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The Macroprudential Toolkit: Effectiveness and Interactions

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Abstract

We use a DSGE model with financial frictions, leverage limits on banks, loan-to-value limits and debt-service ratio (DSR) limits on mortgage borrowing, to examine: i) the effects of different macroprudential policies on key macro aggregates; ii) their interaction with each other and with monetary policy; and iii) their effects on the volatility of key macroeconomic variables and on welfare. We find that capital requirements can nullify the effects of financial frictions and reduce the effects of shocks emanating from the financial sector on the real economy. LTV limits, on their own, are not sufficient to constrain household indebtedness in booms, though can be used with capital requirements to keep debt-service ratios under control. Finally, DSR limits lead to a significant decrease in the volatility of lending, consumption and inflation, since they disconnect the housing market from the real economy. Overall, DSR limits are welfare improving relative to any other macroprudential tool.

Keywords: Macroprudential Policy, Monetary Policy, Leverage Ratio, Affordability Constraint, Collateral Constraint.

JEL classification: E44, E58, G21, G28.

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1 Introduction and Motivation

Since the 2008 global financial crisis, policymakers have designed macroprudential policies that help stabilise debt and prevent or lessen the impact of future financial shocks. However, with many of these policies still untested, policymakers are facing the challenge of understanding their interactions with monetary policy or with the rest of the macroprudential toolkit. The task is even harder when, unlike for monetary policy, the objectives of macroprudential policy are much broader in nature and cannot be defined numerically. For example, the Bank of England's Financial Stability Objective is 'to protect and enhance the stability of the financial system of the United Kingdom'. It does this via its Financial Policy Committee whose responsibility 'in relation to the achievement by the Bank of its Financial Stability Objective relates primarily to the identification of, monitoring of, and taking of action to remove or reduce systemic risks with a view to protecting and enhancing the resilience of the UK financial system'[1].

The ample range of potential risks to be monitored and addressed as well as the availability of multiple macroprudential tools adds complexity to the task of choosing optimal policy by central bankers. For example, there are different macroprudential household tools designed to mitigate risks on household balance sheets. They have to be set in conjunction with tools that address risks for the financial sector, which can also have implications for housing markets and household debt. Additionally, household behaviour can also affect the wider economy via aggregate demand effects, hence the composition of household balance sheets may also be of interest to the monetary policymaker. This raises the importance of policy interactions, not only between different macroprudential tools, but also between macroprudential and price stability tools.

This paper contributes to the existing literature on the optimal use of monetary and macroprudential policy by considering a comprehensive macroprudential toolkit that includes collateral constraints, capital requirements for banks and affordability constraints on mortgage borrowers. Most previous papers on the topic have looked at the interactions of one tool at a time with monetary policy but not at a broader macroprudential toolkit. It is of vital importance to also assess the relative effectiveness of different available measures and their interactions with each other as well as their individual interactions with monetary policy in order to design the appropriate set of policy actions. Our setup allows us to explore a rich set of interactions between policies acting on bank balance sheets, household balance sheet and firms' production decisions. To the standard DSGE model of Smets and Wouters (2007), we follow Iacoviello (2015) and add household borrowing subject to a collateral constraint in the form of a loan-to-value (LTV) limit. We also add an endogenous leverage constraint on banks, resulting from the possibility of bankers absconding with their assets a la Gertler and Karadi (2105). The financial and real frictions in the model give rise to meaningful roles for macroprudential policy and monetary policy. However, unlike the existing academic literature, which focuses on a very limited set of tools, we model the actual policy toolkit used by central banks at the

[1] See the Remit and Recommendations for the Financial Policy Committeein the UK.

moment. We do this by augmenting the model in two important ways.

First, we add capital requirements on banks. We do this via a maximum leverage ratio set by the policy maker. Further, we assume that banks see leverage limits as an absolute maximum and they will expend effort (i.e., incur costs) to avoid reaching it. This approach ties in with the data, as in practice banks keep excess capital buffers over and above their capital requirements.

Second, in addition to the LTV limit, we examine the role of affordability constraints on mortgage lending and their interaction with monetary policy. Most of the existing literature on household and bank leverage has considered the policy design of either LTV limits or capital requirements. But affordability constraints can be used to stress test households' debt levels. We follow the current macroprudential framework in the UK and model affordability constraints as stressed debt-service ratios (DSR)[2] on households' balance sheets. We augment the standard DSR measure which captures debt repayments as a proportion of labour income, by adding a fixed buffer on top of the mortgage interest rate. This tests whether borrowers can still afford their mortgage payments should credit conditions tighten. Additionally, a change in the monetary policy rate will have a direct effect on DSR ratios by increasing interest repayments. As such, adding this tool in the model introduces an additional channel of monetary and macroprudential policy interaction, which is missing in the literature with just collateral constraints.

Affordability constraints were introduced in the UK in June 2014 (Financial Stability Report, 2017). The FPC argued that this tool allows them to guard against an increase in the number of highly-indebted households. A high proportion of highly leveraged households can lead to demand externalities if they are forced to deleverage following a negative aggregate shock, cutting back on spending and amplifying the economic bust. The FPC did not expect their recommendation to restrain housing market activity unless lending standards declined. We interpret this as implying that the LTV limit will be the usual binding constraint on lending but that the affordability constraint would 'kick in' if lending rose too strongly relative to income.

There are two key issues we examine in this paper. First, we investigate the interaction of macroprudential tools with each other and with monetary policy. Second, we examine the gains from adding each policy to the macroprudential toolkit in terms of reducing the volatility of key macroeconomic variables. In order to assess the impact of the different macroprudential policy tools and their interaction with each other, we adopt the following approach. We first develop a baseline model in which we have frictions in the banking and the housing sectors. We then consider the impact of adding a maximum leverage ratio on banks imposed by the macroprudential policymaker. Next, we examine the impact of introducing DSR limits on household borrowing either as a sole macroprudential policy, or together with capital requirements. In each case, we examine the volatilities of household borrowing, house prices, output and inflation as well as welfare. To

[2]We use affordability constraints and debt-service ratios limits interchangeably throughout the paper.

understand the interaction between different tools, we examine the responses of macroeconomic variables to productivity, housing demand, and monetary policy shocks.

We find that capital requirements can nullify the effects of financial frictions, reducing the impact of various shocks on the spread between lending and deposit rates, and reduce the effects of shocks emanating from the financial sector on the real economy. LTV limits, on their own, are not sufficient to constrain household indebtedness in booms, though can be used with capital requirements to keep debt-service ratios under control. DSR limits, on the other hand, lead to a significant decrease in the volatility of lending, consumption and inflation, since they disconnect the housing market from the real economy. Overall, DSR limits turn out to be welfare improving relative to any other macroprudential tool.

The remainder of the paper is structured as follows. In the next section we briefly review the literature that is most relevant to our paper before going on to describe the model in Section 3 and its calibration in Section 4. Section 5 describes our quantitative experiments, examining the effects of the various macroprudential tools and their interactions with each other and with monetary policy. Section 6 derives a welfare-based loss function against which we assess our macroprudential policy tools. Section 7 concludes.

2 Literature Review

In this section, we review some of the existing literature on macroprudential policy tools that is most relevant to this paper.

A substantial corpus of evidence establishes the existence of quantitatively relevant channels through which macroprudential tools might influence aggregate demand and through which monetary policy might have an effect on bank profitability and risk-taking (e.g., Woodford (2011), Curdia and Woodford (2009), Korinek and Simsek (2014), Farhi and Werning (2016) and Aguilar et al. (2019)). In particular some authors (Angelini et al. (2014), Rubio and Carrasco-Gallego (2015), Rubio and Yao (2019), De Paoli and Paustian (2017) and Carrillo et al. (2017)) have explicitly turned to the question of how monetary and macroprudential policies should be coordinated in a world featuring both nominal rigidities and financial frictions. These papers evaluate the optimal policy response of monetary policy and macroprudential actions either on LTV limits or on capital requirements when the economy is faced with aggregate shocks, such as to productivity or monetary policy. In most of these papers, the objective of macroprudential policy is to avoid excessive lending, that is, to minimize the variances of total lending or the ratio of loans to output. The extent to which policies are complementary or substitutes for each other, depends on the nature of the shock. For example, shocks to net worth or productivity create no tension between policies targeting output and inflation on the one side and bank lending on the other. However, there are welfare losses when the committees are non-cooperative in the case of cost-push shocks. In this case, monetary and macroprudential policies become strategic complements with both policies tightened more than in the case of coordination.

Our model contributes to this literature in two important ways. First, we introduce DSR limits on household balance sheets to cap mortgage borrowing. This tool acts to reduce the overall indebtedness of the household sector relative to nominal income. It is different from collateral constraints because it is imposing constraints relative to borrowers' income rather than to the value of their house. By modeling this tool relative to a regulatory stress rate buffer on existing mortgage rates rather than a standard loan-to-income limit, we introduce additional interactions between macroprudential and monetary policy. Second, we consider the interaction of monetary policy with a rich macroprudential toolkit including collateral requirements for banks and households as well as DSR limits. This allows us to examine not only the coordination between macroprudential and monetary policy tools, but also the optimal interaction of policies within the macroprudential toolkit.

To our knowledge, affordability constraints have not been addressed in the literature so far, although some authors have examined tools acting on limiting household debt relative to income. Ingholt (2017) compares LTV limits on mortgage lending with LTI limits in terms of smoothing responses to shocks. Greenwald (2018) examines a mortgage-payments-to-income limit in a DSGE model, and finds that it amplifies the transmission mechanism from policy rates to debt, house prices and economic activity. The paper also finds that a relaxation of payments-to-income standards is essential to match the recent boom. Fazio et al. (2019) study the impact of debt limits on housing markets and find that they might have distributional effects. However, unlike our model, neither of these papers have a banking sector. The introduction of a banking sector in our model opens up a new transmission channel that the above-mentioned papers are not able to capture.

In terms of model setup, there are two papers that use a similar model to ours in the literature on policy coordination. First, Ferrero et al. (2018) introduce a DSGE model with housing, heterogeneous households, loan-to-value limits on mortgage lending and capital requirements on financial intermediaries, to study how monetary and macroprudential policies should optimally respond to shocks. The authors derive a welfare-based loss function containing five (quadratic) terms. Two of them stem from the standard NK model where the policymaker seeks to stabilize the output gap and inflation. The remaining terms come from the desire of the policymaker to stabilize the distribution of non-durable consumption and housing consumption between borrowers and savers. Monetary policy is constrained by the zero bound. In a similar fashion, Rubio and Yao (2019) also study optimal macroprudential and monetary policy in a low interest-rate environment.

Second, Gelain and Ilbas (2017) study the implications of macroprudential policy in the context of an estimated Smets and Wouters (2007) type DSGE model for the United States, featuring a financial intermediation sector, subject to Gertler and Karadi (2011) financial frictions. Macroprudential policy aims at stabilizing nominal credit growth and the output gap by setting a lump-sum levy on bank capital. Monetary policy pursues a standard inflation targeting mandate using the short term interest rate. The paper focuses on testing how the variations in the macroprudential objectives affect the coordination between macro and monetary policies. In addition, the paper derives optimal policy rules and optimal weights under the assumption that the two policy makers cannot coordinate. In both papers macroprudential policy is always binding and the interaction between various macroprudential policy tools is not considered.

Finally, Hinterschweiger et al. (2020) uses a DSGE model with default to assess various macroprudential tools. However, unlike us, the paper does not consider affordability constraints within the macroprudential toolkit, nor does it consider the interaction of macroprudential policy with monetary policy. We also concentrate on the ability of macroprudential policy to reduce the volatility of lending and house prices, rather than default. And, by assuming an efficient steady state, we are able to derive a utility-based welfare measure that does not arbitrarily weight steady-state utility against its volatility.

3 Model

We start by describing our baseline model. The household and housing sectors follow Iacoviello (2015). We have two types of households: patient ones, who save via bank deposits, and impatient ones, who borrow from banks against housing collateral. Patient households have a higher discount factor than impatient households. Hence, they value future consumption relative to current consumption by more than the impatient households. Both types of households obtain utility from consumption, housing and leisure. In line with typical new Keynesian models (e.g., Smets and Wouters (2007)), we have a perfectly competitive final-goods sector whose firms combine intermediate goods to produce the final good. Intermediate-goods-producing firms combine the labour of patient and impatient householdsl to produce intermediate goods. They face price adjustment costs and have to borrow from banks to finance their working capital (ie, wage payment) needs. Finally, we have a banking sector that accepts deposits from the patient households and lends money to impatient households and firms. Following Gertler and Karadi (2011), banks face a costly enforcement problem. Specifically, we assume that banks are able to divert a fraction of their assets to their owners, albeit at the expense of not being able to continue as a bank. To stop this from happening, it must always be more profitable for the banks to continue operating than to divert funds. This incentive constraint acts as a friction in the banking sector that limits leverage and creates a spread between loan and deposit rates.

3.1 Patient Households

We start by describing the problem faced by patient households. We assume that there is a unit continuum of these households and that they maximise the present discounted value of their current and future streams of utility, subject to a budget constraint. They obtain utility from consumption, housing and leisure - i.e.

obtain disutility from working. We can write the problem facing patient household i mathematically as:

$$Maximise\ E_0 \sum_{t=0}^{\infty} \beta_P^t \left[ln(c_{i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right]$$

$$Subject\,to:\ D_{i,t} + Q_t H_{i,t} = Q_t H_{i,t-1} + R_{t-1} D_{i,t-1} + W_{P,t} h_{i,t} + \Pi_t - P_t c_{i,t} - P_t T_P - \tau_H Q_t H_{i,t}$$

Where c_i denotes consumption of household i, H_i indicates housing held by household i, h_i corresponds to hours worked by household i, D_i denotes bank deposits held by household i, Q represents the price of a unit of housing, R corresponds to the interest rate paid on bank deposits (which will be equal to the central bank's policy rate), W_P denotes the wage paid to patient households, P represents the aggregate price level, Π denotes profits of the firms and banks returned to the patient households, who we assume own them, net of money used by patient households to provide initial capital to new banks, and T_P corresponds to lump-sum taxes. In order to deliver an efficient steady state in the housing market, we introduce a constant tax/subsidy on saver's housing denoted by τ_H. To generate volatility in house prices, we add a 'housing demand' shock common to all (ie, both patient and impatient) households, denoted by A_H.

Assuming all patient households are identical, the first-order conditions for this problem imply:

$$\frac{1}{c_{P,t}} = \beta_P R_t E_t \frac{1}{(1+\pi_{t+1}) c_{P,t+1}} \tag{1}$$

$$\frac{(1+\tau_H) q_t}{c_{P,t}} - \frac{jA_{j,t}}{H_{P,t}} = \beta_P E_t \frac{q_{t+1}}{c_{P,t+1}} \tag{2}$$

$$w_{P,t} = h_{P,t}^{\xi} c_{P,t} \tag{3}$$

Where c_P denotes aggregate consumption by patient households, H_P represents the aggregate housing stock owned by patient households, π denotes the rate of inflation, q denotes real house prices and w_P corresponds to the real wage paid to patient households. Equation (1) is the familiar patient household's intertemporal Euler equation, relating consumption today to the real interest rate and expected consumption tomorrow. Equation (2) is the housing demand equation for patient households, which shows that the higher is the real cost of housing, the less housing will be demanded. Finally, equation (3) is the labour supply equation for patient households, which shows that the higher the real wage paid to patient households is, the more hours of labour they will supply.

3.2 Impatient Households

We assume that there is a unit continuum of impatient households, who also maximise the present discounted value of their current and future streams of utility. Again, they obtain utility from consumption, housing and leisure (i.e., obtain disutility from working). In addition to a budget constraint, however, they also face a collateral (loan-to-value) constraint on their borrowing. We assume that this constraint is imposed on them exogenously by the banks for reasons that are not modelled. Following Iacoviello (2015), we assume that impatient households discount the future at a greater rate than the patient households, ie, $\beta_I < \beta_P$. We can write the problem facing impatient household i mathematically as:

$$Maximise\ E_0 \sum_{t=0}^{\infty} \beta_I^t \left[ln(c_{i,t}) + jA_{H,t} ln(H_{i,t}) - \frac{1}{1+\xi} h_{i,t}^{1+\xi} \right]$$

$$Subject\,to:\ L_{i,t} = Q_t(H_{i,t} - H_{i,t-1}) + R_{L,t-1}L_{i,t-1} - W_{I,t}h_{i,t} + P_t c_{i,t} + P_t T_I \tag{4}$$

$$L_{i,t} = \rho_L L_{i,t-1} + (1-\rho_L) LTV\ H_{i,t} E_t Q_{t+1} \tag{5}$$

Where c_i denotes consumption of impatient household i, H_i represents housing held by household i, h_i corresponds to hours worked by household i, L_i denotes bank lending to household i, R_L denotes the interest rate charged on bank loans, w_I denotes the wage paid to impatient households, LTV is the loan-to-value limit targeted by the banks on their lending, and T_I denotes lump-sum taxes, including those used to achieve an efficient allocation of consumption in steady state.[3]Note that, following Iacoviello (2015), we assume that impatient households only adjust slowly to their borrowing limits. The intuitive justification for allowing impatient consumers to adjust slowly to the mortgage borrowing limits is that these limits are typically imposed when mortgages are taken out; thus they will not effectively apply to all mortgage lending. Since, in our model, there are only one-period loans, imposing the LTV limit at all times would mean that the limit was applying counterfactually to all mortgage lending. Given this intuition, we can interpret ρ_L as the proportion of existing mortgages and $1 - \rho_L$ as the proportion of new mortgages.

The first-order conditions for this problem imply:

$$\frac{1}{c_{I,t}}(1-\mu_t) \quad = \quad \beta_I E_t \frac{R_{L,t} - \rho_L \mu_{t+1}}{(1+\pi_{t+1})c_{I,t+1}} \tag{6}$$

[3]In the United Kingdom, the Financial Policy Committee has the power to direct banks to set LTV limits at levels of their choosing for owner-occupier and/or buy-to-let mortgages. But, as the Committee has not used these powers yet, we set the LTV ratio at the average across all UK owner-occupied mortgage lending between 2005-2018.

$$\frac{jA_{j,t}}{H_{I,t}} = \frac{q_t}{c_{I,t}} - \frac{\mu_t(1-\rho_L)LTV\,E_t[q_{t+1}(1+\pi_{t+1})]}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}} \tag{7}$$

$$w_{I,t} = h_{I,t}^{\xi} c_{I,t} \tag{8}$$

Where c_I denotes aggregate consumption by impatient households, H_I represents the aggregate housing stock owned by impatient households and w_I corresponds to the real wage paid to impatient households. Equation (6) is the intertemporal Euler equation for impatient households. Note that in addition to the real interest rate they pay on their borrowing and their expected future consumption, the consumption of impatient households will also depend on the tightness of the loan-to-value constraint on their borrowing, as picked up by the Lagrange multiplier, μ. Equation (7) is the housing demand equation for impatient households. This equation shows that in addition to its utility value, a marginal unit of housing yields extra value to impatient households by loosening their collateral constraint, enabling them to borrow and consume more. This effect is picked up by the term: $\frac{\mu_t(1-\rho_L)LTV\,E_t[q_{t+1}(1+\pi_{t+1})]}{c_{I,t}}$. Equation (8) is the labour supply equation for impatient households showing that the higher the real wage is, the more hours of labour they will supply.

3.3 Firms

As is standard in the new Keynesian literature, we assume that there is a unit continuum of monopolistically-competitive intermediate-goods-producing firms and a representative perfectly-competitive firm that combines intermediate goods to produce a final good. We assume that the intermediate-goods-producing firms face costs of adjusting prices a la Rotemberg (1982). They also have to borrow to finance their working capital needs, creating a direct link between the financial sector and output and inflation. In what follows we present the optimisation problem for the two types of firms.

3.3.1 Final-Goods-Producing Firms

The representative final goods firm operates in a perfectly-competitive market and produces a final good by combining inputs of intermediate goods. These final goods are then consumed or invested. We can write the problem for this firm mathematically as follows:

$$Maximise\, P_t y_t - \int_{l=0}^{1} P_{l,t} y_{l,t} dl$$

$$Subject\, to: \; y_t = (\int_{l=0}^{1} y_{l,t}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$$

Where y denotes final goods output, y_l represents output of intermediate firm l and P_l corresponds to the price of output for the intermediate firm l.

The first-order condition for this firm gives the demand function for the output of individual firms:

$$y_{l,t} \quad = \quad (\frac{P_t}{P_{j,t}})^{\epsilon} y_t \tag{9}$$

3.3.2 Intermediate-goods-producing firms

We assume a unit continuum of firms producing differentiated intermediate goods in a monopolistically-competitive market. These firms face costs of adjusting prices. In addition, they have to borrow to finance their wage bill (what we think of as working capital). As a result of this constraint any shocks that have an effect on the rate of interest on bank lending will have a direct effect on firms' costs and, hence, output and inflation. This is an important channel of transmission for macroprudential policy since, by reducing the effects of shocks on bank lending rates, macroprudential policy can have beneficial effects on the real economy by reducing output and inflation volatility. Since the firms are owned by the patient households, they discount their profits using the patient households' stochastic discount rate. We can write the problem facing the intermediate firm l mathematically as:

$$Maximise \sum_{t=0}^{\infty} \frac{\beta_P^t}{P_t c_{P,t}} [(1+\tau_P) P_{l,t} y_{l,t} + n A_{n,t} - W_{P,t} h_{P,l,t} - W_{I,t} h_{I,l,t}$$
$$+ L_{l,t} - R_{L,t-1} L_{l,t-1} - \frac{\chi}{2} \left(\frac{P_{l,t}}{P_{l,t-1}} - 1 \right)^2 P_t y_t)]$$

Subject to:

$$L_{l,t} = W_{P,t} h_{P,l,t} + W_{I,t} h_{I,l,t} \tag{10}$$

$$y_{l,t} = A_{z,t} h_{P,l,t}^{(1-\sigma)} h_{I,l,t}^{\sigma} \tag{11}$$

$$y_{l,t} = \left(\frac{P_t}{P_{l,t}} \right)^{\epsilon} y_t$$

Where τ_P is a subsidy to make the steady-state production efficient,[4] $h_{P,l}$ is the labour input of patient households within firm l, $h_{I,l}$ is the labour input of impatient households within firm l and L_l is borrowing

[4] This tax/subsidy is a device to calculate the efficient steady state in this economy, so that we can analytically derive the welfare loss function and express variables as gaps with respect to the efficient steady state. The dynamics of the model should not be affected by this choice but it permits obtaining a more rigorous analysis of the welfare implications of the measures.

by firm l. All intermediate firms are subject to an aggregate technology shock, A_Z. Following Iacoviello (2015), we assume that firms default on an exgenous amount nA_n of their loans from banks, where n denotes the steady-state net worth of the banking sector and A_n follows an exogenous process. This will act as an exogenous shock to bank balance sheets.

If we assume a symmetric equilibrium, the first-order conditions for this problem imply:

$$\frac{(1-\sigma)y_t}{h_{P,t}} rmc_t = \frac{R_{L,t}}{R_t} w_{P,t} \tag{12}$$

$$\frac{\sigma y_t}{h_{I,t}} rmc_t = \frac{R_{L,t}}{R_t} w_{I,t} \tag{13}$$

$$\pi_t(1+\pi_t) = \frac{(1-\epsilon)(1+\tau_p)}{\chi} + \frac{\epsilon}{\chi} rmc_t + \frac{1}{R_t} E_t \pi_{t+1}(1+\pi_{t+1})^2 \frac{y_{t+1}}{y_t} \tag{14}$$

Equations (12) and (13) represent the demand for each type of labour; in each case, the lower the wage, the more labour is demanded. Note that the wage is multiplied by the interest rate spread, reflecting the fact that firms have to borrow to pay their wage bill. Again, it is this channel that provides a direct link from the financial sector to firms' costs and, hence, output and inflation. Equation (14) is the new Keynesian Phillips curve, which relates inflation today to expected future inflation, expected future output growth and real marginal cost.

3.4 Banks

Our modeling of the banking sector follows Gertler and Karadi (2011) with an endogenously-generated interest rate spread and leverage ratio. We assume that banks issue loans to impatient households and firms and finance these out of patient household deposits and their own net worth, n. To ensure that banks cannot accumulate retained earnings to achieve full equity finance, we follow Gertler and Karadi (2011) and assume that each period, banks have an *iid* probability $1-\zeta$ of exiting. Hence, the expected lifetime of a bank is $1/(1-\zeta)$. When banks exit, their accumulated net worth is distributed as dividends to the patient households. Each period, exiting banks are replaced with an equal number of new banks which initially start with a net worth of $L\nu$, where L is the steady state value of the banking sector's assets, provided by the patient households. A bank that survived from the previous period – bank b, say – will have net worth, n_b, given by:

$$n_{b,t} = R_{L,t-1} L_{b,t-1}(1+\tau_b) - R_{t-1} D_{b,t-1} - nA_{n,t} \tag{15}$$

where τ_b is a subsidy which ensures a steady-state spread of zero (the efficient level), L_b is the total lending of bank b to impatient households and firms and D_b are deposits from patient households held at bank b. As we explained earlier, $nA_{n,t}$ denotes non-performing loans, acting as an exogenous shock to bank balance sheets.

Total net worth, n of the banking sector will be given by:

$$n_t = \zeta(R_{L,t-1}L_{t-1}(1+\tau_b) - R_{t-1}D_{t-1} - nA_{n,t}) + (1-\zeta)L\nu \tag{16}$$

Each period, banks (whether new or existing) finance their loan book with newly issued deposits and net worth:

$$L_{b,t} = D_{b,t} + n_{b,t} \tag{17}$$

Following Gertler and Karadi (2011), we introduce the following friction into the banks' ability to issue deposits. After accepting deposits and issuing loans, banks have the ability to divert some of their assets for the personal use of their owners. Although the patient households are both the owners of the banks and the depositors in the model, we assume that each household is 'large' enough that we could imagine the banks owners and depositors being separate individuals, with the owners prepared to divert assets towards their own personal use. Specifically, they can sell up to a fraction θ of their loans in period t and spend the proceeds during period t. But, if they do, their depositors will force them into bankruptcy at the beginning of period $t+1$. When deciding whether or not to divert funds, bank b, will compare the franchise value of the bank, V_b, against the gain from diverting funds, θL_b. Hence, depositors will ensure that banks satisfy the following incentive constraint:

$$\theta L_{b,t} \leq V_{b,t} \tag{18}$$

The problem for bank b is to choose L_b and D_b each period to maximise its franchise value subject to its incentive constraint, equation (18), its balance sheet constraint (17) and the evolution of its net worth (15).

$$Maximise\, V_{b,t} = P_t E_t \sum_{j=1}^{\infty} \left(\beta_P^j \zeta^{j-1}(1-\zeta) \frac{1}{c_{P,t+j}P_{t+j}} (R_{L,t+j-1}L_{b,t+j-1}(1+\tau_b) - R_{b,t+j-1}D_{t+j-1} - nA_{n,t+j} \right)$$

We can note that both the objective and constraints of the bank are constant returns to scale. As a result, we can rewrite the optimisation problem for bank b in terms of choosing its leverage ratio, $\varphi_b = \frac{L_b}{n_b}$, to maximise the ratio of its franchise value to net worth, $\psi_b = \frac{V_b}{n_b}$. Given constant returns to scale, we can aggregate up across all banks. Doing so, we obtain the aggregate Bellman equation for the franchise value of the banking sector as a whole:

$$\psi_t = \beta_H E_t(\frac{P_t}{P_{t+1}})\frac{c_{P,t}}{c_{P,t+1}}(1-\zeta+\zeta\psi_{t+1})((R_{L,t}(1+\tau_b)-R_t)\varphi_t + R_t - \frac{n}{n_t}A_{n,t+1})) \tag{19}$$

$$Subject\, to:\, \theta\varphi_t \leq \psi_t \tag{20}$$

where we note that constant returns to scale implies that all banks will choose the same leverage ratio, φ.

3.5 Monetary policy

The central bank operates a Taylor Rule of the form:

$$lnR_t = (1-\rho_R)ln(R) + \rho_R lnR_{t-1} + (1-\rho_R)[\phi_\pi \pi_t + \phi_y ln(\frac{y_t}{y})] + \epsilon_{R,t} \tag{21}$$

where y denotes the steady-state level of output and ϵ_R is a white-noise shock.

3.6 Market clearing

Aggregating the budget constraints for each sector implies the goods market clearing condition:

$$y_t = \frac{c_t}{1-\frac{\chi}{2}\pi_t^2} \tag{22}$$

We assume a fixed stock of housing equal to unity:

$$H_{P,t} + H_{I,t} = 1 \tag{23}$$

And:

$$L_{M,t} + L_{E,t} = L_t \tag{24}$$

where L_M and L_E denote total lending to households and firms, respectively.

3.7 Augmenting the Baseline Model with Additional Macroprudential Tools

Relative to the baseline model described above, we add two more macroprudential tools.

First we consider the effects of adding a maximum leverage ratio constraint on banks as a way of capturing capital requirements. In particular, we suppose that the macroprudential policy maker sets a maximum leverage ratio Lev. Banks regard Lev as an absolute maximum, exerting effort and incurring costs in order to avoid reaching it. These costs get larger, the closer the bank gets to the maximum leverage limit. Specifically, we suppose that banks face the following cost function:

$$\left(\frac{\phi_b}{(Lev-\varphi_t)}-\frac{\phi_b}{(Lev-\varphi)}\right)n_t \tag{25}$$

where φ_t is their leverage in period t and φ is steady-state leverage.

The banking sector net worth will evolve according to:

$$n_t = \zeta\left(R_{L,t-1}L_{t-1}(1+\tau_b)-R_{t-1}D_{t-1}-\left(\frac{\phi_b}{(Lev-\varphi_t)}-\frac{\phi_b}{(Lev-\varphi)}\right)n_{t-1}-nA_{n,t}\right)+(1-\zeta)\nu \tag{26}$$

And the Bellman equation for the banking sector will now be given by:

$$\psi_t = \beta_P E_t \left(\frac{P_t}{P_{t+1}}\frac{c_{P,t}}{c_{P,t+1}}(1-\zeta+\zeta\psi_{t+1})\left((R_{L,t}(1+\tau_b)-R_t)\varphi_t+R_t-\frac{\phi_b}{(Lev-\varphi_t)}+\frac{\phi_b}{(Lev-\varphi)}-\frac{n}{n_t}A_{n,t+1}\right)\right) \tag{27}$$

Subject to equation (20).

The first-order conditions for this problem imply:

$$\varphi_t = Lev-\sqrt{\frac{\phi_b}{R_{L,t}(1+\tau_b)-R_t}}\ \textit{and}\ \theta\varphi_t<\psi_t \tag{28}$$

where we have assumed that the maximum leverage ratio (with associated penalty cost function) has been calibrated such that imposing it results in the diversion risk constraint always being slack. We discuss this in more detail in Section 5.1, below.

Second, we add an affordability constraint on household lending, which in essence is a debt-service-ratio (DSR) limit on impatient households' balance sheets. Specifically, we assume that the representative impatient household i faces the following constraint:

$$L_{i,t} = \rho_L L_{i,t-1} + (1-\rho_L)\frac{DSR\, h_{i,t} w_{I,t}}{R_{L,t} - 1 + stress} \tag{29}$$

where DSR is the maximum debt service ratio - i.e. the proportion of impatient households' wage income being used to pay the interest on a loan - at which the loan would still be considered 'affordable' at the stressed interest rate set by the macroprudential policy maker. *stress* denotes by how much the interest rate is stressed when considering affordability. Intuitively, the constraint checks whether a borrower would still be able to afford the interest payments on their loan if the interest rate they had to pay were to rise by the amount implied by the *stress* parameter. Given that mortgage loans in our model are all assumed to be for one period only, this affordability constraint is equivalent to a loan-to-income (LTI) constraint, where the LTI ratio depends on the current interest rate.

In what follows, we consider only those models in which either the LTV constraint binds all the time or the affordability constraint binds all the time, with the other constraint being slack. As such, we assume that the imposition of an affordability constraint on household lending renders the LTV limit slack in all periods. We discuss how these two separate constraints might interact with each other in Section 5.3, below.

The addition of an affordability constraint and the assumed slackness of the LTV constraint results in the following first-order conditions for the impatient households:

$$\frac{1}{c_{I,t}}(1-\mu_t) = \beta_I E_t \frac{R_{L,t} - \rho_L \mu_{t+1}}{(1+\pi_{t+1})c_{I,t+1}} \tag{30}$$

$$\frac{jA_{j,t}}{H_{I,t}} = \frac{q_t}{c_{I,t}} - \beta_I E_t \frac{q_{t+1}}{c_{I,t+1}} \tag{31}$$

$$w_{I,t}\left(1 + \frac{\mu_t(1-\rho_L)DSR}{R_{L,t} - 1 + stress}\right) = h_{I,t}^{\xi} \tag{32}$$

Where μ is now the Lagrange multiplier on the affordability constraint. The housing demand equation is now simplified as impatient borrowers no longer benefit from having more housing to relax their collateral constraint. Against that, impatient households are now prepared to supply more labour for a given wage, since doing so will relax their affordability constraint.

4 Calibration

Before displaying our quantitative experiments, we first discuss our calibration and what this means for the implied steady-state relationships in our model. We calibrate the parameters of the model either to match

Table 1: Parameter values

Parameter	Description	Value
β_P	Discount rate for patient households	0.9925
β_I	Discount rate for impatient households	0.985
j	Weight on housing in utility function	0.1377
ξ	Inverse Frisch elasticity of labour supply	1.83
σ	Proportion of total wage bill going to impatient households	0.33
ε	Elasticity of demand for differentiated intermediate goods	6
χ	Size of price adjustment costs	70
ϕ_π	Coefficient on inflation in Taylor rule	1.5
ϕ_y	Coefficient on output in Taylor rule	0.125
ρ_R	Interest rate smoothing in Taylor rule	0.81
ρ_L	Inertia in loan-to-value constraint	0.7
θ	Proportion of assets that can be diverted	0.1
ζ	Bank survival rate	0.975
ν	Capital of newly-formed banks as a fraction of bank assets	0.05
ϕ_b	Scale parameter of penalty cost function	0.0526
φ_{max}	Maximum leverage ratio	20
LTV	LTV limit	0.4833
DSR	Debt-service ratio	0.1323
$stress$	Stress rate (annualised)	3pp

the previous literature or to hit steady-state targets. Our parameter choices for the baseline model are shown in Table 1.

The discount rate for patient households is 0.9925, implying a risk-free rate of 3% per annum. The discount rate for impatient households is set to 0.985, following Ferrero et al. (2018). The steady-state version of equation (6), implies the following steady-state value for the Lagrange multiplier on the impatient households' borrowing constraint:

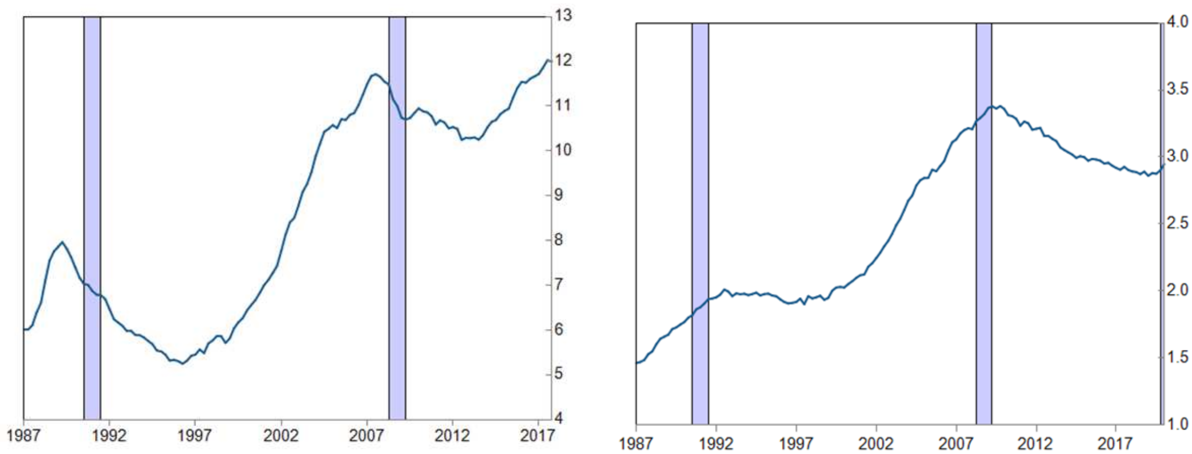
$$\mu = \frac{1-\beta_I R_L}{1-\beta_I \rho_L} \tag{33}$$

Given the calibration of the two discount factors, the impatient households will be constrained in their ability to borrow (ie, the LTV or affordability constraint will bind). However, we set the banking subsidy, τ_b, to ensure a zero spread in steady state.

Based on the estimation results reported in Smets and Wouters (2007), we set the inverse Frisch elasticity to 1.83. Following Iacoviello (2015), we set the inertia in the borrowing constraint equal to 0.7 and the share of the total wage bill going to impatient households equal to 0.33. We set the elasticity of substitution, ϵ, equal to 6. Absent the production subsidy, this would imply a mark-up of 1.2 in the intermediate goods sector, in line with the results in Macallan et al. (2008). We then set the size of the price adjustment costs, χ, such that the coefficient on (log) real marginal cost in the new Keynesian Phillips curve, $\frac{\epsilon-1}{\chi}$, was equal to 0.0852. This is the value that would be obtained in a Calvo (1982) model of price-setting with prices

Figure 1: UK Data

(a) Ratio of housing wealth to GDP (b) Ratio of mortgage borrowing to GDP



assumed to be adjusted once a year, on average. We set the survival rate for banks equal to 0.975, implying an average expected life for a retail bank of 10 years, the proportion of assets that can be diverted to 10% and the amount of capital that new banks start off with equal to 1/20 of the steady-state assets of the banking sector. Finally, we used standard values for the Taylor rule.

We set the maximum leverage ratio to 20 (ie, minimum capital requirement of 5%). Then, by setting the scale parameter on the penalty cost function to 0.0526, we ensure a steady-state leverage ratio of 10, roughly in line with the average leverage in the UK banking sector. Note that this is lower than the steady-state leverage ratio in the baseline model, which equals 11.5. This implies that capital requirements bind in the steady state.

We turn to the data to choose a target for the steady-state housing wealth to output ratio. Figure 1a shows that this ratio has risen over time from about $6\frac{1}{2}$ in the 1980s and 1990s to around 12 in 2019. Hence, we set the weight of housing in the utility function, j equal to 0.1377, which ensures a steady-state value for the housing wealth to output ratio of 12 in the model.

Figure 1b shows that in the UK, the ratio of mortgage borrowing to GDP is currently around 2.9. Given that, we set the LTV ratio to 0.4833, ensuring that the steady-state ratio of mortgage borrowing to GDP in our model is also equal to 2.9. For the affordability constraint, we set the stress buffer to 0.0075. This implies a 3 percentage point buffer per annum on top of the current interest rate when assessing principal and interest repayments for mortgage borrowing relative to labour income. Given that we set the subsidy to firms so as to ensure that real marginal cost is unity in steady state, and the subsidy to banks to ensure that the interest rate spread is zero in steady state, the steady-state versions of equations (13) and (29) imply:

$$\frac{L_M}{y} = \frac{\sigma DSR}{\frac{1}{\beta_P} - 1 + stress} \tag{34}$$

Given our other parameters, we set the DSR limit to ensure that the steady-state ratio of mortgage borrowing to GDP, $\frac{L_M}{y}$, is equal to 2.9, as in the baseline model. This implies a value for the DSR of 0.1323. This value for the DSR is low relative to the value of 0.4 that is applied in the UK in practice. However, this is a result of having only one-period loans in our model. For a long-term mortgage, the DSRs fall over the lifetime of the mortgage as income rises.

5 Macropruential Tools: Effects and Interactions

The novelty of this paper comes from analysing a comprehensive set of macroprudential tools, which includes two housing tools - i.e. an LTV ratio and a DSR limit - as well as capital requirements. This is important given the increased use in recent years in many countries, of macroprudential policies targeted to the household sector. The addition of different housing tools to existing macroprudential tools, such as capital requirements on lending to non-financial corporates, reinforced questions around the conduct of macroprudential policy and its interplay with monetary policy. In particular, policymakers have been keen to understand how different housing tools interact with each other, with capital requirements or with monetary policy.

This section examines these interactions in more detail to assess how different macroprudential tools can be complements or substitutes to each other and how they can smooth cycles and deal with economic shocks. We start by examining the effects of capital requirements, showing that they can nullify the effects of the financial frictions in the model and reduce the effects of shocks on the interest rate spread. We then examine the interactions of all our macroprudential and monetary policy tools by simulating four versions of the model with four different configurations of macroprudential policies in place: i) the baseline model with only an LTV limit in place; ii) a model with both an LTV limit and capital requirements; iii) a model with capital requirements and affordability constraints (but no LTV limit); and iv) a model with affordability constraints only. In each case, we use Dynare to calculate the volatilities of the key macroeconomic variables and their impulse responses to aggregate shocks.

5.1 The Role of Capital Requirements in the Macroprudential Framework

Before analysing the effects of the housing market tools, we first use our model to examine the implications of capital requirements. The purpose of capital requirements is to ensure the resilience of banks in the face of shocks. Minimum capital requirements are normally set by microprudential regulators. However, macroprudential regulators typically have the ability to raise capital requirements above the regulatory minimum, in response to cyclical movements in either aggregate or sector-specific financial risks. The purpose of this additional capital is to ensure the resilience of the banking sector as a whole if risks crystallise. In practice, macroprudential capital requirements help ensure that frictions within the banking sector do not amplify the effects of shocks passing through the banking sector onto the real economy.

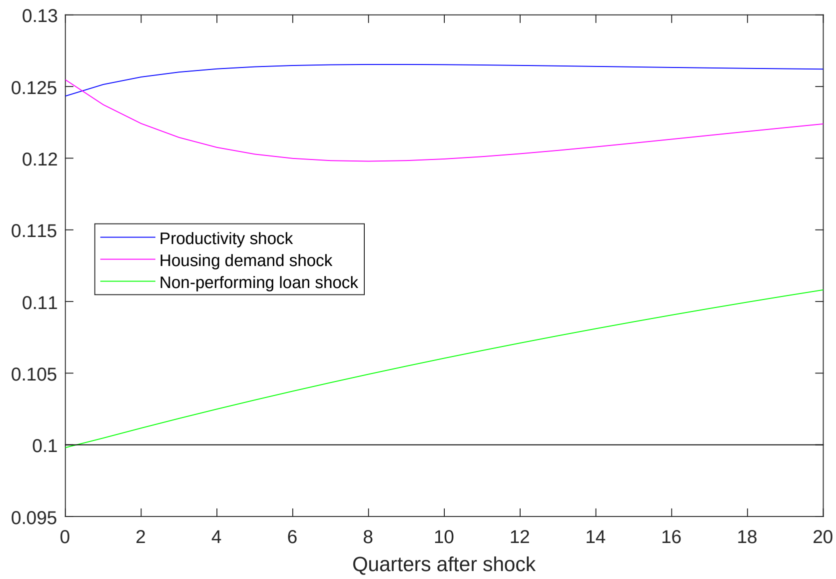
In the context of our model, the key friction is the ability of bankers to divert a proportion of their assets to consumption. This friction gives rise to a spread between lending and deposit rates. By raising the level of capital in the banking system above the level in the baseline economy, capital requirements can prevent this 'diversion constraint' from binding, thus eliminating the key friction in the banking sector. Similarly, the key financial shock affecting the banking sector in our model is an increase in non-performing loans. Capital requirements can make the system more resilient to such a shock by requiring banks to set aside more buffers. In what follows, we compare the baseline model with the model with capital requirements in order to examine the extent to which capital requirements are able to neutralise the financial friction and increase the resilience of the banking sector to negative shocks to non-performing loans.

Figure 2 plots the ratio of banks' stock-market value to assets, i.e., $\frac{\psi}{\varphi}$, following shocks to productivity, housing demand and non-performing loans when capital requirements are switched on in the model. The shocks are deliberately large: amounting to three standard deviations. The reason for doing this was to illustrate the ability of capital requirements to neutralise the effects of financial frictions, even in extremely rare circumstances. The calibration implies that the shocks give a 3.24% fall in productivity, a 7.36% rise in house prices and a 1844 basis point rise in the lending spread in the baseline model. For the persistence of the shocks we set the autocorrelation coefficients to 0.95, 0.98 and 0.02 for the productivity, housing demand and non-performing loans shocks, respectively. [5] With capital requirements in place, the value of divertable assets should lie below the stock-market value of the banking sector and hence the friction should not bind. Figure 2 shows that this generally holds: the model needs a very extreme shock to non-performing loans (i.e. resulting in at least a 1844 basis point rise in the lending spread) to push the ratio of banks' stock-market value to their assets marginally below 0.1. In the model, 0.1 is the baseline calibration for the proportion of assets that are divertable, θ. Figure 2 shows that the introduction of capital requirements has effectively neutralised the effect of the banking sector friction.

Figure 3 plots the behaviour of the spread to each of our three extreme shocks when capital requirements are switched off (in the top plot) versus when they are included in the model (in the bottom plot). This spread is the nearest equivalent in our model to the 'excess bond premium', which Gilchrist and Zakrajsek (2012) found to be a good leading indicator for the risk of a recession in the near term, which we can think of as 'GDP-at-Risk'. (Adrian et al. (2019) and Aikman et al. (2019) suggest that GDP-at-risk can serve as a useful measure of financial instability.) In our model, movements in the spread can be thought of as proxies for the resilience of the banking sector. That is, if the shocks translate into large movements in the

[5] These values, and those for the standard deviations of the shocks are estimated from UK data. We discuss the estimation of the shock processes in Section 6, below.

Figure 2: Ratio of banks stock market value to their divertable assets

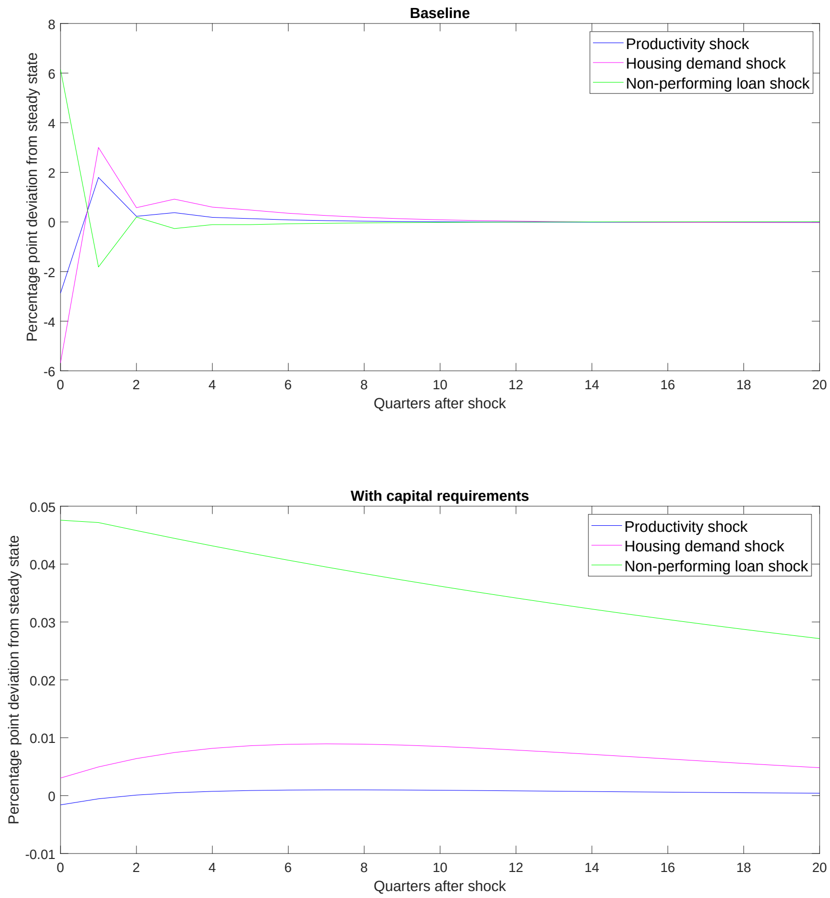


spread, then the financial sector is not shielding the real economy from the negative effects of the shocks. By passing through the impact of negative financial volatility to the real economy via the change in lending spreads, the financial sector can be thought of as being less resilient.

Figure 3 shows that the introduction of capital requirements within the model can greatly dampen the effects of all of our shocks on the lending spread, by a factor of roughly one thousand. The lending spread barely moves (by less than two basis points annualised) in response to either a severe productivity or housing demand shock. A three standard deviation shock to non-performing loans raises the lending spread by 1844 basis points in the baseline model, but only by 14 basis points once capital requirements are imposed in the bottom plot. This implies that capital requirements are able to insulate the real economy from the effects of a financial shock, since the lending spread is the channel through which such a shock leads to real economic effects.

In this subsection, we have shown that capital requirements act to increase the resilience of the banking sector by neutralising the effects of the diversion friction and by substantially reducing the response of the lending spread to shocks. In particular, the introduction of capital requirements enables banks to absorb shocks to their balance sheets without the effects being passed through to the real economy via higher lending spreads.

Figure 3: Behavior of the spread



5.2 The Interaction of Macroprudential and Monetary Policy Tools following Aggregate Shocks

Next, we examine the interaction of all of our macroprudential tools with each other and with monetary policy. To investigate the interaction between different tools, we gradually switch on different policies in our model, and examine their impact on output, lending, inflation, house prices, labour supply variables, financial variables and the interest rate following aggregate economic shocks. In this section, we consider one standard deviation shocks to productivity, housing demand and non-performing loans.

5.2.1 Housing Demand Shock

Figure 4 plots the responses of various macroeconomic variables to a housing demand shock that leads to an approximately 3% rise in house prices. There are two important results coming out of this experiment. First, when lending to households is constrained by DSR limits (i.e. blue and magenta lines), the economy does not respond to the housing demand shock, except for an increase in house prices. Affordability constraints limit the impact of housing market shocks on household borrowing and the real economy since, when borrowing is not linked to housing wealth, a shock to house prices does not influence credit constraints or how much households can borrow. In booms this may impose a cost in terms of lost GDP growth, which does not increase as much as in experiments where DSR limits are switched off, as shown in Panel 1. However, this mechanism also prevents GDP growth from falling due to a negative shock in house prices, limiting the effects of a crisis. As a result, the key benefit of DSR limits arises from limiting the volatility in economic variables.

Second, monetary policy responds less to the housing demand shock when capital requirements complement the LTV ratio, as shown in Plot 5 of Figure 4. This result occurs because capital requirements dampen the effect of the house price shock on lending, which decreases the effect of the shock on GDP and inflation. Hence, macroprudential policy acting through capital requirements contributes to price stability in the face of a housing demand shock, helping monetary policy achieve its primary objective.

5.2.2 Technology Shock

We also investigate the responses of variables to a positive technology shock, shown in Figure 5. The plots show a significant difference in impulse responses conditional on which of the two housing tools is switched on. However, adding capital requirements to either LTV or DSR policies leads only to marginal changes in variable responses. In the models with LTV ratios in place (i.e. black and red lines), the productivity shock leads to positive changes in output. This incentivises borrowers to purchase more housing, driving up house prices and lending. However, when affordability constraints are switched on (i.e. the blue and magenta lines), the link between house price movements and borrowing is muted, leading to very modest effects of the productivity shock on the economy.

Figure 4: Responses to a housing demand shock (≈3% rise in prices)

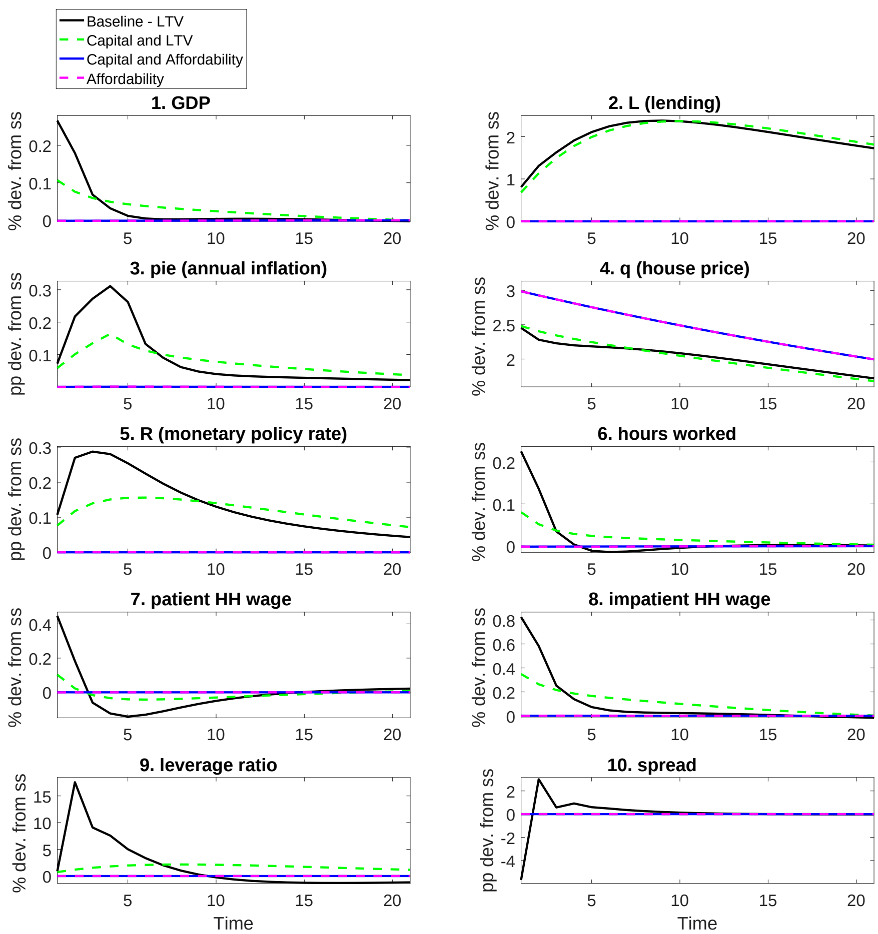
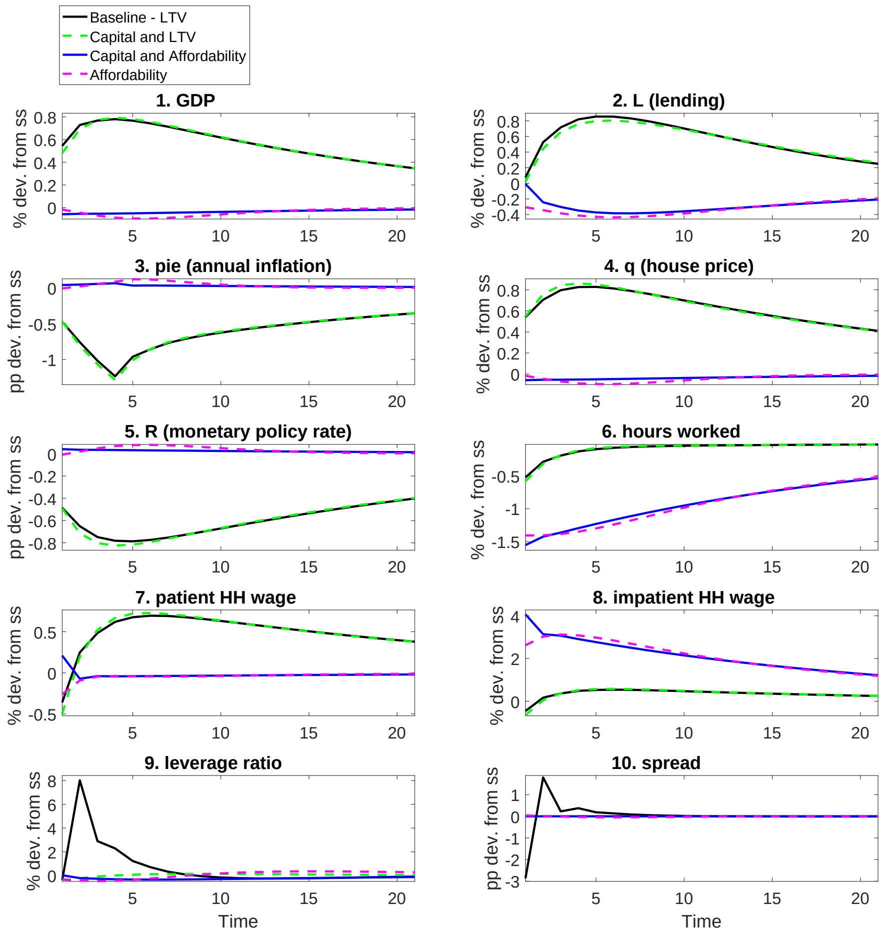


Figure 5: Technology shock



Plot 5 of Figure 5 shows the interaction of macroprudential tools with monetary policy. When affordability constraints are switched on, interest rates move very little. This suggests that, when faced with a technology shock, housing policies implemented via DSR ratios may support the objectives of the monetary policymaker. However, if housing policy is instead introduced via LTV ratios, monetary policy has to be more active in order to bring inflation back to target.

5.2.3 Financial Shock

We also examine the responses of variables to the financial (non-performing loans) shock, which lowers the net worth of the banks. Figure 6 shows that in all the simulations, the shock increases the bank spread upon impact, which lowers bank lending. Absent a monetary policy response, a rise in spreads would imply higher costs for firms and, hence, lower output and higher inflation. However, the resulting monetary policy response makes the impact on most real economy variables modest in all but the simulation with just the affordability constraints switched on (i.e. the magenta line). When DSR limits are the only macroprudential policy used, GDP, house prices and labour supply increase significantly relative to the other simulations, while inflation and wages are reduced. These effects are mostly driven by the interaction between the monetary policy behaviour and the DSR constraints on households. The initial rise in spreads makes lending more expensive, increasing both the real cost of production for firms and the cost of borrowing for households. As a result, workers' wages are reduced, leading to a drop in income. When borrowing is linked to DSR limits, a drop in income tightens budget constraints and incentivises workers to supply more labour to compensate for the wage loss. This results in higher output at lower costs, leading to the drop in inflation and the subsequent decrease in the policy rate. In turn, the lower base rate acts to loosen DSR constraints, supporting household borrowing and their demand for housing. These effects are also present but are substantially more muted when capital requirements are added to DSR limits. That is because leverage limits on commercial banks make financial variables less sensitive to financial shocks, as previously discussed in Section 5.1.

These results suggest that macroprudential policy implemented only through affordability constraints may aggravate the effect of financial shocks on the real economy, due to the feedback loops between real economic variables and DSR limits. In this case, macroprudential DSR limits and monetary policy have conflicting objectives and become strategic substitutes (i.e. as macroprudential policy is tightened through the introduction of affordability constraints, the monetary policy is loosened).

5.2.4 Monetary Policy

To further examine the interaction between macroprudential and monetary policy tools, we investigate the responses of macro variables to a monetary policy shock, which leads to a 1% rise in annualised rates. Figure 7 shows that the real economy behaves almost identically in all four models, with output, inflation and labour supply variables all falling by roughly similar magnitudes regardless of which macroprudential policy is switched on. However, the impact of the monetary policy shock on the financial sector depends significantly on macroprudential tools. For instance, the monetary policy shock leads to a large contraction in lending when affordability constraints are switched on and this is further aggravated if capital requirements are active as well (i.e. blue and magenta dotted lines) . This effect occurs for two reasons. First, the monetary policy contraction leads to a drop in GDP which results in lower household income. As borrowing is backed by household earnings, a loss of income leads to an immediate tightening of credit constraints and of overall lending. Second, the rise in risk-free rates leads to a subsequent rise in the mortgage lending rate. This further tightens households' credit constraints by increasing the proportion of interest payments that households have to pay back for any given loan size - i.e. increases the denominator in equation (29). These results suggest that capital requirements, DSR limits and monetary policy can have important spill-overs

Figure 6: Financial shock

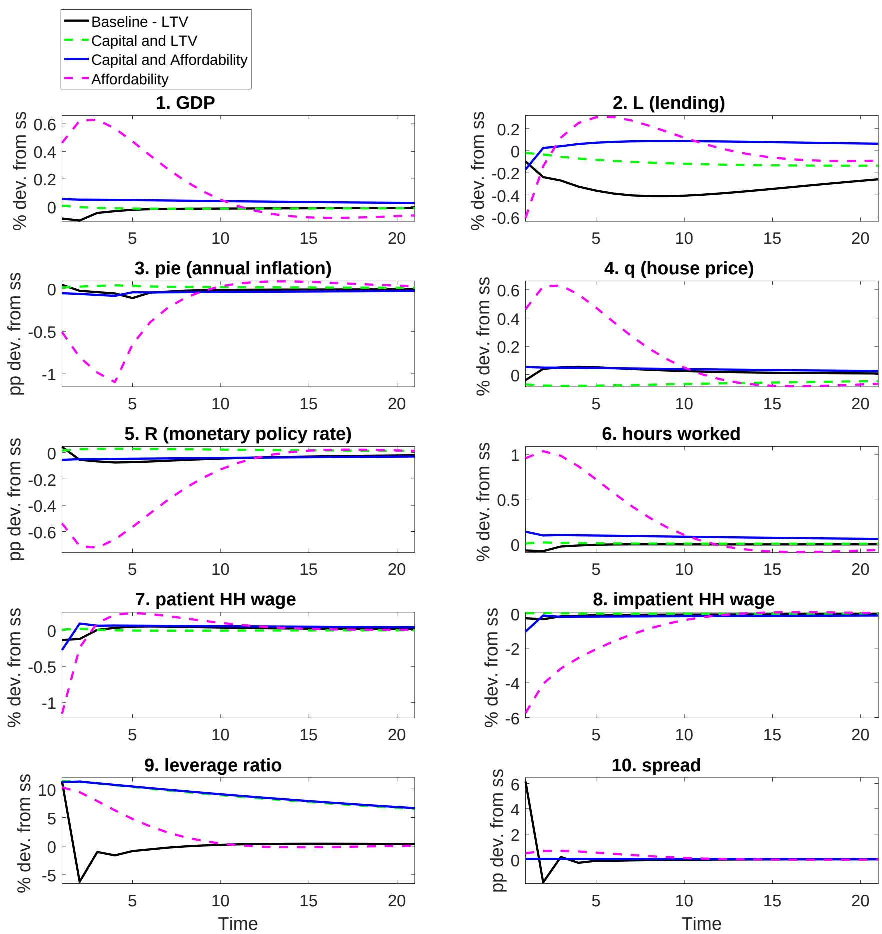
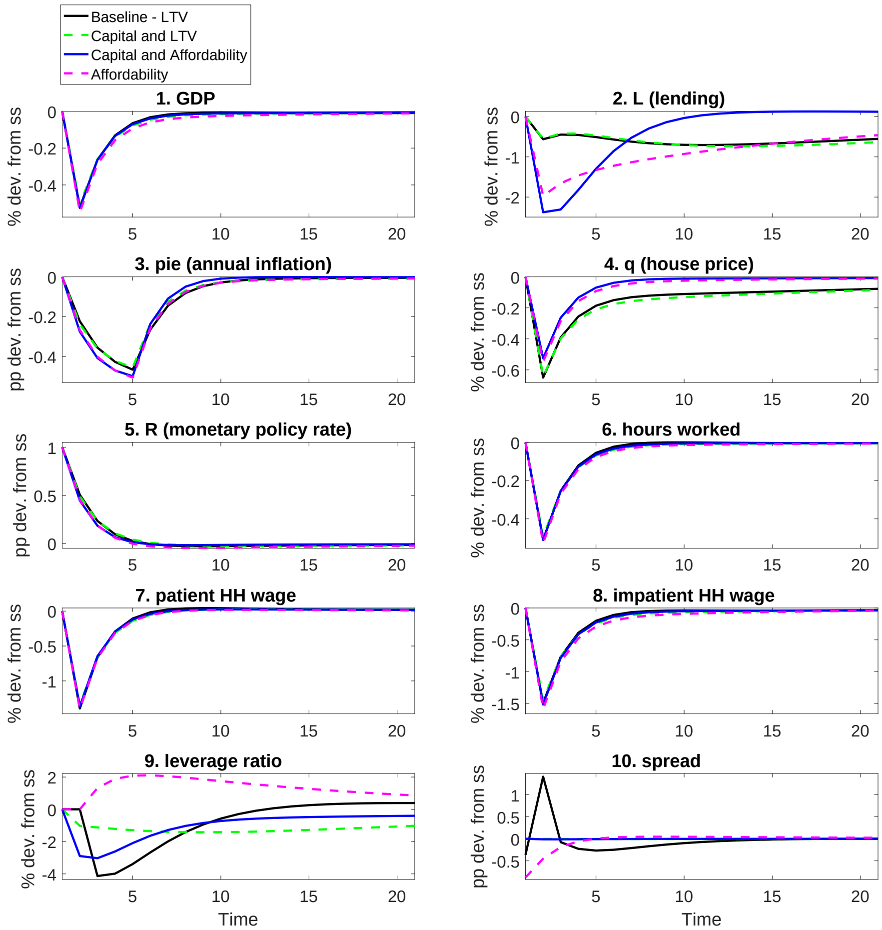


Figure 7: Monetary policy shock (1% rise in rates)



on each other, highlighting the importance of coordination between policymakers. However, although the macroprudential policies and monetary policy can have important spill-overs on each other, the consequence for the real economy of these spill-overs is limited.

5.3 The Interaction of Housing Tools with Each Other

The previous section described the interaction of our two housing tools - i.e. LTV ratios and affordability constraints - with capital tools and with monetary policy. This section provides more details on how the two housing tools may interact with each other.

To understand how LTV and DSR ratios evolve following economic shocks and how imposing macroprudential limits on one affects the other, we conduct the following experiment. For the versions of the model where the LTV limit is switched on - i.e the baseline with and without capital requirements - we calculate the prevailing DSR in the economy. Similarly, for the versions of the model where the DSR limit is switched on, we calculate the prevailing LTV ratio in the economy. For each shock, we then examine these prevailing ratios relative to our calibrations in Section 4- i.e. a 48.33% limit for the LTV ratio and a 0.1323 limit for the DSR ratio.

This exercise allows us to investigate whether different housing tools are complements to each other - i.e. they are both more binding or tighter at the same time - or substitutes to each other - i.e. when one is looser the other one is tighter. This is an important exercise for policymaking. For instance, if we find that the two housing tools are complements, then a collateral constraint (i.e. an LTV limit) will interact with and have positive spill-overs for borrowers' debt-service ratios in which case the macroprudential policymaker can address risks coming from the housing market using either one of these housing tools. However, if collateral constraints and affordability tools are substitutes, then they will respond to boom-bust cycles differently and hence the policymaker may need to assess the effectiveness of each tool separately. This case is more likely to occur in boom periods, when rising house prices reduces LTV ratios but not DSR constraints, since the latter is linked to the borrowers' incomes rather than to their collateral values. Greenwald (2018) finds that a cap on debt-to-income ratios, as opposed to LTV ratios, is more effective in limiting boom-bust cycles and such a policy would have reduced the size of the 2007 boom by nearly 60%. And Ingholt (2019) finds that a lower LTV limit could not have prevented the 2007 boom since soaring house prices slackened the constraint.

The implied DSR ratio for models where the LTV tool is switched on for macroprudential reasons, is calculated as:

$$DSR = \frac{L_{M,t}(R_{L,t} - 1 + stress)}{h_{I,t}w_{I,t}} \tag{35}$$

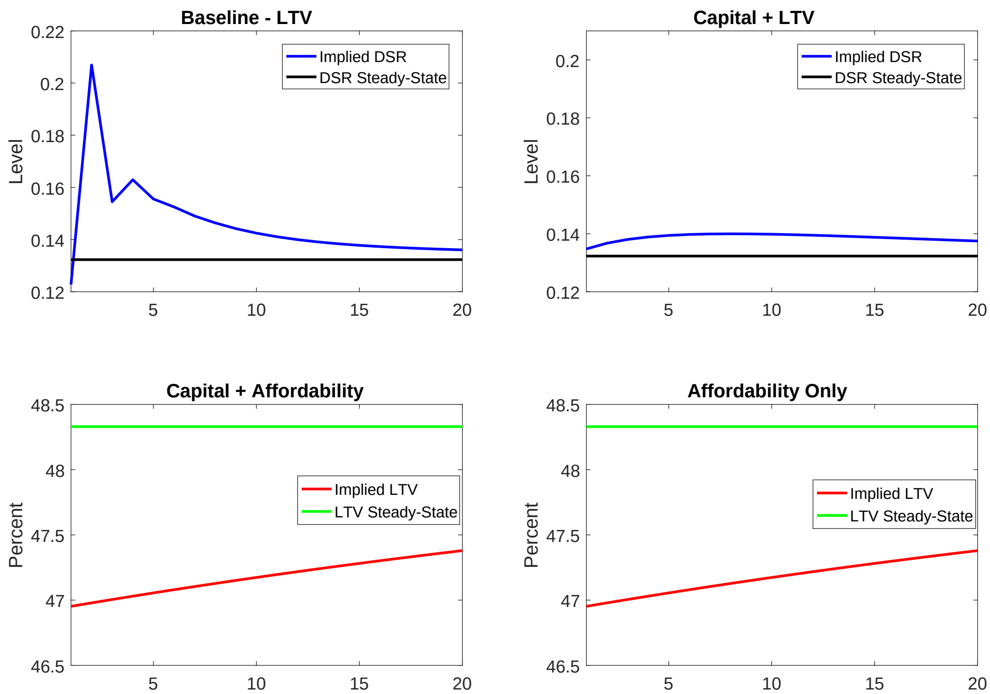
The implied LTV ratio for the models where the DSR limits are imposed as a macroprudential tool is given by:

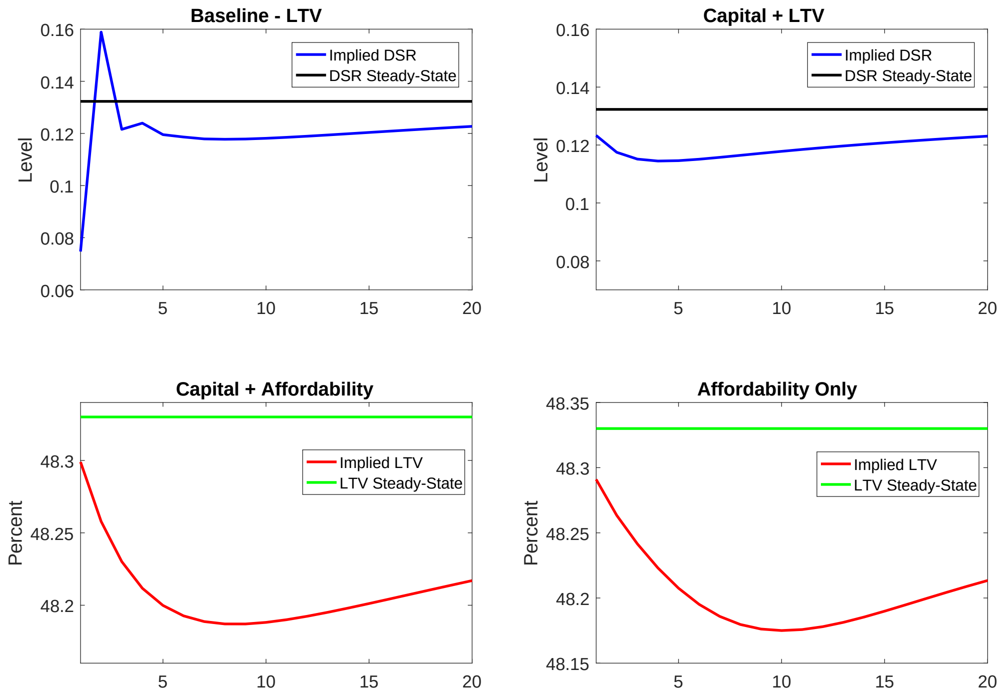
$$LTV = \frac{L_{M,t}}{H_{I,t}E_tQ_{t+1}} \tag{36}$$

Figure 8 shows the implied responses of the DSR and LTV ratio in each of the four cases described in the previous section following the housing demand shock, the technology shock and the financial shock. For each shock, the blue lines in the top panels show the implied DSR in the baseline simulation and in the baseline simulation with capital requirements. The red lines in the two bottom panels show the implied LTV ratio in the simulation with affordability constraints switched on and the one with both affordability constraints and capital requirements.

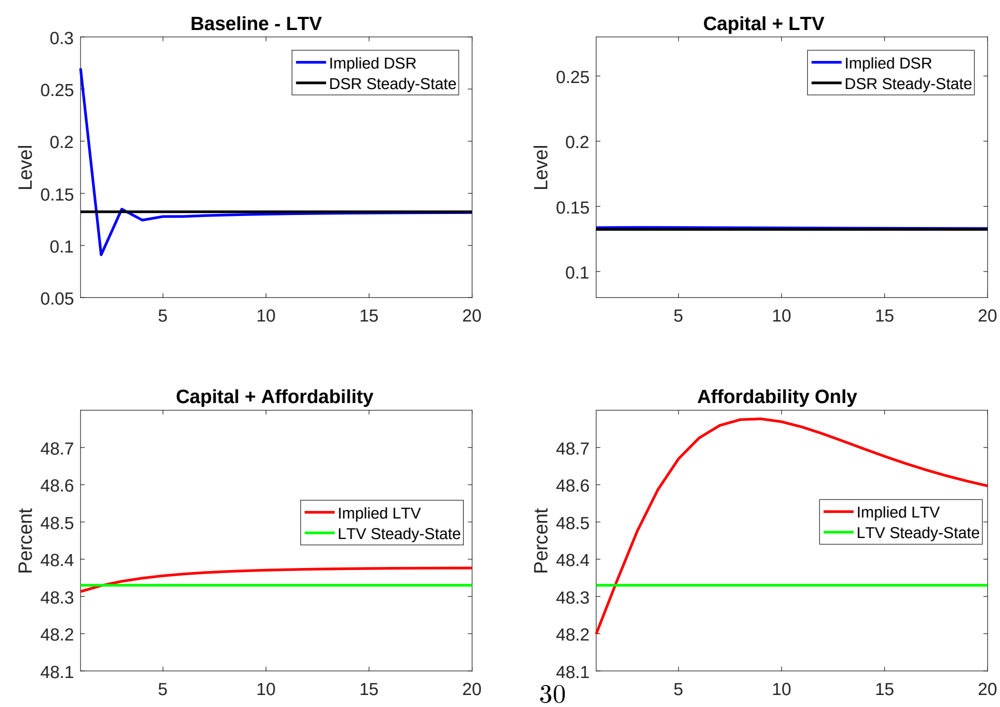
Figure 8: Performance of housing tools following aggregate shocks

(a) Housing demand shock





(b) Technology shock



(c) Financial shock

Figure 8a shows the results for the housing demand shock. In the baseline simulation (i.e. top-left panel), the implied DSR increases to 0.2 following the housing shock. This response exceeds the 0.1323 macroprudential DSR limit calibration that we impose in the versions of the model where the affordability constraint is used as a macroprudential tool. This increase in DSR is due to a large increase in borrowing and in the base rate in the baseline simulation. The house price appreciation of nearly 3%, relaxes LTV constraints and allows households to access more debt. The LTV ratio in the economy remains constant over time due to movements in house prices, but the additional debt in the economy raises debt service ratios. As a result, a macroprudential LTV tool is not sufficient on its own to constrain debt levels when the economy is hit by a housing demand shock. The feedback mechanism between house prices and borrowing implies that the LTV tool acts procyclically. Instead, a macroprudential constraint on DSRs would lean against the wind in a countercyclical manner.

DSRs are however less responsive to the housing demand shock when the macroprudential LTV limit is augmented by capital requirements in the top-right panel. This occurs because lending and interest rates respond less to the shock when capital requirements are imposed on banks, as shown in Figure 4. This limits the fluctuations in DSRs.

The bottom two panels of Figure 8a show the implied LTV ratio when macroprudential policy operates through affordability constraints with or without capital requirements. In both panels, the implied LTV ratios increase marginally following the house price shock but remain close to their steady-state value of 48.33%. This result occurs because macroprudential DSR tools break the link between collateral values and mortgage borrowing. As a result, the LTV ratios remain fairly constant.

Figure 8b shows the results for the technology shock. As before, the DSR nearly doubles and becomes very volatile in the baseline case. This occurs because, the technology shock increases household borrowing and decreases hours worked by the impatient household. Higher debt is thus serviced by lower labour income leading to a rise in DSRs.

In contrast, adding capital requirements to the baseline model in the top-right panel of Figure 8b, leads to DSRs that decrease marginally following the shock. As shown in Figure 5, complementing LTV limits with capital requirements leads to a larger loosening in monetary policy and to a more muted decrease in labour supply. These effects outweigh the initial increase in aggregate borrowing and weigh down on DSRs. This suggests that a capital requirement may constrain household leverage in the face of a technology shock. Similar to Figure 8a, the LTV ratios remain mostly stable over time when affordability constrains are imposed in the bottom panels.

Finally, Figure 8c shows the implied LTV and DSR ratios for the financial shock. The implied DSR nearly doubles in the baseline simulation shown in the top-left panel. As shown in Figure 6, this result is caused by the negative implications of the shock on borrowers' incomes and labour supply decisions, which tightens DSRs. However, similar to the previous shock, adding capital requirements to the baseline simulation results in constant and unresponsive implied DSRs. Additionally, in the bottom panels, LTV ratios remain relatively stable over time when affordability constraints are used as macroprudential housing tools, regardless of whether capital requirements are in place.

Putting these results together suggests that DSR and LTV tools are substitutes to each other in booms (i.e. DSR limits get tighter when LTV limits become looser). LTV ratios, on their own, are not sufficient to constrain household indebtedness when house prices rise. In contrast, capital requirements and DSR limits seem to have similar effects on debt, suggesting a complementary between these two macroprudential tools. Having capital requirements in place, if calibrated tightly enough, could augment LTV ratios to keep debt-service ratios under control, making macroprudential DSR limits obsolete. However, increasing capital requirements in the face of a housing demand or a technology shock could be costly since it is a blunt tool that affects all types of lending, not just mortgage lending.

6 The Impact of Macroprudential Tools on the Volatility of Key Macroeconomic Variables and Welfare

In this section, we examine the extent to which the adoption of macroprudential policy tools can improve welfare by stabilising output, inflation, lending and house prices. First, we derive the welfare-based loss function for our model against which we evaluate the performance of the various macroprudential tools. Our discussion of the loss function follows Ferrero et al. (2018) and Rubio and Yao (2019). We derive the loss function by taking a weighted-average of the per-period utility functions of patient and impatient households where the savers are given an arbitrary weight of ω. We assume that the planner discounts the future at the discount rate of the savers, β_P. A second-order approximation of the resulting objective function around a zero-inflation steady state in which the loan-to-value constraint is assumed to bind gives:

$$L \approx \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta_P^t (\hat{y}_t^2 + \lambda_\pi \pi_t^2 + \lambda_c \tilde{c}_t^2 + \lambda_H \tilde{H}_t^2) \tag{37}$$

where $\hat{y}$ denotes the log deviation of output from its efficient steady-state level, $\tilde{c}$ represents the consumption gap, defined as the log difference in consumption between patient and impatient households relative to the log difference between their consumption levels in the efficient steady state, and $\tilde{H}$ corresponds to the housing gap, defined as the log difference in housing held by patient and impatient households relative to the log difference between their housing levels in the efficient steady state. The efficient steady state is defined and derived in Annex 1 of this paper.

The weights on inflation, the consumption gap and the housing gap are derived in Annex 2 of this paper and are given by:

$$\lambda_\pi = \frac{\chi}{1+\xi},\ \lambda_c = \frac{1+\xi-4\sigma(1-\sigma)}{4(1+\xi)^2}\ and\ \lambda_H = \frac{j}{4(1+\xi)}$$

As in Ferrero et al. (2018), the loss function adds terms in the consumption and housing gaps to the standard output gap and inflation terms found in standard New Keynesian macroeconomic models. These terms are generated by incomplete financial markets where households are unable to completely share consumption and housing risk between them. Risk-sharing is further limited by the collateral constraint faced by impatient households. The goal of macroprudential policy in this set-up is to limit the welfare losses that arise out of incomplete risk-sharing.

Since the performance of different tools in smoothing financial and real economic variables is likely to depend on the relative importance of each of the shocks in driving the economy, it is important that we have a good estimate of the relative volatilities of the three shocks, as well as their persistence. As such, we estimate our shock processes: productivity, A_z, housing demand, A_H, and non-performing loans, A_n. In each case, we assume that the (log of the) shock follows an AR(1) process[6]. We estimate the standard deviations and first-order autocorrelation coefficients of the shocks using Bayesian techniques and quarterly UK data for GDP growth, real house prices and the spread of effective mortgage interest rates over the Bank of England base rate for the period 1999 – 2018. Table 2 shows the priors and the full results from the estimation. We then set our parameter values in line with the mean estimated values. As such, the standard deviation of the productivity shock is set to 1.08% and its autocorrelation to 0.95, which is in line with existing literature (e.g. Smets and Wouters, 2007). We set the standard deviation of the housing demand shock to 6.93% and its autocorrelation to 0.98. Finally, we set the standard deviation of the financial shock to 11.69% and its autocorrelation to 0.02.

Table 3 shows the results of stochastically simulating the model. For each of the four versions of the model considered earlier, we show the standard deviations of total bank lending, L, output, y, inflation, π and real house prices, q. In addition, we show the implied welfare loss based on our loss function. Relative to the baseline model, imposing capital requirements leads to reductions in the volatilities of macro variables. However, these are marginal and hence have no significant effect on the welfare loss. Switching on affordability constraints leads to an increase in the volatility of real house prices, but to a large decrease in the volatilities of lending, inflation and output. This results in a substantial improvement in welfare. Combining DSR limits with capital requirements leads to an even more substantial decrease in the volatilies of lending output and inflation, which implies a further substantial improvement in welfare.

[6] We acknowledge that estimation of our shocks is subject to limitations. First, estimating the shocks using AR(1) processes may be too simplistic. Second, there are limitations in identifying the three shocks based on the information in the time series of real GDP growth, real house prices and the mortgage spreads. For example, these economic variables may be driven by shocks other than just technology, housing or financial shocks which are unaccounted for by our model and may be driving the results. However, although not perfect, we argue that a Bayesian estimation approach still provides a more accurate magnitude of the shocks compared to a simple calibration. As these shock magnitudes feature in welfare calculations, it is desirable to produce estimates that are closer to reality rather than making simple guesses.

Table 2: Estimation of shock processes

	Prior			Estimated Max Posterior		Posterior
Parameter	**Type**	**Mean**	**Std. error**	**Mode**	**Std. error**	**Mean**
σ productivity shock	Inv gamma	0.01	∞	0.0105	0.0010	0.0108
σ housing demand shock	Inv gamma	0.035	∞	0.0503	0.0235	0.0693
σ financial shock	Inv gamma	0	∞	0.1174	0.0106	0.1169
ρ productivity shock	Beta	0.5	0.2	0.9551	0.0229	0.9472
ρ housing demand shock	Beta	0.5	0.2	0.9857	0.0114	0.9766
ρ financial shock	Beta	0.5	0.2	0.0089	0.0101	0.0155

Table 3: Volatility of macro variables

	$\sigma_{House\ Prices}$ (%)	$\sigma_{Lending}$ (%)	σ_π (%)	σ_y (%)	Welfare loss
Baseline: 60% LTV ratio	13.08	11.92	3.30	2.92	0.0239
Baseline and CR	13.04	11.85	3.31	2.90	0.0239
DSR	15.08	1.92	1.96	1.4	0.0083
CR and DSR	15.02	1.59	0.27	0.35	0.00017

Table 4: Variance decomposition

	LTV			**LTV and CR**			**DSR**			**CR and DSR**		
	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}	ε_{Az}	ε_{Aj}	ε_{An}
Lending	5.51	92.37	2.11	5.2	94.25	0.55	73.82	0	26.18	92.17	0	7.83
Output	98.30	1.39	0.31	99.45	0.54	0.01	3.24	0	96.75	27.13	0.02	72.85
Inflation	96.67	3.10	0.23	98.44	1.46	0.1	1.68	0	98.32	30.78	0	69.21
House prices	6.11	93.88	0.01	6.27	93.66	0.07	0.03	99.14	0.83	0.01	99.97	0.02

To investigate these results further, we decompose the variance in lending, real house prices, output and inflation into the proportions driven by each of our shocks. The results are shown in Table 4. The introduction of affordability constraints wipes out any effect of the housing demand shock on all variables other than house prices. This is because affordability constraints ensure that borrowing is no longer linked to house prices. The introduction of capital requirements reduces the contribution of the financial shock to lending, output and inflation volatility. That is, capital requirements can help protect the real economy from financial shocks.. These results suggest that capital requirements are a good addition, from a macroprudential standpoint, to housing tools in the face of financial shocks.

7 Conclusion

In this paper, we examine three macroprudential policies: LTV ratios, capital requirements on banks and affordability constraints on mortgage borrowing. We consider the interaction of macroprudential policies with each other as well as with monetary policy. Additionally, we assess the effects of each policy on macroeconomic stability, as measured by the standard deviations of output and inflation, on financial stability, as measured by the standard deviations of bank lending and house prices, and on welfare.

We first showed that capital requirements can nullify the effects of financial frictions, reducing the effects of various shocks on the spread between lending and deposit rates, and reduce the effects of financial sector shocks on the real economy. We also found that LTV limits, on their own, are not sufficient to constrain household indebtedness in booms, though could be used with capital requirements to keep debt-service ratios under control. However, increasing capital requirements in the face of a housing demand or a technology shock could be costly since it is a blunt tool affecting all types of lending, and not just mortgage lending. Instead, setting DSR limits in booms, can lead to a significant decrease in the volatility of lending, consumption and inflation, since they disconnect the housing market from the real economy. Additionally, with DSR limits in place, the average LTV ratio hardly moves in response to shocks. Overall, DSR limits are welfare improving relative to any other macroprudential tool.

In terms of interactions with monetary policy, we found that interest rate movements had stronger effects on lending with DSR limits in place rather than with collateral constraints. The effects are strengthened further when capital requirements are switched on. These results suggest that monetary policy can act as a complement to macroprudential policy. At the same time, the effects of a monetary policy change on output and inflation are not much affected by macroprudential policy. That is, monetary policymakers are still able to achieve their goals in the presence of macroprudential policy.

Future research on the interaction between policies should consider allowing policy tools to vary over the cycle and work out the welfare implications of optimal simple macroprudential policy rules. For instance, it is important to examine the optimal degree of countercyclicality in capital requirements or in the DSR stress buffer. This would better inform macroprudential policymakers on the effectiveness of different tools in smoothing aggregate shocks over the business cycle.

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Annex 1: The efficient steady state

In this annex, we define the conditions under which a zero-inflation steady state is efficient and show that we can obtain an efficient steady state in our decentralised economy by setting taxes and subsidies.

Consider a social planner who maximises a weighted average of patient and impatient households' period utility function, subject to the aggregate resource constraint and market clearing in the housing and labour markets. Price adjustment costs are zero in a zero inflation steady state.

Maximise:

$$U = \omega U(c_P, H_P, h_P) + (1-\omega)U(c_I, H_I, h_I)$$

Subject to

$$h_P^{(1-\sigma)} h_I^{\sigma} = c_P + c_I$$

And

$$H_P + H_I = 1$$

Let μ_1 and μ_2 be the Lagrange multipliers on the resource and housing constraints, respectively. Then the first-order conditions will imply:

$$\omega U_{c,P} = \mu_1 \tag{38}$$

$$(1-\omega)U_{c,I} = \mu_1 \tag{39}$$

$$\omega U_{H,P} = \mu_2 \tag{40}$$

$$(1-\omega)U_{H,I} = \mu_2 \tag{41}$$

$$\omega U_{h,P} = -\mu_1(1-\sigma)\frac{y}{h_P} \tag{42}$$

$$(1-\omega)U_{h,I} = -\mu_1 \frac{\sigma y}{h_I} \tag{43}$$

Where Uc, U_H and U_h are the marginal utilities of consumption, housing and hours worked, respectively, for household type j. Combining equations 39, 40, 41 and 42 gives:

$$\frac{U_{c,P}}{U_{H,P}} = \frac{U_{c,I}}{U_{H,I}} = \frac{\mu_1}{\mu_2} \tag{44}$$

In addition, equations 43 and 44 imply that the marginal rate of substitution between consumption and each type of labour is equal to the marginal rate of transformation between each type of labour and output.

$$\frac{U_{h,P}}{U_{c,P}} = (1-\sigma)\frac{y}{h_P} \tag{45}$$

$$\frac{U_{h,I}}{U_{c,I}} = \sigma\frac{y}{h_I} \tag{46}$$

Furthermore, if Pareto weights are set to match the population weights, i.e. $\omega = \frac{1}{2}$, then in the efficient steady state:

$$c_P = c_I = \frac{y}{2} \tag{47}$$

$$H_P = H_I = \frac{1}{2} \tag{48}$$

Next, we show that by choosing taxes and subsidies we can achieve the efficient steady state in the decentralised economy. We set the subsidy to firms, τ_P, equal to $\frac{1}{(\varepsilon-1)}$. The zero-inflation steady-state version of the New Keynesian Phillips curve now implies:

$$rmc = \frac{(\varepsilon-1)(1+\tau_p)}{\varepsilon} = 1 \tag{49}$$

This implies:

$$R_L = \frac{1}{\beta_P}\frac{(\beta_P + \zeta(\varphi-1) - (1-\zeta)\varphi\nu\beta_P)}{\zeta\varphi(1+\tau_b)} \tag{50}$$

If we set the subsidy to banks, τ_b, equal to $\frac{\beta_P}{\zeta\varphi^*}(1-\frac{\zeta}{\beta_P}-(1-\zeta)\varphi^*\nu)$ where φ^* is the degree of leverage in the efficient steady state, then:

$$R = R_L = \frac{1}{\beta_P} \tag{51}$$

And:

$$\theta\varphi^* = (1 - \zeta + \zeta\theta\varphi^*)\left(\frac{\beta_P}{\zeta}(1 - \frac{\zeta}{\beta_P} - (1-\zeta)\varphi^*\nu) + 1\right) \tag{52}$$

Which can be used to solve for φ^*.

The steady-state versions of equations 3, 8, 12 and 13 imply:

$$\frac{U_{h,P}}{U_{c,P}} = w_P = (1-\sigma)\frac{y}{h_P} \tag{53}$$

$$\frac{U_{h,I}}{U_{c,I}} = w_I = \sigma\frac{y}{h_I} \tag{54}$$

Evaluating the Euler equation for impatient households at the efficient steady state gives:

$$\mu = \frac{1 - \frac{\beta_I}{\beta_P}}{1 - \beta_I\rho_L} \tag{55}$$

The Lagrange multiplier will be positive in the efficient steady state so long as $\beta_P > \beta_I$. Hence, the housing demand equation for impatient households in steady state implies:

$$\frac{c_I}{H_I} = \frac{(1 - \beta_I - \mu(1-\rho_L)LTV)\,q}{j} = \frac{\left(1 - \beta_I - \frac{1-\frac{\beta_I}{\beta_P}}{1-\beta_I\rho_L}(1-\rho_L)LTV\right)q}{j} \tag{56}$$

Similarly, for patient households we obtain:

$$\frac{c_P}{H_P} = \frac{(1 + \tau_H - \beta_P)q}{j} \tag{57}$$

Equation 45 then implies that to obtain an efficient steady state, we need to set the housing tax equal to:

$$\tau_H = \beta_P - \beta_I - \frac{(1 - \frac{\beta_I}{\beta_P})}{(1 - \beta_I\rho_L)}(1-\rho_L)LTV \tag{58}$$

The LTV constraint then implies the efficient household debt to GDP ratio:

$$\frac{L_M}{y} = LTV\frac{q}{2y} \tag{59}$$

From the steady-state budget constraint for the impatient households we have:

$$\sigma = \frac{1-\beta_P}{\beta_P}\frac{L_M}{y} + \frac{c_I}{y} + \frac{T_I}{y} \Rightarrow \frac{T_I}{y} = -(\frac{1-\beta_P}{\beta_P}LTV\frac{q}{2y} + \frac{1}{2} - \sigma) \tag{60}$$

The impatient households need to receive a subsidy (net of taxes) proportional to GDP given by the term in brackets on the right-hand side of equation 60. Given such a subsidy, they will enjoy the same consumption and housing as the patient households, in line with our efficiency conditions 47 and 48.

Annex 2: Derivation of the loss function

This annex describes the derivation of the loss function shown in Section 4 of the paper. Following Ferrero et al. (2018), the welfare objective of the policymaker is defined as the present discounted value of the utility of the two types of household, weighted by arbitrary weights, ω and $1-\omega$, and discounted at the patient households' discount rate, β_P:

$$W_0 = E_0 \sum_{t=0}^{\infty} \beta_P^t(\omega U_{P,t} + (1-\omega)U_{I,t})$$

Given the functional forms:

$$U_{P,t} = lnc_{P,t} + jlnH_{P,t} - \frac{1}{(1+\xi)}h_{P,t}^{1+\xi}$$

$$U_{I,t} = lnc_{I,t} + jlnH_{I,t} - \frac{1}{(1+\xi)}h_{I,t}^{1+\xi}$$

A second order approximation of U around the efficient steady state gives:

$$\begin{aligned}U_t - U \approx & \omega U_c\left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{P,t} - \frac{y}{2})^2\right) + (1-\omega)U_c\left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{I,t} - \frac{y}{2})^2\right) + \\ & \omega U_H\left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2\right) + (1-\omega)U_H\left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{I,t} - \frac{1}{2})^2\right) + \\ & \omega U_h\left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2\right) + (1-\omega)U_h\left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2\right)\end{aligned}$$

Using the first-order conditions for the efficient steady state derived in Annex 1 we obtain:

$$U_t - U \approx \mu_1 \left(c_{P,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{P,t} - \frac{y}{2})^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} + \frac{1}{2}\frac{U_{cc}}{U_c}(c_{I,t} - \frac{y}{2})^2 \right) +$$
$$\mu_2 \left(H_{P,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{P,t} - \frac{1}{2})^2 \right) + \mu_2 \left(H_{I,t} - \frac{1}{2} + \frac{1}{2}\frac{U_{HH}}{U_H}(H_{I,t} - \frac{1}{2})^2 \right) -$$
$$\mu_1(1-\sigma)\frac{y}{h_P} \left(h_{P,t} - h_P + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{P,t} - h_P)^2 \right) - \mu_1 \sigma \frac{y}{h_I} \left(h_{I,t} - h_I + \frac{1}{2}\frac{U_{hh}}{U_h}(h_{I,t} - h_I)^2 \right)$$

Given the functional form for preferences, we note that:

$$\frac{U_{cc}}{U_c} = -\frac{2}{y}$$

$$\frac{U_{HH}}{U_H} = -2$$

$$\frac{U_{hh}}{U_h} = \frac{\xi}{h}$$

Substituting in gives:

$$U_t - U \approx \mu_1 \left(c_{P,t} - \frac{y}{2} - \frac{1}{y}(c_{P,t} - \frac{y}{2})^2 \right) + \mu_1 \left(c_{I,t} - \frac{y}{2} - \frac{1}{y}(c_{I,t} - \frac{y}{2})^2 \right) +$$
$$\mu_2 \left(H_{P,t} - \frac{1}{2} - (H_{P,t} - \frac{1}{2})^2 \right) + \mu_2 \left(H_{I,t} - \frac{1}{2} - (H_{I,t} - \frac{1}{2})^2 \right) -$$
$$\mu_1(1-\sigma)\frac{y}{h_P} \left(h_{P,t} - h_P + \frac{1}{2}\frac{\xi}{h}(h_{P,t} - h_P)^2 \right) - \mu_1 \sigma \frac{y}{h_I} \left(h_{I,t} - h_I + \frac{1}{2}\frac{\xi}{h}(h_{I,t} - h_I)^2 \right) \quad (61)$$

Now the aggregate resource constraint is given by:

$$c_{P,t} + c_{I,t} = y_t(1 - \frac{\chi}{2}\pi_t^2) \quad (62)$$

We can approximate any variable x using $x_t = x(1 + \hat{x_t} + \frac{1}{2}\hat{x_t}^2)$. Taking a second-order approximation of equation 63 and ignoring terms independent of policy gives:

$$c_{P,t} + c_{I,t} - y = y(\hat{y_t} + \frac{1}{2}\hat{y_t}^2 - \frac{1}{2}\chi\pi_t^2) \quad (63)$$

We can also note that:

$$H_{P,t} - \frac{1}{2} + H_{I,t} - \frac{1}{2} = 0 \tag{64}$$

Substituting equations 64 and 65 into equation 62 gives:

$$U_t - U \approx \mu_1 y \left(\hat{y_t} + \frac{1}{2}\hat{y_t}^2 - \frac{1}{2}\chi\pi_t^2 \right) - \frac{\mu_1}{y}\left((c_{P,t} - \frac{y}{2})^2 + (c_{I,t} - \frac{y}{2})^2 \right) -$$
$$\mu_2 \left((H_{P,t} - \frac{1}{2})^2 + (H_{I,t} - \frac{1}{2})^2 \right) - \mu_1(1-\sigma)y\left(\frac{h_{P,t} - h_P}{h_P} + \frac{\xi}{2}(\frac{h_{P,t} - h_P}{h})^2 \right) -$$
$$\mu_1 \sigma y \left(\frac{h_{I,t} - h_I}{h_I} + \frac{\xi}{2}(\frac{h_{I,t} - h_I}{h_I})^2 \right) \tag{65}$$

To eliminate the remaining first order terms from equation 66, we express variables in terms of log-deviations from the efficient steady-state values and drop terms of order 3 and higher:

$$U_t - U \approx \mu_1 y \left(\hat{y_t} + \frac{1}{2}\hat{y_t}^2 - \frac{1}{2}\chi\pi_t^2 \right) - \frac{\mu_1 y}{4}(\hat{c}_{P,t}^2 - \hat{c}_{I,t}^2) -$$
$$\mu_1 y \left((1-\sigma)\hat{h}_{P,t} + \sigma \hat{h}_{I,t} \right) - \mu_1 y \left(\frac{(1-\sigma)}{2}\hat{h}_{P,t}^2 + \frac{\sigma}{2}\hat{h}_{I,t}^2 \right) -$$
$$\frac{\mu_1 \xi y}{2}\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2 \right) - \frac{\mu_2}{4}(\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2) \tag{66}$$

Log-linearising the production function around the efficient steady state implies:

$$\hat{y_t} = \hat{A}_{z,t} + (1-\sigma)\hat{h}_{P,t} + \sigma \hat{h}_{I,t}$$

Substituting into equation 67 and dropping the term in $\hat{A}_{z,t}$, as it is independent of policy, implies:

$$U_t - U \approx \frac{\mu_1 y}{2}(\hat{y}_t^2 - \chi\pi_t^2) - \frac{\mu_1 y}{4}(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) -$$
$$\frac{\mu_1(1+\xi)y}{2}\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma \hat{h}_{I,t}^2 \right) - \frac{\mu_2}{4}(\hat{H}_{P,t}^2 + \hat{H}_{I,t}^2) \tag{67}$$

The log-linearised version of the housing market equilibrium condition around the efficient steady state implies:

$$\hat{H}_{P,t} = -\hat{H}_{I,t} \Rightarrow \hat{H}_{P,t}^2 + \hat{H}_{I,t}^2 = \frac{1}{2}(\hat{H}_{P,t} - \hat{H}_{I,t})^2$$

Substituting back into equation 68 and collecting the output, consumption and labour terms implies:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1}{2}(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) - \hat{y}_t^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2\right)\right)$$
$$-\frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{(\mu_1 y\chi)}{2}\pi_t^2 \tag{68}$$

Next, use:

$$\begin{aligned}
\frac{1}{2}(\hat{c}_{P,t}^2 + \hat{c}_{I,t}^2) - \hat{y}_t^2 &= \frac{1}{2}(\hat{c}_{P,t}^2 - \hat{y}_t^2) + \frac{1}{2}(\hat{c}_{I,t}^2 - \hat{y}_t^2) \\
&= \frac{1}{2}\left((\hat{c}_{P,t} + \hat{y}_t)(\hat{c}_{P,t} - \hat{y}_t) + (\hat{c}_{I,t} + \hat{y}_t)(\hat{c}_{I,t} - \hat{y}_t)\right) \\
&= \frac{1}{2}\left((\frac{3}{2}\hat{c}_{P,t} + \frac{1}{2}\hat{c}_{I,t})(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t}) - (\frac{3}{2}\hat{c}_{I,t} + \frac{1}{2}\hat{c}_{P,t})(\frac{1}{2}\hat{c}_{P,t} - \frac{1}{2}\hat{c}_{I,t})\right) \\
&= \frac{1}{4}(\hat{c}_{P,t} - \hat{c}_{I,t})^2
\end{aligned}$$

Substituting back into equation 69 implies:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1}{4}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\left((1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2\right)\right)$$
$$-\frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{\mu_1 y\chi}{2}\pi_t^2 \tag{69}$$

Next, the labour supply equations imply:

$$\frac{w_{P,t}h_{P,t}}{w_{I,t}h_{I,t}} = \frac{1-\sigma}{\sigma}$$

Combining implies:

$$h_{I,t} = (\frac{\sigma}{1-\sigma})^{\frac{1}{1+\xi}} h_{P,t} (\frac{c_{P,t}}{c_{I,t}})^{\frac{1}{1+\xi}}$$

Combining with the production function implies:

$$\begin{aligned}
y_t &= A_{z,t}h_{P,t}(\frac{\sigma}{1-\sigma}\frac{c_{P,t}}{c_{I,t}})^{\frac{\sigma}{1+\xi}} \\
\Rightarrow \hat{h}_{P,t} &= \hat{y}_t - \hat{A}_{z,t} - \frac{\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t}) \\
\Rightarrow \hat{h}_{I,t} &= \hat{y}_t - \hat{A}_{z,t} - \frac{1-\sigma}{1+\xi}(\hat{c}_{P,t} - \hat{c}_{I,t})
\end{aligned}$$

Hence:

$$(1-\sigma)\hat{h}_{P,t}^2 + \sigma\hat{h}_{I,t}^2$$
$$= (1-\sigma)\left(\hat{y}_t - \hat{A}_{z,t} - \frac{\sigma}{(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})\right)^2 + \sigma\left(\hat{y}_t - \hat{A}_{z,t} - \frac{1-\sigma}{(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})\right)^2$$
$$= (\hat{y}_t - \hat{A}_{z,t})^2 + \frac{\sigma(1-\sigma)}{(1+\xi)^2}(\hat{c}_{P,t} - \hat{c}_{I,t})^2$$

Substituting back into equation 70 and ignoring terms independent of policy gives:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\hat{y}_t^2\right) - \frac{\mu_2}{8}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \frac{\mu_1 y\chi}{2}\pi_t^2 \tag{70}$$

Using the first-order conditions for the efficient steady state to express μ_2 in terms of $\mu_1 y$:

$$\mu_2 = \frac{\mu_1 U_{H,I}}{U_{C,I}} = \mu_1 y j$$

Substituting into equation 71 gives:

$$U_t - U \approx \frac{-\mu_1 y}{2}\left(\frac{1+\xi+4\sigma(1-\sigma)}{4(1+\xi)}(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + (1+\xi)\hat{y}_t^2\right) - \frac{j}{4}(\hat{H}_{P,t} - \hat{H}_{I,t})^2 - \chi\pi_t^2$$

The welfare-based loss function can be expressed in terms of quadratic and gap variables as:

$$W_0 = \frac{-\mu_1 y}{2}(1+\xi)E_0\sum_{t=0}^{\infty}\beta_P^t\left(\hat{y}_t^2 + \lambda_1\pi_t^2 + \lambda_2(\hat{c}_{P,t} - \hat{c}_{I,t})^2 + \lambda_3(\hat{H}_{P,t} - \hat{H}_{I,t})^2\right)$$

Where $\lambda_1 = \frac{\chi}{(1+\xi)}$, $\lambda_2 = \frac{(1+\xi+4\sigma(1-\sigma))}{4(1+\xi)^2}$ and $\lambda_3 = \frac{j}{4(1+\xi)}$.

Annex 3: Log-linear equations of the model

This annex presents the log-linearised version of the model based on a Taylor series expansion of the equations of the model around the efficient non-stochastic steady state derived in Annex 1.

$$\hat{c}_{P,t} = E_t\hat{c}_{P,t+1} - (\hat{R}_t - E_t\pi_{t+1})$$

$$\hat{H}_{P,t} = \frac{\beta_P}{(1+\tau_H-\beta_P)}E_t(\hat{q}_{t+1} - \hat{c}_{P,t+1}) - \frac{(1+\tau_H)}{(1+\tau_H-\beta_P)}(\hat{q}_t - \hat{c}_{P,t}) + \hat{A}_{j,t}$$

$$\hat{w}_{P,t} = \xi \hat{h}_{P,t} + \hat{c}_{P,t}$$

$$\sigma(\hat{w}_{I,t} + \hat{h}_{I,t}) + \frac{L_M}{y}\left(\hat{L}_{M,t} - \frac{1}{\beta_P}(\hat{L}_{M,t-1} + \hat{R}_{L,t-1} - \pi_t)\right) - \frac{q}{2y}(\hat{H}_{I,t} - \hat{H}_{I,t-1}) = \frac{1}{2}\hat{c}_{I,t}$$

$$\hat{L}_{M,t} = \rho_L(\hat{L}_{M,t-1} - \pi_t) + (1-\rho_L)E_t(\hat{q}_{t+1} + \pi_{t+1} + \hat{H}_{I,t})$$

$$\hat{c}_{I,t} = E_t\hat{c}_{I,t+1} - \left(\frac{1}{1-\beta_P\rho_L\mu}\hat{R}_{L,t} - \frac{\beta_P\rho_L\mu}{(1-\beta_P\rho_L\mu)}\hat{\mu}_{t+1} - E_t\pi_{t+1} + \frac{\mu}{(1-\mu)\hat{\mu}_t}\right)$$

$$\begin{aligned}\hat{H}_{I,t} = &\frac{\beta_I}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{q}_{t+1} - \hat{c}_{I,t+1}) \\ &+ \frac{\mu(1-\rho_L)LTV}{(1-\mu(1-\rho_L)LTV-\beta_I)}E_t(\hat{\mu}_t + \hat{q}_{t+1} + \pi_{t+1} - \hat{c}_{I,t}) \\ &- \frac{1}{(1-\mu(1-\rho_L)LTV-\beta_I)}(\hat{q}_t - \hat{c}_{I,t}) + \hat{A}_{j,t}\end{aligned}$$

$$\hat{w}_{I,t} = \xi \hat{h}_{I,t} + \hat{c}_{I,t}$$

$$\hat{L}_{E,t} = (1-\sigma)(\hat{w}_{P,t} + \hat{h}_{P,t}) + \sigma(\hat{w}_{I,t} + \hat{h}_{I,t})$$

$$\hat{y}_t = \hat{A}_t + (1-\sigma)\hat{h}_{P,t} + \sigma\hat{h}_{I,t}$$

$$\hat{w}_{P,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{P,t} + \hat{R}_t - \hat{R}_{L,t}$$

$$\hat{w}_{I,t} = r\hat{m}c_t + \hat{y}_t - \hat{h}_{I,t} + \hat{R}_t - \hat{R}_{L,t}$$

$$\pi_t = \beta_P E_t\pi_{t+1} + \frac{\varepsilon}{\chi}r\hat{m}c_t$$

$$\hat{n}_t = \frac{\zeta\varphi(1+\tau_b)}{\beta_P}(\hat{R}_{L,t-1} + \hat{L}_{t-1}) - \zeta\frac{(\varphi-1)}{\beta_P}(\hat{R}_{t-1} + \hat{D}_{t-1}) - \frac{\zeta}{\beta_P}(1+\varphi\tau_b)\pi_t$$

$$\hat{n_t} = \varphi\hat{L}_t - (\varphi-1)\hat{D}_t$$

$$\hat{\varphi}_t = \hat{L}_t - \hat{n_t}$$

$$\hat{\psi}_t = \hat{\varphi}_t$$

$$\hat{\psi}_t = \frac{\varphi\tau_b}{(\varphi\tau_b+1)}\hat{\varphi}_t + \frac{\varphi(1+\tau_b)}{(\varphi\tau_b+1)}(\hat{R}_{L,t} - \hat{R}_t) + \frac{\zeta\psi}{(1-\zeta+\zeta\psi)}\psi\hat{_{t+1}}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(\nu_\pi\pi_t + \nu_y\hat{y}_t) + \varepsilon_{R,t}$$

$$\hat{y}_t = \frac{1}{2}(\hat{c}_{P,t} + \hat{c}_{I,t})$$

$$\hat{H}_{P,t} + \hat{H}_{I,t} = 0$$

$$\frac{L}{y}\hat{L}_t = \frac{L_M}{y}\hat{L}_{M,t} + \hat{L}_{E,t}$$