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Working Paper 22/01

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Produced By:

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Fiscal and Macroprudential Policies in a Monetary Union*

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May 2022

Abstract

In the European Monetary Union (EMU), monetary policy is decided by the European Central Bank (ECB). This can create some imbalances that can potentially be corrected by national policies. So far, fiscal policy was the natural candidate to adjust those imbalances. Nevertheless, after the global financial crisis (GFC), a new policy candidate has emerged, namely national macroprudential policies, with the mission of reducing financial risks. This issue gives rise to an interesting research question: how do macroprudential and fiscal policies interact? By affecting real interest rates and the level of activity, a discretionary macroprudential policy alters the evolution of public debt and can impose a fiscal cost when the government is forced to increase tax rates to stabilize the public debt-to-GDP ratio. In a monetary union, a domestic macroprudential shock creates substantial cross-border financial effects and also influences the foreign country fiscal stance. Moreover, a discretionary government spending policy affects housing prices, so the strenght with which macroprudential policy reacts to a change in the price of houses has an impact on the fiscal multiplier.

Keywords: Monetary union, macroprudential policy, fiscal policy, monetary policy

JEL Classification: E32, E44, F45

*This paper is part of the project PID2020-116242RB-I00 funded by MCIN/AEI/ 10.13039/501100011033. We thank to conference and seminar participants at the ASSA Annual Meeting, 2021, IX Meeting on International Economics, IFABS, 46th SAEe, 14th Annual Meeting of the Portuguese Economic Journal, ICMAIF 2021 (Crete), XXIII Encuentro de Economia Aplicada, University of Paris Nanterre, Banque de France, Tor Vergata University of Rome and Banca d'Italia. Special thanks to Stefan Gebauer for his very useful suggestions. Jose E. Bosca and Javier Ferri acknowledge the financial support of the Generalitat Valenciana grant GVPROMETEO2020-083, Fundacion Rafael del Pino and BBVA Research.

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1 Introduction

The global financial crisis (GFC) introduced new challenges for the implementation of macro-financial policies. Risks to financial stability made it clear the necessity of policies to stabilize the financial system, namely macroprudential policies. However, these kind of policies need to coexist with existing policies such as monetary and fiscal policy, making their coordination an important topic of research. This issue becomes especially interesting in the context of a monetary union, in which monetary policy is centralized by a common central bank and fiscal policy is decentralized to national authorities. How to set up macroprudential policies in this kind of setting has been a question of concern.

In the European Union (EU), after a long policy debate, a new macroprudential framework has emerged. Although some macroprudential tools such as capital requirements are under the umbrella of the Basel Committee and are set up internationally, other tools such as the loan-to-value (LTV) are decided at a national level under the supervision of the European Systemic Risk Board (ESRB). These national tools are left at discretion of national authorities, which need to decide on their setting, depending on both national and European financial markets developments.

In the European Monetary Union (EMU), monetary policy is decided by the European Central Bank (ECB). Countries have lost their capacity to decide on monetary policy at a national level plus the exchange rate is no longer a tool of adjustment. This can create some imbalances than can potentially be corrected by national policies. So far, fiscal policy was the natural candidate to adjust those imbalances. Nevertheless, after the GFC, a new policy candidate has emerged, namely national macroprudential policies. This issue gives rise to important research questions: what is the fiscal impact of macroprudential policies? Do national macroprudential policies impose a fiscal cost on foreign countries - and hence provide an additional argument for cross-country coordination? How can coordination between macroprudential, fiscal and monetary discretionary policies within a country improve macroeconomic outcomes? How should macroprudential rules be designed in order to optimize the impact of fiscal and monetary surprises? Are trade-offs present in the design of the macroprudential rules according to targets? Which are the existing trade-offs among different shocks?

In the present paper, we develop a simple two country DSGE model in a monetary union. We calibrate these two countries to match the features of Spain and Germany. There are two types of agents in each economy. These two agents differ in their temporal discount rate, becoming borrowers and lenders. We allow lenders to have different preferences for national and foreign borrowers debt which illustrates an important empirical feature in an open economy; the financial transactions between the agents inside the economy and between those across economies. In doing so, we follow Sargent (1987) or, more recently Krishnamurthy and Vissing-Jorgensen (2015) or Reis (2020). Our utility-maximizer individuals have debt in the utility function. Moreover, the individuals with lower temporal discount rate have a positive asset position. This structure allows us to introduce country-specific bond preferences that summarizes imperfect financial integration. The bond market consists of four kind of bonds: lenders in both countries have access to (public and private) national and foreign bonds. Our model generates a downward-sloping demand for different types of bonds.

We also consider an endogenous risk premium that varies over time, as in Blagov (2013) and Hansen (2015). In our model, the risk premium arises as an external effect that creates an additional wedge between domestic and foreign issued bonds. Borrowers from relatively more indebted countries will have to pay higher interest rates.

The paper is related to the general equilibrium literature that allows for international lending, like in Stähler and Thomas (2012) or Kolasa (2009). In those papers, one country becomes a net borrower. In our paper, the borrowing country displays agents who are lending home and abroad, as also there are agents in the lending country who borrow from the foreign borrowing country. Our paper is also related to research that looks at policy interactions, i.e. monetary-fiscal (some examples on this link are Beetsma and Jensen, 2005; Ferrero, 2009, Farhi and Werning, 2017; Demid, 2018; or Bonam and Lukkezen, 2019) and monetary-macroprudential (see Farhi and Werning, 2016 for a unified approach and Bussière et al., 2020, for a recent survey). However, papers have been silent about the fiscal-macroprudential interaction and above all the monetary-fiscal-macroprudential policy mix. We aim at filling this gap in the literature.

Recently, Reis (2020) has explored the fiscal footprint of macroprudential policies. He identifies three channels through which this effect travels: first, an increase in the price of government bonds which eases the government budget constraint; second, a reduction on activity and, hence, on government revenues, which makes the government constraint more tight; third, a lower bailout costs. In our paper we present results with a fully developed dynamic general equilibrium model. The model accounts for the first two channels adding a cross-border lending/borrowing channel, but abstracting from defaults and bailouts.

Results show that a discretionary macroprudential policy in one country produces substantial cross-border effects on financial assets, real activity and the fiscal footprint. However, the spillover effects of the fiscal policy are much more limited. We also find that for symmetric macroprudential shocks affecting both countries, there is a monetary surprise that can replicate the results, up to the effects on private debt. That means that a simultaneous discretionary expansionary monetary policy by the central bank can neutralize three gaps that arise after a macroprudential policy (in output, inflation and taxes) without killing the effect on private debt. When the reduction of inflation becomes an objective itself, coordination between macroprudential and fiscal discretionary policies can give rise to better outcomes by improving output results, without killing the inflation drop or imposing a cost in terms of higher taxes in the short run. In terms of the design of policy rules, we conclude that a combination of tight macroprudential policy and loose monetary policy maximizes the effects of fiscal policy, but minimizes the impact of technology shocks. Moreover, a tight macroprudential rule helps fiscal policy to achieve the target of a larger output, and monetary policy to reduce inflation. After a productivity shock, however, a tight macroprudential rule diminishes the positive impact on GDP, but reinforces the negative impact on inflation.

The remainder of the paper goes as follows. Section 2 presents the model setup. Section 3 shows the financial and fiscal impact of macroprudential policy. Section 4 studies the effects of fiscal policy. Section 5 compares the effects of coordinated discretionary macroprudential, monetary and fiscal policies. Section 6 discusses how committing to a looser or tighter rule by the different policy institutions can alter the results of discretionary policies. Section 7 concludes.

2 The Model

We consider two countries; A (domestic) and B (foreign), that trade consumption and bonds in a monetary union. Agents in each country have the option to choose between different assets. Bonds are tradable, whereas houses are non tradable. The difference between the discount rates among households endogenously divides consumers into borrowers and savers in each economy. Governments can also borrow. Both, public and private borrowers are able to borrow from national or foreign lenders. By the same token, lenders choose between lending to national or foreign (public or private) borrowers. Therefore, there are four different bonds in the monetary union. We add country-specific preferences for bonds to account for bond-specific demand functions. Borrowing entails an external effect in the form of a risk premium, which evolves according to relative total debt-to-GDP.

Households also buy domestic and foreign goods. However, for simplicity, we do not consider home bias in this market. Labor hired in a competitive market is the only factor used in production. Prices of final goods are sticky, in a Calvo fashion (Calvo, 1983).

There is a progressive tax scheme, with a flat rate and an exempt labor income. The fiscal authority moves the flat rate in order to stabilize the government debt-to-GDP ratio. Macroprudential institutions monitor housing prices and decide the LTV. The central bank chooses the reference interest rate on the basis of the monetary union inflation.

Both countries are symmetrical in terms of the equations characterizing their economies. The relative weights of the population between countries are represented by ω and $(1-\omega)$ for country A and B, respectively. Within each country the share of borrowers is represented by τ and τ^*, respectively. That is, if we designate by N the total population in the monetary union, then $N = N_A + N_B$, where N_A and N_B stand for population in economy A and B, respectively. Then, we define $\omega = \frac{N_A}{N}$ and $(1-\omega) = \frac{N_B}{N}$. If we assume that the patient and impatient population in economy A are N_l and N_r then $\tau = \frac{N_r}{N_A}$ and $(1-\tau) = \frac{N_l}{N_A}$. Similarly for country B, $\tau^* = \frac{N_r^*}{N_B}$ and $(1-\tau^*) = \frac{N_l^*}{N_B}$.

Next, we sketch the model, introducing the most relevant equations and leaving the rest to Appendix 1. We also omit the equivalent equations of country B.

2.1 Households

2.1.1 Patient Households

Patient households discount the future at a lower rate $\left(\frac{1}{\beta^l}\right)$ than impatient households. This fact drives them to be the lenders in the economy because they value relatively more future consumption than the group of borrowers in the population.

In general, lower-case letters stand for real variables whereas capital letters are reserved for nominal variables. Patient households solve the following optimization problem, maximizing their utility function:

$$U^l = E_0 \sum_{i=0}^{\infty} \beta^{li} \left(\begin{array}{c} \ln c_{t+i}^l + \gamma_h \ln h_{t+i}^l + \\ +\gamma_{b_{At}}(\bullet) \ln(b_{At}^l) + \chi_B \ln(b_{Bt}^l) + \\ \gamma_{b_{At}}^g(\bullet) \ln(b_{At}^g) + \chi_B^g \ln(b_{Bt}^g) - \frac{\left(n_{t+i}^l\right)^{1+\eta}}{1+\eta} \end{array} \right).$$

Variables are written relative to the population of lenders N_l, that is, c_t^l can be interpreted as total consumption of lenders divided by the total amount of lenders in economy A. In the same vein, n^l stands for per capita working hours and h^l represents the stock per capita of houses owned by lenders. Parameters η, γ_h, relate to the elasticity of labor supply with respect to wages and the preferences for housing, respectively. The utility function distinguishes between four types of bonds that capture different financial alternatives for lenders. Bonds issued by national borrowers in hands of national lenders are denoted by b^l_{At}, where $b^l_{At} = \frac{B^l_{At}}{P_{At}}$ and B^l_{At} are nominal bonds, which pay a gross nominal interest rate R_{At}. Similarly, bonds bought by national lenders from the home government are called b^g_{At}, whereas b^l_{Bt} and b^g_{Bt} stand for bonds issued by foreign households and government.

Preferences differ between public and private bonds because of differences in safety and/or liquidity, and also differ between domestic and foreign bonds, due to imperfect financial market integration. The assumption here is that different assets offer non-pecuniary services to households, a generalization of the model's utility function in Sargent (1987) or, more recently, in Krishnamurthy and Vissing-Jorgensen (2015) or Reis (2020). In this framework, as the total debt-to-output ratio in economy A increases with respect to economy B, bonds in economy B become more attractive. In particular, preferences for bonds change over time as a function of the ratio of total debt-to-output in economy A with respect to economy B. That is,

$$\gamma_{b_{At}}(\bullet) = \chi_A + \vartheta \left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right),$$

and

$$\gamma^g_{b_{At}}(\bullet) = \chi^g_A + \vartheta \left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right),$$

where b_t and b^*_t represent aggregate debt (to be defined more precisely below) in country A and B, respectively, and ϑ is a positive parameter. Notice that the difference $\chi^g_A - \chi_A$ captures the existence of a public-private bond bias, whilst the presence of a country bias would be reflected by $\chi^g_A - \chi^g_B$ and $\chi_A - \chi_B$.

We assume that patient households do not internalize the effect that their lending decision has on the terms $\frac{b_t}{y_t}$ and $\frac{b^*_t}{y^*_t}$, i.e. they do not consider how their decided amount of lending may affect the aggregates b_t and b^*_t.

The budget constraint, in real terms, can be written as follows:

$$\begin{aligned} & c^l_{At} + \frac{P_{Bt}}{P_{At}} c^l_{Bt} + q_t \left(h^l_t - h^l_{t-1} \right) + b^l_{At} + \frac{P_{Bt}}{P_{At}} b^l_{Bt} + b^g_{At} + \frac{P_{Bt}}{P_{At}} b^g_{Bt} \\ & \leq \left(1 - x^l_t \right) w_t n^l_t + \frac{R_{At-1}}{\pi_{At}} b^l_{At-1} + \frac{P_{Bt}}{P_{At}} \frac{R_{Bt-1}}{\pi_{Bt}} b^l_{Bt-1} \\ & + \frac{R^g_{At-1}}{\pi_{At}} b^g_{At-1} + \frac{P_{Bt}}{P_{At}} \frac{R_{t-1}}{\pi_{Bt}} b^g_{Bt-1} + d_t. \end{aligned} \tag{1}$$

c^l_{At} and c^l_{Bt} are, respectively, per capita consumption of home and foreign produced goods. P_{At} and P_{Bt} are the producer price indexes in countries A and B. The numeraire is P_{At}, which we use to deflate all the nominal variables, including loans, wages and profits. $\frac{P_{Bt}}{P_{At}}$ is the inverse of the terms of trade and q_t the real price of houses, h^l_t. A positive value for b^l_{At} and b^l_{Bt} corresponds to a lending amount, whereas a negative value implies a borrowing amount. For patient households both are positive. Bonds yield an interest rate, which depends on each particular type, with bonds issued by government in country B yielding an interest rate R_t that coincides with the policy rate. d_t are profits stemming from the monopolistically competitive firms that lenders own. Finally, we assume that labor income ($w_t n^l_t$, i.e. wages times per capita working hours) is taxed, while, for the sake of simplicity, profits and bond returns are not taxed. The average tax rate on lenders labor income x^l_t is defined as $\frac{t^l_{At}}{w_t n^l_t}$, where t^l_{At} represents the per capita amount of taxes paid by these households,

$$t^l_{At} = m_{At}(w_t n^l_t - \overline{t_A}). \tag{2}$$

The previous expression assumes a progressive tax scheme by means of the introduction of a per capita tax-exempt income, equal to $\overline{t_A}$, and a flat tax rate on labor income, m_{At}. Hence, the average tax rate increases with income, helping to strengthen the automatic stabilizer feature of the tax. As will be evident below, m_{At} is going to be the instrument used by the fiscal authority to keep the ratio of public debt-over-GDP constant in the long run. A higher m_{At} implies a more progressive tax scheme.

The inflation rate on the domestically produced goods, π_{At}, and foreign goods, π_{Bt}, are defined as

$$\pi_{At} = \frac{P_{At}}{P_{At-1}}, \tag{3}$$

$$\pi_{Bt} = \frac{P_{Bt}}{P_{Bt-1}}. \tag{4}$$

The consumption basket for lenders, in terms of utility, is defined as

$$c^l_t = (c^l_{At})^{\omega}(c^l_{Bt})^{1-\omega} \tag{5}$$

Patient households maximize with respect to c^l_{At}, c^l_{Bt}, h^l_t and n^l_t (leading to standard first order conditions, as can be seen the Appendix 1), and also over the four financial assets (b^l_{At}, b^l_{Bt}, b^g_{At}, and

b^g_{Bt}). Optimal decisions regarding bonds should satisfy the following conditions:

$$\lambda^l_t = \beta^l E_t \left[\lambda^l_{t+1} \frac{R_{At}}{\pi_{At+1}} \right] + \frac{\chi_A}{b^l_{At}} + \frac{\vartheta}{b^l_{At}} \left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right), \tag{6}$$

$$\lambda^l_t = \beta^l E_t \left[\lambda^l_{t+1} \frac{R_{Bt}}{\pi_{At+1}} \right] + \frac{P_{At}}{P_{Bt}} \frac{\chi_B}{b^l_{Bt}}, \tag{7}$$

$$\lambda^l_t = \beta^l E_t \left[\lambda^l_{t+1} \frac{R^g_{At}}{\pi_{At+1}} \right] + \frac{\chi^g_A}{b^g_{At}} + \frac{\vartheta}{b^g_{At}} \left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right), \tag{8}$$

$$\lambda^l_t = \beta^l E_t \left[\lambda^l_{t+1} \frac{R_t}{\pi_{At+1}} \right] + \frac{P_{At}}{P_{Bt}} \frac{\chi^g_B}{b^g_{Bt}}, \tag{9}$$

where λ^l_t is the Lagrangian multiplier associated with the restriction (1). From these conditions, we can obtain non-arbitrage conditions among the four assets. Differences in interest rates between different bonds depend on three factors. First, the amount of bonds held by households, as captured by the second term in the right hand side of the above expressions. Other things equal, an increase in the amount of one type of bonds reduces its price and rises its interest rate vis-à-vis other bonds. Thus, there is a downward-sloping demand for bonds. Second, the term related to the endogenous risk premium. Third, a factor capturing the terms of trade, $\frac{P_{At}}{P_{Bt}}$. An increase in the last two factors augments the yield on domestic bonds vis-à-vis foreign bonds. From equations (8) and (9) let us define this risk premium term as

$$\phi_t \equiv -\frac{\pi_A \vartheta}{b^g_A \beta^l \lambda^l} \left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right), \tag{10}$$

which represents the increase in $R^g_{At} - R_t$ to a given change in $\left(\frac{b^*_t}{b_t} \frac{y_t}{y^*_t} - 1 \right)$.Notice that, *caeteris paribus*, it increases with the ratio of total debt-over-output in country A. In (10) we keep some variables constant at their initial steady-state value to emphasize the contribution of changes in the ratios of household debt-to-GDP.

Together with the demand for bonds from foreign households, the above expressions produce induced effects of economic policies on the desired composition of bonds in the portfolio of the lenders households, which translate into changes in the decisions of private/public and national/ foreign borrowers.

2.1.2 Impatient Households

Impatient households have a higher discount rate than patient households $\left(\frac{1}{\beta^r} > \frac{1}{\beta^l}\right)$, which drives them to be the borrowers of the economy. These agents sell bonds to the lenders of both economies, paying for these the corresponding interest rates. Impatient households solve the following optimization problem, maximizing the utility function

$$U_0^r = E_0 \sum_{i=0}^{\infty} \beta^{ri} \left(\ln c_{t+i}^r + \gamma_h \ln h_{t+i}^r - \frac{\left(n_{t+i}^r\right)^{1+\eta}}{1+\eta} \right),$$

subject to

$$c_{At}^r + \frac{P_{Bt}}{P_{At}} c_{Bt}^r + q_t \left(h_t^r - h_{t-1}^r\right) + \frac{R_{At-1}}{\pi_{At}} b_{t-1}^r \le \left(1 - x_t^r\right) w_t n_t^r + b_t^r. \tag{11}$$

c_t^r can be interpreted as total consumption of borrowers divided by the total amount of borrowers in economy A (N_r). $b_t^r = \frac{B_t^r}{P_{At}}$ expresses total real private borrowing in country A from domestic and foreign lenders, and is defined as a positive variable. Similarly to impatient households, x_t^r is defined as $\frac{t_{At}^r}{w_t n_t^r}$, where

$$t_{At}^r = m_{At}(w_t n_t^r - \overline{t_A}). \tag{12}$$

Hence, the average tax rate will differ between borrowers and lenders given that, although wages will be common across households, working hours may be different.

Additionally, these consumers face a borrowing constraint of the form

$$E_t \left[\frac{R_{At}}{\pi_{At+1}} b_t^r \right] \le E_t \left[k_{At} q_{t+1} h_t^r \right], \tag{13}$$

where k_{At} can be interpreted as a loan-to-value ratio (LTV) and will be the instrument for the macro-prudential policy. Notice that keeping constant the rest of variables, a rise in the price of b_t^r (a fall in R_{At}) will increase the supply of private bonds.

The consumption basket for borrowers is defined as

$$c_t^r = (c_{At}^r)^{\omega} (c_{Bt}^r)^{1-\omega}. \tag{14}$$

The impatient household maximizes with respect to c^r_{At}, c^r_{Bt}, h^r_t, n^r_t, and b^r_t. The derivative with respect to b^r_t yields

$$\lambda^r_t = \beta^r E_t \left[\lambda^r_{t+1} \frac{R_{At}}{\pi_{At+1}} \right] + \xi_t R_{At}, \tag{15}$$

where λ^r_t is the Lagrangian multiplier associated with the restriction (11). Borrowers in country A will pay the interest rate R_{At}, regardless who provides the funds (domestic or foreign lenders).

2.2 Firms

We have J firms of mass 1. Each firm j produces a differentiated good and takes decisions subject to three constraints: a constant returns production technology; a downward sloping demand curve, and a perfect competition labor market. In all that follows all variables are represented in per capita terms of total population in the economy. The optimization problem can be written as:

$$\min \; W_t n_t(j),$$

subject to:

$$y_t(j) = z_t n_t(j), \tag{16}$$

$$y_t\left(j\right) = \left(\frac{P_{At}\left(j\right)}{P_{At}} \right)^{-\varepsilon} y_t, \tag{17}$$

where W_t is the nominal wage and z_t is the technical level, both common to all firms.

Optimization with respect to employment yields the following standard labor demand in real terms:

$$w_t = mc_t \frac{y_t}{n_t}. \tag{18}$$

Optimal prices are obtained assuming a Calvo scheme:

$$\max_{p_{At}(j)} \Pi_0 = E_t \sum_{i=0}^{\infty} \lambda^l_{t+i} (\beta^l \theta)^i \left[\left(\prod_{r=1}^{i} \frac{(\pi_{At+r-1})^{\zeta}}{\pi_{At+r}} p_{At}(j) - mc_{t+i} \right) y_{t+i}\left(j\right) \right],$$

subject to the variety demand function,

$$y_{t+i}(j) = \left(\prod_{r=1}^{i} \frac{(\pi_{At+r-1})^{\zeta}}{\pi_{At+r}} p_{At}(j) \right)^{-\varepsilon} y_{t+i}. \tag{19}$$

A proportion θ of firms do not reset prices optimally at t and adjust them according to a simple indexation rule to catch up with lagged inflation: $P_{jt} = (\pi_{t-1})^{\zeta} P_{jt-1}$. We are assuming that firms that are not allowed to change prices optimally reset prices each period according to inflation. $p_{At}(j)$ stands for the relative price $\frac{P_{At}(j)}{P_{At}}$. Taking into account that all firms will set the same optimal price, the solution to the above problem renders the following New Keynesian Phillips curve,

$$\left(\frac{1-\theta(\pi_{At})^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{E_t\sum_{i=0}^{\infty}\lambda_{t+i}^{l}(\beta^{l}\theta)^{i}mc_{t+i}y_{t+i}\left(\prod_{r=1}^{i}\frac{(\pi_{At+r-1})^{\zeta}}{(\pi_{At+r})}\right)^{-\varepsilon}}{E_t\sum_{i=0}^{\infty}\lambda_{t+i}^{l}(\beta^{l}\theta)^{i}y_{t+i}\left(\prod_{r=1}^{i}\frac{(\pi_{At+r-1})^{\zeta}}{(\pi_{At+r})}\right)^{1-\varepsilon}}. \tag{20}$$

2.3 Aggregation

There are some restrictions linking debt. In the domestic economy, bonds issued by domestic borrowers (total private debt) may be in hands of either domestic or foreign lenders:

$$b_t^r = \frac{(1-\tau)}{\tau}b_{At}^l + \frac{(1-\omega)}{\omega}\frac{(1-\tau^*)}{\tau}b_{At}^{*l}, \tag{21}$$

where b_{At}^{*l} is economy B's lenders holdings of bonds issued by economy A's borrowers.

With respect to public debt, in the domestic economy:

$$b_t^g = (1-\tau)\,b_{At}^g + (1-\tau^*)\,\frac{(1-\omega)}{\omega}b_{At}^{*g}. \tag{22}$$

where b_{At}^{*g} is economy B's holdings of bonds issued economy A's government.

Total debt, i.e. the sum of public and private debt, in economy A can be defined as:

$$b_t = b_t^g + \tau b_t^r \tag{23}$$

Aggregation over housing results in:

$$\tau h_t^r + (1-\tau)\,h_t^l = h, \tag{24}$$

where h is an exogenous variable representing the per capita stock of housing in economy A.

Consumption can also be aggregated using the shares of lenders and borrowers in each economy.

Consumption in country A of goods produced in country A:

$$c_{At} = \tau c_{At}^{r} + (1 - \tau)\, c_{At}^{l}. \tag{25}$$

Consumption in country A of goods produced in country B (imports (exports) by country A (B)):

$$c_{Bt} = \tau c_{Bt}^{r} + (1 - \tau)\, c_{Bt}^{l}. \tag{26}$$

Total consumption in country A can be defined as

$$c_t = \tau c_t^{r} + (1 - \tau)\, c_t^{l} = c_{At} + \frac{P_{Bt}}{P_{At}} c_{Bt}, \tag{27}$$

where

$$c_t^{l} = c_{At}^{l} + \frac{P_{Bt}}{P_{At}} c_{Bt}^{l}, \tag{28}$$

and

$$c_t^{r} = c_{At}^{r} + \frac{P_{Bt}}{P_{At}} c_{Bt}^{r}. \tag{29}$$

Employment is aggregated as

$$n_t = \tau n_t^{r} + (1 - \tau)\, n_t^{l}. \tag{30}$$

Total government revenues are characterized by,

$$t_t = (1 - \tau) t_{At}^{l} + \tau t_{At}^{r} = m_A (w_t n_t - t_{At}). \tag{31}$$

Finally, the aggregate production function for the domestic economy is,

$$y_t = z_t n_t. \tag{32}$$

2.4 Fiscal, Monetary and Macroprudential Rules

We assume an exogenous amount of non-productive government consumption g_t for the domestic economy and g_t^* for economy B. Public debt evolves according to

$$b_t^{g} = \frac{R_{At-1}^{g}}{\pi_{At}} b_{t-1}^{g} + (g_t - t_t). \tag{33}$$

$$b_t^{*g} = \frac{R_{t-1}}{\pi_{Bt}} b_{t-1}^{*g} + (g_t^* - t_t^*). \tag{34}$$

Common monetary policy is characterized by a simple Taylor rule. The central bank takes into account the weighted average of the monetary union countries' inflation:

$$R_t = R_{t-1}^{\rho} \left[\left(\pi_{At}^{\omega} \pi_{Bt}^{1-\omega} \right)^{\Phi} \overline{R} \right]^{1-\rho}. \tag{35}$$

Each country uses the flat tax as the instrument to stabilize the ratio of total public debt-over-GDP in the long run. Then, the fiscal policy rule can be represented as:

$$m_{At} = m_{At-1} + \psi_1 f_t \left(\frac{b_t^g}{y_t} - \overline{\left(\frac{b^g}{y} \right)} \right) + \psi_2 f_t \left(\frac{b_t^g}{y_t} - \frac{b_{t-1}^g}{y_{t-1}} \right), \tag{36}$$

where the parameter ψ_1 captures the speed of adjustment from the current ratio to the desired ratio, and f_t is a dummy variable that controls for the time period in which the fiscal rule is initially inactive. Similarly, for country B,

$$m_{Bt} = m_{Bt-1} + \psi_1^* f_t^* \left(\frac{b_t^{*g}}{y_t^*} - \overline{\left(\frac{b^{*g}}{y^*} \right)} \right) + \psi_1^* f_t^* \left(\frac{b_t^{*g}}{y_t^*} - \frac{b_{t-1}^{*g}}{y_{t-1}^*} \right). \tag{37}$$

As an approximation for a realistic macroprudential policy, we consider a Taylor-type rule for the loan-to-value ratio, in the spirit of a Taylor rule for monetary policy. In standard models, the LTV ratio is a fixed parameter which is not affected by economic conditions. However, we can think of regulations of LTV ratios as a way to moderate credit booms. When the LTV ratio is high, the collateral constraint is less tight. And, since the constraint is binding, borrowers will borrow as much as they are allowed to. Lowering the LTV tightens the constraint and therefore restricts the loans that borrowers can obtain. Recent research on macroprudential policies has proposed Taylor-type rules for the LTV ratio so that it reacts inversely to variables such that the growth rates of GDP, credit, the credit-to-GDP ratio or house prices. These rules can be a simple illustration of how a macroprudential policy could work in practice. We consider a decentralized macroprudential policy in which each country can implement its own rule for the LTV, as it is the case within the EU:

$$k_{At} = k_{SSA} \left(\frac{q_t}{\overline{q}} \right)^{-\phi_{Aq}^k}, \tag{38}$$

$$k_{Bt} = k_{SSB} \left(\frac{q_t^*}{\overline{q}^*} \right)^{-\phi_{Bq}^k}, \tag{39}$$

where k_{SS} and $\overline{q}$ are the steady-state values for the loan-to-value ratio and house prices in country A. $\phi_{Aq}^k \geq 0$ measures the response of the loan-to-to value to house prices in country A. This kind of rule would deliver a lower LTV ratio in booms, when house prices are high, therefore restricting the credit in the economy and avoiding a credit boom derived from good economic conditions.

2.5 GDP and Balance of Payments

The total resource constraint should satisfy the condition that total production y_t should be equal to the sum of factor incomes or total final demand in the economy. That is,

$$y_t = w_t n_t + (1-\tau)\, d_t, \tag{40}$$

or

$$y_t = c_{At} + \frac{(1-\omega)}{\omega} c_{At}^* + g_t. \tag{41}$$

To find an expression for aggregate firm's profits in economy A, we can combine expressions (40) and (18) to obtain:

$$(1-\tau)\, d_t = \left(\frac{1}{mc_t} - 1 \right) w_t n_t. \tag{42}$$

To obtain an expression for the balance of payments, first, we multiply the household budget constraints (1) and (11) by their respective shares in population $(1-\tau)$ and τ and aggregate them. Then, we substitute $g_t = y_t - c_{At} - \frac{(1-\omega)}{\omega} c_{At}^*$ into the previous expression to reach:

$$\begin{aligned}
(1-\tau)\frac{P_{Bt}}{P_{At}}\left(b_{Bt}^l + b_{Bt}^g\right) + (1-\tau)\, b_{At}^g - (1-\tau^*)\frac{(1-\omega)}{\omega} b_{At}^{*l} = \\
\left(\frac{(1-\omega)}{\omega} c_{At}^* - \frac{P_{Bt}}{P_{At}} c_{Bt} \right) + (g_t - t_t) + \\
+ (1-\tau)\frac{P_{Bt}}{P_{At}}\left(\frac{R_{Bt-1}}{\pi_{Bt}} b_{Bt-1}^l + \frac{R_{t-1}}{\pi_{Bt}} b_{Bt-1}^g \right) + (1-\tau)\frac{R_{At-1}^g}{\pi_{At}} b_{At-1}^g \\
- (1-\tau^*)\frac{(1-\omega)}{\omega}\frac{R_{At-1}}{\pi_{At}} b_{At-1}^{*l},
\end{aligned} \tag{43}$$

with $x_t = \frac{t_t}{w_t n_t}$. Notice that all the previous variables are in terms of total population in economy A (they are divided by N_A).

A clear intuition of the balance of payment condition can be grasped if we obtain the steady state version of (43)

$$
\begin{aligned}
&(1-\tau)\frac{P_B}{P_A}b_B^l\left(1-\frac{R_B}{\pi_B}\right)-(1-\tau^*)\frac{(1-\omega)}{\omega}b_A^{*l}\left(1-\frac{R_A}{\pi_A}\right) \\
&+(1-\tau)\frac{P_B}{P_A}b_B^g\left(1-\frac{R}{\pi_B}\right)-(1-\tau^*)\frac{(1-\omega)}{\omega}b_A^{*g}\left(1-\frac{R_A^g}{\pi_A}\right)+(t-g)= \\
&\left(\frac{(1-\omega)}{\omega}c_A^*-\frac{P_B}{P_{At}}c_B\right),
\end{aligned}
\tag{44}
$$

where the the LHS in expression (44) represents the variation in the net foreign asset position, as the difference between bonds owned abroad (both private and public) and domestic bonds (private and public) owned by foreigners. The RHS is the current account balance.

2.6 Calibration

The values assigned to the parameters of the model use information for Spain (country A) and Germany (country B). We impose some parameters and use the equations of the steady state of the model to calibrate the rest. In this way, we are able to reproduce relevant observable economic facts of the two countries. Table 1 reflects the parameters that have been initially set and the sources used, while Table 2 reproduces the calibrated parameters and the targets obtained with the model.

The weight of Spain, ω, has been set according to Eurostat population statistics. The parameter ϑ, the reaction of the risk premium to changes in the ratio of debt-to-GDP, takes a value of 0.09. It comes from an estimation of a 4.5 basis points increase in the risk premium (the difference between interest rates on public debt in Spain and Germany) for every one percentage point increase in the debt-to-GDP ratio.[1] To obtain the value of 0.09 we use equation (10). The Taylor rule parameters Φ and ρ are the standard ones. The first value is consistent with the original parameters proposed by Taylor in 1993. The latter value reflects a realistic degree of interest-rate smoothing (see McCallum, 2001). They are also consistent with recent estimations of Taylor rules (see for example, Sauer and Sturm, 2007). The inverse of Frisch elasticities, the Calvo price probabilities, and the inflation indexation are

[1] See European Commission (2018) and IMF (2017).

the ones estimated for Spain and Germany by Casares and Vázquez (2018). As regards the share of credit constrained consumers, we guess a value of $\tau = 0.5$ for Spain.[2] To set τ^* we correct τ using the relationship between the ratio of household debt-over-GDP in Germany and the same ratio in Spain. The value of the elasticity of substitution among different goods in the German monopolistic sector has been calculated for a price-cost margin of 39%, according to the Deutsche Bundesbank (2017). We assume that the price-cost margin in Spain is halfway between the estimates for Germany and Italy in this study. We use the European Mortgage Federation information (Westig and Bertalot, 2017) to set the value for the LTV. Also, we assume the same intensity of reaction between Spain and Germany in the fiscal rule.

Table 2 shows the values of some parameters related to the targeting of different empirical facts. In our model, steady-state interest rates depend on the discount rate and preference parameters for different types of bonds. We chose β^{*l} so that, given the value of the bond preference parameters, the annual interest rate of the German government bond is 1.9%.

Then, we set the impatient discount rate for Spanish lenders at the same figure. Notice that this does not imply that the interest rate of the Spanish public debt is the same as that of Germany, due to the different country preferences for bonds and the existence of a risk premium. We consider that the borrower's discount rate is 2 percentage points lower, in the range of values observed in the literature (see Iacoviello, 2005 for a discussion on the calibration of this parameter). There are four preference parameters that affect the demand for bonds in Spain (χ_A, χ_A^g, χ_B, χ_B^g) and the respective ones for Germany (χ_A^*, χ_A^{*g}, χ_B^*, χ_B^{*g}). We calibrate these values so that the steady-state model solution reproduces two sets of facts: *(a)* a set of interest rates (the 1 to 2 year Spanish government bond, R_A^g, and the mortgage interest rates in Spain and Germany, R_A and R_b); *(b)* a set of debt ratios (the ratio of Spanish public debt in the hands of Spanish households, the ratio of German public debt in German households' hands, the ratio of German private debt in German households' hands, total Spanish debt-over-GDP, and total German debt-over-GDP). The housing preference parameter for Spain has been chosen to obtain the residential stock-over-GDP according to Bank of Spain (BdE). Following Deloittte (2016) report, the number of dwellings per citizen in Spain is roughly the same as in Germany. Thus, we set the value of this parameter for Germany to replicate the same ratio as in Spain. The values for tax-exempt income $\overline{t_A}$ and $\overline{t_b}$ guarantee that the flat rates for Spain and Germany are 0.31 and 0.37,

[2] Galí, López-Salido and Vallés (2004), in a model with only Ricardian and rule-of-thumb consumers, consider that the best guess for the share of non Ricardian consumers is in the neighbourhood of 1/2.

respectively.[3]

As shown in Table 3, we normalize the steady-state per capita aggregate income in Spain to 1, and the German per capita income to 1.3, according to Eurostat. Government consumption represents 18.5% of GDP in Spain and 18.8% in Germany. The targeted long-run public debt-over-GDP is set to an annual 60 and 85 per cent in Germany and Spain, respectively. The German figure is in accordance with the the EU's Stability and Growth Pact, while the Spanish one is close to the average along the last decade. Table 4 shows the model steady-state values for aggregate demand and bonds.

Table 1. Parameters imposed in the model

Parameter	Value	Description	Source
ω	0.35	Weight domestic country	Eurostat
ϑ	0.09	Risk premium reaction to the debt/GDP	European Commission, IMF
Φ	1.5	Inflation parameter Taylor rule	Sauer & Sturm (2007)
ρ	0.8	Persistence interest rate Taylor rule	Sauer & Sturm (2007)
η	1.74	Inverse Frisch elasticity A	Casares & Vázquez (2018)
η^*	1.59	Inverse Frisch elasticity B	Casares & Vázquez (2018)
τ	0.5	Share of borrowers A	Galí, López-Salido, Vallés (2004)
τ^*	0.36	Share of borrowers B	FRED
θ	0.86	Price Calvo probability A	Casares & Vázquez (2018)
θ^*	0.56	Price Calvo probability B	Casares & Vázquez (2018)
ε	3.17	Monopolistic competition elasticity A	Deutsche Bundesbank (2017)
ε^*	3.56	Monopolistic competition elasticity B	Deutsche Bundesbank (2017)
k_{SSA}	0.80	LTV A	Westig and Bertalot, 2017
k_{SSB}	0.76	LTV B	Westig and Bertalot, 2017
$\psi_1 = \psi_1^*$	$\frac{1}{6}$	Fiscal reaction to SS deviation	
$\psi_2 = \psi_2^*$	1.1	Fiscal adjustment speed	
$\phi_{Aq}^k = \phi_{Bq}^k$	0	Macropru reaction parameter	
ζ	0.44	Inflation indexation A	Casares & Vázquez (2018)
ζ^*	0.21	Inflation indexation B	Casares & Vázquez (2018)

[3] These tax rates are approximated using data on taxation from the European Commission.

Table 2. Parameters calibrated from model equations

Parameter	Value	Target	Source
β^{*l}	0.985	$(R-1)*4*100=1.9\%$	Bundesbank
β^{l}	0.985		Assumed
β^{*r}	0.965		Iacoviello (2005)
β^{r}	0.965		Iacoviello (2005)
χ_A	0.0901	$(R_A-1)*4*100=3.2\%$	ECB
χ_A^g	0.0606	$(R_A^g-1)*4*100=2.2\%$	BdE
χ_B	0.0375	$\frac{b_B^{*l}}{b^{*r}}=0.71$	Bundesbank
χ_B^g	0.0413	$\frac{b^*}{y^*}=8.8$	OECD
χ_A^*	0.0532	$\frac{b}{y}=12.7$	OECD
χ_A^{*g}	0.0399	$\frac{b_A^g}{b_t^g}=0.58$	BdE
χ_B^*	0.0454	$(R_b-1)*4*100=3.5\%$	ECB
χ_B^{*g}	0.0189	$\frac{b_B^{*g}}{b_t^{*g}}=0.48$	Bundesbank
$\overline{t_A}$	0.04	$m_A=0.31$	European Commission
$\overline{t_B}$	0.24	$m_B=0.37$	European Commission
γ_h	0.6785	$\frac{h}{4y}=0.65$	BdE
γ_h^*	0.7156	$\frac{h}{4y}=0.65$	Deloitte

Table 3. Normalizations and exogenous variables

Variable	Value	Source
y	1	Normalization
y^*	1.3	Eurostat
$\frac{g}{y}$	0.185	World Bank
$\frac{g^*}{y^*}$	0.188	World Bank
$\frac{b^g}{4y}$	0.85	EU's Stability and Growth Pact corrected
$\frac{b^{*g}}{4y^*}$	0.6	EU's Stability and Growth Pact

Table 4. Steady state (in country per capita values on a quarterly basis)

Spain		
y	1.00	GDP in country A
c_A	0.29	Consumption in A of goods produced in A
c_A^*	0.53	Exports of A
b^g	3.40	Government bonds issued in A
τb^r	9.22	Private bonds issued in A
$(1-\tau)b_A^l$	5.09	Private bonds issued in A held by A's lenders
$(1-\tau)b_B^l$	4.46	Private bonds issued in B held by A's lenders
$(1-\tau)b_A^g$	1.97	Public bonds issued in A held by A's lenders
$(1-\tau)b_B^g$	3.01	Public bonds issued in B held by A's lenders
Germany		
y^*	1.30	GDP in country B
c_B^*	0.69	Consumption in country B of goods produced in country B
c_B	0.37	Exports of country B
b^{*g}	3.12	Government bonds issued in B
$\tau^* b^{*r}$	8.28	Public bonds issued in B
$(1-\tau^*)b_A^{*l}$	2.23	Private bonds issued in A held by B's lenders
$(1-\tau^*)b_B^{*l}$	5.88	Private bonds issued in B held by B's lenders
$(1-\tau^*)b_A^{*g}$	0.77	Public bonds issued in A held by B's lenders
$(1-\tau^*)b_B^{*g}$	1.50	Public bonds issued in B held by B's lenders

3 The Financial and Fiscal Impact of Macroprudential Policy

Macroprudential policy aims at stabilizing financial risks, mainly affecting the amount of private debt in the economy. However, by doing so, it can also affect public debt and taxes. In this section, we present the dynamics shown by the model after a macroprudential policy consisting of permanently lowering the LTV in Spain by two percentage points, from 0.80 to 0.78. Although this is an asymmetric policy intervention, we also study cross-border effects on the German economy. We assume here that the macroprudential rule is not active, by setting $\phi_{Aq}^k = \phi_{Bq}^k = 0$.

Figures 1 to 4 show the dynamics of some macroeconomic variables, in percentage deviations with respect to their steady state, following the policy. The macroprudential policy reduces largely the amount of private debt in the economy (Figure 1). A less indebted economy generates a reduction in the risk premium (Figure 2), pushing up the demand for public and private Spanish bonds. Figure 1 illustrates the effects on bond holdings. While German holdings of Spanish government bonds boost, this is no the case for Spanish lenders, who find it more profitable to invest in housing. Figure 3 shows that there is a diversion from borrowers' housing demand, which is heavily punished by a tighter collateral constraint, to housing demand by lenders. It induces a change in the Spanish lenders portfolio of assets, from bonds to houses. Houses are non-tradable goods, and this housing effect is absent in Germany. The reduction in the supply of Spanish private bonds affects negatively the equilibrium interest rate of household bonds, as is evident in equation (13). Overall, Figure 2 reveals that these movements in the supply and demand of Spanish bonds lowers the private bonds interest rate by 26 basis points (bp) on impact and the government bond interest rate by 33 bp. Figure 2 also displays a reduction in the inflation rate in both countries, which is due to the fall in GDP (Figure 3) provoked by the macroprudential policy.

The lower inflation rates and a weaker level of activity move government debt up at period t (Figure 1). The increase in the government-debt-to-GDP ratio is even higher, so the fiscal authority reacts by rising the marginal tax according to its fiscal rule. Hence, using Reis' semantics (Reis, 2020), the macroprudential policy intervention makes a positive fiscal footprint in our model economy. Interestingly, the policy produces substantial cross-border effects not only in terms of financial assets, as we have seen, but also on real activity and fiscal outcomes. German GDP falls almost as much as it does in Spain during 5 quarters, but contrary to Spain, GDP recovers gradually afterwards. The main reason for that is the pronounced drop in German exports (Figure 3). The rise in public-debt-to-GDP compels the German government to lift the tax rate (Figure 4), a policy which is relatively more harmful in terms of income for borrowers than for lenders.

The tightening of macroprudential policy in Spain triggers tighter fiscal policies in response to higher debt to GDP ratios in both countries. So fiscal policy is not acting countercyclically, but procyclically in this scenario. This result seems different from standard findings of other papers on, for example, monetary/fiscal policy coordination, which usually find that a contractionary shock of one policy might trigger some countercyclical response by the other. In fact, this is what happens also in our model when fiscal policy is tightened, provoking lower output and inflation and a monetary countercyclical response by lowering interest rates via the Taylor rule.

The fact that using Spanish macroprudential policy to reduce the stock of private debt alters in such a way the macroeconomic outcomes of the other country in the monetary union calls for a supranational coordination of these policies. Our results indicate that macroeconomic policy in Spain does not only cause a positive fiscal footprint in Spain but also in Germany, imposing thus an economic and possibly political cost on the neighboring country. If the German government is aware of this circumstance it could decide to react in two ways. First, it could partially abate borrowing constraints, which in our model consists of rising the LTV. In this way, it would penalize financial stability to relieve the fiscal burden. Second, it might decide to punish Spanish government through a tighter macroprudential policy in a similar game to a trade war, in which case both economies would end up with a much higher tax rate.

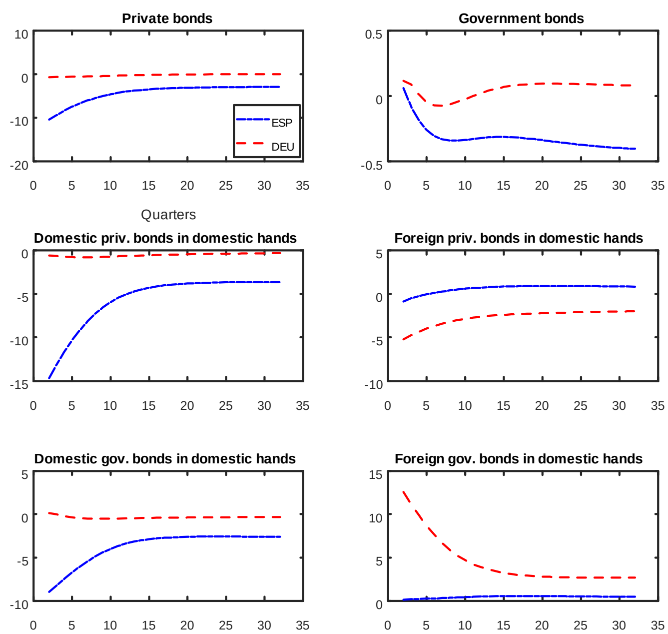

Figure 1: Macroprudential policy in Spain: bonds

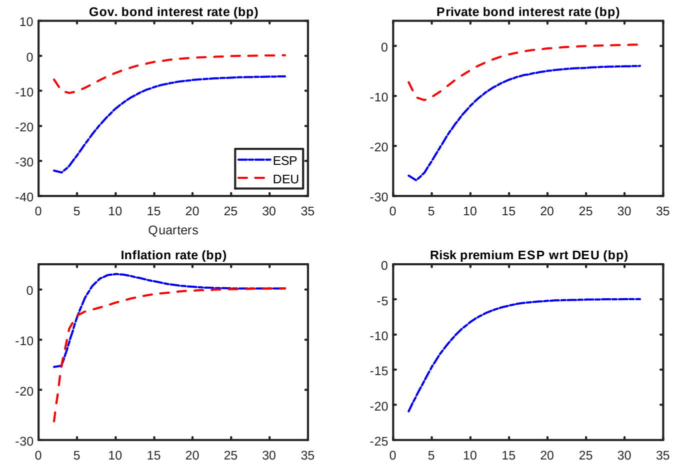

Figure 2: Macroprudential policy in Spain: interest rates

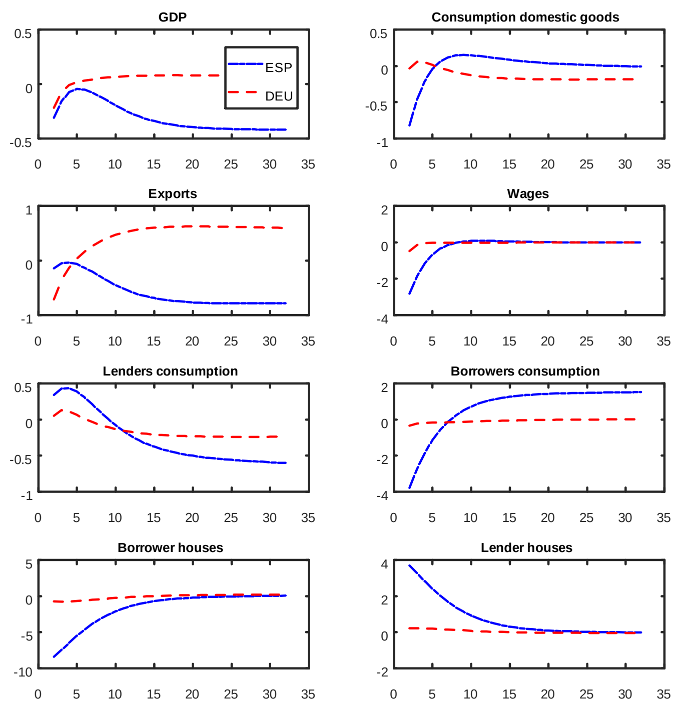

Figure 3: Macroprudential policy in Spain: real activity

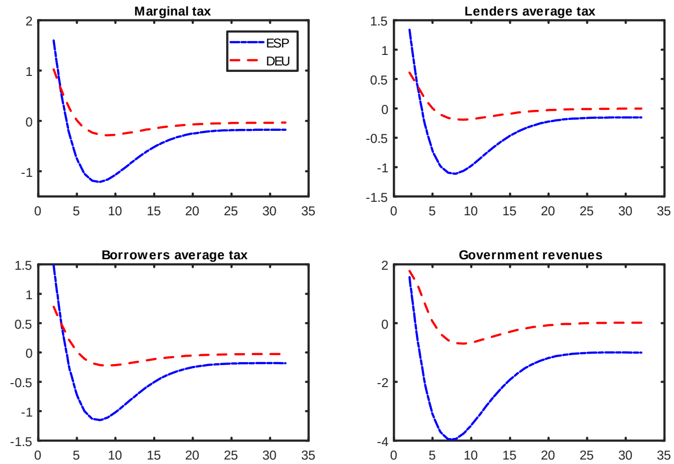

Figure 4: Macroprudential policy in Spain: taxes

4 Discretionary Symmetric Policies

In this section, we study the possibilities that arise in our model for policy coordination among policy makers in both countries. To start with, we assume that governments in the two countries coordinate their actions in such a way that the direction and intensity in the use of macroprudential and fiscal policy in both countries is the same. We call this a symmetric macroprudential and fiscal policy intervention. Also, we introduce a discretionary monetary policy conducted by the central bank.

The left-hand column of Figure 5 shows the response after a permanent 2 percentage point drop in LTV in Germany and Spain, which is the symmetrical equivalent of the macroprudential policy in Spain studied above. Compared to the one-country policy, the simultaneous intervention of macroprudential authorities in both economies produces a non-linear increment in the fiscal footprint of both countries, understood as the increase in the tax rate necessary to accomplish the fiscal rule. This result can be explained by cross spillover effects.

In the middle column we show the response of macroeconomic variables to a contractionary monetary policy shock. More specifically, we assume that the central bank changes the interest rate in away that replicates the same impact effect as macroprudential policy on the Spanish GDP.[4] As compared to macroprudential policy, the monetary shock generates very similar results for GDP, inflation and government bonds. It also produces an almost identical fiscal footprint (increase in the tax rate), the

[4] This shock is exactly a 44 *bp* positive surprise in the interest rate lasting three years. However, due to the perfect foresight assumption we use to solve the model, interest rates actually fall on impact.

main difference being in the size of the effects on private bonds, which is more pronounced in the case of a macroprudential symmetric intervention.

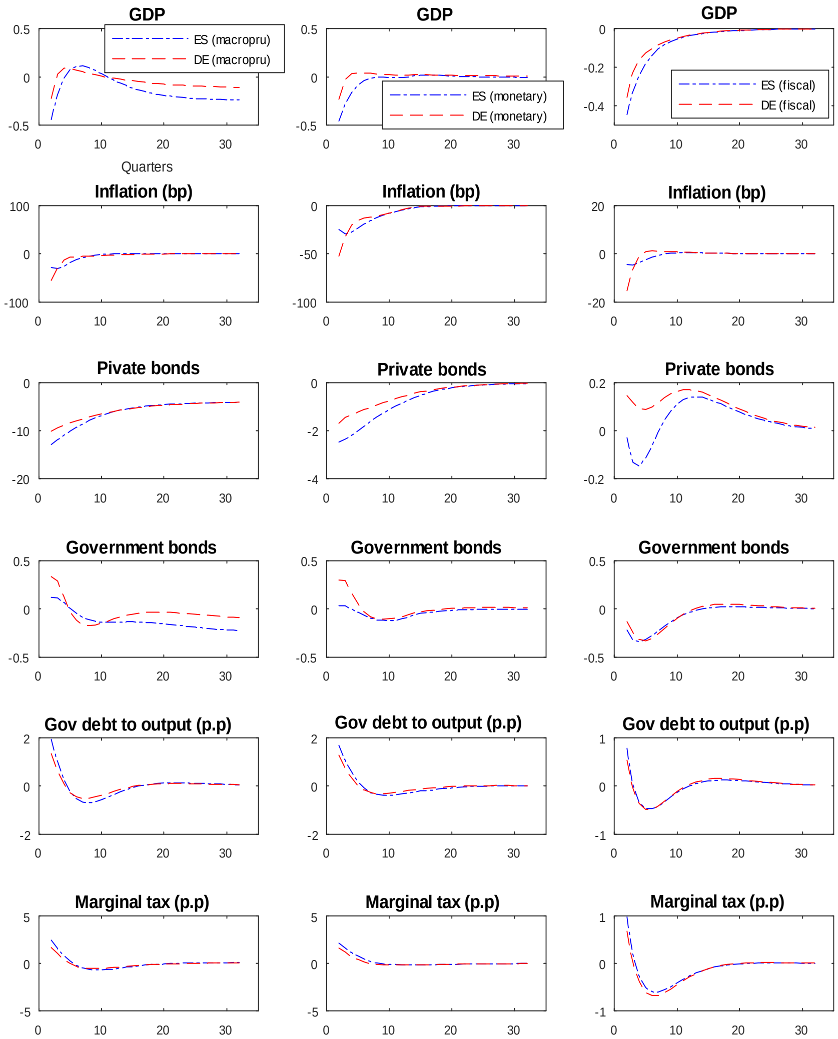

Figure 5: Symmetric shocks: macroprudential, monetary and fiscal

In the right column, we represent a contractionary fiscal policy.[5] For the same drop in GDP, fiscal policy entails a smaller inflation reduction, a more modest effect on the tax rate, which soon turns negative, and virtually no effect on private debt. The analogies and contrasts in the effects among the three policies bring the possibility of using a different policy mix for the achievement of different combinations of targets. Consider that policy-makers pursue four targets. Add to the three economic

[5]The permanent drop on government spending amounts to 0.5 pertentage points of GDP in both countries.

goals (output, inflation and financial stability) one pure political target, and assume that governments dislike the increase in the tax rate to which they are forced when government debt to output goes up. Assume that macroprudential institutions detect a need for a financial correction and lower the LTVs in the two countries. This comes at the cost of lower output, a downward deviation of inflation rates targets, and higher taxes. A simultaneous discretionary expansionary monetary policy by the central bank can neutralize the three gaps (output, inflation and taxes) without killing the effect on private debt.

Assume now that, in addition to the financial correction, the central bank sees as necessary to reduce the inflation rate. The central bank may consider a no-surprise policy, letting the monetary rule and the macroprudential shock to work. This will reproduce again the effects of the first column. The central bank fulfills the target of a lower inflation, the macroprudential authority gets the goal of less private debt, and the cost is expressed in terms of weaker output and a stronger tax bit. However, a coordination with governments can improve the outcomes if they carry out an expansionary fiscal policy. A simultaneous macroprudential-fiscal policy mix in this way can improve output without killing the inflation drop or imposing a cost in terms of taxes in the short run.

The takeaway message of the previous discussion is that the different policy makers (macroprudential, fiscal and monetary authorities) can take advantage of the discretionary margin they have to coordinate their actions in order to improve macroeconomic outcomes.

5 Policy Rules and Policy Interplay

In all the discussion so far we have kept constant the policy rule parameters at their benchmark values. This implies that the fiscal and monetary policy rules were operative, but the macroprudential rule was not playing any role given that benchmark values are $\phi^k_{Aq} = \phi^k_{Bq} = 0$. In this section we relax these assumptions and study how committing to a looser or tighter rule by the different policy institutions can alter the results of discretionary policies. Figure 10 shows the results.

Figure 10 depicts impact effects in Spain under different parameterizations of monetary, macroprudential and fiscal rules. A higher value of the parameters mean, respectively, a more intense reaction in the interest rate, given a change in inflation, a bigger movement in the LTV for a given variation in the price of housing, or a greater change in the tax rate after deviations in the public debt over GDP from its long-run target. Thus, larger values of Φ, ϕ^k_{Aq}, and ψ_1 are associated with tighter monetary, macroprudential and fiscal rules.

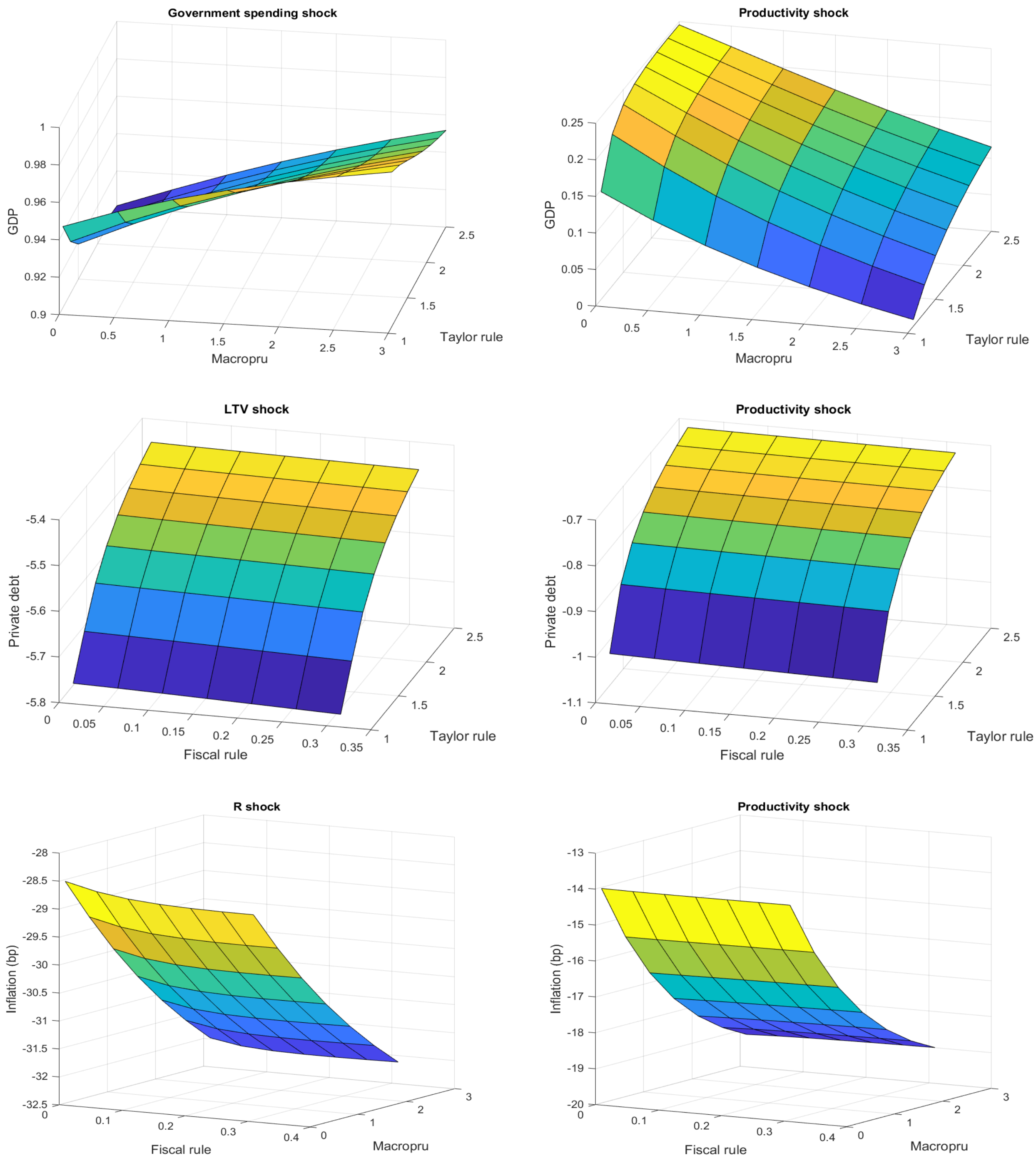

Figure 10: Policy shocks and policy rules: impact effects

Figure 10 studies the effectiveness of fiscal, macroprudential and monetary discretionary policies to influence their main targets: GDP, private debt, and inflation. More particularly, we analyze the interaction between a given discretionary policy and the policy rules related with the other two policies. In all three cases we compare the results with those we observe when the economy is affected by a permanent increase in productivity of one percentage point. Both discretionary policies or productivity increases, as well as induced changes in rules, only occur in Spain.

For example, in the case of a government consumption policy in Spain (first row in Figure 10), both monetary and macroprudential rules interact with the effect of the policy or the productivity shock. After a permanent increase in government spending of one percentage point of GDP, there is an increase in output, but the multiplier is less than one. The monetary and the macroprudential rule interact with the effect of the policy. A combination of a loose monetary policy and a tight macroprudential rule maximizes the fiscal multiplier. In our plot, the multiplier goes up by 9 percent with respect to the minimum value that is reached when the macroprudential rule is inactive and the monetary rule is the tightest of all those considered. However, the combination of policies that maximizes the effect of a fiscal shock is the same that minimizes the impact effect of a productivity shock on GDP. Therefore, there is a kind of trade-off between the policy-mix that would be preferred by the fiscal authority in order to enlarge the effectiveness of its policy, and the production benefits the economy would receive from a technology shock with that combination.

In the next row, we examine the impact of a discretionary macroprudential policy (consisting of a 1 percentage point cut in the Spanish LTV) on private debt. In this case, we focus on the interplay with fiscal and monetary policies, as reflected in their policy rules. As regards private debt, the LTV shock does not interact with the fiscal rule, and mildly interacts with the monetary rule. Actually, the difference in the effectiveness of the policy is about 5 percent, depending whether the monetary policy is loose (the maximum effect) or tight (the minimum one). In this case the preferred policy-mix by the macroprudential institution is the same that maximizes the impact on private debt of a technology shock.

Finally, we inspect the effectiveness of a monetary surprise of 50 basis point to reduce inflation. We confront this policy with the fiscal and macroprudential rules. Again, the strength of the reaction of the tax rate measured by the fiscal rule only plays a marginal role in shaping the impact on inflation. More important is the task of the macroprudential policy. In our model, a tighter macroprudential policy may contribute to increase the success of the monetary surprise to lower inflation by 13 percent. This policy is also aligned with the most adequate one to take advantage in terms of inflation of a productivity shock.

From the above discussion we can conclude that a tight macroprudential rule seems to help in achieving the targets of discretionary fiscal and monetary policies. After a productivity shock, however, a tight macroprudential rule diminishes the positive impact on GDP and reinforces the negative impact on inflation. The role of the fiscal rule to interact with other policies and change their outcomes is only marginal.

6 Conclusions

In this paper, we develop a simple two-country general equilibrium model in a monetary union with borrowers and lenders in each economy. Public and private borrowers face a downward sloping bond-specific demand function. We calibrate the two countries to proxy Spain and Germany, paradigmatic examples of periphery and core countries. We use the model to study the interactions between macroprudential and fiscal policies.

After a macroprudential policy in Spain there is a change in the Spanish lenders portfolio of assets, from bonds to houses. There is also a deviation of Spanish bond holdings from national to foreign investors. Lower inflation rates and a weaker level of activity move government debt up. The increase in the government-debt-to-GDP ratio is even higher, so the fiscal authority reacts by rising the marginal tax according to a fiscal rule. German exports fall due to weaker Spanish demand, and so does aggregate output. The rise in public-debt-to-GDP compels the German government to lift the tax rate Thus, an asymmetric macroprudential intervention creates a non trivial positive footprint in the whole monetary union.

The effects of a symmetric macroprudential intervention in the monetary union, including the fiscal footprint, can be replicated, up to the effect on private debt, by means of a positive shock on the interest rates by the European Central Bank. It provides an argument for a coordination between macroprudential and monetary policy. A European coordinated macroprudential and monetary intervention allows for financial stabilization but neutralizes the fiscal footprint in the European countries and unwanted effects on output and inflation.

There is also a link between discretionary fiscal policy and macroprudential policy, as captured by a rule. A negative public spending policy positively affects housing prices, so when macroprudential policy is tight, private debt is more affected and borrowers' consumption falls more. Thus, a tight macroprudential policy increases the fiscal multiplier. Moreover, a tight macroprudential rule also helps to a monetary surprise to achieve a higher impact on inflation. After a permanent productivity shock, however, a tight macroprudential rule diminishes the positive impact of productivity on GDP and reinforces the negative impact on inflation.

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Appendices

Appendix 1

In this Appendix, we show all the equations of the model

National economy households equations

The Patient Households

$$\gamma_{b_{At}}(\bullet) = \chi_A + \vartheta\left(\frac{b_t^*}{b_t}\frac{y_t}{y_t^*} - 1\right) \tag{45}$$

$$\gamma_{b_{At}}^g(\bullet) = \chi_A^g + \vartheta\left(\frac{b_t^*}{b_t}\frac{y_t}{y_t^*} - 1\right) \tag{46}$$

$$\begin{aligned} & c_{At}^l + \frac{P_{Bt}}{P_{At}} c_{Bt}^l + q_t\left(h_t^l - h_{t-1}^l\right) + b_{At}^l + \frac{P_{Bt}}{P_{At}} b_{Bt}^l + b_{At}^g + \frac{P_{Bt}}{P_{At}} b_{Bt}^g \\ & \leq \left(1 - x_t^l\right) w_t n_t^l + \frac{R_{At-1}}{\pi_{At}} b_{At-1}^l + \frac{P_{Bt}}{P_{At}} \frac{R_{Bt-1}}{\pi_{Bt}} b_{Bt-1}^l \\ & + \frac{R_{At-1}^g}{\pi_{At}} b_{At-1}^g + \frac{P_{Bt}}{P_{At}} \frac{R_{t-1}}{\pi_{Bt}} b_{Bt-1}^g + d_t, \end{aligned} \tag{47}$$

$$t_{At}^l = m_{At}(w_t n_t^l - \overline{t_A}) \tag{48}$$

$$x_t^l = \frac{t_{At}^l}{w_t n_t^l} \tag{49}$$

$$\pi_{At} = \frac{P_{At}}{P_{At-1}} \tag{50}$$

$$c_t^l = c_{At}^l + \frac{P_{Bt}}{P_{At}} c_{Bt}^l \tag{51}$$

$$\lambda_t^l = \frac{\omega}{c_{At}^l} \tag{52}$$

$$\frac{c_{At}^{l}}{c_{Bt}^{l}} = \frac{\omega P_{Bt}}{(1-\omega)\,P_{At}} \tag{53}$$

$$\frac{\gamma_h}{h_t^l} = \lambda_t^l q_t - \beta E_t\left[\lambda_{t+1}^l q_{t+1}\right] \tag{54}$$

$$w_t = \frac{c_{At}^l}{\omega}\left(n_t^l\right)^{\eta} \tag{55}$$

$$\lambda_t^l = \beta^l E_t\left[\lambda_{t+1}^l \frac{R_{At}}{\pi_{At+1}}\right] + \frac{\chi_A}{b_{At}^l} + \frac{\vartheta}{b_{At}^l}\left(\frac{b_t^*}{b_t}\frac{y_t}{y_t^*} - 1\right) \tag{56}$$

$$\lambda_t^l = \beta^l E_t\left[\lambda_{t+1}^l \frac{R_{Bt}}{\pi_{At+1}}\right] + \frac{P_{At}}{P_{Bt}}\frac{\chi_B}{b_{Bt}^l} \tag{57}$$

$$\lambda_t^l = \beta^l E_t\left[\lambda_{t+1}^l \frac{R_{At}^g}{\pi_{At+1}}\right] + \frac{\chi_A^g}{b_{At}^g} + \frac{\vartheta}{b_{At}^g}\left(\frac{b_t^*}{b_t}\frac{y_t}{y_t^*} - 1\right) \tag{58}$$

$$\lambda_t^l = \beta^l E_t\left[\lambda_{t+1}^l \frac{R_t}{\pi_{At+1}}\right] + \frac{P_{At}}{P_{Bt}}\frac{\chi_B^g}{b_{Bt}^g} \tag{59}$$

$$\phi_t \equiv -\vartheta\left(\frac{b_t^*}{b_t}\frac{y_t}{y_t^*} - 1\right) \tag{60}$$

The Impatient Households

$$\begin{aligned} &c_{At}^r + \frac{P_{Bt}}{P_{At}}c_{Bt}^r + q_t\left(h_t^r - h_{t-1}^r\right) + \frac{R_{At-1}}{\pi_{At}}b_{t-1}^r \\ &\leq \left(1 - x_t^r\right) w_t n_t^r + b_t^r. \end{aligned} \tag{61}$$

$$t_{At}^r = m_{At}(w_t n_t^r - \overline{t_A}) \tag{62}$$

$$x_t^r = \frac{t_{At}^r}{w_t n_t^r} \tag{63}$$

$$E_t\left[\frac{R_{At}}{\pi_{At+1}}b_t^r\right] \leq E_t\left[k_{At} q_{t+1} h_t^r\right], \tag{64}$$

$$c_t^r = c_{At}^r + \frac{P_{Bt}}{P_{At}} c_{Bt}^r \tag{65}$$

$$\lambda_t^r = \frac{\omega}{c_{At}^r} \tag{66}$$

$$\frac{c_{At}^r}{c_{Bt}^r} = \frac{\omega P_{Bt}}{(1-\omega) P_{At}} \tag{67}$$

$$\frac{\gamma_h}{h_t^r} = q_t \lambda_t^r - E_t \left[\beta^r q_{t+1} \lambda_{t+1}^r + \xi_t k_{At} q_{t+1} \pi_{At+1}\right] \tag{68}$$

$$w_t = \frac{c_{At}^r}{\omega} \left(n_t^r\right)^\eta \tag{69}$$

$$\lambda_t^r = \beta^r E_t \left[\lambda_{t+1}^r \frac{R_{At}}{\pi_{At+1}}\right] + \xi_t R_{At} \tag{70}$$

The foreign country's households

The Patient Households

$$\gamma_{b_{At}}^*(\bullet) = \chi_A^* + \vartheta \left(\frac{b_t^*}{b_t} \frac{y_t}{y_t^*} - 1\right) \tag{71}$$

$$\gamma_{b_{At}}^{*g}(\bullet) = \chi_A^{*g} + \vartheta \left(\frac{b_t^*}{b_t} \frac{y_t}{y_t^*} - 1\right) \tag{72}$$

$$\begin{aligned}
&\frac{P_{At}}{P_{Bt}} c_{At}^{*l} + c_{Bt}^{*l} + q_t^* \left(h_t^{*l} - h_{t-1}^{*l}\right) + \frac{P_{At}}{P_{Bt}} b_{At}^{*l} + b_{Bt}^{*l} + \frac{P_{At}}{P_{Bt}} b_{At}^{*g} + b_{Bt}^{*g} \\
&\leq \left(1 - x_t^{*l}\right) w_t^* n_t^{*l} + \frac{P_{At}}{P_{Bt}} \frac{R_{At-1}}{\pi_{At}} b_{At-1}^{*l} + \frac{R_{Bt-1}}{\pi_{Bt}} b_{Bt-1}^{*l} \\
&+ \frac{P_{At}}{P_{Bt}} \frac{R_{At-1}^g}{\pi_{At}} b_{At-1}^{*g} + \frac{R_{t-1}}{\pi_{Bt}} b_{Bt-1}^{*g} + d_t^*
\end{aligned} \tag{73}$$

$$t_{Bt}^l = m_{Bt}(w_t^* n_t^{*l} - \overline{t_B}) \tag{74}$$

$$x_t^{*l} = \frac{t_{Bt}^l}{w_t^* n_t^{*l}} \tag{75}$$

$$c_t^{*l} = \frac{P_{At}}{P_{Bt}} c_{At}^{*l} + c_{Bt}^{*l} \tag{76}$$

$$\pi_{Bt} = \frac{P_{Bt}}{P_{Bt-1}} \tag{77}$$

$$\lambda_t^{*l} = \frac{1-\omega}{c_{Bt}^{*l}} \tag{78}$$

$$\frac{c_{At}^{*l}}{c_{Bt}^{*l}} = \frac{\omega}{(1-\omega)} \frac{P_{Bt}}{P_{At}} \tag{79}$$

$$\frac{\gamma_h^*}{h_t^{*l}} = \lambda_t^{*l} q_t^* - \beta^{*l} E_t \left[\lambda_{t+1}^{*l} q_{t+1}^* \right] \tag{80}$$

$$w_t^* = \frac{c_{Bt}^{*l}}{1-\omega} \left(n_t^{*l} \right)^{\eta^*} \tag{81}$$

$$\lambda_t^{*l} = \beta^{*l} E_t \left[\lambda_{t+1}^{*l} \frac{R_{At}}{\pi_{Bt+1}} \right] + \frac{P_{Bt}}{P_{At}} \frac{\chi_A^*}{b_{At}^{*l}} + \frac{\vartheta}{b_{At}^{*l}} \frac{P_{Bt}}{P_{At}} \left(\frac{b_t^*}{b_t} \frac{y_t}{y_t^*} - 1 \right) \tag{82}$$

$$\lambda_t^{*l} = \beta^{*l} E_t \left[\lambda_{t+1}^{*l} \frac{R_{Bt}}{\pi_{Bt+1}} \right] + \frac{\chi_B^*}{b_{Bt}^{*l}} \tag{83}$$

$$\lambda_t^{*l} = \beta^{*l} E_t \left[\lambda_{t+1}^{*l} \frac{R_{At}^g}{\pi_{Bt+1}} \right] + \frac{P_{Bt}}{P_{At}} \frac{\chi_A^{*g}}{b_{At}^{*g}} + \frac{P_{Bt}}{P_{At}} \frac{\vartheta}{b_{At}^{*g}} \left(\frac{b_t^*}{b_t} \frac{y_t}{y_t^*} - 1 \right) \tag{84}$$

$$\lambda_t^{*l} = \beta^{*l} E_t \left[\lambda_{t+1}^{*l} \frac{R_t}{\pi_{Bt+1}} \right] + \frac{\chi_B^{*g}}{b_{Bt}^{*g}} \tag{85}$$

Impatient Households

$$\frac{P_{At}}{P_{Bt}} c_{At}^{*r} + c_{Bt}^{*r} + q_t^* \left(h_t^{*r} - h_{t-1}^{*r} \right) + \frac{R_{Bt-1}}{\pi_{Bt}} b_{t-1}^{*r} \\ = \left(1 - x_t^{*r}\right) w_t^* n_t^{*r} + b_t^{*r} \tag{86}$$

$$t_{Bt}^r = m_{Bt}(w_t^* n_t^{*r} - \overline{t_B}) \tag{87}$$

$$x_t^{*r} = \frac{t_{Bt}^r}{w_t^* n_t^{*r}} \tag{88}$$

$$E_t \left[\frac{R_{Bt}}{\pi_{Bt+1}} b_t^{*r} \right] \leq E_t \left[k_{Bt} q_{t+1}^* h_t^{*r} \right] \tag{89}$$

$$c_t^{*r} = \frac{P_{At}}{P_{Bt}} c_{At}^{*r} + c_{Bt}^{*r} \tag{90}$$

$$\lambda_t^{*r} = \frac{1 - \omega}{c_{Bt}^{*r}} \tag{91}$$

$$\frac{c_{At}^{*r}}{c_{Bt}^{*r}} = \frac{\omega P_{Bt}}{(1 - \omega) P_{At}} \tag{92}$$

$$\frac{\gamma_h^*}{h_t^{*r}} = q_t^* \lambda_t^{*r} - E_t \left[\beta^{*r} q_{t+1}^* \lambda_{t+1}^{*r} + \xi_t^* k_{Bt} q_{t+1}^* \pi_{Bt+1} \right] \tag{93}$$

$$w_t^* = \frac{c_{Bt}^{*r}}{(1 - \omega)} \left(n_t^{*r}\right)^{\eta^*} \tag{94}$$

$$\lambda_t^{*r} = \beta^{*r} E_t \left[\lambda_{t+1}^{*r} \frac{R_{Bt}}{\pi_{Bt+1}} \right] + \xi_t^* R_{Bt} \tag{95}$$

The National Firms

$$w_t = mc_t \frac{y_t}{n_t} \tag{96}$$

$$\left(\frac{1-\theta\left(\frac{\pi_{At}}{\pi_{At-1}^{\zeta}}\right)^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\frac{V_t}{F_t} \tag{97}$$

$$V_t = \lambda_t^l mc_t y_t + E_t(\beta^l\theta)\left(\frac{\pi_{At}^{\zeta}}{\pi_{At+1}}\right)^{-\varepsilon} V_{t+1} \tag{98}$$

$$F_t = \lambda_t^l y_t + E_t(\beta^l\theta)\left(\frac{\pi_{At}^{\zeta}}{\pi_{At+1}}\right)^{1-\varepsilon} F_{t+1} \tag{99}$$

The Foreign Firms

$$w_t^* = mc_t^*\frac{y_t^*}{n_t^*} \tag{100}$$

$$\left(\frac{1-\theta\left(\frac{\pi_{Bt}}{\pi_{Bt-1}^{\zeta}}\right)^{\varepsilon^*-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon^*}} = \frac{\varepsilon^*}{\varepsilon^*-1}\frac{V_t^*}{F_t^*} \tag{101}$$

$$V_t^* = \lambda_t^{*l} mc_t^* y_t^* + E_t(\beta^{*l}\theta^*)\left(\frac{\pi_{Bt}^{\zeta}}{\pi_{Bt+1}}\right)^{-\varepsilon} V_{t+1}^* \tag{102}$$

$$F_t^* = \lambda_t^{*l} y_t^* + E_t(\beta^{*l}\theta^*)\left(\frac{\pi_{Bt}^{\zeta}}{\pi_{Bt+1}}\right)^{1-\varepsilon} F_{t+1}^* \tag{103}$$

Aggregation

$$b_t^r = \frac{(1-\tau)}{\tau}b_{At}^l + \frac{(1-\omega)}{\omega}\frac{(1-\tau^*)}{\tau}b_{At}^{*l}, \tag{104}$$

$$b_t^{*r} = \frac{\omega}{(1-\omega)}\frac{(1-\tau)}{\tau^*}b_{Bt}^l + \frac{(1-\tau^*)}{\tau^*}b_{Bt}^{*l}, \tag{105}$$

$$b_t^g = (1-\tau)\,b_{At}^g + (1-\tau^*)\,\frac{(1-\omega)}{\omega}b_{At}^{*g}. \tag{106}$$

$$b_t^{*g} = (1-\tau)\,\frac{\omega}{(1-\omega)}b_{Bt}^g + (1-\tau^*)\,b_{Bt}^{*g}, \tag{107}$$

$$b_t = b_t^g + \tau b_t^r \tag{108}$$

$$b_t^* = b_t^{*g} + \tau^* b_t^{*r} \tag{109}$$

$$\tau h_t^r + (1-\tau)\, h_t^l = h \tag{110}$$

$$\tau^* h_t^{*r} + (1-\tau^*)\, h_t^{*l} = h^* \tag{111}$$

$$c_{At} = \tau c_{At}^r + (1-\tau)\, c_{At}^l \tag{112}$$

$$c_{Bt}^* = \tau^* c_{Bt}^{*r} + (1-\tau^*)\, c_{Bt}^{*l} \tag{113}$$

$$c_{Bt} = \tau c_{Bt}^r + (1-\tau)\, c_{Bt}^l \tag{114}$$

$$c_{At}^* = \tau^* c_{At}^{*r} + (1-\tau^*)\, c_{At}^{*l} \tag{115}$$

$$c_t = \tau c_t^r + (1-\tau)\, c_t^l = c_{At} + \frac{P_{Bt}}{P_{At}} c_{Bt} \tag{116}$$

$$c_t^* = \tau c_t^{*r} + (1-\tau)\, c_t^{*l} = c_{Bt}^* + \frac{P_{At}}{P_{Bt}} c_{At}^* \tag{117}$$

$$n_t = \tau n_t^r + (1-\tau)\, n_t^l \tag{118}$$

$$n_t^* = \tau^* n_t^{*r} + (1-\tau^*)\, n_t^{*l} \tag{119}$$

$$t_t = (1-\tau) t_{At}^l + \tau t_{At}^r \tag{120}$$

$$t_t^* = (1-\tau^*) t_{Bt}^l + \tau^* t_{Bt}^r \tag{121}$$

$$y_t = z_t n_t \tag{122}$$

$$y_t^* = z_t^* n_t^* \tag{123}$$

Fiscal, monetary and macroprudential policies

$$b_t^g = \frac{R_{At-1}^g}{\pi_{At}} b_{t-1}^g + (g_t - t_t) \tag{124}$$

$$b_t^{*g} = \frac{R_{t-1}}{\pi_{Bt}} b_{t-1}^{*g} + (g_t^* - t_t^*) \tag{125}$$

$$R_t = R_{t-1}^{\rho} \left[\left(\pi_{At}^{\omega} \pi_{Bt}^{1-\omega} \right)^{\Phi} \overline{R} \right]^{1-\rho} \tag{126}$$

$$m_{At} = m_{At-1} + \psi f_t \left(\frac{b_t^g}{y_t} - \overline{\left(\frac{b^g}{y} \right)} \right) \tag{127}$$

$$m_{Bt} = m_{Bt-1} + \psi^* f_t^* \left(\frac{b_t^{*g}}{y_t^*} - \overline{\left(\frac{b^{*g}}{y^*} \right)} \right) \tag{128}$$

$$k_{At} = k_{SSA} \left(\frac{q_t}{\overline{q}} \right)^{-\phi_{Aq}^k} \tag{129}$$

$$k_{Bt} = k_{SSB} \left(\frac{q_t^*}{\overline{q}^*} \right)^{-\phi_{Bq}^k} \tag{130}$$

GDP and balance of payments restriction

$$\begin{gathered}
(1-\tau) \frac{P_{Bt}}{P_{At}} \left(b_{Bt}^l + b_{Bt}^g \right) + (1-\tau) b_{At}^g - (1-\tau^*) \frac{(1-\omega)}{\omega} b_{At}^{*l} = \\
\left(\frac{(1-\omega)}{\omega} c_{At}^* - \frac{P_{Bt}}{P_{At}} c_{Bt} \right) + (g_t - t_t) + \\
+ (1-\tau) \frac{P_{Bt}}{P_{At}} \left(\frac{R_{Bt-1}}{\pi_{Bt}} b_{Bt-1}^l + \frac{R_{t-1}}{\pi_{Bt}} b_{Bt-1}^g \right) + (1-\tau) \frac{R_{At-1}^g}{\pi_{At}} b_{At-1}^g \\
- (1-\tau^*) \frac{(1-\omega)}{\omega} \frac{R_{At-1}}{\pi_{At}} b_{At-1}^{*l}.
\end{gathered} \tag{131}$$

$$y_t = c_{At} + \frac{(1-\omega)}{\omega} c_{At}^* + g_t \tag{132}$$

$$(1-\tau)\, d_t = \left(\frac{1}{mc_t} - 1 \right) w_t n_t \tag{133}$$

$$y_t^* = c_{Bt}^* + \frac{\omega}{(1-\omega)} c_{Bt} + g_t^* \tag{134}$$

$$(1-\tau^*)\, d_t^* = \left(\frac{1}{mc_t^*} - 1 \right) w_t^* n_t^* \tag{135}$$