Idiosyncratic Uncertainty, Asymmetric Information, and Private Credit

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Abstract

We propose firm-specific (idiosyncratic) uncertainty as a key cross-country determinant of the total credit allocated to the private sector. We show that in the presence of informational asymmetry in the credit market, theory suggests that higher uncertainty lowers the ratio of private credit to output by reducing the former proportionally more than the latter. Output falls because the higher uncertainty enlarges economic distortions and reduces aggregate capital accumulation. Credit falls proportionally more because the higher uncertainty allows firms to earn larger rents and increases their internal funds, while it reduces the overall financing required in the lower output environment. Thus, a country with higher idiosyncratic uncertainty is characterized by a lower credit-to-output ratio. We show that this theoretical prediction is supported by regression analysis.

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1 Introduction

What drives cross-country differences in the amount of credit extended to the private sector? Given that the ratio of private credit to output is a standard indicator of financial development (e.g. Beck et al. (2000), and Beck et al. (2007)), a number of works have considered various determinants of this ratio. For example, the extent to which creditor rights are protected has been analyzed as a determinant (e.g., La Porta et al. (1997) and La Porta et al. (1998)). If, for instance, creditors are paid first out of the proceeds of liquidating bankrupt firms, they would extend more credit than otherwise. Relatedly, the degree of law enforcement has also been studied (e.g., Jappelli et al. (2005), Safavian and Sharma (2007), and Djankov et al. (2008)). Even when laws on creditor rights protection are on books, if their enforcement is time-consuming and costly, creditors may not be encouraged to lend. Further, the role of credit bureaus has been examined (e.g., Jappelli and Pagano (2002) and Pagano and Jappelli (1993)). When information on, say, debtors’ credit history and current indebtedness is shared among creditors, informational problems such as adverse selection problem can be mitigated, thus inducing lenders to offer more credit.

In this paper, we propose firm-specific (idiosyncratic) uncertainty as another key cross-country determinant of private credit. First, we report suggestive evidence on cross-country variations in the degree of uncertainty to show its relevance as a potential determinant. We then show theoretically how in the presence of credit frictions due to informational asymmetry, a country with higher uncertainty may have a lower credit-to-output ratio. Finally, we conduct a regression analysis to show that even after controlling for other cross-country determinants discussed in the literature such as creditor rights, law enforcement, and information sharing, uncertainty is still negatively associated with the ratio, thus providing a support to our theoretical prediction.

To illustrate cross-country variations in the degree of idiosyncratic uncertainty, we use firm-level data from the World Bank’s (2000) World Business Environment Survey (WBES). Utilizing the fact that firms report both estimated past sales growth rates and predicted future rates, we first estimate a change in the sales growth rates for each firm after controlling for firm-specific characteristics and country-level cyclical factors. We then obtain the ‘conditional’ volatility of the sales growth rates at a country level as a proxy of the degree of idiosyncratic uncertainty of the country. Roughly speaking, this procedure follows Castro et al. (2009), who estimate the sector-specific conditional volatility of firms’ sales growth (for the US) as a proxy of the sector-level

\[ \text{The past (future) sales growth rate is the one over the last (next) three years.} \]
idiosyncratic uncertainty.

Next, we show theoretically how the cross-country variations in uncertainty can explain differences in the credit-to-output ratio across countries. The key factor linking uncertainty and the ratio is asymmetric information in the credit market. Specifically, our analysis is based on Carlstrom and Fuerst (1998), a model that incorporates the costly state verification (CSV) setting developed by Townsend (1979). In this setting, firms raise external funds from financial intermediaries to undertake production, but are exposed to idiosyncratic shocks during production, whose realized values are their own private information. Financial intermediaries, however, observe these shocks only with a cost.

When this informational asymmetry is present, higher idiosyncratic uncertainty, reflected in a higher variance of idiosyncratic shocks, lowers the credit-to-output ratio in the long run. This occurs since it reduces credit proportionally more than output. Output falls because higher idiosyncratic uncertainty, increasing economic distortions caused by the credit frictions, lowers aggregate capital accumulation. However, higher uncertainty, increasing firms’ bargaining power, gives them larger economic rents. This in turn allows them to accumulate more capital despite the fall in aggregate capital. Put differently, the higher uncertainty reallocates capital from households (original lenders with informational disadvantage) to firms (borrowers with informational advantage). The resulting rise in firms’ internal funds, coupled with the lower total finance required in the lower output environment, causes a proportionally larger fall in credit than output, thus lowering the credit-to-output ratio. From a cross-country perspective, this implies that a country with higher idiosyncratic uncertainty, other things equal, should be characterized by a lower credit-to-output ratio.

Finally, we test if this theoretical prediction is consistent with the data. Specifically, we base our analysis on Djankov et al. (2007), who study a number of factors including creditor rights, law enforcement and information sharing as cross-country determinants of the credit-to-output ratio. Adding the country-specific conditional volatility of firms’ sales growth (as a proxy of idiosyncratic uncertainty) to their empirical specification, we show that even after controlling for those key determinants, uncertainty is still negatively associated with the credit-to-output ratio. Moreover,

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2Meanwhile, Carlstrom and Fuerst (1998) study how the amplification and propagation of technology shocks are altered by the asymmetric information problem within the business cycles. A large number of other papers also consider how credit frictions amplify and propagate various shocks over the business cycles (e.g., Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke et al. (1999)).
we find that the role of uncertainty may be quantitatively important.

Broadly speaking, this paper is related to works in the ‘financial deepening’ literature, which advocates the close tie between financial and economic developments since Goldsmith (1969). Using the ratio of private credit to output as an indicator of financial development, recent empirical works such as Beck et al. (2000) and Beck et al. (2007) show that countries with higher credit-to-output ratios grow faster and reduce poverty at faster rates. Our paper, showing that countries with higher idiosyncratic uncertainty are associated with lower credit-to-output ratios, suggests the possibility that these countries may experience slower economic development.

Besides, this paper is a part of ongoing research on the role of uncertainty in macroeconomics. While our interest here is the cross-country variations in idiosyncratic uncertainty and its role as a determinant of private credit, a number of recent papers consider its time-varying nature and its role in the business cycles. The key observation there is that uncertainty exhibits a countercyclical pattern in a number of developed countries (e.g., Higson et al. (2004), Bloom et al. (2010) and Bachmann and Bayer (2011)). Motivated by this, several works study how the time-varying uncertainty may cause business-cycle fluctuations.

The rest of the paper is organized as follows. Section 2 presents suggestive evidence on cross-country variations in the degree of idiosyncratic uncertainty. Section 3 introduces the theoretical framework with credit frictions. Section 4 shows how a higher degree of idiosyncratic uncertainty lowers the credit-to-output ratio. Section 5 presents the regression analysis to study uncertainty as a cross-country determinant of the ratio. Section 6 concludes.

2 Cross-country variations in idiosyncratic uncertainty

In this section, we report suggestive evidence on the cross-country variations in the level of firm-specific (idiosyncratic) uncertainty. We do so based on the World Business Environment Survey (WBES), which collects various firm-level data from more than 10,000 firms in 80 countries between

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3 Examples of theoretical works in this literature include Greenwood and Jovanovic (1990), King and Levine (1993), Kiyotaki and Moore (2005), and Greenwood et al. (2010).

4 For instance, these authors show the counter-cyclicality for the UK, US and Germany, respectively.

5 While some works such as Bloom et al. (2010) and Bachmann and Bayer (2011) analyze this by focusing on non-convex adjustment costs as a propagation mechanism (earlier contributions include Bernanke (1983) and Dixit and Pindyck (1994)), others such as Arellano et al. (2010), Christiano et al. (2010), and Gilchrist et al. (2010) focus on credit frictions (earlier contributions include Williamson (1987)).
Our empirical methodology is roughly based on Castro et al. (2009), who estimate the sector-specific ‘conditional’ volatility of firms’ sales growth for the US as a proxy of the sector-level idiosyncratic uncertainty; it is conditional since the volatility is obtained after controlling for both observable and unobservable firm characteristics and time-varying sectoral factors. Similarly, we calculate the country-specific conditional volatility of sales growth as a proxy of the country-level idiosyncratic uncertainty. To do this, we utilize the fact that the WBES asks firms to estimate their sales growth in real terms over the past three years and also to predict it in real terms over the next three years.\(^7\)

We assume that the log of sales growth rate of firm \(i\) in country \(j\) from year \(t-3\) to \(t\) (i.e., from 1996 (1997) to 1999 (2000)), \(g_{i,j,t}\) is given as:

\[
g_{i,j,t} = \alpha_i + \delta_{j,t-3} + \beta_j \ln(size_{i,j,t-3}) + \epsilon_{i,j,t},
\]

where \(\alpha_i\) is the firm-specific fixed effects, capturing the firm-specific unobserved characteristics affecting its sales growth and \(\delta_{j,t-3}\) is the country-specific fixed effects, controlling for changes in the sales growth (between year \(t-3\) and \(t\)) caused by country-level factors including business cycle fluctuations. Based on the empirical evidence that firms’ growth declines with a firm size (e.g. Evans (1987) and Hall (1987)), the log of the number of workers is included as an observable variable with its coefficient varying across countries. The residual, \(\epsilon_{i,j,t}\), is thus the component of firm’s growth rate between periods \(t-3\) and \(t\) which is not accounted for by those factors. We also assume that the predicted sales growth of the firm from year \(t\) to \(t+3\) (i.e., from 1999 (2000) to 2002 (2003)), \(g_{i,j,t+3}\) is expressed similarly with the same firm-specific fixed effects and the same coefficient on a firm size as Eq.1:

\[
g_{i,j,t+3} = \alpha_i + \delta_{j,t} + \beta_j \ln(size_{i,j,t}) + \epsilon_{i,j,t+3},
\]

Next, we take a difference of Eqs.1 and 2 to obtain the predicted change in firms’ growth rates, \(\Delta g_{i,j,t+3} (\equiv g_{i,j,t+3} - g_{i,j,t})\):

\[
\Delta g_{i,j,t+3} = \Delta \delta_{j,t} + \beta_j \Delta \ln(size_{i,j,t}) + u_{i,j,t+3},
\]

\(^6\)Data from this survey were previously used by, for example, Djankov et al. (2003) and Acemoglu and Johnson (2005).

\(^7\)Since Castro et al. (2009) use US firm-level panel data from COMPUSTAT, they estimate the conditional volatility using the actual sales growth rates.
where ∆δ_{j,t} and ∆ln(size)_{i,j,t} represent changes in those variables. Since firms report changes in the number of their workers over the previous three years, we can estimate the residual, u_{i,j,t+3}(≡ ϵ_{i,j,t+3} − ϵ_{i,j,t}), which is an idiosyncratic component of the expected change in sales growth. We then assume that when u_{i,j,t+3} exhibits high variance at a country level, it implies that the country is subject to a high degree of idiosyncratic uncertainty.

Then, to estimate the country-level variance of u_{i,j,t+3}, σ_j^2, we adopt the following functional form: σ_j^2 = σ^2 exp(θ_j). The log of σ_j^2 is thus estimated as the coefficients on the dummy variables in the following equation:

\[ \ln \hat{u}_{i,j}^2 = \theta_j + v_{i,j}, \] (4)

where \( \hat{u}_{i,j} \) is the estimated residual from Eq.3. Subsequently, the standard deviation of u_{i,j,t+3} is estimated as \( \sqrt{\exp(\hat{\theta}_j)} \). In what follows, we use this measure of conditional volatility of firms’ sales growth as a proxy of the country-level idiosyncratic uncertainty.

After selecting only firms which actually report both previous and coming years’ sales growth rates (i.e., g_{i,j,t} and g_{i,j,t+3}) and also report a change in the firm size, ∆ln(size)_{i,j,t}, we are left with 6286 firms from 74 countries (thus giving an average of 85 firms per country). 9,10 Figure 1 displays the cross-country variations in idiosyncratic uncertainty, proxied by the country-level conditional volatility of firms’ sales growth rates. While countries such as Egypt, Germany, Portugal, Italy, and US exhibit least idiosyncratic uncertainty (represented by the leftmost 5 bars) at the time of the survey, countries such as Estonia, Botswana, Turkey, Ukraine, and Uzbekistan show largest uncertainty (represented by the rightmost 5 bars).11

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8 Castro et al. (2009) use the same specification to calculate the sector-specific volatility.
9 Since our focus is credit extended to private firms, we first exclude government-owned firms from the original WBES sample. The number of countries is less than the number covered in the original data (80), because we dropped a few countries for which only a small number of firms are left. We chose the cut-off value of 30, but the choice of this value does not affect our main results below.
10 Those 74 countries are: Albania, Argentina, Armenia, Azerbaijan, Belarus, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Bulgaria, Cameroon, Canada, Chile, China, Colombia, Costa Rica, Cote d’Ivoire, Croatia, Czech Republic, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, France, Georgia, Germany, Ghana, Guatemala, Haiti, Honduras, Hungary, India, Indonesia, Italy, Kazakhstan, Kenya, Kyrgyz Republic, Lithuania, Madagascar, Malaysia, Mexico, Moldova, Namibia, Nicaragua, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Portugal, Romania, Russia, Singapore, Slovak Republic, Slovenia, South Africa, Spain, Sweden, Tanzania, Thailand, Trinidad and Tobago, Turkey, Uganda, Ukraine, United Kingdom, United States, Uruguay, Uzbekistan, Venezuela, Zambia, and Zimbabwe.
11 The former group of countries exhibits a standard deviation of 6 or less, while the latter shows a standard deviation of 22 or more.
This attempt to measure a country-level idiosyncratic uncertainty is new to our knowledge. However, one obvious question that can be raised about this measure is that we ignored the potentially important sectoral differences in the level of uncertainty within a country. Using the US data, Castro et al. (2009) show that firms producing capital goods face higher level of idiosyncratic uncertainty than ones producing consumption goods. This implies the existence of a potentially important role of sectoral composition when measuring idiosyncratic uncertainty at a national level. That is, to the extent that firms from certain sectors are included in the sample unproportionately, the resulting measure may be biased. However, since the WBES targets adequate distribution of firms in various aspects including sectoral composition\footnote{Other aspects include size and location}, the sample may still reflect the population distribution to some extent.

3 The theoretical framework

Having shown some suggestive evidence on cross-country variations in idiosyncratic uncertainty, we present a theoretical framework to explain how those variations may account for cross-country differences in the credit-to-output ratio. The model closely follows Carlstrom and Fuerst (1998), in which there are two types of agents: entrepreneurs (firms) and households. Entrepreneurs
own a technology to produce final goods using labor and capital. To undertake production, they finance the cost of acquiring these inputs using both internal and external funds. Households, on the other hand, do not have access to the production technology, but they have funds to spare. In equilibrium, funds flow from households to entrepreneurs through financial intermediaries. However, frictions arise in credit markets, since idiosyncratic shocks to entrepreneurs’ production are privately observed by them. The detailed model specification is given below.

3.1 Entrepreneurs

3.1.1 Production function

There is a continuum of entrepreneurs in the interval $(0, 1)$. Each one, represented by the subindex $j$, produces a final good, $Z_{j,t}$ with the Cobb-Douglas constant returns to scale technology:

\[ Z_{j,t} = \omega_{j,t} A_t F(H_{j,t}, K_{j,t}) = \omega_{j,t} A_t H_{j,t}^\sigma K_{j,t}^{1-\sigma}. \]

(5)

Entrepreneur $j$ employs labor provided by households, $H_{j,t}$, and capital provided by both households and entrepreneurs, $K_{j,t}$. $\omega_{j,t}$ is an iid stochastic productivity parameter specific to each entrepreneur, such that $E(\omega_{j,t}) = 1$, while $A_t$ is the aggregate technology parameter.

3.1.2 Costly state verification

Regarding the financing process, entrepreneur $j$, with an initial net worth given by $N_{j,t}$, requires a loan of size $TF_{j,t} - N_{j,t}$, where $TF_{j,t}$ is the total finance required for the project. $TF_{j,t}$ is given by the costs of hiring labor and renting capital:

\[ TF_{j,t} = w_t H_{j,t} + r^k_t K_{j,t}, \]

(6)

where $w_t$ and $r^k_t$ are the real wage and the real capital rental rate, respectively.\textsuperscript{13} External funding required by entrepreneurs is provided by households through a financial intermediary. However, credit markets are imperfect. Specifically, the friction is modeled following the costly state verification (CSV) approach. In this context, the realized value of the idiosyncratic shock $\omega_{j,t}$ is entrepreneur $j$’s private information; if the financial intermediary wants to observe the realization of $\omega_{j,t}$, it needs to pay a monitoring cost.

\textsuperscript{13}Following Carlstrom and Fuerst (1997, 1998), we implicitly assume that all input costs are paid before the production takes place.
Under the CSV approach, the optimal contract takes the form of a standard debt contract (see Townsend (1979)). The total amount to be repaid is expressed as $\Psi_{j,t}(TF_{j,t} - N_{j,t})$, where $\Psi_{j,t}$ is the gross interest rate on the loan. Given that entrepreneur $j$ is subject to idiosyncratic shocks, $\omega_{j,t}$, there exists a cut-off value, $\omega_{j,t}$, such that

$$\omega_{j,t}A_h H_{j,t}^\sigma K_{j,t}^{1-\sigma} = \Psi_{j,t}(TF_{j,t} - N_{j,t}).$$

That is, if $\omega_{j,t}$ exceeds $\omega_{j,t}$, she pays its debt and keeps the rest. Otherwise, she defaults on the debt and simultaneously the financial intermediary pays the monitoring cost and seizes the realized value of the project. The contract is made before the idiosyncratic shock is realized, which determines $\omega_{j,t}$ and the size of the project, i.e., the amount of each input used in production, $H_{j,t}$ and $K_{j,t}$. Entrepreneurs are risk neutral, so that they maximize their expected profits in the contract. Perfect competition is assumed among financial intermediaries, and thus they simply recoup the total amount lent to entrepreneurs. Notice that by lending to a large number of entrepreneurs, financial intermediaries can perfectly diversify idiosyncratic risk.

Under the standard debt contract, the expected share of the production that goes to entrepreneur $j$ is expressed as

$$f(\overline{\omega}_{j,t}) \equiv \int_{\omega_{j,t}}^\infty \omega \phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t},$$

whereas the fraction going to the lender is

$$g(\overline{\omega}_{j,t}) \equiv \int_{0}^{\overline{\omega}_{j,t}} \omega \phi(\omega)d\omega + [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t} - \mu \int_{0}^{\overline{\omega}_{j,t}} \omega \gamma \phi(\omega)d\omega.$$

In these expressions, $\phi$ and $\Phi$ represent probability and cumulative density functions, respectively, which are common to all entrepreneurs. The last term in Eq.9, $\mu \int_{0}^{\overline{\omega}_{j,t}} \omega \gamma \phi(\omega)d\omega$, indicates the monitoring costs that financial intermediaries pay on average, in which $\mu$ is a parameter common to all entrepreneurs. $\gamma = 0$ implies that the cost of monitoring bankrupt entrepreneurs does not depend on the size of the realized production outcome, while $\gamma = 1$ implies that this cost increases linearly in the size of the realized outcome. In the intermediate case of $\gamma$ between 0 and 1, monitoring technology exhibits economies of scale. When $\mu = 0$, monitoring costs disappear regardless of the value of $\gamma$: the realized value of the idiosyncratic shock is effectively public information. Also,

We also assume for simplicity that financial intermediaries do not incur any cost in their operations.

This generalization is one of the differences of our framework from Carlstrom and Fuerst (1998), who focus on the case where $\gamma = 0$. 

14
notice that \( f(\omega_{j,t}) \) and \( g(\omega_{j,t}) \) add to

\[
f(\omega_{j,t}) + g(\omega_{j,t}) = 1 - \mu \int_0^{\omega_{j,t}} \omega^\gamma \phi(\omega) d\omega. \tag{10}
\]

This clarifies that \( \mu \int_0^{\omega_{j,t}} \omega^\gamma \phi(\omega) d\omega \) represents a deadweight loss due to credit-market frictions: on average this fraction of the total production \( A_t F(H_{j,t}, K_{j,t}) \) is lost in the bankruptcy process.

### 3.1.3 Distortion

Formally, the optimal contract is determined as the solution of the following problem:

\[
\max \{H_{j,t}, K_{j,t}, \omega_{j,t}\} f(\omega_{j,t}) A_t F(H_{j,t}, K_{j,t}) \tag{11}
\]

subject to

\[
g(\omega_{j,t}) A_t F(H_{j,t}, K_{j,t}) = TF_{j,t} - N_{j,t}. \tag{12}
\]

The first-order conditions (FOCs) are given as:

\[
w_t = \left( g(\omega_{j,t}) + f(\omega_{j,t}) / \lambda(\omega_{j,t}) \right) A_t F_{H_{j,t}} \quad \text{and} \quad r^k_t = \left( g(\omega_{j,t}) + f(\omega_{j,t}) / \lambda(\omega_{j,t}) \right) A_t F_{K_{j,t}},
\]

where \( F_{H_{j,t}} \) and \( F_{K_{j,t}} \) are the partial derivatives of \( F(H_{j,t}, K_{j,t}) \) with respect to each of its arguments and \( \lambda(\omega_{j,t}) = f'(\omega_{j,t}) / g'(\omega_{j,t}) \) is the Lagrange multiplier of the problem. Since those FOCs imply that \( F_{K_{j,t}} \) and \( F_{H_{j,t}} \) are common across \( j' \)s,\(^{16}\) we know that \( \omega_{j,t} = \bar{\omega}_t \) for all \( j \) and that \( F_{K_{j,t}} = F_{K_t} \) and \( F_{H_{j,t}} = F_{H_t} \), where \( F_{K_t} \) and \( F_{H_t} \) are the partial derivatives of \( F(H_t, K_t) \).\(^{17}\) Then, the FOCs become:

\[
w_t = \Theta(\omega_t)^{-1} A_t F_{H_t} \tag{13}
\]

and

\[
r^k_t = \Theta(\omega_t)^{-1} A_t F_{K_t}, \tag{14}
\]

where \( \Theta(\omega_t) \equiv (g(\omega_t) + f(\omega_t) / \lambda(\omega_t))^{-1} \).

To understand what constitutes the wedge between the factor prices and the marginal product of those inputs, it is convenient to express \( \Theta(\omega_t) \) as

\[
\Theta(\omega_t) = \left( 1 - \mu \int_0^{\omega_t} \omega^\gamma \phi(\omega) d\omega \right) \left( 1 - \frac{1}{\lambda(\omega_t)} \right)^{-1}. \tag{15}
\]

The wedge is composed of two key factors. The first component, \( \mu \int_0^{\omega_t} \omega^\gamma \phi(\omega) d\omega \), is the deadweight loss due to the fact that financial intermediaries incur costs to monitor bankrupt entrepreneurs.

\(^{16}\)This is evident since the ratio of capital to labor employed is common: \( \frac{K_{j,t}}{H_{j,t}} = \frac{1-\sigma w_t}{\sigma r^k_t} \).

\(^{17}\)\( H_t = \int_0^1 H_{j,t}dj \) and \( K_t = \int_0^1 K_{j,t}dj \)
The second component, $\left(1 - \frac{1}{\lambda (\varpi_t)}\right) f(\varpi_t)$, is the distortion due to the economic rents obtained by entrepreneurs in the presence of informational asymmetry. To see why, notice that the Lagrange multiplier, which can be interpreted as the shadow price of net worth, exceeds unity only when $\mu > 0$:

$$
\lambda (\varpi_t) = \frac{1}{1 - \mu \varpi_t^\gamma \frac{\phi(\varpi_t)}{1 - \Phi(\varpi_t)}}.
$$

(16)

That is, with informational asymmetry present, an increase in net worth (internal funds) by 1 unit leads to an increase in the maximized expected return by more than 1 unit, representing the rents earned by entrepreneurs. Notice also that this component of the distortion becomes larger as the share of revenues going to entrepreneurs, $f(\varpi_t)$, increases.

Finally, we confirm for later reference that this wedge makes a gap between total finance required and aggregate production. Since the production function exhibits constant returns to scale, the Euler’s theorem implies that $F(H_t, K_t) = H_t F_{H_t} + K_t F_{K_t}$. Then, using Eqs.13 and 14, we obtain

$$
TF_t = \Theta(\varpi_t)^{-1} A_t F(H_t, K_t),
$$

(17)

where $TF_t = w_t H_t + r_t^k K_t$.

### 3.2 Households

The representative household obtains utility from the consumption of the final good, $C^h$, and leisure (implied by the disutility of working in the production of the final good, $H_t$). The lifetime utility of the household takes the following functional form:

$$
U_t = \sum_{t=0}^{\infty} \beta^t [\log C^h_t - \chi H^\eta_t], \chi > 0, \eta \geq 1.
$$

This utility is maximized subject to the following budget constraint:

$$
C^h_t + K^h_{t+1} = w_t H_t + r_t^k K^h_t + (1 - \delta) K^h_t,
$$

(18)

where $K^h_t$ is the capital holding by the household at the beginning of period $t$. While the sources of funding in period $t$ include wage earnings, $w_t H_t$, the net return from capital holding at the beginning of the period, $r_t^k K^h_t$, and undepreciated capital, $(1 - \delta) K^h_t$, these resources are used to purchase the final good, $C^h_t$ or to invest in capital for the next period, $K^h_{t+1}$.

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18Note that $f'(\varpi_t) = -(1 - \Phi(\varpi_t))$ and $g'(\varpi_t) = (1 - \Phi(\varpi_t))(1 - \mu \varpi_t^\gamma \frac{\phi(\varpi_t)}{1 - \Phi(\varpi_t)})$

19The return from lending to entrepreneurs (through financial intermediaries) does not appear in the constraint. This is because the net return they obtain within a period is zero in equilibrium.
The associated FOCs can be expressed as:

\[
\frac{C_{t+1}^h}{C_t^h} = \beta \left( 1 + r_{t+1}^k - \delta \right) \quad (19)
\]

and

\[
w_t = \chi \eta C_t^h H_t^{\eta-1}. \quad (20)
\]

Eq.19 is a consumption Euler equation, indicating that households are indifferent at the optimum between consuming and investing in capital goods. Eq.20 represents a standard positively-sloped labor supply function.

### 3.3 Aggregate net worth and entrepreneurs’ consumption

In the discussion of the contracting problem entrepreneurs’ initial net worth was taken as given. We now specify how it is actually determined. We also clarify how entrepreneurs make their consumption and saving decisions in case they do not default on their debt.\(^{20}\)

#### 3.3.1 Aggregate net worth

Entrepreneur \(j\)’s net worth in period \(t\), \(N_{j,t}\), is given as the gross return from the capital that she holds at the beginning of the period, \(K_{j,t}^e\): \(N_{j,t} = (r_t^k + 1 - \delta)K_{j,t}^e.\)\(^{21}\) Aggregating across entrepreneurs, we obtain:

\[
N_t = (r_t^k + 1 - \delta)K_t^e, \quad (21)
\]

where \(N_t\) and \(K_t^e\) are aggregate net worth and entrepreneurial capital, respectively.

#### 3.3.2 Entrepreneurs’ consumption and saving decision

If entrepreneur \(j\) is solvent after production in period \(t\), she then decides how much of the profits to consume and how much to save for the next period. To avoid a situation in which entrepreneurs

\(^{20}\)In case of default, they consume and save nothing.

\(^{21}\)Since entrepreneurs who went bankrupt in the previous period do not hold any capital at the beginning of the current period, we need to assume, for example, that they obtain wages at the beginning of the period so that the net worth of each entrepreneur is always positive. (This point also applies to entrepreneurs who newly enter the economy.) However, in general, the entrepreneurs’ wage share in production is assumed to be very small. For example, in Bernanke et al. (1999) it is set to be 0.01. Thus, following Carlstrom and Fuerst (2001), we ignore this component of net worth to simplify the analysis.
ultimately become self financed (by accumulating enough net worth) and thus the agency problem becomes irrelevant, we assume that a constant fraction, \( \nu \) of entrepreneurs die each period\(^{22}\) and consume all the accumulated wealth just before their death. However, the population of entrepreneurs remains constant by the introduction of new entrepreneurs that replace those exiting the economy.\(^{23}\)

In this setting, aggregate entrepreneurial consumption, \( C^e_t \), is simply given as:

\[
C^e_t = \nu A_t f(\overline{\omega}_t) F(H_t, K_t). \tag{22}
\]

Thus, \( \nu \) also represents a constant share of aggregate entrepreneurial return consumed each period. Similarly, entrepreneurs’ aggregate capital accumulation is expressed as:

\[
K^e_{t+1} = (1 - \nu) A_t f(\overline{\omega}_t) F(H_t, K_t). \tag{23}
\]

It follows that the survival rate of \( 1 - \nu \) represents the entrepreneurs’ aggregate saving rate.

### 3.4 Equilibrium conditions

#### 3.4.1 Goods market equilibrium

Taking account of the deadweight loss from monitoring, the supply of the final good is thus given by

\[
Y_t = (1 - \mu) \int_0^{\overline{\omega}_t} \omega^\gamma \phi(\omega) d\omega A_t F(H_t, K_t). \tag{24}
\]

Its associated market clearing condition thus takes the following form,

\[
Y_t = C_t + K_{t+1} - (1 - \delta) K_t, \tag{25}
\]

where \( C_t = C^h_t + C^e_t \) and \( K_t = K^h_t + K^e_t \).

#### 3.4.2 Labor market equilibrium

The aggregate demand for labor is given by Eq.13, while the supply is obtained from Eq.20. Combining them, we obtain the equilibrium level of labor as \( H_t = \left( \frac{\alpha}{\chi^*} A_t \Theta(\overline{\omega}_t)^{-1} \frac{K^{1-\sigma}}{C_t^\sigma} \right)^{1/(\eta - \sigma)} \).

\(^{22}\)This implies that their expected survival periods are \( 1/\nu \) periods.

\(^{23}\)This strategy is adopted by Bernanke et al. (1999), for example. Another popular strategy to avoid a situation in which entrepreneurs become self-financed is to assume that they maximize their intertemporal utility with a higher discount rate than households. This approach is followed, for instance, by Carlstrom and Fuerst (1997, 1998).
3.4.3 Capital market equilibrium

The aggregate demand for capital is obtained from Eq.14 and can be expressed as \( r_t^k = (1 - \sigma)\Theta(\omega_t)^{-1}A_t \left( \frac{H_t}{K_t} \right)^\sigma \), whereas the aggregate supply is given at each period \( t \). This market clears when the two schedules coincide.

4 Idiosyncratic uncertainty and the credit-to-output ratio

Using the above framework, we study how a change in idiosyncratic uncertainty, modeled by a mean preserving change in the dispersion of firms’ idiosyncratic shocks, affects the credit-to-output ratio in the long run. To simplify, we focus on the steady state equilibrium in which there is no technological growth (i.e., \( A_t = 1 \)). Since this implies that all the endogenous variables are constant in the steady state, we drop the time subscripts.

To consider a change in the dispersion of shocks, we first need to decide which distribution function to use. Conventionally, models with credit frictions based on the CSV framework (including Carlstrom and Fuerst (1998)) use a log-normal distribution. However, one potentially undesirable feature with this function is that a mean preserving change in standard deviation necessarily involves a change in skewness. To focus on the effects of different degrees of dispersion, we prefer to abstract away from potential impacts caused by the changes in skewness. This is the reason why we use a beta distribution. This distribution, as Figure 2 shows, allows us to keep the distribution symmetric when changing its dispersion. (The vertical line in the left tail will be explained below.) Specifically, a beta distribution exhibits unit mean and zero skewness when the probability function takes the following form:

\[
\phi(\omega, \rho) = \frac{1}{Beta(\rho, \rho)} \frac{\omega^{\rho-1} (2 - \omega)^{\rho-1}}{2^{2\rho-1}},
\]

where \( Beta(\rho, \rho) = \int_0^1 t^{\rho-1} (1 - t)^{\rho-1} dt \). In this setting, an increase in the standard deviation of \( \omega \), denoted as \( s \), corresponds to a decrease in \( \rho \):

\[
\frac{ds}{d\rho} < 0.
\]

The general form is given as \( \phi(\omega, \alpha, \beta, a, b) = \frac{1}{Beta(\alpha, \beta)} \frac{(\omega - a)^{\alpha-1} (b - \omega)^{\beta-1}}{(b - a)^{\alpha+\beta-1}} \) where \( Beta(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1 - t)^{\beta-1} dt \). The distribution exhibits symmetry and has a unit mean by setting \( \alpha = \beta (= \rho) \) and \( a = 0 \) and \( b = 2 \).
4.1 The credit-to-output ratio in the long run

In the model, the credit-to-output ratio is defined as \((TF - N)/Y\), where \(TF\) is total finance required, \(N\) is net worth, and \(Y\) is output. To see how a mean-preserving change in the dispersion of idiosyncratic shocks affects this ratio, we first clarify its key determinants.

Since financial intermediaries are perfect competitors and do not use inputs to operate, aggregate credit, \(TF - N\) equals their return from the financial contract, \(g(\varpi)F(H, K)\) (cf. Eq.12):

\[
TF - N = g(\varpi)F(H, K),
\]

where total finance required, \(TF\) is given as (cf. Eq.17):

\[
TF = \Theta(\varpi)^{-1}F(H, K),
\]

and net worth, \(N\) is thus:

\[
N = \frac{1}{\lambda(\varpi)}f(\varpi)F(H, K).
\]

The aggregate output, after subtracting the deadweight loss, is given as (cf. Eq. 24):

\[
Y = \left(1 - \mu \int_0^\varpi \omega^\gamma \phi(\omega) d\omega \right) F(H, K).
\]

Combining Eqs.28 and 31 yields \((TF - N)/Y = g(\varpi)/(1 - \mu \int_0^\varpi \omega^\gamma \phi(\omega) d\omega)\). Equivalently, rewriting it using entrepreneurs’ share, \(f(\varpi)\) yields:

\[
\frac{TF - N}{Y} = 1 - \frac{f(\varpi)}{1 - \mu \int_0^\varpi \omega^\gamma \phi(\omega) d\omega}.
\]
That is, a larger entrepreneurs’ share of production net of deadweight loss corresponds to a smaller credit-to-output ratio. It is thus clear that the ratio is decreasing in the deadweight loss and entrepreneurs’ share.

4.2 The effect of idiosyncratic uncertainty on deadweight loss and entrepreneurs’ revenue share

Acknowledging that the deadweight loss, \( \mu \int_0^{\bar{\omega}} \omega^\gamma \phi(\omega) d\omega \) and entrepreneurs’ share, \( f(\bar{\omega}) \) are the key determinants of the ratio, we study the effects of the standard deviation of idiosyncratic shocks, \( s \) on these variables. To do this, we decompose its effects into the direct and indirect effects: the former is the effect of \( s \) on these for a given cut-off value, \( \bar{\omega} \), while the latter is the effect of \( s \) through the change in \( \bar{\omega} \). Algebraically, denoting the deadweight loss as \( dl \) and entrepreneurs’ share as \( f \) for convenience, the decomposition is seen as:

\[
\frac{ddl}{ds} = \frac{\partial dl}{\partial s} + \frac{\partial dl}{\partial \bar{\omega}} \frac{d\bar{\omega}}{ds},
\]

(33)

and

\[
\frac{df}{ds} = \frac{\partial f}{\partial s} + \frac{\partial f}{\partial \bar{\omega}} \frac{d\bar{\omega}}{ds},
\]

(34)

where the first terms on the RHS of the equations capture the direct effects while the second capture the indirect ones. In what follows, we first consider the indirect effects.

4.2.1 Indirect effect

First, the effects of the cut-off value on the deadweight loss and entrepreneurs’ share are:

\[
\frac{\partial dl}{\partial \bar{\omega}} = \mu \bar{\omega}^\gamma \phi(\bar{\omega}),
\]

(35)

and

\[
\frac{\partial f}{\partial \bar{\omega}} = -(1 - \Phi(\bar{\omega})),
\]

(36)

where \( \frac{\partial dl}{\partial \bar{\omega}} > 0 \) and \( \frac{\partial f}{\partial \bar{\omega}} < 0 \). In words, while a fall in the cut-off value decreases the deadweight loss by lowering the default rate, it increases the entrepreneurs’ share by easing the terms of their debt repayment.

Next, to see how a change in the standard deviation affects the cut-off value, \( \frac{d\bar{\omega}}{ds} \), we derive the
long-run shadow price of net worth (cf. Eq.16):

\[ \lambda(\omega) = \frac{1}{1 - \mu \frac{\phi(\omega)}{1 - \Phi(\omega)}} = \frac{1}{\beta^{-1}(1 - \upsilon)} \]  

(37)

The shadow price falls in the entrepreneurs’ aggregate saving rate, \(1 - \upsilon\), and the steady state gross return on capital, \(\beta^{-1}\).\(^{26}\) Intuitively, when the saving rate and the gross return are lower, entrepreneurs’ net worth becomes scarcer relative to the size of production, which increases the value of net worth. In Eq.37, the standard deviation, \(s\) affects the hazard rate, \(\phi(\omega) / (1 - \Phi(\omega))\) for a given value of \(\omega\). We thus know that \(\omega\) is an implicit function of \(s\) for given \(\mu, \beta, \upsilon\) and \(\gamma\).

Then, denoting \(\omega(\frac{\phi(\omega)}{1 - \Phi(\omega)})\) as \(h(\omega, s)\), the implicit function theorem indicates that

\[ \frac{d\omega}{ds} = -\frac{h_s(\omega, s)}{h_\omega(\omega, s)}. \]  

(38)

What is the sign of \(\frac{d\omega}{ds}\)? Though analytical derivation is difficult due to the complexity of the distribution function (cf. Eq.26), we can deduce that it is positive for reasonable parameter values. To see how, observe from Figure 2 that as long as the default rate takes a plausible value, the cut-off value should be located on the left tail of the distribution.\(^{27}\) Then, a rise in \(s\) should increase the hazard rate for a given value of \(\omega\), i.e., \(h_s(\omega, s) > 0\). For instance, this is certainly the case at the vertical line in the figure.\(^{28}\) Notice also that the hazard rate is increasing in \(\omega\) at least in the left half of the distribution, implying that \(h_\omega(\omega, s) > 0\). Thus, we expect from Eq.38 that \(\frac{d\omega}{ds} < 0\).

Overall, indirect effects are expected to be \(\frac{\partial dl}{\partial \omega} \frac{d\omega}{ds} < 0\) and \(\frac{\partial f}{\partial \omega} \frac{d\omega}{ds} > 0\). In words, an increase in the dispersion, through the indirect channel, should exert a downward pressure on the deadweight loss and an upward pressure on the entrepreneurs’ share.

\(^{25}\)This is obtained as follows. First, notice that rewriting Eq.30 yields \(f(\omega)F(H, K) = \lambda(\omega)N\), which says that entrepreneurs’ aggregate return is given by their net worth valued at its shadow price. Second, Eq.23 implies \(K^e = (1 - \upsilon)f(\omega)F(H, K)\), which indicates that a fraction \(1 - \upsilon\) of the return is accumulated in the form of capital. Third, Eqs.21 and 19 give \(N = (\beta^{-1})K^e\), which shows that the gross return on the accumulated capital stock determines the steady state level of entrepreneurs’ net worth. Combining these steady state relations lead to Eq.37.

\(^{26}\)We assume that \(\beta^{-1}(1 - \upsilon) < 1\), since the shadow price becomes less than unity otherwise. However, this assumption seems plausible. To see this, let us rewrite the assumption as \(\upsilon > 1 - \beta\) and suppose that the time unit is a quarter and \(\beta\) (the discount factor) is 0.99. Then, this inequality becomes \(\upsilon > 0.01\). Since \(1/\upsilon\) is entrepreneurs’ average survival periods, for this to be violated, the survival periods need to exceed 100 quarters, which seem to be unrealistically long.

\(^{27}\)For example, by defining that a period is a quarter, Carlstrom and Fuerst (1997, 1998) calibrate the model’s parameters such that the default rate is 0.974%.

\(^{28}\)This line represents a default rate of 3.00% at the initial steady state cut-off value, which turns out to be our calibrated default rate.
4.2.2 Direct effect

The direct effects, $\frac{\partial dl}{\partial s}$ and $\frac{\partial f}{\partial s}$, are not easy to see analytically. However, we can deduce from Figure 2 that the sign of the former is positive since a rise in $s$ increases the default rate for a given $\omega$, although the sign of the latter is still uncertain.

4.2.3 Calibration

Therefore, the total effects of standard deviation on deadweight loss and entrepreneurs' share (Eqs.33 and 34) are difficult to pin down analytically. While the effect on the former is uncertain because the signs of the direct and indirect effects contradict, the effect on the latter is inconclusive because the sign of the direct effect is not clear. We thus calibrate the model to clarify the total effects.

The time unit is a quarter. We set the discount factor $\beta$ equal to 0.99, a conventional value in the literature. As for $\gamma$, following Carlstrom and Fuerst (1997, 1998) we here set $\gamma = 0$. Another extreme value of $\gamma = 1$ (taken by Bernanke et al. (1999)) is considered below as a robustness check. Next, since the monitoring costs parameter $\mu$ is difficult to determine, we set the reference value as 0.24, later changing it to 0.12 and 0.36 to check robustness. Regarding the entrepreneurs’ exit probability $\upsilon$, we again consider a range of values. As a reference value, we set $\upsilon = 0.1$, which corresponds to their expected survival periods of 10 quarters and entrepreneurs’ aggregate saving rate of 0.9. We then adjust to $\upsilon = 0.15$ and $\upsilon = 0.05$ below.

Having set $\gamma$, $\mu$, and $\upsilon$, we then tie down the steady state values of $\rho$ and $\omega$ from Eq.37 in combination with an empirically plausible value for the credit spread. Since lending takes place within a period in the model and thus the gross risk-free interest rate is equal to unity, we define the spread as $\Psi(\omega) - 1$ (i.e., the gross interest rate on the loan minus the gross risk-free rate). Then, denoting the spread as $\psi(\omega)$, Eqs.28 and 29 yield $\psi(\omega) = \omega/g(\omega) - 1$. Regarding its plausible value, we look at the World Bank’s indicator (World Development Indicator) called “risk premium on lending (prime rate minus treasury bill rate, %)”. Taking an average of all the countries for which the figure is available for 2006, we have an annual rate of 5.90%. Since our time unit is a quarter, we consider a range of values to check robustness. As a reference value, we set $g(\omega) = 0.1$, which corresponds to their expected survival periods of 10 quarters and entrepreneurs’ aggregate saving rate of 0.9. We then adjust to $g(\omega) = 0.15$ and $g(\omega) = 0.05$ below.30

30For example, Carlstrom and Fuerst (2001) introduce a lower bound of 0.04 and an upper bound of 0.36 after looking at some empirical studies (for the lower bound, Warner (1977), for the upper bound, Alderson and Betker (1995)).

31Bernanke et al. (1999), focusing on the US economy, set the exit probability as 0.0272. Thus, our reference value is higher.

31Zimbabwe is excluded since an extraordinarily high number of 174.1% is reported for that year. If this is
quarter, we set $\psi(\omega) = 0.015$ (1.5%). We then obtain the reference steady state values of $\omega$ and $\rho$ as 0.62 and 11.87 respectively.\footnote{Subsequently, we can calculate all the steady state values of the contract-related variables, including the deadweight loss and entrepreneurs' revenue share. They are given as: $f(\omega) = 0.38; \mu \int_0^\omega \omega^n \phi(\omega)d\omega = 0.007; \Phi(\omega) = 0.03; \lambda(\omega) = 1.1; \Theta(\omega) = 1.04; \psi(\omega) = 0.015$.} (This value of $\rho$ corresponds to the standard deviation, $s$ of 0.20.)

### 4.2.4 Total effect

At the calibrated steady state, we find that a rise in the standard deviation increases the deadweight loss and entrepreneurs’ revenue share:

$$\frac{d\text{dl}}{ds} > 0, \quad (39)$$

and

$$\frac{df}{ds} > 0. \quad (40)$$

Decomposing the effect on $dl$, the indirect and direct effects are, as expected, $\frac{\partial dl}{\partial \omega} \frac{d\omega}{ds} < 0$\footnote{In particular, we obtain $h_s(\omega, s) > 0$ and $h_\omega(\omega, s) > 0$ thus $\frac{d\omega}{ds} > 0$ (see Eq.38) \footnote{In fact, 96% of the increase in the share is explained by the indirect effect. Specifically, we obtain $\frac{df}{dp} = -0.0118$ and and $\frac{\partial f}{\partial p} \frac{dp}{ds} = -0.0113.$}} and $\frac{d\omega}{ds} > 0$. Thus, the direct effect is dominant. That is, the fact that a rise in the standard deviation makes the ‘left tail’ of the distribution thicker ensures a rise in the default rate and thus the deadweight loss. Turning to the the effect on $f$, a rise in $s$, as expected, puts an upward pressure on it through the indirect channel, i.e., $\frac{\partial f}{\partial \omega} \frac{d\omega}{ds} > 0$. It turns out that the direct effect also raises the share, $\frac{df}{ds} > 0$. However, we find that the indirect effect is overwhelmingly dominant.\footnote{In fact, 96% of the increase in the share is explained by the indirect effect. Specifically, we obtain $\frac{df}{dp} = -0.0118$ and and $\frac{\partial f}{\partial p} \frac{dp}{ds} = -0.0113.$} That is, a rise in the standard deviation increases entrepreneurs’ share mainly by lowering the cut-off value and thus entrepreneurs’ debt repayment costs.

### 4.3 The effect of uncertainty on the credit-to-output ratio

Having acknowledged the total effects of the dispersion of idiosyncratic shocks on deadweight loss and entrepreneurs’ share of production, we know from Eq.32 that a rise in idiosyncratic uncertainty decreases the credit-to-output ratio:

$$\frac{dTF-N}{ds} < 0. \quad (41)$$

What are the effects of uncertainty on the individual components of the ratio, i.e., $TF$, $N$, and $Y$? To see these, we derive the expression of gross output, $F(H, K)$ (i.e., output without included, the average jumps up to 7.95%).
deadweight loss being subtracted) as (see Appendix):

\[ F(H, K) = \left( \frac{\chi \eta}{\sigma} \alpha^{-\frac{2}{\sigma}} \Theta(\bar{\omega})^{\frac{\sigma+\eta}{\sigma}} \left( 1 - \mu \int_{0}^{\bar{\omega}} \omega^{\gamma} \phi(\omega)d\omega - \nu f(\bar{\omega}) - \frac{\delta \alpha}{\Theta(\bar{\omega})} \right) \right)^{-\frac{1}{\eta}}, \quad (42) \]

where \( \alpha \equiv (1 - \sigma)/(\beta^{-1} - 1 + \delta) \). Then, after setting a few more parameter values,\(^{35}\) we find at the calibrated steady state that (cf. Eqs. 29, 30, and 31):

\[ \frac{dT F}{d s} < 0, \quad (43) \]

\[ \frac{dN}{d s} > 0, \quad (44) \]

and

\[ \frac{dY}{d s} < 0. \quad (45) \]

We thus know that a rise in idiosyncratic uncertainty lowers the credit-to-output ratio by reducing credit proportionally more than output.

To see the intuition, we look at the expression of aggregate capital, \( K \) (see Appendix for derivation):

\[ K = \left( \frac{\chi \eta}{\sigma} \alpha^{-\frac{2}{\sigma}} \Theta(\bar{\omega})^{\frac{\sigma+\eta}{\sigma}} \left( 1 - \mu \int_{0}^{\bar{\omega}} \omega^{\gamma} \phi(\omega)d\omega - \nu f(\bar{\omega}) - \frac{\delta \alpha}{\Theta(\bar{\omega})} \right) \right)^{-\frac{1}{\eta}}. \quad (46) \]

To clarify how idiosyncratic uncertainty affects this variable, we simplify Eq. 46 by imposing some assumptions\(^{36}\):

\[ K = \left( (1 - \sigma) \left( \Theta(\bar{\omega}) \right)^{-1} \right)^{\frac{1}{\sigma}}. \quad (47) \]

Given that the wedge, \( \Theta(\bar{\omega}) \) is increasing in the deadweight loss and entrepreneurs’ rents (cf. Eq. 15), Eq. 47 implies that uncertainty lowers aggregate capital by enlarging economic distortions. This fall in aggregate capital ensures lower output and total finance required. However, despite the fall in aggregate capital, net worth increases because of the larger rents they earn.

To summarize, the intuition behind why higher idiosyncratic uncertainty lowers the credit-to-output ratio is given as follows. A rise in uncertainty, increasing the deadweight loss and also

\(^{35}\)First, we set the share of income spent on labor as \( \sigma = 0.67 \), a widely used value in the literature. We then choose for simplicity \( \chi = 1 \). Regarding the households’ preference parameter, \( \eta \), we follow Ascari (2000) to set \( \eta = 4.5 \). Finally, the depreciation rate of capital, \( \delta \) is given as \( \delta = 0.025 \) as in Faia and Monacelli (2007). These parameter values, combined with the values set earlier, yield \( TF = 1.97 \), \( N = 0.71 \) and \( Y = 2.04 \).

\(^{36}\)In particular, we assume that the capital stock fully depreciates within the period, \( \delta = 1 \); households supply labor inelastically, \( \eta = 1 \); the subjective discount factor is equal to one, \( \beta = 1 \); the coefficient of the disutility of work in the representative household’s utility function is unity, \( \chi = 1 \).
entrepreneurs’ economic rents, enlarges economic distortions. This, in turn, lowers aggregate capital accumulation and thus aggregate output in the long-run. However, because the higher uncertainty provides entrepreneurs with larger rents, they accumulate more capital despite the fall in aggregate capital. Put differently, a rise in idiosyncratic uncertainty reallocates capital from households to entrepreneurs (i.e., from agents with informational disadvantage to ones with advantage). The resulting rise in entrepreneurs’ net worth, coupled with the fall in whole finance required in the lower output environment, causes a proportionally larger fall in credit than output, thus lowering the credit-to-output ratio.

4.4 Robustness

We here check robustness of the results. As mentioned above, we particularly check it in terms of monitoring technology parameter, γ, monitoring cost parameter, µ, and entrepreneurs’ exit rate, υ, all of which are the key parameters determining the values of contract related variables. The reference case studied above takes γ = 0, µ = 0.24, and υ = 0.10. First, we set γ = 1 while keeping the other parameter values the same as the reference values (including µ = 0.24 and υ = 0.10). In this case, we still obtain \( \frac{d\ln}{ds} > 0 \) and \( \frac{df}{ds} > 0 \), thus \( \frac{d(TF-N)/Y}{ds} < 0 \). That is, an increase in the degree of idiosyncratic uncertainty causes an increase in both deadweight loss and entrepreneurs’ share in production, thus decreasing the credit-to-output ratio. Moreover, this again happens because credit falls proportionally more than output, which is caused by a fall in total finance, a rise in net worth, and a fall in output. Second, we try different values of µ, i.e., µ = 0.12 and µ = 0.36 (while keeping the other parameter values at the reference values). For both values, the results are the same. Finally, the results are also robust to the different values of υ, i.e., υ = 0.05 and υ = 0.15.

5 Regression analysis

We have seen that in the presence of credit frictions, idiosyncratic uncertainty reduces the credit-to-output ratio in the long run. In the cross-country context, this implies that a country with higher idiosyncratic uncertainty, other things equal, should be characterized by a lower credit-to-output ratio. We now test this theoretical implication by using the conditional volatility of firms’ sales growth rates as a proxy of idiosyncratic uncertainty (See Figure 1).
Figure 3a presents a simple plot of this proxy against the ratio of private credit to output.\textsuperscript{37} Consistent with the theoretical prediction, we observe the negative relation between them. Moreover, there appears to be a negative relation between idiosyncratic uncertainty and the log of output per capita in real terms (see Figure 3b), which is also consistent with the theory.\textsuperscript{38}

However, to highlight the role of idiosyncratic uncertainty particularly as a determinant of the ratio of private credit to output, we now control for the other determinants that have been identified in the literature. As explained in the introduction, determinants extensively discussed in previous works include creditors’ rights, law enforcement, and information sharing.\textsuperscript{39} Acknowledging this, we base our analysis on Djankov et al. (2007) (DMS hereafter) who empirically test the explanatory power of those variables simultaneously. By conducting a cross-country analysis covering 129 countries, they demonstrate that greater creditors right, higher quality of law enforcement, and more developed information sharing all lead to a larger fraction of private credit relative to output.\textsuperscript{40,41}

To test the role of idiosyncratic uncertainty as a potential determinant of this ratio, we add the

\textsuperscript{37}The ratio is the World Development indicator (World Bank), “Domestic credit to private sector (% of GDP)”. To be in line with the regression analysis conducted below, this ratio is the average from 1999 to 2003.

\textsuperscript{38}The GDP per capita figure is the World Development indicator (World Bank), “GDP per capita (current US$)” averaged between 1996 and 2002.

\textsuperscript{39}See the introduction for examples of works which have studied these determinants.

\textsuperscript{40}In DMS, their focus is given on creditor rights and information sharing. But, they consider the potential role of law enforcement too.

\textsuperscript{41}Making use of their large sample, DMS further show that creditor rights and law enforcement are more important as determinants in rich countries, while information sharing is more relevant in poor countries.
volatility of firms’ sales growth rates to their regression model as an extra explanatory variable. The rest of our empirical specification closely follows theirs (in particular, Table 6 on page 314 of DMS) including the choice of conditioning variables and the period each variable spans. Algebraically, the regression model we test is given as:

\[ CR_j = \alpha + \beta V_j + \sum_{i=1}^{m} \gamma_i X_{i,j} + \epsilon_j, \]  

(48)

where \( CR_j \) is the credit-to-output ratio in country \( j \), \( V_j \) is the volatility of firms’ sales growth (i.e., the proxy of idiosyncratic uncertainty) in country \( j \), and \( X_{i,j} \) contain the conditioning variables including creditors rights, law enforcement, information sharing, total GDP, GDP per capita growth, and inflation. The rationale behind the inclusion of the latter three variables is as follows. Total GDP is expected to increase the ratio based on the hypothesis that proper functioning of the credit market requires a fixed institutional cost, which is covered only when the total GDP is high enough. Per capita GDP growth is controlled since when economy is growing faster, more credit may be required. Finally, inflation may decrease credit since it can devalue the stock of outstanding debt, thus undermining the debt contract.\(^{42}\)

The number of countries in our regression analysis is 70, including both developed and developing countries. The number is lower than DMS due to the limited availability of our measure of idiosyncratic uncertainty.\(^{43}\) The other variables are from the two following data sources. Data on creditors’ rights, law enforcement, and information sharing are taken from DMS; remaining data are from the World Bank (see Table 1 for detailed description of each variable). As mentioned, we follow DMS regarding the periods these variables span. Notice that they are around 1999-2000, being consistent with the period when the WBES data was collected (see Table 1 for the details).

\(^{42}\)DMS show that the coefficients on total GDP, per capita GDP growth, and inflation have expected signs and apart from inflation, they are statistically significant. (Inflation is statistically significant when regression covers only poor countries.)

\(^{43}\)Compared to the 74 countries for which we calculated the proxy, 4 countries (Estonia, Haiti, Trinidad and Tobago, and Uzbekistan) are missing, since for those countries, not all the variables for the regression analysis are available.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP per capita growth</td>
<td>“GDP per capita growth (annual %)” (based on constant local currency), average between 1979 and 2003. Source: World Development Indicators.</td>
</tr>
<tr>
<td>Creditor rights</td>
<td>Ranging from 0 to 4, this index measures various powers of secured lenders in bankruptcy, which include whether they can seize their collateral smoothly (e.g. without asset freeze imposed by the court) and whether they are paid first out of the proceeds of liquidating firms. The index figure for 1999 is used for the regression analysis. Source: Djankov et al. (2007) (This index, in turn, is constructed based on La Porta et al. (1997).)</td>
</tr>
<tr>
<td>Information sharing</td>
<td>The information sharing index takes 1 if either a public registry or a private bureau operates. The former is a public authorities’ database which collects borrowers’ finance-related information (e.g. their outstanding loans) and shares it with financial institutions; the latter is a private firm which facilitates the flow of the borrowers’ information among financial institutions. The index figure for 1999 is used for the regression. Source: Djankov et al. (2007)</td>
</tr>
<tr>
<td>Law enforcement</td>
<td>This is measured by the number of days to resolve a payment dispute on unpaid debt (worth 50% of the country’s GDP per capita) through courts. Since this variable is constructed as in 2003, the 2003 figure is used for the regression. Source: Djankov et al. (2007) (This variable, in turn, is based on the methodology in Djankov et al. (2003).)</td>
</tr>
</tbody>
</table>
Table 2 presents the results of OLS with White-corrected standard errors with and without idiosyncratic uncertainty. For simplicity, the table only shows the coefficients on creditor rights, law enforcement, information sharing, and idiosyncratic uncertainty. The first specification is the one without uncertainty. First, creditor rights index, ranging from 0 to 4 with a higher index corresponding to stronger rights, has a statistically significant effect on the credit-to-output ratio. An increase in the index by 1, for example, is associated with 9.4 percentage points increase in the ratio. Second, the quality of law enforcement, proxied by the log of number of days to resolve a payment lawsuit in courts, is also significant. With the smaller number corresponding to a higher quality, it is negatively associated with the ratio. Third, the information sharing index, taking 0 or 1 (1 indicating the availability of either a public registry or a private bureau in a country; 0 indicating the availability of neither), also has a significant effect on the ratio. When information is shared (i.e., when the index is 1), the ratio is higher by 16 percentage points.

The second specification is the one with idiosyncratic uncertainty as an additional explanatory variable. The coefficient on the uncertainty is negative and significant at a 5% level. This is consistent with our theoretical prediction. Moreover, it suggests that idiosyncratic uncertainty may be a quantitatively important as a determinant of the credit-to-output ratio. Specifically, an increase in the uncertainty (i.e., the volatility of sales growth) by 1 unit corresponds to a fall in the ratio by 1.5 percentage points. To grasp its importance, notice from Figure 3a that the uncertainty measures of most countries range approximately from 5 to 30. This difference (25) is then associated with a difference in the credit-to-output ratio of 37.5 percentage points. For example, the values of the volatility of sales growth for Italy and Botswana are 5.2 and 29.4, respectively (corresponding to the difference in the ratio of 36.3 percentage points). Meanwhile, the actual difference in the credit-to-output ratios between these countries is 61.8 percentage points. This suggests that the role of idiosyncratic uncertainty may be quantitatively important as a determinant of the credit-to-output ratio.

44 The effects of the rest of explanatory variables, i.e., GDP, GDP per capita growth, and inflation, are in line with the results shown in DMS (page 314).

45 Italy (Botswana) takes 77.6 (15.8) percent.
Table 2: Determinants of private credit to output

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creditor rights</td>
<td>9.40***</td>
<td>8.70***</td>
</tr>
<tr>
<td></td>
<td>(3.08)</td>
<td>(3.08)</td>
</tr>
<tr>
<td>Information sharing</td>
<td>16.24**</td>
<td>12.57*</td>
</tr>
<tr>
<td></td>
<td>(7.05)</td>
<td>(7.26)</td>
</tr>
<tr>
<td>Law enforcement</td>
<td>-16.48***</td>
<td>-15.09***</td>
</tr>
<tr>
<td></td>
<td>(5.03)</td>
<td>(4.74)</td>
</tr>
<tr>
<td>Idiosyncratic uncertainty</td>
<td></td>
<td>-1.53**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.75)</td>
</tr>
<tr>
<td>Constant</td>
<td>-194.9***</td>
<td>-155.7***</td>
</tr>
<tr>
<td></td>
<td>(58.05)</td>
<td>(56.36)</td>
</tr>
<tr>
<td>Observations</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.60</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses; ***p < 0.01, **p < 0.05, *p < 0.1.
Coefficients on total GDP, GDP per capita growth, and Inflation are suppressed.

6 Conclusion

We have studied the effect of cross-country variations in firm-specific (idiosyncratic) uncertainty on the private credit to output ratio. Motivated by the suggestive evidence on cross-country differences in the degree of idiosyncratic uncertainty, we have shown theoretically that in the presence of asymmetric information in the credit market, a rise in the level of uncertainty causes a fall in the credit-to-output ratio. This takes place because credit falls proportionally more than output. Output falls because a rise in uncertainty, enlarging economic distortions, reduces aggregate capital accumulation. However, the higher uncertainty, increasing firms’ bargaining power and thus providing them with larger economic rents, increases their internal funds. This, coupled with the fall in the total financing required in the lower output environment, gives rise to a larger fall in credit relative to output. From a cross-country perspective, this result implies that a country with
higher idiosyncratic uncertainty, other things equal, should be characterized by a lower credit-to-output ratio. We have shown that even after controlling for other key determinants of this ratio, idiosyncratic uncertainty appears to be negatively associated with it, thus providing a support to our theoretical prediction.

One idea for future work is to endogenize entrepreneurs’ aggregate saving rate in the theoretical model, which is currently given exogenously by their survival rate. By doing this, one would obtain a more flexible environment to consider the long-run effects of idiosyncratic uncertainty on the allocation of credit to the private sector. This is because in the face of a change in the severity of the asymmetric information problem, entrepreneurs will have an incentive to adjust their saving rate and thus their net worth.
A Derivation of the steady state of the model

The following relations characterize the steady state of the model, where $A = 1$ is assumed:

\[ Y = (1 - \mu \int_0^\omega \phi(\omega)d\omega)F(H, K) \quad (A-1) \]

\[ w\Theta(\omega) = \sigma \left( \frac{K}{H} \right)^{1-\sigma} \quad (A-2) \]

\[ r^k\Theta(\omega) = (1 - \sigma)\left( \frac{K}{H} \right)^{-\sigma} \quad (A-3) \]

\[ f(\omega)H^\sigma K^{1-\sigma} = \lambda(\omega) N \quad (A-4) \]

\[ K^e = (1 - v)f(\omega)H^\sigma K^{1-\sigma} \quad (A-5) \]

\[ N = (r^k + 1 - \delta)K^e \quad (A-6) \]

\[ r^k + 1 - \delta = \beta^{-1} \quad (A-7) \]

\[ C^e = \frac{v}{1 - \nu}K^e \quad (A-8) \]

\[ H^{\eta-1} = \frac{(C^h)^{-1}}{\chi \eta}w \quad (A-9) \]

\[ Y = C^h + C^e + \delta K \quad (A-10) \]

where in Eqs.A-2, A-3, and A-4, $\Theta(\omega)^{-1} = g(\omega) + \frac{f(\omega)}{\lambda(\omega)}$ and $\lambda(\omega) = -\frac{f'(\omega)}{g'(\omega)}$.

We now derive the steady state expression for output. First, notice from Eqs.A-3 and A-7 that

\[ H = \left( \frac{\beta^{-1} - 1 + \delta}{1 - \sigma} \Theta(\omega) \right)^{\frac{1}{\sigma}} K. \quad (A-11) \]

Eqs.A-2 and A-9 yields the labor market equilibrium condition. Rewriting this yields

\[ C^h = \frac{\sigma}{\chi \eta \Theta(\omega)}H^{\sigma - \eta}K^{1-\sigma}. \quad (A-12) \]
We obtain from Eqs. A-5 and A-8 that

\[ C^e = \upsilon_f(\omega)H^\alpha K^{1-\sigma}. \]  

(A-13)

Using Eq. A-1, the market clearing condition for the final good (Eq. A-10) is expressed as:

\[ (1 - \mu \int_0^\omega \omega^\gamma \phi(\omega) d\omega) H^\alpha K^{1-\sigma} = C^h + C^e + \delta K. \]  

(A-14)

Incorporating Eqs. A-11, A-12 and A-13 into Eq. A-14 and rearranging, we have

\[ K = \left( \frac{\chi \eta}{\sigma} \alpha^{-\frac{\eta}{\sigma}} \Theta(\omega)^{\frac{\sigma}{\sigma-\eta}} \left( 1 - \mu \int_0^\omega \omega^\gamma \phi(\omega) d\omega - \upsilon_f(\omega) - \frac{\delta \alpha}{\Theta(\omega)} \right) \right)^{-\frac{1}{\eta}}, \]  

(A-15)

where \( \alpha \equiv (1 - \sigma)/(\beta^{-1} - 1 + \delta) \). Substituting Eqs. A-11 and A-15 into Eq. A-1, we obtain

\[ Y = \left( 1 - \mu \int_0^\omega \omega^\gamma \phi(\omega) d\omega \right) \left( \frac{\chi \eta}{\sigma} \alpha^{-\frac{\eta}{\sigma}} \Theta(\omega)^{\frac{\sigma}{\sigma-\eta}} \left( 1 - \mu \int_0^\omega \omega^\gamma \phi(\omega) d\omega - \upsilon_f(\omega) - \frac{\delta \alpha}{\Theta(\omega)} \right) \right)^{-\frac{1}{\eta}}. \]  

(A-16)

The gross output, \( F(H, K) \) is given as:

\[ F(H, K) = \left( \frac{\chi \eta}{\sigma} \alpha^{-\frac{\eta}{\sigma}} \Theta(\omega)^{\frac{\sigma}{\sigma-\eta}} \left( 1 - \mu \int_0^\omega \omega^\gamma \phi(\omega) d\omega - \upsilon_f(\omega) - \frac{\delta \alpha}{\Theta(\omega)} \right) \right)^{-\frac{1}{\eta}}. \]  

(A-17)

References


