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Abstract

A number of studies have found a negative relationship between macroeconomic volatility and economic growth. We show this may be explained by a portfolio effect within a finite horizon model, where a safer asset, for example, public debt, is less productive than capital.

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1. Introduction

A number of authors, including Ramey and Ramey (1995), Kneller and Young (2001) and Rafferty (2004), have found a negative correlation between the variance of output and economic growth.[2] An inverse economic relationship may be attributed to the adverse effect of uncertainty on irreversible investment in capital or technology, as shown earlier in papers by Bernanke (1983), Pindyck (1988) and Ramey and Ramey (1991). Other authors, Bean (1990) and Saint-Paul (1993), have posited a positive relationship between volatility and growth, because temporary downturns may be exploited as opportunities to prepare for higher productivity in the future through allowing less time in the present to be spent on normal activity. Rafferty (2004) seems to reconcile both these views by finding that economic growth is correlated negatively with the variance of unexpected output movements, as a measure of uncertainty, but positively with the variance of predictable output changes, which suggests some scope for firms to plan productivity improvements. This present paper continues the former line of enquiry into the uncertainty aspect of macroeconomic volatility but focuses instead on the supply rather than the demand for investment funds.

Reconsidering the saving function will not necessarily be helpful in this regard, because uncertainty of future earnings implies extra, precautionary saving, and thus higher rather than lower rates of economic growth.[3] While the effect of interest uncertainty may be opposite in sign [Sandmo (1970)], equally it may not, because the sign of the relationship depends on the curvature of the utility function [Leland (1968)].[4] Furthermore, specific effects that are theoretically unambiguous may also very small when it comes to measurement.[5] Consequently, we abstract probably ambiguous and insubstantial saving effects in favour of a pure portfolio analysis. Such a model may be obtained by specifying a logarithmic utility function, the absence of future incomes and the presence of interest uncertainty.

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[2] As often noted, this challenges the validity of analyzing business cycles as entirely separate from trends and vice versa. A point that is also generally made is that a successful stabilization policy might then lead to growth dividends.

[3] The well known condition is that the third derivative of the utility function is positive, which appears to embrace all but the quadratic form.

[4] The presence of earned along with unearned income is also germane to the result, and any covariance between the two incomes is also of importance.

[5] With only interest uncertainty, Smith (1996) finds that increasing the relative risk aversion (also, the elasticity of intertemporal substitution inverse) by a unit raises the growth rate by less than a $1/50^{th}$ of a percentage point.

The Diamond (1962) overlapping generations model is also presented, where young households can save by acquiring both capital and public debt. Bonds are considered as perpetuities with a long-term price determined within a separate market rather than by a preset redemption value. This gives rise to an additional portfolio equilibrium condition besides the macroeconomic equilibrium one of the investment-saving equality. The main result of the model is then determined by two key but fairly standard features. First, the holding of public debt constitutes, in Tirole's terminology (1985), a form of "non-productive saving", which crowds-out "productive saving" in the accumulation of capital. Secondly, capital is considered to be the riskier of the two assets, because its return is deemed to be prone to unpredictable macroeconomic shocks. An increase in macroeconomic uncertainty, thus, causes an output-reducing, portfolio switch in favour of the non-productive asset. A third feature that the production function has the property of constant returns in a more general measure of the capital stock - either, through a Romer (1986) learning-by-doing externality, as considered here, or through a complementary process of human capital accumulation, as in Lucas (1988) - then establishes an inverse relationship from macroeconomic volatility to long-run economic growth. The following *Section* presents the model and a final one briefly provides some qualifications.

2. Model

2.1 Household saving and asset demands

A representative young household derives expected utility from consumption in two periods,

$$U_t = \theta \ln c_t^Y + (1-\theta) E\left(\ln c_{t+1}^O\right), \qquad 0 < \theta < 1 \tag{1}$$

where θ is a time-preference factor. The periodic budget constraints are

$$c_t^Y + b_t + k_{t+1} = (1-\tau) w_t \qquad c_{t+1}^O = R_{t+1}^B b_t + R_{t+1}^K k_{t+1} \tag{2}$$

where w_t is the wage and τ is the rate at which it is taxed. Some of first period income is consumed, c_t^Y, while the rest is saved by acquiring public debt, b_t, and deposits, which, through an implicit process of financial intermediation, determine the next period's capital stock, k_{t+1}. Second period consumption, c_{t+1}^O, depends on saving in the first period and on the respective returns on public debt and deposits/capital of R_{t+1}^B and R_{t+1}^K.

The returns to capital are assumed to be uncertain, which is indicative of equity capital, although the model could be extended to include safer corporate bonds without removing the underlying issue of uncertainty.[6]

In order to obtain an open-form solution in the presence of uncertain, we specify a binomial distribution of returns, and simplify further by imposing symmetric outcomes,

$$prob\left(R_{t+1}^{K} = \overline{R}^{K}(1+\sigma)\right) = 1/2 \qquad \overline{R}^{K}(1-\sigma) \leq \overline{R}^{B} \leq \overline{R}^{K}(1+\sigma) \tag{3}$$
$$prob\left(R_{t+1}^{K} = \overline{R}^{K}(1-\sigma)\right) = 1/2$$

There are 50% probabilities of high and low returns to capital, $\overline{R}^{K}(1+\sigma)$ and $\overline{R}^{K}(1-\sigma)$, while the return to public debt, $\overline{R}^{B}$, is certain. The expanded form of the utility function is

$$U = \theta \ln\left((1-\tau)w_t - s_t\right)$$
$$+(1-\theta)\left(\frac{1}{2}\ln\left((1-\lambda)\overline{R}^{K}(1+\sigma)+\lambda\overline{R}^{B}\right)+\frac{1}{2}\ln\left((1-\lambda)\overline{R}^{K}(1-\sigma)+\lambda\overline{R}^{B}\right)+\ln s_t\right)$$

Saving is unaffected by uncertainty, because of the preclusion of future earnings in the household budget and because of the specification of a logarithmic function, where the intertemporal elasticity of substitution and the rate of relative risk-aversion are collapsed into a single parameter valued at unity. It is then determined at certainty-equivalent level as

$$s_t = (1-\theta)(1-\tau)w_t, \tag{4}$$

which implies a separable portfolio-choice. This is derived from the first- and second-order conditions,

$$\frac{\partial U}{\partial \lambda} = -\frac{\beta}{2}\left(\frac{\left(\overline{R}^{K}(1+\sigma)-\overline{R}^{B}\right)}{(1-\lambda)\overline{R}^{K}(1+\sigma)+\lambda\overline{R}^{B}}+\frac{\left(\overline{R}^{K}(1-\sigma)-\overline{R}^{B}\right)}{(1-\lambda)\overline{R}^{K}(1-\sigma)+\lambda\overline{R}^{B}}\right)=0$$

$$\frac{\partial^2 U}{\partial \lambda^2} = -\frac{\beta}{2}\left(\frac{\left(\overline{R}^{K}(1+\sigma)-\overline{R}^{B}\right)^2}{\left((1-\lambda)\overline{R}^{H}(1+\sigma)+\lambda\overline{R}^{B}\right)^2}+\frac{\left(\overline{R}^{K}(1-\sigma)-\overline{R}^{B}\right)^2}{\left((1-\lambda)\overline{R}^{K}(1-\sigma)+\lambda\overline{R}^{B}\right)^2}\right)<0$$

The first-order condition may be restated as

$$\lambda\left((\overline{R}^{K}-\overline{R}^{B})^2-\overline{R}^{K^2}\sigma^2\right)-\overline{R}^{K}(\overline{R}^{K}-\overline{R}^{B})+\overline{R}^{K^2}\sigma^2=0. \tag{5}$$

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[6] Including safer corporate bonds would be beneficial in allowing portfolios to reflect heterogeneous risk-preferences away from the present case of a representative saver.

Thus, the assumption of two-state, symmetric outcomes allows uncertainty to be succinctly represented by a single parameter, σ^2, representing the variance (normalized in terms of the squared mean). [7]

We note that in partial equilibrium, where all asset returns are given, there are very restrictive ranges of values for which the portfolio-balance condition, $0 < \lambda < 1$, may hold; and the smaller is σ^2, the more restrictive these are. In the limit, $\sigma^2 \to 0$, while $R^B \to \overline{R}^K$, the value of λ is indeterminate. However, we find that in general equilibrium, where the return factors, R^K and $\overline{R}^B$, are a part of the overall solution, the portfolio-share, λ, is generally determined even in the certainty case. The demands for capital and public debt are expressed as shares of saving,

$$\begin{aligned} k_{t+1} &= (1-\lambda)s_t \\ b_t &= \lambda s_t \end{aligned}, \tag{6}$$

2.2 Production

There is a unit measure of firms, each indexed z and with a Cobb-Douglas production function,

$$y_t(z) = X_t l_t(z)^{1-\alpha} k_t(z)^{\alpha}, \tag{7}$$

where $l_t(z)$ and $k_t(z)$ are its labor and capital inputs. There is also a Romer (1986) learning-by-investing knowledge externality, causing total factor productivity to depend on the average capital stock, k_t,

$$X_t = A(1+u_t)k_t^{1-\alpha}, \tag{8}$$

In addition, total factor productivity is prone to a common, stochastic productivity shock u_t, $u_t > -1$, with zero mean $E(u_t) = 0$, which is the source of macroeconomic volatility.

Firms choose the level of labour input by maximizing expected profit,

$$E(\pi_t(z)) = Ak_t^{1-\alpha} l_t(z)^{1-\alpha} k_t(z)^{\alpha} - E(R_t^K)k_t(z) - w_t l_t(z)$$

[7] This will also reflect the effects of all higher-order squared moments, including kurtosis.

where $E(R_t^K)$ is the expected cost of capital and where wage w_t is a wage that is certain since in being predetermined before the shock realization on the basis of expected productivity. Labor is thus absolved from the effects of uncertainty, receiving $w_t = (1-\alpha)Ak_t^{1-\alpha} l_t(z)^{-\alpha} k_t(z)^{\alpha}$. After imposing symmetric equilibrium $k_t(z) = k_t$ and the normalization $l_t(z) = 1$, this becomes

$$w_t = (1-\alpha)Ak_t \tag{9}$$

Equations (4), (6) and (9) are combine to give a solution for the economic growth factor,

$$G \equiv k_{t+1}/k_t = (1-\lambda)(1-\theta)(1-\tau)(1-\alpha)A, \tag{10}$$

where $1-\alpha$ is the share of wage income out of the total, $1-\tau$ is the proportion of wage income that workers take home, $1-\theta$ is their savings rate out of disposable income and $1-\lambda$ is the portfolio share of saving that goes into deposits and thence into capital. Economic growth is non-stochastic because the wage, the income base for saving, is certain and because the shock distribution is constant over time. This is turn means the future wage can be known with certainty, $w_{t+1} = Gw_t$, and so is any revenue raised by taxing labor.

2.3 The return on capital

The return on capital, however, is stochastic because it depends on what remains after the wage bill has been paid out of the stochastic realisation of output,

$$R_t^K = A(1+u_t)k_t^{1-\alpha} l_t(z)^{1-\alpha} k_t(z)^{\alpha-1} - w_t l_t(z) k_t^{-1},$$

which reduces to $R_t^K = (\alpha + u_t)A$ under the above assumptions. The owners of the capital stock, the old, bear the full brunt of uncertainty. The outcome is consistent with the specification of the uncertainty faced by households in equation (3), where $u_t = \pm\alpha\sigma$ or that

$$R_t^K = \alpha(1+\sigma_t)A \quad \text{where} \quad \sigma_t = \pm\sigma \tag{11}$$

2.4 The return on public debt

We now determine the return on government bonds. Allowing for a fixed coupon payment z and a variable price bond price, v_t, the return factor is

$$R_{t+1}^B = \left(z + v_{t+1}\right)/v_t \tag{12}$$

The government budget constraint in the absence of other primary expenditure requires the new issues of bonds whenever there is shortfall of tax receipts below debt-servicing expenditures. Designating n_t is the number of bonds extant in period t , new issues, are made in the following period, $n_{t+1} - n_t$, to cover the deficit and at a speed that is inversely related to the current selling price,

$$(n_{t+1} - n_t)v_{t+1} = zn_t - \mathrm{T}_{t+1}, \tag{13}$$

Equations (12) and (13) give $R^B_{t+1} = \left(T_{t+1} + n_{t+1}v_{t+1}\right)/n_t v_t$; and since, by definition, $b_t \equiv n_t v_t$, the return factor is

$$R^B_{t+1} = \left(T_{t+1} + b_{t+1}\right)/b_t \tag{14}$$

We assume that the revenue is raised by only taxing labour income,

$$\mathrm{T}_{t+1} = \tau w_t , \tag{15}$$

where τ is the only exogenous policy instrument. The alternative of endogenous taxes to satisfy the government budget constraint for a given level of debt-servicing is also possible but probably less tenable at very higher levels of public debt where there may be an unwillingness to pay commensurately higher taxes. Equations (13), (14) and (9) then give

$$R^B_{t+1} = \left(\tau(1-\alpha)Ak_{t+1} + b_{t+1}\right)/b_t \tag{16}$$

In a steady-state where λ is constant, so that $k_{t+1}/b_t = (1-\lambda)/\lambda$ and $k_{t+1}/k_t = G$,

$$\overline{R}^B = \tau(1-\alpha)A(1-\lambda)/\lambda + G \tag{17}$$

Thus, the return on public debt is specified to be certain.

2.4 Solution

The model is solved by combining equations (5), (6), (11), (17) and (10) for the asset demands, their returns and economic growth. In the absence of uncertainty, $\varepsilon = 0$, λ is solved as a quadratic with a single economic solution and a degenerate one.[8] Introducing uncertainty generalizes this to a quartic equation, where of the four mathematical solutions the one ranked third is unique in making economic sense by satisfying all three of the following properties: $\lambda \geq 0$, $\forall \tau \geq 0$; $\lambda = 0$, if $\tau = 0$; and $\partial\lambda/\partial\tau > 0$.

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[8] The determinacy of the solution depends on *strategic substitutability*, whereby a general movement into a particular asset reduces its relative rate of return, rather than on *imperfect asset substitutability*.

2.5 Parameterization

We assume that in the absence of taxation, of public debt and of uncertainty, where economic growth is at its highest, it as high as the rate of return on capital (and on public debt), implying the parameter restriction: $(1-\theta)(1-\alpha)=\alpha$. If the capital share takes its stylized value of one-third, $\alpha = 1/3$, then $\theta = 1/2$ and the general quartic solution is

$$(1-\tau^L)^2\lambda^4 + 6\tau^L(1-\tau^L)\lambda^3 + \left(13\tau^2 - 3\tau - 1 - \sigma^2\right)\lambda^2 - \left(12\tau^2 + 3\tau - \sigma^2\right)\lambda + 2\tau + 4\tau^2 = 0 \quad (18)$$

The numerical values for the economic solution for the share of public debt are presented in the following *Table* alongside the implied solutions for economic growth as annualized rates (from 35 year periods).

Table: The wealth share of public debt and the annualized rate of economic growth as functions of the debt-servicing tax rate (τ) and macroeconomic volatility (σ^2)

	$\sigma^2 = 0$	$\sigma^2 = 0.2$	$\sigma^2 = 0.4$	$\sigma^2 = 0.6$
$\tau = 0$	*0,* 3.00%	*0,* 3.00%	0, 3.00%	0, 3.00%
$\tau = 5\%$	*0.255,* 1.98%	*0.320,* 1.72%	*0.394,* 1.39%	0.474, 0.98%
$\tau = 10\%$	*0.333,* 1.51%	*0.386,* 1.27%	*0.444,* 0.98%	0.507, 0.64%
$\tau = 15\%$	*0.386,* 1.10%	*0.431,* 0.88%	*0.481,* 0.62%	*0.534,* 0.31%
$\tau = 20\%$	*0.425,* 0.74%	*0.465,* 0.53%	*0.509,* 0.28%	- -

Moving down the columns, higher labour taxes reduce economic growth both directly through diminishing the amount that young households can save and indirectly through increasing the proportion of that saving going into public debt. Moving along the rows, it is also apparent that macroeconomic uncertainty has an unambiguously negative effect on economic growth, because this raises the demand for public debt for any level of saving. There are no entries in the bottom right-hand corner, because the combination of high taxes for debt-servicing and of increased uncertainty cause the economy to decline or the non-existence of a steady-state.[9]

[9] The case where economic growth becomes negative relates to the concept of a maximum sustainable debt considered by Rankin and Roffia (2003), but it is not equivalent for two reasons. First, Rankin and Roffia's maximum depends on the property of a diminishing marginal product of capital, which does not apply to the

3. Further comments

A simplified portfolio model with overlapping generations has been presented in order to produce an analytic solution to make the point that an inverse relationship between macroeconomic volatility and economic growth may be explained if assets that are more productive are also relative risky. This is analogous to Tobin's (1965) model of money and capital, where inflation instead of risk plays takes center stage.

Various modifications could be made to the model, but the main result is robust so long as the core assumption that the more productive asset is riskier is maintained. One fundamental change would be to introduce separate pro-cyclical taxes on the two assets to possibly reverse their relative uncertainty. A second would be where the acquisition by households of the equity of a non-competitive financial sector is considered to constitute another form of non-productive saving as opposed to their holding of its deposits [Roberts (2009)]. The logic of the present paper then suggests that if risk-averse shareholders - rather than the depositors - take the hits, macroeconomic uncertainty would raise economic growth by rebalancing portfolios towards deposits. A final caveat is that high levels of public debt may also lead to the additional risk of default. However, this is unclear whether this would reverse relative uncertainty, both because default risk is also likely to depend on macroeconomic risk and because public debt and private deposits become jointly vulnerable when they appear as balancing items in the same financial sector accounts. These issues suggest gains from pursuing this analysis further by taking into account a wider array of assets and their patterns of uncertainty.

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present model. Secondly, the Rankin and Roffia analysis is with reference only to a macroeconomic equilibrium condition rather than to a portfolio equilibrium as now considered.

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