# National labor markets, international factor mobility and macroeconomic instability

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September 2009

#### Abstract

We consider a standard two-country environment, where one of the countries has rigid wages and unemployment, and analyze how factor markets' integration affects the economy with respect to expectationsdriven fluctuations. We demonstrate that by allowing free capital mobility, indeterminacy is exported to the world economy. If further liberalization is permitted, by allowing free movements of labor, the scope for indeterminacy is reduced and open labor markets may produce a stabilizing effect on the global macroeconomy. Whether this also implies higher welfare in the long run depends on differentials in average firm size across countries.

*Keywords:* Indeterminacy, Factor Movements, Globalization, Efficiency Wages

JEL: F15, F20, E24, E32

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## 1 Introduction

The present paper analyses whether, in a world where goods and capital markets are highly integrated, the liberalization of labor movements between countries, subject to different labor market characteristics, may stabilize the economies (with respect to expectation driven fluctuations) and, in addition, raise welfare (with respect to the steady state). Although several studies have investigated the link between capital mobility and macroeconomic performance, the existing macroeconomic literature has not, as yet, properly addressed the implications of international labor movements. Our work aims at filling this gap.

The question of migration, in a world where capital markets are highly integrated, is taking centre stage in the debate about globalization. A peculiarity of the recent wave of globalization is that increased integration in goods and capital markets is accompanied by increased restrictions in labor movements. However, is such an asymmetric process of integration of world markets beneficial or harmful for macroeconomic stability and efficiency? As regards stability, one major concern is that increased financial openness can produce unwanted disturbances to economies in so far as domestic capital becomes more responsive to expected future changes in international prices and, correspondingly, magnifies the amplitude of fluctuations in real wages or employment levels.<sup>1</sup> As regards efficiency, one major concern is that labor movement liberalization, between countries with different labor market institutions, may exacerbate unemployment and reduce world output.<sup>2</sup>

The effect of increased integration of world capital markets on macroeconomic fluctuations depends on the structural characteristics of countries and on the nature of shocks. As suggested by Obstfeld and Taylor (2004), the often unpredictable direction of capital flows in international markets, points towards expectation driven shocks. Accordingly, in the present paper we build a model in which endogenous fluctuations in economic variables are

 $<sup>^1 \</sup>mathrm{See},$  e.g., Azariadis and Pissarides (2007), Bhagwati (1998), Prasad et al. (2003), Rodrik (1997).

 $<sup>^{2}</sup>$ Workers' concern about the impact of globalization is a worldwide phenomenon (see e.g., Edwards and Lusting 1997, ILO 1999 and Scheve and Slaughter 2001).

driven by self-fulfilling changes in expectations, and we interpret macroeconomic instability (stability) as local dynamic indeterminacy of the steady state.<sup>3</sup> Differences in the operation of labor markets, on the other hand, may account for divergences in wage levels and for the existence of migration flows between economies in which product and capital markets are highly integrated.

A well established result, at least in the closed economy literature, is that the existence of market distortions, in otherwise standard dynamic general equilibrium models, can induce indeterminacy (see, for instance, Farmer 1999). In the present paper, we extend this type of analysis to the case of open economies. Specifically, we develop a two-country dynamic overlapping generations model, in which agents may only work when young (with a constant marginal labor disutility), and consume when young and old (under a Cobb Douglas utility function). Each country produces one identical good (hence there is no trade based on comparative advantages) under a private Cobb Douglas technology with labor externalities. Markets for output and capital services are perfectly competitive, and the economies differ only with respect to their labor markets. We assume that one country operates under full employment (with perfectly competitive labor markets), the autarkic equilibrium converging to a determinate steady state; while the other country is characterized by involuntary unemployment (with efficiency wages). The latter, under not particularly stringent parameter restrictions, displays indeterminacy at the autarkic equilibrium and thereby endogenous fluctuations in output, employment, wages and interest rates may emerge. We then analyze how free trade in capital and labor affects the local stability properties of the world economy and the steady state levels of unemployment, wages and output.<sup>4</sup>

<sup>&</sup>lt;sup>3</sup>The occurrence of *local indeterminacy* implies that there is a continuum of deterministic trajectories all converging to the steady state. In this case there are also infinitely many stochastic fluctuations with rational expectations, close to the steady state, driven by self fulfilling volatile expectations (see Guesnerie and Woodford, 1992). For this reason, the occurrence of local indeterminacy is frequently associated to macroeconomic instability.

<sup>&</sup>lt;sup>4</sup>Bertocchi (2003) also considers differential labor market structures within a dynamic general equilibrium set up. However, the focus in her paper is on small open economies and the analysis is concerned with the impact of capital market liberalization and unionization

Our set up allow to focus on how differences in labor markets, *per se*, affect the outcome for the world economy of letting free movements of labor across countries.<sup>5</sup> We first show that opening up the economy to free capital movements enlarges the scope for indeterminacy in the rigid wage country and brings indeterminacy to world markets. Thereby, the country with perfectly competitive labor markets too experiences output fluctuations and, depending on expectations of future interest rates, capital flows reversals across countries can be observed. This result is not entirely surprising and is consistent with earlier works by Lahiri (2001), Sakuragawa and Hamada (2001), Weder (2001) and Meng and Velasco (2003) among others, although these authors disregard distortions in labor markets. From a steady state point of view, allowing for capital mobility does not affect wages, capital, unemployment or output levels.<sup>6</sup>

Allowing for labor mobility alongside capital mobility may eliminate indeterminacy and, therefore, has stabilizing effects at the global macroeconomic level. From a steady state point of view, world economic integration does not affect wage and employment levels nor output in the rigid wage country, while it affects its unemployment level. Whether liberalization of labor movements also implies higher welfare in the long run, however, crucially depends on the gap in employment per firm (that is on average firm size) between the two countries under autarky. In some cases labor mobility helps in achieving both stability and efficiency. In particular, this occurs if, prior to full integration, the level of employment per firm in the fully employed competitive country falls sufficiently short of the corresponding level in the rigid wage country. In this case, under free labor movements,

on cross-country income convergence and distribution.

<sup>&</sup>lt;sup>5</sup>Indeed, a relevant and intensively debated source of difference across countries lies precisely in labor market institutions. See, for instance, Davis (1998), Freeman (1998) and Bertola and Boeri (2002).

<sup>&</sup>lt;sup>6</sup>An advantage of the simple structure we choose to consider - with identical Cobb Douglas preferences and technologies across countries - is that the steady state interest rates are always the same and identical between economies, even at the autarkic equilibrium. As a result, at the steady state, there are no incentives for capital movements across countries, and capital flows are induced by expectation shocks. In contrast, as we shall see, differences in labor markets across countries create, at the steady state, differences in expected labor returns conducive of labor flows across countries.

workers migrate towards the competitive country, unemployment decrease and world output expands. If, instead, the competitive country has larger levels of employment per firm than the rigid wage country, then, the rigid wage country experiences net inflows of workers and higher unemployment, and world output shrinks.

These results provide new insights on the long run welfare effects of workers' migration and on its implications for macroeconomic instability linked to globalization of capital markets. If workers are free to move, workers' movements follow the direction of capital movements, which weakens the conditions under which expectation driven fluctuations may occur; therefore, we should expect less variability in macro-aggregates linked to changes in expectations. In the long run, net migration flows can go both ways, depending on initial differentials in average firm size. Accordingly, economic integration between a rigid and a competitive wage country does not necessarily exacerbate unemployment in the rigid wage country.<sup>7</sup> Although our model is rather stylized, and abstracts from asymmetries across countries that are not linked to labor markets, it can be related to recent labor integration experiments particularly in Europe. For instance, the latest wave of migration from Poland into the UK demonstrates that the liberalization of labor movements, between countries with highly integrated capital and product markets but different labor markets characteristics, can be beneficial for both the destination country and the country of origin.<sup>8</sup>

<sup>&</sup>lt;sup>7</sup>This outcome is in sharp contrast with results obtained in Davis (1998) for a static model. Our steady state results are also of relevance to dual-economy models à la Harris and Todaro (1970). Contrary to Harris and Todaro, we show that liberalizing labor movements, under internationally (or sectorally) mobile capital, may actually reduce unemployment.

<sup>&</sup>lt;sup>8</sup>It should be bared in mind that, as a result of the transition to market economies, Eastern European countries experienced tremendous changes in their labor markets. By the end of the 1990s, however, labor markets in these countries assumed characteristics broadly similar to other EU members, including lack of flexibility when compared to the UK or Ireland (see Riboud et al., 2002).

The UK was the largest of the three countries (the others being Ireland and Sweden) among the EU-15 that did not impose strict restrictions to workers coming from the eight EU Accession countries (Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia). Poland represents by far the largest of the Accession countries and it also had the largest outflow of workers. At the time of the EU enlargement in 2004 the unemployment rate in Poland was almost 20%, while unemployment in the UK was

The remainder of the paper is as follows. In Section 2 we present the structure of the model and obtain the perfect foresight equilibrium for the closed economies. Section 3 focuses on the equilibrium of the world economy under free capital movements, while equilibrium under both capital and labor mobility is analyzed in Section 4. Section 5 concludes.

## 2 Autarky

The world is made up of two countries, A and B, that share the same consumption and production structure.<sup>9</sup> Both countries have perfectly competitive markets for output and capital services, and only differ in the functioning of labor markets. In country A there is involuntary unemployment, while country B operates at full employment. All agents have rational expectations and, in what follows, we study the perfect foresight intertemporal equilibria.

#### 2.1 The model

In each period  $t = 1, ..., \infty$ , a single output, used as consumption or as capital good, is produced and it is taken as the numeraire. Within each country there is a given number  $m_i$ , i = A, B, of identical profit maximizing firms. The production function of a typical firm in country i is given by

$$\mathcal{A}\overline{l}_{i,t}^{\nu}k_{i,t}^{\theta}l_{i,t}^{1-\theta}, \, \mathcal{A} > 0 \text{ and } 0 < \theta < 1 \tag{1}$$

where  $l_{i,t}$  represents the number of units of effective labor employed by each firm,  $k_{i,t}$  is the amount of capital rented by each firm at the rate  $r_{i,t}$  and  $\overline{l}_{i,t}^{\nu}$  represents productive labor externalities:<sup>10</sup>  $\overline{l}_{i,t}$  being the average level

at its natural rate (5%). By 2007, unemployment in Poland halved, while in the UK the employment rate increased.

<sup>&</sup>lt;sup>9</sup>Since we focus on fluctuations driven by volatile expectations, we assume that preferences and technologies are time invariant.

 $<sup>^{10}</sup>$ The existence of social increasing returns to scale, is a feature which is not peculiar to our set up. See, for instance, Barinci and Chéron (2001), Coimbra et al. (2005) and Lloyd Braga et al. (2007). As in Lloyd Braga et al. (2007), we use (country specific) labor externalities to allow for the existence of indeterminacy with a positive interest rate. Labor external effects can be interpreted as coming from thick labor market effects or from knowledge spillovers.

of employment in the country (which is taken as given by each firm) and v being the degree of labor externalities  $(1 + v \text{ measuring the degree of social returns to scale}).<sup>11</sup> Given (1), for a profit maximizing firm <math>k_{i,t}$  must satisfy the following, at a symmetric equilibrium ( $\bar{l}_{i,t} = l_{i,t}$ )

$$\theta \mathcal{A} k_{i,t}^{\theta-1} l_{i,t}^{1-\theta+\nu} = r_{i,t}, \qquad (2)$$

where  $r_{i,t}$  is the rental rate of capital.

Population is constant over time and agents live for two periods. In each period t, there is a continuum of identical young agents, with a constant given mass  $N_i$ , i = A, B, native from each country. Preferences of a typical individual born at t are described by the following lifetime utility function

$$c_{i,t}^{\alpha} c_{i,t+1}^{1-\alpha} - a e_{i,t}, \ 0 < \alpha < 1 \ \text{and} \ a > 0 \tag{3}$$

where  $c_{i,t}$  and  $c_{i,t+1}$  are consumption in the young and old age, respectively, and  $e_{i,t} \in \{0,1\}$  represents the number of units of effective labor (or effort) supplied.

A young employed worker at t (that does not shirk) contributes with one unit of effective labor (i.e.,  $e_{i,t} = 1$ ), receives a positive wage  $w_{i,t}$  and saves  $k_{i,t+1}^h$  in the form of productive capital goods, which are rented to firms in the next period and used for consumption in the old age. We assume that capital is totally depreciated in one period, so that  $r_{i,t}$  is also the interest factor. Accordingly, he/she chooses  $k_{i,t+1}^h$ ,  $c_{i,t}$ ,  $c_{i,t+1}$  to maximize his/her expected lifetime utility subject to the following constraints:  $k_{i,t+1}^h = w_{i,t} - c_{i,t}$  and  $c_{i,t+1} = r_{i,t+1}k_{i,t+1}^h$ . From the first order conditions we obtain

$$k_{i,t+1}^{h} = (1 - \alpha)w_{i,t} \tag{4}$$

$$c_{i,t} = \alpha w_{i,t}$$

$$c_{i,t+1} = r_{i,t+1}(1-\alpha)w_{i,t} ,$$
(5)

<sup>&</sup>lt;sup>11</sup>Note that this technology exhibits constant returns to scale at the individual firm private level. Also, note that, at a symmetric equilibrium (where  $\bar{l}_{i,t} = l_{i,t}$ ), the aggregate marginal productivity of labor curve is a standard downward sloping curve when  $v < \theta$ .

where  $r_{i,t+1}$  is the expected value, evaluated at period t, of the interest rate in the next period, which under perfect foresight is identical to its realized value. Using (3) and (5), the indirect utility of a young employed worker (that does not shirk) is given by

$$V = \alpha^{\alpha} (1 - \alpha)^{1 - \alpha} r_{i,t+1}^{1 - \alpha} \left( w_{i,t} - \bar{w}_{i,t} \right), \text{ where}$$
(6)

$$\bar{w}_{i,t} \equiv \frac{a}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} r_{i,t+1}^{1-\alpha}}.$$
(7)

Expression (7) defines the reservation wage. Note that, if a worker is unemployed, then, V = 0; accordingly, all workers are willing to work and to supply  $e_{i,t} = 1$  for  $w_{i,t} > \bar{w}_{i,t}$ , while for  $w_{i,t} < \bar{w}_{i,t}$  the labor supply is zero.

Consider a perfectly competitive labor market. Then, the labor market supply faced by each identical firm is infinitely elastic at  $w_{i,t} = \bar{w}_{i,t}$ , as long as the employment level does not exceed the mass  $n_i^R$  of young agents (per firm) resident in country *i*. Equilibrium may then lead to full employment or to the existence of some unemployment. The final outcome depends on the marginal productivity of labor, which in turn is affected by the level of capital accumulated through past savings. However, under perfect competition *involuntary* unemployment would not emerge since, if  $l_{i,t} < n_i^R$  wages would be identical to the reservation wage. To account for the existence of involuntary unemployment, we impose some form of labor market rigidity in country A; while for country B we consider full employment and, to simplify, a perfectly competitive labor market. In what follows we describe in detail the labor market equilibrium in each country and obtain the corresponding (general) equilibrium dynamic equations.

## 2.2 Equilibrium with unemployment and efficiency wages: Country A

**2.2.1 The labor market.** For country A, we use a simplified version of the Shapiro-Stiglitz (1984) efficiency wage model, which accounts for the

existence of involuntary unemployment.<sup>12</sup> If a worker employed in a firm is caught shirking (i.e.,  $e_{A,t} = 0$ ) he/she is fired, that is the firm does not pay the wage and he/she is forced to enter the unemployment pool and his/her indirect utility is zero. However, employers can only imperfectly monitor workers. The monitoring technology is not made explicit, we simply assume that the ex-ante probability of shirking and being caught is given by  $0 < \lambda < 1$ . A young agent faces three possibilities: being unemployed, being employed and not shirking or being employed and shirking  $(e_{A,t} = 0,$  $w_{A,t} > 0$ ). Using (3) and (5), the indirect utility of an employed worker that shirks is  $V = (1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}r_{A,t+1}^{1-\alpha}w_{A,t}$ . Then, using (6) it can be easily checked that employed workers have no incentives to shirk when  $w_{A,t} \geq \frac{w_{A,t}}{\lambda}$ . The latter is the non-shirking condition (NSC). Firms choose  $w_A$ ,  $l_A$  and  $k_A$ such that profits are maximized. Since the output of a worker who shirks is zero, at equilibrium the positive wage paid by firms should be such that it induces workers not to shirk, that is each employed worker supply  $e_t = 1$ . Accordingly, the problem solved by the firm is the following

$$\max_{w_{At}, l_{At}, k_{At} \in \Re^{3}_{++}} \left( \mathcal{A}\bar{l}^{v}_{A,t} k^{\theta}_{A,t} l^{1-\theta}_{A,t} - w_{A,t} l_{A,t} - r_{A,t} k_{A,t} \right) , \quad s.t. \ NSC$$

Obviously the NSC is binding: thus from the first order conditions we obtain, at a symmetric equilibrium, expression (2) and

$$(1-\theta) \mathcal{A}k_{A,t}^{\theta} l_{A,t}^{-\theta+\nu} = w_{A,t}$$
(8)

$$\frac{\bar{w}_{A,t}}{\lambda} = w_{A,t} , \qquad (9)$$

where the reservation wage  $\bar{w}_{A,t}$  is defined in (7).

We assume that the level of employment satisfying (8) verifies  $l_{A,t} < n_{A,t}^R$ , so that we obtain a symmetric equilibrium with unemployment.<sup>13</sup> Expression (8) shows that the level of employment is such that the wage is identical

 $<sup>^{12}</sup>$ It should be stressed that, although we focus on efficiency wages as the source of rigidity in country A labor market, analogous results would apply if we consider monopoly unions or search generated unemployment.

<sup>&</sup>lt;sup>13</sup>This amounts to assume that the marginal productivity at full employment is lower than  $\frac{\bar{w}_{A,t}}{\lambda}$ . See also footnote 18, where conditions on the parameters ensure that a steady state with unemployment satisfies this assumption.

to its respective marginal product, as in perfectly competitive markets. Also, given the existence of private constant returns to scale, profits are zero.<sup>14</sup> In contrast to perfect competition, though, wages are set as a mark up over the reservation wage (see 9); thus workers are better off when employed, and unemployment is involuntary in nature.<sup>15</sup>

Using (8)-(9) we may characterize the labor market equilibrium in country A through the following Lemma.

**Lemma 1** . A symmetric equilibrium in the labor market with unemployment in country A is characterized by

$$l_{A,t} = \left(\frac{\bar{w}_{A,t}}{\lambda(1-\theta)\mathcal{A}k_{A,t}^{\theta}}\right)^{\overline{-\theta+v}} < n_{A,t}^{R},$$
  
$$w_{A,t} = (1-\theta)\mathcal{A}k_{A,t}^{\theta}l_{A,t}^{-\theta+v} > \bar{w}_{A,t}, \text{ where } \bar{w}_{A,t} \text{ is given by (7).}$$

As long as international labor movements are not allowed, all young residents in country A are native from A and, therefore, in Lemma 1 we have  $n_{A,t}^R = N_A/m_A \equiv n_A$ . Also,  $\bar{w}_{A,t}$  depends on  $r_{A,t+1}$ , implying that the equilibrium level of wages and employment is influenced by expectations of future interest rates.<sup>16</sup> As we shall see, this opens the door to fluctuations in wages and employment driven by self-fulfilling volatile expectations (endogenous fluctuations).<sup>17</sup>

Endogenous fluctuations may have relevant welfare implications. To see how, consider an equilibrium with volatile expectations and fluctuations

<sup>&</sup>lt;sup>14</sup>Note that, if the respective marginal product of labor was higher than the wage set by the firm (according to 9) we would obtain the full employment equilibrium, and profits would be positive. However, if profits were distributed to the young generations (the workers) it turns out that the general equilibrium dynamic equations would be identical to the case of full employment and perfect competition. In this situation the two countries would be symmetric - both characterized by full employment - and no interesting dynamics would occur.

<sup>&</sup>lt;sup>15</sup>In the limit case of  $\lambda = 1$  unemployment would no longer be involuntary. Moreover, the case  $\lambda = 1$  with unemployment represents simultaneously a monopsony and a perfectly competitive labor market, since the labor supply curve is infinitely elastic for  $l_{A,t} < n_{A,t}^R$ .

<sup>&</sup>lt;sup>16</sup>Note that the unemployment rate in any period t is given by  $\mu_t = 1 - l_t/n_A$ . Hence its value is determined by the value of  $l_t$  and also depends on expectations of future interest rates.

<sup>&</sup>lt;sup>17</sup>It should be stressed that, irrespective of being voluntary or involuntary, it is the existence of unemployment *per se* that creates the possibility of fluctuations in employment. In fact, employment per firm would be fixed at  $n_A$  under full employment.

around a steady state. In a period where the expected future interest rate is low (relative to its steady state level) workers face a higher unemployment rate; whereas, in a period where the expected future interest rate is high the reverse would happen. Since it is likely that some generations benefit while others are harmed by fluctuations, we cannot *a priori* establish whether an equilibrium with fluctuations driven by expectations is welfare improving or not for the economy as a whole; it depends on the social welfare function. If the latter is sufficiently concave in utility of different generations, equilibrium fluctuations may become quite costly from an inter-generational equity point of view. Moreover, since in any period t wages are set as a mark up over the reservation wage, employed workers are better off than unemployed workers, implying that employment fluctuations affect not only inter-generational but also intra-generational equity.

**2.2.2 Equilibrium dynamic system.** At equilibrium the aggregate demand for capital services,  $m_A k_{t+1}$ , must be identical to its aggregate supply,  $k_{t+1}^h m_A l_{A,t}$ . Hence, using (4) we have that  $k_{t+1} = (1 - \alpha) l_t w_t$ . Combining the latter and expression (8), we obtain the following capital accumulation equation for country A

$$k_{A,t+1} = \gamma k_{A,t}^{\theta} l_{A,t}^{1-\theta+\nu}, \qquad (10)$$

where  $\gamma \equiv (1 - \alpha) (1 - \theta) \mathcal{A}$ . Combining (2), (8), (9) and (10), we obtain

$$l_{A,t+1}^{(1-\alpha)(1-\theta+\nu)} = \delta k_{A,t}^{-\theta(1-(1-\alpha)(1-\theta))} l_{A,t}^{(1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu}, \qquad (11)$$

where  $\delta \equiv \frac{a(1-\alpha)(1-\theta)^{1-\alpha}}{\lambda\alpha^{\alpha}\theta^{1-\alpha}\gamma^{1+\theta(1-\alpha)}}$ . Equations (10) and (11) define a two dimensional dynamic model, and characterize the equilibrium in terms of the two state variables  $(k_A, l_A)$ . In period  $t, k_A$  is a predetermined variable whose value is given by past saving. By contrast,  $l_A$  is a non predetermined variable whose value is influenced by expectations of future interest rates via the workers' reservation wage.

**Definition 1** . An intertemporal perfect foresight equilibrium under autarky for country A is a sequence  $(k_{A,t}, l_{A,t}) \in \Re^2_{++}$ ,  $t = 1, ..., \infty$ , such that (10) and (11) are satisfied, where  $0 < \lambda < 1$  is the ex-ante probability of shirking and being caught. **2.2.3 Steady state.** In country A, the interior steady state  $(k_A, l_A) \in \Re^2_{++}$ , verifying the dynamic system (10)-(11), with  $k_A = k_{A,t} = k_{A,t+1} > 0$  and  $l_A = l_{A,t} = l_{A,t+1} > 0$  satisfies

$$k_A = \gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}},\tag{12}$$

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}}.$$
(13)

Equation (13) gives a unique solution for the steady state value  $l_A$ .<sup>18</sup> Substituting the latter into (12) we obtain the steady state value of  $k_A$ .

The steady state values of the wage and interest rate are, accordingly, given by:  $w_A = \left[ (1-\alpha)^{\theta} (1-\theta) \mathcal{A} l_A^{\nu} \right]^{\frac{1}{1-\theta}}$  and  $r_A = \theta/(1-\alpha) (1-\theta)$ . Therefore, this economy has a positive interest rate at the steady state equilibrium, i.e., r > 1, if and only if the propensity to consume when young satisfies the following restriction,  $\alpha > (1-2\theta)/(1-\theta)$ . To ensure this, and other results to follow, from now on we impose the following restrictions on parameter values

$$\frac{1+2\theta}{1+3\theta} > \alpha > Max\left\{\frac{1-2\theta}{1-\theta}, \frac{1}{2}\right\},\tag{14}$$

$$0 < \theta < 1/2 . \tag{15}$$

These restrictions cover most empirically relevant values of  $\theta$  and  $\alpha$ .<sup>19</sup>

2.2.4 Equilibrium dynamics and indeterminacy. Local indeterminacy occurs when the number of stable eigenvalues is higher than the number of predetermined variables. The system (10) and (11) is loglinear and the associated 2x2 Jacobian matrix is provided in Appendix B. Since in our model there is only one predetermined variable,  $k_A$ , indeterminacy arises

<sup>&</sup>lt;sup>18</sup>By Lemma 1, at equilibrium  $l_A < n_A$ . In order for this condition to be fulfilled at the steady state, the parameter *a* has to satisfy the following restriction:  $0 < a < \lambda n_A^{\frac{\nu}{1-\theta}} \theta^{1-\alpha} \alpha^{\alpha} (1-\alpha)^{\frac{\theta}{1-\theta}} \mathcal{A}^{\frac{1}{1-\theta}} (1-\theta)^{\frac{\theta+\alpha(1-\theta)}{1-\theta}}$ . This restriction ensures that, at the steady state defined in (12)-(13), there is unemployment, i.e.  $l_A < n_A$ , and it also ensures that  $l_{A,t} < n_A$  along trajectories sufficiently close to the steady state.

<sup>&</sup>lt;sup>19</sup>Estimates from national accounting for OECD countries are usually in accordance with values of the capital share of output,  $\theta$ , that belong to the 0.25-0.4 range, and to values of the propensity to consume when young,  $\alpha$ , usually higher than 0.5.

when both eigenvalues (in absolute value) are lower than 1. The proposition below dictates parameter conditions under which local indeterminacy occurs.<sup>20</sup>

**Proposition 1**. Assume that (14)-(15) are satisfied, and define  $\underline{v}_{au} \equiv (\alpha(1-\theta) - (1-2\theta))/(1-\alpha)$  and  $\overline{v}_{au} \equiv 2(1-\alpha(1-\theta))/(2\alpha-1)$ . Then, country A exhibits indeterminacy under autarky if and only if  $\underline{v}_{au} < v < \overline{v}_{au}$ .

**Proof.** See Appendix B.  $\blacksquare$ 

Note that assuming  $\alpha > \frac{1-2\theta}{1-\theta}$  implies that  $\underline{v}_{au} > 0$ . Therefore indeterminacy, with a positive interest rate and capital accumulation, requires a minimum degree of labor externalities bounded away from zero.<sup>21</sup> Nevertheless, indeterminacy still occurs with a sufficiently small degree of externalities, consistent with empirical evidence<sup>22</sup> and with a standard negatively sloped (aggregate) labor demand curve, i.e.,  $v < \theta$ . For instance, indeterminacy prevails when  $\alpha = 0.6$ ,  $\theta = 1/3$  and v = 0.18.<sup>23</sup> This means that, under empirically relevant values of the propensity to consume, capital share in output and externalities, there are stochastic endogenous fluctuations, whereby employment and wages fluctuate due to self-fulfilling volatile expectations.

To gain an intuition of why indeterminacy requires a lower bound on externalities, consider the case of an economy that, at period t, is at its steady state for some time and that, for some reason, experiences an increase in the *expected* future interest rate. Then, the reservation wage,  $\bar{w}_{A,t}$ , will decrease (see 7) and (see Lemma 1) the current level of employment per firm,  $l_{A,t}$ , will increase (assuming  $\nu < \theta$ ), leading to an increase in capital accumulation  $k_{A,t+1}$  (driven by the wage bill). This, by itself, would produce a tendency for a decrease in the interest rate at t + 1. However, the increase in  $k_{A,t+1}$  increases the marginal productivity at t + 1, inducing *per se* an

<sup>&</sup>lt;sup>20</sup>Note that the occurrence of indeterminacy is not caused by the existence of efficiency wages, that is  $\lambda$  does not influence the conditions for indeterminacy (see footnote 16).

<sup>&</sup>lt;sup>21</sup>Lloyd-Braga et al. (2007) discuss this property at length. Note also that  $\alpha < \frac{1+2\theta}{1+3\theta}$  ensures  $\bar{\nu}_{au} > y_{au}$ , and that the same restriction on  $\alpha$  applies in Lloyd-Braga et al. (2007) for an infinitely elastic labor supply.

<sup>&</sup>lt;sup>22</sup>The degree of externalities found in empirical works is quite small, usually below 0.3. See, Basu and Fernald (1997) and Burnside (1996).

<sup>&</sup>lt;sup>23</sup>For  $\alpha = 0.6$  and  $\theta = 1/3$  we obtain  $\underline{v}_{au} = 0.17$  and  $\overline{v}_{au} = 6$ .

increase in employment per firm at t+1 (since  $\partial \log l_{t+1}/\partial \log k_{t+1} = \theta/(\theta - v)$ ). Higher employment triggers in turn an increase in the interest rate at t+1. If employment per firm at t+1 rises by a sufficient amount, that is if v is sufficiently high, its positive effect on the realized interest rate at t+1 will off set the negative effect due to the increase in  $k_{A,t+1}$ . As a result, the initial expectation of an increase in future interest rates can be self-fulfilling. Indeterminacy also implies that equilibrium trajectories will eventually return back to the steady state. In our case, we should observe a reversal in the future capital stock, that is the future wage bill should decrease. The latter will only be possible if  $l_{A,t+1}$  does not increase too much. Note, however, that the required increase in  $l_{A,t+1}$  needed for  $r_{A,t+1}$  to rise is lower the higher the labor externality. Accordingly, a necessary condition for indeterminacy to occur is the existence of a lower bound on labor externalities (i.e.,  $\nu > \underline{v}_{au}$ ).

# 2.3 Equilibrium with full employment and a perfectly competitive labor market: Country B

**2.3.1 The labor market**. In country B, we consider a perfectly competitive labor market with full employment.<sup>24</sup> The firm labor demand in each period t is, therefore, determined by the identity between wages and the marginal productivity of labor,  $w_{B,t} = (1 - \theta) \mathcal{A} \bar{l}_{B,t}^v k_{B,t}^\theta l_{B,t}^{-\theta}$ . Full employment is obtained when the corresponding marginal product of labor exceeds the reservation wage. Denoting by  $n_B^R$  the mass of young agents per firm resident in country B, it is straightforward to obtain the following Lemma.

**Lemma 2** . A symmetric equilibrium in the labor market with full employment in country B is characterized by

$$l_{B,t} = n_B^R,$$
  
$$w_{B,t} = (1-\theta) \mathcal{A}k_{B,t}^{\theta} l_{B,t}^{\nu-\theta} \ge \bar{w}_{B,t}, \text{ where } \bar{w}_{B,t} \equiv \frac{a}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}r_{B,t+1}^{1-\alpha}}.$$

 $<sup>^{24}</sup>$ To simplify the exposition and avoid concerns on the distribution of profits (see footnote 14), we characterize the economy with full employment as perfectly competitive. Note, also, that full employment in the perfectly competitive country *B* is needed to ensure the existence of a two country equilibrium when both capital and labor are free to move. Later on, in Section 4, we provide a comprehensive discussion of the issue.

As long as international labor movements are not allowed, all young residents in country B are native from B and, therefore, in Lemma 2 we have  $n_B^R = N_B/m_B \equiv n_B$ .

**2.3.2 The equilibrium dynamic system.** In country *B* the equilibrium dynamics is summarized by a first-order difference equation; namely, the capital accumulation equation,  $k_{B,t+1} = (1 - \alpha)w_{B,t}n_B$ . Using Lemma 2, we derive a one dimensional dynamic equilibrium model in  $k_B$ , i.e.,

$$k_{B,t+1} = \gamma k_{B,t}^{\theta} n_B^{1-\theta+\nu}.$$
(16)

Therefore,

**Definition 2** . An intertemporal perfect foresight equilibrium under autarky for country B is a sequence  $k_{B,t} > 0$ , such that (16) is satisfied.

**2.3.4 Steady state.** The steady state,  $k_B$ , verifying the dynamic equation (16), where  $k_B = k_{B,t} = k_{B,t+1} > 0$ , is given by,

$$k_B = \gamma^{\frac{1}{1-\theta}} n_B^{1-\theta+\nu}.$$
(17)

Using (2) and Lemma 2, the steady state values of the interest rate and wage are, accordingly, given by  $r_B = \theta/(1-\alpha) (1-\theta)$  and  $w_B = \left[(1-\alpha)^{\theta} (1-\theta) \mathcal{A} n_B^{\nu}\right]^{\frac{1}{1-\theta}}$ .<sup>25</sup>

2.3.4 Equilibrium dynamics and (in)determinacy. Since there is full employment and capital is a predetermined variable, there is no indeterminacy at the autarkic equilibrium. Moreover, since the equilibrium dynamics, given by (16), is loglinear and  $\theta < 1$ , all equilibrium trajectories converge to the steady state. The following proposition restates the result.

**Proposition 2**. Under autarky, country B has a (stable) determinate steady state.

<sup>&</sup>lt;sup>25</sup>We assume that  $0 < a < \theta^{1-\alpha} \alpha^{\alpha} (1-\alpha)^{\frac{\theta}{1-\theta}} \mathcal{A}^{\frac{1}{1-\theta}} (1-\theta)^{\frac{\theta+\alpha(1-\theta)}{1-\theta}} n_B^{\frac{\nu}{1-\theta}}$ . This restriction implies that  $w_B > \bar{w}_B$  at the steady state and therefore  $w_{B,t} \ge \bar{w}_{B,t}$  close to the steady state, as required by Lemma 2.

## 3 Free capital mobility

In this section we first study the equilibrium dynamic system for the case of free international capital mobility. We then show that a steady state exists, study the occurrence of local indeterminacy, and derive relevant comparative static results.

#### 3.1 Equilibrium dynamic system

Liberalization of capital movements between both countries implies a no arbitrage condition in the world capital market, that is interest rates must be identical in every period. Hence, the equilibrium world capital stock  $(K_t)$ , available for production in every period t, must be distributed across firms of both countries in a way such that

$$K_t = m_A k_{A,t} + m_B k_{B,t} \tag{18}$$

$$r_t \equiv r_{A,t} = r_{B,t} \,. \tag{19}$$

These equations, together with (2), can be used to obtain the level of capital rented by a representative firm in each country, that is  $k_{A,t}$  and  $k_{B,t}$ , as a function of  $K_t$  and of the level of employment per firm in each country

$$k_{A,t} = (K_t/m_A) \frac{1}{1+z_t}$$
(20)

$$k_{B,t} = (K_t/m_B) \frac{z_t}{1+z_t}$$
(21)

where,

$$z_t = \frac{m_B}{m_A} \left(\frac{l_{B,t}}{l_{A,t}}\right)^{\frac{1-\theta+\nu}{1-\theta}}.$$
(22)

Capital accumulation in the world is driven by the sum of saving (i.e., labor income) in both countries,

$$K_{t+1} = (1 - \alpha) \left( w_{A,t} m_A l_{A,t} + w_{B,t} m_B l_{B,t} \right).$$
(23)

From this equation, substituting the expressions for  $w_A$  and  $w_B$  as given in Lemma 1 and 2, and using (20)-(21), we obtain the following dynamic equation

$$K_{t+1}/m_A = \gamma \left( K_t/m_A \right)^{\theta} l_{A,t}^{1-\theta+\nu} H_t^{1-\theta} , \qquad (24)$$

where,

$$H_t \equiv H(l_{A,t}) = 1 + z_t, \quad z_t \text{ satisfying (22) with } l_{B,t} = n_B. \tag{25}$$

Combining (2), (20), (25), (24) and Lemma 1, we obtain the other dynamic equation governing employment in country A,

$$l_{A,t+1}^{(1-\alpha)(1-\theta+\nu)}H_{t+1}^{(1-\alpha)(1-\theta)} = \delta \left(K_t/m_A\right)^{-\theta(1-(1-\alpha)(1-\theta))} l_{A,t}^{(1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu}H_t^{(1-\alpha)(1-\theta)^2+\theta}.$$
(26)

Equations (24) and (26) define the equilibrium dynamic system written in terms of two variables: K, whose value is determined by the world past savings, and  $l_A$ , whose value is influenced by current expectations of future rental rates.<sup>26</sup>

#### 3.2 Steady state

A steady state,  $(K, l_A)$ , for the system (24) and (26) is a solution of the following system of equations

$$K/m_A = (1+z) \gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}}, \text{ with } z = (m_B/m_A) (n_B/l_A)^{\frac{1-\theta+\nu}{1-\theta}}$$
 (27)

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}}.$$
 (28)

Note that steady state level of employment per firm in country A, as given in (28), is identical to the steady state level of employment per firm under autarky, as given in (13). By use of (28), (27), (20) and (21), it can be checked that the steady state values  $k_A$  and  $k_B$  are the same as in the autarkic equilibrium. Hence, wages and interest rates at the steady state are also the same.

To gain an intuition of why steady state values are unchanged note that, by use of (21), country *B* capital share of world capital, evaluated at the steady state, is given by  $s_B^k \equiv m_B k_B/K = z/(1+z)$ . While, by use of Lemma 1 and 2 and (20)-(22), country *B* saving share of world savings,

<sup>&</sup>lt;sup>26</sup>Note that, although K is predetermined,  $k_A$  and  $k_B$  are non-predetermined variables. The values of  $k_A$  and  $k_B$ , given in (20)-(21) with z as given in (25), are influenced by employment per firm in country A, which depends on the reservation wage and thereby on expectations of future interest rates.

evaluated at the steady state, is given by  $s_B^s \equiv w_B n_B/(w_A l_{A+} w_B n_B) = z/(1+z)$ . Therefore, at the steady state, the amount of capital goods used in production in country B is equal to investment in capital goods through savings ( $s_B \equiv s_B^k = s_B^s$ ); the same applying to country A ( $s_A^k = s_A^s \equiv 1 - s_B$ ). This means that, at the steady state, there are no net exports or imports of capital services between countries, and the values of  $l_A$ ,  $k_A$  and  $k_B$  are the same irrespective of capital mobility. However, as discussed in the following section, indeterminacy may occur and in this case there are stochastic equilibrium trajectories, driven by self-fulfilling volatile expectations, along which net capital flows between the two countries are observed.

#### **3.3** Local Equilibrium dynamics and indeterminacy

The dynamic system (24)-(26) implicitly defines a two dimensional non linear map G, such that  $(K_{t+1}/m_A, l_{A,t+1}) = G (K_t/m_A, l_{A,t})$  around the steady state. We follow the usual procedure of (log)linearizing around the steady state and studying the eigenvalues of the associated 2x2 Jacobian matrix evaluated at the steady state. Details are in Appendix B. The following proposition dictates parameter conditions under which indeterminacy occurs.

**Proposition 3** . Assume that (14)-(15) are satisfied, and define  $\underline{v}_k \equiv \frac{(\alpha(1-\theta)-(1-2\theta))(1-s_B)}{(1-\alpha)+s_B(\alpha-(1-2\theta)/(1-\theta))} \text{ and } \overline{v}_k \equiv \frac{2(1-\alpha(1-\theta))(1-s_B)}{(2\alpha-1)+2s_B(1/(1-\theta)-\alpha)}.$  Then, the

world economy exhibits local indeterminacy under free capital mobility if and only if  $\underline{v}_k < v < \overline{v}_k$ .

**Proof.** See Appendix B. ■

In view of Proposition 3, liberalizing capital movements entails that 'local' shocks to expectations may now affect other countries and render the latter also susceptible to equilibrium fluctuations driven by self-fulfilling volatile expectations. Indeed, an increase in expected future interest rates, for instance, induces an increase (if  $v < \theta$ ) in the current level of employment per firm in country A and, thereby, an increase in its current interest rate,  $r_{A,t}$ , which, becoming higher than  $r_{B,t}$ , induces capital flows from country B to country A, leading to fluctuations in wages and output in country B as well. Hence, the direction of capital flows is entirely determined by expectations and, depending on the latter, capital flows reversals across countries can be observed.

Note, moreover, that the lower bound on externalities, needed for indeterminacy, is now smaller than that required for indeterminacy to prevail in country A under autarky, i.e.,  $\underline{v}_k < \underline{v}_{au}$  since  $0 < s_B < 1$ . For instance, when  $\alpha = 0.6, \ \theta = 1/3 \ \text{and} \ s_B = 1/2 \ \text{indeterminacy prevails when} \ v = 0.09^{27}$ Therefore, under capital mobility it is easier to obtain fluctuations driven by self-fulfilling expectations with small values of  $\nu$  consistent with empirical evidence. To gain intuition, consider the sequence of events following an increase in the expected future interest rate analyzed under autarky. As discussed above, the initial increase in  $l_{A,t}$  will tend to trigger, in period t, inflows of capital from country B into country A; which, will further increase current saving in country A, while it will decrease current saving in country B. Hence, ceteris paribus, a differential between future returns in the two countries would arise, which cannot be sustained under perfect capital mobility. Indeed, the differential in future returns induces a reversal in capital movements from country A to country B, which re-establish the no arbitrage condition in capital markets at t + 1. The capital inflows in country B at t+1, off setting the initial outflow at t, help bringing the equilibrium trajectories back to the steady state, as required when indeterminacy occurs. Therefore, due to these additional effects linked to capital movements, externalities need not to be as high as under autarky.

## 4 Integrated equilibrium

Assuming free capital mobility between countries we now allow for free international labor mobility. Workers can seek employment in either country A or country B, and can only be employed in the country of residence. To simplify, we ignore the travel costs of migration and assume that firms do

<sup>&</sup>lt;sup>27</sup>With  $\alpha = 0.6$ ,  $\theta = 1/3$  and  $s_B = 1/2$  we have that  $\underline{v}_k = 0.08$ , while  $\overline{v}_k = 0.55 > \theta$  (see footnote 23). Remark that  $\overline{v}_k < \overline{v}_{au}$ ; however, this is of little relevance because  $\overline{v}_k = 0.55$  is still well above empirically plausible values of  $\nu$ .

not discriminate workers by their origin. As in previous sections, we focus on (two-country) equilibria characterized by efficiency wages and unemployment in country A, and perfect competition and full employment in country B. Therefore, Lemma 1 and Lemma 2 still apply, where the mass of young resident per firm in each country,  $n_{i,t}^R$ , can now differ from the mass of young native per firm,  $n_{i,t}$ , i = A, B.

#### 4.1 Equilibrium Dynamic System

Equilibrium in the world labor market is attained when, in each period t, the expected utility of working in country A is identical to the expected utility of working in country B. Under free capital movements (19) applies so that, by use of (7), we have  $\bar{w}_{A,t} = \bar{w}_{B,t} = \bar{w}_t \equiv \frac{a}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}r_{t+1}^{1-\alpha}}$ . Hence, in view of (6) the following no arbitrage condition in the world labor market must hold in every period

$$w_{B,t} - \bar{w}_t = \frac{l_{A,t}}{n_{A,t}^R} \left( w_{A,t} - \bar{w}_t \right),$$
(29)

where  $l_{A,t}/n_{A,t}^R$  is the probability of being employed in country A.

Notice that, from Lemma 1, we have  $w_{A,t} > \bar{w}_t$  and  $l_{A,t} < n_{A,t}^R$ . Therefore, a two-country equilibrium in the world labor market implies that  $w_{A,t} > w_{B,t} > \bar{w}_t$ , that is wages do not equalize.<sup>28</sup> Dividing both sides of (29) by  $w_{A,t}$ , and recalling that, by (9),  $\bar{w}_t/w_{A,t} = \lambda$ , the above no arbitrage condition can be re-written as,

$$\frac{w_{B,t}}{w_{A,t}} = \lambda + (1-\lambda) \frac{l_{A,t}}{n_{A,t}^R}.$$
(30)

From (30) it can be seen that, for a given  $l_{A,t}$ , the lower is *net migration* into country A, that is the lower is  $n_{A,t}^R - n_A$ , the lower is the wage gap  $(w_{B,t}/w_{A,t})$ 

<sup>&</sup>lt;sup>28</sup>Note that the no arbitrage condition in world labor market, requiring  $w_{B,t} > \bar{w}_t$ , is not compatible with an equilibrium where unemployment exists in country B. In fact, a two-country equilibrium would not be possible if unemployment prevailed in the perfectly competitive country B. In our model, if unemployment prevailed in country B, wages would be identical to the reservation wage  $(w_{B,t} = \bar{w}_t)$  and the expected utility of a worker living in B would be zero, which is identical to the utility of being unemployed. On the other hand, the expected utility derived by moving to country A would be positive (since  $w_{A,t} > \bar{w}_t$ ); hence, no young agents will be willing to live and work in country B.

moves closer to 1). Combining Lemma 1, Lemma 2 and (20)-(22), equation (30) becomes,

$$n_{B,t}^{R} = l_{A,t} \left( \lambda + (1-\lambda) \frac{l_{A,t}}{n_{A,t}^{R}} \right)^{\frac{1-\theta}{v}}.$$
 (31)

From Lemma 2 recall that  $l_{B,t} = n_{B,t}^R$ , hence  $n_{B,t}^R$  also represents the level of employment per firm in Country *B*. By definition, the world young population *N* satisfies,

$$N \equiv m_A n_A + m_B n_B = m_A n_{A,t}^R + m_B n_{B,t}^R.$$
 (32)

Using (32), expression (31) can be re-written as,

$$n_{B,t}^{R} = C(l_{A,t}, n_{B,t}^{R}) \equiv l_{A,t} \left( \lambda + (1-\lambda) \frac{m_{A} l_{A,t}}{N - m_{B} n_{B,t}^{R}} \right)^{\frac{1-\theta}{v}}.$$
 (33)

Combining Lemma 1, Lemma 2 and (23) - and recalling that the no arbitrage condition in the world capital market implies that (20)-(22) must be satisfied - the dynamic world capital accumulation equation and the dynamic equation for employment per firm in country A are given, respectively, by

$$K_{t+1}/m_A = \gamma \left( K_t/m_A \right)^{\theta} l_{A,t}^{1-\theta+\nu} H_t^{1-\theta} , \qquad (34)$$

$$l_{A,t+1}^{(1-\alpha)(1-\theta+\nu)}H_{t+1}^{(1-\alpha)(1-\theta)} = \delta \left(K_t/m_A\right)^{-\theta(1-(1-\alpha)(1-\theta))} l_{A,t}^{(1-\alpha)(1-\theta)(1-\theta+\nu)+\theta-\nu}H_t^{(1-\alpha)(1-\theta)^2+\theta}$$
(35)

where,

$$H_t \equiv \tilde{H}(l_{A,t}) = 1 + z_t, \ z_t \text{ satisfying (22) with } l_{B,t} = n_{B,t}^R, \ n_{B,t}^R \text{ satisfying (33).}$$
(36)

Note that the only difference between this dynamic system and that with free capital mobility (24-26) lies in the expression for  $z_t$ , as employment per firm in country B is now affected by changes in employment per firm in country A through (33).

#### 4.2 Steady state

The steady state levels of K,  $l_A$  and  $n_B^R$  must satisfy

$$K/m_A = (1+z) \gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}}$$
, with  $z = (m_B/m_A) \left(n_B^R/l_A\right)^{\frac{1-\theta+\nu}{1-\theta}}$  (37)

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}}$$
(38)

$$n_B^R = \left(\lambda + (1-\lambda)\frac{l_A m_A}{N - m_B n_B^R}\right)^{\frac{1-\theta}{\nu}} l_A.$$
(39)

**4.2.1 Existence.** First note that equation (38) gives us the unique solution for the steady state value  $l_A$ . Then given  $l_A$ , and using (32), equation (39) can be re-written as an identity between two functions in  $n_A^R/l_A$ , i.e.,

$$LHS(\frac{n_A^R}{l_A}) \equiv \frac{N}{m_A l_A} - \frac{n_A^R}{l_A} = \frac{m_B}{m_A} \left(\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right)^{\frac{1-\theta}{v}} \equiv RHS(\frac{n_A^R}{l_A}).$$
(40)

A steady state value for  $n_A^R/l_A$  is thus a solution of (40) and, by Lemma 1, it must also satisfy  $n_A^R/l_A > 1$ . Since equation (40) is non linear multiple steady states may exist. In what follows we state necessary and sufficient conditions on the world level of young population, N, for the existence of a unique steady state  $n_A^R/l_A > 1$ .

**Proposition 4**. Assume that  $v < 1 - \theta$  and define  $N_1 \equiv (m_A + m_B) l_A$ , where  $l_A$  satisfies (38). Then, under Lemma 1, a unique steady state  $n_A^R/l_A > 1$  exists if and only if  $N > N_1$ .

**Proof.** See Appendix A.  $\blacksquare$ 

Using the steady state values of  $l_A$  and  $n_A^R/l_A$  we can then determine the corresponding steady state value of  $n_A^R$  and of net migration into country A, i.e.,  $n_A^R - n_A$ . The associated steady state level of employment per firm in country B is, by use of (32),  $n_B^R = N/m_B - (m_A/m_B) n_A^R$ . Finally, given  $l_A$  and  $n_B^R$ , the steady state value for K is obtained through (37).

**4.2.2 Welfare** We now evaluate welfare properties of the steady state, by analyzing how migration flows affect economic activity in both countries. From expression (13), (28) and (38) it can be seen that the level of employment per firm in country A is identical to the level obtained under autarky or free capital mobility. Also by use of (2), (8), (37), (38) and (20), it can be checked that r,  $w_A$  and  $k_A$  remain the same.<sup>29</sup> Using Lemma 2, (21)-(22) and

<sup>&</sup>lt;sup>29</sup>Note that, as in the case of perfect capital mobility, there are no net capital movements at the steady state (hence  $s_B^s = s_B^k = s_B$ ).

(37), and given the steady state value of  $n_B^R$ , we can derive the steady state level of capital per firm in country B,  $k_B = \gamma^{\frac{1}{1-\theta}} n_B^{R\frac{1-\theta+\nu}{1-\theta}}$ , and the steady state level of wages in country B,  $w_B = \left[(1-\alpha)^{\theta}(1-\theta)\mathcal{A}(n_B^R)^{\nu}\right]^{\frac{1}{1-\theta}}$ , which are both increasing in  $n_B^R$ . Hence, using (32), it can be established that the lower is  $n_A^R$ , the lower is net migration in country A, the higher is  $n_B^R$  and thereby  $k_B$  and  $w_B$ , and the smaller the wage gap between the two countries (see 30). Since wages, capital and employment per firm in country A do not vary with respect to migration flows, a steady state with a lower value for  $n_A^R$ is superior welfarewise than one with higher values of  $n_A^R$ , under a utilitarian social welfare function for the world. The following proposition summarizes.

**Proposition 5**. Assume that conditions stated in Proposition 4 hold. Then, the steady state level of net migration into country A is negatively correlated with the wage, capital and employment per firm in country B, and with world welfare (according to a utilitarian social welfare function); and it is positively correlated with the wage gap between the two countries.

We now analyze how unemployment in country A, wages, capital and employment per firm in country B, and world output compare with the corresponding steady state levels realized under autarky or free capital mobility. Note that, although the steady state level of  $l_A$  does not change with respect to the free capital or autarky case, unemployment may increase or decrease according to whether net migration into country A is positive or negative.

Indeed, if there are net migration flows into country A (i.e.,  $n_A^R > n_A$ ) unemployment, measured by  $m_A(n_A^R - l_A)$ , will be higher than the corresponding level under autarky and free capital mobility, measured by  $m_A(n_A - l_A)$ . Since, from (32), the identity  $m_A(n_A^R - n_A) = m_B(n_B - n_B^R)$  must hold, then, positive net migration in country A corresponds to a decrease in the level of employment per firm in country B relative to the cases of free capital mobility and autarky, i.e.,  $n_B^R < n_B$ . As a result  $w_B$  and  $k_B$  will also be lower, and the steady state level of output at the world level also decreases relative to the case of autarky or free capital mobility. Denoting by  $n_B^*$  the level of employment per firm in country B at which the return of working in country B is identical to that of working in country A at the steady state under autarky (and free capital mobility), then the conditions under which there is positive or negative net migration into country A can be summarized as follows.

**Proposition 6**. Assume that the conditions stated in Proposition 4 hold and define  $n_B^* \equiv (\lambda + (1 - \lambda)(l_A/n_A))^{\frac{1-\theta}{v}} l_A$ . Then, compared to the steady state under autarky or free capital mobility, the steady state of the fully integrated economy exhibits:

(i) Positive net migration and higher unemployment in country A, lower wages, capital and employment per firm in country B and lower output in the world, if and only if  $n_B > n_B^*$ .

(ii) Negative net migration and lower unemployment in country A, higher wages, capital and employment per firm in country B and higher output in the world, if and only if  $n_B < n_B^*$ .

**Proof.** See Appendix A.  $\blacksquare$ 

Propositions 5 and 6 (i) imply that eliminating barriers to international factor movements may induce a decrease in world output and welfare.<sup>30</sup> Propositions 5 and 6 (ii) imply that low employment per firm in the perfectly competitive full employment economy relative to the rigid wage country, before integration, induce positive net migration into the competitive economy and creates a more efficient world. Indeed, this is the outcome one would expect in partial equilibrium, or in a static model, if both countries were characterized by perfectly competitive labor markets and full employment and factor movements were liberalized. In this case workers would move to the country offering a higher wage, that is from the labor abundant country to the labor scarce country, until real wages are equalized. As a result there would be a world redistribution of workers to the advantage of the more productive country and an expansion in world output. In our dynamic general equilibrium model, where one of the two countries is characterized

<sup>&</sup>lt;sup>30</sup>Given that, here, we are analyzing economies operating under several market distortions, this result is consistent with the theory of the second best, according to which the correction of one market failure does not necessarily improve welfare.

by rigid wages and unemployment, however, the process is different. Recall that, irrespective of liberalization of factor movements, in our economy we have  $w_i = \left[ (1 - \alpha)^{\theta} (1 - \theta) \mathcal{A} l_i^{\nu} \right], i = A, B$ ; that is wages at the steady state are positively related with employment per firm or, equivalently, with the average firm size.<sup>31</sup> Accordingly if, under autarky or free capital mobility, country B is sufficiently less labor abundant than country A, the wage in country B is much lower than in country A and expected income of working in B is relatively low (see 29). To ensure no arbitrage in the world labor market, then, the wage in country B has to be higher under full integration. As a result, wages and the average firm size (i.e.,  $n_B^R$ ) in the competitive country increase, unemployment in the rigid wage country (i.e.,  $n_A^R - l_A$ ) decreases and world output expands. Moreover, although wages do not equalize, the wage gap between countries is reduced (see 30).

#### 4.3 Local dynamics and (in)determinacy

Compared to the case of free capital mobility, the conditions for indeterminacy now are also function of an additional parameter  $\eta \in (1, +\infty)$  that represents the elasticity of employment per firm in country B with respect to employment per firm in country A, evaluated at the steady state.<sup>32</sup> The proposition below summarizes the conditions under which local indeterminacy prevails.

**Proposition 7**. Assume that the conditions in Proposition 4 hold and define  $\underline{v}_{l} \equiv \frac{(\alpha - (1-2\theta)/(1-\theta))(1-s_{B}(1-\eta))}{(1-\alpha)+(\alpha-(1-2\theta)/(1-\theta))s_{B}(1-\eta)}, \quad \overline{v}_{l} \equiv \frac{2(1-\alpha(1-\theta))(1-s_{B}(1-\eta))}{(2\alpha-1)+2(1/(1-\theta)-\alpha)s_{B}(1-\eta)}, \quad \eta^{1} \equiv 1 + \frac{(1-\alpha)}{(\alpha-(1-2\theta)/(1-\theta))s_{B}} \text{ and } \eta^{2} \equiv 1 + \frac{(2\alpha-1)}{2(1/(1-\theta)-\alpha)s_{B}}. \quad Then, the world economy exhibits local indeterminacy under full integration if and only if one of the following set of conditions hold:$ 

- (a)  $\bar{v}_l > v > \underline{v}_l$  and  $1 < \eta < \eta^2$ , or
- (b)  $v > \underline{v}_l$  and  $\eta^2 < \eta < \eta^1$

<sup>&</sup>lt;sup>31</sup>This happens because in our economy capital, being driven by the wage bill, is increasing in employment per firm. Hence, when the latter increases, the level of capital per firm increases as well, which, by increasing the marginal productivity of labor, leads to an increase in wages at steady state. Note that, in our model, this is true even for very small levels of labor externalities.

 $<sup>^{32} \</sup>mathrm{See}$  Appendix A, Lemma 4.

#### **Proof.** See Appendix B. $\blacksquare$

Since the minimum bound on labor externalities required for indeterminacy is higher than that required under autarky  $(\underline{v}_l > \underline{v}_{au})$ , while the upper bound is lower  $(\bar{v}_l < \bar{v}_{au})$ , the range of values under which indeterminacy occurs is smaller in the integrated equilibrium than in autarky. Accordingly, indeterminacy is more difficult to obtain with free international movements of capital and labor. To explain why, we refer to the same example analyzed previously, where we considered an increase in the expected future interest rate. Earlier we saw that, following an increase in the expected future interest rate at time t,  $l_{A,t}$  increases. Since the elasticity of employment per firm in country B with respect to employment per firm in country A (i.e,  $\eta$ ) is positive, higher  $l_{A,t}$  implies migration into country B and an increase in current employment per firm in country  $B^{33}$ . Moreover, capital now flows out of country A into country B. Indeed, since  $\eta > 1$ ,  $n_{Bt}^R/l_{A,t}$  and  $z_t$ increase with  $l_{A,t}$  (see 22), therefore  $k_{B,t} = K_t \frac{z_t}{1+z_t}$  also increases (see 21). This implies in turn that, for a given predetermined value  $K_t$ , capital flows from country A to country B. As a result, under free labor movements, current savings tend to increase in both countries and so does world capital at t + 1.<sup>34</sup> The latter renders more difficult the occurrence of an increase in  $r_{t+1}$ , as initially expected at time t. Also, the increase in world capital tends to increase labor productivity and employment per firm, which induces a further increase in world savings at t + 1, and thereby of world capital in t+2, reinforcing the initial swerving away from the steady state observed at time t. Hence the reversal of equilibrium trajectories, required for indeterminacy, is now more difficult to obtain. Finally, note that migration flows at t are higher the higher is the responsiveness of employment per firm in country B to changes in employment per firm in country A; therefore, for indeterminacy to occur the elasticity  $\eta$  needs to be bounded from above.

<sup>&</sup>lt;sup>33</sup>Note that, due to the existence of unemployment in country A, migration into country B does not imply changes in employment per firm in country A.

<sup>&</sup>lt;sup>34</sup>It should be stressed that the current capital outflow from country A may partially damp the increase in savings due to the rise in employment  $l_{A,t}$ .

## 5 Conclusions

In this paper we have shown that labor movement liberalization, between economies with integrated capital markets and different labor market characteristics, helps to achieve higher aggregate stability by reducing the scope for expectation driven fluctuations. Our results also suggest that, if the competitive country operating at full employment becomes a net importer of workers, then moving to a fully integrated world economy brings about both macroeconomic stability and efficiency. If instead, as a result of labor movement liberalization, the rigid wage country becomes a net importer of workers, then the world economy faces a trade off between efficiency and stability. Whether the rigid wage country becomes a net importer or exporter of workers depends on how large is the gap in the average firm size between the rigid wage and competitive countries before integration.

# Appendix A: Steady State Integrated Equilibrium - Proofs

**Proof of Proposition 4** (Existence and uniqueness of the steady state). In Figure 1 we plot the functions  $LHS(\frac{n_A^R}{l_A}) \equiv \frac{N}{m_A l_A} - \frac{n_A^R}{l_A}$  and  $RHS(\frac{n_A^R}{l_A}) \equiv \frac{m_B}{m_A} \left[\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right]^{\frac{1-\theta}{v}}$  for  $\frac{n_A^R}{l_A} \in (0,\infty)$ , considering  $l_A$  fixed and given by (38). Assuming that  $v < 1 - \theta$ ,  $RHS(n_A^R/l_A)$  defines a convex negatively sloped function, going to infinity as  $n_A^R/l_A$  tends to 0, and going to  $(m_B/m_A) \lambda^{(1-\theta)/\nu}$  as  $n_A^R/l_A$  tends to  $\infty$ . Its slope is given by

$$\operatorname{RHS}'\left(n_A^R/l_A\right) = -\frac{1-\theta}{v}(1-\lambda)\frac{m_B}{m_A} \left[\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right]^{\frac{1-\theta}{v}-1} \left(\frac{l_A}{n_A^R}\right)^2 < 0.$$
(A1)

Since  $RHS'(n_A^R/l_A) \in (-\infty, 0)$  is a continuous function, there must exist a value  $(n_A^R/l_A)^*$  such that  $RHS'((n_A^R/l_A)^*) \equiv -1$ . See Figure 1 where RHS represents this function.

#### [Figure 1]

The function  $LHS(n_A^R/l_A)$  is represented by a line with a constant slope identical to -1. The higher is N the more outwards the line LHS is. See Figure 1, where three different lines represent the function  $LHS(\frac{n_A^R}{l_A})$  for three different values of N:  $N_2$ ,  $N_1$  and  $N > N_1$ . The value  $N_2$  is such that the line LHS is tangent to the RHS curve. By inspecting Figure 1, we can see that for  $N > N_2$  the line LHS always crosses the RHS twice: at a lower value  $n_A^R/l_A \equiv a^N < (n_A^R/l_A)^*$ , with  $RHSt(a^N) < -1$ , and at a higher value  $n_A^R/l_A \equiv b^N > (n_A^R/l_A)^*$ , with  $RHSt(b^N) > -1$ . However,  $a^N$  and  $b^N$  can only be a steady state for a given N, if they take a value higher than 1. Let  $N_1$  be the value of N such that either  $a^{N_1}$  or  $b^{N_1}$  take the value 1, i.e.,  $(1 + m_B/m_A) m_A l_A \equiv N_1$ . Note that  $a^N$  decreases with Nand  $b^N$  increases with N. Hence, for  $N > N_1$ , only  $b^N$  will be a steady state satisfying  $(n_A^R/l_A)^{ss} > 1$ . For  $N_2 < N < N_1$  there are no steady states if  $b^{N_1} = 1$  (and  $a^{N_1} < 1$ ). Two steady states would exist for  $N_2 < N < N_1$  if  $a^{N_1} = 1$  (and  $b^{N_1} < 1$ ). Finally for  $N < N_2$  there are no steady states.

We now illustrate some other related results, which are important for

proofs of Proposition 6 and Proposition 7. We highlight them in two Lemmas.

**Lemma 3** Under the conditions stated in Proposition 4 and given (A1), the unique steady state satisfies  $0 > RHS' ((n_A^R/l_A)^{ss}) > -1$ .

*Proof.* From observation of Figure 1, notice that when  $N > N_1$ , the unique steady state must satisfy  $(n_A^R/l_A)^{ss} > (n_A^R/l_A)^*$ , where  $(n_A^R/l_A)^*$  is such that  $RHS'((n_A^R/l_A)^*) = -1$ . Given convexity of the  $RHS(n_A^R/l_A)$ , this implies that  $RHS'((n_A^R/l_A)^{ss}) > -1$ .

The other result of relevance is linked to the domain of  $\eta$ , that is the elasticity of employment per firm in country B with respect to employment per firm in country A, in accordance with (33), and evaluated at the steady state.

**Lemma 4** Assume that Lemma 3 holds and define  $\eta \equiv \left[\frac{dn_{B,t}^R}{dl_{A,t}}\frac{l_{A,t}}{n_{B,t}^R}\right]_{n_B^R, l_A}$ , then  $\eta > 1$  at the unique steady state  $\left(n_A^R/l_A\right)^{ss} > 1$ .

Proof. Using expression (33) we obtain  $\eta = \frac{1 + \frac{1-\theta}{\nu}(1-\lambda)\frac{s_P(1-s_B)}{s_B}}{1 - \frac{1-\theta}{\nu}(1-\lambda)\frac{s_P^2(1-s_B)}{s_B}}$ , where  $s_P \equiv n_B^R m_B/n_A^R m_A > 0$  represents the ratio of residents in country B and in country A, evaluated at the steady state under analysis, and  $s_B \equiv z/(1+z) \in (0,1)$  represents, as under perfect capital mobility, country B share of world capital where now  $z \equiv (m_B/m_A) \left(n_B^R/l_A\right)^{\frac{1-\theta+\nu}{1-\theta}} > 0$ . Using (A1) we can rewrite  $\eta = \frac{1-RHS'(n_A^R/l_A)^{ss}}{1+RHS'((n_A^R/l_A)^{ss})}$ . By Lemma 3, we then have that, under Proposition 4,  $\eta > 1$  at the unique steady state  $\left(n_A^R/l_A\right)^{ss}$ .

**Proof of Proposition 6** (Net migration) To prove the result of Proposition 6, we just have to show that under condition (i) there is negative net migration into country B and that under condition (ii) there is positive net migration into country B. Note that, using (32), expression (39) can be rewritten as,

$$n_B^R = \left(\lambda + (1-\lambda)\frac{l_A}{n_A + n_B \frac{m_B}{m_A} - \frac{m_B}{m_A} n_B^R}\right)^{\frac{1-\theta}{v}} l_A.$$
(A2)

From (A2) it can be seen that if  $n_B = n_B^* \equiv \left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$ , then net migration is zero, since  $n_B^R = n_B$ .

Differentiating (A2) with respect to  $n_B^R$  and  $n_B$ , we obtain that  $\frac{\partial n_B^R}{\partial n_B} = \frac{RHS'(n_A^R/l_A)}{1+RHS'(n_A^R/l_A)}$ , where  $RHS'(n_A^R/l_A)$  is given by (A1). Since, by Lemma 3,  $0 > RHS'((n_A^R/l_A)^{ss}) > -1$  at the unique steady state under Proposition 4, we obtain that  $\frac{\partial n_B^R}{\partial n_B} < 0$ . Therefore, if  $n_B$  decreases from  $\left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$  then  $n_B^R$  increases from  $\left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$ , so that  $n_B^R - n_B > 0$ , that is positive net migration into country B. This proves (*ii*) of Proposition 6. A symmetric argument proves (*i*).

# Appendix B: Local Indeterminacy - Proofs

**Proof of Proposition 1** (Autarky). The system (10) and (11) is loglinear and deviations from the steady state,  $dLogK_{A,t+1} = LogK_{A,t+1} - LogK_A$ , and  $dLogl_{A,t+1} = Logl_{A,t+1} - Logl_A$  are determined as follows

$$\begin{bmatrix} dLogk_{A,t+1} \\ dLogl_{A,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \theta & 1-\theta+v \\ \frac{-\theta(1-(1-\alpha)(1-\theta))}{(1-\alpha)(1-\theta+\nu)} & \frac{((1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu)}{(1-\alpha)(1-\theta+\nu)} \end{bmatrix}}_{J} \begin{bmatrix} dLogk_{A,t} \\ dLogl_{A,t} \end{bmatrix}$$

the trace, T, and the determinant, D, of the associated Jacobian matrix, J, being given, respectively, by  $T = \frac{(1-\theta)-\alpha(1-\theta+\nu)}{(1-\alpha)(1-\theta+\nu)}$  and  $D = \frac{\theta(1-\theta)}{(1-\alpha)(1-\theta)(1-\theta+\nu)}$ . The eigenvalues of the 2x2 Jacobian matrix J are the roots of the char-

The eigenvalues of the 2x2 Jacobian matrix J are the roots of the characteristic polynomial  $P(\lambda) \equiv \lambda^2 - T\lambda + D$ . Since there is only one predetermined variable (capital), indeterminacy arises when both eigenvalues (in absolute value) are lower than 1. This case will be obtained when, simultaneously, D > T - 1, D < 1 and D > -T - 1. The condition  $D > T - 1 \Leftrightarrow \nu (1 - \theta) > 0$ , is always verified given that  $\nu > 0$  and  $\theta < 1$ .  $D < 1 \Leftrightarrow (1 - \theta) (\alpha(1 - \theta) - (1 - 2\theta)) < \nu(1 - \theta)(1 - \alpha)$ . Since  $\alpha(1 - \theta) > (1 - 2\theta)$  under (14)-(15), both left and right hand side of the latter inequality are positive. Therefore this inequality is satisfied iff  $v > \frac{(\alpha(1 - \theta) - (1 - 2\theta))}{(1 - \alpha)} \equiv \underline{v}_{au}$ . Finally,  $D > -T - 1 \Leftrightarrow 2(1 - \theta) (1 - \alpha(1 - \theta)) > \nu(1 - \theta) (2\alpha - 1)$ . The left and right hand side of this inequality are both positive, since  $2\alpha - 1 > 0$  under (14). Hence the latter inequality is verified iff  $\nu < \frac{2(1 - \alpha(1 - \theta))}{(2\alpha - 1)} \equiv \bar{\nu}_{au}$ , with  $\bar{\nu}_{au} > v_{au}$  if and only if  $\alpha < \frac{1 + 2\theta}{1 + 3\theta}$ , the latter inequality being satisfied under (14).

**Proof of Proposition 3** (*Capital mobility*). Under perfect capital mobility, the Jacobian matrix, associated with the linearized system (24)-(26) around the steady state, is given by

$$\begin{bmatrix} \theta & (1-\theta+\nu)+(1-\theta)\frac{d\ln H}{d\ln l_A} \\ \frac{-\theta(1-(1-\alpha)(1-\theta))}{(1-\alpha)(1-\theta+\nu)+(1-\alpha)(1-\theta)\frac{d\ln H}{d\ln l_A}} & \frac{((1-\alpha)(1-\theta)(1-\theta+\nu)+(\theta+(1-\alpha)(1-\theta)^2)\frac{d\ln H}{d\ln l_A}}{(1-\alpha)(1-\theta+\nu)+(1-\alpha)(1-\theta)\frac{d\ln H}{d\ln l_A}} \end{bmatrix}$$
(B1)

where, by use of (25),

$$\frac{d\ln H}{d\ln l_A} = -\frac{1-\theta+\nu}{1-\theta}s_B, \text{ with } s_B = z/(1+z) \in (0,1).$$
 (B2)

Using (B2), the trace and determinant of the Jacobian matrix (B1) are, respectively, equal to

$$T = \frac{(1-\theta) - \alpha(1-\theta+\nu)(1-x)(1-\theta) - (1-\theta+\nu)x}{(1-\alpha)(1-\theta)(1-\theta+\nu)(1-x)}$$
(B3)

$$D = \frac{\theta(1-\theta) - \theta(1-\theta+\nu)x}{(1-\alpha)(1-\theta)(1-\theta+\nu)(1-x)},$$
 (B4)

where  $x \equiv s^B \in (0, 1)$ .

Since 1 - x > 0, the denominator of both T and D is positive. The condition  $D > T - 1 \Leftrightarrow \nu(1 - \theta) > 0$ , is always verified given that  $\nu > 0$  and  $\theta < 1$ .  $D < 1 \Leftrightarrow (1 - \theta) (1 - s^B) (\alpha(1 - \theta) - (1 - 2\theta)) < \nu(1 - \alpha) (1 - \theta) + s^B (\alpha (1 - \theta) - (1 - 2\theta))$ . Since  $\alpha(1 - \theta) > (1 - 2\theta)$  under (14)-(15), both left and right hand side of the latter inequality are positive. Therefore, this inequality is satisfied iff  $v > \frac{(\alpha(1 - \theta) - (1 - 2\theta))(1 - s^B)}{(1 - \alpha) + s^B(\alpha - (1 - 2\theta))(1 - s^B)} \equiv \underline{v}_k$ . Finally,  $D > -T - 1 \Leftrightarrow (1 - s^B)(1 - \theta)2(1 - \alpha(1 - \theta)) > \nu((1 - \theta)(2\alpha - 1) + 2s^B(1 - \alpha(1 - \theta)))$ . Both left and right hand sides of this inequality are positive, since  $2\alpha - 1 > 0$  under (14). Hence the latter inequality is verified iff  $\nu < \frac{2(1 - \alpha(1 - \theta))(1 - s^B)}{(2\alpha - 1) + 2s^B(1/(1 - \theta) - \alpha)} \equiv \frac{1}{2} + \frac{1}$ 

 $\bar{\nu}_k$ , with  $\bar{\nu}_k > \underline{v}_k$  if and only if  $\alpha < \frac{1+2\theta}{1+3\theta}$ , the latter inequality being satisfied under (14).

**Proof of Proposition 7** (*Capital and labor mobility*). Since, equations (34)-(35) are analogous to (24) and (26), the matrix given in (B1) still represents the Jacobian matrix, associated with the linearized system (34)-(35). However, using (36), the elasticity  $\frac{d \ln H}{d \ln l_A}$ , as given in (B2), must now be substituted by  $\frac{d \ln \tilde{H}}{d \ln l_A}$ , that is by the elasticity of  $\tilde{H}$  with respect to  $l_A$  evaluated at the steady state. Differentiating (36) and (33) we obtain

$$\frac{d\ln\tilde{H}}{d\ln l_A} = -\frac{1-\theta+\nu}{1-\theta}s^B(1-\eta)$$
(B5)

By Lemma 4,  $\eta > 1$ , hence  $s^B(1 - \eta) < 0$ . Using (B1) and (B5) it can be checked that, under labor mobility, the trace and determinant of the Jacobian matrix are given by (B3) and (B4) with  $x \equiv s^B(1 - \eta) < 0$ .

Since  $x \in (-\infty, \mathbf{0})$ , 1-x > 0 and the denominator of both T and D is positive. Hence,  $D > T-1 \Leftrightarrow \nu(1-\theta) > 0$ , which is always verified. Also  $D < 1 \Leftrightarrow (\alpha - (1-2\theta)/(1-\theta))(1-x) < \nu((1-\alpha) + (\alpha - (1-2\theta)/(1-\theta))x)$ . Since  $x \equiv s^B(1-\eta) < 0$  and  $\alpha(1-\theta) > (1-2\theta)$  under (14)-(15), the left hand side of the latter inequality is always positive. If  $s^B(1-\eta) < -\frac{(1-\alpha)(1-\theta)}{\alpha(1-\theta)-(1-2\theta)} \Leftrightarrow \eta > \eta^1 \equiv 1 + \frac{(1-\alpha)}{(\alpha-(1-2\theta)/(1-\theta))s_B}$ , the right hand side is negative and D > 1. If  $\eta < \eta^1$ , then D < 1 for  $v > \frac{(\alpha-(1-2\theta)/(1-\theta))(1-s^B(1-\eta))}{(1-\alpha)+(\alpha-(1-2\theta)/(1-\theta))s^B(1-\eta)} \equiv v_l$ . Turning to the condition D > -T - 1, we have  $D > -T - 1 \Leftrightarrow 2(1-\alpha(1-\theta))(1-x) > \nu((2\alpha-1)+2x(1/(1-\theta)-\alpha))$ . The left hand side of this inequality is always positive. Since, under  $(14), 2\alpha - 1 > 0$  the right hand side can be negative or positive. If  $s^B(1-\eta) > -\frac{(2\alpha-1)}{2[1/(1-\theta)-\alpha]} \Leftrightarrow \eta < \eta^2 \equiv 1 + \frac{(2\alpha-1)}{2(1/(1-\theta)-\alpha)s_B}$ , the right hand side is positive. Hence, if  $\eta < \eta^2$  and  $\nu < \frac{2(1-\alpha(1-\theta))(1-s^B(1-\eta))}{(2\alpha-1)+2(1/(1-\theta)-\alpha)s^B(1-\eta)} \equiv \bar{\nu}_l$ , then D > -T - 1. Also if  $\eta > \eta^2$ , then D > -T - 1.

Finally note that  $\eta^1 > \eta^2 \Leftrightarrow \alpha < \frac{1+2\theta}{1+3\theta}$ , the latter inequality being verified under (14). Moreover, when  $\alpha < \frac{1+2\theta}{1+3\theta}$  and  $\eta < \eta^2$ , then  $\bar{\nu}_l > \underline{v}_l$ . Accordingly, the conditions for indeterminacy, D > T - 1, D < 1 and D > -T - 1, are simulataneously satisfied if and only if one of the conditions stated in Proposition 7 is verified.  $\blacksquare$ 

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