Switching-track after the Great Recession

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Abstract

Data suggests that GDP shifted to a permanently lower trend following the Great Recession for most advanced countries, and researchers have not yet reached a consensus concerning the drivers of this phenomenon. We contribute to this literature by suggesting a DSGE model with financial frictions and endogenous growth through learning-by-doing. With an aggregate AK technology, a negative shock to the capital stock has the effect of moving the economy to a lower trend. A Taylor rule policy designed to reduce the output gap may counterbalance the shock, bringing the economy back to the past trend. However, when the recession is deep, persistent and the ZLB binds, a revision of potential output measures may weaken the recoverying role of monetary policy, making the economy converge to a lower trend. We calibrate the model to the U.S. economy and find that GDP can fully recover from a textbook TFP shock under a standard Taylor rule, whilst persistently large demand shocks can affect the supply side permanently. Our framework is thus consistent with episodes of economic recovery as well as episodes of no-recovery. Results rely on the observation that the measurement of U.S. potential output switched track as the Great Recession unfolded, because the severe and prolonged slump put downward pressure on estimates. As a consequence, the output gap closed following the switching-track of potential output, rather than faster GDP growth.

Keywords: Great Recession, Economic Recovery, Endogenous Growth, Hysteresis, Trend Shift, Switching-track

JEL Codes: E12, E22, E32, O41, E52

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1 Introduction

The Great Recession had a profound impact on the economic performance of OECD countries. Crucially, the crisis had a persistent level effect on GDP, which still remains below its pre-crisis trend for the vast majority of advanced economies. Focusing on the United States, past recessions were typically followed by a temporary acceleration of growth, with GDP converging back to its original trend, in line with the pre-recession path of potential output. This is commonly referred to as a V-shaped recovery. During the last twelve years, however, we saw potential output switching-track in conjunction with the crisis, as the deep and persistent fall in economic activity led to a downward swerve of the GDP trend from its original path. Consequently, the output gap closed following the switching-track of potential output, rather than faster GDP growth, giving rise to a so-called L-shaped recovery (see Figure 1).

![Figure 1: US GDP per capita](image)

This fact challenged the general consensus on the distinct relationship between trend, growth and business cycles. If recessions are not followed by recoveries, downturns will affect the
long run path of GDP, implying that potential output cannot be represented by a stable trend in the productive capacity of the economy.

In the words of Janet Yellen: “Are there circumstances in which changes in aggregate demand can have an appreciable, persistent effect on aggregate supply? Prior to the Great Recession, most economists would probably have answered this question with a qualified ‘no.’ (...) This conclusion deserves to be reconsidered in light of the failure of the level of economic activity to return to its pre-recession trend in most advanced economies. This post-crisis experience suggests that changes in aggregate demand may have an appreciable, persistent effect on aggregate supply—that is, on potential output.”

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In this sense, we see the Great Recession as a large demand shock deeply reducing economic activity for an unusually long period, then translating into a permanent supply-side effect. As a consequence, GDP moved down to a lower trend, gradually inducing a change in measured potential output. The economy then converged to its new lower potential output path, ending the recession without a full recovery. We interpret the measure of potential output as policy makers’ beliefs about the level of GDP that can feasibly be sustained in the long run, and a key indicator affecting monetary policy interventions in the recovery process. Our model can reproduce the dynamics of GDP after the Great Recession, relying on the combination of four key assumptions. First, we embed learning-by-doing à la Romer (1986) into a DSGE model with financial frictions à la Christiano, Motto, and Rostagno (2014). Under this assumption, aggregate technology faces constant returns to capital. Consequently, a negative shock to the capital stock would not allow the economy to return to its previous growth trajectory, affecting the level of GDP permanently. Second, in line with Christiano et al. (2014), the Great Recession is modelled as a large negative shock to aggregate demand that induces a surge in bankruptcy. Third, the depreciation rate is assumed to be endogenous and positively related to the bankruptcy rate. A sudden increase in bankruptcy will then deplete the capital stock, inducing a permanent decline in output. Finally, the monetary

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authority follows a standard Taylor rule, but it is also assumed to revise potential output when recessions are deep and persistent. This way, we introduce the switching-track in the model. The large and persistent decline in GDP associated with the Great Recession then induces a downward revision of potential output, with the output gap shrinking until both GDP and potential output stabilize on a lower trend. Conversely, the Taylor rule makes the economy converge to the past trend in normal times, despite aggregate constant returns to capital.

Let us explain the mechanics of the model in more detail. First, we model the Great Recession as a demand shock that combines higher aggregate risk and lower consumer confidence, reducing investment and consumption demand. Like in Christiano et al. (2014), an increase in credit risk reduces credit and raises the probability of bankruptcy at equilibrium.

Second, the rise in bankruptcies that follows the demand shock causes capital destruction and thus a permanent supply effect. To generate this result, we augment the framework in Christiano et al. (2014) by positively linking the probability of bankruptcy to the depreciation rate of capital. During the Great Recession, a surge of bankruptcy leads to a depletion of the capital stock. The recession is then driven by a large shock to aggregate demand inducing supply effects. Indirect evidence on capital destruction following the Great Recession is provided by Lanteri (2018), who analyses the dynamics of the second-hand market for equipment in the United States.\(^2\) Firstly, he finds that quantities sold vary procyclically. Procyclicality of the second-hand market of capital goods is an indirect evidence of an increase of the depreciation rate of capital during recessions. More importantly, Lanteri (2018) finds that the price of second hand equipment is procyclical and more volatile than the price of new equipment, showing a significant decline at the beginning of the Great Recession (see Figure 2).

\(^2\)Lanteri (2018) collects data on quantities and prices from sales of second hand capital, including commercial aircrafts, ships, and construction equipment, among others.
Figure 2: Log-deviation from trend of the price of used compared to new capital goods. Data made available by Lanteri (2018).

Figure 3: Capital quality. Constructed following methodology suggested by Kozlowski et al. (2015), all details in the Appendix.
Additional evidence is provided by Kozlowski et al. (2015), who construct a measure of the quality of corporate non-financial assets, which fundamentally captures variations in the market value of structures. As it can be observed in Figure 3, their measure of capital quality was unusually low in conjunction with the Great Recession, which can be interpreted in their framework as an increase in the depreciation rate of capital. Overall, these results suggest that the productive capacity of capital goods fell during the Great Recession, and constitutes additional evidence indicating that downturns can be associated with increasing capital depreciation rates.

Third, monetary policy interventions aimed at reducing the output gap systematically induce a recovery. We model monetary policy interventions as a standard Taylor rule, including inflation and the output gap, but we measure potential output after the Great Recession as a moving average of past GDP values. This is meant to capture the observed switching-track of potential output, resulting from the unprecedented revision process of potential output estimates conducted by the Congressional Budget Office (Coibion et al., 2017b). Figure 4 shows that the output gap closed following the potential output switching-track process instead of faster growth, and thus the strength of the policy intervention weakened over time. In our framework, a persistent increase in risk raises the bankruptcy rate, destroying capital for a period long enough for the monetary authority to revise potential output down, inducing a potential output switching-track and, consequently, making the economy converge to a lower trend. In the model, the policy rule induces a severe reduction of the nominal interest rate as the shock hits. The economy reaches the zero lower bound for a while, exacerbating the severity of the recession. We also show that the Taylor rule is sufficient to drive a V-shaped recovery when shocks are small, by modelling normal times business cycles as standard TFP shocks and small demand shocks.
The rest of the paper is organized as follows: Section 2 describes how the paper fits into the literature, Section 3 explains the model, Section 4 reports the calibration, Section 5 contains results for the Great Recession simulations, Section 6 describes normal times, Section 7 relaxes the AK assumption and Section 8 concludes.

## 2 Context and Motivation

“Extreme economic events have often challenged existing views of how the economy works and exposed shortcomings in the collective knowledge of economists. (...) The financial crisis and its aftermath might well prove to be a similar sort of turning point.”

The slow recovery that followed the Great Recession caught the profession by surprise,
thus many prominent researchers and policy makers have offered their contribution to the ongoing debate. Empirical work has documented large long-term negative effects on output for most OECD countries (Ball, 2014). Hall (2014, 2016) focuses on the United States’ experience, and identifies the shortfalls in business capital and total factor productivity as the main drivers of the deviation of GDP from its previous trend. This paper follows this idea by associating the Great Recession with a substantial decline in the stock of capital and, through learning-by-doing, in total factor productivity.

In recent years, the consequences of the Great Recession are motivating a general rethinking of the underlying mechanisms of growth and two hypothesis have become object of debate: secular decline vs endogenous response to the crisis. The former is advocated by Fernald et al. (2017), who argue that the slowdown in TFP growth prior to the Great Recession casts doubt on the hypothesis of a causal link between downturns and changes in potential output. Such findings are consistent with ideas expressed by Gordon (2015), who points to supply-side headwinds to explain the change in GDP trend.

On the other hand, studies that advocate the endogenous response hypothesis stress that evidence of the unfading effects of recessions was already available prior to the Great Recession. For example, some noteworthy empirical papers argued that financial crises tend to lead to shortfalls in aggregate supply across developed and developing countries (Cerra & Saxena, 2008) (Reinhart & Rogoff, 2014). Moreover, the idea that demand-driven recessions could have permanent supply effects, i.e. hysteresis, had already been an object of debate in the 1980s, as Blanchard and Summers (1986) argued this possibility. However, this strand of literature was mostly abandoned due to the advent of the so-called Great Moderation, a period characterized by reduced macroeconomic volatility. Negative shocks were small in size and the GDP trend remained stable. Many studies have been conducted on the drivers of this phenomenon, which is considered to be the result of good luck and/or good policy.4

In the current debate, the hypothesis of output hysteresis is gaining new traction, lead-

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4See for example Clarida, Gali, and Gertler (2000) and Stock and Watson (2002).
ing to a re-evaluation of the role of policy interventions as well (Summers, 2014). Benigno and Fornaro (2017) build a Schumpeterian framework with nominal rigidities in the spirit of Aghion and Howitt (1992), and present a theory to show that a fall in demand can lead to a stagnation trap, i.e. an equilibrium with low growth and employment. Pessimistic expectations, the ZLB and the procyclicality of innovation investment are crucial to generate their results. They show that the existence of the unemployment steady state is a consequence of the ZLB constraint on monetary policy, which would otherwise restore the initial level of employment and the growth rate. Similarly to our paper, they identify monetary policy as a key element in determining the new steady state after a shock. However, they don’t introduce a mechanism to undo the negative level effect of the shock on technology, and therefore on GDP, in the case of a temporary stagnation trap. Output thus becomes permanently lower in their framework, irrespectively of the policy constraints, as the Taylor rule stabilizes employment, rather than the output gap. Singh and Garga (2018) take a similar approach, but focusing on the dynamic response of the model to demand shocks, rather than on steady state analysis, and highlight the permanent level effect of demand shocks on output, with a Taylor rule targeting employment. They stress that an alternative policy rule could counteract the negative effects of demand shocks, but their result does not hold in the case of TFP shocks.

Other papers focus on explaining the lack of recovery following financial crises in particular, and thus bring together endogenous growth and financial shocks. Bianchi, Kung, and Morales (2019) also propose a Shumpeterian model, adding financial frictions to explore the properties of different kinds of financial shocks, and their long lasting effects on the economy. Cozzi et al. (2017) propose an estimated Shumpeterian DSGE model and stress how the inclusion of endogenous growth leads to the amplification of financial shocks.

Anzoategui et al. (2019) build on the work of Comin and Gertler (2006), employing endogenous growth models à la Romer (1990) with financial frictions, with the aim to generate an endogenous response of TFP to aggregate demand shocks through the R&D channel.
They show that a shock equivalent to a risk shock can generate permanent effects on the level of TFP due to a temporary slowdown in productivity enhancing investments. The paper contributes to the debate on the drivers of the slowdown in aggregate TFP, but does not directly address the permanent level effect on output, as the model predicts a quick, but incomplete recovery after the shock. It thus cannot replicate the dynamics of GDP data after the Great Recession, as the level of economic activity fell and remained lower, growing in parallel with the previous trend. Ikeda and Kurozumi (2019) choose a similar framework, and, just like us, they propose a model where financial shocks can generate permanent shortfalls in GDP, resulting in a parallel downward shift in the level of economic activity. However, their framework does not provide a mechanism to generate V-shaped recoveries in normal times. Monetary policy does not induce a recovery, as the class of Taylor rules they consider employs the growth rate of GDP rather than the output gap. Queralto (2019) also contributes to this literature, by focusing on banking crises in an open economy expanding product variety framework. The paper shows that financing frictions can affect the introduction of new varieties, and thus endogenous TFP.

Overall, these models provide insights on the possible mechanisms through which endogenous growth can lead to output shortfalls, but such theories tend to remain silent concerning the drivers of recovery in the past and/or in different scenarios. This paper, indeed, aims to reconcile episodes of economic recovery in periods characterized by small shocks, such as the Great Moderation, with episodes of no recovery, such as the Great Recession. To achieve this, we propose a novel way of incorporating endogenous growth into DSGE models with financial frictions, employing a learning-by-doing technology à la Romer (1986), where knowledge is crucial for economic growth, and its accumulation is directly linked to the capital stock. Moreover, we introduce a new mechanism for capital depreciation and a Taylor rule that allows for switching-track. One of the main contributions of this paper is the introduction of a new channel for of capital destruction in recessions. The latter is notoriously difficult to identify in the data. National Accounts estimates of the capital stock are con-
structured using the perpetual inventory method, and typically employ geometric depreciation rates, held constant over time.\footnote{For the United States, see Fraumeni (1997).} As a consequence, the measurement of the capital stock in National Accounts reflects movements of investment, but cannot reflect cyclical capital destruction. Nonetheless, the idea that recessions destroy capital is not new, and we aim to provide new insights to the existing literature. For example, in a vintage capital framework, 

\textit{Caballero and Hammour (1994)} provided a rational to the evidence stated by \textit{Davis and Haltiwanger (1990)} that job destruction is much more cyclically responsive than job creation. Which bring them to understand “\textit{recessions as times of ‘cleansing,’ when outdated or relatively unprofitable techniques and products are pruned out of the productive system.}” In their view, the associated physical capital becomes obsolete and is then scrapped. More recently, \textit{Gourio (2012)} built a business cycle model with disaster risk, characterizing disasters as episodes of large capital destruction. He noted that “\textit{economic crises often lead to microeconomic volatility and large reallocation, implying that some specialized capital goods may become worthless,}” and stresses that “\textit{intangible capital (customer and employee value) \ldots is destroyed during prolonged economic depressions.}”

\textit{Kozlowski et al. (2015)} build on the Gourio framework to explain the persistent effect of the Great Recession on output. They introduce capital quality shocks that cause capital destruction and a learning process on the probability of tail events, which keeps investment low after the shock, even with a Neoclassical technology. \textit{Lanteri (2018)} also contributes to this literature, by building a model where firms face idiosyncratic productivity shocks and adjust their capital stock through investment and disinvestment, by operating in both the new and second hand capital markets. Assuming that new and used capital goods are imperfect substitutes, \textit{i.e.} investment is partially irreversible, implies that a fall in aggregate productivity will reduce the value of second-hand capital more than newly-produced capital due to its specificity. As a consequence, the least productive firms will downsize less that what would be optimal, causing misallocation in the economy and thus a shortfall in the
productive capacity of capital.

3 The Model

We build a New Keynesian model of endogenous growth by introducing a learning-by-doing technology à la Romer (1986) in a Dynamic Stochastic General Equilibrium model with financial frictions. We build on the financial accelerator framework introduced by Bernanke, Gertler, and Gilchrist (1998) and draw from Christiano et al. (2014) for risk shocks. Households gain utility from consumption, dis-utility from labour, and are subject to confidence shocks, i.e. reductions in their marginal utility of consumption. The production sector is comprised of a labour union, final good producers, intermediate good producers, capital producers and entrepreneurs. The latter face financial frictions, since they need to fund part of their capital expenditure with external resources, exposing themselves to the possibility of bankruptcy. We model a risk shock as an increase in the probability of bankruptcy for entrepreneurs. The financial sector acts as an intermediary between the production side and households, by providing loans to entrepreneurs and selling bonds to savers. The monetary authority sets the nominal interest rate according to a Taylor rule, factoring in the deviation of inflation from its target and the output gap. We also assume that potential output is measured as an average of past GDP values, so that large negative shocks lead to downward revisions. The following sections describe the behavior of all agents in the model.

3.1 Labour Packer

Households, indexed by $\ell \in (0,1)$, supply differentiated labour input to a labour packing firm, i.e. a union, which then supplies an homogeneous labour input $h^p_t$ to the production sector. The bundling technology is

$$ h^p_t = \left( \int_0^1 h_t(\ell) \frac{\ell}{\epsilon} \, d\ell \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (1) $$
The parameter $\varepsilon > 1$ represents the elasticity of substitution across types of labour, which are assumed to be gross substitutes, and $h_t(\ell)$ represents hours worked by household $\ell$. Labour demand by the labour packer is

$$h_t(\ell) = \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} h_t^p,$$

where $W_t(\ell)$ is the wage of labor variety $\ell$. Substituting (2) into (1), we obtain the aggregate wage index

$$W_t^{1-\varepsilon} = \int_0^1 W_t(\ell)^{1-\varepsilon} d\ell. \quad (3)$$

Thus total employment (measured in hours worked) is given

$$h_t \equiv \int_0^1 h_t(\ell)d\ell = h_t^p \int_0^1 \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} h_t^p d\ell. \quad (4)$$

Defining $\int_0^1 \left( \frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} d\ell$ as a measure of wage dispersion across varieties implies that if the latter is larger than 1, then aggregate labour used in production is smaller than total employment. However, in this paper we will only consider symmetric equilibria, implying that the total labor supply $h_t^p = h_t$.

### 3.2 Households

There is a continuum of households of measure one. The $\ell$-type household consumes, supplies labour, buys bonds and owns firms subject to wage frictions à la Rotemberg (1982);\(^6\) when changing its wage, it incurs in a cost assumed to be proportional to aggregate output. Since households are assumed to be identical, differing only on the type of labor they offer, labor market equilibrium is symmetric. We will then omit index $\ell$ to simplify notation.

A representative household, offering a particular type of labor, maximises utility subject

\(^6\)We choose this specification for price and wage frictions because they are better suited in the context of large shocks compared to the standard Calvo style. The letter imply a constant probability of resetting prices, which does not account for size effects. For a detailed discussion see Reiff and Karadi (2012).
to its budget constraints

$$\max_{c_t, w_t, B_{t+1}} \quad u_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \epsilon_t^c \left( \log \left( c_{t+j} - \chi c_{t-j} \right) - \psi \frac{h_{t+j}}{1 + \nu} \right) \right]$$  \hspace{1cm} (5)

s.t. $B_{t+1} + P_t c_t = R_{t-1} B_t + P_t w_t h_t - \frac{\chi \omega}{2} \left( \frac{w_t}{w_{t-1}} - 1 \right)^2 P_t Y_t + D_t - \tau_t,$  \hspace{1cm} (6)

and the labor demand (2), where $c_t$ is consumption and $h_t$ is labor, $\beta \in (0, 1)$ is the time discount factor, $\chi > 0$ regulates the degree of habit formation, $\nu > 0$ is the inverse of labour supply elasticity, $\psi > 0$ is a parameter regulating labour hours in steady state and $\chi^w > 0$ regulates price frictions. $B_{t+1}$ represent riskless one period nominal bonds, purchased at time $t$, earning the risk-less nominal interest rate $R_t$. Variable $w_t$ represents the real wage rate, $P_t$ is the price of the final good, $D_t$ are profits redistributed to households by firms and $\tau_t$ are taxes. $\epsilon_t^c$ is a confidence shock, affecting the marginal utility of consumption, which follows an AR(1) process of the form:

$$\log(\epsilon_t^c) = \rho_c \log(\epsilon_{t-1}^c) + \epsilon_{c,t}$$  \hspace{1cm} (7)

where $\rho_c \in (0, 1)$ and $\epsilon_{c,t} \sim N(0, \sigma_c^2)$ . The FOCs for $c_t$ and $B_{t+1}$ in real terms are:

$$\lambda_t = (c_t - \chi c_{t-1})^{-1} \epsilon_t^c + \beta \chi E_t (c_{t+1} - \chi c_t)^{-1} \epsilon_{t+1}^c$$  \hspace{1cm} (8)

$$\lambda_t = \beta E_t R_t \frac{\lambda_{t+1}}{\pi_{t+1}},$$  \hspace{1cm} (9)

where $\lambda_t$ is the Lagrange multiplier associated to the budget constraint and $\pi_t = \frac{P_t}{P_{t-1}}$.

The FOC for $w_t$ gives the wage Phillips curve:

$$w_t = \frac{\epsilon}{\epsilon - 1} \psi h_t \frac{\epsilon_t^c}{\lambda_t} + E_t \left[ \frac{\beta \lambda_{t+1} Y_{t+1} Y_t}{\lambda_t h_t} - \Omega_t \pi_t \frac{Y_t}{h_t} \right]$$  \hspace{1cm} (10)

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*All nominal variables in this paper are defined in an arbitrary numeraire.*
\[ \Omega_t = \frac{\chi_t^w}{\varepsilon - 1}(\pi_t^w - 1)\pi_t^w \]  
(11)

\[ \frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_{t-1}} , \]  
(12)

where \( \varepsilon \) is the degree of substitution across labour types. The RHS of (10) shows that wages depend on the wage markup, the marginal rate of substitution and expectations. The expectation term implies that the labour supply is forward looking, and therefore will be less sensitive to contemporaneous shocks. Equation (12) is an identity to pin down the equilibrium.

### 3.3 Final Good Sector

The final good \( Y_t \) is produced under perfect competition and can be turned into consumption or investment, as well as used to cover the costs associated with financial, price and wage frictions. Production uses intermediate goods as inputs according to the following CES technology

\[ Y_t = \left( \int_0^1 y_t(i) \frac{\theta - 1}{\theta} \, di \right)^{\frac{\theta}{\theta - 1}} \]  
(13)

with the elasticity of substitution \( \theta > 1 \). The associated demand function for intermediate goods is

\[ y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t \]  
(14)

with aggregate price index

\[ P_t = \left( \int_0^1 p_t(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}} . \]  
(15)

### 3.4 Intermediate Good Sector

Each intermediate firm \( i, i \in (0,1) \) operates under monopolistic competition. It employs capital services and labour by the mean of the following technology

\[ y_t(i) = a_t K_t^\eta k_t(i)^{\alpha} h_t(i)^{1-\alpha}, \quad \alpha \in (0,1) \]  
(16)
where $K_t$ represents a measure of knowledge, freely available to all firms and acquired through learning-by-doing. We assume that $K_t = k_t$, where $k_t = \int_0^1 k_t(i)di$. This implies that $K$ is a pure externality that comes from the aggregate level of capital employed in the economy, and $0 \leq \eta \leq 1 - \alpha$ represents the strength of the spillovers. $a_t$ is an aggregate productivity shock, following

$$\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t},$$

$\rho_a \in (0, 1)$ and $\epsilon_{a,t} \sim N(0, \sigma^2)$. This is a moderate shock, typical of business cycle dynamics.

Solving the firms’s problem, we find labour and capital services demand

$$w_t = (1 - \alpha)s_t \frac{y_t(i)}{h_t(i)}$$  \hspace{1cm} (17)

$$r^k_t = \alpha_s t \frac{y_t(i)}{k_t(i)}$$  \hspace{1cm} (18)

where $s_t$ is the real marginal cost function

$$s_t = \frac{1}{\alpha^a(1 - \alpha)^{1-\alpha}} \frac{1}{a_t K_t^\eta} w_t^{1-\alpha} (r^k_t)^\alpha.$$  \hspace{1cm} (19)

Intermediate good producers are monopolistically competitive and they face an adjustment cost when changing prices. Following Rotemberg (1982), the adjustment cost increases with the magnitude of the change in prices and the size of the economy. It is given by

$$\phi_p \frac{2}{2} \left( \frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t,$$  \hspace{1cm} (20)

where $\phi_p \geq 0$ is a measure of price rigidities.

Using the demand function for intermediate goods and assuming symmetry, the first order condition of the optimization problem yields the New Keynesian Phillips curve:

$$(1 - \theta) + \theta s_t - \pi_t \phi_p(\pi_t - 1) + \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1} \phi_p(\pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \right] = 0.$$  \hspace{1cm} (21)
3.5 Capital Producers

There is a unit mass of identical perfectly competitive capital producers. Each period \( t \), the representative capital producer buys undepreciated capital \((1 - \delta_t)k_t\) at nominal price \( q_t \), uses it to produce new capital \( k_{t+1} \) by combining it with investment \( i_t \), and then sells \( k_{t+1} \) units to entrepreneurs at the same price, implying that the price of second-hand capital units relative to new capital units is \((1 - \delta_t) < 1\). The behaviour of the endogenous depreciation rate \( \delta_t \) is modelled in the following subsection. The evolution law of raw capital reads

\[
    k_{t+1} = i_t \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) + (1 - \delta_t)k_t. \tag{22}
\]

As in Christiano et al. (2014), investment is subject to the adjustment cost function \( S \left( \frac{i_t}{i_{t-1}} \right) \). This assumption helps reducing the volatility of investment and tames inflation as the reaction of investment to shocks is smoother.

Capital producers maximise their flow of profits subject to the evolution law of capital above. The FOC reads

\[
    1 = \hat{q}_t \left( -S' \left( \frac{i_t}{i_{t-1}} \right) \left( \frac{i_t}{i_{t-1}} \right) + 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) + \beta E_t \lambda_{t+1} \hat{q}_{t+1} S' \left( \frac{i_{t+1}}{i_{t+1}} \right) \left( \frac{i_{t+1}}{i_t} \right)^2, \tag{23}
\]

where \( \hat{q}_t = q_t / P_t \) is the real price of capital.

3.6 Entrepreneurs and Financial Intermediation

This section closely follows Christiano et al. (2014), with a few differences designed to generate an endogenous depreciation rate depending on the fraction of firms going bankrupt. There is a unit mass of perfectly competitive entrepreneurs. At the end of any period \( t \), each entrepreneur has a net worth \( N, N > 0 \).\(^8\) Even if the net worth of a particular entrepreneur

\(^8\)As for financial frictions, the model mainly draws from Bernanke et al. (1998), so we assume that the entrepreneur employs its own net worth \( N \) as well as loans from financial intermediaries, \textit{i.e.} mutual funds, to finance his venture.
is changing over time, we omit index \( t \) to simplify notation. At equilibrium, net worth is distributed \( f_t(N) \) across entrepreneurs, with total net worth

\[
\bar{N}_{t+1} = \int_0^\infty N f_t(N) \, dN. \tag{24}
\]

At the end of period \( t \), entrepreneurs use their net worth \( N \) and loans \( B_{t+1}^N \), \( B_{t+1}^N \geq 0 \), to acquire capital \( k_{t+1}^N \) from capital producers.\(^9\) They pay price \( q_t \) for any capital unit they buy such that

\[
q_t k_{t+1}^N = N + B_{t+1}^N. \tag{25}
\]

At equilibrium, all capital is allocated to entrepreneurs, s.t.,

\[
k_{t+1} = \int_0^\infty k_{t+1}^N f_t(N) \, dN. \tag{26}
\]

As shown below, like in Christiano et al. (2014), the equilibrium debt-to-net-worth ratio does not depend on \( N \), implying that loans and capital are proportional to it. At \( t + 1 \), entrepreneurs use capital \( k_{t+1}^N \) to produce capital services \( \omega k_{t+1}^N \) that they sell to intermediate firms at price \( r_{t+1}^k P_{t+1} \), where \( \omega \) is an entrepreneur specific productivity shock and \( P_{t+1} \) is the aggregate price index. Entrepreneurs draw the idiosyncratic productivity \( \omega \) at period \( t \) after buying capital \( k_{t+1}^N \). Idiosyncratic productivity \( \omega \) is assumed to be i.i.d. across time and firms, drawn at \( t \) from the c.d.f. \( F_t(\omega) \), lognormally distributed, with unit mean and standard deviation \( \sigma_t \).

An entrepreneur with net worth \( N \) obtains a loan \( B_{t+1}^N \) from mutual funds at the interest factor \( Z_{t+1} \). The interest factor is contingent on the state of the economy in \( t + 1 \) and, as shown below, it is independent of \( N \) at equilibrium. For this reason, index \( N \) is omitted. On top of aggregate risks, the debt contract has to take into account the presence of idiosyncratic risk, since entrepreneurs facing low realizations of the shock \( \omega \) may be unable to repay the

\(^9\)As usual, for an arbitrary asset \( X \), \( X_{t+1} \) refers to the amount of this asset transferred from \( t \) to \( t + 1 \).
loan, and go bankrupt. Conversely, entrepreneurs with sufficiently high returns on their capital will repay their loans and make positive cash flow. For a given state contingent interest factor $Z_{t+1}$, let us define $\bar{\omega}_{t+1}$ as the state contingent productivity $\omega$ that zeroes the entrepreneur’s cash flow at $t+1$, i.e.,

$$\Pi^N_{t+1}(\omega_{t+1}) = \bar{\omega}_{t+1} R^k_{t+1} q_t k^N_{t+1} - B^N_{t+1} Z_{t+1} = 0,$$

where $\Pi^N_{t+1}(\omega)$ represents the entrepreneur’s cash flow.\(^\text{10}\) If $\omega > \bar{\omega}_{t+1}$, then $\Pi^N_{t+1}(\omega) > 0$, whilst if $\omega < \bar{\omega}_{t+1}$, the entrepreneur goes bankrupt. The fraction of firm that go bankrupt at period $t$ is then $F_{t-1}(\bar{\omega}_t)$.

At $t+1$, after production takes place, an $\omega$-type successful entrepreneur sells its undepreciated capital $(1 - \hat{\delta}_{t+1})\omega$ back to capital producers at price $q_{t+1}$. The depreciation rate of capital is $\hat{\delta}_{t+1} = \delta \left( \frac{F_{t-1}(\bar{\omega}_t)}{F(\bar{\omega})} \right)^{a_\omega}$, $a_\omega \geq 0$, where $\delta \in (0, 1)$ is a parameter representing the depreciation rate at steady state. This specification includes negative network spillovers from entrepreneurs that went bankrupt in the previous period. It is a simplified way of representing the negative network effects on capital value associated with the disruptions in the production process generated by economic downturns and bankruptcy. A larger bankruptcy rate will make the capital of surviving firms less productive in the future due to the destroyed links. When the probability of bankruptcy is at its steady state value $F(\bar{\omega})$, the impact of spillovers is normalized to 1. The ex-post return at $t+1$ to a unit of capital bought at $t$ for an $\omega$-type successful entrepreneur is $\omega R^k_{t+1}$, with

$$R^k_{t+1} = \frac{r^k_{t+1} P_{t+1} + (1 - \hat{\delta}_{t+1}) q_{t+1}}{q_t}.$$

In case of bankruptcy, the mutual fund pays a monitoring cost $\mu$, $\mu \in (0, 1)$, to appropriate the payments generated by the capital services provided to intermediate firms as well as the capital stock, which is then liquidated, subject to physical depreciation and obsolescence.

\(^\text{10}\)Like $Z_{t+1}$, $\bar{\omega}_{t+1}$ is independent of $N$ at equilibrium. For this reason, index $N$ is omitted.
Moreover, when an entrepreneur goes bankrupt, the steady state depreciation rate of capital is \( \kappa, \kappa \in (\delta, 1) \) which is meant to capture both physical depreciation and obsolescence, obsolescence being measured by the difference \( \kappa - \delta \). Negative network spillovers affect bankrupt entrepreneurs too, so that \( \hat{\kappa}_{t+1} = \kappa \left( \frac{F_{t+1}(\bar{\omega})}{F(\bar{\omega})} \right)^{a_{\omega}} \), \( a_{\omega} \geq 0 \). The ex-post return to capital for bankrupt \( \omega \)-type entrepreneur is \( \omega R_{t+1}^{f} \), with

\[
R_{t+1}^{f} = \frac{r_{t+1}^{k} P_{t+1} + (1 - \hat{\kappa}_{t+1})q_{t+1}}{q_{t}} \quad (29)
\]

which in this case is appropriated by mutual funds.

At time \( t \) mutual funds issue bonds to households at the risk-less factor \( R_{t} \) to raise the resources needed to finance entrepreneurs. They also receive a transfer from the Monetary Authority. The transfer is financed by a tax on household’s profits from entrepreneurial activity, and is thus proportional to the return to capital of successful entrepreneurs. The parameter \( \xi \) regulates the size of the transfer. This is a measure implemented to relax the cash constraint, aiming to promote credit provision in bad times, and will assure steady state values in line with U.S. data. For simplicity, let us assume that mutual funds specialize in entrepreneurs with net worth \( N \) and operate under perfect competition. Since the interest factor \( Z_{t+1} \) is state contingent, at each state of nature a zero profit condition holds, \( i.e.: \)

\[
(1 - \mu)q_{t}k_{t+1}^{N} \int_{0}^{\bar{\omega}_{t+1}} \omega R_{t+1}^{f} dF(t) + (1 - F(t(\bar{\omega}_{t+1}))) B_{t+1}^{N} Z_{t+1} + \xi \int_{0}^{\bar{\omega}_{t+1}} \omega dF(t) R_{t+1}^{f} q_{t} k_{t+1}^{N} = B_{t+1}^{N} R_{t}. \quad (30)
\]

Divide both sides by \( R_{t+1}^{k} q_{t} k_{t+1}^{N} \), use (27) and define leverage \( L_{t} = \frac{N+R_{t+1}^{N}}{N} \) to get

\[
(1 - \mu) \frac{R_{t+1}^{f}}{R_{t+1}^{k}} \int_{0}^{\bar{\omega}_{t+1}} \omega dF(t) + \bar{\omega}_{t+1}(1 - F(t(\bar{\omega}_{t+1}))) + \xi \int_{0}^{\bar{\omega}_{t+1}} \omega dF(t) = \frac{R_{t}}{R_{t+1}^{k}} \frac{L_{t} - 1}{L_{t}}. \]

The zero profit condition above can then be used to find an expression for leverage

\[
L_{t} = \left( 1 - \frac{R_{t+1}^{k}}{R_{t}} \left( \bar{\omega}_{t+1}(1 - F(t(\bar{\omega}_{t+1}))) + (1 - \mu)H(t(\bar{\omega}_{t+1}) + \xi G(t(\bar{\omega}_{t+1}) \right) \right)^{-1}. \quad (31)
\]
where

\[ H_t(\bar{\omega}_{t+1}) = \frac{R_{t+1}^f}{R_{t+1}^k} \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) \]

\[ G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega). \]

\( G(\bar{\omega}), G(\bar{\omega}) < 1, \) represents unsuccessful entrepreneurs’ contribution to the average \( \omega, \) and \( H(\bar{\omega}), H(\bar{\omega}) < G(\bar{\omega}), \) is corrected by the ratio of unsuccessful to successful returns.

Notice that the loan contract \((Z_{t+1}, B_{t+1}^N)\) can be also written as a contract on \((\bar{\omega}_{t+1}, L_t)\). Any pair \((\bar{\omega}_{t+1}, L_t)\) that satisfies (31) is a \((t+1)\)-state contingent contract offered to entrepreneurs. As it will become clear below, at equilibrium, the conditions of the loan contract \((\bar{\omega}_{t+1}, L_t)\) are the same for all entrepreneurs irrespective of their net worth \(N\). On one side, for a given net worth \(N\), choosing loan \(B^N\) is equivalent to choosing leverage \(L\). On the other side, setting the nominal interest rate \(Z\) determines the cutoff productivity \(\bar{\omega}\).

At time \(t+1\), for any realization of the aggregate shocks, the debt contract \((\bar{\omega}_{t+1}, L_t)\) for an entrepreneur with net worth \(N\) is expected to generate the cash flow

\[ \int_{\bar{\omega}_{t+1}}^{\infty} \Pi_{t+1}^N(\omega) dF(\omega) = \left(1 - \Gamma_t(\bar{\omega}_{t+1})\right) R_{t+1}^k L_t N, \quad (32) \]

where \(1 - \Gamma_t(\bar{\omega}_{t+1})\) is the expected share of total revenues retained for successful entrepreneurs, with

\[ \Gamma_t(\bar{\omega}_{t+1}) = G_t(\bar{\omega}_{t+1}) + \bar{\omega}_{t+1} \left(1 - F_t(\bar{\omega}_{t+1})\right) \quad (33) \]

being the expected share going to the mutual fund.

For any state of nature in \(t+1\), the entrepreneur chooses the contract that maximizes expected profit, which is equivalent to

\[ \max_{\bar{\omega}_{t+1}} \frac{1 - G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1} \left(1 - F_t(\bar{\omega}_{t+1})\right)}{1 - \frac{R_{t+1}^k}{R_t^k} \left(\bar{\omega}_{t+1} \left(1 - F_t(\bar{\omega}_{t+1})\right) + (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\right)} = \max_{\bar{\omega}_{t+1}} L_t(\bar{\omega}_{t+1})(1 - \Gamma_t(\bar{\omega}_{t+1})). \quad (34) \]

\(^{11}\)Use (31) to substitute for \(L_t\) in (32), then divide by \(R_{t+1}^k N\) to get (34).
Thus the FOC pinning down the equilibrium $\bar{\omega}_{t+1}$ reads:

$$
1 - F_t(\bar{\omega}_{t+1}) = \frac{R^k_{t+1} [1 - F_t(\bar{\omega}_{t+1}) - G_t(\bar{\omega}_{t+1}) (1 - R^F_{t+1} (1 - \mu) - \xi)]}{1 - R^k_{t+1} (\bar{\omega}_{t+1} (1 - F_t(\bar{\omega}_{t+1})) + (1 - \mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1}))}.
$$

(35)

This shows that the loss of accepting a higher threshold for entrepreneurs equals the benefit of higher leverage. The LHS is the elasticity of the share the entrepreneur keeps w.r.t. $\bar{\omega}_{t+1}$, whilst the RHS is the elasticity of leverage w.r.t. $\bar{\omega}_{t+1}$.\footnote{The FOC also shows that our framework reduces to the standard model in Christiano et al. (2014) in the case $R^k = R^F$ and $\xi = 0.$} Also, notice that this specification implies that the elasticity of leverage is affected by credit subsidies.

It is easy to see that the equilibrium $\bar{\omega}_{t+1}$ does not depend on $N$. From (31), leverage does not depend on it either. Consequently, from (27), mutual funds set the same $(t+1)$-state contingent interest factor $Z_{t+1}$ irrespective of net worth.

Let us assume the standard deviation of $F_t(\omega)$ follows

$$
\log \left( \frac{\sigma_t}{\bar{\sigma}} \right) = \rho_\sigma \log \left( \frac{\sigma_{t-1}}{\bar{\sigma}} \right) + \epsilon_{\sigma, t},
$$

with $\bar{\sigma} > 0$, $\rho_\sigma \in (0, 1)$ and the risk shock $\epsilon_{\sigma, t}$ being i.i.d. Notice that a higher value of $\sigma_t$ implies a higher probability of drawing a low value of $\omega$. As the variance of the shock increases, the tails of the distribution get thicker, increasing the probability of tail events and modifying the threshold for $\bar{\omega}$, i.e., increasing the probability of bankruptcy.

Let us finally assume that at the end of period $t+1$ (after entrepreneurs pay back to mutual funds their period $t$ debt) a fraction $(1 - \gamma)$ of successful entrepreneur’s cash flow $\Pi^N_{t+1}(\omega)$ gets transferred to households, $\gamma \in (0, 1)$. Moreover, each entrepreneur receives a transfer $w^e P_{t+1} k^N_{t+1}$ from households, $w^e \in (0, 1)$, as a form of insurance, to compensate for risk taking, and ensuring that bankrupt entrepreneurs will keep a strictly positive net worth allowing them to buy some capital for the following period.

Since liquidating the capital of failed entrepreneurs generates physical capital depletion, the depreciation of capital is endogenous and depends on the fraction of entrepreneurs going bankrupt.
The aggregate depreciation rate at time $t$ reads

$$1 - \delta_t = (1 - \hat{\kappa}_t) F_{t-1}(\bar{\omega}_t) + (1 - \hat{\delta}_t)(1 - F_{t-1}(\bar{\omega}_t)).$$

The intuition is that when bankruptcy happens some capital (tangible or intangible) is destroyed in the process, reducing the overall value of capital. The externality aims to capture negative disruptive spillovers generated by bankruptcy. These could be the result of some specialized machines not being reallocated as effectively as in normal times, or some important network links being destroyed, or knowledge embedded in intangible capital being lost.

### 3.7 Aggregate Economy

The quantity of capital produced by capital producers must be equal to the capital purchased by entrepreneurs:

$$k_{t+1} = \int_0^\infty k_{t+1}^N f_t(N) dN.$$  

From (25), (31) and the definition of leverage, $L_t = \frac{N_t + B_{t+1}^N}{N_t}$, the equation above becomes

$$q_t k_{t+1} = \frac{1}{1 - \frac{R_{t+1}^N}{R_t} \left( \bar{\omega}_{t+1}(1 - F_{t}(\bar{\omega}_{t+1})) + (1 - \mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1}) \right)} \bar{N}_{t+1}. \quad (36)$$

Consequently, the level of capital in the economy depends on aggregate net worth, as defined in (24), and financial conditions.

All intermediate firms face the same wage and capital cost, therefore, by symmetry:

$$\frac{k_t(i)}{h_t(i)} = \frac{k_t}{h_t}, \quad (37)$$

for all $i \in (0, 1)$. Also, market clearing for capital and labour implies $\int_0^1 k_t(i) di = k_t$ and $\int_0^1 h_t(i) di = h_t$. Notice that if firms could change their prices in every period, they will choose the same price and produce the same quantity. In which case, $P_t(i) = P_t$ and $Y_t(i) = Y_t$, hence aggregate production would become

$$Y_t = a_t k_t^{\alpha+\eta} h_t^{1-\alpha}. \quad (38)$$
Therefore, if $\eta = 1 - \alpha$ aggregate technology has an AK structure. Aggregate profits of all entrepreneurs at the end of time $t$ are $[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^k q_{t-1}k_t$, so that aggregate net worth at $t + 1$ is:

$$\bar{N}_{t+1} = \gamma[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^k q_{t-1}k_t + P_t w^e k_t$$  \hspace{1cm} (39)$$

Using the aggregate production function and $K_t = k_t$:

$$r_t^k = \alpha s_t a_t k_t^{\alpha + \eta - 1} h_t^{1 - \alpha}$$  \hspace{1cm} (40)$$

$$w_t = (1 - \alpha)s_t a_t k_t^{\alpha + \eta} h_t^{-\alpha}$$  \hspace{1cm} (41)$$

$$s_t = \frac{1}{\alpha^\alpha (1 - \alpha)^{1 - \alpha}} \frac{1}{a_t k_t^{\eta}} w_t^{1 - \alpha}(r_t^k)^\alpha$$  \hspace{1cm} (42)$$

All bonds held by households must be equal to the amount of loans in aggregate, and transfers to mutual funds must equal taxes:

$$q_t k_{t+1} - \bar{N}_{t+1} = B_{t+1}$$

$$\xi \int_0^{\omega_{t+1}} \omega dF_t(\omega) R_t^k q_t k_{t+1} \bar{N} = \tau_t$$

Output is allocated to consumption and investment, but also to intermediary production aimed to cover price and wage adjustment costs as well as monitoring costs, i.e.,

$$Y_t = \underbrace{c_t + i_t}_{\text{GDP}} + \mu \int_0^{\omega_t} \omega dF(\omega) R_t^k \hat{q}_{t-1} k_t + \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t + \frac{\chi}{2} (\pi_t - 1)^2 Y_t.$$  \hspace{1cm} (43)$$

GDP is then defined as the sum of consumption and investment, measuring aggregate demand.
3.8 Monetary Authority

The monetary authority uses the Taylor rule to set the nominal interest rate, subject to the zero lower bound constraint:

$$R_t^m = \bar{R}^m + \rho_\pi (\pi_t - \bar{\pi}^m) + \rho_y \log \left( \frac{\hat{GDP}_t}{y^p_t} \right)$$  \hspace{1cm} (44)

$$R_t = \max(1, R_t^m),$$ \hspace{1cm} (45)

where $\bar{R}^m$ and $\bar{\pi}$ are target values, $\rho_\pi > 0$ and $\rho_y > 0$ are policy parameters, and $y^p_t$ is a measure of potential output. It is computed as a moving average of past GDP values, detrended by the stationary growth rate of the economy $g_z$, so that $\hat{GDP}_t = GDP_t (1+g_z)^t$.

We define this measure to take into account the revisions carried out by central banks in the context of the Great Recession. We implicitly assume that the monetary authority does not have full information about the functioning of the economy, and it infers the underlying path from observed values of GDP. In particular, our measure takes into account the lag in potential output revisions, as we disregard the most recent 4 periods. This implies that the estimate of potential output is not very sensitive to negative shocks if they are small in size and duration. We set $n$ equal to 10, to allow the central bank to consider a long enough period of time in the estimation of the underlying trend, but also react relatively quickly to shocks.

As we will show in the following sections, the Taylor rule provides stimulus to the economy, inducing it to close the output gap after a negative shock. Despite the fact that the aggregate technology is AK, and because of the monetary policy intervention, GDP appears to fluctuate around a stable linear trend in normal times, appearing consistent with diminishing returns to capital. However, if the shock is large and prolonged, potential output will be revised downwards and the stimulus to the economy will lose strength over time. As a result, the economy will reveal its AK structure and the recovery will fail to materialize.
4 Baseline Calibration

We calibrate the model on quarterly data for the United States, selecting parameters commonly used in the literature. We target a labour share of 60.8%, which was calculated as the average value of the share of labour compensation in GDP between 2004 and 2007. We select this period to capture the state of the economy in the years preceding the Great Recession, taking into account the downward trend in the labour share and the rise in markups (Autor et al., 2017). This is achieved by setting \( \alpha = 0.24 \) and assuming a price mark-up of 25%. The latter implies a value of 5 for the elasticity of substitution across intermediate good, \( \theta \). We also calibrate the model to have zero inflation in steady state and a quarterly growth rate of 0.6%. This is obtained by setting labour hours in steady state to 0.2, in line with the average of annual hours worked by persons engaged between 2004 and 2007, choosing \( \gamma = 0.966 \) (close to Bernanke et al. (1998)), with \( \delta \) set to 0.028 and \( \kappa \) to 0.04. These are somewhat higher than standard values, but we make this choice because our definition of capital is wider compared to National Accounts, including all categories of intangible capital. Although these stocks are not fully included in GDP measures, they do affect the outcome of production, and they depreciate faster than traditional measures of capital, so we account for their depreciation in our calibration. Monitoring costs are set to \( \mu = 0.21 \), in line with Christiano et al. (2014) and within the range suggested by Carlstrom and Fuerst (1997). We set \( \xi = (1 - \mu)(1 - \frac{R_c}{R_k}) \) to obtain a steady state bankruptcy rate of approximately 1%, similarly to Christiano et al. (2014). We set \( \eta = 1 - \alpha \) so that the aggregate technology is AK. We then detrend equilibrium conditions using the stationary growth rate, so that \( \tilde{c}_t = \frac{\alpha}{1+g_z}c_t \) and \( \tilde{y}_t = \frac{1}{1+g_z}Y_t \), for example. For Rotemberg adjustment costs, we follow the approaches proposed by Ascari and Rossi (2012) and Born and Pfeifer (2016), calculating them as follows:

\[
\phi^p = \frac{(\theta - 1)\theta^p}{(1 - \theta^p)(1 - \beta p^p)} \tag{47}
\]

\[
\chi^w = \frac{(\epsilon - 1)\frac{\theta + 1}{\theta} (1 - \alpha) \theta^w}{(1 - \theta^w)(1 - \beta w^w)} \tag{48}
\]

where \( \theta^w = \theta^p = 0.75 \) represent the probabilities of not being able to reset prices and wages. \( \epsilon \) is set to 21, implying a wage mark-up of 1.05, and at the high end of values used within the literature,
to capture the lower bargaining power workers enjoy in bad times.

<table>
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<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
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<td>$\kappa$</td>
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<td>$\rho_c$</td>
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<tr>
<td>$a_\omega$</td>
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<td>strength of depreciation spillovers</td>
</tr>
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</table>

Table 1: Quarterly calibration parameters

Finally, we set $a_\omega$ to 0.15, which implies that a 1% deviation of the probability of bankruptcy from its normal value leads to an increase in the depreciation rate of approximately 0.1%. This is considerably smaller compared to the values in Lanteri (2018)’s data and capital quality shocks in Kozlowski et al. (2015), but our choice is conservative as the model is not expected to represent all dimension of the data. Our view is that there is sufficient evidence to support the capital destruction channel as quantitatively relevant, but a precise estimate to base the calibration on is not available. We therefore choose the parameter $a_\omega$ to bring our model simulations close to the fall in GDP observed in the data. The persistence of the Great Recession shocks, $\rho_c = 0.9$ and $\rho_\sigma = 0.97$ is taken from Christiano et al. (2014).
5 The Great Recession

Demand Shock. We model the Great Recession as the reaction of the economy to a large and persistent demand shock that combines a risk shock and a confidence shock, hitting both at the same time.\textsuperscript{13} The magnitude of the risk shock is set to increase the bankruptcy probability from a baseline value of 1% to 4%, in line with the estimation conducted by Christiano et al. (2014). This is achieved by setting the magnitude of the risk shock to 15.7%.

As for the confidence shock, it aims to make the economy hit and remain in the ZLB region for a few quarters, consistently with the behavior of the US economy. Notice that our model does not capture unconventional monetary policies implemented by the FED, nor forward guidance, hence we calibrate the size and persistence to generate a zero lower bound episode in line with what a standard Taylor rule would have implied. In order to quantify this, we build a measure of the alternative policy, using U.S. data.\textsuperscript{14} We calculate the nominal interest rate (in annual terms) emerging from the Taylor rule as follows:

\[
TR = \max \left\{ 0, TR + 1.5 (\pi_t - \bar{\pi}) + 0.5 \log \left( \frac{GDP_t}{GDP_t} \right) \right\},
\]

setting the target nominal interest rate $\bar{TR}$ to the 2004-2007 average of the Federal Funds Rate ($\bar{TR} = 3.64\%$) and the inflation target $\bar{\pi}$ to its official value of 2\%. Quarterly GDP is measured in real terms and potential output $GDP_t$ is constructed from CBO data (in 2012 dollars), employing the estimates published at the beginning of each year, and thus capturing the unfolding of all revisions. Inflation is measured as the GDP deflator.\textsuperscript{15} The resulting Taylor rule predicts a much shorter ZLB episode compared to the Federal Funds Rate, as displayed in Figure 5. The predicted interest rate stays at the ZLB for a few quarters. To generate results in line with this, we calibrate the size of the confidence shock to 13.345\%.

\textsuperscript{13}In order to appropriately account for the size of the shock and the ZLB constraint, we solve the model non-linearly, employing a Newton-type method (Juillard, 1996).
\textsuperscript{14}A similar exercise was also conducted by the Board of Governors of the Federal Reserve System, among others, with comparable results. See link here.
\textsuperscript{15}Additional Details on data series used can be found in the Appendix.
Data Target. In order to compare the simulations with key data dynamics in the U.S., we follow the approach of Christiano, Eichenbaum, and Trabandt (2015) to estimate data targets. We measure the deviation of variables from the path they would have followed, had the recession not happened, by calculating the percentage difference from a linear trend, fitted on past data. In order to select time intervals, we aim to follow the CBO’s methodology for projections of potential output as closely as possible. “Typically, a trend is considered to extend from at least one previous business cycle through the most recent quarter of data (because the peak of the current cycle is not known at the time of a forecast).”\footnote{Revisions of CBO’s potential output since 2007’, February 2014 report.} They consider full peak-to-peak business cycles, so we build our estimates by including data from the business cycles peaks preceding the Great Recession, up to the period before the downturn. We construct min-max targets by considering the intervals $[x : 2008Q2]$, with $x = (1990Q3, 2001Q1)$. We consider aggregate series retrieved from the FRED database for GDP, consumption, investment, credit and the Federal Funds Rate. Finally, in order to compare the switching-track of potential output in our model and in the data, we employ the measure of potential output we built to calculate the Taylor rule in Figure 5. More details on data
Figure 6: Model impulse responses to Great Recession shocks vs U.S. data targets

sources can be found in the Appendix.

**Great Recession.** Figure 6 shows impulse responses for some key variables in our model to the Great Recession shock. The fall in confidence affects consumption demand and the sharp rise in risk depresses investment demand and raises the depreciation rate. As bankruptcy rates increase, capital depreciates faster resulting in capital destruction. Even when the shock subsides and financial variables (credit, FFR and the bankruptcy rate) converge back to the previous steady state values, the negative effects on GDP, consumption and investment do not dissipate and the economy displays an L-shaped recovery. Our model fits the data quite well, especially for GDP and consumption, which both fall by almost 10% at the new steady state, converging monotonically in the same way as the data. As for investment, the simulation does not capture the full depth of the total shortfall, which was partially driven by the the real estate market. Nonetheless, results are in line with private non-residential investment data. The model fits the data for credit well at the beginning of the sample, set aside the collapse of the credit boom that we do not model, and performs worse for later periods. This is likely because we are not modelling unconventional
monetary policies providing credit easing. The behavior of the nominal interest rate is in line with the Taylor rule estimated in Figure 5, and thus does not replicate the Federal Funds Rate after 2010, which remains for a long while at the ZLB, likely due to unconventional monetary policies.

**Switching-track.** Figure 7 clearly shows that the model economy replicates quite well the *switching-track* of potential output in the data. The estimated output gap shrinks with time, thus reducing its impact on the Taylor rule. Our simulations are in line with the U.S experience, and our specification can also approximate the timing of output gap dynamics observed in the data.

In order to better understand the mechanics of the two shocks, Figure 17 in the Appendix includes the impulse response of other variables to the Great Recession shocks. As it is common in financial crises, credit declines and net worth contracts, mirroring a stock market crash. Monetary policy is in place to mitigate the shock, but the ZLB constraint limits its effectiveness and, most

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17The figure shows a slight increase in potential output in 2013, which is the result of the comprehensive revision of the national income and product accounts (NIPAs), to include new investment categories. This led to an increase in GDP value. We adjusted the potential output measures published, following the methodology indicated by the CBO in their 2014 report, but a small margin of error remains.
importantly, the severity and the length of the recession put downward pressure on potential output estimates, giving rise to a switching-track. Without a strong policy intervention, the economy moves to a lower growth path, revealing its AK nature. In Figure 20 in the Appendix, we compare the results of our baseline calibration with the case in which the central bank’s measure of potential output is much less sensitive to past observed data. In this case, the policy remains much stronger, and the negative level effects on GDP trend are considerably reduced, showing the policy’s potential effect on recovery in the model. A slower revision of potential output (inducing a weaker switching-track) sustains consumption and investment demand, generating stronger inflation, and allowing the real interest rate to rise faster, promoting savings and a stronger recovery compared to the baseline simulation. The policy then induces households to consume and save more, and entrepreneurs to invest more, with the economy moving to a higher trend.

**Zero-Lower-Bound.** Figure 18 in the Appendix highlights the role of the ZLB constraint in our results, showing that the level effect on GDP would have been less severe had the constraint not been binding. We also show that the risk shock is the main driver of these results (Figure 19), but the economy does not reach the ZLB in this scenario, as the severe supply effect of the shock drives inflation up.

**Welfare Losses.** In order to evaluate the impact of the policy, we calculate consumption equivalent welfare losses, by comparing the path of utility following the Great Recession shocks to the path of utility at the initial steady state (where the economy would have stayed had the Great Recession not happened). We conduct the analysis for 500 periods. Moreover, we calculate welfare for different intensities of potential output revision (switching-track). The Baseline column in Table 2 gives the welfare losses produced by the Great Recession demand shock when output gap is revised following the baseline revision rule. The other two columns do the same for alternative revision rules. Comparing columns, we conclude that faster revision rules generate larger losses, which implies that a weaker policy intervention in the model wouldn’t have been optimal. This is likely because the growth rate of a learning-by-doing economy is suboptimal, and thus some additional effort leading to a faster growth rate is Pareto improving.
<table>
<thead>
<tr>
<th>Intensity of revision</th>
<th>Faster than baseline</th>
<th>Baseline</th>
<th>Slower than baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>0.7</td>
<td>0.04</td>
</tr>
<tr>
<td>Welfare losses</td>
<td>-7.3%</td>
<td>-6.8%</td>
<td>-6.4%</td>
</tr>
</tbody>
</table>

Table 2: Welfare losses for different intensity of potential output revision

In an AK world, when capital is destroyed output moves to a lower path. The optimal reaction to such a shock is to remain in the new lower balanced growth path. A monetary policy forcing households to save more than optimal in order to make capital return to its previous trend has to be suboptimal. Why is then the case that a Taylor rule forcing output to go back to its previous trend generates welfare gains? The fundamental reason has to be related to the learning-by-doing nature of the model. Since in this framework private returns to capital are smaller than social returns, the equilibrium growth rate is sub-optimally low. As a result, monetary policy improves welfare by increasing the growth rate.

**Capital Destruction Channels.** The model has two capital destruction channels: the direct effect on the capital stock of entrepreneurs going bankrupt or *liquidation costs*, and the negative externality that bankruptcy has in the capital value of all entrepreneurs or *disruption spillovers*. The first captures the role of entrepreneurs exiting the economy through bankruptcy, as we assume a liquidation cost in terms of capital such that $\kappa > \delta$. These firms are hit by the risk shock the hardest, and their exit affects the aggregate economy, but not enough to generate a downturn as large as the Great Recession. Figure 8 shows that the same demand shock generates a permanent level effect on GDP even with no disruption spillovers, but the magnitude of the impact is considerably smaller. The second channel is the result of disruption spillovers. This modelling choice is motivated by Lanteri (2018) data, which shows that the price of used capital goods relative to new ones fell in the whole secondary market, and not just for bankrupt firms. Moreover, this aims to capture the idea that in downturns most firms are worse off, not just the ones that exit. This second channel is key to replicate the magnitude of the Great Recession in the model, as it is clear from Figure 8.\textsuperscript{18}

\textsuperscript{18}In a previous version of the model we only considered the spillover channel and this was sufficient to generate the dynamics presented in this manuscript.
Figure 8: Baseline simulation vs case without disruption spillovers

6 The Great Moderation: Normal Times

In this section, we aim to analyse the model’s reaction to moderate shocks, consistently with pre-Great Recession downturns.\textsuperscript{19} All along this section, we hold the value of potential output constant, so that the output gap measures the deviation of GDP from the balanced growth path trend.\textsuperscript{20} We do not update the potential output measure because the revision procedure adopted after the Great Recession started was the first of its kind in the U.S. experience, as documented by Coibion, Gorodnichenko, and Ulate (2017a). In normal times, “Recessions typically have little effect on historical estimates of potential output because the methodology aims to exclude cyclical effects” CBO (2014).

\textsuperscript{19}Since in normal time the ZLB is never binding, we solve the model with perturbation methods.

\textsuperscript{20}An alternative practice, perhaps even more common in DSGEs, is to model potential output as the level of output that would prevail without nominal rigidities. We do not adopt this measure because all frictions affect the impact of the policy intervention in our framework. Without them, monetary policy would not be able to influence the real interest rate, hence they are important to determine the new equilibrium after a shock. In Figure 22 in the Appendix, we show that removing price and wage rigidities results in monetary policy losing all power. Moreover, measures of potential output as deviations of output from trend tend to be the instrument of choice for central banks. For example, see Edge et al. (2008) for a discussion of the FED’s FRB/US model or Vetlov et al. (2011), for the ECB’s NAWM.
6.1 TFP Shocks

In order to show that our model can also be consistent with fast recovery episodes, we consider a textbook 1% TFP shock. In Figure 9, we plot simulation results for our baseline calibration as well as for a Taylor rule with no weight on the output gap. This figure shows that small shocks have permanent effects on GDP when $\rho_y = 0$, whilst the economy recovers when $\rho_y = 0.125$. The persistence of the shock is in line with the literature, and was selected to illustrate a case where the economy quickly recovers to the previous steady state.\textsuperscript{21} A positive weight on the output gap implies that the monetary authority will respond with stimulus to aggregate demand when the output gap is negative, by offering a lower interest rate for each level of inflation. A negative TFP shock reduces the supply of output, which puts pressure on prices to increase, raising inflation. The presence of the output gap in the Taylor rule, compared to a scenario where the weight of the output gap is zero, puts additional pressure on prices by leading to higher consumption and investment.

The impact of the counter-cyclical policy intervention on investment is even more important to drive the recovery. The stimulus affects demand by lowering the wedge between the return to capital for successful entrepreneurs and the risk-less rate, thus effectively reducing the influence of financial frictions on the economy (see Figure 23 in the Appendix). The condition for leverage (31), clearly shows that a fall in the wedge implies that financial intermediaries will offer contracts with lower leverage for each feasible value of $\bar{\omega}$. In equilibrium, the entrepreneur will then find it optimal to pick a contract with lower leverage and lower $\bar{\omega}$, keeping a larger share of her returns. Bankruptcy is thus reduced, leading to less capital destruction. The spread between the interest rate on loans and the risk-less rate is also reduced as a consequence of the policy intervention, so we conclude that a positive weight on the output gap tames risk in the economy overall, sustains aggregate demand and prevents capital destruction, keeping output and savings higher until the output gap closes and the economy displays a V-shaped recovery.

In order to compute welfare, we run the simulation with second-order perturbation methods, and we find that the policy intervention unequivocally improves welfare (Table 3).

\textsuperscript{21}The Taylor rule systematically generates a full recovery after a TFP shock, but Figure 21 in the Appendix shows that the recovery is fast as long as the persistence of the shock is not too high.
Figure 9: Effects of a 1% negative TFP shock

<table>
<thead>
<tr>
<th>Weight on the output gap</th>
<th>$\rho_y = 0$</th>
<th>$\rho_y = 0.125$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 periods</td>
<td>-0.2%</td>
<td>-0.1%</td>
</tr>
</tbody>
</table>

Table 3: Welfare losses of TFP shock compared to steady state for different weights on the output gap in the Taylor rule.
6.2 Demand Shocks

In this section, we aim to represent demand shocks in normal times, which we characterize as periods subject to shocks that are not as severe and persistent as during the Great Recession. We target an increase in the bankruptcy probability of approximately 0.5 percentage points, to mirror Christiano’s estimation for the recessions preceding the financial crisis. Thus, we opt for a 5% increase in risk combined with a 4.5% confidence shock and we reduce the persistence of shocks to 0.7. This is close to pre-Great Recession estimates on confidence shocks documented by Angeletos, Collard, and Dellas (2018) and with the analysis of Christiano et al. (2015), who illustrate the increased persistence of financial shocks during the Great Recession compared to previous downturns. Figure 10 shows that monetary policy drives a V-shaped type of recovery in this case as well, although the economy will not converge back to the previous steady state within the 50 quarters period. Since we assume that potential output is never revised, the economy will eventually converge to the previous trend. In any case, the remaining output gap is small enough not to be perceived as a change of trend resulting from the demand shock. In our baseline calibration, we choose strong depreciation spillovers, but it is possible for the impact of spillovers to be non-linear in the size of shocks. As we reduce the parameter $\alpha_{\omega}$ in Figure 11, we see that the model can generate a full recovery even with demand shocks.

In order to understand the mechanics behind the recovery role of monetary policy, it is useful to think about the policy in place as a force counteracting the negative shocks. As the demand shock hits and GDP falls, a larger output gap gives the policy strength, but as the recovery starts to materialize and inflation recovers, the monetary authority faces a trade-off between above target inflation and a negative output gap. The higher weight on inflation in the Taylor rule results in a slowdown in the recovery, although GDP will eventually go back to the initial steady state. As we reduce the strength of the capital destruction spillovers, the dynamics do not change, but a smaller shock allows a full recovery within the time-frame in which policy makers do not face the trade-off.
Figure 10: Effects of a small demand shock

Figure 11: Effects of a small demand shock, with different intensity of depreciation spillovers
7 What if We Relax the AK Assumption?

In the previous sections we assumed $\eta = 1 - \alpha$, which implies an aggregate AK technology. Is such a strong assumption necessary to generate our results? To test this, we simulate the model for lower values of $\eta$. We pick $\eta = 0.06$ to get a quasi standard Cobb-Douglas technology. Figure 24 in the Appendix shows that when we subject this version of the model to comparable shocks to simulate the Great Recession, and we keep potential output constant to reproduce a more standard DSGE, the model generates a quick and full recovery. This shows that the AK technology, and not the nature of the shocks, is the source of the parallel downward shift of GDP in this model.

We then try an intermediate case, setting $\eta = 0.6$ to get closer to an AK model, and allowing potential output revisions as in our baseline. Figure 12 shows that the shocks result in a persistence

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22 Note that in this case the model does not grow endogenously any more, implying that the growth rate of the economy converges to zero.

23 The model would also generate a similar recovery if we reduced the weight of the output gap in the Taylor rule, as the recovery is a consequence of diminishing returns to capital.
recession of reduced depth. Although this version of the model could be a good representation of GDP dynamics in the presence of additional shocks, we prefer our baseline specification, as it captures key aspects of the Great Recession in a parsimonious way. Simulations for normal times also show that the intensity of knowledge spillovers affects the severity of shocks as well as the speed of recovery, even with monetary policy in place (see Figure 25 and 26 in the Appendix).

8 Conclusion

Our paper contributes to the literature by showing that an endogenous growth model can reproduce the dynamics of the US GDP well, once the role of monetary policy is taken into account. In our framework, the differentiating factors between the Great Recession and the Great Moderation were the size and persistence of the shocks, the subsequent capital destruction, the binding of the zero lower bound and the introduction of the switching-track. We argue that an AK model can be a good representation of aggregate GDP because of the growing importance of intangible capital, human capital and knowledge spillovers for production in OECD countries, as documented by Haskel and Westlake (2018): “Because intangible investments, on average, behave differently from tangible investments, we might reasonably expect an economy dominated by intangibles to behave differently too.”

We believe it is possible that the economy has evolved from a Neoclassical structure to an AK structure due to technological advances, but we leave the evaluation of this hypothesis to future work. Finally, this paper does not fully address the role of unconventional monetary policies implemented following the financial crisis. It would be interesting to explore this aspect, by evaluating further the impact of central bank credit policies on the recovery process in our model, following the example of Gertler and Karadi (2011).

24(Haskel & Westlake, 2018) page 10.
References


with forward variables through the use of a relaxation algorithm (Tech. Rep.).


Appendix

A Data sources

**Taylor rule:** GDP is Real Gross Domestic Product, Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, inflation is Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Seasonally Adjusted (percentage change from a year ago), FFR is the Effective Federal Funds Rate, Percent, Not Seasonally Adjusted. Potential output historical estimates were retrieved from the Congressional Budget Office website.

**Simulation targets:** all data was retrieved from the FRED database. GDP is Real Gross Domestic Product, consumption is Real Personal Consumption Expenditures, investment is Real Gross Private Domestic Investment and private non-residential investment is Private Nonresidential Fixed Investment, divided by its deflator (retrieved from BEA). All variables are expressed in billions of chained 2012 dollars, quarterly, seasonally adjusted annual rates and in per capita terms. The bankruptcy probability is the estimated risk shock in Christiano et al. (2014). Credit is Total Credit to the Non-Financial Corporations, adjusted for breaks, in billions of US Dollars, divided by the GDP deflator. The nominal interest rate is the Effective Federal Funds Rate (quarterly averages).

B Taylor rule mechanics

With a positive weight in the output gap, the Taylor rule implies that a negative output gap will put pressure on the nominal interest rate to fall. However, in our simulations we find that a positive weight on the output gap generates a higher nominal interest rate in equilibrium, compared to pure inflation targeting (see Figures 9 and 10). This appears to be a puzzle. In order to understand how a positive weight on the output gap can generate an increase in both the nominal interest rate and inflation, we propose a simple exercise of comparative statics. In our model, as in the standard New Keynesian model, the nominal interest rate and inflation are determined by the intersection of the Euler equation and the Taylor rule. These can be represented as linear relationships between
inflation and the nominal interest rate, so that their intersection represents the steady state, i.e. the point in which $\pi_t = \bar{\pi} = 1$ and the nominal interest rate is $R_t = \bar{R} = \frac{1+gz}{\beta}$: Ignoring the ZLB constraint, the two equations can be written as:

Taylor Rule: $R_t = \frac{1+gz}{\beta} + 1.5(\pi_t - \bar{\pi}) + 0.125 \log \left( \frac{\hat{GDP}_t}{\bar{GDP}} \right)$ \hspace{1cm} (49)

Euler equation: $R_t = \frac{1+gz}{\beta} \frac{\hat{\lambda}_t}{\hat{\lambda}_{t+1}} \frac{\pi_{t+1}}{\pi_t} \pi_t$ \hspace{1cm} (50)

Figure 13 plots them for a grid of $\pi_t$ and $R_t$ close to the steady state. We then ask: how would the equilibrium change if we considered the response of the economy to a TFP shock? We use values of $\frac{\hat{\lambda}_t}{\hat{\lambda}_{t+1}}$ and $\frac{\pi_{t+1}}{\pi_t}$ from our baseline TFP shock at time 1, 2, 5 and 10, with no weight on the output gap. Figure 14 shows that the Euler equation rotates upwards as the shock hits, and then gradually comes back to the initial value. The transition is consistent with the IRFs in figure 9, as the TFP shock results in a temporary increase in the nominal interest rate and inflation.

Then we repeat the exercise for the scenario in which the weight on the output gap is positive,
considering data for $\frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{t+1}}{\pi_t}$ and $\frac{\Delta GDP_t}{GDP}$ from our simulation. Figure 15 shows that in this case the Taylor rule shifts to the right as the output gap pushes the intercept down, so that $R_t$ and $\pi_t$ are higher in equilibrium at time 1, compared to the pure inflation targeting case, consistently with our results in Figure 9. This simple exercise shows that a positive weight on the output makes the monetary authority offer a lower nominal interest rate for each level of inflation, stimulating substitution from savings to consumption through the Euler equation. The equilibrium will be the result of the interaction of the Taylor rule and the Euler equation, in turn effected by aggregate shocks.
Figure 15: TFP shock mechanics with positive weight on the output gap
C Capital quality

In this section we compare the depreciation rate in our model for the Great Recession to a measure of capital quality constructed by Kozlowski et al. (2015). They employ the FED’s Flow of Funds data for non-financial assets held by corporations, at market value (MV) and historical cost (HC). We replicate their methodology with quarterly series, and assuming the depreciation rate to be at our steady state value. They define a capital quality shock as a decline in the productive value of installed capital, which is equivalent to a rise in depreciation in our model. Assuming: $k_t = \phi_t \hat{k}_t$, the capital used in production $k_t$ depends on the installed capital $\hat{k}_t$ and the shock $\phi_t = 1$, equal to 1 in normal times.

Then assuming $P^k_t k_t = MV_t$ and $P^k_{t-1} k_t = HC_t$ we can retrieve an estimate of $P^k_{t-1} \hat{k}_t$ (capital previously installed, at historical cost, at time t) by using the standard capital accumulation equation, with $X_t$ as investment:

$$P^k_{t-1} \hat{k}_t = (1 - \delta)P^k_{t-1} k_{t-1} + P^k_{t-1} X_t = (1 - \delta)MV_{t-1} + HC_t - (1 - \delta)HC_{t-1}$$

(51)

then using the non-residential investment deflator from BEA:

$$\phi_t = \frac{k_t}{\hat{k}_t} = \frac{P^k_t k_t}{P^k_{t-1} k_t} \frac{PriceIndex_{t-1}}{PriceIndex_t}$$

(52)

The resulting measure has an average value of 1.01 in the interval we consider, and displays a large negative realization in conjunction with the Great Recession. This mostly captures variations in the market value of structures, as the methodology behind the data adjusts the market value of commercial real estate. In this sense, this series complements Lanteri (2018)’s data, which was informative as to the value of equipment.

Mapping into our model, $\phi_t = \frac{1-\delta}{1-\delta}$. We normalize the data to average 1 to ease comparison, and figure 16 shows that our model is consistent with the dynamics of the data. The magnitude of the model’s fall in capital value is considerably smaller compared to the constructed measure, confirming our conservative approach.
Figure 16: Capital quality $\phi_t$ in the data and in our model.
Figure 17: The Great Recession (baseline): quarterly model simulations
Figure 18: The Great Recession: quarterly model simulations, without the ZLB constraint
Figure 19: The risk shock: quarterly model simulations
Figure 20: The Great Recession for different values of potential output revision intensity.

Note that GDP, consumption and investment fall more as the shock hits when the revision of potential output is slower than baseline, i.e. the policy intervention is stronger. This is a consequence of the ZLB binding, as stronger demand stimulus generates higher inflation compared to the baseline, resulting in a sharp fall in the real interest rate with a binding ZLB. The latter provides stimulus to the marginal utility of consumption, but results in a sharper contraction in the labour supply, negatively affecting the level of economic activity. As soon as the nominal interest rate leaves the ZLB, the real interest rate re-bounces and monetary policy generates a partial recovery.
Figure 21: TFP shock for different values of persistence, baseline calibration.
Figure 22: TFP shock: Baseline vs No-wage-no-price frictions
Figure 23: TFP shock: additional variables
Figure 24: The Great Recession shocks with a quasi standard Cobb-Douglas technology and no potential output revisions
Figure 25: Relaxing the AK assumption: small demand shock for different intensities of knowledge spillovers
Figure 26: Relaxing the AK assumption: TFP shock for different intensities of knowledge spillovers