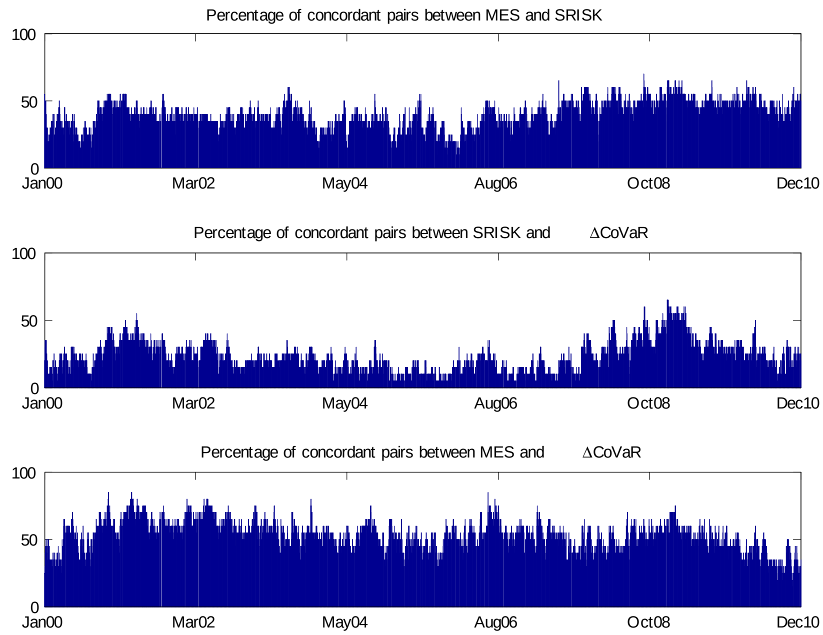


Figure G1: **Different Risk Measures, Different SIFIs (Top 20 Firms):** These figures show the daily percentage of concordant pairs between the top ten financial institutions based one MES and SRISK (top panel), the top 20 financial institutions based on SRISK and ΔCoVaR (middle panel), and the top 20 financial institutions based on ΔCoVaR and MES (bottom panel).

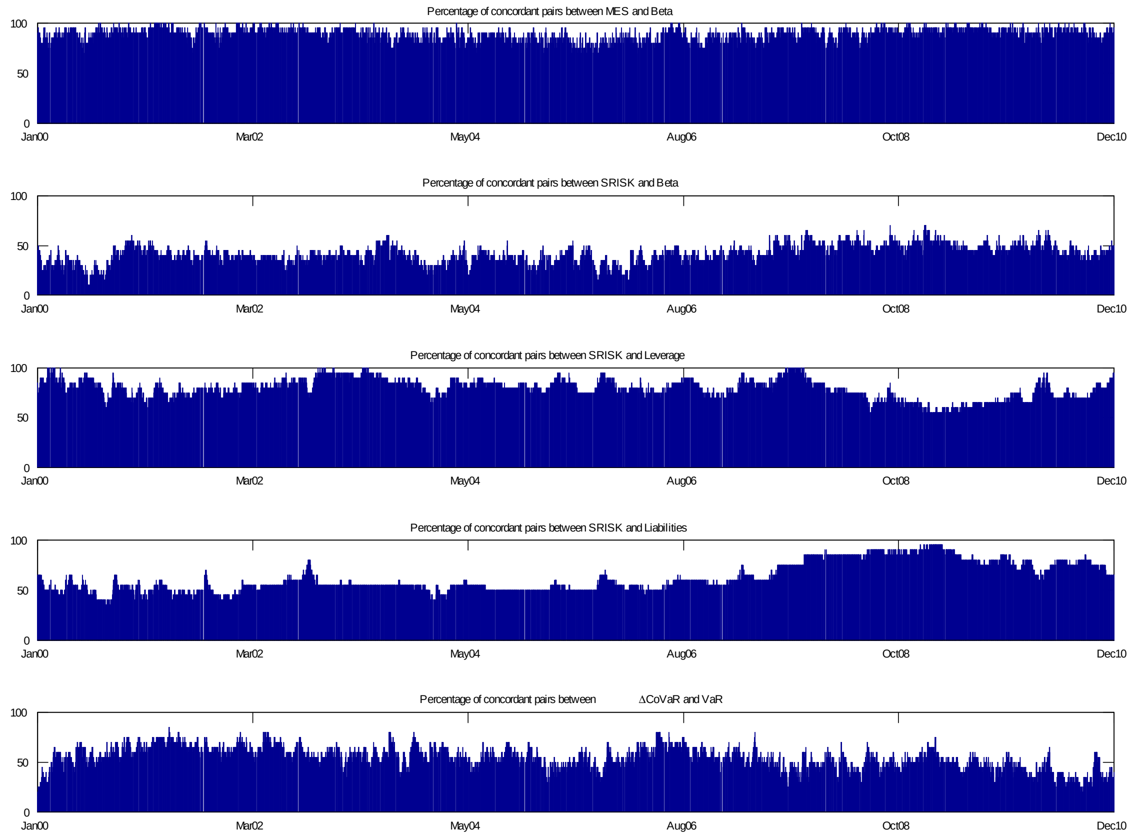


Figure G2: **Driving Forces of Systemic Risk Rankings (Top 20 Firms):** The top figure shows the daily percentage of concordance between the top 20 financial institutions given the MES and the top 20 financial institutions given the beta. The next three figures show the daily percentage of concordance between the first 20 financial institutions given the SRISK and the first 20 financial institutions given the beta, leverage or liabilities. The bottom figure shows the daily percentage of concordance between the top 20 financial institutions given the ΔCoVaR and the top 20 financial institutions given the VaR.