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Labor Market Dynamics and Growth*

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Abstract

We embody a technological diffusion process into the canonical search and matching model of the labor market. New matches imitate the production process of incumbents. The resulting model retains the features of a labor search model whilst also generating endogenous growth through creative destruction. The model is calibrated to standard moments from the US labor market and generates, consistent with data, an order of magnitude more amplification in unemployment than a similarly calibrated model without endogenous growth. The model provides a natural framework to decompose the sources of growth based on labor market flows. Using cross-country data for 32 countries across a broad range of development, we find that growth via creative destruction is quantitatively more important for developing countries.

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1 Introduction

Over the last thirty years, the Diamond-Mortensen-Pissarides (DMP) (Diamond (1982), Mortensen (1982), and Pissarides (1985)) model has become the workhorse model of the labor market in macroeconomics. We make a small perturbation to this framework by allowing new jobs to imitate the technology of incumbent matches. The resulting model generates endogenous growth via creative destruction and allows us to explore the interaction between growth and labor market dynamics. The model retains the tractability of the DMP model, with closed form solutions for the equilibrium growth rate and labor market tightness. The model provides a new resolution to the Shimer (2005) puzzle — consistent with data, the strong cyclicality of unemployment can be reconciled with small movements in average labor productivity. Finally, the simplicity of the framework lends itself to a novel cross-country growth decomposition exercise.

Despite a seemingly innocuous modification to the DMP model, the economics of the two models are quite different. When differences in productivity between matches is sufficiently small, there is no endogenous growth, and the baseline model is isomorphic to the standard DMP model. Free entry implies that the value of a vacancy must be zero. Without endogenous growth, the value of labor market tightness is equilibrated through the free entry condition. Thus the *average* match productivity is consequential for equilibrium tightness. Conversely, when the dispersion in match productivity is sufficiently large, labor market tightness equilibrates an exit condition, such that the wage of the least productive match is equal to the productivity and the profits are zero. Here, rather than the *average* match being important, it is instead the *marginal* match productivity that dictates the level of vacancies in the economy. Where the *marginal* match is defined as the least productive match in the economy. This equilibrium features the exit of unproductive matches who are replaced by, on average, more productive entrants. It is through this channel, of creative destruction, that growth is generated in equilibrium. In the presence of endogenous growth, the free entry condition is instead equilibrated via the rate of creative destruction.

The differences in equilibration across the two models provide interesting distinctions in our economic understanding of the labor market. First, under endogenous growth, labor market tightness varies much more with aggregate productivity, since the marginal match profitability varies much more than the average match profitability. The model thereby provides a novel reconciliation of the Shimer (2005) puzzle with large vacancy costs. Second, comparative statics with respect to model parameters are qualitatively different. For example, in the standard DMP model, changes in the exogenous separation rate and the discount rate both lower the profitability of the average match and therefore lower labor market tightness. Under endogenous growth by contrast, labor market tightness equilibrates the wage of the least productive match and is therefore independent of the exogenous separation or discount rates. Instead, the exogenous separation rate and the discount rate both lower the endogenous growth rate.

To perform a quantitative evaluation, we perform a calibration based on standard labor market moments used in the literature. In line with the data, the model with endogenous growth does indeed generate more volatility in unemployment than a standard DMP model. The additional amplification can be attributed to three channels. First, labor market tightness depends on the marginal rather than average match quality. Second, in a model with growth, measured average productivity is an endogenous object that depends both on the current aggregate productivity as well as past accumulated endogenous growth. In the calibration, creative destruction and growth are larger in recessions resulting in a moderated response of measured labor productivity. Larger aggregate productivity shocks are therefore required in order to match the same movement in measured average labor productivity. Third, the endogenous growth model generates countercyclical separations, which further increases fluctuations in the unemployment rate.

The model suggests a tight link between the degree of labor market mobility and growth via imitation in an economy. We use this feature to derive a cross-country growth decomposition. The analytical tractability of the model enables for a simple expression linking growth to the log of the separation rate and changes in the level of the job finding rate. Exploiting harmonized labor force data provided by Donovan et al. (2022), we decompose growth into growth via imitation, and an exogenous component. We find that growth via imitation is far more important for countries in a lesser state of economic development. For more developed nations, the exogenous component accounts for a larger share of total growth, reflecting features likely omitted from the model, such as research and development, for example.

Present in the model is a trade-off, in order to generate growth, one requires a high rate of job turnover. Increasing job turnover, through an elevated separation rate, a labor search model predicts that this will in turn increase unemployment. Aghion and Howitt (1994) explore this mechanism by introducing exogenous growth into an otherwise standard search model of the labor market. However, without explicitly modeling how new technologies are adopted, it is difficult to understand how growth and employment interact. Mortensen and Pissarides (1998) show that whether growth and unemployment are positively or negatively related depends crucially on whether all, or just new jobs, are subject to technological advancement. In the subsequent years the literature has progressed, with the incorporation of capital into the model (Pissarides and Vallanti, 2007), and on-the-job search (Michau, 2013). Yet until recently, explicit modeling of what fundamentally drives growth in a search model has been conspicuously absent. Although quite distinct from the mechanism in this model, Martellini and Menzio (2020) provide a potential rationale. In a DMP world with growing matching efficiency, Martellini and Menzio (2020) show there exists a balance growth path equilibrium. In their environment, although workers meet firms at a greater frequency, growth and unemployment are constant as the increase in meetings is perfectly offset by the decline in the proportion of job offers that are accepted.[1]

Although the underlying economics is quite different, Bilal et al. (2021) also model technological imitation in a labor search environment.[2] In their framework, new firms imitate the technology of existing firms in a frictional labor market. Rather than thinking of a technology embodied within a firm, we instead make the interpretation that the technology is unique to a particular job. With that distinction in mind, we model technological diffusion in the economy in the spirit of Luttmer

[1]See also, Menzio (2021) and Martellini and Menzio (2021) which further study balanced growth path with declining search frictions for the product market and the specialization of workers, respectively.

[2]In our model, labor market tightness equilibrates the wage in the marginal match via the outside option of workers. In turn, wages rise which causes the endogenous exit of less productive matches. For this reason, in the special case of our model with no worker bargaining power, as in Bilal et al. (2021), there is no endogenous growth (Proposition 6). By contrast, in their model, the outside option of workers is independent of labor market tightness and the endogenous exit of unproductive firms is instead generated by a fixed operating cost.

(2007) and Lucas (2009). Innovators, in our context are the newly posted jobs, imitate existing jobs' technology in a random fashion. Given an unbounded, and sufficiently dispersed distribution of productivity, growth is generated through selection. In a similar vein to Lucas and Moll (2014) and Perla and Tonetti (2014), low productivity agents exit the market and are replaced, with random imitation, by the average technology in the economy. Consequently, the distribution of productivity evolves over time, shifting more weight to higher productivity jobs, and thus generating aggregate growth in the economy.

In relation to the literature discussed, the novel theoretic contributions of this paper are: a new model merging technological diffusion and growth into a search model of the labor market; the incorporation of aggregate shocks; the identification of a new mechanism for the equilibration of labor market tightness and its role in explaining unemployment fluctuations; and the development of a growth decomposition based on labor market flows.

The rest of the paper is structured as follows. To provide intuition, we first consider a model absent any aggregate shocks. Section 2 presents the general environment and studies the model along a balanced growth path. In section 3 aggregate shocks are incorporated, and the model is calibrated to the U.S. labor market. Here, we evaluate quantitatively the business cycle properties of the model. Section 4 performs our growth decomposition exercise, and section 5 concludes. The proofs are collected in the appendix.

2 Stationary Model

To make the underlying economic intuition as transparent as possible we begin by presenting a model absent of any aggregate shocks. After formulating and solving the model, we provide comparative statics for changes in the aggregate level of productivity of the balanced growth path equilibrium. In order to make a quantitative evaluation, we extend the model to incorporate aggregate shocks in section 3.

2.1 Environment

Time is continuous and the economy is populated by infinitely lived, risk neutral workers and firms who discount the future at a constant rate $r > 0$. The labor market is characterized by search frictions whereby unemployed workers meet firms with open vacancies according to a constant returns to scale Cobb-Douglas matching function. While in unemployment, a worker receives a flow payoff denoted by $B(t)$. A firm posts a vacancy subject to a flow vacancy cost $K(t)$. After a worker and firm meet they draw their technology from a distribution with associated cumulative distribution function $\Phi(Z,t)$ and produce an amount pZ, where p is the total factor productivity at time t.[3] While matched, the firm pays the worker a wage that is continually renegotiated via Nash bargaining, during which the worker obtains a share $\beta \in (0,1)$ of the total surplus. The productivity of the match Z grows deterministically at a constant rate μ and at a Poisson rate $\delta > 0$ becomes permanently unprofitable, to ensure positive effective discounting we assume that $r + \delta > \mu$.

Key to the mechanism of this paper is the sampling distribution of technology $\Phi(Z,t)$. It is assumed that new matches draw their productivity from the productivity distribution of incumbent jobs. Hence technology diffuses from one generation of matches to the next via imitation. It is through this channel that the model generates endogenous growth. The flow income of the unemployed $B(t)$ and the flow cost of posting a vacancy $K(t)$ are assumed proportional to the average technology in the economy at large $\overline{Z}(t)$, such that

$$\overline{Z}(t) = \int_{M(t)}^{\infty} Z d\Phi(Z,t), \quad B(t) = b_0\overline{Z}(t) \quad , \text{and} \quad K(t) = k_0\overline{Z}(t).$$

The minimum productivity $M(t)$ is an equilibrium object and b_0 and k_0 are primitive parameters that represent the relative benefit and cost of unemployment and vacancy posting compared to the mean technology in the economy. The specification for the flow incomes above imply that home production grows proportionally with production in the economy at large. Incorporated within the

[3] The parameter p will be the source of propagation when aggregate shocks are introduced. In the stationary environment it will prove useful to have it as a reference point. That said, all derivations go through assuming p equal to unity.

cost of creating a job $K(t)$, is a cost of imitating an existing technology. As technology improves, the cost of learning and imitating it also increases. Finally, the distribution of technology at time t will evolve with entry and exit. At time zero, we assume that the $\Phi(Z,0)$ has a Pareto distribution with shape parameter α.

Assumption 1 *The distribution of technology Z is initialized with a Pareto distribution with an arbitrary lower bound $M(0)$ and shape parameter $\alpha > 1$, such that the cdf is given by*

$$\Phi(Z,0) = 1 - \left(\frac{M(0)}{Z}\right)^{\alpha} \quad \text{for, } Z \geq M(0).$$

Given that home production is proportional to mean labor productivity a further assumption is required. Assumption 2 assumes that the value of home production is not too large. The purpose of Assumption 2 is that this will guarantee that the output produced by the least productive match will exceed the output produced in home production.

Assumption 2 *The value of the parameters b_0, α and p satisfy*

$$p > \frac{\alpha}{\alpha - 1} b_0.$$

2.2 Value Functions

Workers can be in one of two states, either unemployed or employed, which we denote by $s \in \{u, e\}$. Similarly a firm can have its vacancy either filled or unfilled. The present value for a worker in state s at time t is given below.

$$rV_e(Z,t) = W(Z,t) + \delta(V_u(t) - V_e(Z,t)) + \mu Z \frac{\partial V_e(Z,t)}{\partial Z} + \frac{\partial V_e(Z,t)}{\partial t}$$
$$rV_u(t) = B(t) + \theta(t)q(\theta(t)) \int (V_e(Z,t) - V_u(t)) d\Phi(Z,t) + \frac{\partial V_u(t)}{\partial t}$$

An employed worker earns an endogenous flow wage $W(Z,t)$, and at an exogenous Poisson rate δ loses their job and becomes unemployed. The final two terms reflect the changing environment within a match. Firstly, the deterministic drift in the productivity level Z, and secondly, the changing value within a given productivity level Z. The unemployed earn flow income $B(t)$. The rate at which the unemployed find work is governed by a constant return to scale matching function. Workers arrive to firms at a rate $m(u,v)/v = m(u/v,1) = q(\theta) = A\theta^{-\eta}$ and firms to workers at

$m(u,v)/u = \theta q(\theta)$, where $\theta := v/u$ defines the tightness of the labor market. The firm value functions are analogous to the worker value functions. The value to a firm of being matched with technology Z at time t is given by $J(Z,t)$. The flow profit is given by output pZ, net of the wage it pays the worker $W(Z,t)$. An unmatched firm pays a flow cost $K(t)$ to keep the vacancy open and meets a worker at rate $q(\theta(t))$.

$$rJ(Z,t) = pZ - W(Z,t) - \delta\left(J(Z,t) - V_v(t)\right) + \mu Z \frac{\partial J(Z,t)}{\partial Z} + \frac{\partial J(Z,t)}{\partial t}$$
$$rV_v(t) = -K(t) + q(\theta(t)) \int (J(Z,t) - V_v(t)) d\Phi(Z,t) + \frac{\partial V_v(t)}{\partial t}$$

There are three equilibrium conditions imposed on these value functions. First, a free entry condition specifying that firms continue to post vacancies until the value of the marginal vacancy is zero, $V_v(t) = 0$, which implies that $\frac{\partial V_v(t)}{\partial t}$ is also zero. The left hand side of the expression is the flow vacancy cost multiplied by the expected duration of the vacancy. The right hand side is the expected value of a filled vacancy. Second, it is assumed the total surplus of a match is divided between a worker and firm via a Nash bargained wage. The worker retains a share β of the surplus, and the firm a share $1 - \beta$.

$$\frac{K(t)}{q(\theta(t))} = \int J(Z,t) d\Phi(Z,t) \qquad \text{where, } \theta(t) \geq 0 \qquad \text{(Free entry)}$$
$$\beta J(Z,t) = (1-\beta)(V_e(Z,t) - V_u(t)) \qquad \text{(Nash bargained wages)}$$
$$0 = (g(t) - \mu) J(M(t),t) \qquad \text{where, } g(t) \geq \mu \text{ and } J(M(t),t) \geq 0 \qquad \text{(Firm exit)}$$

The growth rate of the economy is defined as $g(t) \geq \mu$ and will be endogenized later. The exogenous growth of the economy is given by μ, the deterministic drift of a match's productivity process. Endogenous growth occurs when less productive matches exit. For $g(t) > \mu$ the least productive match must give the firm no value, and since the new entrants have productivity $Z \geq M(t)$, the economy grows. If there are no endogenous separations and $J(M(t),t) \geq 0$, then all growth will be through the exogenous productivity process and the growth rate $g(t)$ will equal μ.

2.3 Kolmogorov Forward Equation

The Kolmogorov forward equation below describes how the cumulative distribution function (cdf) evaluated at technology level Z evolves over time.

$$\frac{\partial \Phi(Z,t)}{\partial t} = \underbrace{\Phi(Z,t)E(t)}_{\text{entrants}} - \underbrace{\Delta(t)}_{\text{endogenous separations}} - \underbrace{\delta\Phi(Z,t)}_{\text{exogenous separations}} - \underbrace{\mu Z \frac{\partial \Phi(Z,t)}{\partial Z}}_{\text{productivity drift}}$$
$$- \underbrace{\Phi(Z,t)\left(E(t) - \Delta(t) - \delta\right)}_{\text{change in employment}}$$

The distribution evolves through new entrants, matches separating, and the technology of existing matches growing with deterministic drift μ. Since the sampling distribution that entrants draw Z from is identical to the distribution of Z across existing matches, the flow in of new matches below Z is equal to the number of new matches $E(t)$ multiplied by the share of those less than Z, $\Phi(Z,t)$. Matches can separate for one of two reasons. The first is those matches that separate endogenously. Once a firm's technology hits the minimum support of the distribution, the mass of these $\Delta(t)$ are subtracted from the cdf for all $Z \geq M(t)$. By contrast exogenous separation occur uniformly across Z at a rate δ. The next term comprises the deterministic drift in the productivity process, and the final component is zero on a balanced growth path, and reflects the rescaling of the distribution, coming from the net movement of workers into employment.

Appendix A.1 derives the distribution of technology at time t. Proposition 1 summarizes a well-known result, the distribution along a balanced growth path remains Pareto with a constant shape parameter.

Proposition 1 *Given a Pareto initialization (Assumption 1), at any time $t \geq 0$ the distribution of technology retains a Pareto distribution with constant shape parameter α.*

$$\Phi(Z,t) := \Psi(z) = 1 - \left(\frac{1}{z}\right)^{\alpha} \quad \textit{where,} \quad z := \frac{Z}{M(t)}$$

The Proposition states that the distribution function remains Pareto with the same shape parameter along the balanced growth path. This is a property that is unique to the Pareto and, in fact, only the Pareto is consistent with balanced growth.[4] Further, the mass of endogenous

[4] The normalized Kolmogorov Forward Equation under balanced growth satisfies $(E-\delta)\Psi(z) - (\mu - g)z\psi(z) = \Delta$. Further, using that $\Delta = E - \delta$ and $\lim_{z\to\infty}\Psi(z) = 0$ gives the unique solution to the differential equation $\Psi(z) = 1 - z^{-\frac{E-\delta}{\mu-g}}$.

separations is also derived in Appendix A.1 and given by expression (1) below. The number of matches hitting the boundary per unit of time is the product of two terms. The first α, is the density of firms at the boundary. The second $(g(t) - \mu)$, is the relative speed at which the distribution moves in comparison with a given match.

$$\Delta(t) = \alpha \left(g(t) - \mu\right) \tag{1}$$

2.4 Normalizing the Model

The model lends itself to a convenient normalization, where all variables are scaled by the productivity of the least productive match. Capital letters denote variables in levels, and lower case denote variables that have been normalized. For illustrative purposes, take the value function of an employed worker in match Z at time t. We define the normalized productivity $z := \frac{Z}{M(t)}$ and similarly for the value function $v_e(z,t)$ and equilibrium wage $w(z,t)$,

$$v_e(Z/M(t), t) := \frac{V_e(Z,t)}{M(t)}, \text{ and } W(Z,t) := w(z,t)M(t).$$

All the constituent parts of the value function in levels $V_e(Z,t)$, can be redefined as follows:

$$\begin{aligned}
\frac{Z}{M(t)} \frac{\partial \left(V_e(Z,t)\right)}{\partial Z} &= z \frac{\partial \left(v_e(z,t)\right)}{\partial z} \\
\frac{1}{M(t)} \frac{\partial \left(V_e(Z,t)\right)}{\partial t} &= \frac{\partial v_e(z,t)}{\partial t} - \frac{\partial v_e(z,t)}{\partial z} z g(t) + g(t) v_e(z,t)
\end{aligned}$$

Substituting these expressions into the value function presented earlier gives the normalized value function for an employed worker in a match of normalized technology z at time t.

$$(r + \delta - g(t))v_e(z,t) = w(z,t) + \delta v_u(t) - \frac{\partial v_e(z,t)}{\partial z} z(g(t) - \mu) + \frac{\partial v_e(z,t)}{\partial t}$$

Appendix A.2 derives the expression for the joint (normalized) surplus below by performing the same exercise for the other three value functions. The parameter $b := B(t)/M(t)$ is the flow income associated with unemployment relative to the minimum productivity level in the economy.

$$(r + \delta - g(t))s(z,t) = pz - b - \beta\theta(t)q(\theta(t)) \int_1^{\infty} s(z,t) d\Psi(z) - \frac{\partial s(z,t)}{\partial z} z(g(t) - \mu) + \frac{\partial s(z,t)}{\partial t}$$

2.5 Balanced Growth Path

Nash bargaining implies that the firm captures a constant share of the surplus, and the value satisfies

$$J(Z,t) = (1-\beta)S(Z,t).$$

Thus the firm entry and exit conditions outlined previously can be re-written in terms of the normalized joint surplus.

$$\frac{k}{q(\theta(t))} = (1-\beta)\int s(z,t)d\Psi(z) \qquad \text{where, } \theta(t) \geq 0 \qquad \text{(Free entry)}$$

$$0 = (g(t)-\mu)s(1,t) \qquad \text{where, } g(t) \geq \mu \text{ and } s(1,t) \geq 0 \qquad \text{(Firm exit)}$$

We begin by looking for an equilibrium along a balanced growth path (BGP). A balanced growth path equilibrium is defined as one in which the growth rate and labor market tightness are constant. In the stationary environment attention is restricted to equilibria along the BGP.

Assumption 3 *A balanced growth path equilibrium is one in which the endogenous variables $\theta(t) := \theta$ and $g(t) := g$ are constant.*

Since under a BGP g and θ are independent of t and hence the normalized surplus can be written as an ordinary differential equation in just z, dropping all time indices.

$$(r+\delta-g)s(z) = pz - b - \beta\theta q(\theta)\int_1^\infty s(z)d\Psi(z) - \frac{\partial s(z)}{\partial z}z(g-\mu) \tag{2}$$

This differential equation can be solved up to an integrating constant. Optimal firm exit implies that matches separate when they generate negative surplus, therefore $s(1) \geq 0$. Optimal exit also requires that there is no value in waiting to exit or exiting early, i.e., the smooth pasting condition $(g-\mu)s'(1) = 0$ has to be satisfied. Intuitively, this implies that the flow profits for the least productive firm, under endogenous growth are zero.

The solution to the ODE given by (2) with associated smooth pasting condition is derived in Appendix A.3. The equation for the surplus and equilibrium conditions are written compactly below.

$$s(z) = \left(\frac{p}{r+\delta-\mu}\right) z - \left(b + \theta \frac{\beta}{1-\beta} k\right) \left(\frac{1}{r+\delta-g}\right) + \frac{p(g-\mu)}{(r+\delta-\mu)(r+\delta-g)} z^{-\frac{1}{g-\mu}(r+\delta-g)}$$

$$\frac{k}{q(\theta)} = (1-\beta) \int s(z) d\Psi(z) = (1-\beta) \frac{p\frac{\alpha}{\alpha-1} - b - \theta \frac{\beta}{1-\beta} k}{r+\delta-\alpha\mu+g(\alpha-1)} \tag{Free entry}$$

$$0 = (g-\mu)s(1) \tag{Firm exit}$$

$$0 = (g-\mu)\frac{\partial s(1)}{\partial z} \tag{Smooth pasting}$$

where, $g \geq \mu$, $\theta \geq 0$ and $s(1) \geq 0$.

2.6 Equilibrium

To recap, an equilibrium is a balanced growth path such that $\theta(t) = \theta$ and $g(t) = g$, assumption 3. At time $t = 0$, the distribution of productivity among worker-firm matches is Pareto with shape parameter α, assumption 1. The endogenous variables θ and g resolve the firm exit and free entry conditions, respectively. The equilibrium is characterized in Proposition 2. The proposition follows directly from substituting the surplus expression derived above into the firm exit and free entry conditions.

Proposition 2 *Given the Pareto initialization outlined in assumption 1, and a BGP equilibrium taking the form described in assumption 3, there exists a unique solution that satisfies the free entry, firm exit and smooth pasting conditions. The equilibrium growth rate g and labor market tightness θ satisfy:*

$$g = \max\left\{-\frac{r+\delta-\alpha\mu}{\alpha-1} + p\frac{q(\overline{\theta})}{k}\frac{1-\beta}{(\alpha-1)^2}, \mu\right\} \tag{P2:g}$$

where $\overline{\theta} := \frac{1-\beta}{\beta}\frac{p-b}{k}$ and θ solves

1. *if $g > \mu$*

$$\theta = \overline{\theta}, \tag{P2:θ i}$$

2. *and instead, if $g = \mu$*

$$k = q(\theta)(1-\beta)\frac{p\frac{\alpha}{\alpha-1} - b - \theta\frac{\beta}{1-\beta}k}{r+\delta-\mu} \tag{P2:θ ii}$$

Proposition 2 defines two classes of equilibria. The equilibrium is always unique, and which case will depend on specific parameter values. Either, there is an equilibrium in which growth is fully exogenous, $g = \mu$ and labor market tightness θ is defined by (P2:θ ii). Alternatively, there is endogenous growth through creative destruction $g > \mu$. In this scenario, the growth rate is defined by the first term in the max operator of equation (P2:g) and θ is equal to $\bar{\theta}$.

Let us first consider an environment in which there is no endogenous growth, $g = \mu$. In this scenario, the solution is identical to that in the canonical DMP model and labor market tightness is equilibrated through the free entry condition (P2:θ ii) where the term $\frac{\alpha}{\alpha-1}$ reflects the average value of z from the offer distribution. This corresponds to the standard job creation condition of the DMP model. There is a unique positive value of labor market tightness that solves the equation. The reason why this might not be an equilibrium for all parameter values is that the labor market tightness that solves (P2:θ ii) could imply that low productive matches make negative profits and want to exit. In particular, the flow income for the firm in the least productive match is proportional to $p - b - \theta\frac{\beta}{1-\beta}k$. Thus, the least productive firm would want to exit if the value of labor market tightness is larger than $\overline{\theta}$. Hence, $\overline{\theta}$ is the largest value that labor market tightness can take. If the solution to (P2:θ ii) implies a larger value of θ than $\overline{\theta}$, the equilibration occurs instead through endogenous growth and separations.

Turning now to an environment in which there is endogenous growth, $g > \mu$. Endogenous growth occurs through a higher entry rate than the exogenous separation rate and the continual exit of low productive matches. Optimal exit implies that the least productive match has zero associated surplus and smooth pasting is satisfied. This implies that labor market tightness is given by $\overline{\theta}$. Intuitively, the flow profits of the least productive match must be zero which gives the condition for θ. In turn, the level of endogenous growth is such that the free entry condition holds, which gives the endogenous growth rate in (P2:g).

We can now ask, under what parameter restrictions does the model feature endogenous growth? To answer this, it is first useful to show that the equilibrium is unique. Observe that θ must be positive (since $p > b$ by Assumption 2) and the free entry condition must therefore hold with equality. In the case in which there is endogenous growth, the least productive match must make zero profits and exit. Thus, θ is equal to $\overline{\theta}$. Note further, this is the maximum value that θ can take as otherwise the least productive firm would make negative profits. Since the Free entry equation is strictly declining in both θ and g (and θ is weakly increasing in g),[5] there can exist at most one solution. In the limit as $\theta \to 0$ without endogenous growth, $g = \mu$, the cost of a hire is smaller than the benefit under the Free entry condition. Similarly, the value is lower than the cost if $\theta = \bar{\theta}$ as $g \to \infty$. Thus, a unique solution exists as the expression for Free entry is continuous in both g and θ. There is a knife edge case when the two terms in (P2:g) are equal which can be used to characterize conditions under which there is positive endogenous growth. Writing the condition in terms of the shape parameter of the Pareto distribution gives

$$\alpha < 1 + \frac{pq(\bar{\theta})(1-\beta)}{k(r+\delta-\mu)}. \tag{3}$$

The equilibrium features no endogenous growth when the differences in productivity between matches are sufficiently small (i.e., for a large enough value of α). Intuitively, as in the DMP model, the model '*wants*' to equilibrate the labor market via tightness. However, when there is a large difference between the average and marginal match, the labor market tightness that would equilibrate the free entry condition is larger than $\bar{\theta}$. This results in negative flow income for low productive matches who therefore prefer to exit, and thus contradicting the assumed no growth equilibrium. In this instance, equilibration occurs instead through creative destruction and endogenous growth. We will see in the next subsection that this change in the equilibration of the model has large implications for the dynamic properties of the model.

2.7 Amplification

In this section we study amplification of aggregate unemployment in the model and follow Shimer (2005) and much of the subsequent literature by investigating how the equilibrium depends on the

[5] Labor market tightness can take different values only if there is no endogenous growth and therefore labor market tightness will be weakly higher with endogenous growth.

level of aggregate productivity p. To provide clear intuition, we will approximate the response in a dynamic model with a permanent change to the aggregate productivity in the stationary environment presented thus far. While approximation of the dynamic DMP model in a stationary environment is known to work well, it is not clear if the same is true in this model, as measured average labor productivity is now an endogenous process. To resolve this, section 3 extends the model to incorporate aggregate productivity shocks and performs a more thorough quantitative evaluation. In the rest of this subsection, we will explore three distinct channels through which the volatility of unemployment will differ in a model with endogenous growth. First, tightness is equilibrated via the wages of the marginal job, rather than the value of the average job as in canonical models. Second, the model generates a separation rate that will vary with changes to the aggregate state. Finally, measured labor productivity is a quasi-endogenous process as it depends on the accumulation of past endogenous growth and, as a result, a more volatile process for aggregate productivity p is required for the model to generate the same movements in measured labor productivity.

Starting with the job finding rate, Proposition 3 presents how the job finding rate varies with aggregate productivity p. We log linearize the balanced growth path equilibrium with respect to the permanent level of aggregate productivity p.

Proposition 3 *The comparative statics of the job finding rate (jfr) with respect to aggregate productivity p satisfies:*
for parameters such that $g > \mu$

$$\frac{\partial \log(jfr)}{\partial \log(p)} = (1-\eta)\frac{p}{p-b}, \tag{P3: i}$$

and parameters such that $g = \mu$ and $\theta < \overline{\theta}$

$$\frac{\partial \log(jfr)}{\partial \log(p)} = (1-\eta)\frac{\beta\theta q(\theta) + r + \delta - \mu}{\beta\theta q(\theta) + \eta(r+\delta-\mu)}\frac{\frac{\alpha}{\alpha-1}p}{\frac{\alpha}{\alpha-1}p - b} \tag{P3: ii}$$

Equation (P3: ii) presents the comparative static of changes in the job finding rate with respect to a change in aggregate productivity p in an environment in which there is no endogenous growth. In this case, we recover the standard comparative statics of the DMP model. Amplification in this case depends crucially on the cyclicality of the *average* benefit from creating a job captured by the *average* match productivity $\frac{\alpha}{\alpha-1}p$ minus the flow income in unemployment b. Using common values for the parameters from the literature implies a small elasticity and therefore little amplification. A notable exception is when the flow income in unemployment b is large compared to the average productivity $\frac{\alpha}{\alpha-1}p$—i.e., the *small surplus* calibration of Hagedorn and Manovskii (2008).

Under parameter values such that there is endogenous growth, we get the comparative statics described in (P3: i). The size of which instead relates to the productivity of the *marginal* rather than *average* match. Where the *marginal* match productivity is defined as the least productive match in the economy. Since the *marginal* match has a level of productivity that is lower than the *average* by a factor $\frac{\alpha}{\alpha-1}$, there is more amplification under endogenous growth, $g > \mu$, unless productivity differences are small—i.e., α is large. Interestingly, this resonates with an earlier literature that followed Shimer (2005). Hagedorn and Manovskii (2008) calibrate the DMP model and found a large value of flow income in unemployment b compared to the average level of productivity and make the point that the marginal workers could have small surplus. However, as Mortensen and Nagypal (2007) point out: "*[w]hile this argument is correct, it is irrelevant in the context of this model, because job creation depends on the average value of z, not its value for the marginal worker*".

The concept of the *marginal* match in our framework is at the extensive margin. Elsby and Michaels (2013) develop a model with decreasing returns to scale. In which, the marginal surplus, at the intensive margin (within a firm), governs desired hiring, and thereby movements in labor market tightness. The marginal surplus can be small compared to the average surplus allowing the model to generate greater amplitude in response to productivity shocks. In our model, a vacancy meets an average match with potentially large surplus but, when there is endogenous growth, labor market tightness equilibrates the market for marginal jobs, as can be seen from the expression for labor market tightness (P2:θ i). The model can thereby generate a great deal of fluctuations in labor market tightness, even when the average surplus, and vacancy costs are large.

Proposition 3 derives the response of the job finding rate to a permanent change in the level of aggregate productivity p. In contrast to a DMP model, the aggregate productivity p differs from the empirical average labor productivity. Measured labor productivity per worker comprises of both the current exogenous level of productivity p as well as the accumulation of past endogenous growth rates. The endogenous growth rate naturally varies with the level of aggregate productivity p, via equation (P2:g). This has two implications. First, the aggregate shocks have to be different in a model with endogenous growth in order to match the same movements in average labor productivity. Second, the separation rate, which is given by $\delta + \alpha(g - \mu)$ will naturally vary over the cycle. Proposition 4 presents the comparative statics for how the growth rate, and therefore also the separation rate, vary with the level of productivity p. If the endogenous growth rate g is pro-cyclical, then the model will generate a permanent scarring effect of a recession. However, the proposition states that it is also possible that the recession is cleansing, and the growth rate is higher when the level of aggregate productivity is low.

Proposition 4 *The comparative statics of the endogenous separation rate* Δ *and growth rate* g*, with respect to aggregate productivity* p *satisfies:*
for parameters such that $g > \mu$

$$\frac{\partial \Delta}{\partial p}, \frac{\partial g}{\partial p} \begin{cases} > 0 & \text{if } p > b + \eta p \\ < 0 & \text{if } p < b + \eta p \end{cases} \tag{P4: i}$$

and parameters such that $g = \mu$ *and* $\theta < \overline{\theta}$

$$\frac{\partial \Delta}{\partial p} = \frac{dg}{dp} = 0. \tag{P4: ii}$$

Proposition 4 states that provided the flow income in unemployment and the elasticity of the matching function are sufficiently high, the growth rate and separation rate are higher when the level of aggregate productivity is low. Under the normalization $p = 1$, separations and growth are counter-cyclical if $b + \eta < 1$, and pro-cyclical under the reverse. Since the endogenous growth rate varies with aggregate productivity p, via equation (P2:g), measured average labor productivity depends on both the current exogenous shocks to productivity p and the accumulation of past endogenous growth. When endogenous growth is higher in recessions—as will be the case in the calibrated model—the shocks to productivity have to be more persistent to generate the same persistence in average labor productivity and require higher variance to generate the same variability in average labor productivity.

To understand the proposition, note that the growth rate equilibrates the free entry condition. From (Free entry), the growth rate will be high when aggregate productivity is high and when the vacancy filling rate is high. The impact of productivity on the growth rate is therefore ambiguous as there are two opposing effects. When productivity is high, this has a direct positive impact on surplus which, all else equal, would require more endogenous growth for the free entry condition to hold. However, since labor market tightness given by $\bar{\theta}$ is increasing in the level of aggregate productivity p, this results in a lower vacancy filling rate which acts to lower the profitability of a vacancy and therefore the endogenous growth rate in (P2:g). The direct impact of aggregate productivity on growth is therefore counteracted by movements in the vacancy filling rate. Proposition 4 gives the condition under which the direct impact of aggregate productivity p is fully offset by the indirect impact associated with the movement in the vacancy filling rate.

2.8 Comparative statics

In evaluating the degree of amplification in search models, the bulk of the literature has propagated models through shocks to aggregate productivity. However, alternative shocks to the separation rate (see for example, Lilien (1982)), and stochastic discount rate (see for example, Kehoe et al. (2019)) have also been studied. In the quantitative model to follow, we model the aggregate state as changes in total factor productivity and the degree of amplification will relate to the mechanisms presented in the previous section. Here we outline how a modest modification to the canonical labor search model, that new jobs draw their productivity from the distribution of incumbents, leads to qualitatively very different economic outcomes.

Proposition 5 presents the results for how the job finding and separation rates vary with the discount and exogenous separation rates. The Proposition follows directly from differentiating the relevant expressions. Interestingly, the signs of the comparative statics change depending on whether there is endogenous growth.

Proposition 5 *The comparative statics of the job finding (jfr) and separation rate (sep) with respect to the exogenous separation (δ) and discount rate (r) are given by the expressions* (P5: (i)) *and* (P5: (ii)).

for parameters such that $g > \mu$

$$\frac{\partial jfr}{\partial \delta} = \frac{\partial jfr}{\partial r} = 0 \quad \textit{and} \quad \frac{\partial sep}{\partial r} = -\frac{\alpha}{\alpha - 1}, \; \frac{\partial sep}{\partial \delta} = -\frac{1}{\alpha - 1} \tag{P5: (i)}$$

and parameters such that $g = \mu$ and $\theta < \overline{\theta}$

$$\frac{\partial jfr}{\partial \delta} < 0, \; \frac{\partial jfr}{\partial r} < 0 \quad \textit{and} \quad \frac{\partial sep}{\partial r} = 0, \; \frac{\partial sep}{\partial \delta} = 1 \tag{P5: (ii)}$$

Absent endogenous growth, when $g = \mu$, the model is equivalent to the DMP model. In this scenario, increases in the discount rate r or separation rate δ will both decrease the profitability of the average match. The lower profitability is resolved through a fall in labor market tightness and thereby a fall in the effective hiring cost and wages. Separations mechanically increase one-to-one with an increase in the exogenous separation rate and are unchanged by changes in the discount rate.

The picture is quite different when endogenous growth is present, $g > \mu$. An increase in the exogenous separation rate or the discount rate has no impact on labor market tightness. This is because tightness is equilibrated through the wage for the marginal match, instead of the profitability of the average match. Put differently, $\overline{\theta}$ outlined in Proposition 2, is independent of both the discount and separation rate. Additionally, the increase in the exogenous separation rate is offset by a larger fall in the endogenous component of separations such that total separations fall. The increased exogenous separation rate lowers the profitability of a match. Since labor market tightness does not respond, the adjustment occurs fully from movements in the endogenous growth rate (and thereby separations). Since matches that endogenously separate are worse than the average match, a larger fall in endogenous separations is required to offset the increase in exogenous separations. The increase in the discount rate lowers the profitability of a formed match and equilibration therefore occurs through a fall in the endogenous growth and separation rate.

Proposition 6 presents a final comparison to draw when endogenous growth is present in the economy. In the DMP model, for $g = \mu$, vacancy posting is constrained efficient when the share of the surplus attributed to the firm is equal to the elasticity of the matching function with respect to vacancies, $\beta = \eta$—the Hosios condition (Hosios, 1990). Once, one introduces endogenous growth this condition no longer guarantees that the equilibrium is constrained efficient. However, rather than maximizing the joint welfare, it turns out that the same condition maximizes the growth rate of the economy.

Proposition 6 *The bargaining power that maximizes the growth rate corresponds to the Hosios condition, with $\beta = \eta$, and for a sufficiently low or high bargaining power there is never endogenous growth.*

To understand the result intuitively, note that the growth rate is maximized when the effective hiring cost per flow income to the firm is minimized. The effective hiring cost is given by $\frac{k}{q(\theta)(1-\beta)}$, the inverse of which enters additively in the equation for the growth rate, (P2:g). To maximize growth, the trade-off is thus the same as maximizing welfare in the canonical DMP model. If one increases the share of the surplus given to firms $(1-\beta)$, that encourages vacancy posting and the firm filling rate $q(\theta)$ declines. The product of the two is at a maximum, as in the Hosios condition, when $\beta = \eta$. When the bargaining power is sufficiently low or high, there is never endogenous growth. Intuitively, as the bargaining power approaches zero, wages become independent of labor market tightness and $\bar{\theta} \to \infty$. Hence, this implies that the labor market is in the exogenous growth case and $g = \mu$. This result nicely illustrates the differences in our framework compared to that of Bilal et al. (2021). The authors develop a rich model of firm dynamics in which unemployed workers have no bargaining power, $\beta = 0$. Given Proposition 6, their model therefore does not have the property that labor market tightness equilibrates the marginal match, and instead the endogenous exit of unproductive firms results from a fixed operating cost.

3 Model with Aggregate Shocks

This section extends the model presented to incorporate aggregate shocks. The environment is identical to that presented in section 2, with the addition that aggregate productivity p follows a Markov process. After presenting and solving the modified model it is then calibrated and is evaluated quantitatively.

3.1 Environment

In this section, the model is extended to incorporate aggregate shocks. The aggregate state is normalized to be on the unit interval, $a \in [0,1]$ and productivity within the state is a function $p(a)$, with $p'(a) > 0$. At a Poisson rate χ the aggregate state changes. Given the initial state a, the state a' is drawn from the density function $\gamma(a,a')$, where $\gamma : [0,1]^2 \to \mathbb{R}^+$ and $\int_0^1 \gamma(a,a')da' = 1$ for all $a \in [0,1]$. Assumption 1 follows directly in this environment. However, assumptions 2 and 3 have to be adapted to the stochastic setting. To ensure labor production always exceeds home production, we now require assumption 4.

Assumption 4 *The value of the parameters b_0, α and $p(0)$, where $p(0)$ is the minimum possible realization of $p(a)$, satisfy*

$$p(0) > \frac{\alpha}{\alpha - 1} b_0.$$

As in the stationary environment, the endogenous variables of the model are the growth rate $g(a,t)$ and market tightness $\theta(a,t)$. We restrict equilibria according to assumption 5. Following changes to the aggregate state a the equilibrium objects g and θ are assumed to be jump variables. The assumption is verified when solving the model.

Assumption 5 *The equilibrium growth rate and market tightness are jump variables and only depend on time t through the aggregate state variable a. Thus, the two can be defined as*

$$g(a,t) := g(a) \quad \text{and} \quad \theta(a,t) := \theta(a).$$

3.2 Equilibrium

The normalized surplus is as in equation (2), with the exceptions that the parameter p varies with a, and there is an additional option value associated with changes to the aggregate state. The expression for normalized surplus is given by

$$(r + \delta + \chi - g(a,t))s(a,z,t) = p(a)z - b - \beta\theta(a,t)q(\theta(a,t)) \int_1^\infty s(a,z',t)d\Psi(z')$$
$$+ \chi \int_0^1 s(a',z,t)\gamma(a,a')da' - \frac{\partial s(a,z,t)}{\partial z} z(g(a,t) - \mu) + \frac{\partial s(a,z,t)}{\partial t}.$$

Under the candidate equilibrium described in Assumption 5 the value of the surplus only depends on time through the aggregate state a. Hence the normalized surplus equation can be simplified to (4)

$$(r+\delta+\chi-g(a))s(a,z) = p(a)z - b - \beta\theta(a)q(\theta(a))\int_1^\infty s(a,z')d\Psi(z') \\ +\chi\int_0^1 s(a',z)\gamma(a,a')da' - \frac{\partial s(a,z)}{\partial z}z(g(a)-\mu). \tag{4}$$

The equilibrium conditions are as in the stationary environment and given by

$$\frac{k}{q(a,\theta)} = (1-\beta)\int_1^\infty s(a,z)d\Psi(z), \qquad \text{where, } \theta(a)\geq 0 \qquad \text{(Free entry)}$$

$$0 = (g(a)-\mu)s(a,1), \qquad \text{(Firm exit)}$$

$$0 = (g(a)-\mu)\frac{\partial s(a,1)}{\partial z}, \qquad \text{where, } g(a)\geq\mu \text{ and } s(a,1)\geq 0. \qquad \text{(Smooth pasting)}$$

The equilibrium of the system described above is described below in Proposition 7.

Proposition 7 *Given the Pareto initialization outlined in Assumption 1, an equilibrium taking the form described in Assumption 5, and the HJB equation describing the value of normalized surplus (equation (4)), a solution that satisfies the free entry, firm exit and smooth pasting conditions is a fixed point for $\theta(a)$, $g(a)$ and $s(a,1)$ such that:*

$$\bar{\theta}(a) \equiv \frac{1-\beta}{\beta}\frac{1}{k}\left(p(a)-b+\chi\int_0^1 s(a',1)\gamma(a,a')da'\right)$$

and

$$\theta(a) = \min\left\{\bar{\theta}(a), \left(p(a)\frac{\alpha}{\alpha-1}-b\right)\left(\frac{1-\beta}{\beta k}\right) - \frac{(r+\delta+\chi-\mu)}{q(\theta(a))\beta} + \frac{\chi}{\beta}\int_0^1\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da'\right\}$$

and,

$$g(a) = \max\left\langle -\frac{(r+\delta+\chi-\alpha\mu)}{\alpha-1} + \frac{q(\bar{\theta}(a))(1-\beta)}{k(\alpha-1)}\left(\frac{p(a)}{(\alpha-1)} - \chi\int_0^1 s(a',1)\gamma(a,a')da'\right) \right. \\ \left. +\frac{\chi}{\alpha-1}q(\bar{\theta}(a))\int_0^1\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da' \quad , \quad \mu\right\rangle$$

and

$$(r+\delta+\chi-g(a))s(a,1) = p(a) - b - \theta(a)\frac{\beta}{1-\beta}k + \chi\int_0^1 s(a',1)\gamma(a,a')da'.$$

The complexity of the equilibrium described in Proposition 7 arises due to the class of equilibrium potentially changing with the aggregate state. In other words, one could be in a situation where for some aggregate states all growth is exogenous and other states for which there is endogenous growth. There are two other potential classes of equilibrium, one in which there is no endogenous growth in any aggregate state. In this scenario, $g = \mu$ for all a and $\theta(a)$ is pinned down by the free entry condition. Finally, the parameters could be such that there is endogenous growth in all aggregate states, $g(a) > \mu$ for all a. Under this scenario the equilibrium simplifies considerably and can be fully summarized with a closed form analytic solution. When there is always endogenous growth, the expression for tightness and growth simplifies to

$$\theta(a) = \frac{(1-\beta)\,(p(a)-b)}{\beta k}, \qquad \text{(equilibrium, when } g(a) > \mu \ \forall \ a\text{)}$$
$$g(a) = -\frac{(r+\delta+\chi-\alpha\mu)}{\alpha-1} + \frac{q(\theta(a))(1-\beta)}{k(\alpha-1)}\left(\frac{p(a)}{(\alpha-1)}\right) + \frac{\chi}{\alpha-1}q(\theta(a))\int_0^1 \left(\frac{\gamma(a,a')}{q(\theta(a'))}\right) da'.$$

Given functional form assumptions specified in the next section, the above expressions can be computed directly without having to rely on a fixed point or numerical approximation of any kind. The first two terms in the growth rate are the same as in the stationary model explored in section 2, up to the addition of the constant χ. The final term reflects the changing aggregate state. This term includes a component $\frac{q(\theta(a))}{q(\theta(a'))}$ which relates the relative market tightness in the current state a to the next potential state a'. Given an aggregate productivity process that exhibits mean reversion, when productivity is high, market tightness $\theta(a)$ is high relative to tightness in the new aggregate state $\theta(a')$. Hence, the final term in the expression for $g(a)$ is small. In contrast, when the current productivity is low, market tightness is expected to increase in the next state, resulting in a larger value of $\frac{q(\theta(a))}{q(\theta(a'))}$ and therefore a larger term. Consequently, as a result of mean reversion in the productivity process, the growth rate $g(a)$ will therefore generally decline with productivity under weaker conditions than those outlined along the BGP in Proposition 4.

3.3 Parameterization

There are only two aspects of the model that functional forms have yet to be specified for. They are, the transition probability function $\gamma(a, a')$, and the process by which the aggregate state manifests itself to output $p(a)$. We assume that $p(a)$ is log linear in a, such that

$$p(a) = \exp\left[p_0 + p_1(a - 0.5)\right] \quad \text{where, } a \in [0, 1].$$

Where p_0 and p_1 are parameters to be calibrated. The process by which a changes follows Poisson uncertainty. At Poisson rate χ a new aggregate state a' is drawn from $\gamma(a, a')$. The function $\gamma(a, a')$ is assumed locally uniform and we specify as

$$\gamma(a, a') = \begin{cases} \frac{1}{\epsilon} & \text{if } a' \in [a(1-\epsilon), a(1-\epsilon) + \epsilon] \\ 0 & \text{otherwise.} \end{cases}$$

The productivity process outlined has several advantages. It is bounded and stationary. It exhibits mean reversion. The parameter χ captures the frequency of shocks whereas the parameter ϵ captures the expected size of the shock. Hence the implied level of mean reversion depends on both parameters. The process becomes continuous, and the ergodic distribution is asymptotically log normal as $\epsilon \to 0$ (where p_1 and χ can be scaled to preserve the variance and the persistence). These properties are documented in Proposition 8.

Proposition 8 *As the parameter $\epsilon \to 0$ the distribution of $\log(p)$ becomes asymptotically normal. The ergodic variance and conditional mean for any $p_1 > 0$ and $\epsilon \in [0, 1]$ are given by*

$$Var(\log(p(a_t))) = p_1^2 \frac{\epsilon}{12(2-\epsilon)} \quad ,$$
$$\mathbb{E}(\log(p(a_t))|a_0) = p_0 + p_1 \exp\left[-\chi\epsilon t\right](a_0 - 0.5),$$

where a_0 is the value of the aggregate state a at time zero.

3.4 Calibration

Two different calibrations are performed. First, the baseline model is calibrated. Second, to understand the role of endogenous growth in the baseline model, we calibrate the same model in which growth is entirely exogenous. To guarantee all growth is exogenous, the comparative version of the model is calibrated for $\alpha \to \infty$. In this situation, the match quality distribution is degenerate and the mean and minimum technologies are the same. This rules out endogenous growth by construction and all growth is thus generated exogenously through the deterministic drift μ.

The model is calibrated to the US labor market assuming one unit of time corresponds to a month. It relies on a combination of survey data and moments from the literature. The full series of moments used in the calibration is given in Tables 1 and 2. Data are from 1990-2019, inclusive and all series are seasonally adjusted. They include a monthly series of the unemployment rate, job finding rate of the unemployed and separation rate of the employed taken from the Current Population Survey worker flows data (CPS). The stock of vacancies is taken from the Job Openings and Labor Turnover Study (JOLTS) and are again monthly. Finally, we use the Bureau of Labor Statistics (BLS) quarterly data on output per hour in nonfarm business as our measure for labor productivity.

The tenet of this calibration follows that of Shimer (2005). Parameters of the model are calibrated to match aggregate (mean) moments of the labor market. The parameters of the productivity process target dynamics moments of the labor productivity series in the data. Dynamic moments of the labor market are deliberately omitted from the calibration, and we will compare the two models performance in matching them ex post. We will focus on the baseline model's ability to reconcile the relatively large volatility in unemployment, given much less volatility in the underlying labor productivity series.

Targeted moments, and the fit of the two models are reported in Tables 1 and 2. The moments used in the calibration are a combination of labor market flows, unemployment and vacancy rates and labor productivity. In addition, we also use moments from the literature that speak to specific aspects of the model. The moments reported within Table 1 can be computed with the model along its balanced growth path, described in section 2. Table 2's moments rely on the model with aggregate shocks of section 3.

Starting with the moments presented in Table 1, as has been discussed, inspection of equation

(P2:g) highlights the crucial role of cross-sectional productivity dispersion, governed by the Pareto shape parameter, in determining the growth rate. The BLS labor productivity data implies a monthly growth rate of 0.16%, approximately 2% annually. Under the exogenous growth model, growth can only be generated deterministically through the parameter μ which is set to correspond with the monthly growth rate. The mean labor market tightness, defined as the vacancy rate divided by the unemployment rate and the job finding rate are targeted by the normalized flow cost of vacancies k_0 and the efficiency of the matching function A. Since the elasticity of the matching function is imposed to be the same across model and the two models generate the same level of unemployment and vacancies, the efficiency of the matching function is calibrated identically across specification. The relative value of the flow value of unemployment to the mean level of output b_0 is taken to be 71% from Hall and Milgrom (2008). The bargaining parameter β is targeted to match the pass-through of wages following changes to productivity. A key facet of the model is the share of endogenous separations. In the model, high wage workers are the same as high z workers who only separate because of the exogenous separation rate. We therefore pick the exogenous separation rate to match the rate of separations of high wage workers, as documented by Mueller (2017).

Table 1: Aggregate Moments

Parameters		**Moments**	**Source**	**Value**		
Baseline	Exog. Growth			Data	Baseline	Exog. Growth
	$\alpha \to \infty$	By assumption				
$\alpha = 5.71$	$\mu = 0.0016$	Growth rate	BLS	0.0016	0.0016	0.0016
$A = 0.41$	$A = 0.41$	Job finding rate	CPS	0.253	0.253	0.253
$\mu = 0.0005$	$\delta = 0.014$	Separation rate	CPS	0.014	0.014	0.014
$k_0 = 4.54$	$k_0 = 4.55$	Mean tightness	CPS+JOLTS	0.581	0.581	0.581
$b_0 = 0.71$	$b_0 = 0.71$	Unemployment benefit/mean output	Hall and Milgrom (2008)	0.71	0.71	0.71
$\beta = 0.042$	$\beta = 0.042$	Pass-through $\left(\frac{\partial \log(w)}{\partial \log(z)}\right)$	Card et al. (2018)	0.05	0.05	0.05
$\delta = 0.007$	—	Separation rate (high wage)$^{+}$	Mueller (2017)	0.007	0.007	—

The table presents the parameters of the baseline model, the exogenous growth model with $\alpha \to \infty$, and the moments used in calibration. $^{+}$The separation rate of the high wage is defined as the separation rate of those earning above the median wage.

Parameters associated with the dynamics of the productivity process are given in Table 2. The elasticity of the matching function ensures that the relative volatility of the job finding rate and separation rate is consistent with the data. The exogenous growth model exhibits no volatility in the separation rate so targeting this moment is infeasible. Instead, we fix the elasticity in both models to be the 0.13 calibrated in the baseline. The parameter χ, the arrival rate of productivity shocks governs the persistence of labor productivity. This parameter is used to target the autocorrelation of de-trended labor productivity. The parameters ϵ and p_1 both govern the degree of volatility of the series. Since ϵ bounds how much productivity moves in the short run and p_1 over any time horizon, they can be separately identified using two moments related to productivity dispersion. We use the standard deviation of de-trended labor productivity and the first difference in the log of the raw labor productivity series. Finally, we normalize the mean of labor productivity to unity with the parameter p_0. Both specifications yield values of p_0 close to zero. This is a consequence of the magnitude of the shocks being small.

Table 2: Dynamic Moments

Parameters		**Moments**	**Source**	**Value**		
Baseline	Exog. Growth			Data	Baseline	Exog. Growth
$\epsilon = 0.16$	$\epsilon = 0.13$	st. dev. of labor productivity*	BLS	0.0089	0.0089	0.0089
$\chi = 0.40$	$\chi = 0.63$	autocorrelation of labor productivity*	BLS	0.75	0.71	0.71
$p_0 = -9.8 \times 10^{-5}$	$p_0 = -8.9 \times 10^{-5}$	normalize mean productivity	—	1	1	1
$p_1 = 0.16$	$p_1 = 0.19$	st. dev. of first difference log labor productivity	BLS	0.0067	0.0067	0.0067
$\eta = 0.13$	—	relative st. dev. of finding rate / separation rate*	CPS	1.16	1.16	—

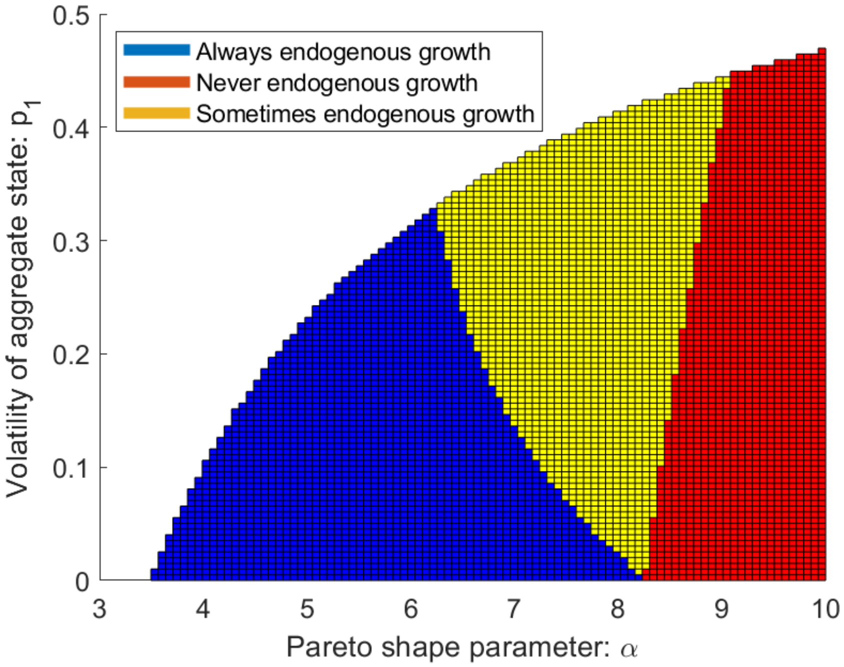
All of the moments in this table are computed as the mean 1000 simulations of the model. For moments indexed by a *, the underlying series has been successively logged and detrended using an HP filter with a smoothing parameter of 1600.

Both models can match every targeted moment perfectly, with the exception of the persistence of labor productivity. The similarity of the parameter estimates across specification is quite striking. Other than a finite α that we impose on the baseline, the only significant differences relate to the values of δ and μ which determines the amount of exogenous separations and growth, respectively. Under the exogenous growth model, growth and separations are all determined exogenously. Unsurprisingly therefore the parameters μ and δ are larger than in the baseline model to reconcile the same growth and separation rates. Despite the similarity of the calibrated parameters, the equilibrium is quite different. As discussed, in the baseline model there are three potential classes of equilibria: (i) one in which there is always endogenous growth $g(a) > \mu$ for all a; (ii) one in which there are states with endogenous growth $g(a) > \mu$ and some without $g(a) = \mu$; (iii) there is never endogenous growth $g(a) = \mu$ for all a. The class of equilibria depends on the degree of dispersion of productivity present in the model, both in the cross-section governed by α, and the time series driven by p_1. Figure 1 plots resimulations of the baseline model, varying the parameters p_1 and α, keeping other parameters fixed to their calibrated values. The one exception is p_0 which also changes to ensure the mean of the series in every simulation is equal to one. The different colors represent the different class of equilibria described, and the absence of color in the north west of the plot are for values in which Assumption 4 is violated.

Our baseline model is calibrated at $(\alpha = 5.71, p_1 = 0.16)$. This point is in the epicenter of the blue region, the region where there is endogenous growth in all aggregate states, and the closed form analytical expressions are available. The variance of productivity in the cross-section and time series are decreasing in the parameter α and increasing in p_1, respectively. Moving in an easterly direction in Figure 1 reduces the cross-sectional dispersion and makes it more and more likely we are in a world without endogenous growth. As one increases p_1, there is more and more variation in the aggregate state. This makes it less likely that there is either only endogenous or only exogenous growth. We saw the extreme case of $\alpha \to \infty$ ensures only exogenous growth. The extreme case of $p_1 = 0$ puts the economy on the balanced growth path explored in section 2. Equation (3) defines the condition, in this situation, for whether or not there is endogenous growth. This corresponds to the point the red and blue areas touch on the origin.

Figure 2 plots the growth of the mean match productivity Z and the separation rate by the aggregate state. In the exogenous growth model, productivity grows at a monthly rate of μ, and the monthly separation rate is δ, irrespective of the aggregate state. The baseline model exhibits counter-cyclical patterns of creative destruction. In low times, a close to zero, the growth in mean Z and the separation rate is highest. Proposition 5 puts forward the condition for which the separation rate and growth rate declines with the aggregate state, which is not satisfied under the baseline calibration. However, as is discussed in section 3.2 a productivity process that exhibits

Figure 1: Class of Equilibrium



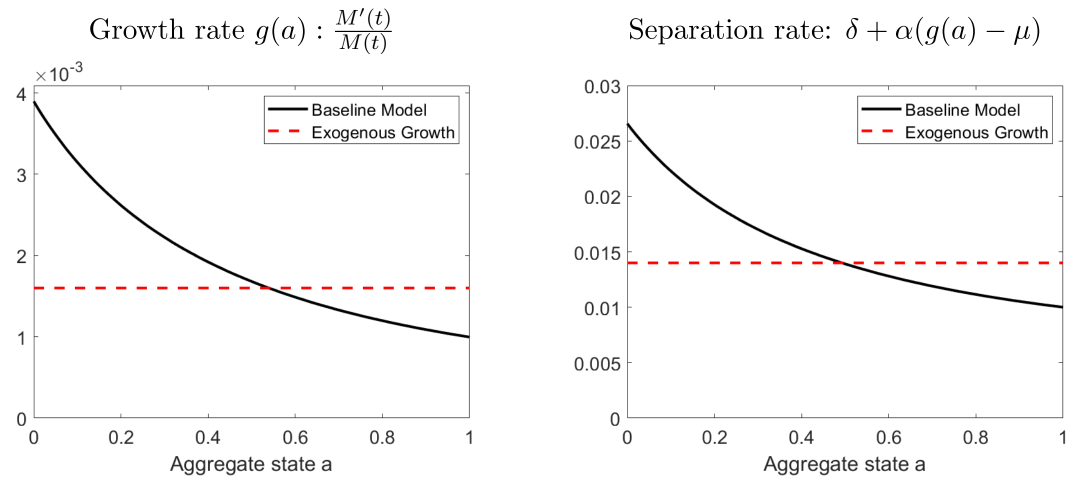
The figure documents the class of equilibria given the variability of match productivity, governed by α, and aggregate productivity dispersion, governed by p_1. The parameter p_0 varies such that the mean aggregate productivity is unity. The remaining parameters are fixed and presented in tables 1 and 2.

mean reversion, as is the case here, requires a less strict condition for this phenomenon.

3.5 Amplification

In addition to the targeted series for labor productivity, Table 3 demonstrates the two models ability in matching dynamic labor market moments. The table shows the standard deviation and autocorrelation of log deviations from an HP trend of unemployment, job finding rate, separation rate and labor productivity. The first rows present the results from the data. This highlights two well-known properties that: (i) unemployment and job finding rates are more variable compared to measured average labor productivity; and (ii) unemployment and the job finding rate are more persistent compared to average labor productivity. The last row reiterates the results from the literature by presenting the results from the exogenous growth model. As emphasized by Shimer (2005), the model fails to generate enough volatility in the unemployment rate or the job finding

Figure 2: The Cyclicality of Creative Destruction



The Figure presents how the growth and separation rates vary with the aggregate state a. The exercise is performed for the baseline and exogenous growth models, the parameter values of which are documented in tables 1 and 2.

rate. The middle rows present the results from the baseline model. The baseline model generates more volatility in unemployment, the job finding rate, and the separation rate. This highlights the results and discussion in section 2.7 that the job finding rate is more variable when there is endogenous growth. The table confirms that this result carries over to a full simulation of the stochastic model. The table further highlights that the model can match well the variability of the separation rate.

There are three mechanisms through which the endogenous growth model generates greater amplification than the exogenous growth model. These are discussed in section 2.7. To summarize, they are: (i) the endogeneity of the labor productivity series; (ii) the job finding rate depends on the marginal, rather than mean productivity, resulting in larger response to changes in aggregate productivity p; and (iii) the endogenous growth model can generate fluctuations in the separation rate. In the exogenous growth model, the series for p and the labor productivity series are one and the same. By comparison, in the model with endogenous growth, the labor productivity series depends both on the exogenous series for p and the accumulation of productivity though endogenous growth. Since endogenous growth is highest when p is lowest, see the first panel of Figure 2. Hence, the endogenous growth model requires a more volatile series for aggregate

Table 3: Unemployment Amplification

	Untargeted			Targeted
	u	f	s	lp
Data				
Standard deviation	0.10	0.074	0.064	0.0089
Quarterly autocorrelation	0.94	0.83	0.75	0.74
Baseline model				
Standard deviation	0.079 (0.011)	0.055 (0.0068)	0.048 (0.0060)	0.0089 (0.0011)
Quarterly autocorrelation	0.86 (0.030)	0.71 (0.059)	0.71 (0.059)	0.71 (0.059)
Exogenous growth (DMP)				
Standard deviation	0.024 (0.0034)	0.031 (0.0036)	0 (−−−)	0.0089 (0.0010)
Quarterly autocorrelation	0.86 (0.029)	0.71 (0.055)	1 (−−−)	0.71 (0.055)

All series are quarterly, seasonally adjusted, and reported as log deviations from an HP trend with smoothing parameter 1600. The standard deviation from 1000 model resimulations are reported in parenthesis.

productivity p to generate the same level of volatility in labor productivity. Substituting the two productivity processes into the ergodic variance expression in Proposition 8 highlights this. The ergodic variance of the p series is approximately 10% higher in the calibrated endogenous growth model, as compared with the exogenous growth model. Introducing movements in the separation rate adds an additional channel to generate movements in unemployment. Since the finding rate is pro-cyclical, θ is increasing in p in Proposition 7, and the separation rate is counter-cyclical, the second panel of Figure 2. Thus, the two exhibit negative covariance and this will thus result in even greater movements in unemployment.

Table 4 attempts to decompose the additional amplification that the endogenous growth model generates, over and above the exogenous growth model. The left hand column presents the cyclicality in the exogenous growth calibration, and the right hand the endogenous growth counterpart. In this example, to be as transparent as possible we neglect from removing any trend and present the simulated data in their raw monthly form. However, the results of the decomposition are quantitatively very similar if we detrend the data using a HP filter. The source of amplification is

Table 4: Decomposition of Unemployment Volatility

	Exogenous Growth		Endogenous Growth	
	Baseline	+ endog. growth prod. param.	- separations	Baseline
std(log(f_t))	0.0463	0.0514	0.0869	0.0869
std(log(s_t))	0	0	0	0.0751
std(log(u_t))	0.0392	0.0440	0.0651	0.1381
% increase in amplification in			log(u_t)	log(f_t)
endogenous productivity			4.8%	12.6%
marginal match			21.3%	87.4%
endogenous separations			73.9%	—

Series are monthly and simulated over a 10,000 year time horizon. The series from left to right are: the calibrated exogenous growth model; the exogenous growth model with the productivity parameters calibrated from the endogenous growth model; the endogenous growth model, assuming a constant separation rate; the calibrated endogenous growth model.

categorized into the three mechanisms discussed. While both the responsiveness of the job finding rate to the marginal match and the quasi-endogeneity of the productivity process will determine the volatility of the separation rate, we treat the three channels distinctly. To understand the amount of amplification from the productivity process, we simulate the exogenous growth model feeding in the productivity process calibrated from the endogenous growth model. To understand the impact of endogenous separations, the endogenous growth model is simulated assuming a constant separation rate. The rest of the amplification can then be attributed to the responsiveness of the job finding rate to the marginal rather than the mean match productivity.

The greater volatility needed in the primitive productivity series explains less than five percent of the volatility in unemployment and one eighth of the volatility in the finding rate. The responsiveness of the job finding rate is quantitatively much more important. Explaining 21.3% of the higher standard deviation of log unemployment and 87.4% of the job finding rate. Finally, the bulk of the unemployment volatility is matched by matching the inflow into unemployment as well as the outflow.

4 Growth Accounting

So far, we have examined in the context of the U.S., the consequences of introducing growth via imitation in amplifying the volatility of employment over the business cycle. A further interesting implication of the model is the cross-country predictions it makes about creative destruction and growth. In this section we develop a sufficient statistic approach that allows us to decompose the share of growth that is generated endogenously by the model and how this share varies with a country's stage of development. We allow all parameters of the model to vary across country except for the Pareto shape parameter α. The approach is two staged where the first step uses the separation rate across country and the second step utilizes growth rates. We find that growth via imitation is far more consequential for countries in earlier stages of development.

To recap, in the model separations can happen for one of two reasons. Either matches exogenously separate at Poisson rate δ. Alternatively, unproductive matches hit the boundary condition and endogenously separate at a rate $\alpha(g - \mu)$. Creative destruction occurs in the model as those endogenously separating are replaced with more productive matches. The first stage of our cross-country approach will look at the rate at which employed workers leave their jobs for unemployment. However, a high separation rate does not necessarily imply a high endogenous growth rate since the high separation rate could be a consequence of a high exogenous separation rate δ. We therefore follow the same identification argument made in section 3.4 and examine how the separation rate varies across the wage distribution. The argument being that the exogenous rate δ is equally likely to hit, regardless of one's position in the wage distribution. Whereas endogenous separations are more likely to impact relatively worse matches, lower in the distribution of wages. Once the separation rate is decomposed into its constituent parts there is one additional hurdle to overcome. Output per worker is a function of the accumulation of past growth $g(t)$ in $\overline{Z}(t)$ as well as the transitory component that depends on the aggregate state $a(t)$. In a second stage we show that the growth rate can be written as a linear function of the separation rate and the change in the log job finding rate.

4.1 Cross-Country Data

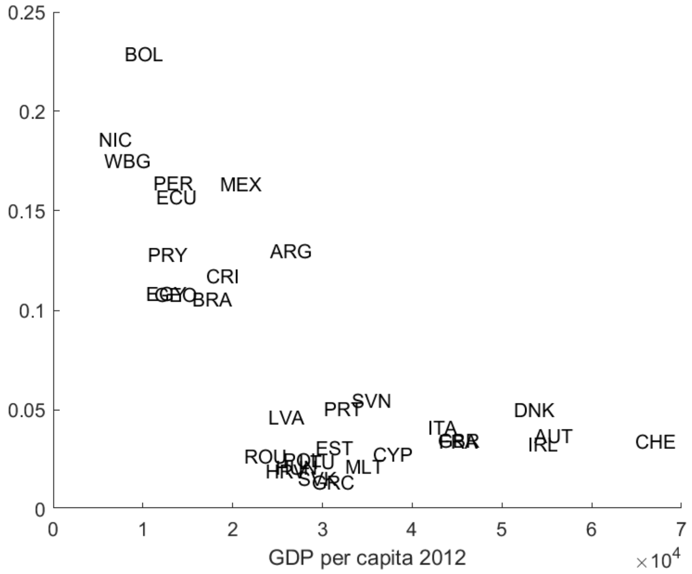
The data used are kindly provided and maintained by Kevin Donovan, Will Lu, and Todd Schoellman and are described in detail in Donovan et al. (2022). The data are produced by harmonizing official government labor market surveys around the world. The raw data covers 75 million observations across 46 countries. For our purposes, we require transitions in and out of employment, and for transitions out of employment by a worker's position in the earnings distribution. Restricting the sample to include these moments leaves us with 32 countries in our analysis.

4.1.1 Separation Rates

The data contains four employment states: out of the labor force; unemployed; paid employment; and self-employment. Donovan et al. (2022) have put a great deal of care to create a consistent definition of employment status across country. Despite this, different states imply different things in different countries. For example, Donovan et al. (2022) document that in less developed countries, self-employment looks like unemployment in the rate at which workers exit to paid employment. To create as consistent a measure of the separation rate, we therefore use the quarterly rate a worker leaves paid employment for any other employment state.

Figure 3 plots the mean quarterly separation rate for each nation in our sample against their gross domestic product per capita in 2012 in U.S. dollars. There is a clear negative correlation between a nation's income and their separation rate. However as was discussed, the model can interpret this finding in one of two ways. Either these differences are driven by exogenous differences in δ and this has no impact on the growth rate of an economy. Alternatively, these differences are driven by endogenous differences in the separation rate, and this would be reflected by higher growth rates for poorer countries. As in the calibration of the structural model, one can tease out the relative size of the two sources of separations by their differential impact over the wage distribution. To examine how the separation rate varies by country at different points in the wage distribution we regress the separation rate in decile d of country i's earnings distribution on the GDP of that country.

Figure 3: Quarterly Separation rate by GDP



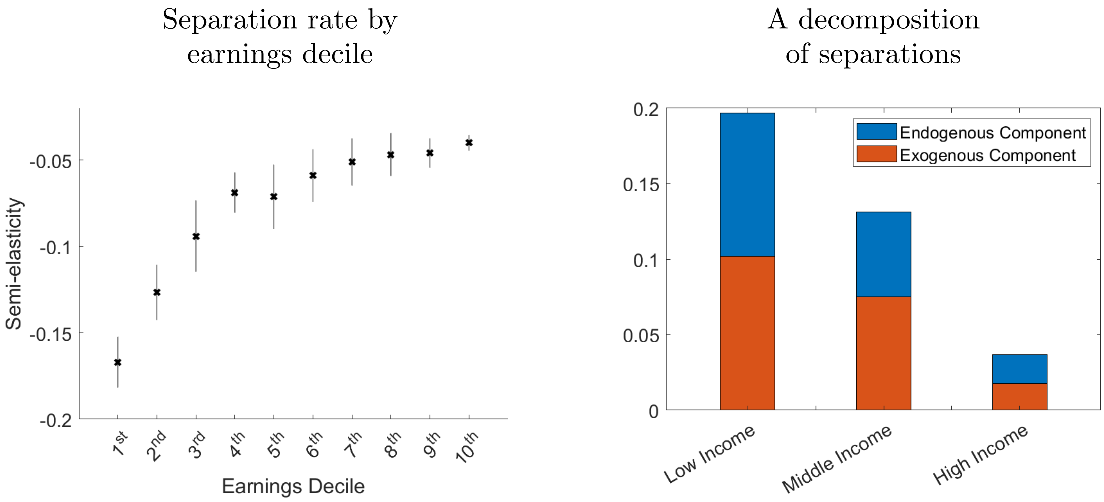
The figure is a scatter plot showing the relationship between a country's separation rate from employment against GDP per capita. The separation rate is defined as the quarterly rate a worker leaves paid employment for any other labor market state. GDP per capita is as of 2012 and measured in US dollars.

$$\text{sep}_{it}^{d} = \alpha^{d} + \beta^{d} \log\left(GDP_{it}\right) + \epsilon_{it}^{d} \tag{5}$$

The regression equation (5) estimates how the separation rate varies by stage of development across a country's income distribution. Inspection of Figure 3 suggests we would expect the estimate of the semi-elasticity β^d to be negative, since richer countries have on average lower separation rates. Results of these elasticities are plotted by decile in the left hand panel of Figure 4. All the estimated βs are negative. Implying that irrespective of your position in your country's earnings distribution, on average, the more developed country will provide more job security. However, the cross-country differences are far greater at the bottom of the earnings distribution in comparison to the top. A ten percentage point increase in a country's GDP reduces the separation rate of the top decile of earners by 4 percentage points. Whereas a worker in the bottom ten percent of the earnings distribution would experience a 17 percentage point fall in the separation rate.

To decompose the relative importance of endogenous and exogenous separations, we follow the

Figure 4: Separation Rates Across Country



The first panel presents the OLS estimates and associated standard errors of β from equation (5). The second panel decomposes the relative importance of exogenous separations governed by the Poisson rate δ with endogenous separations at a Poisson rate $\alpha(g - \mu)$, by income category.

same identification argument as is used in the calibration of the quantitative model. Exogenous separations occur with equal chance, irrespective of the match quality. Hence, the separation rate of the sufficiently well matched will be entirely due to the exogenous rate δ. Defining the sufficiently well matched as those in the top decile of the earnings distribution, allows us to pin down δ directly. Total separations, by three income categories, are presented in the right hand panel of Figure 4. For each category, the separations rates are decomposed into endogenous and exogenous components. Where the separation rate of high earning workers in a country act as a proxy for the rate of exogenous separation.

4.1.2 Growth Rates

Output per employee evolves from period t to period $t + \tau$ according to equation (6). The factor difference is a combination of the cumulative impact of the rate g and transitory movements in aggregate productivity. As one increases the time horizon τ the relative importance of the transitory component diminishes.

$$Y(t+\tau) = Y(t). \underbrace{e^{\int_t^{t+\tau} g(t')dt'}}_{\text{Permanent component}} . \underbrace{\left(\frac{p(t+\tau)}{p(t)}\right)}_{\text{Transitory component}} \tag{6}$$

Indexing a country in our sample by i, equation (6) can be approximated as equation (7). The growth rate is written linearly in the separation rate and change in the log job finding rate. The permanent component can be written in terms of the separation rate because of the tight connection between growth and job destruction in the model. Log-linearizing around the steady-state, one can express transitory changes in the aggregate state p in terms of changes in the log of the job finding rate. Formal derivation of the expression is performed in Appendix A.10.

$$\begin{aligned} &\Delta \log (Y_{it}) \approx \eta_i + \beta \text{sep}_{it} + \gamma_i \Delta \log (\text{jfr}_{it}) \\ &\text{where, } \eta_i = \frac{\alpha\mu_i - \delta_i}{\alpha}\tau, \quad \beta = \frac{\tau}{\alpha} \quad \text{and } \gamma_i = \frac{1-b_i}{1-\eta_i} \end{aligned} \tag{7}$$

The data is annual, and the rates are quarterly. Hence the value of τ is 4. Estimating the linear expression in (7) by ordinary leas square yields

$$\hat{\beta} = \underset{(0.161)}{0.385} \quad \Longrightarrow \quad \hat{\alpha} = \frac{\tau}{\hat{\beta}} = 10.38.$$

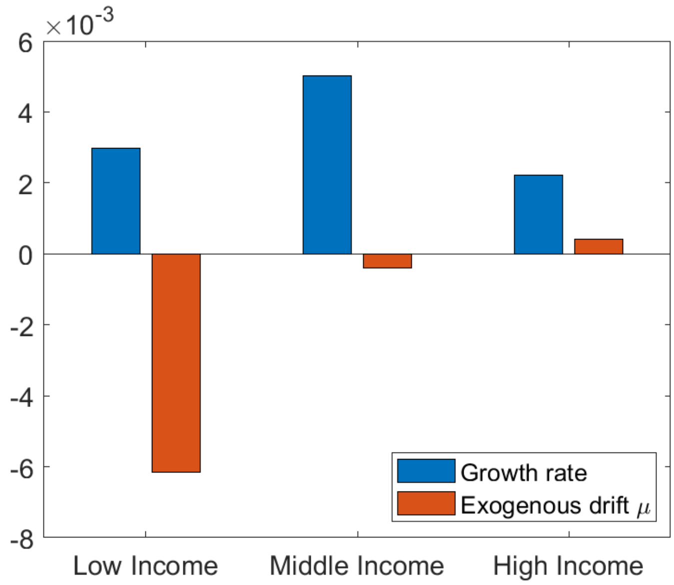
The value of the Pareto parameter α is larger than in our quantitative calibration, implying less cross-sectional dispersion in productivity. For the two estimates to coincide, one would need an estimate of β equal to 0.7, two standard deviations higher than the estimated value.

To decompose growth into the relevant channel, μ_i the exogenous growth rate of country i, is backed out from the average growth rate for country i in our data, $\overline{g}_i$. From the expression for the endogenous separation rate, equation (1), the rate of exogenous growth for country i is computed as

$$\mu_i = \overline{g}_i - \frac{\overline{sep}_i - \delta_i}{\alpha}.$$

Figure 5 plots in blue the mean growth rate of countries in the three categories of development. Over the horizon looked at it, it happens that the middle income countries exhibited the highest growth rate. Next to the aggregate growth rate, in red, the mean exogenous growth rate is plotted.

Figure 5: A decomposition of the quarterly growth rate



The figure decomposes, by a country's income category, the relative importance of exogenous growth, governed by the deterministic drift μ, compared to the total growth rate g.

Note, this can be positive, as in the quantitative section, if matches deterministically grow after formation. Equally however, productivity in a match can fall over time, corresponding to a negative value of μ. For the low income countries, the exogenous growth rate is large and negative. For high income countries, like in the quantitative section, exogenous growth aids total growth and is responsible for approximately one third of the total growth rate.

The findings of this decomposition suggest that the mechanism of growth via imitation is much more important for poorer countries. Although not explicitly modeled in this paper, this is consistent with the mechanism in Acemoglu et al. (2006). A country far from the world technology frontier relies primarily on the adoption of existing technologies, as in the mechanism of this model. As one approaches the frontier, countries rely more on an innovation-based strategy to generate growth. Since research and development is omitted from the model, this is captured in the decomposition through a residual component of the exogenous growth rate μ_i.

5 Conclusion

This paper makes a small modification to the canonical search model of the labor market — new jobs imitate the technology of existing matches. The small change enables the model to generate endogenous growth. Consistent with the data, the model is also able to generate greater volatility in unemployment for a given productivity series. Finally, the model is used to decompose differences in cross-country growth rates. We find that growth via imitation is far more consequential for countries further from the global technological frontier.

A Appendix

A.1 Proof of Proposition 1: Kolmogorov forward equation

The Kolmogorov forward equation satisfies.

$$\frac{\partial \Phi(Z,t)}{\partial t} = \underbrace{\Phi(Z,t)E(t)}_{\text{entrants}} - \underbrace{\Delta(t)}_{\text{endogenous separations}} - \underbrace{\delta\Phi(Z,t)}_{\text{exogenous separations}} - \underbrace{\mu Z\frac{\partial \Phi(Z,t)}{\partial Z}}_{\text{productivity drift}} - \underbrace{\Phi(Z,t)\left(E(t)-\Delta(t)-\delta\right)}_{\text{change in employment}}$$

Look for steady-state distribution, of the form $\Psi(Z/M(t)) = \Phi(Z,t)$, where $M(t)$ is minimum productivity match and $\Psi(1) = 0$ and $\Psi(\infty) = 1$.

$$\begin{aligned}
\frac{\partial \Psi(Z/M(t))}{\partial t} &= -\frac{M'(t)Z}{M(t)^2}\psi(Z/M(t)) \\
&= -g(t)z\psi(z) \\
Z\frac{\partial \Psi(Z/M(t))}{\partial Z} &= \frac{Z}{M(t)}\psi(Z/M(t)) \\
&= z\psi(z)
\end{aligned}$$

Where $z := Z/M(t)$ and $g(t) = M'(t)/M(t)$. Hence KF equation can be expressed as,

$$\Psi(z)E(t) - \Delta(t) - \delta\Psi(z) = (\mu - g(t))\, z\psi(z)$$

Use boundary condition $\Psi(1) = 0$ to pin down measure of endogenous separations.

$$\Delta(t) = (g(t) - \mu)\, \psi(1)$$

Use boundary condition $\Psi(\infty) = 1$ to pin down measure of entrants

$$\begin{aligned}
\Psi(z)E(t) - (g(t)-\mu)\,\psi(1) - \delta\Psi(z) &= -(g(t)-\mu)\, z\psi(z) \\
E(t) &= \delta + (g(t)-\mu)\,\psi(1) \quad \text{assuming, } \lim_{z\to\infty} z\psi(z) = 0
\end{aligned}$$

Substituting back in,

$$
\begin{aligned}
-\left(g(t)-\mu\right) z\psi(z) =& \Psi(z)E(t)-\Delta(t)-\delta\Psi(z) \\
\left(g(t)-\mu\right) z\psi(z) =& \left(g(t)-\mu\right)\psi(1)(1-\Psi(z)) \\
z\psi(z) =& \psi(1)\left(1-\Psi(z)\right) \\
-z\frac{d(1-\Psi(z))}{dz} =& \psi(1)\left(1-\Psi(z)\right) \\
-\int \frac{1}{(1-\Psi(z))} d(1-\Psi(z)) =& \int \frac{\psi(1)}{z} dz \\
-\log(1-\Psi(z)) =& \psi(1)\log(z)+\tilde{C} \\
\frac{1}{1-\Psi(z)} =& cz^{\psi(1)} \quad \text{where, } c=e^{\tilde{C}} \\
\Psi(z) =& 1-cz^{-\psi(1)}
\end{aligned}
$$

To pin down constant c,

$$
\begin{aligned}
\psi(z) =& \psi(1)cz^{-\psi(1)-1} \\
\psi(1) =& \psi(1)c \quad \rightarrow \quad c=1 \\
\Psi(z) =& 1-\left(\frac{1}{z}\right)^{\psi(1)} \\
\Psi(z) =& 1-\left(\frac{1}{z}\right)^{\alpha}
\end{aligned}
$$

Where, α is the Pareto parameter at time zero. Further, $\lim_{z\to\infty} z\psi(z) = \lim_{z\to\infty} z\frac{\alpha}{z^{\alpha+1}} = 0$.

A.2 Normalized surplus function

$$
\begin{aligned}
(r+\delta-g(t))v_e(z,t) &= w(z,t)+\delta v_u(t)-\frac{\partial v_e(z,t)}{\partial z}z(g(t)-\mu)+\frac{\partial v_e(z,t)}{\partial t} \\
(r+\delta-g(t))j(z,t) &= pz-w(z,t)+\delta v_v(t)-\frac{\partial j(z,t)}{\partial z}z(g(t)-\mu)+\frac{\partial j(z,t)}{\partial t}
\end{aligned}
$$

Similarly, for unmatched workers and firms

$$
\begin{aligned}
v_u(t) &= \frac{V_u(t)}{M(t)} \\
\frac{\partial V_u(t)}{\partial t} &= \frac{\partial v_u(t)}{\partial t}M(t)+M'(t)v_u(t) \\
\frac{1}{M(t)}\frac{\partial V_u(t)}{\partial t} &= \frac{\partial v_u(t)}{\partial t}+g(t)v_u(t)
\end{aligned}
$$

$$(r - g(t))v_u(t) = b + \theta(t)q(\theta(t)) \int (v_e(z,t) - v_u(t))d\Psi(z) + \frac{\partial v_u(t)}{\partial t}$$

The overall (normalized) surplus from a match, $s(z,t) := v_e(z,t) - v_u(t) + j(z,t)$

$$(r + \delta - g(t))s(z,t) = pz - b - \theta(t)q(\theta(t)) \int (v_e(z,t) - v_u(t))d\Psi(z) - \frac{\partial s(z,t)}{\partial z} z(g(t) - \mu) + \frac{\partial s(z,t)}{\partial t}$$

Finally, under the Nash bargaining protocol the worker receive a share β of the total surplus, hence

$$(r + \delta - g(t))s(z,t) = pz - b - \beta\theta(t)q(\theta(t)) \int s(z,t)d\Psi(z) - \frac{\partial s(z,t)}{\partial z} z(g(t) - \mu) + \frac{\partial s(z,t)}{\partial t}$$

A.3 Surplus ODE

The HJB equation that defines surplus on a balanced growth path is an ODE of the form

$$(r + \delta - g)s(z) = z - b - \theta \frac{\beta}{1-\beta} k - \frac{\partial s(z)}{\partial z} z(g - \mu)$$
$$\frac{\partial s(z)}{\partial z} + \frac{1}{g-\mu}(r + \delta - g)z^{-1}s(z) = \frac{1}{g-\mu} - \frac{b}{g-\mu} z^{-1} - \frac{\theta}{g-\mu}\frac{\beta}{1-\beta} k z^{-1}$$

Multiplying both sides by the integrating factor

$$e^{\int \frac{1}{g-\mu}(r+\delta-g)z^{-1}dz} = z^{\frac{1}{g-\mu}(r+\delta-g)}$$

gives

$$z^{\frac{1}{g-\mu}(r+\delta-g)} s(z) = \frac{1}{g-\mu} \int \left(z^{\frac{1}{g-\mu}(r+\delta-g)} - \left(b + \theta \frac{\beta}{1-\beta} k\right) z^{\frac{1}{g-\mu}(r+\delta-g)-1} \right) dz.$$

After some rearranging this can be simplified, where $\mathcal{C}$ is a constant of integration.

$$s(z) = \left(\frac{1}{r+\delta-\mu}\right) z - \left(b + \theta \frac{\beta}{1-\beta} k\right) \left(\frac{1}{r+\delta-g}\right) + \mathcal{C} z^{-\frac{1}{g-\mu}(r+\delta-g)}$$

This constant is pinned down using the smooth pasting condition, recall

$$0 = (g - \mu)\frac{\partial s(1)}{\partial z}.$$

Differentiating our expression for surplus with respect to z gives

$$\frac{\partial s(z)}{\partial z} = \left(\frac{1}{r+\delta-\mu}\right) - \frac{1}{g-\mu}(r + \delta - g)\mathcal{C} z^{-\frac{1}{g-\mu}(r+\delta-g)-1}$$

and hence the constant $\mathcal{C}$ can be pinned down as

$$\mathcal{C} = \frac{g-\mu}{(r+\delta-\mu)(r+\delta-g)}.$$

Substituting back into the surplus function yields the expression presented in the paper.

$$s(z) = \left(\frac{1}{r+\delta-\mu}\right) z - \left(b+\theta\frac{\beta}{1-\beta}k\right)\left(\frac{1}{r+\delta-g}\right) + \frac{g-\mu}{(r+\delta-\mu)(r+\delta-g)} z^{-\frac{1}{g-\mu}(r+\delta-g)}$$

A.4 Proof of Proposition 2: Equilibrium

In order to characterize the equilibrium and prove that it is unique, it is useful to first show that there is a unique value for labor market tightness when there is endogenous growth, $g > \mu$. When $g > \mu$, the firm exit conditions imply that $s(1) = \frac{\partial s(1)}{\partial z} 0$. Applying these conditions to the general solution for surplus gives a unique solution for labor market tightness

$$\theta = \bar{\theta} \equiv \frac{1-\beta}{\beta}\frac{p-b}{k}.$$

Intuitively, when there is endogenous growth, the least productive firm must make zero flow profits which gives the unique value for labor market tightness. Turning to an environment without endogenous growth, $g = \mu$, the exit condition $s(1) \geq 0$ implies that $\theta \leq \bar{\theta}$. Integrating the surplus equation over z with the offer density gives the simple free entry condition

$$\frac{k}{1-\beta} = q(\theta)\int s(z)d\Psi(z) = q(\theta)\frac{p\frac{\alpha}{\alpha-1} - b - \theta\frac{\beta}{1-\beta}k}{r+\delta-\alpha\mu+g(\alpha-1)}.$$

Since RHS is continuous and declining in both the endogenous growth rates g and labor market tightness θ, and labor market tightness is is weakly increasing in the growth rate g, at most one solution exists. A solution exists since the RHS goes to infinity as $\theta \to 0$ with $g = \mu$ as $p\frac{\alpha}{\alpha-1} > b$ and zero as $g \to \infty$ with $\theta = \bar{\theta}$. This completes the proof of the equilibrium characterization.

A.5 Proof of Proposition 3

Since we have a Cobb-Douglas matching function with job finding rate $\theta q(\theta) = A\theta^{1-\eta}$, we have that

$$\frac{\partial \log(jfr)}{\partial \log(\theta)} = (1-\eta).$$

Furthermore given the expression for $\bar{\theta}$, we get the following response under endogenous growth

$$\frac{\partial \log(\theta)}{\partial p} = \frac{p}{p-b} d\log(p),$$

and combining gives the first expression in the Proposition.

The free entry condition under exogenous growth (P2:θ ii) can be rewritten as

$$\frac{k}{q(\theta)} = (1-\beta)\frac{\frac{\alpha}{\alpha-1}p - b}{r + \delta - \mu + \beta\theta q(\theta)}.$$

Under this scenario, the response is, in turn, given by

$$\frac{\partial \log(\theta)}{\partial \log(p)} = \frac{\rho + \delta - \mu + \beta\theta q(\theta)}{\eta(r + \delta - \mu) + \beta\theta q(\theta)} \frac{\frac{\alpha}{\alpha-1}p}{\frac{\alpha}{\alpha-1}p - b},$$

which again combined with the expression for the job finding rate gives the second expression in the Proposition.

A.6 Proof of Proposition 4:

The equilibrium growth rate is given by equation (P2:g). For parameters such that $g > \mu$, the expression for the growth rate simplifies to

$$g = -\frac{r + \delta - \alpha\mu}{\alpha - 1} + p\frac{q(\theta)}{k}\frac{1-\beta}{(\alpha-1)^2} \quad \text{where, } \theta = \frac{1-\beta}{\beta}\frac{p-b}{k}.$$

Taking the derivative of the growth rate with respect to aggregate productivity p gives

$$\begin{aligned} \frac{\partial g}{\partial p} &= \mathcal{B}\frac{\partial}{\partial p}\left(p(p-b)^{-\eta}\right) \\ &= \mathcal{A}(p)\mathcal{B}\left(p - b - \eta p\right) \end{aligned}$$

Where the constant,

$$\mathcal{B} = A\frac{\beta^{\eta}(1-\beta)^{1-\eta}}{k^{1-\eta}(\alpha-1)^2} > 0$$

given the parameter restrictions imposed: $k > 0$, $\alpha > 1$, $\eta \in (0, 1)$ and $\beta \in (0, 1)$ and

$$\mathcal{A}(p) = (p-b)^{-\eta-1} > 0$$

given Assumption 2. Hence,

$$\frac{\partial g}{\partial p} > 0 \quad \text{if} \quad p > b + \eta p$$

Since the separation rate is an affine function in g,

$$\Delta = \alpha\left(g - \mu\right)$$

the same condition holds for Δ also.

The second part of the Proposition, when $g = \mu$, is implied by the fact that in this case both separations and growth are exogenous.

A.7 Proof of Proposition 6: Maximum growth rate

The equilibrium growth rate is given by equation (P2:g). Growth is maximized when $g > \mu$, for which (P2:g) simplifies to

$$g = -\frac{r + \delta - \alpha\mu}{\alpha - 1} + p\frac{q(\theta)}{k}\frac{1-\beta}{(\alpha-1)^2} \quad \text{where, } \theta = \frac{1-\beta}{\beta}\frac{p-b}{k}.$$

Taking the derivative of the growth rate with respect to β gives

$$\frac{\partial g}{\partial \beta} = \frac{pk^{\eta-1}}{(p-b)^{\eta}(\alpha-1)^2}\left(\eta\beta^{\eta-1}(1-\beta)^{1-\eta} - (1-\eta)\beta^{\eta}(1-\beta)^{-\eta}\right).$$

The first order condition of which implies that $\beta = \eta$. The second derivative of the growth rate with respect to β is given by

$$\frac{\partial^2 g}{\partial \beta^2} = -\frac{pk^{\eta-1}}{(p-b)^{\eta}(\alpha-1)^2}\left(\frac{\eta(1-\eta)\beta^{\eta-2}}{(1-\beta)^{\eta-1}}\right).$$

Since $p > 0$, $p - b > 0$, $k > 0$, $\alpha > 1$, $\eta \in (0,1)$ and $\beta \in (0,1)$ then $\frac{\partial^2 g}{\partial \beta^2} < 0$ over the domain of β and thus g is maximized when $\beta = \eta$.

To understand the values of β that generate endogenous growth, consider the condition for which $g > \mu$. The condition under which there is endogenous growth is given by

$$-\frac{r + \delta - \mu}{\alpha - 1} + \frac{pAk^{\eta-1}}{(p-b)^{\eta}(\alpha-1)^2}\left(1-\beta\right)^{1-\eta}\beta^{\eta} > 0.$$

Thus, for a sufficiently low or high bargaining power, there is no endogenous growth as the left hand side is smaller than μ as either $\beta \to 1$ or $\beta \to 0$.

A.8 Proof of Proposition 7: Solving for $\theta(a)$ and $g(a)$

A.8.1 Solving for tightness: $\theta(a)$

We begin by substituting in the free entry condition into the HJB equation defining the surplus, this yields

$$(r+\delta+\chi-g(a))s(a,z)=p(a)z-b-\theta(a)\frac{\beta}{1-\beta}k+\chi\int_0^1 s(a',z)\gamma(a,a')da'-\frac{\partial s(a,z)}{\partial z}z(g(a)-\mu).$$

Evaluating the above at the minimum productivity match gives

$$(r+\delta+\chi-g(a))s(a,1)=p(a)-b-\theta(a)\frac{\beta}{1-\beta}k+\chi\int s(a',1)\gamma(a,a')da'.$$

From the condition for firm exit, for an aggregate state a such that there is endogenous growth $g(a)>\mu$ and $s(a,1)=0$. Setting $s(a,1)=0$ in the above gives an expression for tightness when $g(a)>\mu$.

$$\theta(a)=\bar{\theta}(a)\equiv\frac{1-\beta}{\beta}\frac{1}{k}\left(p(a)-b+\chi\int s(a',1)\gamma(a,a')da'\right)\quad\text{for}\quad g(a)>\mu$$

Conversely, when a is such that $g(a)=\mu$, $s(a,1)>0$ and $\theta(a)$ is pinned down by the free entry condition. Substituting $g(a)=\mu$ into the surplus equation and multiplying both sides by $\psi(z)$ and integrating over the full support of z gives an expression for $\theta(a)$ when $g(a)=\mu$.

$$\begin{aligned}
(r+\delta+\chi-\mu)\int s(a,z)\psi(z)dz =& p(a)\int z\psi(z)dz-b-\theta(a)\frac{\beta}{1-\beta}k+\chi\int\int\left(s(a',z)\psi(z)dz\right)\gamma(a,a')da'\\
(r+\delta+\chi-\mu)\left(\frac{k}{q(\theta(a))(1-\beta)}\right) =& p(a)\frac{\alpha}{\alpha-1}-b-\theta(a)\frac{\beta}{1-\beta}k+\chi\int\left(\frac{k}{q(\theta(a'))(1-\beta)}\right)\gamma(a,a')da'\\
\theta(a) =& \left(p(a)\frac{\alpha}{\alpha-1}-b\right)\left(\frac{1-\beta}{\beta k}\right)-\frac{(r+\delta+\chi-\mu)}{q(\theta(a))\beta}+\frac{\chi}{\beta}\int\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da'
\end{aligned}$$

And since $\theta(a)\leq\bar{\theta}(a)$, we get the expression in the Proposition.

A.8.2 Solving for the growth rate: $g(a)$

Multiply the surplus equation by $\psi(z)$ and integrate over the full support of z. This gives,

$$\begin{aligned}
(r+\delta+\chi-g(a))\int_1^\infty s(a,z)\psi(z)dz = p(a)\int_1^\infty z\psi(z)dz-b-\theta(a)\frac{\beta}{1-\beta}k\\
-(g(a)-\mu)\int_1^\infty\frac{\partial s(a,z)}{\partial z}z\psi(z)dz+\chi\int_0^1\int_1^\infty\psi(z)s(a',z)dz\gamma(a,a')da'
\end{aligned}$$

In order to simplify the expression, we go through each integral in succession. The integral on the left hand side and the inner integral in the final term on the right hand side can both be written as, given the free entry condition

$$\int_1^\infty s(a,z)\psi(z)dz = \frac{k}{(1-\beta)q(\theta(a))}.$$

The first integral on the right is the mean of a Pareto distribution and given by $\left(\frac{\alpha}{\alpha-1}\right)$. The second integral on the right hand side is a little more involved but can also be simplified.

$$\begin{aligned}
\int \frac{\partial s(a,z)}{\partial z} z\psi(z)dz =& \alpha \int \frac{\partial s(a,z)}{\partial z}\left(1-\Psi(z)\right)dz \quad \text{(property of a Pareto distribution)}\\
=& \alpha\left[s(a,z)\left(1-\Psi(z)\right)\right]_{z=1}^{\infty} + \alpha\int s(a,z)\psi(z)dz\\
=& \alpha s(a,1) + \frac{\alpha k}{(1-\beta)q(\theta(a))} \quad \text{assuming } \lim_{z\to\infty} s(a,z)(1-\Psi(z)) = 0\\
=& \frac{\alpha k}{(1-\beta)q(\theta(a))}
\end{aligned}$$

Substituting each expression back into the integral equation over normalized surplus gives

$$\begin{aligned}
(r+\delta+\chi-g(a))\left(\frac{k}{(1-\beta)q(\theta(a))}\right) &= p(a)\frac{\alpha}{\alpha-1} - b - \theta(a)\frac{\beta}{1-\beta}k\\
&\quad -(g(a)-\mu)\left(\frac{\alpha k}{(1-\beta)q(\theta(a))}\right) + \frac{\chi k}{1-\beta}\int\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da'.
\end{aligned}$$

Recall $g(a)\geq\mu$, therefore need an expression for when $g(a)>\mu$. Substituting in the expression for $\theta(a)$ when $g(a)>\mu$ gives

$$\begin{aligned}
(r+\delta+\chi-g(a))\left(\frac{k}{(1-\beta)q(\theta(a))}\right) &= \frac{p(a)}{\alpha-1} - \chi\int s(a',1)\gamma(a,a')da'\\
&\quad -(g(a)-\mu)\left(\frac{\alpha k}{(1-\beta)q(\theta(a))}\right) + \frac{\chi k}{1-\beta}\int_{\underline{a}}^{\bar{a}}\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da'.
\end{aligned}$$

After some rearranging one derives an expression for $g(a)$ as

$$\begin{aligned}
g(a) = -\frac{(r+\delta+\chi-\alpha\mu)}{\alpha-1} &+ \frac{q(\bar{\theta}(a))(1-\beta)}{k(\alpha-1)}\left(\frac{p(a)}{(\alpha-1)} - \chi\int_{a^*}^{\bar{a}} s(a',1)\gamma(a,a')da'\right)\\
&+\frac{\chi}{\alpha-1}q(\bar{\theta}(a))\int_{\underline{a}}^{\bar{a}}\left(\frac{\gamma(a,a')}{q(\theta(a'))}\right)da'
\end{aligned}$$

Since for states in which $g(a)=\mu$, $\theta(a)\leq\bar{\theta}(a)$ and $\int_1^\infty s(a,z)\psi(z)dz = \frac{k}{(1-\beta)q(\theta(a))}$, it must be that the above expression is negative and we get the expression in the Proposition.

A.9 Proof of Proposition 8: Productivity process

We begin by assuming that the productivity process is initiated as a draw from the ergodic distribution. That is to say that the initial state at time $t = 0$, denoted by a_0, is the result of an infinite number of draws of ϵ. Thus a_0 is given by,

$$a_0 = \epsilon \sum_{i=1}^{\infty} (1-\epsilon)^{i-1} \varepsilon_i,$$

where ε_i is distributed $U[0,1]$. The logarithm of aggregate productivity is given by

$$\log(p(a_0)) = p_0 + p_1 \epsilon \sum_{i=1}^{\infty} (1-\epsilon)^{i-1} (\varepsilon_i - 0.5),$$

which by the central limit approaches the Normal distribution as $\epsilon \to 0$.

Further, since ε_i is an independent and identically distributed random variable drawn from a standard uniform distribution, with a mean of 1/2 and variance of 1/12, we get that

$$\text{Var}(a_0) = \epsilon^2 \sum_{i=1}^{\infty} (1-\epsilon)^{2i-2} \frac{1}{12} = \frac{\epsilon}{2-\epsilon} \frac{1}{12}, \quad \text{and} \quad \text{Var}(\log(p(a_0))) = p_1^2 \frac{\epsilon}{2-\epsilon} \frac{1}{12}.$$

Given an initial state a_0 and n subsequent aggregate shocks, the new state a' is given by

$$a' = (1-\epsilon)^n a_0 + \epsilon \sum_{i=1}^{n} (1-\epsilon)^{i-1} \varepsilon_i,$$

where, again, ε_i is distributed $U[0,1]$. Taking expectations, we get that

$$E[a'] = (1-\epsilon)^n a_0 + 1/2(1-(1-\epsilon)^n).$$

Given an elapsed duration t, the distribution of the number n of aggregate shocks to occur in the interval $[0, t]$, is Poisson with parameter χt. We can therefore solve for the distribution of the expected value of a_t given any a_0

$$E[a_t|a_0] = \sum_{n=0}^{\infty} (1-\epsilon)^n (a_0 - 0.5) \frac{(\chi t)^n e^{-\chi t}}{n!} + 1/2 = (a_0 - 0.5) e^{-\chi \epsilon t} + 1/2$$

where we use that $\sum_{n=0}^{\infty} \frac{((1-\epsilon)\chi t)^n e^{-\chi t(1-\epsilon)}}{n!} = 1$. The expectation therefore satisfies

$$\mathbb{E}(\log(p(a_t))|a_0) \quad = p_0 + p_1 \exp[-\chi \epsilon t] (a_0 - 0.5).$$

A.10 Cross-country regression specification

To derive our growth accounting regression we begin with equation (6) in the paper which denotes the level of output in $t + \tau$ to that in t.

$$Y(t+\tau) = Y(t). \underbrace{e^{\int_t^{t+\tau} g(t')dt'}}_{\text{Permanent component}} \underbrace{\left(\frac{p(t+\tau)}{p(t)}\right)}_{\text{Transitory component}} \tag{6}$$

Taking logs, equation (6) can be written as

$$\log\left(\frac{Y(t+\tau)}{Y(t)}\right) = \underbrace{\int_t^{t+\tau} g(t')dt'}_{\text{Permanent component}} + \underbrace{\log\left(\frac{p(t+\tau)}{p(t)}\right)}_{\text{Transitory component}} \tag{A.10:1}$$

Looking at the permanent component of equation (A.10:1) first. Employment separations over an interval $[t, t+\tau]$ come about through exogenous separations at Poisson rate δ, and endogenous separations at Poisson rate $\alpha(g(t) - \mu)$. Thus we can define the mean separation rate over an interval $[t, t+\tau]$ as

$$s(t, t+\tau) = \frac{1}{\tau}\int_t^{t+\tau} \delta + \alpha(g(t') - \mu)dt'.$$

Rearranging this expression gives us the permanent component term of equation (A.10:1).

$$\int_t^{t+\tau} g(t')dt' = \left(\frac{s(t, t+\tau) - \delta}{\alpha} + \mu\right)\tau$$

Turning instead to the transitory component, recall that market tightness with endogenous is growth given as,

$$\theta(a) = \frac{1-\beta}{\beta}\frac{1}{k}\left(p(a) - b + \chi\int s(a', 1)\gamma(a, a')da'\right).$$

In a world, as in the calibration where there is always endogenous growth, $s(a, 1) = 0$ for all a and at time t tightness ($\theta(t)$) and the job finding rate ($\mathrm{jfr}(t)$) are given by

$$\theta(t) = \frac{1-\beta}{\beta}\frac{1}{k}(p(t) - b)$$
$$\mathrm{jfr}(t) = A\left(\frac{1-\beta}{\beta}\frac{1}{k}(p(t) - b)\right)^{1-\eta}.$$

Define $\overline{\theta}$ and $\overline{p}$ as the mean values and recall that p_0 is normalized such that $\overline{p} = 1$. Log-linearizing around these values yields an expression for market tightness as

$$
\begin{aligned}
d \log \theta(t) \approx & \frac{\overline{p}}{\overline{p} - b} d \log p(t) \\
\log \theta(t) \approx & \log(\overline{\theta}) + \frac{\overline{p}}{\overline{p} - b} \log \left(\frac{p(t)}{\overline{p}} \right).
\end{aligned}
$$

Performing the same exercise for the job finding rate,

$$
\begin{aligned}
\log(\text{jfr}(t)) &= \log(\overline{\text{jfr}}) + (1 - \eta) \log \left(\frac{\theta(t)}{\overline{\theta}} \right) \\
&\approx \log(\overline{\text{jfr}}) + (1 - \eta) \frac{\overline{p}}{\overline{p} - b} \log \left(\frac{p(t)}{\overline{p}} \right)
\end{aligned}
$$

Thus the log difference in job finding rates between t and $t + \tau$ can be written as,

$$
\log(\text{jfr}(t + \tau)) - \log(\text{jfr}(t)) \approx (1 - \eta) \frac{\overline{p}}{\overline{p} - b} \left(\log(p(t + \tau)) - \log(p(t)) \right)
$$

Rearranging the above, noting that $\overline{p} = 1$, gives an expression for the transitory component part in (A.10:1).

$$
\log \left(\frac{p(t + \tau)}{p(t)} \right) \approx \left(\frac{1 - b}{1 - \eta} \right) \log \left(\frac{\text{jfr}(t + \tau)}{\text{jfr}(t)} \right)
$$

Finally, plugging the expressions for the temporary and permanent component into equation (A.10:1) gives,

$$
\begin{aligned}
\log \left(\frac{Y(t + \tau)}{Y(t)} \right) &\approx \left(\frac{s(t, t + \tau) - \delta}{\alpha} + \mu \right) \tau + \left(\frac{1 - b}{1 - \eta} \right) \log \left(\frac{\text{jfr}(t + \tau)}{\text{jfr}(t)} \right) \\
&= \left(\frac{\alpha\mu - \delta}{\alpha} \right) \tau + \left(\frac{\tau}{\alpha} \right) s(t, t + \tau) + \left(\frac{1 - b}{1 - \eta} \right) \log \left(\frac{\text{jfr}(t + \tau)}{\text{jfr}(t)} \right).
\end{aligned}
$$

The above expression is equivalent to the cross-country regression (7). Where the intercept is given by $\left(\frac{\alpha\mu - \delta}{\alpha} \tau \right)$, the coefficient on the separation rate is $\left(\frac{\tau}{\alpha} \right)$, and the coefficient on the log change in job finding rate is $\left(\frac{1 - b}{1 - \eta} \right)$.

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