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# Cross-Country Interactions, the Great Moderation and the Role of Output Volatility in Growth\*

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This paper investigates the effect of output volatility and the great moderation on growth in a model that simultaneously accounts for cross-country interactions, structural breaks and heterogeneous effects. This is done by augmenting the univariate GARCH-M model of growth for each G7 country with cross-country weighted averages of growth and shift dummies. I find that volatility affects growth positively, that there is a great moderation in five of the G7 countries and that the great moderation has a negative effect on growth in all G7 countries. A simulation exercise shows that cross-country interactions are important in estimating the volatility effect.

**Keywords:** Cross-country interactions, Volatility, Growth, GARCH-M,  
The great moderation.

**JEL:** C32, C5, E32.

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# 1 Introduction

The relationship between output volatility and growth is an important and widely discussed relationship in the economic literature. Research on this relationship is important because an increased understanding of the determinants of growth can have important consequences for human welfare and it can inform policymakers on the growth effects of stabilisation policy. It is particularly important to investigate this relationship empirically as it is theoretically ambiguous. Martin and Rogers (1997), for example, develop a representative agent model where the engine of growth is human capital accumulation through learning-by-doing. In their model, output volatility leads to a lower average labour supply and so less learning-by-doing which affects the growth rate negatively. Aghion and Saint-Paul (1998), in contrast, develop a Schumpeterian model of growth where the opportunity cost of investing in productivity enhancing activities is lower during recessions and so if the frequency of recessions is high, i.e. high volatility, then the growth rate is positively affected.

An important issue related to the relationship between output volatility and growth is the reduction of output volatility in recent decades, the so-called great moderation. The great moderation was first discovered for the United States by Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) and these authors found that the variance of US output growth experienced a break around 1984. Stock and Watson (2005) show that the decline in volatility is not solely a US phenomenon, but that it holds for the G7 countries<sup>1</sup>. Given the existence of a great moderation<sup>2</sup>, it is important to account for structural breaks in the variance of output growth in an econometric model. Moreover, an under-appreciated corollary between a possible effect of output volatility on growth and the great moderation is

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<sup>1</sup>The G7 consist of Canada (CAN), France (FR), Germany (GER), Italy (ITA), Japan (JAP), the United Kingdom (UK) and the United States (US).

<sup>2</sup>There are a number of possible explanations for why output growth became less volatile. Bernanke (2004) explains that the economic literature has suggested three types of explanations, namely (i) structural change, (ii) improved performance of macroeconomic policies and (iii) good luck, i.e. less shocks.

that the great moderation can have growth effects.

In this paper, I investigate both the effect of output volatility and the great moderation on growth with a generalised autoregressive conditional heteroskedasticity in mean (GARCH-M) model, developed by Engle, Lilien, and Robins (1987), that allows for cross-country interactions, structural breaks and heterogeneous effects. I do this by augmenting the standard univariate GARCH-M model with cross-country weighted averages of growth to account for cross-country interactions and shift dummies to account for structural breaks and coin this model the *Global GARCH-M* model. Using cross-country weighted averages in this way is a technique developed in Pesaran, Schuermann, and Weiner (2004) and Déés, Di Mauro, Pesaran, and Smith (2007) in the context of vector autoregressive (VAR) modelling; the so-called Global VAR.

The GARCH-M model is an important tool applied in the literature to uncover the effect of output volatility on growth<sup>3</sup>. In this model, the conditional variance of output growth is allowed to vary over time and depends on previous shocks to output growth. The time-varying conditional standard deviation, which is the measure of output volatility, is also allowed to affect the mean growth rate. Existing studies applying the GARCH-M framework, however, do not take cross-country interactions, structural breaks and heterogeneous effects simultaneously into account and find conflicting results concerning the effect of output volatility on growth. Fang, Miller, and Lee (2008), for example, estimate univariate GARCH-M models for the quarterly growth rate of GDP for the G7 countries<sup>4</sup> augmented with shift dummies to allow for structural breaks, but do not take cross-country interactions into account. They find no effect of output volatility on growth, except for Japan where they find

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<sup>3</sup>Cross-country growth regressions using cross-section and panel methods is another tool used in the literature. These studies generally find that output volatility affects the growth rate negatively (see for example Ramey and Ramey (1995), Martin and Rogers (2000), Badinger (2010)). The measure of output volatility in these studies is, in contrast to studies using a GARCH-M framework, the *unconditional* standard deviation of growth over some period.

<sup>4</sup>They do not include France in the analysis as they do not find evidence for conditional heteroskedasticity.

a negative effect. Lee (2010) estimates a dynamic panel GARCH-M model for the monthly growth rate of industrial production for the G7 countries. This framework accounts for cross-country interactions, but imposes common slopes and does not allow for structural breaks. He finds that output volatility affects growth positively.

In this study, I find that the impact effect of output volatility on growth is positive in four of the G7 countries and that the long-run effect is positive in all G7 countries. A break detection analysis based on Bai and Perron (2003), but adjusted to take cross-country interactions into account, shows that there is evidence for a great moderation in all G7 countries, except for Germany and Japan. The fall in output volatility happened around 1985 for all these countries, except for France where the fall happens around 1980. The long-run effect of the great moderation on growth is found to be negative in all G7 countries.

The paper also assesses the importance of cross-country interactions in estimating the effect of output volatility on growth by performing a simulation exercise. Concerning the impact effect of output volatility, it is shown that cross-country interactions are important in all G7 countries, except for Japan. Concerning the total long-run effect, cross-country interactions are important in all G7 countries.

The paper is organised as follows. Section 2 describes the modelling framework. It presents the Global GARCH-M model, i.e. univariate GARCH-M models of output growth for each of the G7 countries with shift dummies and cross-country weighted averages of growth, and derives the long-run effects of output volatility and the great moderation on output growth. Section 3 discusses the estimation results. Before presenting these results, however, it discusses the data, an exogeneity test, the break detection procedure, the model selection procedure and the estimation method. Section 4 assesses the importance of cross-country interactions in estimating the effect of output volatility on growth using a simulation exercise. Section 5 concludes. Throughout the paper, the word ‘volatility’ implies ‘output volatility’.

## 2 Modelling the effect of volatility on output growth;

### The Global GARCH-M model

In this section, I will describe the Global GARCH-M model used to investigate the effect of volatility and the great moderation on growth for the G7 countries. The Global GARCH-M model for the G7 consists of a standard univariate GARCH-M model for each G7 country, but modified in two ways. The first modification is that I allow for cross-country interactions by including cross-country weighted averages of growth to the model. The use of cross-country weighted averages in this way was first applied in the Global VAR literature (see Pesaran et al. (2004) and Déés et al. (2007)). As a country's growth rate is affected by events in other countries through trade and financial links, it is important to allow for growth spillovers from different countries.

The second adjustment is that I include shift dummies in the mean and variance equation of the univariate GARCH-M model to take structural breaks into account. The break detection procedure is based on Bai and Perron (2003), but adjusted to account for cross-country interactions. Allowing for breaks is important because a well-documented property of post-war output growth data is the occurrence of structural breaks in both the mean, as demonstrated by the literature on productivity slowdowns (see for example Nordhaus (2004)) and the variance, as demonstrated by the literature on the great moderation (see for example Bernanke (2004)). The great moderation is of particular interest in a study on the relationship between volatility and growth because, given that volatility affects growth, the great moderation can have growth effects. Note also that allowing for a great moderation together with cross-country interactions will be especially important if changes in volatility are synchronised across countries so that there could be subsequent cumulated effects. It is also important to account for structural breaks for econometric reasons. As Perron (1989) shows, structural breaks biases the autoregressive parameters in the

mean equation towards 1. Ignoring structural breaks in the variance has a similar effect (see Lamoureux and Lastrapes (1990) and Hillebrand (2005)).

In what follows, I first formally describe the Global GARCH-M model used to model output growth for the G7 countries. Then, I derive the potential long-run effects of volatility and of the great moderation on growth.

## 2.1 The Global GARCH-M model

The univariate GARCH-M model augmented with shift dummies in the mean and variance equation and cross-country weighted averages of growth, i.e. the Global GARCH-M model, is given by

$$\Delta y_{it} = c_{i0} + \sum_{k=1}^l c_{ik} DM_{ikt} + \lambda_i \sigma_{it} + \sum_{k=0}^s \beta_{ik} \Delta y_{it-k}^* + \sum_{k=1}^p \phi_{ik} \Delta y_{it-k} + \varepsilon_{it} \quad (1)$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt} + \eta_i \varepsilon_{it-1}^2 + \gamma_i \sigma_{it-1}^2 \quad (2)$$

where  $\Delta y_{it}$  is the growth rate for country  $i$  at time  $t$ ,  $DM_{ikt}$  is a possible shift dummy  $k$  in the conditional mean which is equal to 0 before the break date and equal to 1 on and after the break date,  $\sigma_{it}$  is the conditional standard deviation of growth, which is the measure of volatility,  $\Delta y_{it-k}^*$  are the cross-country weighted averages of growth and  $\varepsilon_{it}$  is the error term.

The cross-country weighted averages of growth are defined as

$$\Delta y_{it}^* = \sum_{j=1}^7 w_{ij} \Delta y_{jt}, \quad \sum_{j=1}^7 w_{ij} = 1 \quad \text{and} \quad w_{ii} = 0, \quad (3)$$

where I have chosen to measure  $w_{ij}$  by the average share of total trade of country  $i$  with country  $j$ . Total trade is defined as the sum of exports and imports between country  $i$  and the other countries. Note that the weight  $w_{ij}$  is assumed to be constant

over time. As will be shown below, a constant weight is needed in order to calculate the long-run effects of volatility and the great moderation on growth. Estimating the model while allowing for time-varying trade weights, however, is possible. As a robustness check, I will also estimate the model with time-varying trade weights and check if the results are different compared to the results with constant trade weights.

The conditional variance of country  $i$  at time  $t$ ,  $\sigma_{it}^2$ , depends on a constant  $\alpha_{i0}$ , possible shift dummies,  $DV_{ikt}$ , which are equal to 0 before the break date and equal to 1 on and after the break date, the squared lagged error term and a lag of the variance. A permanent moderation in volatility would be captured by a shift dummy with a negative coefficient. Note that equation (2) reduces to an ARCH(1) model if  $\gamma_i$  equals zero.

As usual, the conditional mean of the error term is assumed to be equal to 0, so  $\varepsilon_{it} \sim (0, \sigma_{it}^2)$ . Assuming that the cross-country weighted averages account for all the cross-country interactions, it follows that the error term of each country only consist of an idiosyncratic component and so the error term across countries is uncorrelated, i.e.  $COV[\varepsilon_{it}, \varepsilon_{jt}] = E[\varepsilon_{it}\varepsilon_{jt}] = 0$  for  $i \neq j$ .

## 2.2 Measuring the long-run effect of volatility and the great moderation on growth

To find the long-run effect of volatility and the great moderation on output growth, I first calculate the short-run effects and then premultiply these effects with a multiplier that scales up to take account of the short-run dynamics and cross-country interactions.

The short-run effect of volatility on growth is equal to

$$\frac{\partial \Delta y_{it}}{\partial \sigma_{it}} = \lambda_i \equiv \lambda_i^{SR}. \quad (4)$$

To find the short-run effect of the great moderation on economic activity, equation (2) is first substituted into (1) and then the derivative is taken with respect to the particular variance dummy or dummies,  $DV_{ikt}$ , that relates or relate to the great moderation, i.e. a variance break dummy or dummies with a negative coefficient. These dummies are denoted as  $DV_{iGM}$  and their effects as  $\alpha_{iGM}$ . Thus,

$$\frac{\partial \Delta y_{it}}{\partial DV_{iGM}} = \lambda_i \frac{1}{2} \left( \alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt} + \eta_i \varepsilon_{it-1}^2 + \gamma_i \sigma_{it-1}^2 \right)^{-\frac{1}{2}} \alpha_{iGM} \quad (5)$$

$$= \frac{\lambda_i \alpha_{iGM}}{2 \sqrt{\sigma_{it}^2}} \quad (6)$$

which depends on the effect of volatility on growth,  $\lambda_i$ , the conditional variance of the errors at time  $t$ ,  $\sigma_{it}^2$ , and the effect of the great moderation on the conditional variance,  $\alpha_{iGM}$ . As we are interested in the long-run effect of the great moderation, the conditional variance of the errors at time  $t$  in equation (6) is replaced with the long-run or unconditional variance of the errors, denoted as  $\bar{\sigma}_i^2$ , so that the expression of the long run effect of the great moderation does not depend on  $t$ . Thus,

$$\frac{\partial \Delta y_{it}}{\partial DV_{iGM}} = \frac{\lambda_i \alpha_{iGM}}{2 \sqrt{\bar{\sigma}_i^2}} \quad (7)$$

$$\equiv \alpha_{iGM}^{SR} \quad (8)$$

where  $\bar{\sigma}_i^2 = \left( \frac{\alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt}}{1 - \eta_i - \gamma_i} \right)$ .

In order to take account of cross-country interactions, I first stack the model, i.e.

$$\Delta \mathbf{y}_t = \mathbf{c}_0 + \sum_{k=1}^l \mathbf{c}_k DM_{kt} + \lambda \boldsymbol{\sigma}_t + \sum_{k=0}^s \beta_k \Delta \mathbf{y}_{t-k}^* + \sum_{k=1}^p \phi_k \Delta \mathbf{y}_{t-k} + \boldsymbol{\varepsilon}_t \quad (9)$$

$$\boldsymbol{\sigma}_t^2 = \boldsymbol{\alpha}_0 + \sum_{k=1}^f \boldsymbol{\alpha}_k DV_{kt} + \eta \boldsymbol{\varepsilon}_{t-1}^2 + \gamma \boldsymbol{\sigma}_{t-1}^2 \quad (10)$$

$$\Delta \mathbf{y}_t^* = \mathbf{W} \Delta \mathbf{y}_t \quad (11)$$

where  $\Delta \mathbf{y}_t = (\Delta y_{1t}, \dots, \Delta y_{7t})'$  and similarly for other emboldened vectors. All the coefficient matrices are  $7 \times 7$  matrices, except the constants  $\mathbf{c}_0$  and  $\boldsymbol{\alpha}_0$  which are  $7 \times 1$  vectors. The diagonal elements of these matrices are equal to the country-specific estimates and the off-diagonal elements are equal to 0. The country specific trade weights,  $w_{ij}$ , are collected in a weight matrix,  $\mathbf{W}$ , which is equal to

$$\mathbf{W} = \begin{bmatrix} 0 & w_{12} & \cdots & w_{17} \\ w_{21} & 0 & \vdots & w_{27} \\ \vdots & \vdots & \ddots & \vdots \\ w_{71} & w_{72} & \cdots & 0 \end{bmatrix}. \quad (12)$$

Substituting (11) in (9) gives

$$\Delta \mathbf{y}_t = \mathbf{c}_0 + \sum_{k=1}^l \mathbf{c}_k D M_{kt} + \lambda \boldsymbol{\sigma}_t + \sum_{k=0}^s \beta_k \mathbf{W} \Delta \mathbf{y}_{t-k} + \sum_{k=1}^p \phi_k \Delta \mathbf{y}_{t-k} + \boldsymbol{\varepsilon}_t \quad (13)$$

so that

$$\Delta \mathbf{y}_t = \left( \mathbf{I}_7 - \mathbf{B}(1)\mathbf{W} - \boldsymbol{\Phi}(1) \right)^{-1} \left( \mathbf{c}_0 + \sum_{k=1}^l \mathbf{c}_k D M_{kt} + \lambda \boldsymbol{\sigma}_t + \boldsymbol{\varepsilon}_t \right) \quad (14)$$

where  $\mathbf{B}(1) = \beta_0 + \beta_1 L + \beta_2 L^2 + \dots + \beta_s L^s$  and  $\boldsymbol{\Phi}(1) = \phi_1 L + \phi_2 L^2 + \dots + \phi_p L^p$  are polynomials in the lag operator  $L$ , so  $\mathbf{B}(1)\mathbf{W} = \sum_{k=0}^s \beta_k \mathbf{W}$  and  $\boldsymbol{\Phi}(1) = \sum_{k=1}^p \phi_k$ . It follows that the multiplier is equal to  $\left( \mathbf{I}_7 - \mathbf{B}(1)\mathbf{W} - \boldsymbol{\Phi}(1) \right)^{-1}$  and so the long-run effects of volatility and the great moderation on output growth are found by premultiplying the respective short-run effects with the multiplier. This gives

$$\boldsymbol{\lambda}^{LR} \equiv \left( \mathbf{I}_7 - \mathbf{B}(1)\mathbf{W} - \boldsymbol{\Phi}(1) \right)^{-1} \boldsymbol{\lambda}^{SR} = \begin{bmatrix} \lambda_{(1,1)}^{LR} & \lambda_{(1,2)}^{LR} & \cdots & \lambda_{(1,7)}^{LR} \\ \lambda_{(2,1)}^{LR} & \lambda_{(2,2)}^{LR} & \cdots & \lambda_{(2,7)}^{LR} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{(7,1)}^{LR} & \lambda_{(7,2)}^{LR} & \cdots & \lambda_{(7,7)}^{LR} \end{bmatrix} \quad (15)$$

and

$$\alpha_{GM}^{LR} \equiv \left( \mathbf{I}_7 - \mathbf{B}(1)\mathbf{W} - \Phi(1) \right)^{-1} \alpha_{GM}^{SR} = \begin{bmatrix} \alpha_{(1,1)GM}^{LR} & \alpha_{(1,2)GM}^{LR} & \cdots & \alpha_{(1,7)GM}^{LR} \\ \alpha_{(2,1)GM}^{LR} & \alpha_{(2,2)GM}^{LR} & \cdots & \alpha_{(2,7)GM}^{LR} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{(7,1)GM}^{LR} & \alpha_{(7,2)GM}^{LR} & \cdots & \alpha_{(7,7)GM}^{LR} \end{bmatrix}, \quad (16)$$

respectively. Because the model takes cross-country interactions into account through the weight matrix  $\mathbf{W}$ , the off-diagonal elements of the matrices with the long-run effects are nonzero. The diagonal elements are the long-run effects of country  $i$  on the growth rate of country  $i$ . The off-diagonal elements are the long-run effects of country  $j$ , i.e. the column element in the matrices with the long-run effects, on the growth rate of country  $i$ , i.e. the row element in the matrices with the long-run effects.

Note that for each country there are seven long-run effects. To arrive at the *total* long-run effect, the row elements of the matrices with the long-run effects are summed. Thus the total long-run effects of volatility and the great moderation on the individual countries' output growth are equal to

$$\lambda^{TLR} \equiv \lambda^{LR} \mathbf{S} = \begin{bmatrix} \sum_j \lambda_{(1,j)}^{LR} \\ \sum_j \lambda_{(2,j)}^{LR} \\ \vdots \\ \sum_j \lambda_{(7,j)}^{LR} \end{bmatrix} \quad \text{and} \quad \alpha_{GM}^{TLR} \equiv \alpha_{GM}^{LR} \mathbf{S} = \begin{bmatrix} \sum_j \alpha_{(1,j)GM}^{LR} \\ \sum_j \alpha_{(2,j)GM}^{LR} \\ \vdots \\ \sum_j \alpha_{(7,j)GM}^{LR} \end{bmatrix} \quad (17)$$

respectively, where  $\mathbf{S}$  is a  $7 \times 1$  vector of ones. The elasticities in  $\lambda^{TLR}$  show the effect on growth in each country of a one percentage point increase in volatility in all G7 countries. The elasticities in  $\alpha_{GM}^{TLR}$  show the effect on growth in each country of a once and for all shift in volatility experienced across all countries at the same time.

### 3 Measuring the effect of volatility and the great moderation on output growth in the G7, 1961–2013

I now turn to the empirical analysis of the paper, estimating the model of (1)-(3) for the G7 economies between 1961–2013 and evaluating the elasticities of (17). Before estimating the model itself, I start this section with an overview of the data, an exogeneity test for the contemporaneous cross-country weighted average in the model and a discussion of the possible presence of structural breaks in the mean and the variance of output growth. As will be shown, estimating GARCH-M models is not straightforward as they can face weak identification problems. After discussing how these are overcome and the relatively sophisticated specification search rule, I describe the estimated Global GARCH-M model in Subsection 3.5.

#### 3.1 Properties of output growth in the G7

The measure of output growth in this study is the seasonal adjusted monthly growth rate of industrial production from the OECD’s Main Economic Indicators (MEI) database<sup>5</sup>. The analysis is done over the period February 1961–May 2013 and Figure 1 plots the G7 output growth rates over time<sup>6</sup>. Table 1 presents the summary statistics of the monthly growth rates over this period and shows that Japan grew at the highest rate of 0.38% per month and France grew at the lowest rate of 0.04%

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<sup>5</sup>To check if this measure is a good measure of economic activity, the industrial production index is compared with GDP in Figure 3 in the appendix. This shows that the two series follow the same path, except for France and the United Kingdom where the growth rate of industrial production is slower than GDP from 1975 and 1970 onwards, respectively. Panel A and B of Table 7 in the appendix show the correlation coefficient and the standard deviation of the two series, respectively. The correlation of the two series is high, irrespective if the series are in levels, in logs or in growth rates. The IPI series is much more volatile than GDP for all G7 countries.

<sup>6</sup>France and Japan have extreme observations and these are replaced by the median growth rate of the original data over the full sample. The extreme observations are March–April 1963 for France due to a miner strike, May–July 1968 for France due to the May ’68 uprising and March–June 2011 for Japan due to an earthquake.

**Table 1:** Summary statistics of the monthly growth rates of industrial production for the G7 (in %), 1961:02–2013:05

	N	Median	Mean	Sta. Dev.	Min	Max
CAN	628	0.19	0.24	1.10	-3.76	4.13
FR	628	0.04	0.14	1.35	-4.70	5.66
GER	628	0.23	0.20	1.75	-9.46	12.31
ITA	628	0.18	0.18	2.16	-14.78	13.51
JAP	628	0.38	0.34	1.54	-8.38	6.60
UK	628	0.10	0.09	1.36	-7.88	9.73
US	628	0.29	0.24	0.76	-4.21	3.09

per month. Also, in terms of standard deviation of growth, Italy is the most volatile country, whereas the United States is the least volatile country. Other properties of the monthly growth rates of industrial production for the G7 is that they are stationary, serially correlated, conditional heteroscedastic and not normally distributed<sup>7</sup>.

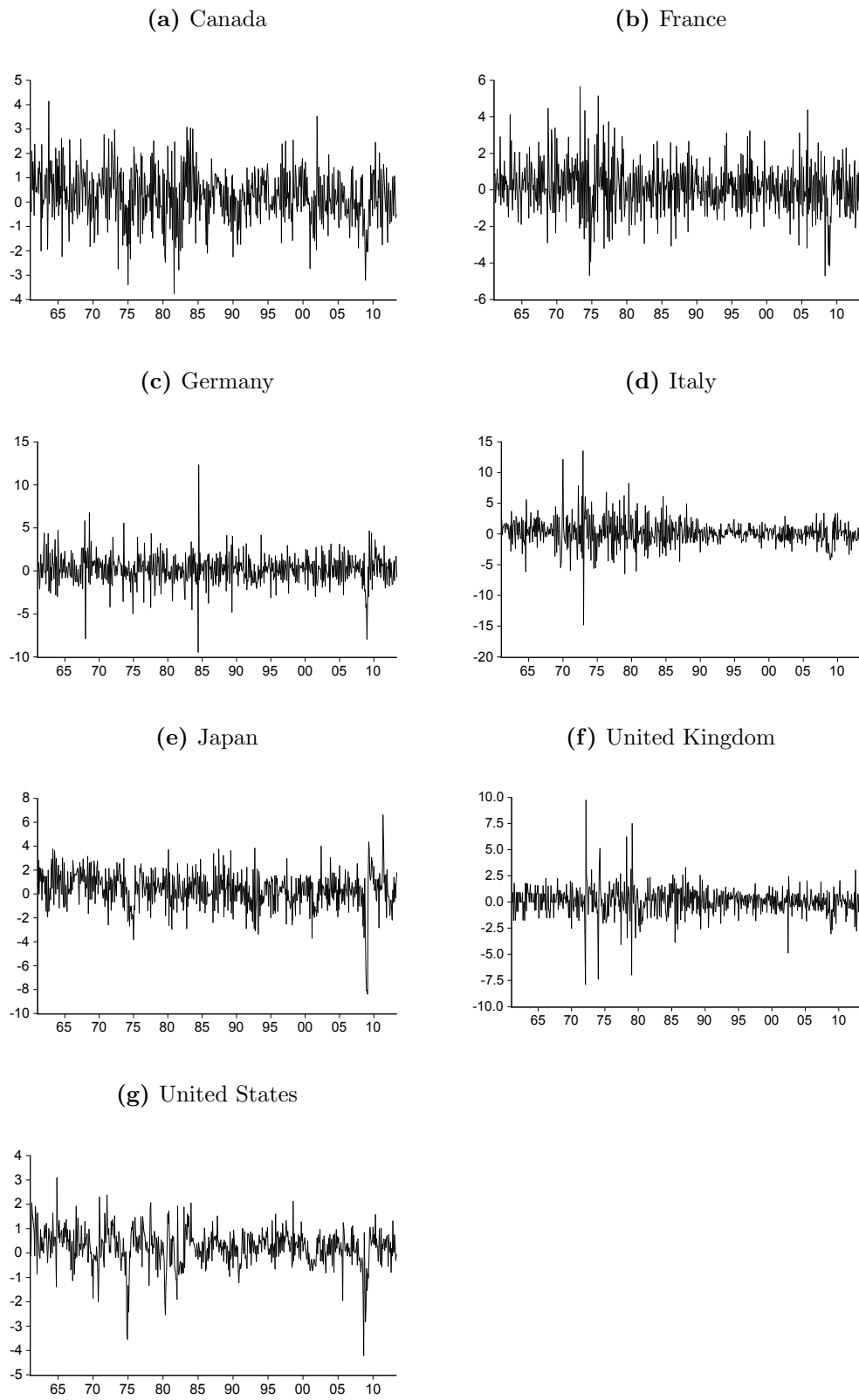
The data used to calculate the fixed trade weights in the definition of the cross-country weighted averages,  $w_{ij}$  in equation 3, is import and export data from the IMF’s Direction of Trade Statistics<sup>8</sup>. As discussed earlier, fixed trade weights are needed in order to calculate the long-run effects of volatility and the great moderation on growth. As the sample period is quite long, however, it is possible that the trade pattern between country  $i$  and  $j$  changes over time. Figures 6-12 in the appendix plots the trade weights over time together with the average trade weight for each G7 country with the other members of the G7 and shows that for some countries this is indeed the case. Therefore, I also estimate the model in (1)-(3) with time-varying trade weights as a robustness check<sup>9</sup>.

<sup>7</sup>The unit root test is the Elliott, Rothenberg, and Stock (1996) test. This test has more power than the original augmented Dickey-Fuller unit root test. This test has also another advantage. When a break is present in the data, the Dickey-Fuller test is biased towards non-rejection of a unit root (Perron, 1989). The ERS DF-GLS unit root test, in contrast, is asymptotically robust to level breaks (Elliott et al., 1996, p. 816). To test for serial correlation and conditional heteroscedasticity in the data I calculate the Ljung-Box Q-statistics for the data and the Ljung-Box Q-statistics for the squared data, respectively. To test if the data is normally distributed I use the Jarque-Bera test statistic. Table 8 in the appendix presents all the results of these tests.

<sup>8</sup>The trade weight matrix can be found in Table 9 in the appendix. The cross-country weighted averages of output growth are plotted in Figure 5 in the appendix.

<sup>9</sup>As the data of the trade flows is yearly, the trade weight can only change every year.

**Figure 1:** Monthly growth rates of industrial production for the G7, 1961:02–2013:05



### 3.2 Exogeneity test of $\Delta y_{it}^*$

In order to get valid estimates for the model in (1)-(3), an important assumption is that the contemporaneous cross-country weighted average of growth can be taken to be exogenous, i.e. the country specific growth rate at time  $t$ ,  $\Delta y_{it}$ , does not affect the cross-country weighted average of growth at time  $t$ ,  $\Delta y_{it}^*$ . In statistical terms this assumption implies that  $\Delta y_{it}^*$  and the error term are uncorrelated, i.e.  $COV(\Delta y_{it}^*, \varepsilon_{it}) = 0$ .

One justification for assuming that the contemporaneous cross-country weighted average is exogenous is when a country is small. This is related to the small country assumption in open macroeconomics where, for example, a small country cannot affect the world price of a commodity. In this study, however, where  $N = 7$ , this argument is implausible and so it is important to test the assumption statistically.

A standard procedure to test if a certain variable is exogenous, proposed by Hausman (1978), is to test if OLS and 2SLS estimates differ significantly. If they do not, then the variable is indeed exogenous. As GARCH models cannot be estimated with OLS and 2SLS, I need to exclude the variance equation and the conditional standard deviation in the mean equation of the model in (1)-(3) in order to apply the test. Using this adjusted model, I can test if the error term is correlated with  $\Delta y_{it}^*$  through a regression test based on the reduced form of  $\Delta y_{it}^*$  and involves two steps. First, I estimate the reduced form of  $\Delta y_{it}^*$  and save the residuals. The reduced form of  $\Delta y_{it}^*$  includes all the variables as in the adjusted model plus an extra autoregressive and cross-country weighted average lag. Then, I add these residuals to the adjusted model and test if the coefficient on these residuals, denoted as  $\zeta$ , is significant. If they are not, then this implies  $\Delta y_{it}^*$  is exogenous.

Table 2 shows the results of the above procedure. For all G7 countries, the null hypothesis that the estimated coefficient  $\zeta$  is 0 cannot be rejected at the 5% level and so this gives evidence that  $\Delta y_{it}^*$  can be treated as exogenous.

**Table 2:** Exogeneity test of  $\Delta y_{it}^*$ 

	CAN	FR	GER	ITA	JAP	UK	US
$\zeta$	-1.303 (-1.73)	1.102 (0.82)	-0.702 (-0.99)	-0.935 (-0.64)	-1.467 (-0.48)	-0.062 (-0.19)	0.514 (0.76)

*Note:* -  $t$ -statistics based on White heteroskedasticity-consistent standard errors are in brackets.

### 3.3 Determining potential structural breaks in the mean and variance of growth

In Section 2, I discussed the importance of allowing for structural breaks in the mean and variance of output growth and therefore include shift dummies in the model of (1)-(3). The exact number of breaks and break dates, however, are unknown. To obtain the number of breaks and the exact break dates, I use a procedure developed by Bai and Perron (1998) and Bai and Perron (2003). Bai and Perron (1998) develop various procedures to identify multiple structural breaks and Bai and Perron (2003) discuss practical issues for the empirical application of these procedures. The potential break dates in the mean and the variance of output growth found here are then used to create shift dummies and these are then included in the Global GARCH-M model.

The particular procedure used in this study to determine structural breaks in the mean and variance of the output growth rate of country  $i$  is based on Bai and Perron (2003, p.15-16). To find breaks in the mean of a series  $x_t$ , they recommend to first regress  $x_t$  on a constant and accounting for potential serial correlation via non-parametric adjustment<sup>10</sup>. Based on this regression, the  $UDmax$  and  $WDmax$  test statistics are used to see if at least one break is present. If the  $UDmax$  test statistic, the  $WDmax$  test statistic or both are larger than the relevant critical value, then we can reject the null hypothesis of 0 breaks in favour of at least one

<sup>10</sup>The non-parametric adjustment is to estimate the model using a quadratic spectral kernel based HAC covariance estimation using prewhitened residuals. The kernel bandwidth is determined automatically using the Andrews AR(1) method.

break. If there is evidence of at least one break, then the  $\text{Sup}F(\ell+1|\ell)$  statistic, where we test the null hypothesis of  $\ell+1$  breaks given  $\ell$  breaks, is used sequentially to determine higher order breaks where the maximum number of possible breaks is 5<sup>1112</sup>.

The approach to find breaks in the variance of a series  $x_t$ , following Herrera and Pesavento (2005) and Fang and Miller (2009), is to first regress  $x_t$  on a constant and on the mean break dummy or dummies found previously, saving the estimated residuals, say  $\hat{\mu}_{it}$ , and then using the  $UDmax$ ,  $WDmax$  and  $\text{Sup}F(\ell+1|\ell)$  statistics in the same way as discussed above based on the regression of the absolute values of  $\hat{\mu}_{it}$  on a constant.

To find breaks in the mean and variance of output growth of country  $i$ , I apply the above general method but adjust it to account for cross-country interactions. In particular, I do not apply the procedure directly on the output growth rates, but on the growth rates where the effects of cross-country interactions are taken out. To find the breaks in the mean of output growth of country  $i$ , I first find the filtered growth rates by regressing the output growth rate of country  $i$  on a constant and cross-country weighted averages of growth, i.e.

$$\Delta y_{it} = c_{i0} + \sum_{k=0}^s \beta_{ik} \Delta y_{it-k}^* + \xi_{it} \quad (18)$$

where  $\xi_{it}$  is the error term. The inclusion of the contemporaneous and the number of lagged cross-country weighted averages is based on the Akaike Information Criterion (AIC) and I allow for up to six lags. I then calculate the  $UDmax$ ,  $WDmax$  and  $\text{Sup}F(\ell+1|\ell)$  statistics based on the regression of the filtered output growth rates,

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<sup>11</sup>This procedure allows for serial correlation and different variances of the residuals across regimes.

<sup>12</sup>Throughout the procedure I use a trimming of 0.15. The sample period in this study is 1961:02–2013:05 and so there are 628 observation. Thus a trimming of 0.15 implies that each segment has a minimum of 94 observations, i.e. 7.85 years.

i.e. the estimated residuals of equation (18),  $\hat{\xi}_{it}$ , on a constant, i.e.

$$\hat{\xi}_{it} = \alpha_{i0} + e_{it} \quad (19)$$

where  $e_{it}$  is the error term. Note that the above procedure does not allow for volatility to effect the growth rate and therefore it is possible for the procedure to interpret a change in volatility as a break. It is, however, better to find too many breaks here, and to include potentially insignificant break dummies in the Global GARCH-M model, than to miss breaks that should be included.

To find break in the variance of the output growth rate of country  $i$ , I first regress  $\Delta y_{it}$  on a constant, the mean break dummy or dummies found previously and the cross-country weighted averages of growth to filter out the effects of cross-country interactions, i.e.

$$\Delta y_{it} = c_{i0} + \sum_{k=1}^l c_{ik} DM_{ikt} + \sum_{k=0}^s \beta_{ik} \Delta y_{it-k}^* + \mu_{it} \quad (20)$$

where  $\mu_{it}$  is the error term<sup>13</sup>. Then, I regress the absolute values of the estimated residuals of equation (20),  $\hat{\mu}_{it}$ , on a constant, i.e.

$$|\hat{\mu}_{it}| = \alpha_{i0} + \nu_{it} \quad (21)$$

where  $\nu_{it}$  is the error term and I calculate the  $UDmax$ ,  $WDmax$  and  $SupF(\ell+1|\ell)$  statistics based on this regression in order to find the number of breaks and the break dates for the variance of output growth of country  $i$ .

Table 3 shows the results of the break detection procedure discussed above, where panel A and B show the results for the mean and variance of output growth, respectively. The emboldened figures are statistics that are larger than their respective critical value. For the breaks in the mean of output growth, the results indicate that

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<sup>13</sup>Here too, the inclusion of the contemporaneous and the number of lagged cross-country weighted averages is based on the AIC and I allow for up to six lags

**Table 3:** Results of the break detection procedure

Panel A: Breaks in the mean								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	4.96	3.48	4.14	<b>7.47</b>	<b>19.99</b>	6.00	<b>13.91</b>	7.46
$WDmax$	5.03	3.97	5.41	7.47	<b>22.40</b>	6.00	<b>15.59</b>	8.20
$SupF(2 1)$	1.88	2.19	0.76	1.61	3.94	2.34	<b>20.63</b>	8.51
$SupF(3 2)$	4.50	2.05	1.38	4.55	<b>10.19</b>	1.67	1.33	9.41
$SupF(4 3)$	0.66	1.11	2.09	1.35	4.98	1.15	0.36	10.04
$SupF(5 4)$	0.00	0.16	0.00	1.61	0.00	0.00	0.00	10.58
Break 1				1996:01	1971:01		1991:04	
Break 2					1991:06		1999:06	
Break 3					2001:12			
Panel B: Breaks in the variance								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	<b>17.62</b>	<b>9.04</b>	4.91	<b>45.84</b>	1.99	<b>23.03</b>	<b>17.63</b>	7.46
$WDmax$	<b>17.62</b>	<b>11.94</b>	5.42	<b>45.84</b>	3.17	<b>24.75</b>	<b>17.63</b>	8.20
$SupF(2 1)$	7.96	<b>11.47</b>	3.78	<b>13.22</b>	1.58	<b>11.73</b>	3.43	8.51
$SupF(3 2)$	6.94	7.44	1.09	2.36	1.39	1.73	1.88	9.41
$SupF(4 3)$	4.86	1.76	0.66	9.78	2.39	5.44	1.12	10.04
$SupF(5 4)$	0.00	0.16	0.00	0.00	0.00	0.00	0.00	10.58
Break 1	1984:11	1973:04		1969:08		1972:01	1984:02	
Break 2		1981:04		1985:12		1987:08		

Note: - C.V. stands for the 5% critical value.

there is at least one break for Italy, Japan and the United States. As there is at least one level break for these countries, we also look at the  $SupF(\ell+1|\ell)$  statistics sequentially. This procedure indicates that Italy experienced a break in January 1996, Japan in January 1971, June 1991 and December 2001 and the United states in April 1991 and June 1999.

The  $UDmax$  and the  $WDmax$  statistics for the breaks in the variance of output growth show that there is at least one break for Canada, France, Italy, the United Kingdom and the United States. Two countries, namely Germany and Japan, did not experience any break in the variance. For the countries with at least one break the  $SupF(\ell+1|\ell)$  statistics is again used sequentially to determine

higher order breaks. This procedure shows that Canada has a break in November 1984, France in April 1973 and April 1981, Italy in August 1969 and December 1985, the United Kingdom in January 1972 and August 1987 and the United States in February 1984.

### 3.4 Model selection procedure and estimation method

In this subsection, I will discuss the relative sophisticated specification search to find the most parsimonious model of (1)-(3) and the various issues that complicate this process. I will first discuss the possibility of the Zero-Information-Limit Condition (ZILC) highlighted by Nelson and Startz (2007) and then I will review three further important issues related to estimating GARCH model. I end this subsection by discussing the estimation method.

Nelson and Startz (2007) noted that in many econometric models the asymptotic variance of a parameter estimate depends on the value of another structural parameter and that for estimates of the structural parameter in a particular range the asymptotic variance of the variable of interest is very large and the model is weakly identified. More formally, Nelson and Startz (2007, p.49) argue that ZILC holds for an estimator  $\hat{\theta}$  if there is a value of  $v$ , say  $v_0$ , such that  $\lim_{v \rightarrow v_0} I_{\hat{\theta}} = 0$ , where  $I_{\hat{\theta}}$  is the inverse of the variance of  $\hat{\theta}$ . Nelson and Startz (2007) introduce ZILC as a way to identify such models where the above leads to spurious inference.

Ma, Nelson, and Startz (2007) show that ZILC can hold for GARCH models. They show that if the true ARCH effect,  $\eta_i$  in (2), is small, then the GARCH(1,1) model is weakly identified. The effect of this is that the GARCH coefficient,  $\gamma_i$  in (2), is biased upward and the corresponding standard error is too small. Thus the results point to persistence of the conditional variance where in fact this is not the case. Ma, Nelson, and Startz (2007, p16-17) also propose a procedure to detect ZILC. To check for ZILC in the GARCH(1,1) model, the implied autocorrelation function of the conditional variance from the GARCH(1,1) should be compared with

the one implied by an ARCH( $q$ ) model. If they differ a lot, then this is evidence that ZILC holds. Alternatively, if the estimated conditional variance of the GARCH(1,1) and an ARCH( $q$ ) models are very different, then this also gives evidence for ZILC. Ma, Nelson, and Startz (2007) propose to model the variance as an ARCH model if ZILC is detected and so in the case of ZILC, I replace equation (2) with

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt} + \eta_i \varepsilon_{it-1}^2. \quad (22)$$

There are, however, three other issues that also need to be taken into account in order to find the most parsimonious model for each country. *First*, ignoring the possibility of a nonnormal error process leads to inconsistent standard errors<sup>14</sup>. As there is evidence that the G7 output growth rates are not normally distributed, but closer to being  $t$ -distributed, this is a relevant issue. To account for this, I estimate the models assuming a normal and assuming a  $t$ -distribution as error distribution. This procedure makes it possible to test which distribution fits the data better. *Second*, the Hessian of the log-likelihood function of the GARCH-M model is not block diagonal and so the mean and variance parameters are correlated. Therefore, all the parameters need to be estimated simultaneously. And *third*, the models should be well-specified. The models are well-specified if the mean and variance of the standardized residuals are equal to 0 and 1, respectively, the distribution of the residuals corresponds to the one assumed in the estimation procedure and there is no evidence of serial correlation and conditional heteroscedasticity in the standardized residuals.

Taking the above issues into account, the most parsimonious model for each country is determined by estimating up to a GARCH(1,1) model with the  $t$ -distribution as error distribution and up to six lags of all the possible lagged variables in the

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<sup>14</sup>Previous studies remedied this by estimating GARCH models with the normal distribution but applying the consistent variance-covariance estimator developed by Bollerslev and Wooldridge (1992).

mean model. The well-specified model that maximises the Akaike Information Criteria (AIC) where ZILC does not holds is then picked as the best representation of the data.

The models are estimated with maximum likelihood using the Marquardt optimization algorithm. As with any iterative process, however, the algorithm could stop at a local maximum instead of the global maximum. To counter this problem, all the models are estimated with various initial values and choosing the initial value with the largest log likelihood<sup>15</sup>.

### 3.5 Estimation results

After examining the data, showing that  $\Delta y_{it}^*$  is found to be exogenous, finding the break dates in the mean and variance of the output growth rates and discussing various issues that complicate the specification search, I can finally present the estimation results of the model developed in (1)-(3) and the evaluation of the total long-run effect of volatility and the great moderation in (17).

Table 4 shows the estimation results of the well-specified model of (1)-(3) for each G7 country where panel A and B show the estimates for the mean and variance equation, respectively<sup>16</sup>. A first thing to note is that the cross-country weighted averages of growth show up significantly across all the equations. This implies that the model captures some sophisticated dynamics with lagged own country growth and cross-country weighted averages of growth.

Looking at the estimates on volatility,  $\lambda$ , we see that the impact effect of volatility is positive for all countries and range from 0.061 for Italy to 0.679 for Canada. The average impact effect is equal to 0.364. The impact effect, however, is only statistically significant at conventional levels in four of the G7 countries, namely

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<sup>15</sup>I use 10 different starting values. In particular, I estimate the models with the estimates of the OLS regression for the mean equation as starting values and various fractions of these OLS estimates. The fractions of the OLS estimates are 0.9, 0.8, ..., 0.1.

<sup>16</sup>A series of residual diagnostic tests are shown in Table 10 in the appendix and Figure 13 in the appendix for plots of the distribution of the residuals together with the theoretical distribution assumed in the estimation process.

**Table 4:** Estimation results

Panel A: Mean Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$c_0$	-0.594** (0.243)	-0.366 (0.225)	-0.160 (0.347)	0.035 (0.189)	-0.250 (0.338)	-0.241* (0.129)	-0.240** (0.113)
$c_1$				-0.337*** (0.117) [1996:01]	-0.331** (0.135) [1971:01]		0.247*** (0.057) [1991:04]
$c_2$					-0.370*** (0.126) [1991:06]		-0.181*** (0.054) [1999:06]
$c_3$					0.312** (0.141) [2001:12]		
$\lambda$	0.679** (0.271)	0.323 (0.224)	0.189 (0.246)	0.061 (0.122)	0.578** (0.256)	0.236* (0.135)	0.480*** (0.184)
$\beta_0$	0.586*** (0.059)	0.520*** (0.043)	0.475*** (0.066)	0.538*** (0.064)	0.340*** (0.077)	0.423*** (0.050)	0.269*** (0.026)
$\beta_1$	0.195** (0.078)	0.353*** (0.052)	0.327*** (0.070)	0.484*** (0.071)	0.215*** (0.083)	0.251*** (0.050)	0.150*** (0.027)
$\beta_2$	0.054 (0.071)	0.367*** (0.054)	0.065 (0.070)	0.282*** (0.069)	0.072 (0.080)		0.029 (0.030)
$\beta_3$	0.085 (0.068)	0.259*** (0.055)	0.189*** (0.062)	0.151** (0.072)	0.097 (0.082)		-0.042 (0.032)
$\beta_4$	0.109* (0.066)	0.056 (0.053)	0.106* (0.063)	0.042 (0.072)			0.091*** (0.029)
$\beta_5$		0.087* (0.051)		0.134* (0.070)			
$\beta_6$				-0.147** (0.066)			
$\phi_1$	-0.240*** (0.045)	-0.494*** (0.043)	-0.355*** (0.045)	-0.380*** (0.044)	-0.277*** (0.046)	-0.275*** (0.042)	0.026 (0.044)
$\phi_2$	-0.054 (0.044)	-0.268*** (0.047)	-0.143*** (0.034)	-0.139*** (0.048)	0.126*** (0.041)	-0.127*** (0.033)	0.088** (0.036)
$\phi_3$	0.133*** (0.039)	-0.177*** (0.047)		-0.040 (0.044)	0.272*** (0.039)		0.104*** (0.032)
$\phi_4$		-0.153*** (0.045)		-0.027 (0.044)	0.116*** (0.036)		0.039 (0.035)
$\phi_5$		-0.126*** (0.044)		0.015 (0.043)			-0.062** (0.031)
$\phi_6$		0.092*** (0.035)		0.101*** (0.038)			

**Table 4:** Continued

Panel B: Variance Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$\alpha_0$	1.093*** (0.097)	1.286*** (0.180)	1.554*** (0.159)	1.087** (0.451)	0.842*** (0.148)	0.819*** (0.182)	0.375*** (0.054)
$\alpha_1$	-0.509*** (0.106) [1984:11]	1.139** (0.486) [1973:04]		1.190** (0.583) [1969:08]		1.286*** (0.358) [1972:01]	-0.204*** (0.054) [1984:02]
$\alpha_2$		-1.639*** (0.473) [1981:04]		-1.873*** (0.705) [1985:12]		-1.542*** (0.329) [1987:08]	
$\eta$	0.095** (0.047)	0.061 (0.045)	0.329*** (0.094)	0.199*** (0.069)	0.266*** (0.075)	0.212*** (0.072)	0.347*** (0.089)
$\gamma$				0.476*** (0.148)	0.224** (0.092)		
$\nu$		14.132 (8.623)	6.267*** (1.174)	10.532** (4.797)	7.436*** (2.162)	6.793*** (1.557)	5.857*** (1.333)

Notes: - The model is:

$$\Delta y_{it} = c_{i0} + \sum_{k=1}^l c_{ik} DM_{ikt} + \lambda_i \sigma_{it} + \sum_{k=0}^s \beta_{ik} \Delta y_{it-k}^* + \sum_{k=1}^p \phi_{ik} \Delta y_{it-k} + \varepsilon_{it}$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt} + \eta_i \varepsilon_{it-1}^2 + \gamma_i \sigma_{it-1}^2$$

$$\Delta y_{it}^* = \sum_{j=1}^7 w_{ij} \Delta y_{jt}, \quad \sum_{j=1}^7 w_{ij} = 1 \quad \text{and} \quad w_{ii} = 0,$$

where  $\Delta y_{it-k}^*$  are the cross-country weighted of growth and  $w_{ij}$  is the average share of total trade of country  $i$  with country  $j$  and total trade is defined as the sum of exports and imports between country  $i$  and the G7 other countries.

- $DM_{ikt}$  and  $DV_{ikt}$  are country-specific shift dummies in the mean and variance equation, respectively. The break dates are in square brackets.
- All models are estimated with the  $t$ -distribution as error distribution where  $\nu$  are the estimated degrees of freedom, except for Canada.
- Standard errors are in brackets and \*, \*\*, \*\*\* denotes statistical significance at the 10%, 5% and 1% level, respectively.

**Table 5:** Long-run elasticities

	CAN	FR	GER	ITA	JAP	UK	US
$\lambda^{TLR}$	2.078*** (0.538)	1.173*** (0.318)	1.195*** (0.388)	1.348*** (0.390)	2.287*** (0.701)	0.897*** (0.236)	1.722*** (0.486)
$\alpha_{GM}^{TLR}$	-0.361*** (0.097)	-0.268*** (0.096)	-0.198*** (0.062)	-0.254*** (0.086)	-0.245*** (0.083)	-0.217*** (0.075)	-0.259*** (0.075)

*Notes:* - The standard errors for the total long-run effects are obtained with the delta method.

Canada, Japan, the United Kingdom and the United States. For these countries, the effect is also economically important. A one unit increase in volatility in the United States at time  $t$ , for example, increases output growth in the United States at time  $t$  with 0.480 percentage points.

The total long-run effects of volatility,  $\lambda^{TLR}$ , shown in Table 5, are statistical significant at the 1% level for all G7 countries<sup>17</sup>. The elasticities range from 0.897 for the United kingdom to 2.287 for Japan, which are all larger than the corresponding impact effect. The average long-run effect of total volatility for the G7 is 1.529 which is about 4 times larger than the average impact effect. These effects are also economically significant. A one unit increase in volatility in all G7 countries increases the growth rate in the United States, for example, by 1.722 percentage points in the long-run.

The coefficients on the included mean shift dummies all show up statistically significant at the 5% level. By estimating the effect of the shift dummies on output growth, we can also find out the sign of the shift. The results indicate that Italy experienced a negative shift in the mean of output growth in January 1996. Japan experienced three breaks in the mean of output growth. A negative one in January 1971, another negative one in June 1991 and a positive one in December 2001. Finally, the United States experienced a positive break in April 1991 and a negative one in

<sup>17</sup>The standard errors for the long-run effect of total volatility are obtained with the delta method.

June 1999.

Panel B of Table 4 shows that the variance model is the ARCH(1) model for all countries, except for Italy and Japan where the variance model is a GARCH(1,1) model<sup>18</sup>. Also, all models are estimated with the  $t$ -distribution as error distribution, except for Canada.

As with the mean shift dummies, all the variance shift dummies included in the models are statistically significant at the 5% level. The results show that Italy and the United Kingdom experienced an increase in volatility around 1970. The results also show that five countries experience a negative shift in volatility, i.e. a great moderation. These countries are Canada, France, Italy, the United Kingdom and the United States. The negative shifts happened around 1985 for all these countries, except for France where the break happened around 1980. The effect of the great moderation,  $\alpha_{i_{GM}}$ , corresponds to the first variance dummy for Canada and the United States. For France, Italy and the United Kingdom the great moderation corresponds to the second variance dummy. The shifts in volatility can also be seen in Figure 2 which plots the estimated conditional standard deviation for all G7 countries over time.

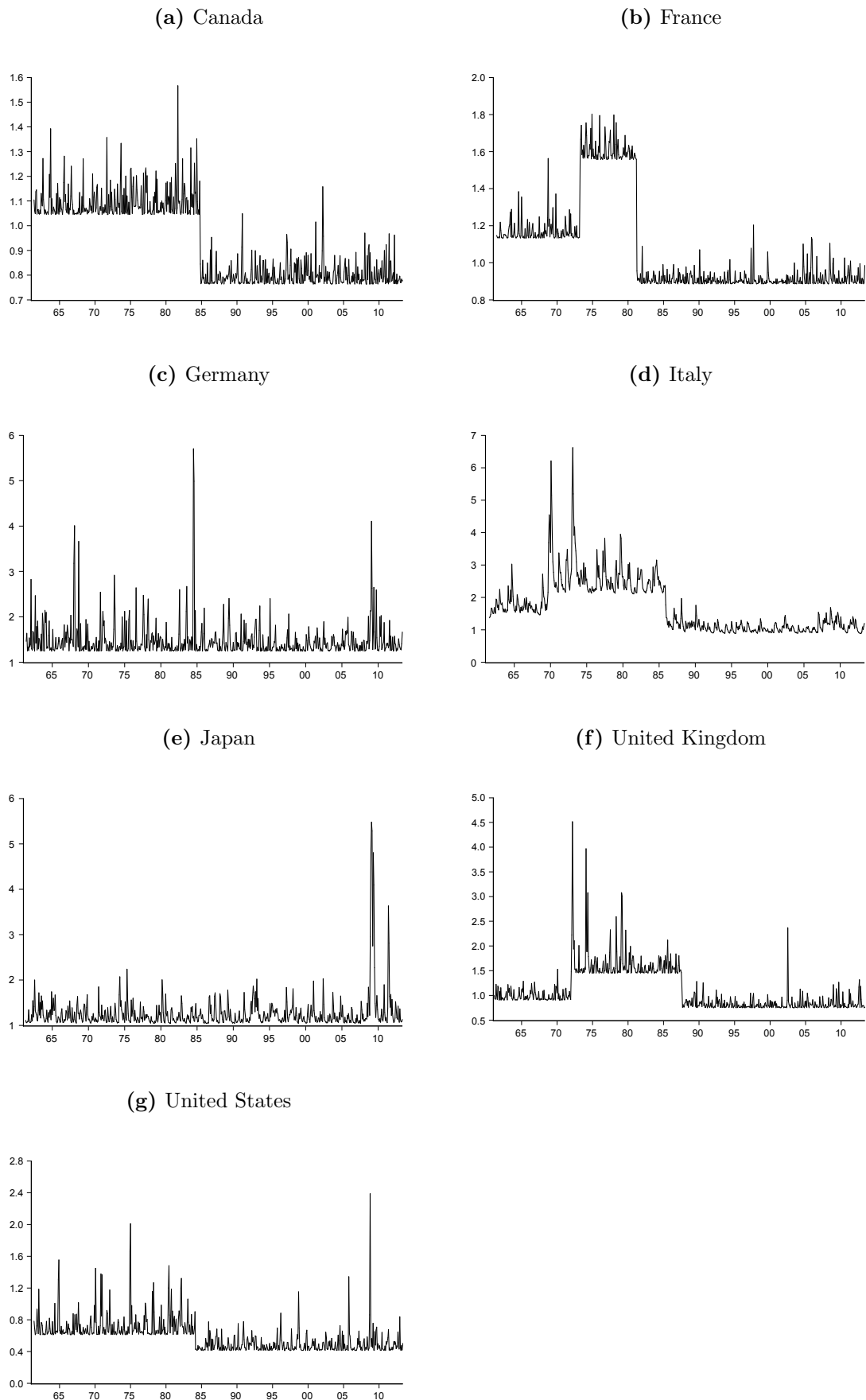
As the effect of volatility on growth is positive and the great moderation signifies a drop in volatility, the great moderations has a negative effect on growth. And because the Global GARCH-M model takes cross-country interactions into account, this also holds for countries that did not experience a great moderation themselves. Table 5 shows that the total long-run effect of the great moderation on growth is statistically significant at the 1% level in all G7 countries<sup>19</sup>. The effects range from -0.198 for Germany to -0.361 for Canada. The average effect is equal to -0.257.

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<sup>18</sup>Figure 14 in the appendix shows that for the model of Italy and Japan there is no evidence for ZILC as the implied autocorrelation function of the conditional variance and the estimated conditional standard deviation for the GARCH(1,1) and ARCH( $q$ ) model are not different. The number of lags  $q$  in the ARCH( $q$ ) model is determined by estimating up to lag 6 and picking the highest lag order while ignoring models where one or more coefficients on the lagged residuals have a negative estimate.

<sup>19</sup>The standard errors are again obtained with the delta method.

**Figure 2:** Estimated conditional standard deviation for the G7, 1961:02–2013:05



These effects on growth are also economically significant. For the United Kingdom, for example, a once and for all drop in the volatility experienced in all G7 countries at the same time decreases the growth rate for the United Kingdom with 0.217 percentage points.

As shown in Section 2, the trade weights need to be constant over time in order to calculate the long-run effects of volatility and the great moderation on growth. As discussed earlier, however, the trade weights are not constant (see Figure 6-12 in the appendix). Therefore, as a robustness check, I also estimate the model where the cross-country weighted averages are calculated using time-varying weights as discussed in Subsection 3.1. This exercise shows that the estimates do not change much compared to the estimates obtained using constant weights<sup>20</sup>. The average impact effect of volatility is now equal to 0.357 compared to 0.364 when using fixed trade weights. Also, there is evidence for a great moderation in the same countries with similar break dates as when fixed trade weights are used.

## 4 Testing the importance of cross-country interactions

One of the key findings in the previous section is the apparent accumulation of the effects of volatility on growth as output effects spillover across countries. This section focuses on the importance of the cross-country interactions in the relationship between volatility and growth with a simulation exercise. In particular, this section considers the implications for the volatility-growth relationship under the null hypothesis that cross-country interactions are unimportant for measuring the effect of volatility on growth.

The simulation exercise is constructed as follows. For each G7 country, I first estimate a model under the null hypothesis of no cross-country interactions, i.e.

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<sup>20</sup>Figure 11-14 and Table 15-16 in the appendix present all the results related to this exercise.

the model does not include cross-country weighted averages of growth<sup>21</sup>. Thus, I estimate a model of the form

$$\Delta y_{it} = \tilde{c}_{i0} + \tilde{c}_{i1} \tilde{D}M_{it} + \tilde{\lambda}_i \tilde{\sigma}_{it} + \sum_{k=1}^p \tilde{\phi}_{ik} \Delta y_{it-k} + \tilde{\varepsilon}_{it} \quad (23)$$

$$\tilde{\sigma}_{it}^2 = \tilde{\alpha}_{i0} + \sum_{k=1}^f \tilde{\alpha}_{ik} \tilde{D}V_{ikt} + \tilde{\eta}_i \tilde{\varepsilon}_{it-1}^2 + \tilde{\gamma}_i \tilde{\sigma}_{it-1}^2 \quad (24)$$

The break detection procedure is applied in the same way as for the model with country interactions, except that there are no cross-country weighted averages in the relevant equation, i.e. setting the  $\beta_{ik}$ 's in (18) and (20) in Subsection 3.3 to 0. The estimation and model selection procedure is also the same as for the model with cross-country interactions (see Subsection 3.4). The model and the estimated parameters obtained under the null of no cross-country interactions are then used to generate 5000 simulated series for each country.

The simulated series are used to gauge the size of the total volatility measure in the original estimation exercise. For each of the 5000 simulated series, I estimate a corresponding model with the cross-country weighted averages, as in equation (1)-(3), and a corresponding model without the cross-country interactions, as in equation (23)-(24). The same estimation method, model selection procedure and break detection procedure is used for the simulation exercise as in the respective original estimation exercise. For some simulated series, however, the model is not well-specified or there is evidence of ZILC and so I delete these simulated series from the analysis.

The next step is to calculate, for each simulation, the difference between the estimated effect of volatility in the model with and the model without cross-country interactions providing a measure of the contribution of cross-country feedbacks. This is done for the impact effect of volatility,  $\lambda_{(s)} - \tilde{\lambda}_{(s)}$ , and the total long-run effect of volatility,  $\lambda_{(s)}^{TLR} - \tilde{\lambda}_{(s)}^{TLR}$ , where  $s = 1, \dots, 5000$  is the number of the simulation.

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<sup>21</sup>See Table 15–18 and Figure 17–18 for all the results related to this exercise.

**Table 6:** Comparing the long-run parameters

Panel A: Impact effect							
	CAN	FR	GER	ITA	JAP	UK	US
$\lambda - \tilde{\lambda}$	0.531	0.293	0.221	0.203	0.006	0.279	0.249
Rank	3717	3084	3854	3353	2627	3483	2790
Total simulations	3804	3153	3855	3369	3334	3489	2856
P-value	0.0229	0.0219	0.0003	0.0047	0.2121	0.0017	0.0231
Panel B: Total long-run effect							
	CAN	FR	GER	ITA	JAP	UK	US
$\lambda^{TLR} - \tilde{\lambda}^{TLR}$	2.016	1.281	1.437	1.719	1.438	1.036	1.396
Rank	3786	3151	3854	3369	3331	3489	2855
Total simulations	3804	3153	3855	3369	3334	3489	2856
P-value	0.0047	0.0006	0.0003	0.0000	0.0009	0.0000	0.0004

Sorting all the differences obtained from the simulations gives the distribution of the cross-country contributions under the null that no cross-country interactions exist in the underlying data generating process. Comparing the contributions found in the original data with the critical value at some significance level with the simulated distribution gives a test of the significance of the cross-country interactions for measuring the effect of volatility. In other words, if the difference implied by the data is extreme compared to the simulated differences, then we can rule out that this is due to chance alone.

Panel A and B of Table 6 show the result for the impact and the total long-run effect of volatility, respectively. The first row of Panel A shows the difference of the estimated impact effect of volatility using the data. The second row shows the place where the estimated difference lies in the sorted vector of cross-country contributions obtained with the simulated data, i.e. the rank. The third row gives the total number of valid simulations. Dividing the rank by the total number of simulations and subtracting this from 1 gives the p-value. The results show that we can reject the null hypothesis that cross-country interactions are not important in measuring the impact effect of volatility at the 5% level for all countries, except for

Japan. Panel B shows the same statistics for the total long-run effect of volatility. Here we can reject the null hypothesis at the 1% for all G7 countries. These results indicate that cross-country interactions are indeed important in measuring the effect of volatility on growth.

## 5 Conclusion

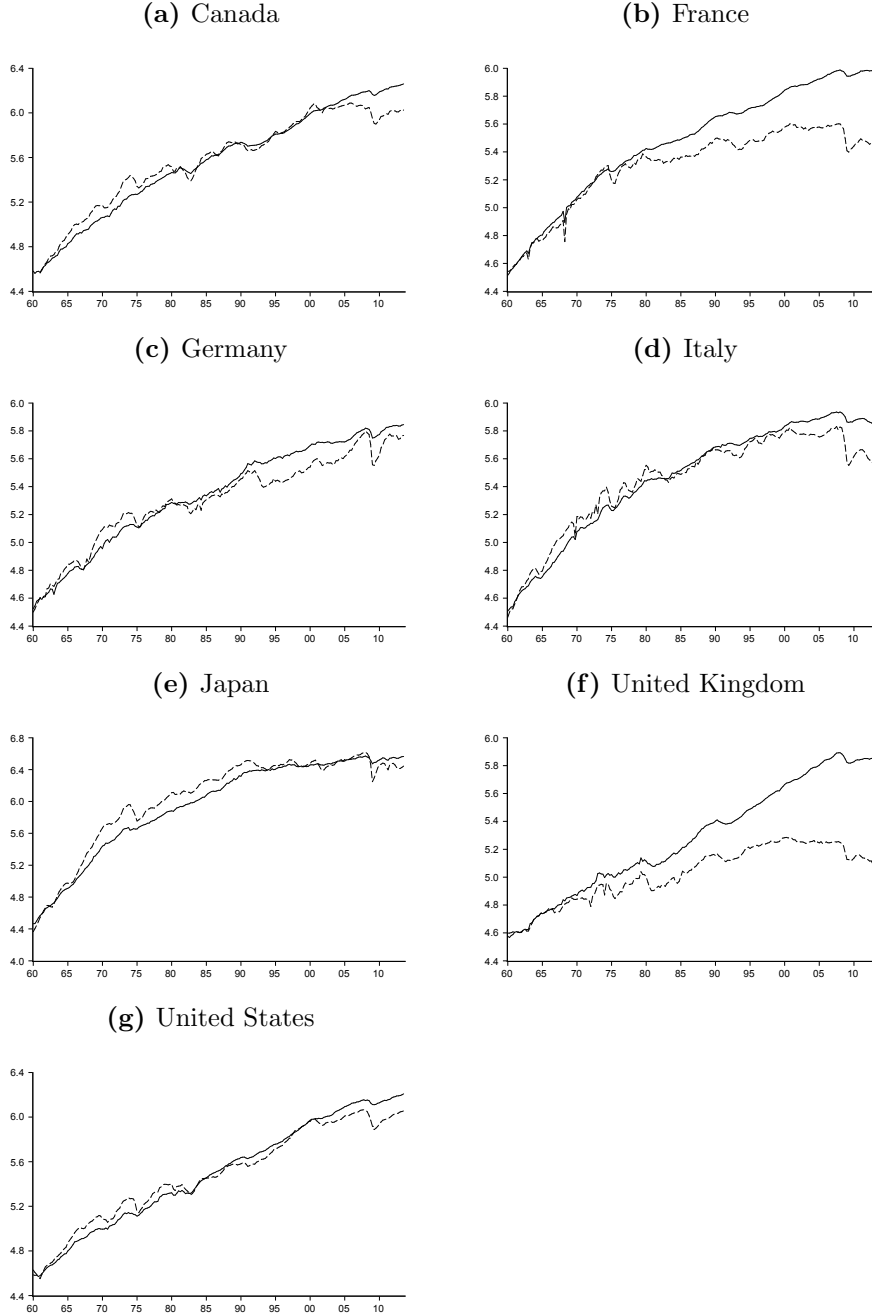
The GARCH-M framework is a widely used tool to uncover the relationship between volatility and growth. Existing studies that apply this framework, however, have a few shortcomings. In particular, no econometric specification applied in the literature accounts simultaneously for cross-country interactions, structural breaks and for heterogeneous effects. This paper developed an econometric model, namely the *Global GARCH-M* model, which is able to incorporate all three issues simultaneously. This is done by estimating a standard univariate GARCH-M model of output growth for each G7 country augmented with cross-country weighted averages of growth to account for cross-country interactions and shift dummies in the mean and variance of growth to account for structural breaks .

This framework is able to answer a number of research questions. First, what is the effect of volatility on growth? The analysis showed that volatility has a positive and statistically significant effect on economic activity in four of the G7 countries, namely Canada, Japan, the United Kingdom and the United States. The total long-run effect of volatility, where all short-run dynamics are allowed to propagate, is found to be positive and statistically significant for all G7 countries. Second, is there evidence of a great moderation and what is its effect on growth if the econometric model takes cross-country interactions into account? A careful structural breaks analysis found evidence for a great moderation in five of the G7 countries, namely Canada, France, Italy, the United Kingdom and the United States. All countries experienced a fall in volatility around 1985, except for France where the

fall happened around 1980. Moreover, the long-run effect of the great moderation on economic activity is found to be negative and statistically significant in all G7 countries. Third, are cross-country interactions important in measuring the effect of volatility on growth? To asses this issue I set up a simulation and found that they are important.

# Appendix

**Figure 3:** Log of IP (dashed) and log of GDP (solid) for the G7, 1961:02–2013:05



*Notes:* Data on industrial production (IP) is the monthly industrial production index over the period 1960m01–2013m09. The data on gross domestic product (GDP) is quarterly GDP over the period 1960Q1–2013Q3. Both series are from the OECD. To get the two series in the same time frame, I transform the monthly IP into quarterly frequencies by taking the average of the respective months. Once the two series have the same frequencies, I transform the GDP series into an index with base year 1961. I also change the base year of the IP series to 1961. The base number is for both series 100.

**Table 7:** Properties of GDP and IPI series

Panel A: Correlation coefficient of GDP and IPI							
	CAN	FR	GER	ITA	JAP	UK	US
Series in levels	0.97	0.92	0.98	0.96	0.98	0.88	0.99
Series in logs	0.99	0.96	0.98	0.98	0.99	0.93	0.99
Series in log differences	0.73	0.89	0.66	0.75	0.64	0.68	0.76
Panel B: Standard deviation of GDP growth and IPI growth							
	CAN	FR	GER	ITA	JAP	UK	US
GDP Growth	0.88	1.16	1.11	1.04	1.33	0.99	0.84
IPI Growth	1.73	2.42	2.07	2.62	2.86	1.68	1.59

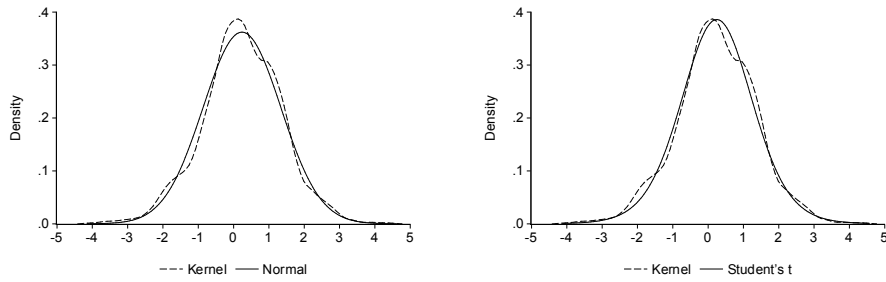
**Table 8:** Unit root, distributional, serial correlation and conditional heteroskedasticity test results for the G7 monthly growth rates of industrial production for the period 1961:02–2013:05

Panel A: Unit root test							
	CAN	FR	GER	ITA	JAP	UK	US
ERS DF-GLS	-8.25	-7.69	-2.45	-8.03	-4.02	-3.08	-6.59
C.V. at 5%	-1.94	-1.94	-1.94	-1.94	-1.94	-1.94	-1.94
Panel B: Distribution							
	CAN	FR	GER	ITA	JAP	UK	US
Skewness	-0.133	0.129	0.058	0.197	-0.856	0.026	-0.899
Kurtosis	3.496	4.391	9.431	10.704	7.538	12.882	7.357
Jarque-Bera	8.28 [0.0159]	52.34 [0.0000]	1082.62 [0.0000]	1556.96 [0.0000]	615.55 [0.0000]	2555.16 [0.0000]	581.39 [0.0000]
Panel C: Tests for serial correlation and conditional heteroscedasticity							
	CAN	FR	GER	ITA	JAP	UK	US
Q(3)	48.23 [0.0000]	59.65 [0.0000]	67.52 [0.0000]	49.14 [0.0000]	92.04 [0.0000]	22.04 [0.0000]	170.51 [0.0000]
Q(9)	81.39 [0.0000]	100.63 [0.0000]	76.37 [0.0000]	89.99 [0.0000]	117.89 [0.0000]	35.94 [0.0000]	250.32 [0.0000]
Q <sup>2</sup> (3)	14.49 [0.0020]	25.11 [0.0000]	87.63 [0.0000]	109.95 [0.0000]	377.60 [0.0000]	109.25 [0.0000]	128.05 [0.0000]
Q <sup>2</sup> (9)	27.68 [0.0010]	66.56 [0.0000]	89.46 [0.0000]	138.60 [0.0000]	399.19 [0.0000]	113.75 [0.0000]	140.66 [0.0000]
ARCH(3)	12.36 [0.0063]	23.11 [0.0000]	104.95 [0.0000]	104.45 [0.0000]	235.24 [0.0000]	114.04 [0.0000]	96.84 [0.0000]
ARCH(9)	21.33 [0.0113]	48.49 [0.0000]	105.51 [0.0000]	114.38 [0.0000]	241.40 [0.0000]	118.48 [0.0000]	100.05 [0.0000]

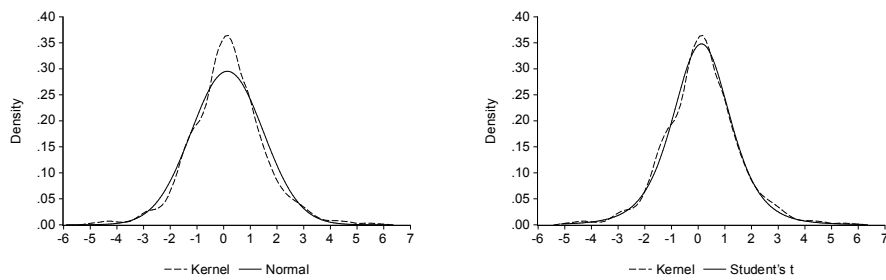
*Notes:* - The unit root test is the Elliott et al. (1996) test.  
- p-values are in square brackets.  
- Q( $p$ ): Ljung-Box Q-statistics at lag  $p$  for the data.  
- Q<sup>2</sup>( $p$ ): Ljung-Box Q-statistics at lag  $p$  for the squared data.  
- ARCH( $p$ ): ARCH LM test at lag  $p$  (Engle, 1982).

**Figure 4:** Kernel estimates of the G7 monthly growth rates of industrial production over the period 1961:02–2013:05 compared with the theoretical normal and student's  $t$ -distribution

*Canada*



*France*



*Germany*

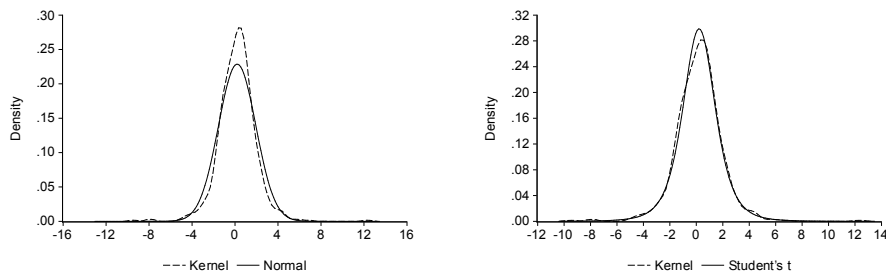
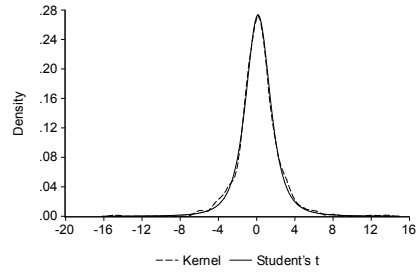
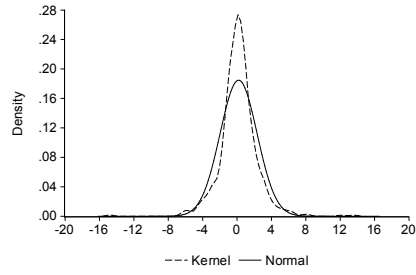
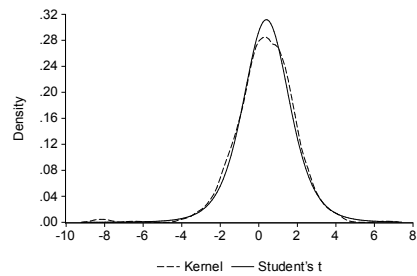
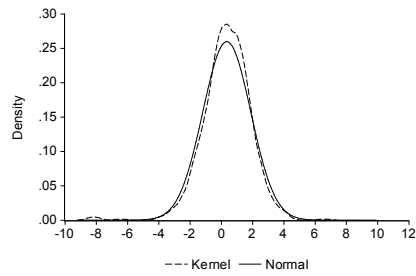


Figure 4: Continued

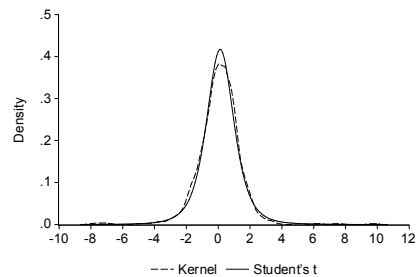
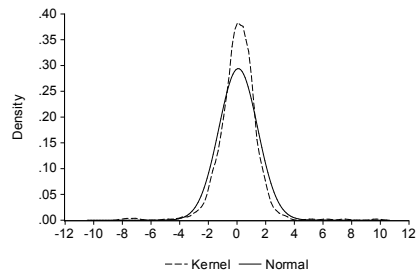
*Italy*



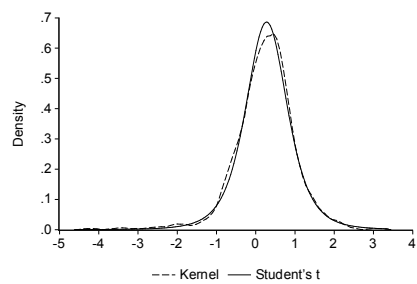
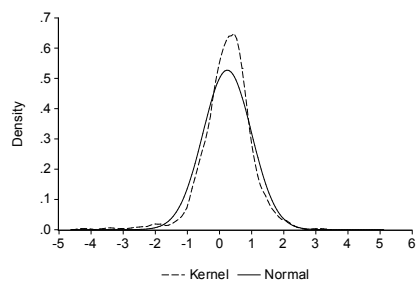
*Japan*



*United Kingdom*



*United States*

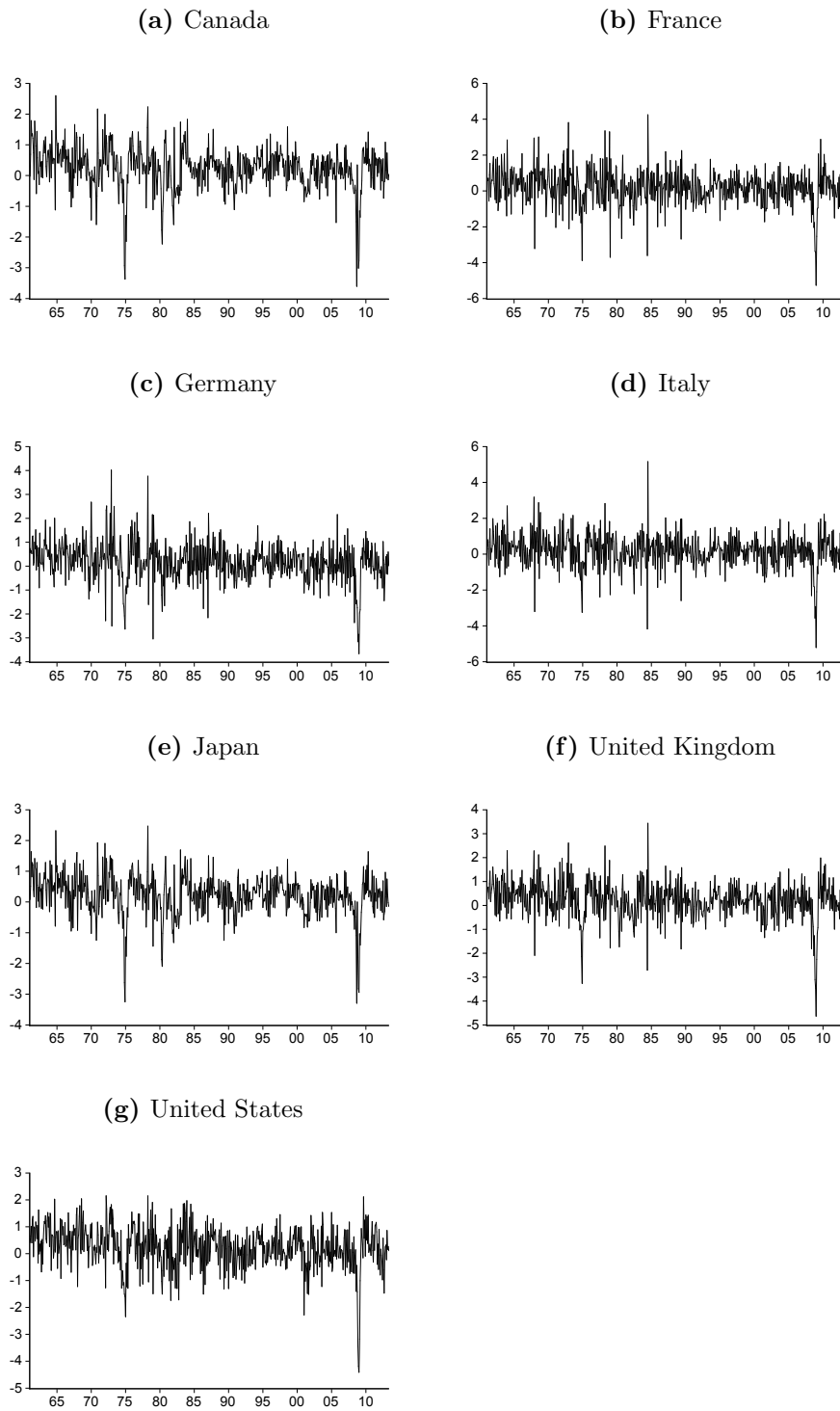


**Table 9:** Average trade share of country  $i$  with country  $j$  as a percentage of total trade

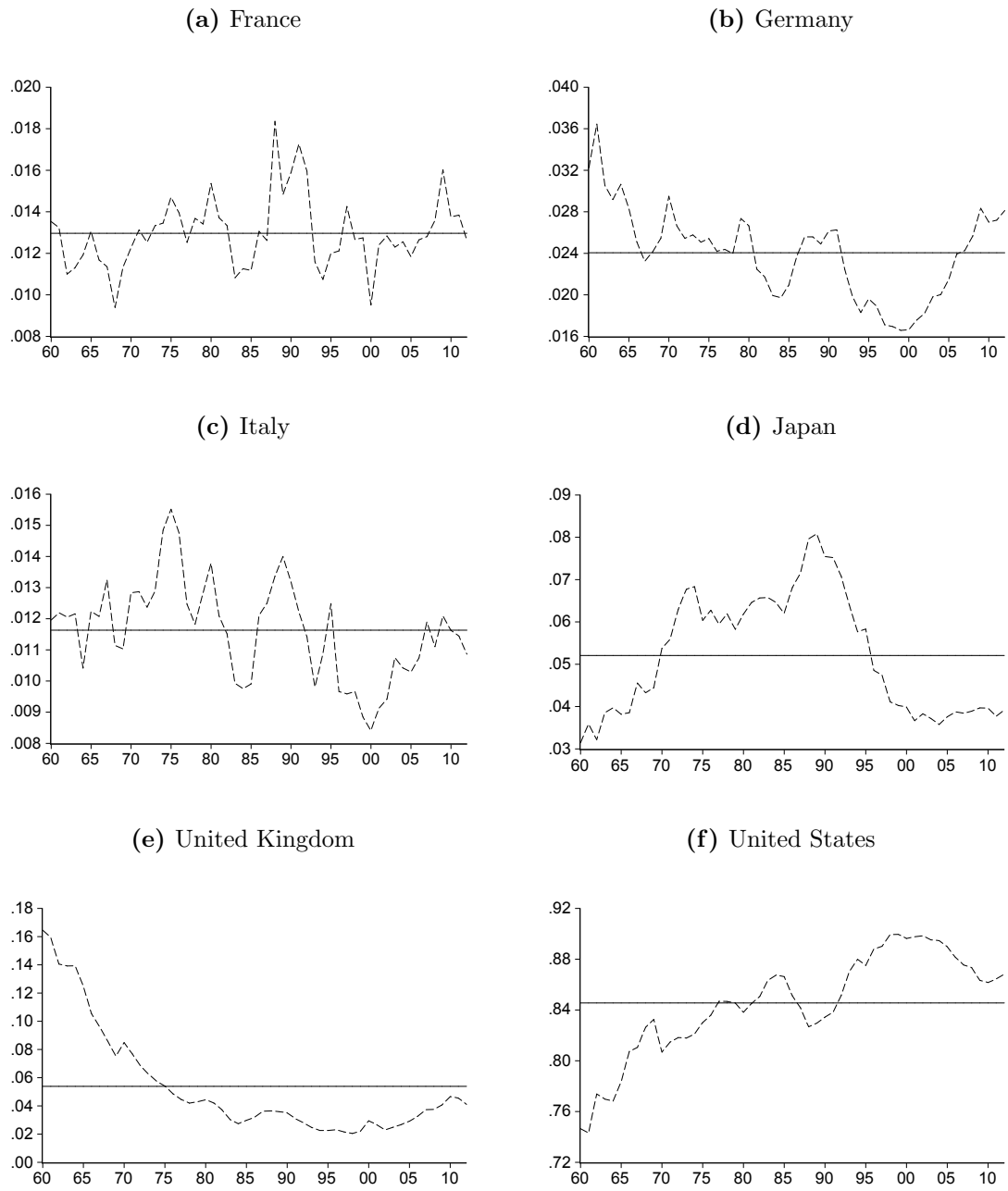
$i \backslash j$	CAN	FR	GER	ITA	JAP	UK	US
CAN	0.00	1.30	2.40	1.16	5.21	5.38	84.54
FR	1.83	0.00	41.25	21.53	3.70	15.59	16.10
GER	2.34	30.80	0.00	21.26	6.51	16.92	22.17
ITA	1.99	27.64	38.79	0.00	3.14	11.63	16.81
JAP	7.72	3.83	9.87	2.78	0.00	6.74	69.05
UK	8.60	18.06	26.24	10.18	6.63	0.00	30.28
US	43.82	5.79	11.39	5.04	23.90	10.07	0.00

*Note:* - The table corresponds to the trade weights matrix  $\mathbf{W}$  in equation (12).

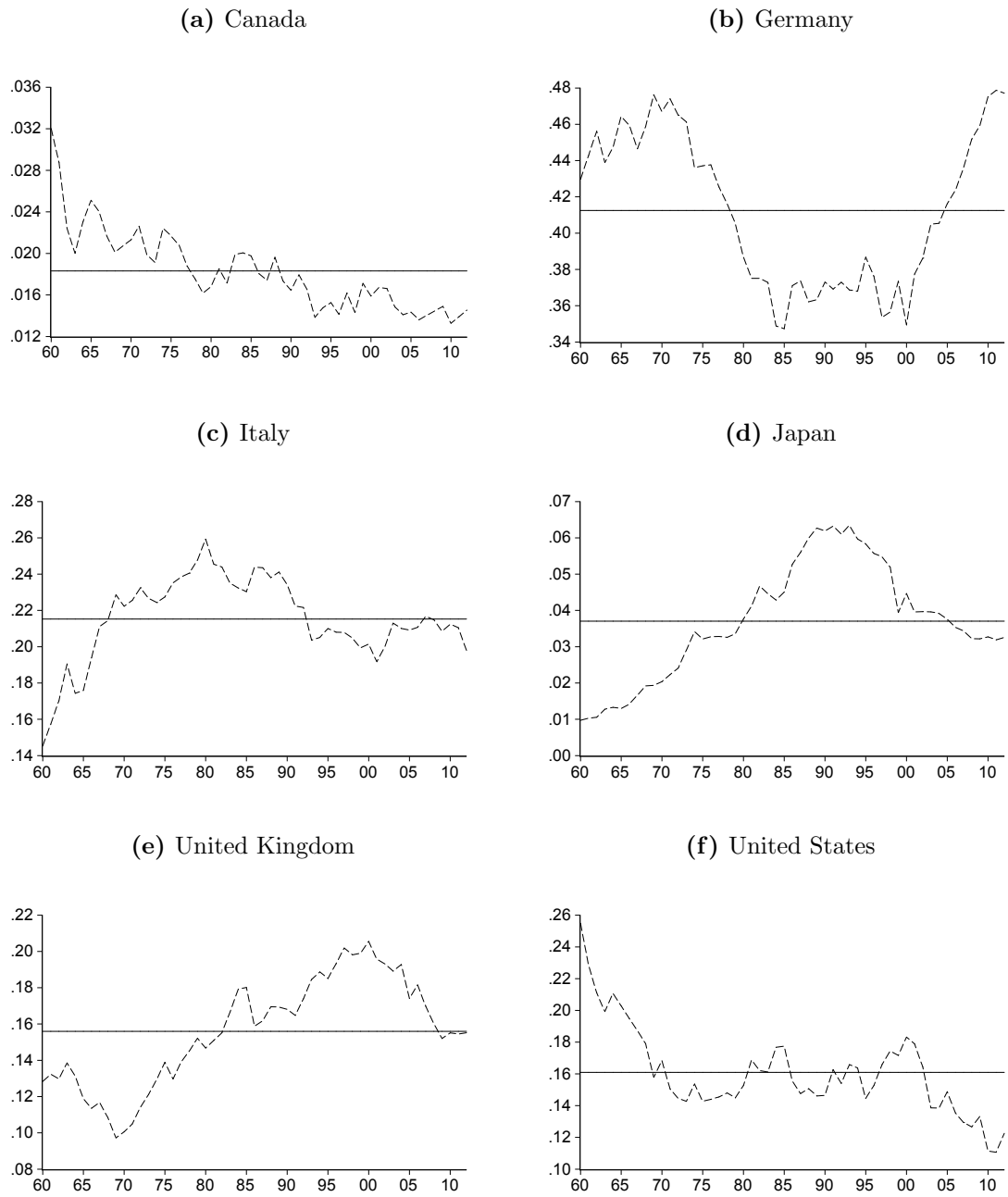
**Figure 5:** Cross-country fixed trade weighted averages of the G7 monthly growth rates of industrial production, 1961:02–2013:05



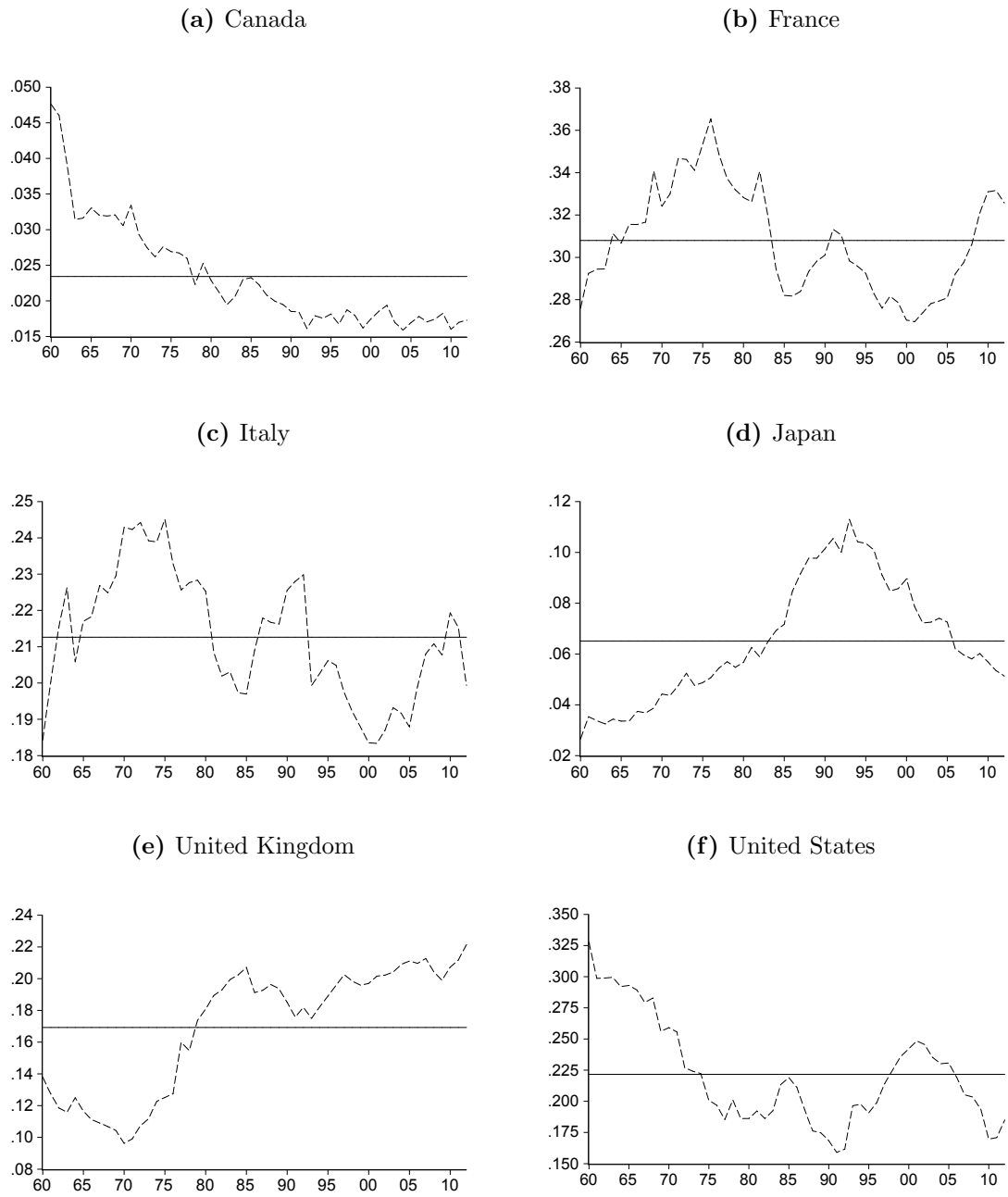
**Figure 6:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of Canada, 1961–2013



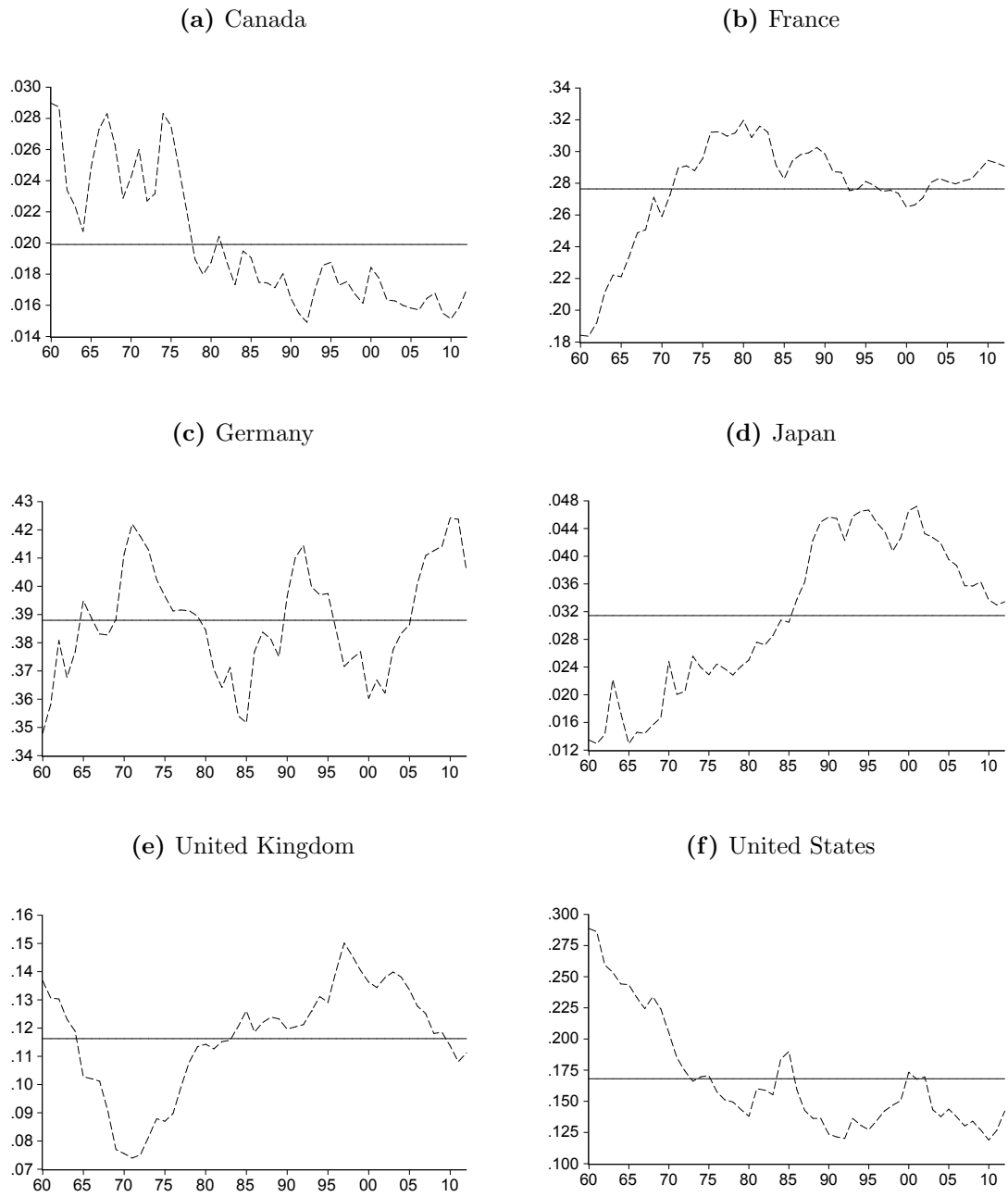
**Figure 7:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of France, 1961–2013



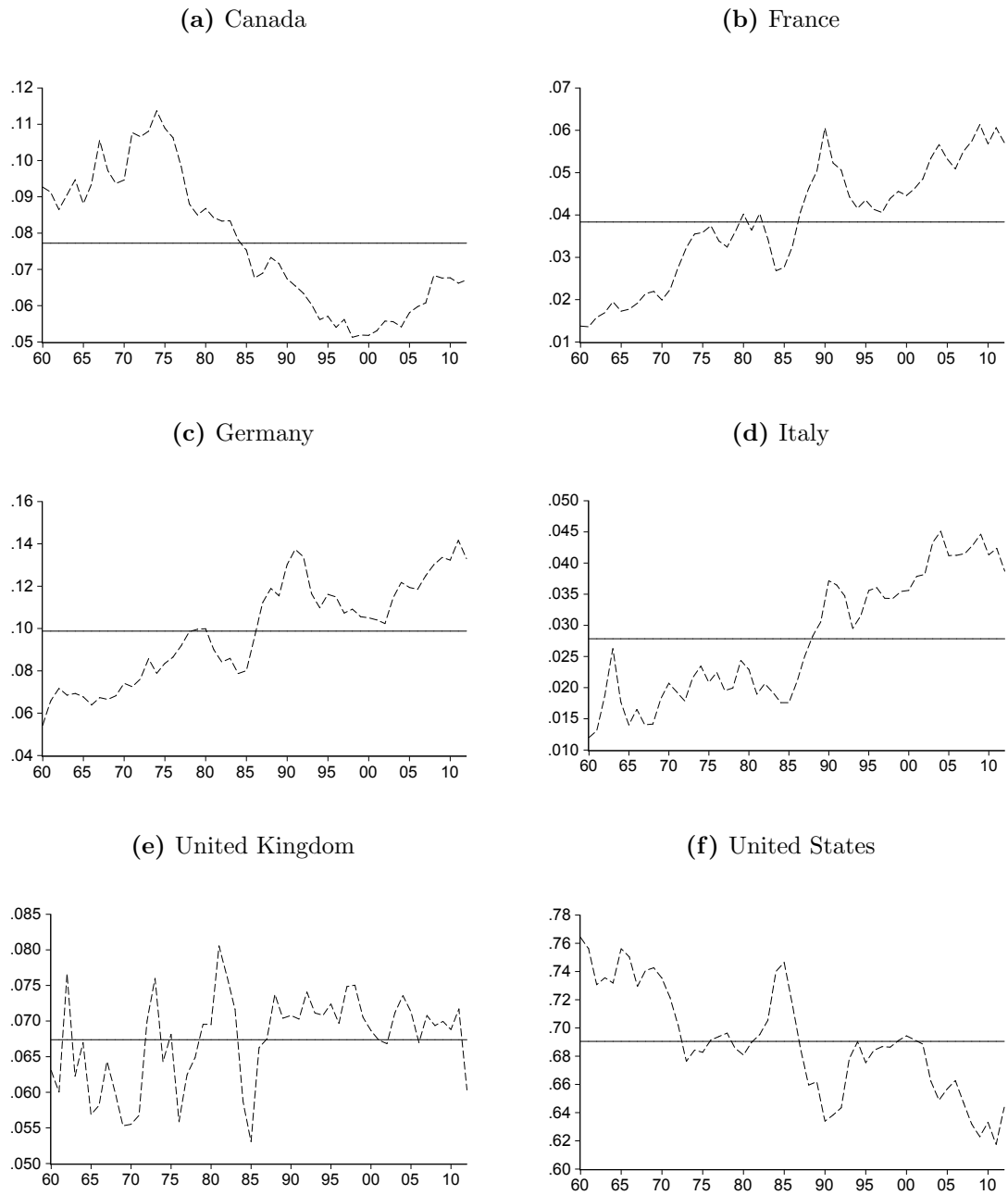
**Figure 8:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of Germany, 1961–2013



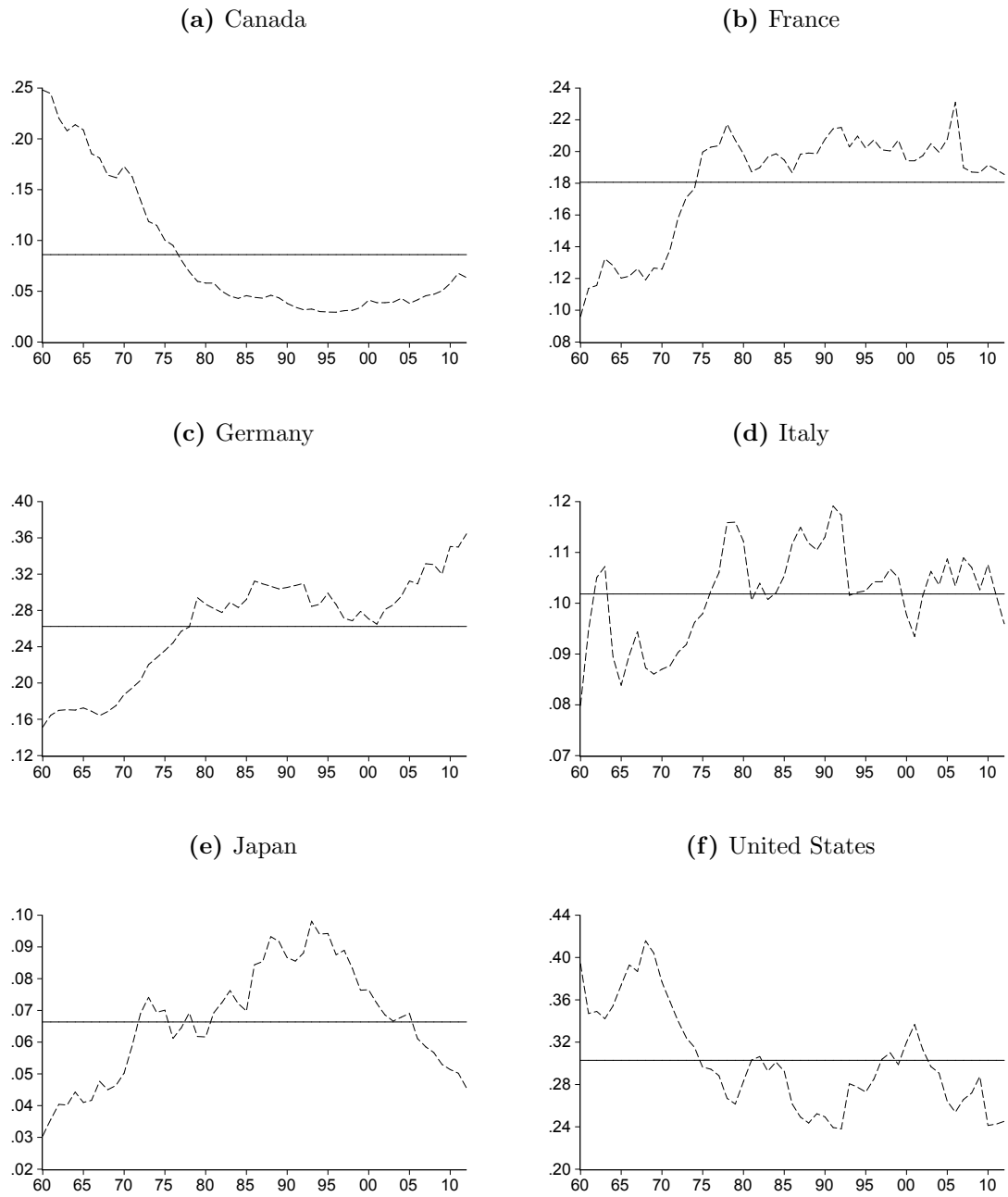
**Figure 9:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of Italy, 1961–2013



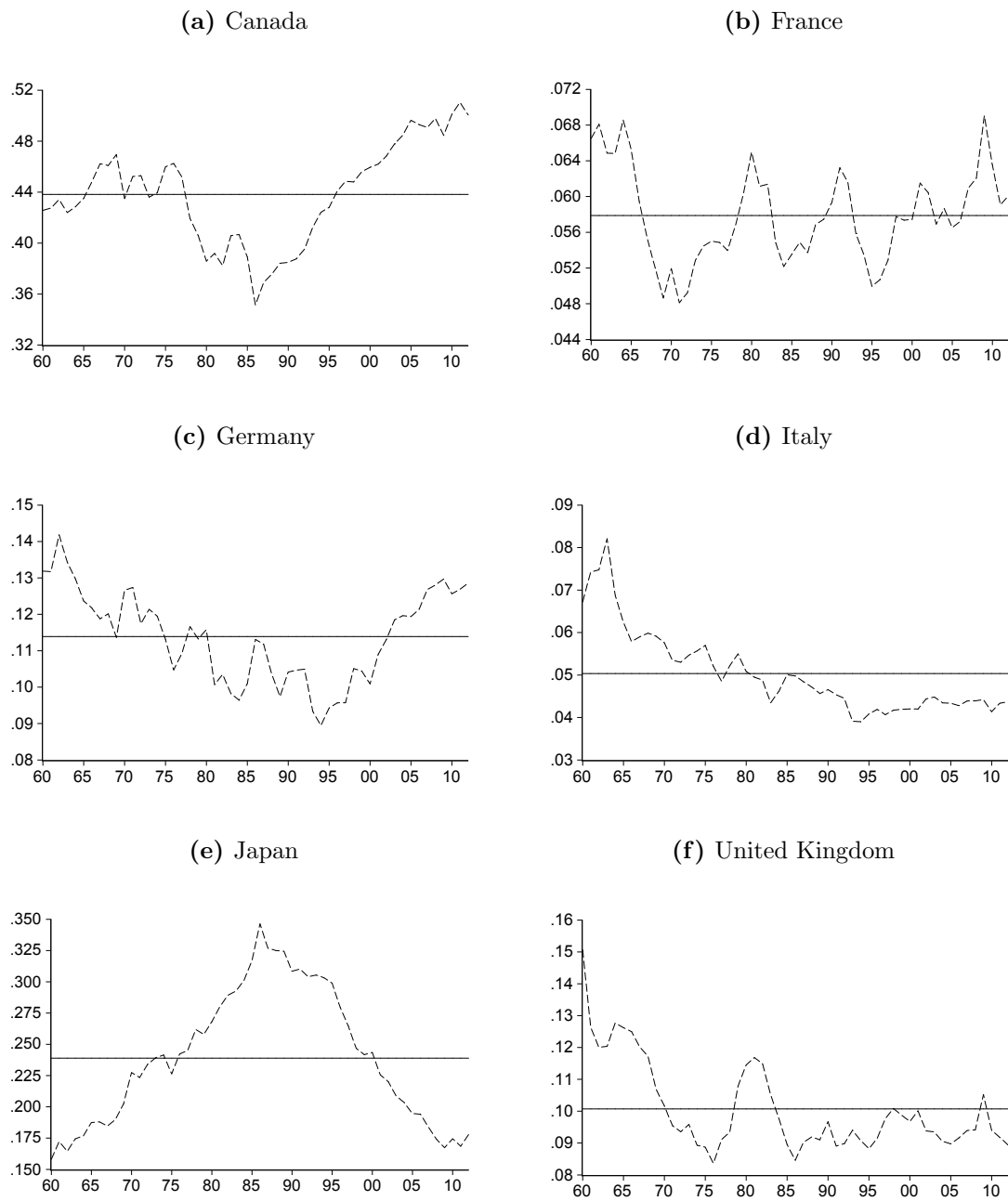
**Figure 10:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of Japan, 1961–2013



**Figure 11:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of United Kingdom, 1961–2013



**Figure 12:** Trade weights over time (dashed) and average trade weight (solid) of the G7 trade partners of United States, 1961–2013



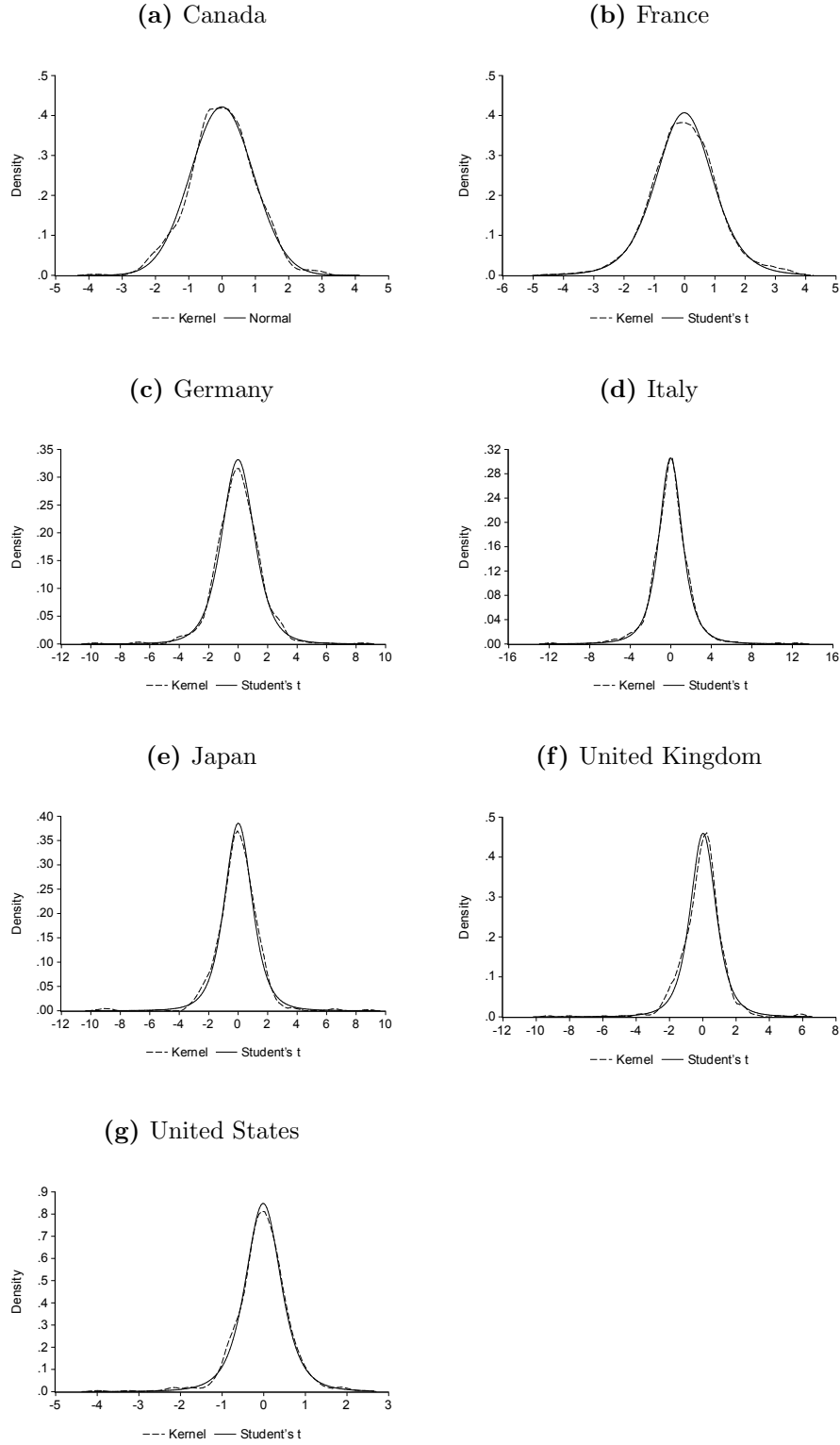
**Table 10:** Residual diagnostics for the models with cross-country weighted averages

	Mean	Variance	Q(3)	Q(9)	Q <sup>2</sup> (3)	Q <sup>2</sup> (9)	ARCH(3)	ARCH(9)
CAN	-0.007 [0.871]	1.002 [0.482]	0.024 [0.988]	5.171 [0.739]	1.629 [0.653]	5.059 [0.829]	1.664 [0.645]	5.425 [0.796]
FR	-0.004 [0.921]	1.001 [0.488]	0.023 [0.989]	0.927 [0.999]	3.864 [0.277]	6.747 [0.663]	3.794 [0.285]	6.752 [0.663]
GER	-0.013 [0.747]	1.038 [0.249]	0.518 [0.772]	8.580 [0.379]	3.727 [0.292]	4.976 [0.836]	3.696 [0.296]	4.850 [0.847]
ITA	-0.022 [0.582]	0.998 [0.494]	0.055 [0.973]	4.865 [0.772]	1.120 [0.772]	9.286 [0.411]	1.122 [0.772]	8.239 [0.510]
JAP	-0.014 [0.735]	1.000 [0.489]	0.913 [0.633]	3.473 [0.901]	5.302 [0.151]	12.496 [0.187]	5.749 [0.125]	12.972 [0.164]
UK	-0.039 [0.337]	1.013 [0.401]	0.585 [0.746]	6.220 [0.623]	3.361 [0.339]	5.915 [0.748]	3.450 [0.327]	6.044 [0.736]
US	-0.044 [0.274]	0.997 [0.489]	0.685 [0.710]	4.184 [0.840]	0.793 [0.851]	6.099 [0.730]	0.770 [0.857]	6.430 [0.696]

*Notes:* - p-values are in square brackets.

- The mean and variance is of the standardized residuals, i.e.  $(\hat{\varepsilon}_{it} - \bar{\hat{\varepsilon}}_{it})/\hat{\sigma}_{it}$ .
- $Q(p)$ : Ljung-Box Q-statistics at lag  $p$  for the residuals.
- $Q^2(p)$ : Ljung-Box Q-statistics at lag  $p$  for the squared residuals.
- $ARCH(p)$ : ARCH LM test at lag  $p$  (Engle, 1982).

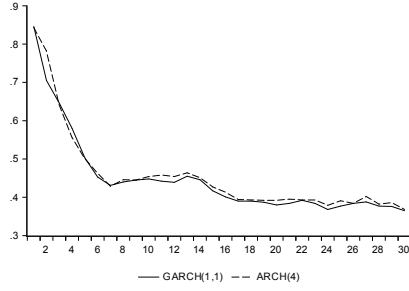
**Figure 13:** Kernel estimate of the residuals and the relevant theoretical distribution for the model with cross-country weighted averages



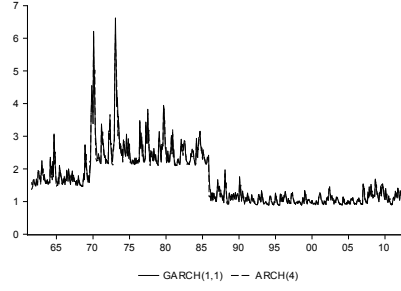
**Figure 14:** ZILC: The model with cross-country weighted averages

*Italy*

(a) Autocorrelation of conditional variance

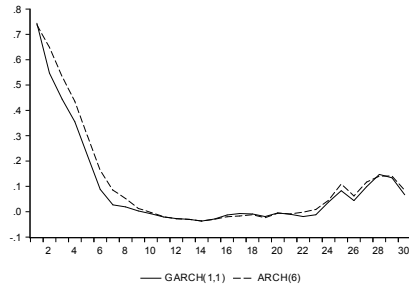


(b) Conditional Standard deviation

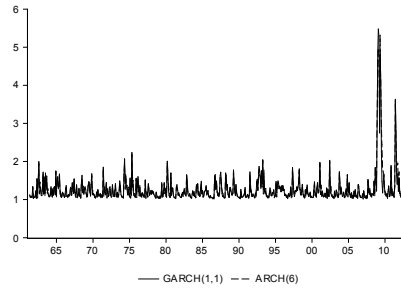


*Japan*

(c) Autocorrelation of conditional variance



(d) Conditional Standard deviation



**Table 11:** Results of the break detection for the model with cross-country weighted averages calculated with time-varying trade weights

Panel A: Breaks in the mean								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	6.02	3.72	3.70	7.31	<b>18.90</b>	6.35	<b>13.77</b>	7.46
$WDmax$	6.02	4.21	5.34	7.31	<b>21.19</b>	6.35	<b>15.44</b>	8.20
$SupF(2 1)$	1.76	2.35	0.79	1.80	3.64	2.38	<b>20.56</b>	8.51
$SupF(3 2)$	4.22	1.25	2.11	4.74	<b>10.18</b>	0.75	2.01	9.41
$SupF(4 3)$	0.62	1.22	2.15	1.24	4.93	1.12	0.26	10.04
$SupF(5 4)$	0.00	0.16	0.00	1.80	0.00	0.00	0.00	10.58
Break 1					1971:01		1991:04	
Break 2					1991:06		1999:06	
Break 3					2001:12			
Panel B: Breaks in the variance								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	<b>15.85</b>	<b>9.07</b>	4.94	<b>45.24</b>	1.98	<b>22.03</b>	<b>17.21</b>	7.46
$WDmax$	<b>15.85</b>	<b>12.26</b>	5.54	<b>45.24</b>	3.16	<b>23.97</b>	<b>17.21</b>	8.20
$SupF(2 1)$	<b>8.85</b>	<b>11.96</b>	3.90	<b>12.79</b>	1.68	<b>12.47</b>	3.49	8.51
$SupF(3 2)$	7.42	7.57	1.26	2.58	1.29	1.85	1.52	9.41
$SupF(4 3)$	5.13	1.92	0.67	<b>10.39</b>	2.53	5.71	1.06	10.04
$SupF(5 4)$	0.00	0.10	0.00	0.00	0.00	0.00	0.00	10.58
Break 1	1973:07	1973:04		1969:08		1972:01	1984:02	
Break 2	1984:11	1981:04		1980:12		1987:08		
Break 3				1990:02				
Break 4				2001:08				

*Note:* - C.V. stands for the 5% critical value.

**Table 12:** Estimation results for the model with cross-country weighted averages calculated with time-varying trade weights

Panel A: Mean Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$c_0$	-0.558** (0.229)	-0.324 (0.216)	-0.128 (0.340)	-0.359** (0.142)	-0.129 (0.270)	-0.236* (0.130)	-0.251*** (0.110)
$c_1$					-0.310** (0.134) [1971:01]		0.254** (0.057) [1991:04]
$c_2$					-0.322*** (0.124) [1991:06]		-0.176*** (0.053) [1999:06]
$c_3$					0.253* (0.139) [2001:12]		
$\lambda$	0.638** (0.257)	0.293 (0.214)	0.169 (0.240)	0.236** (0.108)	0.428** (0.182)	0.236* (0.136)	0.496*** (0.180)
$\beta_0$	0.586*** (0.058)	0.517*** (0.043)	0.450*** (0.064)	0.512*** (0.064)	0.328*** (0.077)	0.425*** (0.048)	0.275*** (0.026 )
$\beta_1$	0.208*** (0.078)	0.339*** (0.052)	0.326*** (0.067)	0.445*** (0.071)	0.209** (0.083)	0.224*** (0.050)	0.157*** (0.027)
$\beta_2$	0.055 (0.072)	0.361*** (0.054)	0.073 (0.070)	0.253*** (0.067)	0.079 (0.080)		0.037 (0.030)
$\beta_3$	0.097 (0.069)	0.239*** (0.055)	0.177*** (0.061)	0.140* (0.073)	0.091 (0.079)		-0.052 (0.032)
$\beta_4$	0.115* (0.068)	0.029 (0.050)	0.112* (0.063)	0.042 (0.069)			0.084*** (0.029)
$\beta_5$				0.128* (0.070)			
$\beta_6$				-0.159** (0.063)			
$\phi_1$	-0.238*** (0.046)	-0.487*** (0.043)	-0.358*** (0.045)	-0.367*** (0.045)	-0.247*** (0.043)	-0.271*** (0.042)	0.023 (0.044)
$\phi_2$	-0.056 (0.045)	-0.247*** (0.046)	-0.142*** (0.034)	-0.124*** (0.047)	0.144*** (0.037)	-0.127*** (0.033)	0.088** (0.035)
$\phi_3$	0.131*** (0.040)	-0.151*** (0.045)		-0.023 (0.042)	0.283*** (0.037)		0.112*** (0.031)
$\phi_4$		-0.124*** (0.043)		0.000 (0.044)	0.115*** (0.035)		0.040 (0.035)
$\phi_5$		-0.092** (0.038)		0.035 (0.043)			-0.062** (0.031)
$\phi_6$		0.111*** (0.035)		0.119*** (0.040)			

**Table 12:** Continued

Panel B: Variance Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$\alpha_0$	0.931*** (0.114)	1.284*** (0.187)	1.555*** (0.158)	1.418*** (0.509)	1.180*** (0.128)	0.816*** (0.176)	0.372*** (0.053)
$\alpha_1$	0.338* (0.186) [1973:07]	1.250** ( 0.520 ) [1973:04]		2.604*** (1.012) [1969:08]		1.275*** (0.352) [1972:01]	-0.205*** (0.052) [1984:02]
$\alpha_2$	-0.691*** (0.165) [1984:11]	-1.751*** (0.510) 1981:04]		-2.692*** (1.034) [1980:12]		-1.523*** (0.325) [1987:08]	
$\alpha_3$				-0.899** (0.380) [1990:02]			
$\alpha_4$				0.122 (0.131) [2001:08]			
$\eta$	0.100** (0.049)	0.067 (0.045)	0.332*** (0.094)	0.224*** (0.080)	0.298*** (0.086)	0.216*** (0.073)	0.360*** (0.089)
$\gamma$				0.327** (0.152)			
$\nu$		12.339* (6.981 )	6.389*** (1.192)	12.959* (7.452)	6.671*** (1.803)	6.764*** (1.558)	6.010*** (1.383)

Notes: - The model is:

$$\Delta y_{it} = c_{i0} + \sum_{k=1}^l c_{ik} DM_{ikt} + \lambda_i \sigma_{it} + \sum_{k=0}^s \beta_{ik} \Delta y_{it-k}^* + \sum_{k=1}^p \phi_{ik} \Delta y_{it-k} + \varepsilon_{it}$$

$$\sigma_{it}^2 = \alpha_{i0} + \sum_{k=1}^f \alpha_{ik} DV_{ikt} + \eta_i \varepsilon_{it-1}^2 + \gamma_i \sigma_{it-1}^2$$

$$\Delta y_{it}^* = \sum_{j=1}^7 w_{ijt} \Delta y_{jt}, \quad \sum_{j=1}^7 w_{ij} = 1 \quad \text{and} \quad w_{ii} = 0,$$

where  $\Delta y_{it-k}^*$  are cross-country weighted averages of growth and  $w_{ijt}$  is the time-varying share of total trade of country  $i$  with country  $j$  and total trade is defined as the sum of exports and imports between country  $i$  and the G7 other countries.

- $DM_{ikt}$  and  $DV_{ikt}$  are country-specific shift dummies in the mean and variance equation, respectively. The break dates are in square brackets.
- All models are estimated with the  $t$ -distribution as error distribution where  $\nu$  are the estimated degrees of freedom, except for Canada.
- Standard errors are in brackets and \*, \*\* and \*\*\* denotes statistical significance at the 10%, 5% and 1% level, respectively.

**Table 13:** Exogeneity test of  $\Delta y_{it}^*$  for the model with cross-country weighted averages calculated with time-varying trade weights

	CAN	FR	GER	ITA	JAP	UK	US
$\zeta$	-1.227 (-1.59)	1.193 (1.06)	-0.794 (-1.11)	-0.790 (-0.51)	-2.202 (-0.81)	-0.054 (-0.18)	0.455 (0.69)

*Note:* -  $t$ -statistics based on White heteroskedasticity-consistent standard errors are in brackets.

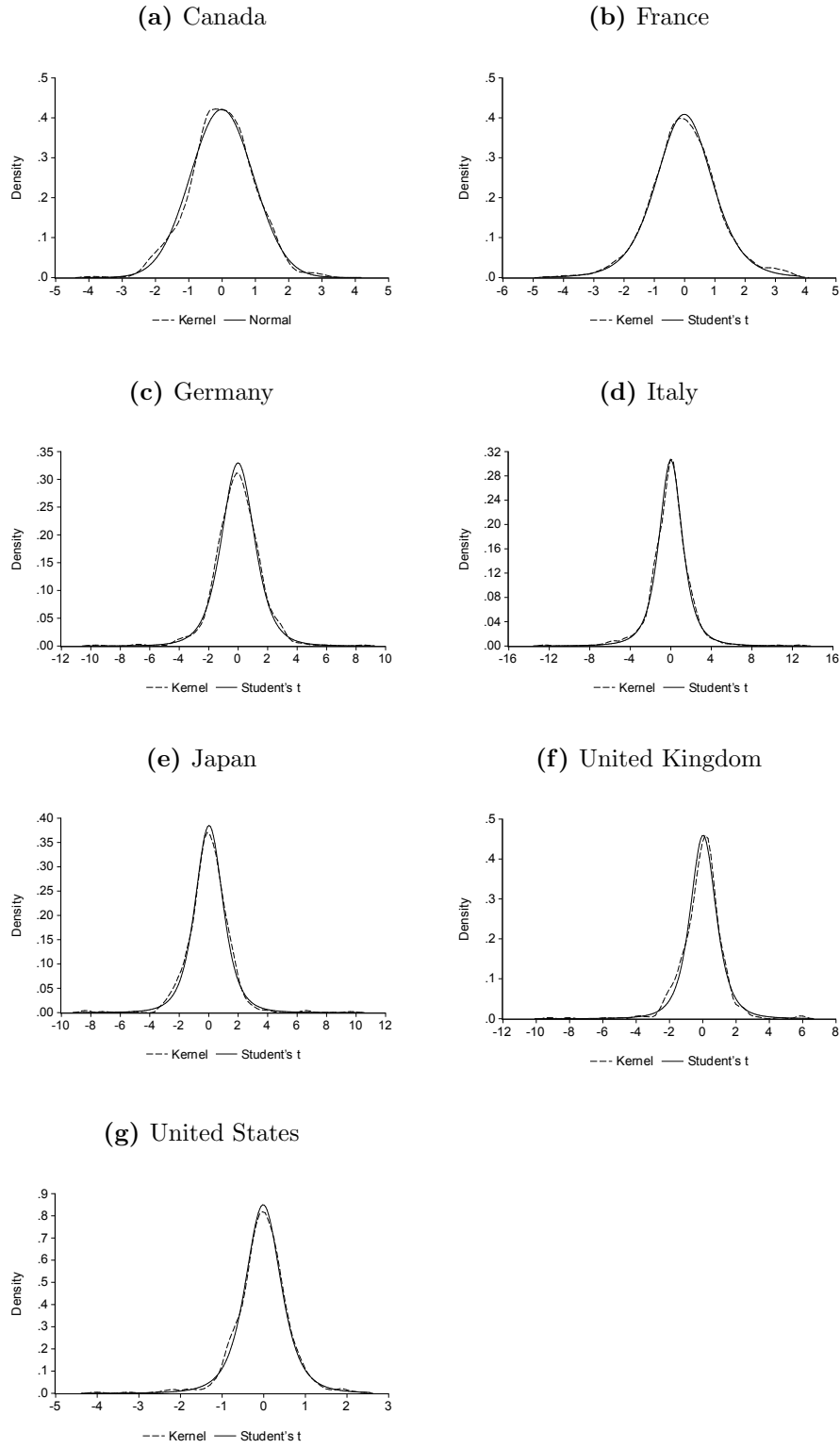
**Table 14:** Residual diagnostics for the model with cross-country weighted averages calculated with time-varying trade weights

	Mean	Variance	Q(3)	Q(9)	Q <sup>2</sup> (3)	Q <sup>2</sup> (9)	ARCH(3)	ARCH(9)
CAN	-0.007 [0.862]	1.001 [0.482]	0.007 [0.996]	5.043 [0.753]	1.812 [0.612]	7.338 [0.602]	1.902 [0.593]	8.494 [0.485]
FR	-0.003 [0.932]	0.999 [0.498]	0.177 [0.915]	1.324 [0.995]	3.609 [0.307]	6.623 [0.676]	3.545 [0.315]	6.794 [0.659]
GER	-0.012 [0.772]	1.040 [0.239]	0.472 [0.790]	9.500 [0.302]	3.392 [0.335]	4.678 [0.861]	3.362 [0.339]	4.546 [0.872]
ITA	-0.019 [0.632]	0.999 [0.497]	0.061 [0.970]	4.034 [0.854]	0.712 [0.870]	8.268 [0.507]	0.694 [0.875]	8.154 [0.519]
JAP	-0.005 [0.901]	1.005 [0.460]	0.436 [0.804]	1.720 [0.988]	4.365 [0.225]	33.992 [0.000]	4.318 [0.229]	32.798 [0.000]
UK	-0.039 [0.337]	1.012 [0.408]	0.536 [0.765]	6.025 [0.644]	3.478 [0.324]	5.883 [0.752]	3.578 [0.311]	5.928 [0.747]
US	-0.046 [0.248]	0.998 [0.493]	0.805 [0.669]	4.202 [0.839]	0.741 [0.864]	6.220 [0.718]	0.718 [0.869]	6.243 [0.715]

*Notes:* - p-values are in square brackets.

- The mean and variance is of the standardized residuals, i.e.  $(\hat{\varepsilon}_{it} - \bar{\hat{\varepsilon}}_{it})/\hat{\sigma}_{it}$ .
- $Q(p)$ : Ljung-Box Q-statistics at lag  $p$  for the residuals.
- $Q^2(p)$ : Ljung-Box Q-statistics at lag  $p$  for the squared residuals.
- $ARCH(p)$ : ARCH LM test at lag  $p$  (Engle, 1982).

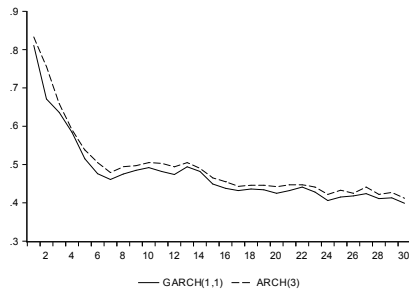
**Figure 15:** Kernel estimate of the residuals and the relevant theoretical distribution for the model with cross-country weighted averages calculated with time-varying trade weights



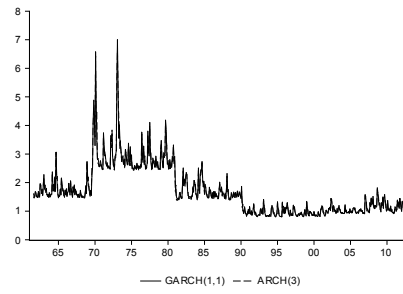
**Figure 16:** ZILC: The model with cross-country weighted averages calculated with time-varying trade weights

*Italy*

(a) Autocorrelation of conditional variance



(b) Conditional Standard deviation



**Table 15:** Results of the break detection without cross-country weighted averages

Panel A: Breaks in the mean								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	<b>25.26</b>	<b>19.33</b>	6.34	<b>9.24</b>	<b>44.46</b>	<b>10.07</b>	<b>22.88</b>	7.46
$WDmax$	<b>25.26</b>	<b>19.33</b>	6.81	<b>12.02</b>	<b>44.46</b>	<b>10.07</b>	<b>22.88</b>	8.20
$SupF(2 1)$	5.68	3.28	1.26	5.96	2.43	1.16	4.36	8.51
$SupF(3 2)$	8.74	1.42	4.49	0.62	1.97	1.25	7.86	9.41
$SupF(4 3)$	2.46	0.24	0.70	0.49	1.59	3.49	4.31	10.04
$SupF(5 4)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.58
Break1	1973:08	1974:08		1974:07	1973:09	2000:07	1969:04	
Panel B: Breaks in the variance								
	CAN	FR	GER	ITA	JAP	UK	US	C.V.
$UDmax$	<b>18.29</b>	<b>8.71</b>	5.41	<b>38.61</b>	2.03	<b>19.89</b>	<b>10.85</b>	7.46
$WDmax$	<b>18.29</b>	<b>12.96</b>	6.07	<b>38.61</b>	2.99	<b>19.89</b>	<b>10.87</b>	8.20
$SupF(2 1)$	<b>11.24</b>	<b>10.96</b>	1.07	<b>15.65</b>	2.79	<b>10.06</b>	5.06	8.51
$SupF(3 2)$	5.97	8.65	0.94	3.20	0.62	<b>16.53</b>	2.44	9.41
$SupF(4 3)$	0.14	2.80	0.95	5.98	2.99	3.83	2.29	10.04
$SupF(5 4)$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.58
Break1	1973:07	1971:09		1968:12		1972:01	1984:02	
Break2	1984:11	1979:08		1985:10		1980:05		
Break 3						1990:08		

*Note:* - C.V. stands for the 5% critical value.

**Table 16:** Estimation results for the model without cross-country weighted averages

Panel A: Mean Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$\tilde{c}_0$	0.176 (0.308)	0.277 (0.290)	0.083 (0.298)	0.595** (0.257)	-0.010 (0.334)	0.122 (0.144)	0.079 (0.154)
$\tilde{c}_1$	-0.251** (0.112) [1973:08]	-0.375*** (0.119) [1974:08]		-0.556*** (0.171) [1974:07]	-0.434*** (0.121) [1973:09]	-0.248*** (0.085) [2000:07]	-0.129 (0.081) [1969:04]
$\tilde{\lambda}$	0.172 (0.272)	0.180 (0.220)	0.098 (0.202)	0.025 (0.116)	0.438* (0.253)	0.078 (0.133)	0.239 (0.179)
$\tilde{\phi}_1$	-0.105** (0.045)	-0.359*** (0.044)	-0.300*** (0.045)	-0.308*** (0.044)	-0.220*** (0.045)	-0.219*** (0.046)	0.138*** (0.044)
$\tilde{\phi}_2$	0.063 (0.042)	-0.105** (0.043)	-0.053 (0.038)	-0.053 (0.046)	0.192*** (0.040)	-0.079** (0.037)	0.175*** (0.039)
$\tilde{\phi}_3$	0.241*** (0.042)	0.021 (0.042)	0.136*** (0.032)	0.053 (0.041)	0.327*** (0.038)	0.062* (0.037)	0.153*** (0.035)
$\tilde{\phi}_4$	0.097** (0.042)	-0.021 (0.038)	0.086*** (0.031)	0.077* (0.043)	0.138*** (0.037)	0.033 (0.037)	0.086** (0.036)
$\tilde{\phi}_5$	0.052 (0.040)	-0.011 (0.038)	0.056 (0.034)	0.110*** (0.042)		0.079** (0.034)	
$\tilde{\phi}_6$		0.152*** (0.035)	0.079** (0.034)	0.144*** (0.038)			

**Table 16:** Continued

Panel B: Variance Equation							
	CAN	FR	GER	ITA	JAP	UK	US
$\tilde{\alpha}_0$	1.052*** (0.133)	1.150*** (0.190)	1.668*** (0.168)	0.534** (0.257)	0.872*** (0.170)	0.778*** (0.179)	0.414*** (0.098)
$\tilde{\alpha}_1$	0.579** (0.259) [1973:07]	1.509*** (0.573) [1971:09]		1.207** (0.566) [1968:12]		1.979*** (0.587) [1972:01]	-0.215*** (0.071) [1984:02]
$\tilde{\alpha}_2$	-0.911*** (0.231) [1984:11]	-1.649*** (0.551) [1979:08]		-1.409** (0.609) [1985:10]		-1.468** (0.634) [1980:05]	
$\tilde{\alpha}_3$						-0.762*** (0.296) [1990:08]	
$\tilde{\eta}$	0.084* (0.046)	0.190*** (0.061)	0.367*** (0.099)	0.208*** (0.068)	0.257*** (0.076)	0.317*** (0.085)	0.272*** (0.083)
$\tilde{\gamma}$				0.594*** (0.119)	0.248** (0.107)		0.110 (0.125)
$\tilde{\nu}$		8.498** (3.497)	5.932*** (1.152)	7.355*** (2.476)	7.222*** (1.926)	7.880*** (2.593)	5.027*** (1.124)

Notes: - The model is:

$$\Delta y_{it} = \tilde{c}_{i0} + \tilde{c}_{i1} \tilde{D}M_{it} + \tilde{\lambda}_i \tilde{\sigma}_{it} + \sum_{k=1}^p \tilde{\phi}_{ik} \Delta y_{it-k} + \tilde{\varepsilon}_{it}$$

$$\tilde{\sigma}_{it}^2 = \tilde{\alpha}_{i0} + \sum_{k=1}^f \tilde{\alpha}_{ik} \tilde{D}V_{ikt} + \tilde{\eta}_i \tilde{\varepsilon}_{it-1}^2 + \tilde{\gamma}_i \tilde{\sigma}_{it-1}^2$$

- $\tilde{D}M_{it}$  and  $\tilde{D}V_{ikt}$  are country-specific shift dummy in the mean and variance equation, respectively. The break dates are in square brackets.
- All models are estimated with the  $t$ -distribution as error distribution where  $\tilde{\nu}$  are the estimated degrees of freedom, except for Canada.
- Standard errors are in brackets and \*,\*\* and \*\*\* denotes statistical significance at the 10%, 5% and 1% level, respectively.

**Table 17:** Long-run elasticities for the model without cross-country weighted averages

	CAN	FR	GER	ITA	JAP	UK	US
$\tilde{\lambda}^{TLR}$	0.2642 (0.420)	0.136 (0.169)	0.098 (0.204)	0.026 (0.119)	0.778 (0.471)	0.069 (0.119)	0.533 (0.413)
$\tilde{\alpha}_{GM}^{TLR}$	-0.109 (0.177)	-0.083 (0.102)	0.000 .	-0.008 (0.039)	0.000 .	-0.037 (0.066)	-0.075 (0.061)

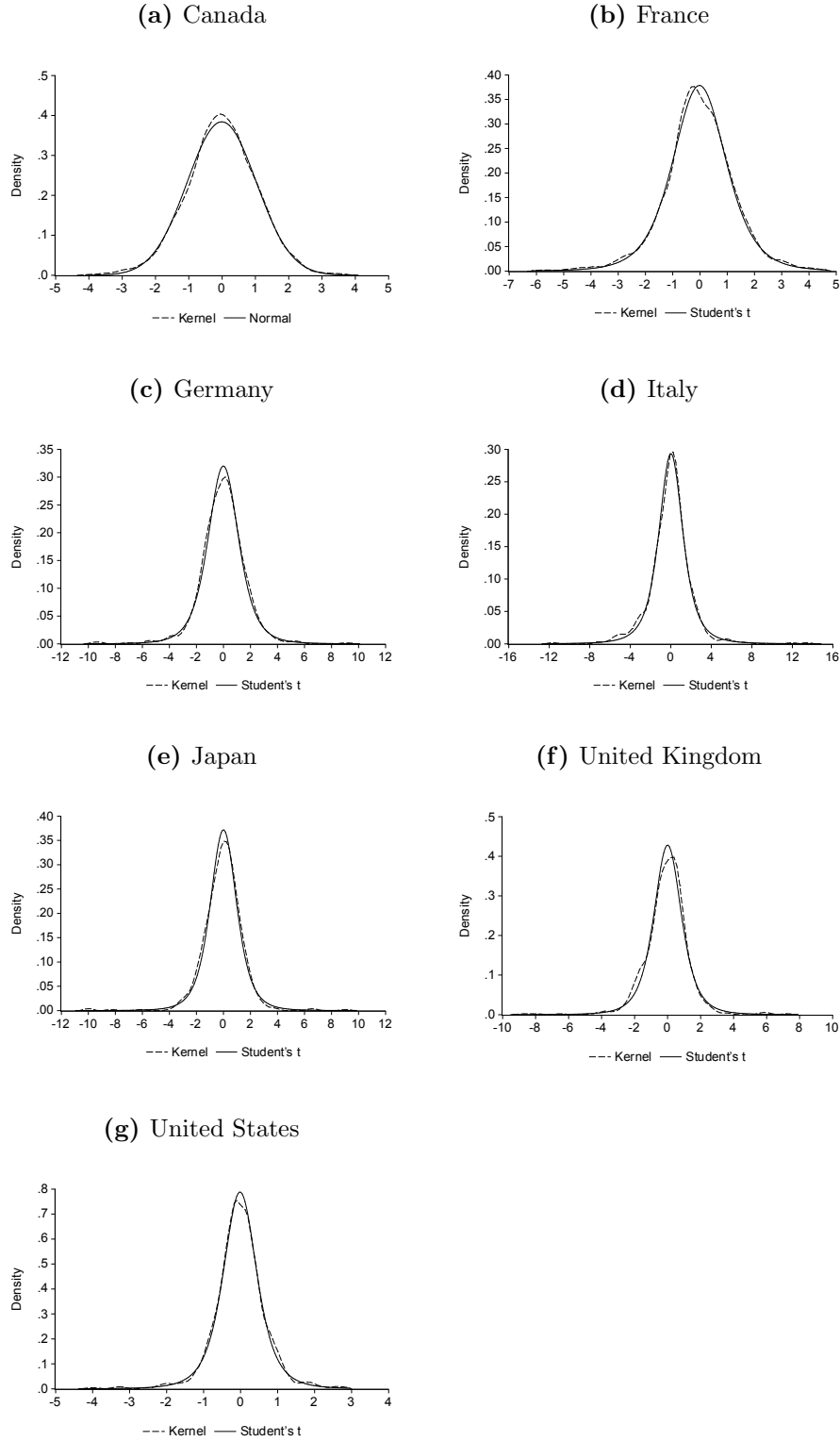
**Table 18:** Residual diagnostics for the models without cross-country weighted averages

	Mean	Variance	Q(3)	Q(9)	Q <sup>2</sup> (3)	Q <sup>2</sup> (9)	ARCH(3)	ARCH(9)
CAN	-0.011 [0.781]	1.001 [0.482]	0.062 [0.996]	4.607 [0.867]	0.687 [0.876]	5.112 [0.824]	0.682 [0.877]	4.948 [0.839]
FR	-0.022 [0.587]	0.993 [0.458]	0.998 [0.802]	8.229 [0.511]	2.222 [0.528]	8.209 [0.513]	2.202 [0.532]	8.548 [0.480]
GER	-0.020 [0.625]	1.028 [0.304]	1.669 [0.644]	4.367 [0.886]	3.987 [0.263]	5.804 [0.759]	3.991 [0.263]	5.557 [0.783]
ITA	-0.022 [0.580]	0.993 [0.455]	0.391 [0.942]	8.752 [0.460]	1.622 [0.654]	11.377 [0.251]	1.586 [0.663]	11.049 [0.272]
JAP	-0.026 [0.519]	1.002 [0.479]	1.181 [0.758]	6.250 [0.715]	5.682 [0.128]	13.563 [0.139]	6.266 [0.099]	14.055 [0.120]
UK	-0.045 [0.262]	1.000 [0.494]	0.359 [0.949]	4.492 [0.876]	2.854 [0.415]	6.444 [0.695]	2.860 [0.414]	6.725 [0.666]
US	-0.025 [0.533]	0.982 [0.379]	1.920 [0.589]	6.951 [0.642]	5.469 [0.141]	8.398 [0.495]	5.859 [0.119]	8.759 [0.460]

*Notes:* - p-values are in square brackets.

- The mean and variance is of the standardized residuals, i.e.  $(\hat{\varepsilon}_{it} - \bar{\hat{\varepsilon}}_{it})/\hat{\sigma}_{it}$ .
- $Q(p)$ : Ljung-Box Q-statistics at lag  $p$  for the residuals.
- $Q^2(p)$ : Ljung-Box Q-statistics at lag  $p$  for the squared residuals.
- $ARCH(p)$ : ARCH LM test at lag  $p$  (Engle, 1982).

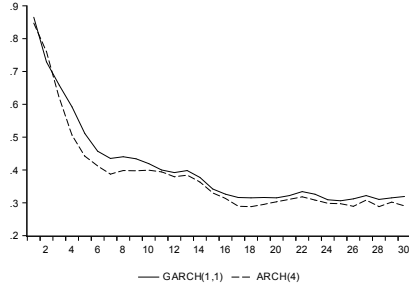
**Figure 17:** Kernel estimate of the residuals and the relevant theoretical distribution for the model without cross-country weighted averages



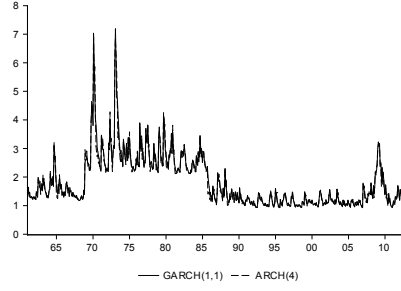
**Figure 18:** ZILC: The model without country interactions

*Italy*

(a) Autocorrelation of conditional variance

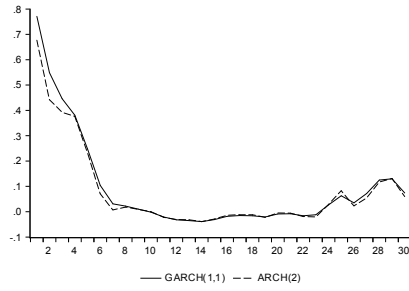


(b) Conditional Standard deviation

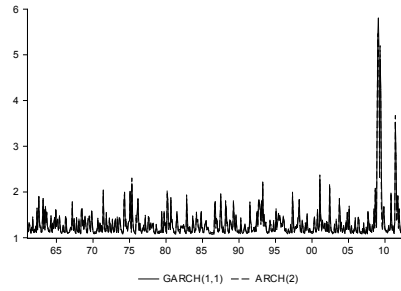


*Japan*

(c) Autocorrelation of conditional variance

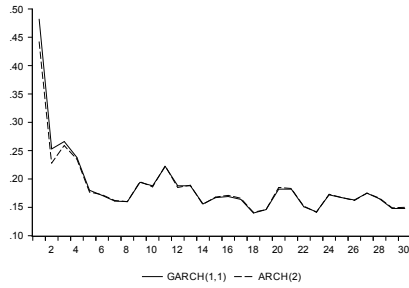


(d) Conditional Standard deviation

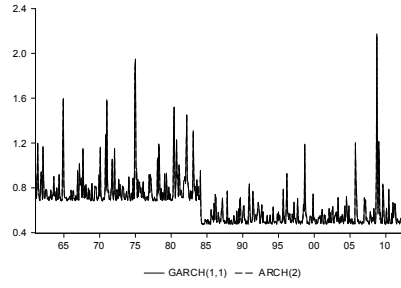


*United States*

(e) Autocorrelation of conditional variance



(f) Conditional Standard deviation



## References

- Aghion, P., & Saint-Paul, G. (1998). Uncovering Some Causal Relationships Between Productivity Growth and the Structure of Economic Fluctuations: A Tentative Survey. *Labour*, 12(2), 279-303.
- Badinger, H. (2010). Output volatility and economic growth. *Economics Letters*, 106(1), 15–18.
- Bai, J. S., & Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, 66(1), 47–78.
- Bai, J. S., & Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, 18(1), 1–22.
- Bernanke, B. S. (2004). The great moderation. *At the meetings of the Eastern Economic Association, Washington, DC February 20, 2004*.
- Bollerslev, T., & Wooldridge, J. (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric reviews*, 11(2), 143-172.
- Dées, S., Di Mauro, F., Pesaran, M. H., & Smith, L. V. (2007). Exploring the international linkages of the euro area: A global var analysis. *Journal of Applied Econometrics*, 22(1), 1–38.
- Elliott, G., Rothenberg, T. J., & Stock, J. H. (1996). Efficient tests for an autoregressive unit root. *Econometrica*, 64(4), 813–836.
- Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity With Estimates of the Variance of United-kingdom Inflation. *Econometrica*, 50(4), 987–1007.
- Engle, R. F., Lilien, D. M., & Robins, R. P. (1987). Estimating Time-varying Risk Premia In the Term Structure - the Arch-m Model. *Econometrica*, 55(2), 391–407.
- Fang, W. S., & Miller, S. M. (2009). Modeling the volatility of real GDP growth: The case of Japan revisited. *Japan and the World Economy*, 21(3), 312–324.
- Fang, W. S., Miller, S. M., & Lee, C. (2008). Cross-country evidence on out-

- put growth volatility: Nonstationary variance and GARCH models. *Scottish Journal of Political Economy*, 55(4), 509–541.
- Hausman, J. (1978). Specification tests in econometrics. *Econometrica*, 46(6), 1251–1271.
- Herrera, A. M., & Pesavento, E. (2005). The decline in US output volatility: Structural changes and inventory investment. *Journal of Business & Economic Statistics*, 23(4), 462–472.
- Hillebrand, E. (2005). Neglecting parameter changes in GARCH models. *Journal of Econometrics*, 129(1-2), 121–138.
- Kim, C., & Nelson, C. (1999). Has the US economy become more stable? A Bayesian approach based on a Markov-switching model of the business cycle. *Review of Economics and Statistics*, 81(4), 608–616.
- Lamoureux, C. G., & Lastrapes, W. D. (1990). Persistence In Variance, Structural-change, and the Garch Model. *Journal of Business & Economic Statistics*, 8(2), 225–234.
- Lee, J. (2010). The link between output growth and volatility: Evidence from a GARCH model with panel data. *Economics Letters*, 106(2), 143–145.
- Ma, J., Nelson, C. R., & Startz, R. (2007). Spurious inference in the GARCH (1,1) model when it is weakly identified. *Studies In Nonlinear Dynamics and Econometrics*, 11(1), 1081–1826.
- Martin, P., & Rogers, C. A. (1997). Stabilization policy, learning-by-doing, and economic growth. *Oxford Economic Papers*, 49(2), 152–166.
- Martin, P., & Rogers, C. A. (2000). Long-term growth and short-term economic instability. *European Economic Review*, 44(2), 359–381.
- McConnell, M., & Perez-Quiros, G. (2000, DEC). Output fluctuations in the United States: What has changed since the early 1980's? *American Economic Review*, 90(5), 1464–1476. doi: 10.1257/aer.90.5.1464
- Nelson, C. R., & Startz, R. (2007). The zero-information-limit condition and spu-

- rious inference in weakly identified models. *Journal of Econometrics*, 138(1), 47–62.
- Nordhaus, W. (2004). Retrospective on the 1970s productivity slowdown. *NBER working paper no. 10950*.
- Perron, P. (1989). The Great Crash, the Oil Price Shock, and the Unit-root Hypothesis. *Econometrica*, 57(6), 1361–1401.
- Pesaran, M. H., Schuermann, T., & Weiner, S. M. (2004). Modeling regional interdependencies using a global error-correcting macroeconometric model. *Journal of Business & Economic Statistics*, 22(2), 129–162.
- Ramey, G., & Ramey, V. A. (1995). Cross-country evidence on the link between volatility and growth. *American Economic Review*, 85(5), 1138–1151.
- Stock, J., & Watson, M. (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association*, 3(5), 968–1006.