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Volatility, Financial Constraints and Trade

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Volatility, financial constraints, and trade

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Abstract

We construct a dynamic monopolistic competition model with heterogeneous firms to study the links between firms' earnings volatility, the degree of financial constraints that they face, their survival probabilities, and their export market participation decisions. Our model predicts that more volatile firms are more likely to face financial constraints and to go bankrupt, need to be more productive to stay in the market, and have more incentives to enter export markets. A further implication is that through market diversification, exports tend to stabilize firms' total sales. We test these predictions, using a panel of 9292 UK manufacturing firms over the period 1993-2003. The data provide strong support to our model.

JEL Classification: D21; F12; G33; L11

Keywords: Firm-level volatility; Financial constraints; Firm survival; Exports

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1. Introduction

A rise in the volatility of firms' earnings streams has been documented throughout the world in recent years[1]. According to Comin and Philippon (2005), this may be due to increased competition in product markets, which might in turn follow from deregulation, increases in R&D investment, and higher use of debt and equity.

A vast literature has dealt with the adverse effects associated with high firm-level earnings volatility. For instance, Burgstahler and Dichev (1997) show that high volatility is costly for firms, as it implies high costs in transactions with stakeholders. Gibbins et al. (1990), Chaney and Lewis (1995), and Fudenberg and Tirole (1995) insist on the adverse reputation effects of high earnings volatility for firm managers. Other papers show that high volatility limits firms' access to external funds, and increases the cost of obtaining credit (Badrinath et al., 1989; Barnes, 2001; Minton and Schrand, 1999)[2]. Consequently, in order to avoid these adverse effects, managers should try to smooth their earnings. One possible way to achieve this goal is by exporting (Hirsch and Lev, 1971; Maloney and Azevedo, 1995; Campa and Shaver, 2002). In this way, firms can diversify their incomes, leading to more stable cash flow and earnings streams (Buch et al. 2006), and, in turn, to a lower degree of financial constraints (Greenaway et al., 2007). To the best of our knowledge, no paper in the literature has analyzed the extent to which firm-level earnings volatility affects firms' survival probabilities and their decisions to enter export markets. Our paper bridges this gap.

We construct a dynamic model of monopolistic competition aimed at studying the impact of firms' income volatility on the degree of financial constraints that they face[3], which in turn affects their probabilities of survival, and of entering export markets. Specifically, our model assumes

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[1] See Chaney et al. (2002) and Comin and Mulani (2004), who document this trend focusing on US firms; Comin and Philippon (2005), who analyze firms in 80 OECD countries; Thesmar and Thoenig (2004), who look at French firms; and Angelidis and Tessaromatis (2005), whose study is based on the UK. A notable exception is Davis et al. (2006) who observe a declining volatility among US privately held firms.

[2] Focusing on a different perspective, Comin et al. (2006) show that increased firm-level earnings volatility in the US explains about 60 percent of the observed increase in the high frequency volatility of wages.

[3] We will hereafter use the terms income volatility and earnings volatility interchangeably.

that firms need to borrow a fixed amount to operate in the market, and that they face an income shock, which determines their earnings volatility. From the point of view of the bank, volatility is perceived as a risk: unless it is compensated with higher interest rates, the bank is less willing to offer funding to firms with high volatility. Keeping other firm's characteristics constant, more volatile firms are therefore more likely to go bankrupt and need to be more productive to stay in the market. In equilibrium three types of firms can coexist in the market: firms with high productivity and low volatility; firms with high productivity and high volatility; and firms with low productivity and low volatility.

When the country opens up to trade, our model predicts that exporters can smooth their income through diversification[4], which leads to an improvement in their financial health. Yet, as in Melitz (2003), trade also increases competition, which decreases the expected profits of some incumbent firms, increases their probability of bankruptcy, and limits their access to external funds. Opening up to trade leads therefore to two possible scenarios. In the first one, the competition effect is relatively weak, and exporters experience a decrease in their financial constraints because, from the point of view of the bank, they have become less risky, due to their reduced total volatility[5]. In this case, there is a selection effect as in Behrens et al. (2007): trade can generate a reallocation of firms in the market, such that less productive and less volatile firms are replaced with highly productive and volatile ones. In the second scenario, the competition effect is strong, and all firms may become more financially constrained due to the decrease in their expected profits[6]. Contrary to Melitz (2003), in this case, trade does not necessarily raise average productivity in the market, as no reallocation of firms takes place. Our analysis also

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[4] Here, by diversification, we mean serving not only the domestic market, but also the foreign market. We do not take into account the further diversification that could take place through exporting to more than one country. Diversification leads to a reduction in firms' total volatility, which encompasses the volatilities in the domestic and the foreign markets.

[5] This scenario is consistent with Chaney (2005), whose model shows that trade can reduce the liquidity constraints faced by firms. Yet, while Chaney (2005) sees exchange rate appreciations as the main determinant of this improvement in firms' financial health, we focus on the reduction in firms' income volatility.

shows that more volatile firms generally have more incentives to trade than their less volatile counterparts, as they take more advantage from the benefit of sales stabilization and associated lower financial costs. Volatility provides therefore a link between trade and the degree of financial constraints faced by firms.

Our model can be seen as an extension of Melitz's (2003) framework, in which firms' earnings volatility, and, consequently, their ability to obtain external credit are included as new elements of firm heterogeneity, in addition to productivity. Our model also links with Manova (2006), who develops a model with heterogeneous firms, in which financial constraints interact with firm heterogeneity to reinforce the selection of only the most productive firms into exporting. Yet, her focus is different from ours: while we consider firms' income volatility as the main determinant of financial constraints, she concentrates on firm-level productivity, industry-level assets tangibility and external finance dependence, and country-level financial development.

We test the predictions of our model using a panel of 9292 UK manufacturing firms over the period 1993-2003. Our choice of the UK in our empirical testing of the model stems from the fact that the UK is the fifth largest exporter of manufactures globally, and within our sample, almost 70 percent of all firms exported in at least one year. Moreover, on average, 30 percent of the total sales of UK exporters are directed abroad. We find empirical evidence showing that more volatile firms are more likely to go bankrupt, need to be more productive to stay in the market, and have more incentives to enter export markets. A further result is that through market diversification, exports tend to stabilize firms' total sales.

The rest of the paper is laid out as follows. In Section 2, we outline our model in the closed economy, focusing on firms' decision to produce and banks' allocation of funding. Section 3 analyzes the open economy case. Specifically, we show how the market equilibrium and the allocation of financial resources change when a country opens up to trade, and describe the factors that may lead firms to start exporting. In Section 4, we summarize the main testable

[6] This scenario is consistent with La Rochelle-Côté (2007) who suggests that financial constraints became

implications that can be derived from our model. Section 5 describes our data, Section 6 tests the main implications of the model, and Section 7 concludes.

2. Closed economy model

2.1 Setup

As in Melitz (2003), we propose a monopolistic competition model with heterogeneous firms, and focus on the stationary equilibrium. In our model, firms differ not only in their productivity, but also in their earnings volatility, which is determined by an income shock. We limit our attention to the case in which firms' productivity and volatility are exogenous, constant, and independent over time. While this assumption is highly stylized, it captures the idea that firms may face systematic risks other than those leading to an increase in productivity[7].

Each firm faces a cash-in advance constraint: at the beginning of every period it needs to incur a fixed cost to enter or stay in the market, and requires the bank to finance it. The fixed cost is assumed to be the same across firms. Both the bank and the borrower are assumed to be risk neutral. Unless it is compensated with higher interest rates, the bank is unwilling to offer funding to firms with highly volatile earnings (which it perceives as facing a high risk of bankruptcy), and firms with low productivity (which typically exhibit lower profits). Hence, the bank charges a higher interest rate to the less productive and more volatile firms.

We assume that within each period the timing of actions is as follows (figure 1). First, after having drawn their initial productivity and volatility from common distributions, new firms enter the market. Next, incumbent firms, new firms, and banks observe the average productivity and average volatility in the market. These market characteristics can change over time. Given these characteristics, firms decide whether to stay in the market, and how much to produce. Each firm can perfectly determine the interest rate that it has to pay during this period. It will opt to

more binding for Canadian firms after trade liberalization.

stay in the market, to ask for a loan, and to produce, if its expected future value is positive. Next, income shocks are realized: firms with negative profits go bankrupt and exit the market, while firms with positive profits repay the loan to the bank and continue to the following period.

INSERT FIGURE 1

It is important to note that as in Melitz (2003), in the stationary equilibrium, the minimum productivity level necessary to earn non-zero profits, the interest rate, and the function determining the types of firms that can obtain external funding and stay in the market are all determined simultaneously.

2.2 Demand

The preferences of a representative consumer are given by the following C.E.S. utility function over a continuum of goods indexed by *j*:

$$U = \left[\int_0^M y_j^{\rho} dj \right]^{1/\rho}, \tag{1}$$

where $0 < \rho < 1$; y_j is the quantity of variety *j* of the differentiated product demanded by the consumer; *M* is the mass of firms in the stationary competitive equilibrium; and $\alpha = 1/(1-\rho)$ is the elasticity of substitution between varieties. We consider that the price of the aggregate good *Y* is: $P_Y = \left[\int_0^M p_j^{1-\alpha} dj \right]^{1/(1-\alpha)}$. The aggregate demand for any of the varieties of the differentiated product is given by $y_j = p_j^{-\alpha} E / P_Y^{1-\alpha}$, where *E* is the total expenditure on the differentiated product, and p_j is the price of the good.

[7] In other papers, productivity and volatility are treated as endogenous variables. For instance, in Comin and Mulani (2005), R&D investments are risky, and consequently affect both firms' productivity and their

2.3 Production

Firms are assumed to be heterogeneous. The first element of heterogeneity between firms is their level of productivity, denoted by $\varphi \in [\hat{\varphi}, \infty)$, where $\hat{\varphi} > 0$ is the minimum level of productivity in the market in equilibrium. Firms' revenues depend on the average productivity in the market $\bar{\varphi}(\hat{\varphi}) > \hat{\varphi}$, which is a function of the minimum productivity[8]. Each of the differentiated goods is produced only with labor, which is assumed to be a linear function of output of the following type: $l(\varphi, \bar{\varphi}) = I + y(\varphi, \bar{\varphi})/\varphi$, where l is labor and I is a fixed cost. This yields the standard pricing rule $p(\varphi) = 1/(\rho\varphi)$, where the numerator represents the common wage rate, normalized to one. Firms with high levels of productivity have lower marginal costs, charge lower prices, and obtain higher revenues than their less productive counterparts[9].

We introduce a second and new element of heterogeneity among firms: their earnings volatility. Volatility affects firms' output through an exogenous firm-specific income shock z_t, which occurs outside the firms. In each period, this shock occurs immediately after firms have chosen prices and quantities. Hence, firms cannot adjust to the change[10]. Specifically, if we consider a firm with productivity φ, selling in a market with average productivity $\bar{\varphi}$, in each period, after the realization of the shock, the firm's operating profits will be given by: $\Pi^{o}(\varphi, \bar{\varphi}) = z_t[p(\varphi)y(\varphi, \bar{\varphi})] - y(\varphi, \bar{\varphi})/\varphi$ [11]. We restrict our attention to distributions of shocks $f(z, \sigma)$, with $z > 0$, that can be completely described by their mean, μ, which we normalize to one, and standard deviation, σ[12]. This assumption ensures that the standard deviation of the shock

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earnings volatility, which are consequently not independent.

[8] For simplicity, in the remaining part of the paper, we omit this dependence from our notation, except when needed in derivations.

[9] As in Melitz (2003), $P_Y = M^{1/(1-\alpha)} p(\bar{\varphi})$. Therefore, $y(\varphi, \bar{\varphi}) = \rho\, M^{-1} \varphi^{\alpha} (\bar{\varphi})^{1-\alpha} E$.

[10] Examples of income shocks of this type could be unexpected exchange rate changes or the unexpected entrance of new competitors in the market.

[11] As in Hendel (1996), this assumption implies that firms cannot perfectly predict future incomes. The results do not change if we assume that the firms' profits face an additive shock as in Chaney (2005), instead of a multiplicative shock.

[12] These distributions could be, for instance, lognormal distribution or folded normal distributions.

unambiguously measures risk, which makes the analysis tractable (Bliss, 2000). The standard deviation $\sigma \in [0,\infty)$ parameterizes the magnitude of the firms' volatility. Note that if $\sigma = 0$ for all firms, the model simplifies to Melitz's (2003) case.

2.4 Existence and uniqueness of equilibrium interest rate and exit function

At the beginning of each period, the firm faces a cash-in-advance constraint: it needs the bank to finance the fixed cost that it has to pay in order to stay in the market. The bank perfectly observes the firm's characteristics. It makes a take-it-or-leave-it offer to the firm and issues funds at an interest rate *r* that differs across firms (Clementi and Hopenhayn, 2006). Both the bank and the firm commit to the contract without a possibility of renegotiation. As in Cooley and Quadrini (2001), only one period debt contracts are signed with the bank. If the firm defaults, the bank liquidates the firm, and the firm immediately exits the market. The bank chooses the interest rate such that the expected repayment from the loan is equal to the return of a riskless loan. This is summarized in the following equation:

$$(1+r_0)I = [1+r(\varphi,\overline{\varphi},\sigma)]\, I\, [1-\delta(\varphi,\overline{\varphi},\sigma)]\,. \tag{2}$$

The left-hand side of equation (2) gives the return of the loan at the riskless interest rate r_0. The right hand side says that with probability $(1-\delta)$, the firm can repay its debts, and with probability δ, it goes bankrupt. For simplicity, we assume that in case of bankruptcy, the bank gets nothing.

At period *t*, after the realization of its shock z_t, the profit of a firm of type (φ,σ) is equal to:

$$\Pi_t(\varphi,\overline{\varphi},\sigma,z_t) = z_t[\,p(\varphi)y(\varphi,\overline{\varphi})] - y(\varphi,\overline{\varphi})/\varphi - \left[1+r(\varphi,\overline{\varphi},\sigma)\right]I\,. \tag{3}$$

The firm will exit the market as soon as it experiences a shock below a certain threshold $\underline{z} < 1$, which is the value of the shock leading to zero profits, i.e.:

$$\underline{z}(\varphi,\overline{\varphi},\sigma) = [1+r(\varphi,\overline{\varphi},\sigma)]I\varphi\rho / y(\varphi,\overline{\varphi}) + \rho\,. \tag{4}$$

It follows that the firm's probability of bankruptcy is given by:

$$\delta(\varphi,\overline{\varphi},\sigma)=\int_0^{z(\varphi,\overline{\varphi},\sigma)} f(z,\sigma)dz \,. \tag{5}$$

Before the realization of the shock, firms and banks observe the market characteristics. Each firm can perfectly determine the interest rate that it has to pay during this period. It will opt to stay in the market, to ask for a loan, and to produce, if its expected future value is positive. Given that the expected future value of a firm of type (φ,σ) at period *t* is equal to:

$$v_t(\varphi,\overline{\varphi},\sigma,1)=\sum_{s=t}^{\infty}(1-\delta((\varphi,\overline{\varphi},\sigma))^{s-t}\Pi_s(\varphi,\overline{\varphi},\sigma,1)=\frac{\Pi(\varphi,\overline{\varphi},\sigma,1)}{\delta(\varphi,\overline{\varphi},\sigma)}=\frac{\Pi^0(\varphi,\overline{\varphi})-[1+r(\varphi,\overline{\varphi},\sigma)]I}{\delta(\varphi,\overline{\varphi},\sigma)}\,, \tag{6}$$

firms with negative expected profits will exit the market. The minimum level of productivity in the market ($\hat{\varphi}$) can be seen as the productivity level of those firms whose profits and volatility are equal to zero, and who consequently pay the riskless interest rate (zero profit condition).

A firm with productivity $\varphi>\hat{\varphi}$ can obtain external finance and stay in the market if it is charged an interest rate lower than or equal to the interest rate leading to zero profits. This firm-specific equilibrium interest rate is unique (see Appendix 1A for the proof), and depends negatively on the firm's productivity, and positively on both the firm's volatility and the minimum productivity in the market. This implies that firms with high volatility can obtain a loan and remain in the market, only if they also have relatively high productivity. The following three types of firms can therefore coexist in the market: firms with low productivity and low volatility; firms with high productivity and low volatility; and firms with high productivity and high volatility.

In order to obtain external funding and stay in the market, firms with productivity φ need to have a volatility lower than $s(\varphi,\hat{\varphi})$. Figure 2 depicts the function $s(\varphi,\hat{\varphi})$, which represents the exit function.

INSERT FIGURE 2

Firms whose volatility and productivity lie on the exit function are charged an interest rate leading to an expected future value equal to zero. These firms are indifferent between staying in the market and exiting it. Firms with productivity and volatility below the exit function are charged an interest rate leading to a positive expected future value. The exit function provides a link between the threshold productivity in the market and the degree of financial constraints faced by firms.

As the threshold productivity increases, the earnings of any given firm decrease. The bank realizes that the probability of bankruptcy increases, which implies that interest rates rise up to the point where some firms cannot obtain credit any longer and exit the market. It follows that the exit function shifts downward. The main results of this section can be summarized in the following propositions, which we prove in Appendix 1B and 1A, respectively.

Proposition 1: *Consider two firms that only differ in their earnings volatility. The firm with higher volatility has a higher probability of bankruptcy than the firm with lower volatility. It is consequently more costly for the more volatile firm to obtain external finance.*

Proposition 1 is in line with the literature that argues that earnings management can be motivated by a desire to decrease earnings volatility. For example, Badrinath et al. (1989) show that investors prefer firms with smooth incomes because firms with high volatility are perceived as more risky. It is also consistent with the empirical result of Minton and Schrand (1999), who show that high cash flow volatility is positively related with the costs of accessing external funds. Our finding also provides some theoretical foundation to the empirical results of Barnes (2001), who find a negative relationship between the volatility of a firm’s earnings and its market valuation.

The interaction between volatility and productivity is summarized in the following proposition:

Proposition 2: *In equilibrium, a firm with high volatility must have a sufficiently high productivity to stay in the market. Hence, three types of firms can coexist in the market: firms with low productivity and low volatility; firms with high productivity and low volatility; and firms with high productivity and high volatility.*

2.5 Firm entry

We now introduce firm entry in the model. We assume that an unbounded pool of identical prospective entrants draw their initial productivity $g(\varphi)$, and volatility $h(\sigma)$ from common distributions with continuous cumulative distributions given by $G(\varphi)$ and $H(\sigma)$ respectively. To enter the market, firms must pay an entry cost denoted by f_e. Firms enter the market if their expected ex-ante value is greater than or equal to the entry cost. To avoid the unbounded entry of firms, we assume that in equilibrium these two values are equal (free entry condition). Denoting with $E_t^a[v_t]$ the expected ex-ante firm value, this condition can be written as:

$$E_t^a[v_t(\hat{\varphi})] - f_e = 0. \tag{7}$$

Furthermore, $E_t^a[v_t(\hat{\varphi})] = P_{in}(\hat{\varphi})\sum_{s=t}^{\infty}(1-\bar{\delta}(\hat{\varphi}))^{s-t}\,\bar{\Pi}_s(\hat{\varphi}) = P_{in}(\hat{\varphi})\bar{\Pi}(\hat{\varphi})/\bar{\delta}(\hat{\varphi})$, where P_{in} is the ex-ante probability of successful entry, $\bar{\delta}$ is the average probability of bankruptcy, and $\bar{\Pi}$ is the average ex-ante profit.

2.6 Equilibrium

A stationary equilibrium is defined by constant aggregate variables over time and free entry for firms into the market. In equilibrium, the distribution of firms can be represented as follows:

$$\mu(\varphi,\sigma) = \begin{cases} [g(\varphi)H(s(\varphi,\hat{\varphi}))]/P_{in}(\hat{\varphi}) & \text{if } \varphi \geq \hat{\varphi}, \text{ and } \sigma \leq s(\varphi,\hat{\varphi}) \\ 0 & \text{otherwise.} \end{cases}$$

It can be shown that if the zero profit condition, the free entry condition, and three stability conditions are satisfied, then there exists a unique stationary equilibrium with a unique exit function (see Appendix 1C for the proof). The first of these stability conditions is Melitz' (2003) aggregate stability condition, which requires that the mass of successful entrants in each period exactly replaces the mass of incumbents who are hit by a bad shock and exit. The other two conditions are specified in Appendix 1C.

3. Open economy model

3.1 Effects of trade on the probability of bankruptcy

We now assume that there are two identical countries, which trade the varieties of *Y*, and which differ in the income shocks affecting their firms. Consistent with the empirical observation that trade can smooth earnings streams, we also assume that exporters can face negatively correlated income shocks across countries. Trade involves two types of costs. First, there is a sunk entry cost into the foreign market, f_{ex}. Every period, the firm pays the amortized per-period portion of this cost, denoted by I_x. As in the closed economy framework, exporters face a cash-in-advance constraint which implies that they have to borrow I_x from the bank. Second, there is a variable per unit cost of product that is transported. This variable cost takes the form of an iceberg cost, so that for one unit of a good to arrive to the final destination, $\tau > 1$ units of the good need to be shipped. While prices in the domestic market are given by: $p_d(\varphi) = 1/(\rho\varphi)$, exporters set higher prices in the foreign market, due to the increase in the marginal cost. These are equal to: $p_x(\varphi) = \tau/(\rho\varphi)$. The profits of an exporter in the domestic market are given by:

$$\Pi_{dt}(\varphi, \bar{\varphi}_F, \sigma_T, \text{cov}) = z_{dt}[p_d(\varphi) y_d(\varphi, \bar{\varphi}_F)] - y_d(\varphi, \bar{\varphi}_F)/\varphi - [1 + r_x(\varphi, \bar{\varphi}_F, \sigma_T, \text{cov})]I,$$

where $y_d(\varphi, \bar{\varphi}_F)$ represents domestic sales; $\bar{\varphi}_F$, the average productivity under free trade; r_x , the interest rate paid by exporters; and z_{dt}, the income shock in the domestic market. Similarly, the profits of an exporter in the foreign market are given by:

$$\Pi_{xt}(\varphi, \bar{\varphi}_F, \sigma_T, \text{cov}) = z_{xt}[p_x(\varphi) y_x(\varphi, \bar{\varphi}_F)] - y_x(\varphi, \bar{\varphi}_F)/\varphi - [1 + r_x(\varphi, \bar{\varphi}_F, \sigma_T, \text{cov})]I_x,$$

where $y_x(\varphi,\overline{\varphi}_F)$ represents sales in the foreign country; and z_{xt} ,the income shock in the foreign market. We assume that z_{xt} is distributed with mean one and variance σ_x^2, while z_{dt} has mean one and variance σ_d^2. In this set-up, if a firm sells in both the domestic and the foreign markets, the two shocks can be aggregated, and modelled as a global shock with total variance σ_T^2 (made up of the sum of the domestic and foreign variances plus twice the covariance between the two shocks, which we denote with *cov*). The probability of bankruptcy of an exporter is therefore given by the following cumulative distribution: $\delta_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov}) = \int_0^{\underline{z}_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov})} f(z,\sigma_T)dz$, where the threshold shock $\underline{z}_x$ is the shock that leads to zero total profits for the exporter, i.e.:

$$\underline{z}_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov}) = (I+I_x)(1+r_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov}))\varphi\ \rho / y_d(\varphi,\overline{\varphi}_F) + \rho(1+\tau^{-\alpha})/(1+\tau^{1-\alpha}) .$$

We assume that the interest rate that the bank charges to exporters (r_x) is such that the expected repayment of the loan equals the return of a riskless loan, i.e.:

$$(I+I_x)(1+r_0) = [1+r_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov})](I+I_x)[1-\delta_x(\varphi,\overline{\varphi}_F,\sigma_T,\text{cov})] .$$

It follows that if the covariance between the domestic and the foreign shocks is negative, and larger in absolute value than $\sigma_x^2/2$, and if the fixed cost of exporting is not excessively high, the exporters' probability of bankruptcy and cost of external financing decrease with trade. The following proposition summarizes the impact of trade on firms' volatility and their probability of bankruptcy.

Proposition 3: *If income shocks in the domestic and foreign markets are negatively correlated, trade can lead to a reduction of the firm's total volatility, through market diversification. If the fixed cost of exporting is not excessively high, trade can also reduce the firm's probability of bankruptcy.*

This result is consistent with an early study by Hirsch and Lev (1971), who find that exports tend to stabilize firms' sales through market diversification. It is also in line with the work of Campa and Shaver (2002), who show that Spanish exporters have more stable cash flows than non-exporters. Finally, proposition 3 provides a theoretical foundation for Greenaway et al.'s (2007) empirical finding that trade improves the financial health of UK firms.

3.2 Exit functions under free trade and incentives to trade

The exit function for non-exporters in the open economy differs from the autarky case because the minimum productivity threshold under free trade, $\hat{\varphi}_F$, differs from the threshold in autarky. We denote with $s(\varphi, \hat{\varphi}_F)$, the exit function for non-exporters; and with $s_x(\varphi, \hat{\varphi}_F)$, the exit function for exporters[13]. The two exit functions determine the allocation of financial resources across firms. In figure 3, we show that the economy can be divided into four types of firms. The first type consists of firms with productivity lower than $\hat{\varphi}_F$, which cannot obtain any loan and cannot produce. The second type consists of firms with productivity between $\hat{\varphi}_F$ and $\hat{\hat{\varphi}}$, and volatility below $s(\varphi, \hat{\varphi}_F)$, which can obtain a loan to sell their products domestically. The third type of firms are firms with productivity between $\hat{\hat{\varphi}}$ and $\hat{\varphi}_x$, and volatility below $s(\varphi, \hat{\varphi}_F)$, which can obtain a loan to export, but find it more profitable to only sell in the domestic market. Finally, firms with productivity higher than $\hat{\varphi}_x$ and volatility below $s_X(\varphi, \hat{\varphi}_F)$ obtain a loan to export, and decide to export. The conditions for the existence of a unique stationary equilibrium in the open economy are illustrated in Appendix 1E.

INSERT FIGURE 3

3.3 Effects of opening up to trade

We now analyze the effects of opening up to trade on the distribution of firms in the market and their access to external finance. When a country opens up to trade, highly productive foreign firms enter the domestic market, increasing the minimum productivity in the market (Melitz, 2003). Due to this increase in competition, it follows that the exit function for non-exporters is shifted right, and becomes flatter than in autarky. This implies that non-exporters with low productivity and low volatility exit the market, as they are now unable to obtain credit, due to the increased competition.

Two possible scenarios can take place. In the first one, the slope of the exit function for exporters under free trade is higher than the slope of the exit function in autarky (figure 4a)[14]. This scenario is likely to occur when opening up to trade leads to a relatively weak increase in competition. Firms with high domestic volatility and high productivity, which were unable to obtain credit before the trade liberalization due to their high volatility, can operate in the new stationary equilibrium. Following the liberalization, these firms can in fact smooth their total earnings as they trade, becoming less volatile. Hence, trade liberalization reallocates firms in the market, in such a way that firms with low/medium productivity and low/medium domestic volatility are replaced with exporters with high productivity and high domestic volatility. This effect tends to increase the average productivity in the market[15].

INSERT FIGURE 4a

In the second scenario, the slope of the exit function for exporters under free trade is lower than or equal to the slope of the exit function in autarky (figure 4b). This scenario is likely to occur if opening up to trade leads to a strong increase in competition, which leads to a large rise in the minimum level of productivity in the market. It is associated with a reduction of credit to all those firms that were to the right of the autarky exit function, but are now to the left of the free trade exit function. These firms are now forced to exit the market, and, because there is no entrance of firms that were unable to produce under autarky, no reallocation of firms takes place.

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[13] The existence of the exit function for exporters is proved in Appendix 1D.
[14] This condition is analytically shown in Appendix 1D.
[15] The magnitude of this effect depends on the assumptions about the distribution of firm characteristics. It can be small if there are few firms with a combination of high domestic volatility and high productivity, but many firms with combinations of low/medium productivity and low/medium domestic volatility. In the extreme case in which there are no firms with high domestic volatility and high productivity, the exit of firms with low productivity and low domestic volatility will not be compensated.

Opening up to trade induces such a decrease in earnings for local firms, that no firm (not even the exporters) experiences a reduction in financial constraints. Thus, there is no gain from trade for the domestic firms. Contrary to Melitz (2003), in this case, trade does not necessarily increase average productivity of the domestic firms in the market. This scenario is consistent with La Rochelle-Côte (2007), whose results suggest that Canadian firms became more financially constrained after trade liberalization.

INSERT FIGURE 4b

3.4 Volatility and the willingness to trade

We now turn to a related question: are firms with high domestic income volatility more willing to trade than firms with low volatility? In our model, this can happen if firms are able to diversify their shocks by exporting. Specifically, firms with high domestic income volatility will have more incentives to trade than firms with low volatility if the differential between exporters' and non-exporters' firm value increases with domestic volatility, i.e. if $\partial(v_x - v_{nx})/\partial\sigma_d > 0$. Denoting with Π_{nx}, r_{nx}, and δ_{nx} the profit, interest rate, and probability of bankruptcy of non-exporters, and with Π_x, r_x, and δ_x, the corresponding variables for exporters, this condition is equivalent to:

$$[-(\partial r_x/\partial\sigma_d)(I + I_x)]/\delta_x - (\partial\delta_x/\partial\sigma_d)\Pi_x/\delta_x^2 + [(\partial r_{nx}/\partial\sigma_d)I]/\delta_{nx} + (\partial\delta_{nx}/\partial\sigma_d)\Pi_{nx}/\delta_{nx}^2 > 0 . \quad (8)$$

As shown in proposition 1, in the absence of trade, an increase in volatility leads to an increase in the interest rate charged to the firm ($\partial r_{nx}/\partial\sigma_d > 0$). However, this is not necessarily the case with trade, due to diversification. If a firm sells to markets characterized by negatively correlated shocks, we can observe two different cases. The first occurs if the interest rate of exporters does not change with domestic volatility ($\partial r_x/\partial\sigma_d = 0$). This may happen if exporters with high domestic volatility exhibit relatively low total volatility, due to diversification, which in turn implies that there is no increase in the probability of bankruptcy following a rise in domestic volatility $(\partial\delta_x/\partial\sigma_d = 0)$. In this situation, it follows from Equation (8), that the differential between exporters' and non-exporters' firm value increases with domestic volatility. Firms with high domestic volatility have therefore stronger incentives to trade than firms with low volatility.

The second case occurs if exporters with high domestic volatility are charged a higher interest rate than exporters with low domestic volatility ($\partial r_x / \partial \sigma_d > 0$). In this situation, firms with high domestic volatility may have, once again, more incentives to export than firms with low domestic volatility provided that two conditions are verified. The first requires that the increase in the interest rate associated with higher domestic volatility is lower for exporters than for non-exporters ($\partial r_{nx} / \partial \sigma_d > \partial r_x / \partial \sigma_d$). This would happen if, following a rise in domestic volatility, the total volatility of exporters remained constant or increased only moderately, thanks to diversification. The second condition requires that this advantage in terms of interest rates is sufficiently large as to counterbalance the rise in the fixed cost necessary to export. In these circumstances, Equation (8) shows that the differential between exporters' and non-exporters' firm value increases with domestic volatility, and firms with higher domestic volatility have more incentives to export than firms with low volatility. Proposition 4 summarizes the relationship between domestic volatility and incentives to trade.

Proposition 4: *Firms with high domestic volatility have more incentives to trade than firms with low domestic volatility, provided that either their interest rate does not increase with domestic volatility, or that the increase in the interest rate associated with a higher domestic volatility is lower for exporters than for non-exporters. In this second case, the advantage in terms of interest rate needs to counterbalance the increase in the amortized per-period portion of the fixed cost that exporters have to face.*

4. Testable implications

From the analysis of our theoretical model, several testable implications emerge. First, the model suggests that firms with higher earnings volatility have a higher probability of bankruptcy. It is consequently more costly for them to obtain external finance. Second, the model implies that there is a positive correlation between volatility and productivity for high levels of volatility, while there is no correlation for low levels: highly volatile firms need to be more productive to stay in the market. Third, the open economy model suggests that, through market diversification, exports tend to stabilize firms' total sales. Exporters' total volatility is therefore lower than their domestic volatility. Finally, firms characterized by high domestic volatility will have more incentives to start exporting than firms with low volatility. In the sections that follow, we will test these implications of the model for a panel of 9292 UK firms over the period 1993-2003.

5. Data and summary statistics

5.1 The dataset

We construct our dataset from profit and loss and balance sheet data gathered by Bureau Van Dijk in the *Financial Analysis Made Easy* (*FAME*) database. This provides information on firms for the period 1993-2003. It includes a majority of firms which are not traded on the stock market, or are quoted on other exchanges such as the Alternative Investment Market (*AIM*) and the Off-Exchange (*OFEX*) market[16]. Unquoted firms are more likely to be characterized by adverse financial attributes such as a short track record, poor solvency, and low real assets compared to quoted firms, which are typically large, financially healthy, long-established firms with good credit ratings.

The firms in our dataset operate in the manufacturing sector. We excluded firms that changed the date of their accounting year-end by more than a few weeks, so that data refer to 12 month accounting periods. Firms that did not have complete records on the variables used in our regressions were also dropped. Finally, to control for outliers, we excluded observations in the

[16] We only selected firms that have unconsolidated accounts: this ensures the majority of firms in our dataset are relatively small. Moreover, it avoids the double counting of firms belonging to groups, which would be included in the dataset if firms with consolidated accounts were also part of it.

one percent tails for each variable[17]. Our panel therefore comprises a total of 51668 annual observations on 9292 firms, covering the years 1993-2003. It has an unbalanced structure, with an average of 7 observations per firm. By allowing for both entry and exit, the use of an unbalanced panel partially mitigates potential selection and survivor bias.

5.2 Summary statistics

Table 1 presents the summary statistics of the main variables used in our empirical analysis. Column 1 refers to the entire sample; column 2 and 3, to surviving and failed firms, respectively. As in Bunn and Redwood (2003), we define a firm as failed (bankrupt) in a given year if its status is in receivership, liquidation, or dissolved[18]. Columns 4 and 5 of Table 1 refer respectively to low and high volatility firms. Firm *i* is classified as a low (high) volatility firm in year *t* if its total volatility in year *t* is in the lowest (highest) half of the distribution of the volatilities of all firms operating in its same industry in year *t*. Columns 6 and 7 respectively refer to non-exporters at time *t*-1 that did not enter (non-starters), and entered (starters) export markets at *t*.

As in Comin and Mulani (2004) and Comin and Philippon (2005), our main volatility measure is calculated as the standard deviation of the firm's total real sales growth, measured over a rolling window of 5 years. Specifically, denoting with $Totalvol_{it}$ this standard deviation for firm *i* at time *t*; with $srgr_{it}$, the growth rate of the real sales of firm *i* at time *t*, and with μ_{it}, its average sales growth rate between *t*-2 and *t*+2, we have:

$$Totalvol_{it} = \left[\frac{1}{5} \sum_{\tau=-2}^{2} \left(srgr_{i(t+\tau)} - \mu_{it} \right)^2 \right]^{1/2}$$

[17] These cut-offs are aimed at eliminating observations reflecting particularly large mergers, extraordinary firm shocks, or coding errors. See Appendix 2 for more information on the structure of our panel and complete definitions of all variables used.

[18] Liquidation and receivership are two types of reorganization procedures, which can take place when a firm becomes insolvent. In liquidation, the assets of the firm are sold so as to meet the claims of creditors. In receivership, the receiver can decide whether it is in the creditors' interests to sell the firm's assets. Generally, it is in the creditors' interests to liquidate if the liquidation value of the firm exceeds its going concern value (Lennox, 1999b).

We also provide measures of volatility only based on domestic sales growth and foreign sales growth, which we denote respectively with $Domesticvol_{it}$ and with $Foreignvol_{it}$[19].

Comparing total sales growth volatility at failed and surviving firms (columns 2 and 3), we can see that the former display a higher volatility (0.207) than the latter (0.199). The difference between the two figures is marginally statistically significant (*t*-statistic: 1.69). In accordance with the first testable implication of our model, there is some evidence that failed firms are more volatile than their surviving counterparts. More formal tests of this hypothesis will be provided in the section that follows.

Focusing now on columns 4 and 5, with emphasis on productivity (*TFP*), which is calculated using the Levinsohn and Petrin (2003) method[20], it appears that both high and low volatility firms display very similar levels of productivity (5.820 and 5.826, respectively). Yet the correlation between *TFP* and volatility is positive for high-volatility firms (0.0650) and negative for low-volatility firms (-0.0655). This seems to support our model's second prediction, according to which one should observe a positive correlation between volatility and productivity for high levels of volatility only.

Next, in relation to our open economy model's predictions, we compare the firm's total, domestic, and foreign sales growth volatility, based on the entire sample (column 1). We can see that, as suggested by our model, the volatility of total sales growth (0.197) is lower than that of domestic sales growth (0.238). The difference between the two means is strongly significant (*t*-statistic = 29.86). Also considering that foreign sales display the highest volatility (0.482), this

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[19] It should be noted that given the way in which we calculate volatility, this variable is not available for the years 1993, 1994, 2002, and 2003. For this reason, all regressions which contain our main measure of volatility are based on the sample 1995-2001. This explains why the number of observations reported in Tables 2 to 4 is lower than that reported in Table 1, which refers to the full sample.

[20] A key issue in the estimation of production functions is the correlation between unobservable productivity shocks and input levels. Profit-maximizing firms respond to positive productivity shocks by expanding output, which requires additional inputs; and to negative shocks, by decreasing output and input usage. Olley and Pakes' (1996) estimator uses investment as a proxy for these unobservable shocks. This could cause problems as any observation with zero investment would have to be dropped from the data. Levinsohn and Petrin (2003), by contrast, introduce an estimator which uses intermediate inputs as proxies,

provides some preliminary support for the hypothesis that, through market diversification, exports tend to stabilize total sales[21, 22].

Finally, focusing on columns 6 and 7 of Table 1, we can see that starters display a much higher domestic sales volatility compared to non-starters (0.309 versus 0.193). The difference between the two means is statistically significant (*t*-statistic = 7.63). Although this comparison is simply based on unconditional means, it provides some strong support for our model's last testable implication. More formal tests of the implications of our model are provided in the next section.

INSERT TABLE 1

6. Specifications and results

6.1 Are more volatile firms more likely to go bankrupt?

In order to test the first implication of our model, namely that more volatile firms are more likely to fail, we will estimate a random-effects Probit specification of the following type:

$$Pr(FAIL_{it}=1) = \Phi(a_0 + a_1\, size_{it} + a_2\, age_{it} + a_3\, group_i + a_4\, TFP_{it} + + a_5\, Collateral_{it}/Debt_{it} + a_6\, Totalvol + u_i + u_j + u_t) \qquad (9)$$

$FAIL_{it}$ is a dummy variable equal to 1 if firm *i* failed in year *t*, and 0 otherwise. $\Phi(.)$ denotes the standard normal distribution function. As typically done in the literature (see for instance Bunn and Redwood, 2003; and Disney et al., 2003), our equation controls for firm's size, age, productivity, and for whether the firm is part of a group. We also include the firm's collateral to debt ratio and the volatility of its total sales growth among the regressors[23]. Since the average

arguing that these (which are generally non-zero) are likely to respond more smoothly to productivity shocks.

[21] If we limit the sample to exporters, the volatility of total sales growth is given by 0.198; that of domestic sales growth, by 0.248; and that of foreign sales growth, by 0.479.

[22] These findings are in line with Buch et al. (2006), who document that for German firms, the volatility of total real sales is lower for exporters than for non-exporters. They are also consistent with Engel and Wang (2008), who show that exports are about twice as volatile as GDP for several OECD countries.

[23] We include the ratio of collateral over debt, as a proxy for the healthiness of firms' balance sheets. All our results were robust to including collateral (measured as the ratio of the firm's total fixed assets to total assets) and total debt (measured as the ratio of the firm's total debt to total assets) separately in the regressions, to including just collateral, to including just debt, and to excluding both collateral and debt.

length of time between the final annual report of a failing firm and its entry into bankruptcy is usually 14 months (Lennox, 1999a), our specification includes regressors evaluated at time *t*. Yet, all our results were robust to using lagged regressors. The error term is made up of three components: u_i, u_t, and u_j. u_i represents a firm-specific effect, and is controlled for by our random-effects estimator. u_t represents a time-specific effect accounting for business cycle conditions, and is taken into account by including a full set of time dummies. u_j represents a sector-specific effect, and is controlled for by including a full set of industry dummies[24].

The estimates of equation (9) are presented in column 1 of Table 2. As typically found in the literature, size and *TFP* have a negative effect on the firm's probability of failure. The coefficient associated with the ratio of the firm's collateral to total debt is poorly determined suggesting that this variable does not play a statistically significant effect on firm survival. In accordance with proposition 1 in our model, the volatility of the firm's total sales growth is positively associated with the chances that the firm will go bankrupt: more volatile firms are therefore more likely to fail.

As an alternative test of this first implication of the model, we make use of the *Quiscore* measure produced by Qui Credit Assessment Ltd., which assesses the likelihood of firm failure in the 12 months following the date of calculation. The lower its *Quiscore*, the more risky the firm, and the higher its chances of failure. We estimate the following fixed-effects regression:

$$QUISCORE_{it} = a_0 + a_1\, size_{it} + a_2\, age_{it} + a_3\, TFP_{it} + a_4\, Collateral_{it}/Debt_{it} + $$
$$+ a_5\, Totalvol + u_i + u_t \qquad (10)$$

The results are presented in column 2 of Table 2. *TFP* is positively associated with *Quiscore*, suggesting that more productive firms are less risky, and less likely to fail[25]. Our

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[24] Firms are allocated to the following industrial groups: metals and metal goods; other minerals, and mineral products; chemicals and man made fibres; mechanical engineering; electrical and instrument engineering; motor vehicles and parts, other transport equipment; food, drink, and tobacco; textiles, clothing, leather, and footwear; and others (Blundell et al., 1992). All our results were robust to including more disaggregated industry dummies as in Bridges and Guariglia (2008).

[25] Industry dummies are not included in this regression, as, being typically time-invariant, they are automatically wiped out in the differencing process undertaken by the fixed-effects estimator.

collateral to debt ratio is also positively associated with *Quiscore*, indicating that the more collateral a firm has relative to its total debt, the less risky it is. Surprisingly, our size variable is negatively associated with *Quiscore*, suggesting that larger firms are more risky. Finally, our volatility measure displays a negative coefficient: more volatile firms are more risky, and therefore more likely to fail, which is in accordance with our proposition 1.

As banks generally look at firms' credit ratings such as *Quiscore* when deciding the terms of the loans they make to firms, it is likely that they will charge higher interest rates to the riskiest firms. Thus, as predicted by our model, more volatile firms are more likely to fail, and to be charged higher interest rates by their lenders.

INSERT TABLE 2

6.2 Is there a positive correlation between productivity and volatility for highly volatile firms only?

The second main implication of our model is that there should be a positive correlation between productivity and volatility for highly volatile firms only. In order to test this implication, we construct the following two dummies: $LOWVOL_{it}$, which is equal to one if firm *i*'s total real sales growth volatility in year *t* is in the lowest half of the distribution of the volatilities of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise; and $HIGHVOL_{it}$, which is equal to one if firm *i*'s volatility in year *t* is in the highest half of the distribution, and 0 otherwise. We then interact our volatility measure with the two dummies and estimate the following equation, using a fixed-effects specification[26]:

$$TFP_{it} = a_0 + a_1 totalvol_{it} * LOWVOL_{it} + a_2 totalvol_{it} * HIGHVOL_{it} + u_i + u_t + e_i \quad (11)$$

The coefficient a_1 can be interpreted as the effect of volatility on *TFP* for firms with low volatility; and a_2, as the effect for firms with high volatility. The estimates are reported in column

1 of Table 3. We can see that only a_2 is statistically significant. This suggests that volatility only affects the productivity of those firms characterized by a high volatility. This is consistent with our model's second prediction. Column 2 reports the results when firms are divided in three categories based on their volatility. There are three interaction terms: one for low-volatility firms, one for medium-volatility firms, and one for high-volatility firms[27]. Once again, there is a positive association between volatility and *TFP* only for high-volatility firms, which supports our model's second proposition.

INSERT TABLE 3

6.3 *Do more volatile firms have more incentives to start exporting?*

Lastly, our model predicts that it is those firms that display highest volatility of domestic sales growth that should be more willing to start exporting. In order to test this prediction, we estimate the following random-effects Probit equation for the probability that a non-exporter at time *t*-1 becomes an exporter at *t*:

$$START_{it} = a_0 + a_1\, size_{it} + a_2\, age_{it} + a_3\, TFP_{it} + a_4\, group_i + a_5\, domesticvol_{it} + u_j + u_t + e_{it} \qquad (12)$$

The dependent variable, $START_{it}$, is equal to one for those firms that exported at *t*, but not at *t*-1, and 0 otherwise. As in the regression for firm failure, our right-hand side variables include the firm's size, its age, its *TFP*, and the dummy indicating whether it is part of a group. To test our model's prediction, we have added the volatility of the firm's domestic sales growth among our regressors. The results are presented in column 1 of Table 4[28]. We can see that larger firms are more likely to enter export markets, and that the volatility of domestic sales growth is also

[26] Once again, industry dummies are not included in this specification.

[27] In this case the interaction dummies are defined as follows: $LOWVOL_{it}$ is equal to one if firm *i*'s total real sales growth volatility in year *t* is in the lowest third of the distribution of the volatilities of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise; $MIDDLEVOL_{it}$ is equal to one if firm *i*'s volatility in year *t* is in the middle third of the distribution, and 0 otherwise; and $HIGHVOL_{it}$ is equal to one if firm *i*'s volatility in year *t* is in the highest third of the distribution, and 0 otherwise.

positively associated with the probability that the firm starts exporting. Yet, it should be noted that our previous measure of volatility, calculated over a rolling window of five years is based on the firm's domestic sales before and after entry in the foreign market. This could introduce bias in the regression. We therefore verify whether our results are robust to using two different measures of domestic sales volatility: the first is the standard deviation of the firm's real domestic sales growth calculated over the five years preceding and including year *t*. The second one is calculated in a similar way but using all years preceding and including year *t*. The results based on these two alternative measures of volatility are presented in columns 2 and 3 of Table 4. We can see that in both cases, a higher volatility is still positively associated with a higher probability to start exporting. Thus, as predicted by the model, those firms displaying high volatility in their domestic real sales are also more likely to start exporting. In summary, the data lend strong support our model.

INSERT TABLE 4

7. Conclusion

We have constructed a dynamic model of monopolistic competition, aimed at studying the impact of firms' volatility on the degree of financial constraints that they face, and on their probabilities of survival and of entering export markets. Our model predicts that high volatility may prevent some firms from obtaining loans, and that more volatile firms are more likely to go bankrupt, and need to be more productive to stay in the market. In equilibrium three types of firms can coexist in the market: firms with high productivity and low volatility; firms with high productivity and high volatility; and firms with low productivity and low volatility.

Extending our model to the open economy, we have shown that trade can enable exporters to smooth their sales, leading to a decrease in the degree of financial constraints that they face, and in their expected average probability of bankruptcy. More volatile firms are therefore more likely to start exporting. Since trade also increases competition, there can be two

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[28] The sample used in this regression is therefore only made up of firms that did not export at time *t*-1. This

possible equilibria in the open economy. If the rise in competition is relatively weak, there is a reallocation of firms in the market, whereby firms with low productivity and low volatility are substituted by exporters with high productivity and high volatility. However, if the competition effect is strong, it is possible that all firms become more financially constrained than in autarky. In this circumstance, trade does not necessary increase the average productivity in the market, as no reallocation of firms takes place.

We have tested our model's predictions, using a panel of 9292 UK manufacturing firms over the period 1993-2003. In line with the model's predictions, we found empirical evidence showing that more volatile firms are more likely to go bankrupt, need to be more productive to stay in the market, and have more incentives to enter export markets. A further result is that exports tend to stabilize firms' total sales, through market diversification.

Our analysis could be extended by considering other channels through which exporters could ameliorate their financial situation, such as improvements in their reputation; enhancements in the quality of their products through the adoption of international standards; and access not only to domestic, but also to foreign credit markets[29]. Our model could also be extended to incorporate the effects of trade on firms' earnings volatility, through a rise in the exchange rate volatility. This effect is particularly important in developing countries that open up to trade (Chaney, 2005; Razin et al., 2003). More research, both empirical and theoretical, is necessary to fully understand the interactions between volatility, financial constraints, and trade.

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explains the low number of observations reported in this Table.

[29] See, for example, Bridges and Guariglia (2008), for a summary of the literature on the reduced effects of liquidity constraints on the behavior of exporters, relative to purely domestic firms.

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Figure 1: Timing of actions

Beginning of period t End of period t

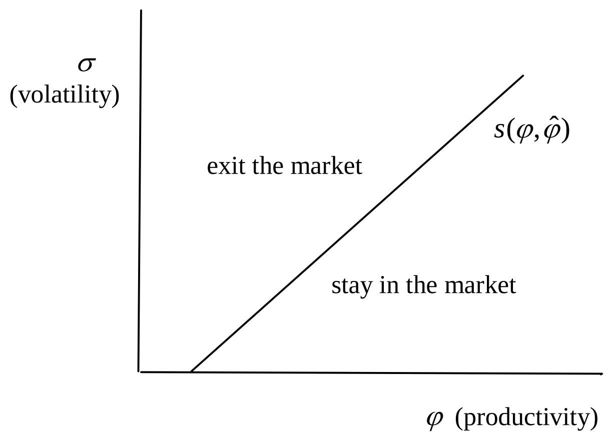
Entry of new firms
(their productivities and
volatilities are drawn)

Firms and banks observe market characteristics → firms with:

- positive expected future value → loans are contracted → firms produce → realization of z → firms that can repay the loan → continue to t+1
- negative expected future value
  ↓ exit

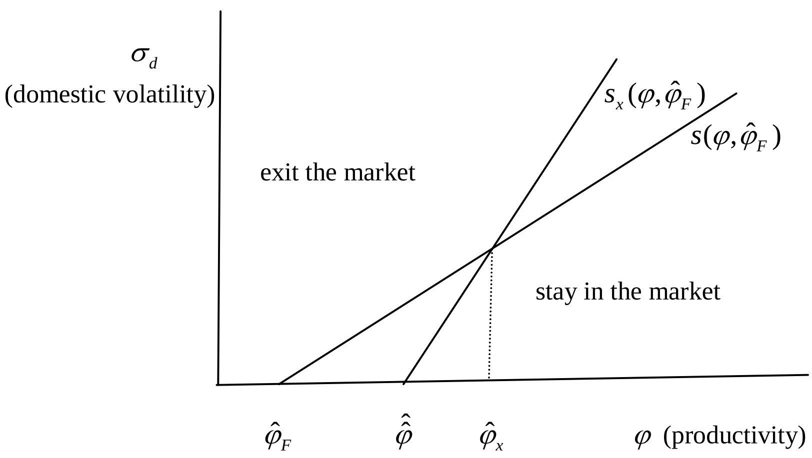
→ firms that cannot repay the loan → are liquidated
↓ exit

FIGURE 2: Exit function in the closed economy



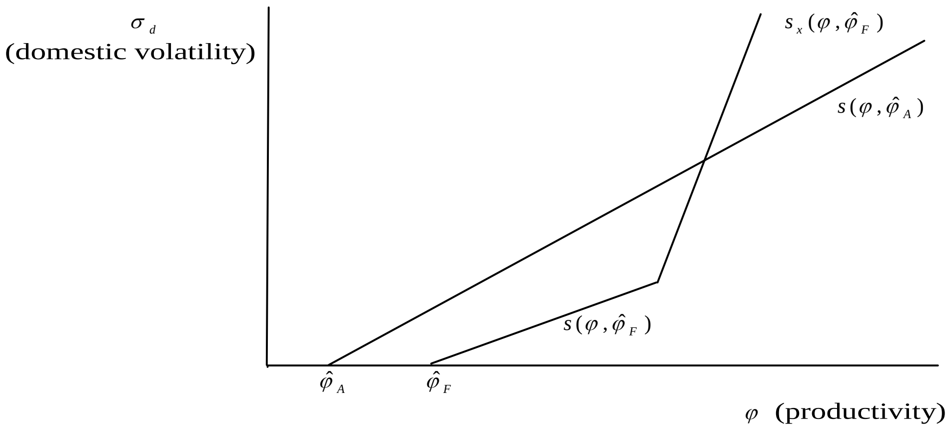
Note: $\hat{\varphi}$ is the productivity threshold in autarky and $s(\varphi,\hat{\varphi})$ is the exit function (firms whose volatility and productivity lie on the exit function are charged an interest rate leading to an expected future value equal to zero).

FIGURE 3: Exit functions in the open economy



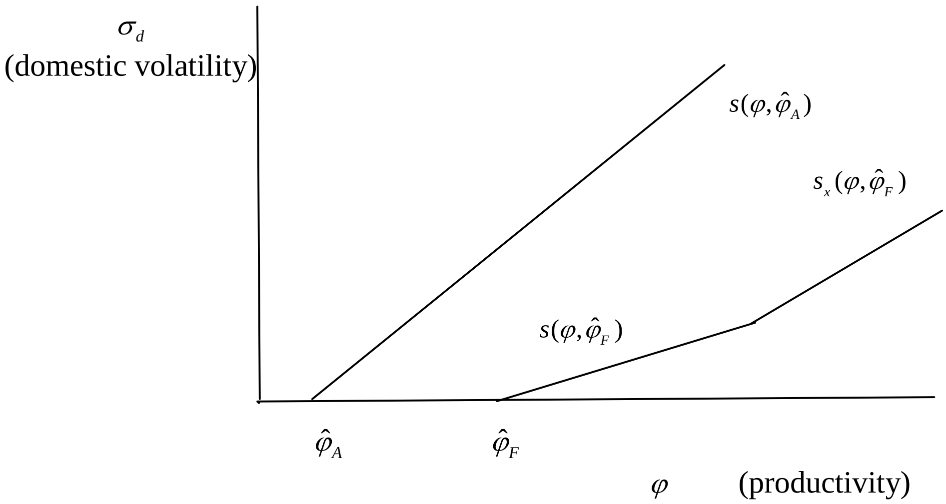
Note 3: $\hat{\varphi}_F$ is the productivity threshold under free trade. Firms with productivity $\varphi \geq \hat{\varphi}_F$ can obtain external funding and remain in the market. Firms with productivity $\varphi \in [\hat{\hat{\varphi}}, \hat{\varphi}_x)$ can obtain external funding to export, but prefer to sell only domestically. Firms with productivity $\varphi \geq \hat{\varphi}_x$ can obtain external funding to export, and decide to sell both domestically and in the foreign market. The function $s(\varphi,\hat{\varphi}_F)$ is the exit function for non-exporters, and $s_x(\varphi,\hat{\varphi}_F)$ is the exit function for exporters.

FIGURE 4a: Effects of opening up to trade: scenario 1



Note: $\hat{\varphi}_A$ is productivity threshold in autarky, $\hat{\varphi}_F$ is the productivity threshold under free trade, $s(\varphi, \hat{\varphi}_A)$ is the exit function in autarky, $s(\varphi, \hat{\varphi}_F)$ is the exit function for non-exporters under free trade, and $s_x(\varphi, \hat{\varphi}_F)$ is the exit function for exporters under free trade.

FIGURE 4b: Effects of opening up to trade: scenario 2



Note: $\hat{\varphi}_A$ is productivity threshold in autarky, $\hat{\varphi}_F$ is the productivity threshold under free trade, $s(\varphi, \hat{\varphi}_A)$ is the exit function in autarky, $s(\varphi, \hat{\varphi}_F)$ is the exit function for non-exporters under free trade, and $s_x(\varphi, \hat{\varphi}_F)$ is the exit function for exporters under free trade.

Table 1: Descriptive statistics

	Total sample (1)	*$FAIL_{it}$ =0* (2)	*$FAIL_{it}$ =1* (3)	*$LOWVOL_{it}$ =1* (4)	*$HIGHVOL_{it}$ =1* (5)	*$START_{it}$ =0* (6)	*$START_{it}$ =1* (7)
$Size_{it}$	8.834	8.881	8.341	9.033	8.972	8.311	8.739
	(1.41)	(1.42)	(1.15)	(1.34)	(1.35)	(1.35)	(1.40)
Age_{it}	27.716	27.956	25.188	33.036	28.105	25.548	23.728
	(24.13)	(24.24)	(22.70)	(24.92)	(23.81)	(22.95)	(21.27)
TFP_{it}	5.650	5.738	4.760	5.826	5.820	5.188	5.489
	(2.44)	(2.48)	(1.84)	(2.40)	(2.40)	(2.27)	(2.51)
$Group_{i}$	0.319	0.340	0.110	0.342	0.331	0.253	0.285
	(0.47)	(0.47)	(0.31)	(0.47)	(0.47)	(0.43)	(0.45)
$Collateral_{it}/Debt_{it}$	1.765	1.781	1.632	1.888	1.587	2.00	1.699
	(3.52)	(3.56)	(3.11)	(3.76)	(3.28)	(4.07)	(2.63)
$Quiscore_{it}$	54.867	55.644	46.720	59.459	54.069	55.138	51.127
	(22.31)	(22.37)	(19.94)	(20.98)	(21.74)	(21.77)	(21.68)
$Totalvol_{it}$	0.197	0.199	0.207	0.092	0.306	0.190	0.238
	(0.19)	(0.19)	(0.17)	(0.04)	(0.22)	(0.21)	(0.31)
$Domesticvol_{it}$	0.238	0.237	0.258	0.131	0.346	0.193	0.309
	(0.27)	(0.27)	(0.30)	(0.17)	(0.31)	(0.23)	(0.42)
$Foreignvol_{it}$	0.482	0.479	0.546	0.370	0.593	0.389	0.771
	(0.50)	(0.50)	(0.53)	(0.42)	(0.54)	(0.31)	(0.84)
Observations	51668	47177	4491	10576	10540	10388	681

Notes: $FAIL_{it}$: dummy variable equal to 1 if firm *i* failed in year *t*, and 0 otherwise. $LOWVOL_{it}$/ $HIGHVOL_{it}$: dummy variable equal to one if firm *i*'s *Totalvol* in year *t* is in the lowest (highest) 50 percent of the distribution of the *Totalvol*s of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise. $START_{it}$: dummy variable equal to 1 if firm *i* exported a positive amount in year *t*, but not in year *t*-1, and 0 otherwise. $Size_{it}$: logarithm of the firm's total real assets. $Collateral_{it}/Debt_{it}$: ratio between the firm's tangible assets and its total (long- and short-term) debt. TFP_{it}: total factor productivity. $Group_{i}$: dummy variable equal to 1 if the firm is part of a group, and 0 otherwise. $Quiscore_{it}$ is a measure of how risky the firm is. The lower its *Quiscore*, the more risky a firm is likely to be. $Totalvol_{it}$: standard deviation of the firm's total real sales growth. The standard deviation is measured over a rolling window of 5 years. $Domesticvol_{it}$: standard deviation of the firm's real domestic sales growth. $Foreignvol_{it}$: standard deviation of the firm's real foreign sales growth.

Table 2: Links between volatility and the probability of bankruptcy

	$FAIL_{it}$ (1)	$QUISCORE_{it}$ (2)
Age_{it}	0.005	0.064
	(1.65)	(0.46)
$Size_{it}$	-0.183	-4.623
	(2.51)*	(7.28)**
$Collateral_{it}/Debt_{it}$	0.001	2.030
	(0.02)	(19.71)**
TFP_{it}	-0.128	3.189
	(3.25)**	(16.30)**
$Group_{i}$	-1.236	
	(8.18)**	
$Totalvol_{it}$	0.762	-4.012
	(2.53)*	(3.21)**
Observations	9610	9934

Notes: t-statistics are reported in parentheses. The estimates in column (1) were obtained using a random-effects Probit specification; those in column (2), using a fixed-effects specification. Time dummies were included in all specifications. Industry dummies were also included in the specification reported in column (1). * denotes significance at 5%; ** denotes significance at 1%. Also see Notes to Table 1.

Table 3: Links between volatility and total factor productivity

	TFP_{it} (1)	TFP_{it} (2)
$Totalvol_{it}*LOWVOL_{it}$	0.028	-0.265
	(0.13)	(0.78)
$Totalvol_{it}*HIGHVOL_{it}/MEDIUMVOL_{it}$	0.222	-0.059
	(3.02)**	(0.33)
$Totalvol_{it}*HIGHVOL_{it}$		0.170
		(2.17)*
Observations	16495	16495

Notes: t-statistics are reported in parentheses. All estimates were obtained using a fixed-effects specification. In column (2), $LOWVOL_{it}$/ $HIGHVOL_{it}$ are dummy variables equal to one if firm *i*'s *Totalvol* in year *t* is in the lowest (highest) 50 percent of the distribution of the *Totalvol*s of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise. In column (3), $LOWVOL_{it}$ is equal to one if firm *i*'s total real sales growth volatility in year *t* is in the lowest 33 percent of the distribution of the volatilities of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise; $MIDDLEVOL_{it}$ is equal to one if firm *i*'s volatility in year *t* is in the middle 33 percent of the distribution, and 0 otherwise; and $HIGHVOL_{it}$ is equal to one if firm *i*'s volatility in year *t* is in the highest 33 percent of the distribution, and 0 otherwise. Time dummies were included in all specifications. * denotes significance at 5%; ** denotes significance at 1%. Also see Notes to Table 1.

Table 4: Links between volatility and the probability to start exporting

	$START_{it}$ (1)	$START_{it}$ (2)	$START_{it}$ (3)
Age_{it}	-0.001	-0.001	-0.001
	(0.54)	(0.38)	(0.34)
$Size_{it}$	0.341	0.272	0.275
	(5.67)**	(5.00)**	(5.06)**
TFP_{it}	-0.072	-0.043	-0.045
	(2.42)*	(1.67)	(1.73)
$Group_{i}$	-0.037	0.047	0.052
	(0.29)	(0.40)	(0.44)
$Domesticvol_{it}$	0.725		
	(4.14)**		
$Predomesticvol_{it}$		0.724	
		(3.60)**	
$Predomesticvol1_{it}$			0.730
			(3.42)**
Observations	3299	3170	3170

Notes: *t*-statistics are reported in parentheses. All estimates were obtained using a random-effects Probit specification. $Predomesticvol_{it}$: standard deviation of the firm's real sales calculated over the five years preceding and including year *t*. $Predomesticvol1_{it}$:standard deviation of the firm's real sales calculated over all years preceding and including year *t*. Time dummies and industry dummies were included in all specifications. * denotes significance at 5%; ** denotes significance at 1%. Also see Notes to Table 1.

APPENDIX 1: Proofs

A. Proof of the existence and uniqueness of an equilibrium interest rate and of the exit function in the closed economy

First, we wish to show the necessary condition for firms to be charged a unique equilibrium interest rate. In equilibrium, the interest rate, the probability of bankruptcy, and the threshold shock are simultaneously determined by the following equations:

$$I(1+r_0) = [1+r(\varphi,\overline{\varphi},\sigma)]\, I\ [1-\delta(\varphi,\overline{\varphi},\sigma)]\,, \tag{A.1}$$

$$\delta(\varphi,\overline{\varphi},\sigma) = \int_0^{\underline{z}(\varphi,\overline{\varphi},\sigma)} f(z,\sigma)\ dz\,, \tag{A.2}$$

$$\underline{z}(\varphi,\overline{\varphi},\sigma)[p(\varphi)y(\varphi,\overline{\varphi})] - y(\varphi,\overline{\varphi})/\varphi - [1+r(\varphi,\overline{\varphi},\sigma)]I = 0\,,\ \text{for}\ \underline{z} < 1\,. \tag{A.3}$$

Substituting the zero profit condition (i.e. $\Pi(\hat{\varphi},\overline{\varphi}) = (\hat{\varphi}/\overline{\varphi})^{\alpha-1} E/(M\alpha) - (1+r_0)I = 0$) in equation (A.3), we obtain the following equation:

$$[\underline{z}(\varphi,\overline{\varphi},\sigma) - \rho](\varphi/\hat{\varphi})^{\alpha-1} I(1+r_0)\alpha - [1+r(\varphi,\overline{\varphi},\sigma)]I = 0\,. \tag{A.4}$$

Using equations (A.1), (A.2), and (A.4), we can determine the threshold shock as follows:

$$\underline{z}^* = [(1-\rho)(\hat{\varphi}/\varphi)^{\alpha-1}] / \int_{\underline{z}^*}^{\infty} f(z,\sigma)dz + \rho\,.$$

Let us define $j_1(\underline{z}) = \underline{z}$, and $j_2(\underline{z},\sigma,\varphi,\hat{\varphi}) = (\hat{\varphi}/\varphi)^{\alpha-1}(1-\rho) / \int_{\underline{z}}^{\infty} f(z,\sigma)dz + \rho$. We wish to show that these two functions intersect at a unique point $\underline{z}^*$. Note that for any $\underline{z}^* \in (0,1]$, there exists a unique probability of bankruptcy and a unique interest rate. Consequently, the conditions needed for the existence of $\underline{z}^* \in (0,1]$ are the same as those that guarantee the existence of a unique equilibrium interest rate. In the interval $[0,\infty)$, j_1 is a monotonically increasing function of $\underline{z}$ from zero to infinity, and j_2 is a monotonically increasing function of $\underline{z}$ from a value less than one to infinity. Note that when $\underline{z} = 0$, the maximum value that j_2 can take occurs when $\varphi = \hat{\varphi}$. In this case, $j_2(0,\sigma,\hat{\varphi},\hat{\varphi}) = 1$. At $\underline{z} = 0$, $j_1(0) > j_2(0,\sigma,\varphi,\hat{\varphi})$. Therefore, by the intermediate value theorem, if $j_1(1) \geq j_2(1,\sigma,\varphi,\hat{\varphi})$, there exists a unique $\underline{z}^* \in (0,1]$ in which both functions intersect.

This is the necessary condition for the existence of a unique threshold shock in the interval $\underline{z}^* \in (0,1]$, and hence for a unique equilibrium interest rate in the model. Q.E.D.

Note that if the minimum productivity, $\hat{\varphi}$, increases, the function j_2 shifts upwards. This implies that the threshold shock, the probability of bankruptcy, and the interest rate charged to the firm will also increase.

Second, we wish to prove the existence and uniqueness of the exit function, which determines the types of firms that can obtain a loan and stay in the market. The condition $j_1(1) \geq j_2(1,\sigma,\varphi,\hat{\varphi})$ can be written as: $(\varphi/\hat{\varphi})^{\alpha-1}\int_1^{\infty} f(z,\sigma)dz - 1 \geq 0$. If this expression holds with equality, i.e. if $(\varphi/\hat{\varphi})^{\alpha-1}\int_1^{\infty} f(z,\sigma)dz - 1 = 0$, we obtain the function representing those combinations of productivity and volatility for a firm that is charged the interest rate leading to zero expected future value. We denote this function by $l(\varphi,\hat{\varphi},\sigma) = 0$. By the implicit function theorem, for any σ, we can write the previous function in the form $s(\varphi,\hat{\varphi}) = \sigma$, for all points where $\int_1^{\infty} (\partial f(z,\sigma)/\partial\sigma)dz \neq 0$. This condition holds for many probability distributions (e.g. the lognormal distribution). It rules out the trivial case in which volatility is irrelevant for equilibrium behavior. The function $s(\varphi,\hat{\varphi}) = \sigma$ represents the exit function expressed in terms of the minimum productivity. Firms whose volatility and productivity lie on this exit function are charged an interest rate leading to zero expected future value, i.e. $\Pi(\varphi,\bar{\varphi},s(\varphi,\hat{\varphi}),1) = 0$.

In the remaining part of the paper, we assume that the slope of the exit function is positive. This slope is given by: $\partial\sigma/\partial\varphi = -(\alpha-1)\int_1^{\infty} f(z,\sigma)dz \Big/ \left[\varphi\int_1^{\infty}(\partial f(z,\sigma)/\partial\sigma)dz\right]$, which is positive if $\int_1^{\infty}(\partial f(z,\sigma)dz/\partial\sigma)dz < 0$. The latter condition holds for a large number of probability distributions, including again the lognormal distribution. Considering that volatility increases the probability of bankruptcy, this condition has a clear implication: in equilibrium, firms with high volatility can obtain a loan and remain in the market only if they are sufficiently productive.

Therefore, in equilibrium, the following three types of firms can coexist in the market: firms with low productivity and low volatility; firms with high productivity and low volatility; and firms with high productivity and high volatility. Q.E.D.

B. Proof of Proposition 1

We wish to show that given two firms (*i* and *j*), which only differ in their volatilities (for example $\sigma_i > \sigma_j$), the more volatile firm has a higher probability of bankruptcy than the less volatile one (i.e. $\delta_i > \delta_j$). To this end, we construct the proof in two parts. We initially show that if the threshold shock $\underline{z}$ is the same for both firms, then the firm with higher volatility has a higher probability of bankruptcy.

The distribution of shocks of the more volatile firm, $f(z,\sigma_i)$, is a Rothschild and Stiglitz´s (1970) mean-preserving increase in spread on the distribution of shocks of the less volatile firm, $f(z,\sigma_j)$. The reason for this is that we have assumed that the distribution of shocks can be completely described by its mean and standard deviation. In this case, the standard deviation unambiguously measures risk as in Bliss (2000). This implies that $f(z,\sigma_j)$ dominates $f(z,\sigma_i)$ according to second order stochastic dominance. Consequently, $\int_0^{\underline{z}} f(z,\sigma_i) - f(z,\sigma_j) dz > 0$. Provided that the threshold shock is the same for both firms, the firm with higher volatility has a higher probability of bankruptcy. This result also holds if the threshold shock is higher for the more volatile firm.

Next, we wish to show that the threshold shock cannot be lower for the more volatile firm. Let us prove this by contradiction. Suppose that the threshold shock is lower for the more volatile firm. This could only happen if $\partial j_2(\underline{z},\sigma,\varphi,\hat{\varphi})/\partial\sigma < 0$, or equivalently if $\int_{\underline{z}}^{\infty} (\partial f(z,\sigma)/\partial\sigma) dz > 0$. Yet, if the distribution of shocks is monotonically increasing with respect to σ, it cannot be the case that the slope of the exit function is positive. Therefore, the threshold shock has to be higher for the more volatile firm than for the less volatile one. Consequently, the former firm has a higher probability of bankruptcy and is charged a higher interest rate than the latter. Q.E.D.

C. Proof of the existence and uniqueness of a stationary equilibrium in the closed economy

From the free entry condition, we know that the average profits can be expressed as follows:

$$\bar{\Pi} = \bar{\delta}(\hat{\varphi}) f_e / P_{in}(\hat{\varphi}) . \tag{A5}$$

In equation (A5), the average probability of bankruptcy is given by $\bar{\delta}(\hat{\varphi}) = \int_{-\infty}^{\underline{\bar{z}}(\hat{\varphi})} f(z, \bar{\sigma}(\hat{\varphi}))dz$, where $\underline{\bar{z}}$ is the average threshold shock, and $\bar{\sigma}$ the average volatility in the market. The ex-ante probability of successful entry is equal to: $P_{in}(\hat{\varphi}) = \text{Prob}(\varphi > \hat{\varphi})\text{Prob}(\sigma \in [0, s(\varphi, \hat{\varphi})]) = \int_{\hat{\varphi}}^{\infty} g(\varphi) \int_0^{s(\varphi,\hat{\varphi})} h(\sigma) d\sigma d\varphi$. We assume that average volatility and average productivity are weighted harmonic means. Therefore, the average volatility is given by $\bar{\sigma}^{\alpha-1} = \int_0^{\infty} \sigma^{\alpha-1} h(\sigma) \int_{\tilde{\varphi}(\sigma,\hat{\varphi})}^{\infty} g(\varphi) d\varphi d\sigma \ / P_{in}(\hat{\varphi})$, where $\tilde{\varphi}$ is the inverse with respect to φ of the exit function $s(\varphi, \hat{\varphi}) = \sigma$, and the average productivity is equal to $\bar{\varphi}^{\alpha-1} = \int_{\hat{\varphi}}^{\infty} \varphi^{\alpha-1} g(\varphi) \int_0^{s(\varphi,\hat{\varphi})} h(\sigma) d\sigma d\varphi / P_{in}(\hat{\varphi})$. Taking into account the zero profit condition, and equation (A1), we can express the average profits as:

$$\bar{\Pi} = \left[\left(\bar{\varphi}(\hat{\varphi}) / \hat{\varphi} \right)^{\alpha-1} - \left(1/(1-\bar{\delta}(\hat{\varphi})) \right) \right] (1+r_0) I \ . \tag{A6}$$

We wish to find a unique $\hat{\varphi}*$ such that equations (A5) and (A6) hold. There exists a unique equilibrium if, as $\hat{\varphi}$ goes from 0 to infinity, equation (A6) decreases monotonically from infinity to zero, and equation (A5) increases from a positive value to infinity. First, as $\hat{\varphi}$ goes to infinity, $\lim_{\hat{\varphi}\to\infty} P_{in}(\hat{\varphi}) \to 0$. Since the probability of bankruptcy is larger than zero, and the average interest rate is bounded, equation (A6) goes to zero and equation (A5) to infinity. Second, as $\hat{\varphi}$ goes to zero, $\lim_{\hat{\varphi}\to 0} P_{in}(\hat{\varphi}) \to 1$, and the average productivity and volatility are constants. Therefore, as $\hat{\varphi}$ goes to zero, equation (A6) goes to infinity and equation (A5) tends to a constant positive value.

Sufficient conditions to ensure that the equilibrium is unique and bounded are that $\left(\overline{\varphi}(\hat{\varphi})/\hat{\varphi}\right)^{\alpha-1}$ is a decreasing function of $\hat{\varphi}$, and that the average probability of bankruptcy is increasing or constant with respect to $\hat{\varphi}$. We know that $\frac{\partial\overline{\delta}(\hat{\varphi})}{\partial\hat{\varphi}} = \frac{\partial\underline{z}(\hat{\varphi})}{\partial\hat{\varphi}} f(\underline{z},\overline{\sigma}) + \int_0^{\underline{z}} \frac{\partial f(z,\overline{\sigma})}{\partial\overline{\sigma}} \frac{\partial\overline{\sigma}(\hat{\varphi})}{\partial\hat{\varphi}} dz$. The derivative $\frac{\partial\underline{z}(\hat{\varphi})}{\partial\hat{\varphi}}$ is increasing if $\left(\overline{\varphi}(\hat{\varphi})/\hat{\varphi}\right)^{\alpha-1}$ is decreasing with respect to $\hat{\varphi}$. Therefore, we only need to determine the conditions under which the average volatility is increasing, constant, or decreases only moderately with respect to $\hat{\varphi}$. Since $\frac{\partial P_{in}(\hat{\varphi})}{\partial\hat{\varphi}} = -g(\hat{\varphi})H(s(\hat{\varphi},\hat{\varphi}) + \int_{\hat{\varphi}}^{\infty} g(\varphi)h(s(\varphi,\hat{\varphi})\frac{\partial s(\varphi,\hat{\varphi})}{\partial\hat{\varphi}}d\varphi < 0$, the derivative $\frac{\partial\overline{\sigma}(\hat{\varphi})}{\partial\hat{\varphi}} = \left[-\int_0^{\infty}\sigma^{\alpha-1}h(\sigma)g(\tilde{\varphi}(\sigma,\hat{\varphi}))\frac{\partial\tilde{\varphi}(\sigma,\hat{\varphi})}{\partial\hat{\varphi}}d\sigma - \frac{\partial P_{in}(\hat{\varphi})}{\partial\hat{\varphi}}\overline{\sigma}^{\alpha-1}\right]/P_{in}(\hat{\varphi})$ is positive or equal to zero if the following condition holds: $\int_0^{\infty}\sigma^{\alpha-1}h(\sigma)g(\tilde{\varphi}(\sigma,\hat{\varphi}))\frac{\partial\tilde{\varphi}(\sigma,\hat{\varphi})}{\partial\hat{\varphi}}d\sigma \le -\frac{\partial P_{in}(\hat{\varphi})}{\partial\hat{\varphi}}\overline{\sigma}^{\alpha-1}$ (condition 1). The economic intuition behind condition 1 is that we wish to avoid situations in which as the minimum productivity increases, the average volatility decreases “too much”, as that could lead to a large decrease of the average probability of bankruptcy, and to an unbounded entry of new firms. The second condition that we need to assume is that as the minimum productivity increases, the average productivity increases relatively less than the increase in the minimum productivity, i.e. $\partial\left(\overline{\varphi}(\hat{\varphi})/\hat{\varphi}\right)^{\alpha-1}/\partial\hat{\varphi} < 0$ (condition 2). The economic intuition behind condition 2 is that we wish to avoid situations in which as the minimum productivity increases, the average productivity in the market is “too high”, as this could again generate an unbounded entry of new firms. Assuming that conditions 1 and 2 hold ensures the existence of an equilibrium. Q.E.D.

D. Proof of the existence and uniqueness of an equilibrium interest rate and of the exit function for exporters

First, we wish to show the necessary condition for exporters to be charged a unique equilibrium interest rate. In equilibrium, the interest rate, the probability of bankruptcy, and the threshold shock are simultaneously determined by the following equations:

$$(I+I_x)(1+r_0)=[1+r_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})]\,(I+I_x)\,[1-\delta_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})], \tag{A.7}$$

$$\delta_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})=\int_0^{\underline{z}_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})} f(z,\sigma_T)\,dz, \tag{A.8}$$

$$\underline{z}_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})[p_d(\varphi)y_d(\varphi,\overline{\varphi}_T)+p_x(\varphi)y_x(\varphi,\overline{\varphi}_T)]-y_d(\varphi,\overline{\varphi}_T)/\varphi-y_x(\varphi,\overline{\varphi}_T)/\varphi$$
$$-[1+r_x(\varphi,\overline{\varphi},\sigma_T,\text{cov})](I+I_x)=0, \qquad \text{for } \underline{z}<1. \tag{A.9}$$

Substituting the zero profit condition under free trade in equation (A.9) and simplifying, we can determine the exporters' threshold shock as follows: $\underline{z}_x{*}=[(1-\rho)(\hat{\varphi}_F/\varphi)^{\alpha-1}(1+I_x/I)]/\int_{\underline{z}_x*}^{\infty} f(z,\sigma_T)dz-\rho(1+\tau^{-\alpha})/(1+\tau^{1-\alpha})$. If $c[(\hat{\varphi}_F/\varphi)^{\alpha-1}]\leq\int_1^{\infty} f(z,\sigma_T)dz$, where $c=\dfrac{(1+I_x/I)(1-\rho)}{1+\rho[(1+\tau^{-\alpha})/(1+\tau^{1-\alpha})]}$, there exists a unique threshold shock in the interval $\underline{z}_x{*}\in(0,1]$, and a unique equilibrium interest rate charged to exporters. Q.E.D.

Second, we wish to show the existence and uniqueness of the exit function for exporters. The function representing the combinations of productivity and volatility for an exporter that is charged the interest rate leading to zero expected future value is: $(\varphi/\hat{\varphi}_F)^{\alpha-1}(1/c)\int_1^{\infty} f(z,\sigma_T(\sigma_d))dz-1=0$. We denote this function by $l_x(\varphi,\hat{\varphi}_F,\sigma_T(\sigma_d))=0$. By the implicit function theorem, we can write the previous function in the form $s_x(\varphi,\hat{\varphi}_f)=\sigma_d$, for all points such that $\int_1^{\infty}\dfrac{\partial f(z,\sigma_T)}{\partial\sigma_T}\dfrac{\partial\sigma_T}{\partial\sigma_d}\,dz\neq 0$. The slope of the exit function for exporters is given by: $\partial\sigma_d/\partial\varphi=-(\alpha-1)\int_1^{\infty} f(z,\sigma_T)dz\Big/\left[\varphi\int_1^{\infty}(\partial f(z,\sigma_T)/\partial\sigma_T)(\sigma_d/\sigma_T)dz\right]$. There will be an equilibrium with exporters and non-exporters, as the one illustrated in figure 3, when the slope of the exit function for exporters is higher than the one for non-exporters. This happens if $(\sigma_d / \sigma_T)\int_1^{\infty} f(z,\sigma_T)dz > \int_1^{\infty} f(z,\sigma)dz$, and if $\hat{\hat{\varphi}} > \hat{\varphi}_F$, where $\hat{\hat{\varphi}}$ is the level of productivity such that $l_x(\hat{\hat{\varphi}}, \hat{\varphi}_F, 0) = 0$. This second condition holds if $I / I_x > \tau^{-\alpha}$ as in Melitz's (2003) model. Q.E.D.

E. Proof of the existence and uniqueness of a stationary equilibrium in the open economy

The free entry condition in the open economy is given by:

$$\overline{\Pi}(\hat{\varphi}_F) = \frac{f_e \ \overline{\delta}(\overline{\varphi}_F(\hat{\varphi}_F), \overline{\sigma}_F(\hat{\varphi}_F))}{P_{in}(\hat{\varphi}_F)}, \quad \text{(A10)}$$

where $P_{in}(\hat{\varphi}_F) = \int_{\hat{\varphi}_F}^{\hat{\varphi}_x(\hat{\varphi}_F)} g(\varphi)H(s(\varphi,\hat{\varphi}_F))d\varphi + \int_{\hat{\varphi}_x(\hat{\varphi}_F)}^{\infty} g(\varphi)H(s_x(\varphi,\hat{\varphi}_F))d\varphi$ is the probability of entry; the variable $\overline{\delta}$ is the average probability of exit under free trade; $\overline{\varphi}_F$ is the average productivity, which is a function of $\hat{\varphi}_F$ (the minimum productivity); $\overline{\sigma}_F$ is the average volatility, which is also a function of $\hat{\varphi}_F$; and $\overline{\Pi}(\hat{\varphi}_F)$ denotes the average profit and is given by:

$$\overline{\Pi}(\hat{\varphi}_F) = \Pi^0(\overline{\varphi}_F, \hat{\varphi}_F) + P_x(\hat{\varphi}_F)\Pi_x^0(\overline{\varphi}_F, \hat{\varphi}_F) -$$

$$-\left[I(1+\overline{r}(\overline{\varphi}_F, \overline{\sigma}_F))(1-P_x(\hat{\varphi}_F)) + (I+I_x)(1+\overline{r}_x(\overline{\varphi}_F, \overline{\sigma}_F, \text{cov}))P_x(\hat{\varphi}_F)\right]. \quad \text{(A11)}$$

In the above equation, $\Pi^0(\overline{\varphi}_F, \hat{\varphi}_F)$ and $\Pi_x^0(\overline{\varphi}_F, \hat{\varphi}_F)$ represent the operating profits in the domestic and foreign markets, respectively; and $P_x(\hat{\varphi}_F) = \frac{1}{P_{in}(\hat{\varphi}_F)} \int_{\hat{\varphi}_x(\hat{\varphi}_F)}^{\infty} g(\varphi)H(s_x(\varphi,\hat{\varphi}_F))d\varphi$ is the probability of exporting, which also represents the percentage of firms in the market that export. The average interest rates of non-exporters and exporters are $\overline{r}$ and $\overline{r}_x$, respectively. We wish to show that, like in the autarky case, as $\hat{\varphi}_F$ goes from 0 to infinity, equation (A10) increases monotonically from a positive value to infinity, and equation (A11) decreases monotonically from infinity to zero.

First, as $\hat{\varphi}_F$ goes to infinity, the probability of entry, and the term $\int_{\hat{\varphi}_F}^{\hat{\varphi}_x(\hat{\varphi}_F)} g(\varphi)H(s_x(\varphi,\hat{\varphi}_F))d\varphi$, all go to zero. Furthermore, the probability of entry is a decreasing function of $\hat{\varphi}_F$. Moreover, as in the autarky case, the average probability of bankruptcy is larger than zero, and the interest rates charged to exporters and non-exporters are bounded. This implies that, as $\hat{\varphi}_F$ goes to infinity, the average interest rates and the probability of bankruptcy are constant. Therefore, equation (A10) goes to infinity and equation (A11) goes to zero. Second, as $\hat{\varphi}_F$ goes to zero, both the probability of entry and the probability of exporting go to one. Additionally, the average volatility tends to a constant, which we denote with $\tilde{\sigma}$. This implies that equation (A10) goes to a constant value and equation (A11) goes to infinity.

As in the autarky case, sufficient conditions for the existence of an equilibrium are that the function $\left(\overline{\varphi}_F(\hat{\varphi}_F)/\hat{\varphi}_F\right)^{\alpha-1}$ is decreasing with respect to $\hat{\varphi}_F$, and that the average volatility is increasing, constant, or decreases only moderately with respect to $\hat{\varphi}_F$ [30]. Q.E.D.

It is important to note that we have focused on situations in which the minimum productivity in autarky is lower than under free trade. We have analysed two possible equilibria. In the first one, free trade leads to a reallocation of firms. In this case, the average profits, given by equation (A11), are higher than in autarky. It follows that in this equilibrium, the minimum productivity threshold is larger than in autarky. In the second case, the average profits of the incumbent firms are lower than in autarky. However, the probability of exit increases, which shifts the free entry condition to the right, increasing again the minimum productivity threshold above the minimum productivity in autarky.

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[30] We do not report the sufficient conditions for brevity. These are available upon request.

Appendix 2: Data

Structure of the unbalanced panel

Number of observations per firm	Number of firms	Percent	Cumulative
1	1306	14.06	14.06
2	918	9.88	23.93
3	870	9.36	33.30
4	825	8.88	42.18
5	752	8.09	50.27
6	703	7.57	57.83
7	650	7.00	64.83
8	757	8.15	72.98
9	1078	11.60	84.58
10	1433	15.42	100.00
Total	9292	100.00	

Definitions of the variables used

$FAIL_{it}$: dummy variable equal to 1 if firm i failed in year t, and 0 otherwise. We define a firm as failed (dead) in a given year if its firm status is in receivership, liquidation, or dissolved.

$Size_{it}$: logarithm of the firm's total real assets. Total assets are given by the sum of fixed (tangible and intangible) assets and current assets, where current assets are defined as the sum of stocks, work-in-progress inventories, trade and other debtors, cash and equivalents, and other current assets.

$Sales_{it}$: includes both UK and foreign turnover.

$Collateral_{it}/Debt_{it}$: ratio between the firm's tangible assets and its total (long- and short-term) debt.

TFP_{it}: total factor productivity calculated using the Levinsohn and Petrin (2003) method.

$Group_i$ dummy variable equal to 1 if the firm is part of a group, and 0 otherwise. A firm is said to be part of a group if it is a subsidiary of one or more holding firms (UK or foreign). Information about whether a firm is part of a group is only provided in the last year of observations available for each firm. We therefore assume that a firm which was part of a group or foreign owned in its last available year was part of a group or foreign owned throughout the period in which it was observed. Given the short sample that we analyze, this is a reasonable assumption.

$Quiscore_{it}$ is given as a number in the range from 0 to 100. The lower its *Quiscore*, the more risky a firm is likely to be. The indicator is constructed taking into account a number of factors, including the presence of any adverse documents appearing against the firm on the public file, and the timeliness of getting the accounts filed. However, the most important factors relate to the financial performance of the firm as evidenced by its balance sheet and profit and loss accounts. The key financial items used include turnover, pre-tax profits, working capital, intangibles, cash and bank deposits, creditors, bank loans and overdrafts, current assets, current liabilities, net assets, fixed assets, share capital, reserves and shareholders funds. The underlying economic conditions are also taken into account.

$START_{it}$: dummy variable equal to 1 if firm *i* exported a positive amount in year *t*, but not in year *t*-1, and 0 otherwise.

$Totalvol_{it}$: standard deviation of the firm's total real sales growth. The standard deviation is measured over a rolling window of 5 years.

$Domesticvol_{it}$: standard deviation of the firm's real domestic sales growth. The standard deviation is measured over a rolling window of 5 years.

$Foreignvol_{it}$: standard deviation of the firm's real foreign sales growth. The standard deviation is measured over a rolling window of 5 years.

$Predomesticvol_{it}$: standard deviation of the firm's real sales calculated over the 5 years preceding and including year *t*.

$Predomesticvol1_{it}$: standard deviation of the firm's real sales calculated over all years preceding and including year *t*.

$LOWVOL_{it}$: dummy variable equal to one if firm *i*'s *Totalvol* in year *t* is in the lowest 50 percent of the distribution of the *Totalvol*s of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise.

HIGHVOL$_{it}$*:* dummy variable equal to one if firm *i*'s *Totalvol* in year *t* is in the highest 50 percent of the distribution of the *Totalvol*s of all firms operating in the same industry as firm *i*'s in year *t*, and 0 otherwise.

Deflators: all variables are deflated using the aggregate GDP deflator.

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