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Preliminary Credit Ratings and Contact Disclosure*

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Abstract

A recent amendment to the European Regulation on credit rating agencies (CRA) requires CRAs to disclose any issuers' request of initial reviews (i.e preliminary ratings). This paper constructs a model of preliminary ratings and uses it to investigate the effect of contact disclosure. A CRA issues a preliminary rating. After receiving this confidential rating, the entrepreneur has the opportunity either to purchase an official rating at a cost or to remain unrated. I identify a tradeoff between the fee and the CRA's reputation. When the project is likely to be good, the CRA issues a good preliminary rating because the risk of loosing reputation is extremely low whereas when the project is likely to be bad the CRA prefers to issue a bad preliminary rating avoiding to risk reputation. I show that when there is disclosure of the contact between the CRA and the entrepreneur, the CRA issues more good preliminary ratings then when there is no evidence of preliminary contact. Disclosure results in a lower probability of stopping high quality projects but also in a higher probability of financing low quality ones.

JEL Classification: D82, D83, G24, G28

Keywords: rating agencies, preliminary ratings, reputation, disclosure

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Introduction

Credit Rating Agencies (CRAs) offer an evaluation service to those who need to obtain finance from the market. Upon request of an issuer, they perform an investigation, providing a rating, their opinion on the creditworthiness of the issuer.[1] Their ratings assist investors making investment decisions.

In the light of the recent financial crises, international bodies, regulators and policy makers have suggested possible ways to overcome some of the industry drawbacks.[2] Both the US and the EU Regulations underwent amendments in conjunction with international bodies such as the FSB and IOSCO[3].

The prevalent modus operandi in the industry, starting from the 1970's, is for the issuer of a financial product to ask for a rating and to pay the rating agency for its services. An issuer can contact several rating agencies for initial reviews. Subsequently the issuer decides whether or not to ask for the official rating from one of the agencies. If it does so, it provides further detailed information to the evaluator. The CRA performs a detailed investigation and issues a report. At the end of the process the rating is published and the agreed fee is paid.

This work focuses on the widespread practice of obtaining an initial indicative evaluation prior to purchasing the published rating. The confidentiality of this so called preliminary ratings (or indicative ratings) is widely regarded as a problem because it can bias ratings and causes poor information provision.[4] Indeed, in October 2009 the US Security and Exchange Commission attempted to impose mandatory disclosure of preliminary ratings[5], even though this proposal is yet to be finalized.

[1] An issuer is defined as an entity selling securities to finance its business. Generally speaking an issuer is a borrower seeking financing from lenders (investors).

[2] The main drawbacks affecting the rating industry relate to the CRAs' conflict of interest, to the lack of competition and to excessive regulatory reliance on ratings.

[3] Financial Stability Board (FSB) and International Organization of Securities Commissions (IOSCO). Both the FSB and IOSCO are international bodies devoted respectively to monitor the global financial system and to develop and promote high standards for securities regulation.

[4] Selective release of ratings when there is no evidence of interaction between the issuer and the CRAs may lead to upward biased ratings.

[5] The proposed rule required issuers disclosure of received preliminary ratings as opposed to the EU proposal which requires CRAs to disclose all the contacts with the issuers. Details of the proposal can be found in SEC Release No. 33-9070 available at `https:\www.sec.gov/rules/proposed/2009/33-9070.pdf`.

Similarly, in Europe[6]

> "*a credit rating agency shall disclose on its website, and notify ESMA on an ongoing basis, information about all entities or debt instruments submitted to it for their initial review or for preliminary rating. Such disclosure shall be made whether or not issuers contract with the credit rating agency for an official rating.*

The OECD hearing held by the Competition Committee[7] points out that built reputation is the main competitive advantage because ratings are experience goods. This highlights the importance of having a model which accounts also for reputation concerns.

The main question here is whether the requirement of disclosure of preliminary rating affects the CRAs' reputation and how their incentives change in response to this different environment. The transparency requirement is likely to alter the rating agency's incentives, since one of the key features of the industry is the fact that a substantial part of a CRAs' payoff is the reputation effect due to the perception that investors have of the rating agency being capable. Before the amendment of the European Regulation, issuer could secretly obtain preliminary ratings. This meant that only through the publication of a proper rating the CRA could modify its reputation by either being proven right or wrong.

In order to understand the theoretical consequences of policy proposals such as those contained in the amended EU Regulation, I develop a game theoretic model of the interaction between the issuers of financial products and the credit rating agency. I investigate how the disclosure of preliminary ratings can affect the credit rating accuracy when reputation is at stake. Here considered is a basic setting with one rating agency, a mass of investors and a mass of entrepreneurs. The latter decide whether to purchase the publishable rating after having seen the preliminary one. The rating agency wants to create and maintain a reputation for accurate ratings. I investigate two different regimes, the first characterised by no disclosure of the contact between the players while the second features compulsory contact disclosure.

[6]In accordance with Point 6 of Part 1 of Section D of Annex 1 to Regulation (EC) No.1060/2009 as amended by Regulation (EU) No 462/2013.

[7]The hearing on Competition and Credit Rating Agencies was held in June 2010. Contributions are due to Prof. John C. Coffee, Columbia University Law School(United States) and Prof. Karel Lannoo, Center for European Policy Studies (CEPS). The document can be found at `http://www.oecd.org/competition/sectors/46825342.pdf`.

I find that imposing disclosure on the credit rating agency leads to more projects being given a good preliminary evaluation and in turn more projects reaching the proper rating stage and accessing funding. This in turn implies preliminary disclosure results in a higher probability of default. The reasoning goes as follows: with disclosure in place the CRA's reputation suffers because all the projects which went through a preliminary evaluation are visible to the market. That is, investors can consistently update their beliefs even in the absence of a published report. Thus the CRA becomes confident more often because by doing so she earns a higher payoff than the achievable one from acting conservatively. In other words, the CRA benefits form the opaqueness of the market.

The financial crises brought about many criticisms of CRAs' conduct, generating renewed attention. The theoretical literature studying rating agencies is relatively new, but has already identified a wealth of interesting questions. This work aims to contribute to filling the gap concerning works dealing with preliminary ratings and reputation, ensuring a better comprehension of the dynamics behind rating agencies' functioning.

Up to now preliminary ratings have been investigated by the economic literature in relation to rating shopping and rating bias[8], but reputation concerns seem to have rarely been considered. It is important to pay specific attention to the consequences of the impact of indicative ratings on CRAs reputation. For instance Sangiorgi and Spatt [2013] investigate how transparency can improve the precision of the information that the market holds regarding the issuer. When the market is aware of the existence of preliminary ratings, voluntary disclosure in equilibrium results in full disclosure whereas when no disclosure requirements exist, selective disclosure arises. I focus on how transparency can alter the perception that the market has of the CRA and relax the assumption that preliminary and official ratings do not diverge. Sangiorgi et al. [2009] conjecture that notching[9] can arise in equilibrium due to the selection effect of rating shopping. Unlike these articles, my setting considers a monopolistic CRA with reputation concerns and a two stage rating process whereby two different evaluations take place and which relates more closely to reality.[10]

[8]Ratings shopping is defined as the practice of asking several CRAs for a rating, seeking the highest rating. Rating inflation (or rating bias) refers to the possible bias that the CRAs have when assessing the creditworthiness of an issuer in order to ensure related and future profits.

[9]Practice that involves the reduction of a competitor's rating when reporting the rating on another scale.

[10]This two stage rating process is well described in IOSCO [November 2016]when explaining Structured

This paper is also related to the strand of the literature on the effect of reputation on the CRAs behaviour. Bolton et al. [2012] show for instance how the exogenous expected reputation cost drives the reporting strategy of the CRA. Unlike their work, my setting allows for endogenous reputation. To capture the potential gains associated with reputation, Camanho et al. [2012], look at an infinite period setting and argue that under monopoly the penalty for lying decreases with reputation because investors tend to attribute failures to bad luck. Bar-Isaac and Shapiro [2013] claim that the value of reputation depends on economic fundamentals that fluctuate over the business cycle. By investigating how CRAs' incentives to provide quality ratings are affected by the business cycle, they find that ratings quality is countercyclical.[11] Mariano [2012] instead shows that in a monopolistic setting, a CRA may disregard its private information, conforming to public information when driven by reputation maximisation and market power protection.[12] The main implication is that reputational concerns may lead to inaccurate and unreliable ratings.

In my setting, I focus only on the reputation that the CRA can gain from the market. Investors update the probability that the CRA is well informed (their measure of the CRA's reputation). I am abstracting from the possibility for the CRA to also gain a different type of reputation among issuers[13]. I do so to concentrate on those market participants that, even though not directly purchasing ratings, still represent one of the categories relying extensively on them. As a result, the CRA wants to be perceived as accurate when evaluating financial products in order to be considered reliable and useful to investors. There are other contexts in which a player may be concerned with the perception that others have on his ability to process information. In this regard Levy [2005] analyses careerist judges who need to signal their ability to interpret correctly the law. Here the reputation motives lead to a creative behaviour whereby previous decision

Finance Preliminary Assessments. This consultation report discusses all types of other CRA products which are not Traditional Credit Ratings.

[11]In Bar-Isaac and Shapiro [2011] the authors show that the CRA's ability to monitor its analysts and investment bank profitability can improve the accuracy of the ratings.

[12]Other contributions aim at explaining why bond and structured products markets are characterized by a different rating behaviour either in the light of reputation spillovers (Rablen [2013]) or "double reputation" (Frenkel [2015]). In the first case the analysis focuses on monitoring while in the second it focuses on market concentration.

[13]In Camanho et al. [2012] both issuers and investors have the same prior on the type of CRA and use Bayesian updating to update believes.

get contradicted more often than what would be welfare maximising. I find that the shift in the behaviour of the CRA is a result of the controversial effect that disclosure has on the CRA's reputation. The revelation of additional information to the investors causes the CRA to suffer from adverse market inference.

The role of credit rating agencies can be seen as to that of an intermediary who delivers information[14]to the market, information that the investors will be relying on either because they want to or because they have to due to regulation[15]. Several articles model information intermediaries such as Ottaviani and Sorensen [2006], where the expert awareness concerning his ability seems to make reputation concerns completely irrelevant or Lizzeri [1999], where a certification intermediary operating in an asymmetric information environment with market power has an incentive to manipulate information when information revelation is a strategic decision. In the first case the experts do not truthfully reveal private information because reputation is updated on the basis of both the reported signal and the realised state, thus they have an incentive to manipulate the report to achieve a better reputation. In the second one, the results show that the monopolist discloses the minimum amount of information needed to allocate the good. Credibility and manipulation are at the heart of the model by Benabou and Larocque [1992], which shows that individuals with private information can manipulate public information, a mix of lies and truth can partially control the effects of actions on reputation. The main implication of the model by Pollrich and Wagner [2016] is that the tendency to push certification to be precise might end up in less reputation building and thus in less resistance to capture[16]. Strausz [2005] analyses the threat of capture in certification markets showing that the price for credibility is high; under full disclosure, in order to retain credibility and for a low discount factor, the certifier charges a fee above the monopoly price.

The reminder of this article is as follows. Section 1 is devoted to the model setup, Section 2 presents the no disclosure assumption and the results. Section 3 deals with the disclosure of preliminary contacts and the result of this model. In Section 4 a discussion of the results, of the possible implications and two extensions are presented.

[14]In this particular industry, information takes the form of an opinion based on private information. CRAs observe a signal about the quality of an issuer and provide uninformed parties with additional information concerning a financial product.

[15]Regulatory reliance has been investigated in Opp et al. [2013], Stolper [2009] and Efing [2013].

[16]According to this view, in the light of the financial crisis, it might be harmful to force rating agencies to issue more precise ratings because they would be more exposed to capture problems.

1 The model

Consider a credit rating agency (CRA), entrepreneurs (ENT) and investors. Investors have a limited role, they observe the published rating, if available, and choose whether to fund the project. After that, they form beliefs on the quality of the CRA. Details about the game are provided in the following sections.

1.1 Agents

There is a continuum of entrepreneurs of mass 1. Each entrepreneur needs to finance a project. Projects differ in quality. The higher the quality of the product the lower the probability of default. For simplicity, denote the quality of a project $\omega \in \{G, B\}$. If financed, a good project always succeeds (s) whereas a bad project always results in a default (f). The quality of the project is not known ex-ante, but there is a common prior belief $\alpha \in (0, 1)$ that the project is good. Whether an investment grade rated project results in a success or in a failure is of no interest to the entrepreneur.

Assumption 1. *Irrespective of its quality the entrepreneur wants to carry out the project.*

To carry out the project the entrepreneur just needs investors to invest in it.[17] Investors observe the published official rating $r \in \{IG, J\}$, if available, and subsequently decide whether to fund the project. The investment takes place when an *investment grade* (IG) rating is published and does not take place either when a *junk* (J) rating is published or when the project remains unrated.[18] The entrepreneur does not know the quality of his project and hence cannot credibly communicate the quality of the project to the market.[19] The market needs a trustworthy source to certify and issue an evaluation.[20] Asking for a rating is not compulsory, however the CRA offers a preliminary rating, $P \in \{G, B\}$, at

[17]A situation where the entrepreneur is not concerned about failure could be when he does not invest his own money

[18]In the following sections I will make a distinction between unrated not visible projects and unrated but visible projects. Regardless of the disclosure regime and their belief about the CRA type, the investors purchase after an IG rating and don't purchase after a J rating.

[19]The unawareness of the entrepreneur concerning the quality of the project is a common feature in the CRA literature. Here the fact that the entrepreneur is uninformed is a simplifying assumption which does not however affect the results.

[20]Here, the CRA is the certifier and the evaluation is the rating.

no cost and thus it is always in the interest of the entrepreneur to ask for one. Knowing the result of the preliminary rating P, the entrepreneur can either ask for a official rating at a fee Φ, or remain unrated and hence abandon the project.

The CRA offers an evaluation of the quality of the project. The certification happens in two stages. First there is a preliminary stage, resulting in a verbal message P. Subsequently there is an official rating stage producing a published official rating $r \in \{IG, J\}$. This stage is only reached if the entrepreneur decides to go ahead and pay the CRA. In the preliminary stage, the CRA gives the initial evaluation, either good ($P = G$) or bad ($P = B$), only to the entrepreneur. When asked to release the official rating, the CRA earns the fee and generates either an investment grade ($r = IG$) or a junk ($r = J$) rating which is published. That is, the entrepreneur cannot prevent a bad rating from being published.[21]

The CRA charges the entrepreneur a fee Φ only in the rating stage. The fee Φ is exogenous and does not depend on the published rating.[22] Hence, the CRA is not paid for the initial evaluation but only for the rating that is disclosed.[23] The quality of the CRA is denoted by $I \in (I, U)$. It can either be of the *informed* type (I) with probability λ or of the *uninformed* type (U) with probability $(1-\lambda)$. The informed type is an exemplary CRA that always produces correct ratings and reveals them honestly.[24] The CRA knows

[21] According to the Parliament and the Council [May 2013], once the official rating is ready, the CRA has to communicate to the issuer the evaluation a full working day before the rating becomes publicly available. The issuer can stop the publication process only if relevant information is contained in the press release or if new relevant information has become available.

[22] A common practice (i.e. Bolton et al. [2012]) is that of assuming that the entrepreneur pays the CRA only if the rating is good. In my model, publishing occurs regardless of whether the rating is good or bad and there is no space for contingent fees. By doing so I implement what is suggested in the Parliament and the Council [May 2013] in terms of independence of the charged fee from the outcome of the performed service and in terms of non discriminatory fees. In Kovbasyuk [2013] instead, of the three regulatory environments that are taken into consideration (publicly disclosed fixed payment, publicly disclosed and rating contingent payment, private rating contingent payment.) a desirable regulation should allow rating contingent fees and require their disclosure.

[23] The decision of having an exogenous fee is dictated by the choice of having results for a non restricted value of the fee. The fee could be determined endogenously by the monopolist, but the value added by this extra step is out of the scope of the present paper where the focus is more on reputation. Moreover, in the presented setting what can influence the entrepreneur is not the fee in absolute terms but its relation to the funding. Therefore this simplifing assumption allows not to restrict the range of possible fees. Notice that in reality the fee is determined as a percentage of the issuance which is different according to the type of product.

[24] I follow the literature in which one type of CRA is behavioural. For instance in Mathis et al. [2009] the exemplary CRA is a truthful one which is committed to always tell the truth; the authors employ a

its type.

1.2 Credit Ratings

In the preliminary stage the informed CRA always has the ability to correctly assess the quality of the project even with little information and will always do so. Furthermore, in the rating stage this type of CRA correctly evaluates the project, as if the received signal was fully revealing.[25] This can be summarised by the following Assumption:

Assumption 2. *The informed CRA correctly evaluates the project throughout the whole rating process.*

The informed CRA is committed to providing the correct evaluations. Given that this type of CRA has the ability to release the correct rating knowing that she is informed, she is behavioural in the sense that sticks to the reputation argument and has no incentive to misbehave. In the preliminary stage the uninformed CRA can wrongfully evaluate the project.[26]. The uninformed CRA is not sufficiently skilled to be sure of whether its initial evaluation is correct or not. Even if uninformed in the first stage, the CRA can become informed when asked to provide an official report. At the official rating stage the CRA has access to an information acquisition technology which generates an information signal $\sigma \in \{G, B\}$ on the quality of the project. In particular, the technology is such that the CRA learns the true quality of the project with probability e and receives an incorrect signal with probability $1 - e$.

$$Pr(\sigma = G|G, U) = Pr(\sigma = B|B, U) = e$$

dynamic model of reputation where a monopolist CRA can mix between lying and truth telling to build reputation, and find two possible unique active equilibria, a truthful and a non-truthful one. Similarly to Camanho et al. [2012] "honest" CRA, the informed CRA in this paper, issues the correct rating to the project.

[25]If the informed CRA has access to an information acquisition technology which generates an information signal $\theta \in \{G, B\}$ on the quality of the project, it is the case that $Pr(\sigma = G|G, I) = Pr(\sigma = B|B, I) = 1$ and $Pr(\sigma = G|B, I) = Pr(\sigma = G|B, I) = 1$. In other words the signal in the rating stage is always sufficient for the informed CRA to correctly assess the project.

[26]In an initial version, the model allowed, in the preliminary stage, for the uninformed CRA to have access to an information acquisition technology generating an information signal $\theta \in \{G, B\}$ on the quality of the project. In particular, the CRA would receive a correct signal in the preliminary stage with probability β whereas it would receive no signal ($\sigma = \varnothing$) with the complementary probability, where β was the signal precision, exogenously given and not costly. This version, simplified by $\beta = 0$, does not affect the results.

$$Pr(\sigma = G|B, U) = Pr(\sigma = B|G, U) = 1 - e$$

where $e \in (\frac{1}{2}, 1)$ is the signal precision, exogenously given and not costly.

1.3 Strategies and Payoffs

As Assumption 2 states, the informed CRA behaves truthfully. I am interested in studying the behaviour of the uninformed CRA. In the preliminary stage the uninformed CRA decides in which circumstances it is optimal to issue a good initial evaluation. The CRA makes a strategic decision $x = 1$ when issuing a positive preliminary ($P = G$) or $x = 0$ when issuing a negative preliminary ($P = B$).[27] The incentives of the CRA are to choose the preliminary rating which leads to the purchase of the official rating from the entrepreneur and to the build of a reputation in the market.

After the entrepreneur receives P, he decides when to go ahead and pay the fee to the CRA. I denote this strategic decision $y_i \in Y = \{0, 1\}$ with $i \in G, B$. More precisely $y_i = 1$ indicates the entrepreneur asking for the official rating at a cost Φ given the preliminary P and $y_i = 0$ indicates the decision to stop and remain unrated. In other words, faced with a good preliminary rating, the entrepreneur will ask for the official rating for $y_G = 1$, whereas he will ask for it when the preliminary is bad for $y_B = 1$.

The entrepreneur's only concern is to carry out the project which requires funding. Recall that, even if the entrepreneur was aware of the quality of his project, the fact that he does not invest money of his own makes him uninterested in success or failure. His payoff is given by the difference between the funding obtained and the cost of the rating. Whenever a project receives an IG official rating the entrepreneur has access to funding V_G, however he also has to bear the cost Φ of hiring a CRA to produce the official rating. In the eventuality that a J official ratings is issued, the entrepreneur doesn't get funded and loses the fee he paid. When the entrepreneur decides not to purchase the official rating, he has to abandon the project, because only an investment grade rating allows the entrepreneur to access financing. Therefore V_G has to be large enough to cover the fee that has to be paid to get rated. Formally

Assumption 3. $V_G - \Phi > 0$.

[27] I introduce x as a notation for mathematical convenience in the computation of payoffs. This is necessary because the decision on whether to issue $P = G$ or $P = B$ is non numerical.

The are three possible outcomes for the entrepreneur

$$\Pi_{ENT} = \begin{cases} V_G - \Phi, & \text{if } r = IG \\ -\Phi, & \text{if } r = J \\ 0, & \text{otherwise} \end{cases} . \tag{1}$$

The CRA earns the fee when the official rating is purchased which is then published and nothing otherwise. In addition, the payoff Π_{CRA} includes a reputation term defined as the perception that investors have of the CRA being of the informed type multiplied by a reputation gain which is here normalised to one. Thus the CRA is motivated by proving its ability to correctly evaluate a project. The updated belief that the investors have about the CRA being of the informed type in the light of the observed rating and of the following result (success or fail of the project) provides the reputation part. In general

$$\Pi_{CRA} = \delta\Phi + \mu \tag{2}$$

Where δ takes the value one if the entrepreneur asks for the official rating and zero otherwise and μ represents the reputation payoff given by the beliefs of the investors.

Summary of Notation	
α	Prior probability of G projects
λ	Prior probability of informed CRAs
e	official rating stage signal precision
Φ	Fee paid to Cra if official rating issued
V_G	Amount of funding
P	Preliminary rating
y_G	ENT's decision after $P = G$
y_B	ENT's decision after $P = B$
x	CRA's strategic decision
r	Official rating
m	Ratio between Φ and V_G

1.4 The Timing

The timing of the game is as follows:

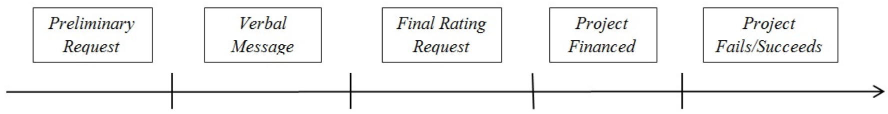


Figure 1: Order of events.

1. Nature chooses state $\omega \in \{G, B\}$ and types $I \in \{I, U\}$;

2. the CRA costlessly verbally discloses its preliminary evaluation $P \in \{G, B\}$;

3. The entrepreneur, knowing the preliminary evaluation decides whether or not to purchase the official report;

4. The CRA issues an official rating $r \in \{IG, J\}$ which is published;

5. Investors invest. IG project which are then carried either succeed or fail, investors update their believes concerning the quality of the CRA and payoffs are realised;

Figure 1 shows the order of events. There is a preliminary rating request followed by an evaluation in the form of a verbal message. The entrepreneur can then request the official rating which leads to the project being financed. Projects that are carried out in the end either succeed or fail.

Figure 2 shows the extensive form of the game. It does not indicate payoffs at the end nodes but those are implied by the type of rating which is published. The end nodes labelled IG, s (from now on IGS) are those in which the entrepreneur gets funded and the uninformed CRA can be perceived as informed. Those labelled IG, f (from now on IGF) represent the cases in which the entrepreneur accesses funding but the project fails. At these nodes the CRA is revealed to be uninformed. The J nodes imply no funding for the entrepreneur and no reputation gain for the CRA whereas the NR nodes represent the cases in which the entrepreneur decides to stop after the preliminary and the associated reputation for the CRA depends on the disclosure regime described in the next section.

The relevant notion of equilibrium here is perfect Bayesian equilibrium (PBE). In this preliminary rating game, a PBE consists of sequentially rational strategy profiles for the

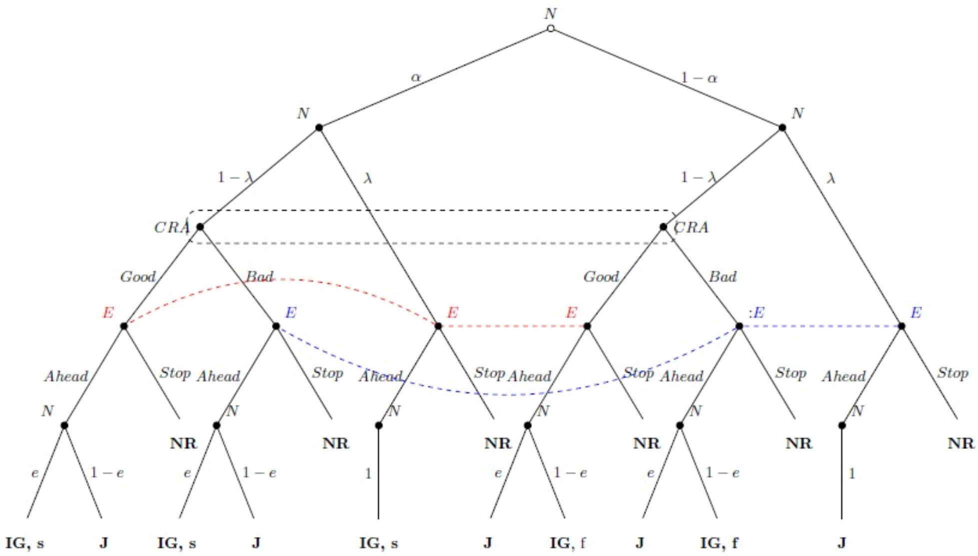


Figure 2: The game in extensive form.

CRA and the entrepreneur consistent with the belief system held by the investors. The investor doesn't observe whether official rating comes from an informed or uninformed CRA. Thus it constructs beliefs based on what has happened prior to publication of the official rating. Such believes do not change the purchasing strategy of the investors which consists in only investing in IG rated projects.[28]

2 Equilibrium Analysis

This section is devoted to the characterisation of the equilibrium in a general disclosure scenario. Subsection 2.1 deals with the choice of the entrepreneur to ask for the official rating, subsection 2.2 deals with the update of investors' beliefs, subsection 2.3 investigates the CRA's choice and presents the results. This section is followed by section 3 where two disclosure requirements are analysed.

[28]The rationale for the relevance of the investors beliefs for the CRA but not for the investors' own decision on whether to fund or not is the following: on the one hand investors are bounded to invest in IG rated projects, thus their belief on the quality of the CRA is irrelevant, on the other however the CRA has an interest in the investors' beliefs for market recognition. Indeed if investors end up not believing in the CRA, the very role of the CRA fails and future regulation could end up removing rating contingent requirements.

2.1 The Entrepreneur's decision to buy the official rating

After having received the preliminary rating, the entrepreneur has to decide whether to buy the official rating or stop and abandon the project. There are three possible outcomes from going ahead. Either the project is valued IG and succeeds or the project is valued IG but then fails or the project is valued J and no funding is obtained. Regardless of the preliminary, asking for the official rating involves a cost Φ to be paid to the CRA. On the one hand, if the entrepreneur buys he gets V_G with some probability q_i, with $i = \{G, B\}$ and can fund the project. Recall that by Assumption 1 the entrepreneur is only concerned about funding. On the other hand if he does not buy the official rating, his payoff is zero.[29]. Let q_G be the probability that a project is certified as IG given a positive preliminary

$$q_G = \frac{\alpha\lambda + \alpha(1-\lambda)xe + (1-\alpha)(1-\lambda)x(1-e)}{\alpha\lambda + \alpha(1-\lambda)x + (1-\alpha)(1-\lambda)x} \tag{3}$$

and similarly let q_B be the probability that a project is certified as IG given a negative preliminary rating

$$q_B = \frac{\alpha(1-\lambda)(1-x)e + (1-\alpha)(1-\lambda)(1-x)(1-e)}{\alpha(1-\lambda)(1-x) + (1-\alpha)\lambda + (1-\alpha)(1-\lambda)(1-x)} \tag{4}$$

The numerator of q_G (q_B) is the sum of the probabilities associated with ending the game with an investment grade rating after receiving a positive (negative) preliminary rating, whereas the denominator accounts for all the possible outcomes following the received preliminary. Notice that both (3) and (4) depend on x because they take into consideration not only the case in which the preliminary comes from an uninformed CRA but also the case in which the CRA is informed.[30]

In the case of $P = G$, the entrepreneur compares the expected payoff from asking for

[29] I assume success or failure do not affect the entrepreneurs decision because he does not invest his own funds.

[30] This is to say that the entrepreneur doesn't know whether the CRA is informed or uninformed. Thus receiving a good preliminary rating doesn't imply that $x = 1$. It could be that $x = 0$ and the good preliminary ratings is due to an informed CRA.

the official rating and paying the fee with zero,[31] and will go ahead if and only if

$$\frac{\alpha\lambda - (\lambda - 1)x(-\alpha + (2\alpha - 1)e + 1)}{\alpha\lambda - \lambda x + x} V_G - \Phi > 0$$

Which can be rewritten as $q_G V_G - \Phi > 0$. When the initial evaluation is bad ($P = B$), the entrepreneur compares the expected payoff for choosing to go ahead with abandoning the project and will ask for the official rating if and only if

$$\frac{(\lambda - 1)(x - 1)(-\alpha + (2\alpha - 1)e + 1)}{-\alpha\lambda + (\lambda - 1)x + 1} V_G - \Phi > 0$$

which can similarly be rewritten as $q_B V_G - \Phi > 0$. For future references, define q_G^1 (q_B^1) and q_G^0 (q_B^0) as q_G (q_B) when $x = 1$ and $x = 0$. Given the choice of the uninformed CRA, these are the probabilities that the entrepreneur gets funded after a certain preliminary.

Lemma 1. *Given a preliminary rating $i = \{G, B\}$, the entrepreneur buys the official rating if*

$$q_i > \frac{\Phi}{V_G} \tag{5}$$

where q_G and q_B are defined in (3) *and* (4) *respectively.*

The entrepreneur will therefore buy if $q_G > \frac{\Phi}{V_G}$ and stop if $q_G < \frac{\Phi}{V_G}$. Similarly, the entrepreneur will choose to buy if $q_B > \frac{\Phi}{V_G}$ and will stop if $q_B < \frac{\Phi}{V_G}$. Notice that $q_G > q_B$, it is more likely to get funded after receiving a good preliminary rating than it is after a bad preliminary has been issued.

Lemma 2. *There cannot be an equilibrium in which the entrepreneur faced with $P = G$ remains unrated ($y_G = 0$) and the entrepreneur faced with $P = B$ buys the official rating ($y_B = 1$).*

Proof. The proof goes as follows;: $y_G = 0$ and $y_B = 1$ require $q_B > \frac{\Phi}{V_G} > q_G$. However, q_G is always greater than q_B. Hence $y_G = 0$ and $y_B = 1$ cannot occur at the same time. □

For the sake of clarity, let me introduce the following definitions[32]

[31] Not going ahead gives a zero payoff because no funding occurs and no fee is paid. This assumption is similar to Camanho et al. [2012] where a bad rating and no rating are considered equivalent outcomes in the model.

[32] The terms confident and conservative indicate a rational behaviour of the CRA, they do not carry any behavioural implication.

Definition 1. *A CRA is (i) confident when she issues $P = G$ ($x = 1$) (ii) conservative when she issues $P = B$ ($x = 0$).*

Lemma 3. *There cannot be an equilibrium in which the CRA is confident and the entrepreneur faced with $P = B$ asks for the official rating with a positive probability ($y_B > 0$).*

Proof. If $x = 1$ then $q_B^1 = 0$. This implies that Lemma 1 cannot hold. □

Intuitively, if the uninformed CRA is overconfident, a $P = B$ result of the preliminary evaluation can only be due to an informed CRA. The entrepreneur knows that the preliminary evaluation will be confirmed in the rating stage if he decides to go ahead and therefore prefers a zero payoff from stopping to paying the fee an surely loosing Φ.

Lemma 4. *There cannot be an equilibrium in which the CRA is conservative and the entrepreneur faced with $P = G$ remains unrated with a positive probability ($y_G < 1$).*

Proof. If $x = 0$, then to $q_G^0 = 1$. This implies that 1 holds by Assumption 3. □

This is intuitive: if the uninformed CRA is conservative, a $P = G$ evaluation can only be due to an informed CRA. The entrepreneur knows this and buys because there is no risk to receive a J rating in the rating stage. The expected payoff of the entrepreneur faced with a good preliminary rating when asking for the official rating is $V_G - \Phi$ which is greater than zero that is the payoff if he remains unrated.

2.2 The investors' beliefs

After funding has occurred and the project is, in case, carried out, the market observes one of three possible outcomes: a project rated investment grade which succeeds (IGS), a project rated investment grade which fails (IGF) and a project rated junk (J) which is not carried out. For each of these possible scenarios, the market forms an updated belief μ of the quality of the rating agency. Recall that an uninformed CRA would like to impress the market by being perceived as informed in order to gain reputation whereas an informed CRA always tells the truth in order to maintain its reputation by assumption.

The investors update their beliefs about the type of the CRA according to the Bayes rule. If they observe a successful investment grade rated project, their belief that the CRA is of the informed type becomes

$$\mu_{\text{IG, s}} = \frac{\lambda y_G}{e(1-\lambda)\left((1-x)y_B + xy_G\right) + \lambda y_G} \tag{6}$$

When they observe a project which received a junk rating, the market posterior probability that the CRA is of the informed type is

$$\mu_J = \frac{(1-\alpha)\lambda y_B}{(1-\lambda)(\alpha - 2\alpha e + e)\left((1-x)y_B + xy_G\right) + (1-\alpha)\lambda y_B} \tag{7}$$

When a project rated IG fails, the market infers that the rating came from an uninformed CRA, $\mu_{\text{IG, f}}$ is therefore zero in this case. I am going to consider two different disclosure assumptions, for now let β be the posterior probability that the market holds about the CRA being informed after no rating is published.[33]

2.3 The Uninformed CRA's decision on the preliminary rating

The CRA, if of the uninformed type, decides whether to issue a good preliminary or a bad one. The CRA does not know whether the project is good or bad. Recall that $\mu_{\text{IG, f}}$, $\mu_{\text{IG, s}}$, μ_J and β are respectively the updated believes of the market when a successful project is given an investment grade rating, when a project which results in a failure was given an investment grade rating, when a project received a junk rating and when a project didn't receive a rating.

The payoff of the CRA from issuing $P = G$ in the preliminary stage is

$$\pi_{G,s}\mu_{IG,s} + \pi_{G,f}\mu_{IG,f} + \pi_{G,J}\mu_J + \pi_{G,NR}\beta + \Phi y_G \tag{8}$$

whereas from issuing $P = B$ in the preliminary stage the CRA gets

$$\pi_{B,s}\mu_{IG,s} + \pi_{B,f}\mu_{IG,f} + \pi_{B,J}\mu_J + \pi_{B,NR}\beta + \Phi y_B \tag{9}$$

Both (8) and (9) consist of two parts, a profit term whereby the CRA gets the fee whenever the entrepreneur decides to go ahead and a reputation term. This term can be explained as follows; with probability $\pi_{G,s}$ ($\pi_{B,s}$) a project initially evaluated as good (bad) by an

[33] In Section 3.1 and 3.2 there are assumptions on disclosure requirements which lead to two different posterior beliefs of the investors. For now, let β be the general posterior belief held by investors.

uninformed CRA succeeds and the CRA gains reputation (the investors now believe that the CRA is informed), with probability $\pi_{G,J}$ ($\pi_{B,J}$) a project initially evaluated as good (bad) gets a J rating, boosting the CRA's reputation because the actual quality cannot be verified. The third term, $\pi_{G,f}\mu_{\text{IG, f}}$ ($\pi_{B,f}\mu_{\text{IG, f}}$), equals 0 an thus there is no reputation gain. With probability $\pi_{G,NR}$ ($\pi_{B,NR}$) initially rated as $P = G$ ($P = B$) remains unrated and the CRA gets β. [34]

Thus, (8) and (9) can be written as

$$(\alpha e y_G)\mu^1_{IG,s} + (\alpha - 2\alpha e + e)y_G\mu^1_J + (1 - y_G)\beta + \Phi y_G \tag{10}$$

$$(\alpha e y_B)\mu^0_{IG,s} + (\alpha - 2\alpha e + e)y_B\mu^0_J + (1 - y_B)\beta + \Phi y_B \tag{11}$$

Where μ^1_J (μ^0_J) and $\mu^1_{IG,s}$ ($\mu^0_{IG,s}$) are respectively (6) and (7) when $x = 1$ ($x = 0$).

In words, if the entrepreneur does not buy, the reputation of the uninformed CRA is only β. If the entrepreneur buys, the CRA earns the fee. In addition, if the project is good and thus successful the reputation becomes $\mu_{IG,s}$ whereas if the project is rated junk the reputation is μ_J. Before stating the result, for $y_G \neq y_B$ define

$$\Phi^* \equiv \frac{\alpha e(y_B\mu^0_{IG,s} - y_G\mu^1_{IG,s}) + (\alpha - 2\alpha e + e)(y_B\mu^0_J - y_G\mu^1_J)}{y_G - y_B} + \beta \tag{12}$$

which leads to

Lemma 5. *Let Assumption 3 hold*

(i) if $y_G > y_B$ then the uninformed CRA strictly prefers to issue a good preliminary rating ($P = G$) if and only if $\Phi > \Phi^*$

(ii) if $y_G = y_B$ the the CRA is indifferent between issuing a good preliminary rating ($P = G$) or a bad preliminary rating ($P = B$)

For greater clarity let me introduce labels for the strategies of the entrepreneur

Definition 2. *An entrepreneur is (i) responsive when he goes ahead after a good preliminary ($y_G = 1$) but stops after a bad one ($y_B = 0$) (ii) pessimistic when he always stops ($y_G = y_B = 0$) (iii) optimistic when he ways goes ahead ($y_G = y_B = 1$)*

[34] Full derivation of these probabilities con be found in Appendix.

This discussion can be summarised in the following proposition

Proposition 1. *If Assumption 3 holds the equilibrium in the model is as follows:*

(i) For $\Phi > \Phi^*(1,0)$ and $0 < \frac{\Phi}{V_G} < q_G^1$ the CRA is confident and the entrepreneur is responsive and goes ahead after a good preliminary ($y_G = 1$) but stops after a bad one ($y_B = 0$).

(ii) For $q_G^1 < \frac{\Phi}{V_G} < 1$ the CRA is confident and the entrepreneur is pessimistic.

(iii) For $0 < \Phi < \Phi^*(1,0)$ and $q_B^0 < \frac{\Phi}{V_G} < 1$ the CRA is conservative and the entrepreneur is responsive.

(iv) For $0 < \frac{\Phi}{V_G} < q_B^0$ the CRA is conservative and the entrepreneur is optimistic.

To show that the above mentioned are the only possible equilibria, the reasoning goes as follows. All the potential equilibria in which $x = 0$ and $y_G = 0$ are ruled out by Lemma 4, similarly all the cases in which $x = 1$ and $y_B = 1$ are ruled out by Lemma 3. Proof is given in the appendix

3 Disclosure requirements

Focusing on the European Regulation, it is possible to investigate whether imposing disclosure of the contact between the CRA and the entrepreneur affects the decisions of the players and thus the resulting equilibrium[35].

I consider two different disclosure assumptions. In the no disclosure model, there is no evidence of the preliminary contact between the CRA and the entrepreneur whereas the disclosure setting allows for the disclosure of the contact. In the no disclosure case, asking for a preliminary rating is a confidential action, known only to the parties involved (i.e the CRA and the entrepreneur. As a result, unrated projects are not visible to the market and investors do not update their beliefs on the quality of the CRA.

[35]For the purpose of the model I refer to mandatory disclosure as a way to increase transparency in the industry while Freixas and Laux [2011] focus on the effectiveness of disclosure during the crisis and discuss proposals and policy implications highlighting the distinction between transparency and disclosure. One of their core suggestions is that more information, for instance in terms of compulsory disclosure, does not necessarely lead to more transparency.

Assumption 4. *The contact between the CRA and the entrepreneur is not seen by the investor. The investors act myopically and do not react to the lack of information.*

The strong assumption in myopic investors aims at representing the real issue concerning the impossibility for the investors to monitor the whole market, keeping an eye on all entrepreneurial projects which could arise. A myopic investor is simply an investor for whom the lack of a published rating is uninformative.[36] Under this assumption, the investors fail to update their belief on the quality of the CRA. Conversely, under contact disclosure, when costlessly checking the project, the CRA must disclose that it has been contacted by the entrepreneur for a preliminary evaluation. As a result, the requirement of making the market aware of who has asked for the services of the CRA results in the visibility of unrated project. This in turn affects the CRA's reputation through the updated belief that the market has of it being of the informed type.

Assumption 5. *The contact between the CRA and the entrepreneur is public information.*

Note that the timing of the game is the same under the two scenarios and that the actual result of the preliminary evaluation is communicated only to the entrepreneur in both cases. If on the one hand the second scenario adds complexity to the payoff of the CRA, on the other it allows the payoff of the entrepreneur to remain unchanged. Indeed, the entrepreneur receives no additional information under the contact disclosure scenario, whereas the CRA's reputation is affected by the different belief that investors hold in the two cases. More details will be given in section 3.1 and section 3.2.

3.1 The Model without Contact Disclosure

This section is devoted to the characterisation of the equilibrium under no disclosure, thus suppose Assumption 4 holds. Without evidence about the contact between the CRA

[36] To justify such an assumption one can think of a situation at the outset of the model characterised by uncertainty on the existence of a project. For instance, if the environment is such that at the outset the investors have a prior τ that there is a project out for evaluation and with the complementary probability there isn't. The investor cannot be certain about the existence of the project whereas CRA and entrepreneur have this information. Thus the updated belief when no rating is published is $\beta = \frac{\tau((1-\alpha)\lambda(1-y_B)+\alpha\lambda(1-y_G))}{y_B(-\alpha\lambda+(\lambda-1)x+1)+y_G(\alpha\lambda-\lambda x+x)-1} + \lambda(1-\tau)$. It can be easily shown that if $\eta = 1$ then $\beta = \mu_{NR}$ whereas if $\eta = 0$ then $\beta = \lambda$. Moreover it is the case that $\mu_{NR} < \beta < \lambda$. This implies that the setting presented in this paper is a simplified version of the one explained here. As long there is some lack or wrongful information on the side of the investors, the results hold.

and the entrepreneur, if the entrepreneur decides not to purchase the official rating the updated reputation is represented by the prior λ on the quality of the CRA[37]. In other words, when the CRA is not mandated to disclose the contact with the entrepreneur, investors fail to update their belief on the quality of the CRA if the project is unrated. Unrated projects are not visible and investors hold the same belief as their prior. Hence (8) > (9) can now be written as

$$y_G[(\alpha - 2\alpha e + e)\mu_J^1 + \alpha e \mu_{IG,s}^1 - \lambda + \Phi] > y_B[\alpha e \mu_{IG,s}^0 + (\alpha - 2\alpha e + e)\mu_J^0 - \lambda + \Phi] \quad (13)$$

Corollary 1. *If the CRA is not required to reveal the contact (Assumption 4) then* $\beta = \lambda$ *and* $\Phi^* = \bar{\Phi}$*, where*

$$\bar{\Phi} \equiv \frac{\alpha e(y_B \mu_{IG,s}^0 - y_G \mu_{IG,s}^1) + (\alpha - 2\alpha e + e)(y_B \mu_J^0 - y_G \mu_J^1)}{y_G - y_B} + \lambda$$

The above implies that $\bar{\Phi}(1,0) = \frac{\lambda(e(\alpha+\lambda-1)-\lambda)}{e(\lambda-1)-\lambda}$, where $\bar{\Phi}(1,0)$ is $\bar{\Phi}$ when $y_G = 1$ and $y_B = 0$.

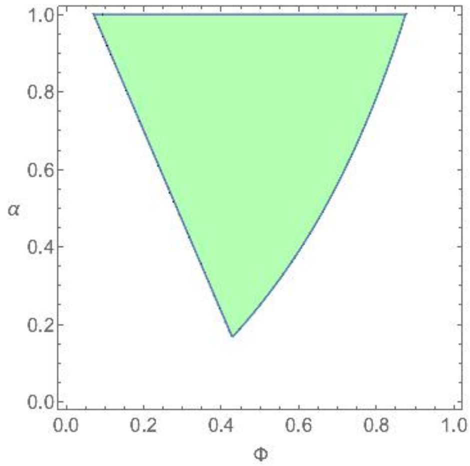
Corollary 2. *There is multiplicity of equilibria when*

(i) $0 < \Phi < \bar{\Phi}$ *and* $q_G^1 < \frac{\Phi}{V_g} < 1$

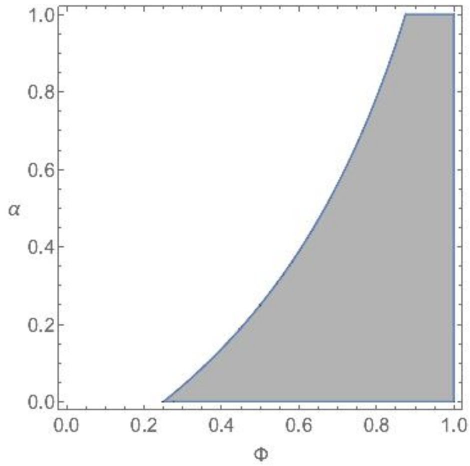
(ii) $\Phi > \bar{\Phi}$ *and* $0 < \frac{\Phi}{V_g} < q_B^0$

Multiplicity occurs when $\frac{\Phi}{V_G}$ is either sufficiently low or sufficiently high with respect to the likelihood of getting funded. If ${q_B}^0$ is big enough the entrepreneur faced with a bad preliminary may be willing to go ahead and risk the fee. The reasoning goes as follows: even though he received a bad preliminary, the fee is low, therefore he is willing to pay the fee for the official rating. Similarly, if ${q_G}^1$ is small enough, the entrepreneur faced with a good preliminary may prefer to stop and remain unrated rather than risking the fee. The reasoning goes as follows: even though he received a good preliminary, the fee is however high, therefore he is not willing to pay and risk the fee for the official rating

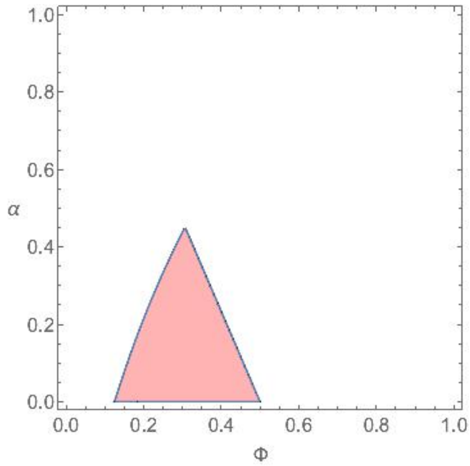
[37]Faure-Grimaud et al. [2009] show that in order for no disclosure to be valuable it is necessary that contracting between the firm and the certifier is secret and that the true value of the firm is unknown also to the firm itself. Here the secrecy of the contact makes the contracting invisible and no disclosure is valuable in the sense that it does not affect the CRA's reputation.



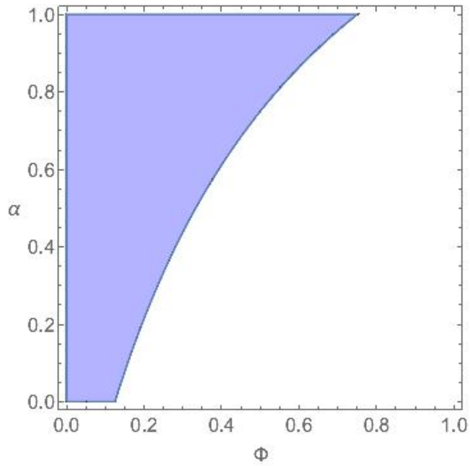
(a) Equilibrium *(i)*



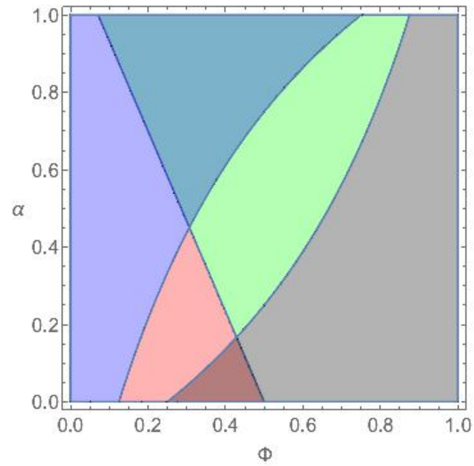
(b) Equilibrium *(ii)*



(c) Equilibrium *(iii)*



(d) Equilibrium *(iv)*



(e) Equilibrium *(i)* to *(iv)*. Top left dark turquoise area of multiplicity (i) and (iv); bottom burgundy area of multiplicity (ii) and (iii)

Figure 3: Equilibrium without disclosure for $e = 3/4$, $\lambda = 1/2$ and $V_G = 1$.

For the sake of the discussion, fix $e = 3/4$, $\lambda = 1/2$ and $\Phi = 1/3$ and rename V the ratio between the fee and the amount of funding. In Equilibrium (i), projects are relatively good and the impact of the fee on the funding is small therefore the CRA prefers to issue a good preliminary $P = G$, the entrepreneur prefers to go ahead if he hears $P = G$ but prefers to stop if he hears $P = B$. Receiving a bad preliminary along the equilibrium path in which the uninformed CRA is issuing a good preliminary implies that the preliminary has to come from the informed CRA. Under these circumstances, asking for the official rating would only result in the loss of the fee.

In equilibrium (iii), projects are more likely to be bad and the impact of the fee on the funding is high therefore the CRA prefers to issue a bad preliminary $P = B$, the entrepreneur prefers to go ahead if he hears $P = G$ but prefers to stop if he hears $P = B$. Receiving a good preliminary along the equilibrium path where the uninformed CRA is issuing a bad preliminary implies that the preliminary is due to the informed CRA and therefore the entrepreneur is sure to get funded.

In equilibrium (ii), unfavourable combinations α and V (i.e. high proportion of good projects and huge impact of the fee on the funding or low proportion of good projects and medium-high impact of the fee) lead the entrepreneur to stop regardless of the opinion of the CRA. Similarly, in the equilibrium (iv), favourable combinations of α and V (i.e. low proportion of good projects and tiny impact of the fee on the funding or high proportion of good projects and medium-low impact of the fee) lead the entrepreneur to ask for the official rating regardless of the opinion of the CRA.

3.2 The Model with Contact Disclosure

Suppose Assumption 5 holds. The fact that a project remains unrated is now a fourth outcome which has the same consequences in terms of possible funding[38] of an $r = J$ outcome but which is radically different in terms of reputation. Now, even when the entrepreneur decides not to purchase the official rating, the market can update its belief on whether the preliminary rating was issued by an informed CRA or not. The market now observes an additional outcome, a project which remains unrated but which is known to have gone through a preliminary evaluation, and can form an updated belief of the quality

[38] As before, unrated projects don't get funded because the investors invest only if they see an IG rating.

of the rating agency. Thus if the investors observe an unrated project, the posterior probability that the CRA is of the informed type becomes

$$\mu_{NR} = \frac{\lambda\left((\alpha-1)y_B - \alpha y_G + 1\right)}{y_B(\alpha\lambda - \lambda x + x - 1) + y_G((\lambda-1)x - \alpha\lambda) + 1} \tag{14}$$

which represents the probability that the CRA is of the informed type given no rating (NR). The presence of a new outcome means that now $\beta = \mu_{NR}$, therefore (8) > (9) can now be written as

$$y_G[(\alpha - 2\alpha e + e)\mu_J^1 + \alpha e\mu_{IG,s}^1 - \mu_{NR}^1 + \Phi] + \mu_{NR}^1 > \\ y_B[\alpha e\mu_{IG,s}^0 + (\alpha - 2\alpha e + e)\mu_J^0 - \mu_{NR}^0 + \Phi] + \mu_{NR}^0 \tag{15}$$

It stands to reason that, (15) differs from (13) only for the μ_{NR} term. The first thing that has to be noticed is that in the model without disclosure the reputation payoff in the no rating case was independent of the preliminary (i.e λ). Now instead giving a good or a bad preliminary implies different payoffs, respectively μ_{NR}^1 and μ_{NR}^0.

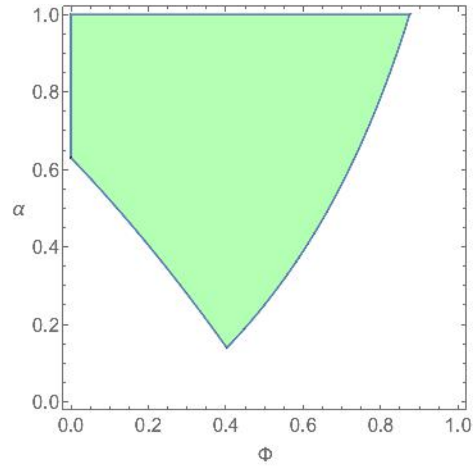
Corollary 3. *outcome. If the CRA is required to reveal the contact (Assumption 5) then* $\beta = \mu_{NR}$ *and* $\Phi^* = \hat{\Phi}$, *where*

$$\hat{\Phi} \equiv \frac{\alpha e(y_B\mu_{IG,s}^0 - y_G\mu_{IG,s}^1) + (\alpha - 2\alpha e + e)(y_B\mu_J^0 - y_G\mu_J^1)}{y_G - y_B} \\ + \frac{\mu_{NR}^0(1-y_B) - \mu_{NR}^1(1-y_G)}{y_G - y_B}$$

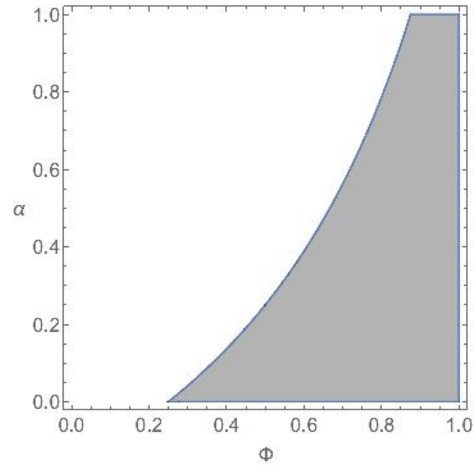
The above implies that $\hat{\Phi}(1,0) = \frac{\lambda\left(-\alpha\lambda + e\left(\left(\alpha^2+\alpha-1\right)\lambda - 2\alpha + 1\right) + \lambda\right)}{(\alpha\lambda-1)(e(\lambda-1)-\lambda)}$, where $\hat{\Phi}(1,0)$ is $\hat{\Phi}$ when $y_G = 1$ and $y_B = 0$. For the sake of the discussion once again fix $e = 3/4$, $\lambda = 1/2$ and $V_G = 1$. $\bar{\Phi}$ and $\hat{\Phi}$ share a common term that has to do with the $r = NR$.

Equilibrium (ii) and equilibrium (iv) resemble those in the no disclosure. This is not surprising given that the introduction of disclosure only affects the CRA's payoff, leaving the entrepreneur with the same payoff.

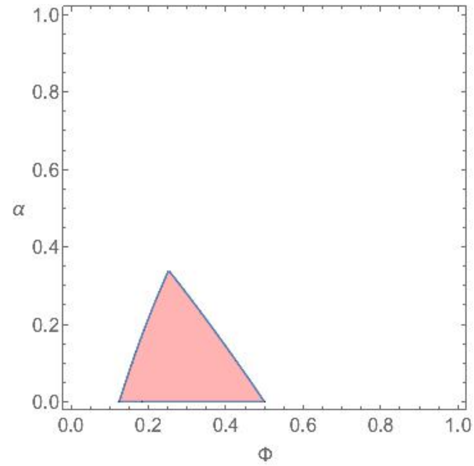
Corollary 4. *(i) For* $0 < \alpha \leq \frac{1}{2}$ *or for* $\frac{1}{2} < \alpha < 1$ *and* $\frac{(2\alpha-1)e}{-\alpha+(\alpha^2+\alpha-1)e+1} < \lambda < 1$ *there is multiplicity of equilibria either when* $0 < \Phi < \hat{\Phi}$ *and* $q_B^0 < V < 1$ *or when* $\Phi > \hat{\Phi}$ *and* $0 < \frac{\Phi}{V_G} < q_G^1$



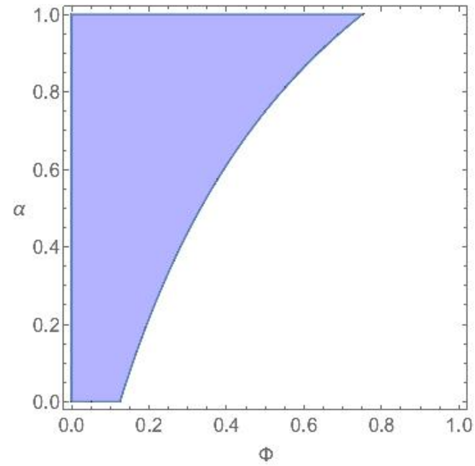
(a) Equilibrium *(i)*



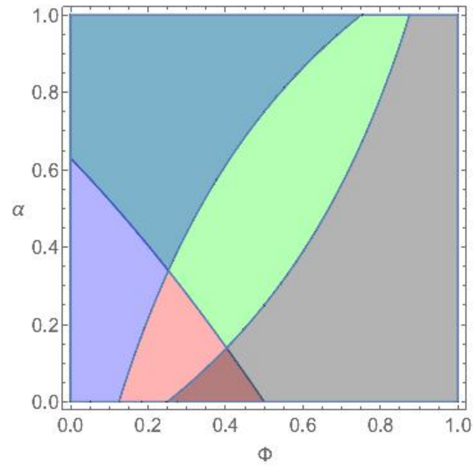
(b) Equilibrium *(ii)*



(c) Equilibrium *(iii)*



(d) Equilibrium *(iv)*



(e) Equilibrium *(i)* to *(iv)*. Top left dark turquoise area of multiplicity (i) and (iv); bottom burgundy area of multiplicity (ii) and (iii)

Figure 4: Equilibrium under contact disclosure for $e = 3/4$, $\lambda = 1/2$ and $V_G = 1$.

(ii) For $0 < \lambda \leq \frac{(2\alpha-1)e}{-\alpha+(\alpha^2+\alpha-1)e+1}$ *there is multiplicity of equilibria when* $0 < \frac{\Phi}{V_G} < q_G^1$

Multiplicity occurs for either low α or for both high α and sufficiently large λ when $\frac{\Phi}{V_G}$ is either sufficiently low or sufficiently high with respect to the likelihood of getting funded. Moreover for low enough λ multiplicity occurs when $\frac{\Phi}{V_G}$ is greater than the likelihood of getting funded.

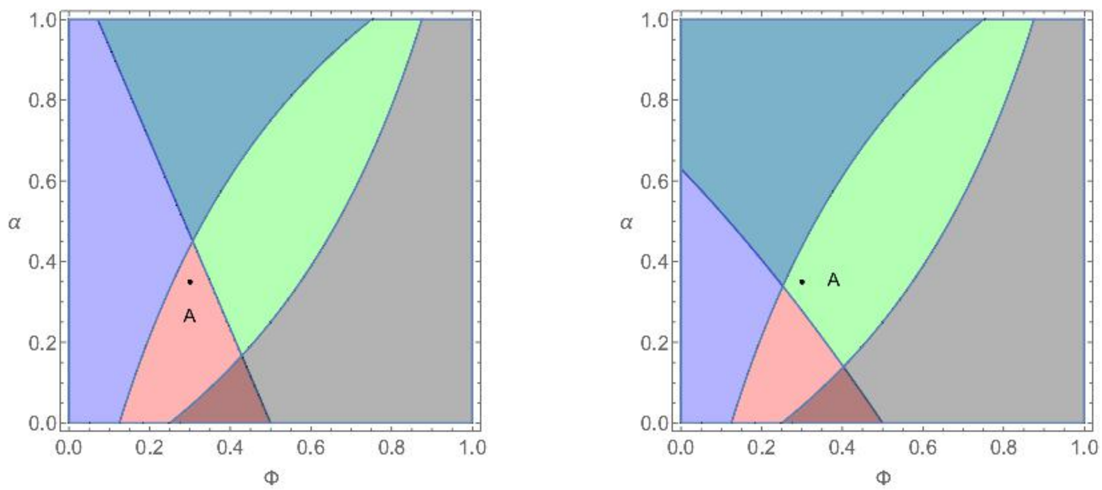
3.3 Discussion

The initial question can be rephrased as follows: does anything change when the CRA is asked to disclose the contact with a possible client? In particular the aim here is to investigate the extent to which such a requirement may affect the behaviour of the CRA through reputation.

Results from the no disclosure model and from the contact disclosure model show that in both settings two scenarios can arise when the uninformed CRA is confident ($x = 1$). It is either the case that the entrepreneur is responsive and faced with a good (bad) preliminary rating asks for the official rating (stops and remains unrated) or that he ignores the CRA's opinion, acts pessimistically and always stops. Similarly, two scenarios arise when the CRA is conservative ($x = 0$). The entrepreneur is either responsive and faced with a good (bad) preliminary asks for the official rating (stops and remains unrated) or he ignores the CRA's opinion, acts optimistically and always purchases the official rating. When the entrepreneur is optimistic and goes ahead ($y_G = 1$, $y_B = 1$), the CRA is conservative and earns the fee for sure. On the contrary when the entrepreneur is pessimistic and stops ($y_G = 0$, $y_B = 0$), the CRA is confident and never earns the fee.

The only difference between the two sets of results stands in the fact that equilibrium (i) now absorbs part of the space previously covered by the equilibrium (iii). In other words, under disclosure the CRA is more willing to issue a good preliminary rather than issuing a bad one. This in turn leads to more requests for official ratings and ultimately to the possibility that more projects get funded.[39] Comparing λ and μ_{NR} should shed some light on this finding. Whether λ is greater or lower of ether μ_{NR}^1 or μ_{NR}^0 crucially depends on y_G and y_B.

[39]Figure 5a and Figure 5b show that when disclosure is in place even for lower values of α the CRA is overconfident.



(a) No disclosure model

(b) Contact disclosure model

Figure 5: Comparison of equilibrium representation where $e = 3/4$, $\lambda = 1/2$ and $V_G = 1$.

Lemma 6. *If the CRA issues a bad preliminary report, the reputation gain associated with $r = NR$ is lower under disclosure whenever $0 \leq y_B < y_G \leq 1$.*

Proof is given in the appendix. With disclosure in place, the CRA receives a lower reputation payoff when issuing a bad preliminary. To compare the two settings, now focus on an example. I have already fixed e, λ and V_G and I now analyse what happens when α is fixed to 0.35 and Φ is fixed to 0.3.

Let A represent this specific set of fixed parameters. In Figure 5a and Figure 5b it can be seen that point A was chosen as an example because it lies in the "conservative-responsive" area in the no disclosure model whereas it is part of the "overconfident-responsive" area in the extension[40]. Provided that for the entrepreneur nothing has changed and that, in the two equilibrium under consideration, receiving a good preliminary corresponds to going ahead while receiving a bad one, results in a stop, the focus is on the CRA's payoff. Note that the payoff for the CRA for issuing a good preliminary is the same in the model with contact disclosure and in the one without disclosure whereas the payoff from issuing a bad preliminary differs, namely it is lower in the contact disclosure model. This is due to the different reputation gain enjoyed by the CRA in the two models when no rating is published. If the CRA knows that the entrepreneur goes ahead

[40]The "conservative-responsive" area is equivalent to Equilibrium *(iii)* whereas the "overconfident-responsive" area is equivalent to Equilibrium *(i)*.

after having been provided with a good preliminary ($y_G = 1$), issuing $P = G$ would give the CRA the same profit in both because it is as if it were avoiding the possibility to end at the no rating situation. (8) in the two regimes differs only with respect to the NR reputation payoff $\beta\pi_{G,NR}$, namely $\mu_{NR}\pi_{G,NR}$ in the contact disclosure model and $\lambda\pi_{G,NR}$ in the model without disclosure. Given that $\pi_{G,NR} = 1 - y_G$, when $y_G = 1$ both terms go to 0. Hence the CRA enjoys the same payoff in the two cases. On the other side, knowing that the entrepreneur faced with a bad preliminary will stop ($y_B = 0$) creates a divergence in the payoff of the two settings. The reputation payoff from saying $P = B$ is higher in the no disclosure setup and this is what causes the shift in the decision of the CRA. Employing Lemma 7, when $y_B = 0$ and $y_G = 1$, the reputation gain associated with $r = NR$ is lower under disclosure. Given $y_B = 0$ and $y_G = 1$, the payoff of the CRA from issuing a bad preliminary is λ in the no disclosure case as opposed to $\frac{\lambda(1-\alpha)}{1-\alpha\lambda}$ in the contact disclosure case. It can be easily seen that the first payoff is greater than the second one, meaning that the reputation gain from an NR rating in the no disclosure case is higher than in the case in which disclosure is in place. This shows that when disclosure is in place the payoff from issuing a bad preliminary is lower than the one related to a good preliminary whereas in the no disclosure model the payoff relation is reversed.

Proposition 1 shows that in Equilibrium (i) and in Equilibrium (iii) the role of the fee is crucial; the tradeoff fee in the no disclosure case always exceeds the tradeoff fee in the contact disclosure one. In other words, when disclosure is in place, the conservative equilibrium loses out to the overconfidence equilibrium.

Lemma 7. *For all parameters values* $\bar{\Phi}(1,0) > \hat{\Phi}(1,0)$.

The proof is given in the appendix. The intuition here works as follows. Recall that below the tradeoff fee the CRA prefers to issue a bad preliminary whereas above this fee the CRA chooses to issue a good preliminary. With disclosure in place we have seen that the CRA gains less in terms of reputation when the entrepreneur decides to stop. If the payoff for saying $P = B$ goes down, the tradeoff fee which identifies the shift from the Equilibrium (iii) to the Equilibrium (i) is lowered as well. Now the CRA is willing to be confident even for lower fees because this still provides a higher payoff than the achievable one from being conservative. More generally it is the case that

Lemma 8. $\bar{\Phi}$ *is always greater than* $\hat{\Phi}$

Proof. $\bar{\Phi} > \hat{\Phi}$ requires $y_G > y_B$. Using a proof by contradiction approach, assume $y_B > y_G$ this implies that $q_B > \frac{\Phi}{V_G} > q_G$. This however is impossible because $q_G > q_B$. Hence, y_G cannot be lower than y_B and the proof follows. $\square$

Provided that $y_G \geq y_B$ always holds, it is easy to see that the returns from $p = B$ are either equal or lower under disclosure. Whenever the CRA is indifferent (i.e. when $y_G = y_B$), the return is equal in the two regimes, whereas whenever $y_G > y_B$ it is lower under disclosure. The intuition works as follows: when the behaviour of the entrepreneur is not preliminary rating dependent, the incentives of the CRA do not change when disclosure is in place. Instead, the incentives for the CRA dampen when $y_G > y_B$ because issuing a bad preliminary is now less profitable. This has to do with the fact that the entrepreneur is generally more likely to proceed to the official rating stage after a good preliminary, minimising the chances for the CRA to end up with $r = NR$. With no official rating and without disclosure the market cannot update its belief and therefore sticks to the prior λ, whereas with disclosure the CRA's reputation is subject to an update. Indeed the market knows that with some probability the entrepreneur stopped because of a $P = B$ indicative rating due to an uninformed CRA. Thus the market updates its belief of whether the issuing CRA is of the informed type. The reputation concerns are not strong enough to lead to a better outcome under disclosure, instead they end up making the CRA more responsive to the fee.

When α is high the CRA has a high chance of evaluating correctly the project if she issues a good preliminary rating. So in this situation if the fee is reasonably high (i.e green area), the CRA issues a good preliminary rating because the risk of loosing reputation is low. When instead α is low, the CRA has a high chance of issuing an incorrect preliminary rating. If the fee is low (i.e. pink area), the CRA prefers to issue a bad preliminary rating avoiding to risk reputation. In the first scenario (α high) the fee is more important than reputation whereas in the second (α low) the driving force is reputation.

3.4 Is contact disclosure beneficial?

The model presented in this paper allows two different types of errors. Indeed it can happen that a G project is stopped and therefore it is not financed or that a B project, which should have been stopped, gets financed and eventually results in a failure. I know from the analysis above that allowing for contact disclosure results in an increase in the overconfidence of the CRA when, after a good preliminary, the entrepreneur buys straight away and when after a bad preliminary the entrepreneur stops. In order to investigate the extent to which this shift is beneficial, I compare the probabilities of the two mistakes happening under both regimes.

On the one hand confident-responsive equilibrium bears a probability $\alpha(1-\lambda)(1-e)$ to dismiss good projects and a probability $(1-\alpha)(1-\lambda)(1-e)$ to wrongly finance bad projects. On the other hand, the conservative-responsive equilibrium avoids wrongful finance of bad projects but causes with probability $\alpha(1-\lambda)$ the stop of good projects. It is also known that the overconfident-responsive equilibrium gains prominance while the conservative-responsive equilibrium shrinks.

Lemma 9. *Under disclosure the probability of dismissing good projects is lower while the probability of financing bad projects is higher with respect to the case where there is no evidence of contact.*

Proof. The probability of stopping good projects is lower in the overconfident-responsive equilibrium than in the conservative-responsive equilibrium. Indeed $\alpha(1-\lambda)(1-e) < \alpha(1-\lambda)$ always holds. Conversely, the probability of funding bad projects is greater in the overconfident-responsive equilibrium than in the conservative-responsive equilibrium. In formula, $(1-\alpha)(1-\lambda)(1-e) > 0$ always holds. So if the overconfident-responsive equilibrium becomes more prominent, disclosure results in fewer stopped good projects and more financed bad ones. □

As an example, now take recessions and booms. In recessions, society is more concerned about not funding good projects rather than financing bad ones whereas in booms the opposite holds. In both cases the idea is that as much as possible money wasting should be avoided. In turn, ideally in recessions a CRA should be confident to boost the economy while she should be conservative during booms to avoid bubbles. This implies that the increased overconfidence of the CRA under disclosure may be beneficial in recessions (allowing more projects to reach the official rating stage) but it may be harmful in booms (allowing too many bad projects to get funded).

In the light of this simple model implication, disclosure requirements seem to be helpful when the economy is facing a recession, but discretion in terms of implementation is needed. One way of doing so in order to avoid adverse effects would be to make the requirement more stringent in times of need and to relax it in good times, for instance introducing time revisions of the requirement every twelve months.

3.5 Why preliminary ratings?

A question which might arise relates to the actual benefit provided by preliminary ratings to CRAs. In this setting, the advantage for the CRA stands only in the possibility for the expert to minimise the chance to end up with an incorrect rating which could penalise her reputation. It is however true (but out of the scope of the present work) that such a structure is more of interest for the entrepreneur which could acquire valuable information on its project before presenting it to the market. The rationale behind the choice to have a two stage evaluation is primarely dictated by the willingness to capture more realistically the structure of the interaction between the players. Here there is no question on the necessity or not of preliminary ratings but a pure representation of how the CRA earns reputation in an opaque or transparent market. This view is reinforced by the regulatory attention devoted to this matter. Merging the two stages (i.e having only the official rating stage) would mean ignoring an important driver, especially when dealing with structured finance.

4 Extensions

This section presents three extensions to the model solved above which add robustness to the findings.

4.1 Availability of partial funding

We now relax this assumption, allowing for partial funding and look at the case in which the entrepreneur can carry out the project even if unrated. Let $V_{NR} > 0$ be a high cost funding which allows alternative funding for the project

Assumption 6. $V_G - \Phi > V_{NR}$

Similarly to Assumption 1 this assumption states that low cost funding net of the fee exceeds the cost funding. Given that the objective of the entrepreneur is to get funded, if Assumption 2 was not satisfied, the entrepreneur would always remain unrated.

The entrepreneur now compares the payoff from asking the official rating and paying the fee with V_{NR}, and will go ahead according to

$$\begin{cases} q_G^x > \frac{\Phi+V_{NR}}{V_G}, & \text{if } P = G \\ q_B^x > \frac{\Phi+V_{NR}}{V_G}, & \text{if } P = B \end{cases} \tag{16}$$

Even though the market now observes more outcomes than in the absence of partial funding,[41] the payoff of the uninformed CRA when disclosure is not in place remains unchanged. The payoff of the CRA from issuing $P = G$ in the preliminary stage is

$$\pi_{G,s}\mu_{IG,s} + \pi_{G,f}\mu_{IG,f} + \pi_{G,J}\mu_J + \lambda\pi_{G,NR,f} + \lambda\pi_{G,NR,s} + \Phi y_G \tag{17}$$

whereas from issuing $P = B$ in the preliminary stage the CRA gets

$$\pi_{B,s}\mu_{IG,s} + \pi_{B,f}\mu_{IG,f} + \pi_{B,J}\mu_J + \lambda\pi_{B,NR,f} + \lambda\pi_{B,NR,s} + \Phi y_B \tag{18}$$

where $\pi_{G,NR,f}$ ($\pi_{B,NR,f}$) and $\pi_{G,NR,s}$ ($\pi_{B,NR,s}$) are respectively the probabilities $(1-\alpha)(1-y_G)$ ($(1-\alpha)(1-y_B)$) and $\alpha(1-y_G)$ ($\alpha(1-y_B)$) that a project rated NR fails or succeeds when the uninformed CRA says $P = G$ ($P = B$). In all four cases the market is unaware of the contact between the CRA and the entrepreneur and therefore sticks to the prior λ. It is straightforward that (17) and (18) perfectly resemble (8) and (9) because $\pi_{G,NR,f} + \pi_{G,NR,s} = \pi_{G,NR}$ and similarly $\pi_{B,NR,f} + \pi_{B,NR,s} = \pi_{B,NR}$.

When disclosure is in place instead, if the investors observe an unrated project which succeeds, their posterior probability that the CRA is of the informed type becomes

$$\mu_{NR,s} = \frac{\lambda\,(y_G - 1)}{(\lambda-1)(x-1)y_B + y_G(\lambda - \lambda x + x) - 1} \tag{19}$$

[41] An unrated project now gets funded, therefore the market will eventually observe whether it succeeds or fails. Thus the possible outcomes are a project rated investment grade which succeeds (IG, s), a project rated investment grade which fails (IG, f) a project rated junk (J), a project not rated which succeeds (NR, s) and a project not rated which fails (NR, f).

Whereas if the investors observe an unrated project which fails, their posterior probability that the CRA is of the informed type becomes

$$\mu_{NR,f} = \frac{\lambda\,(y_B - 1)}{y_B((\lambda-1)x+1) + y_G(x-\lambda x) - 1} \tag{20}$$

Thus the CRA compares the payoff from issuing a good preliminary

$$y_G[(\alpha - 2\alpha e + e)\mu_J^1 + \alpha e \mu_{IG,s}^1 - \alpha\lambda - (1-\alpha)\mu_{NR,f}^1 + \Phi] + \alpha\lambda + (1-\alpha)\mu_{NR,f}^1 \tag{21}$$

with the payoff from issuing a bad preliminary

$$y_B[\alpha e \mu_{IG,s}^0 + (\alpha - 2\alpha e + e)\mu_J^0 - (1-\alpha)\lambda - \alpha\mu_{NR,s}^0 + \Phi] + (1-\alpha)\lambda + \alpha\mu_{NR,s}^0 \tag{22}$$

Corollary 5. *When there is availability of partial funding*

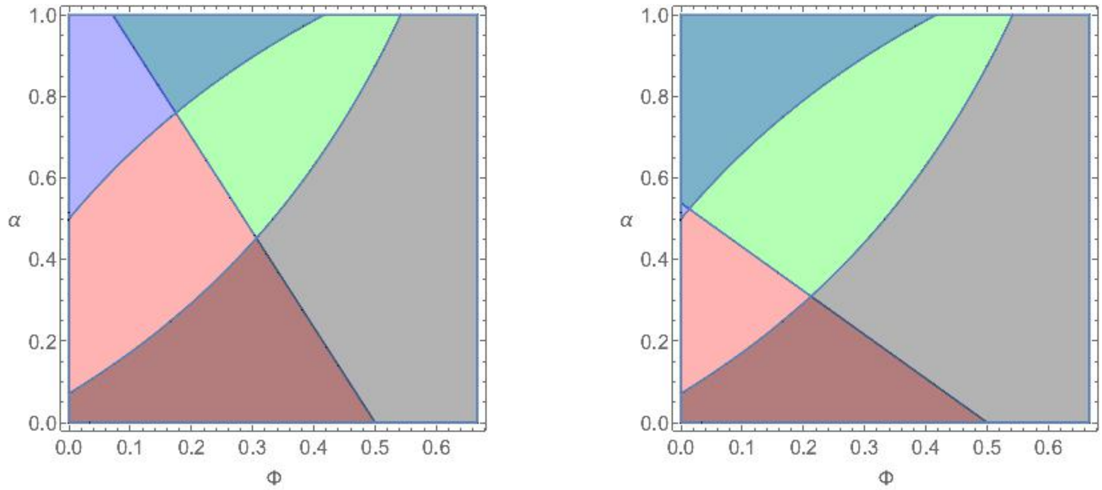
(i) if the CRA is not mandated to disclose the contact (Assumption 4) then Corollary 1 applies;

(ii) if the CRA is required to reveal the contact (Assumption 5), then $\Phi^* = \widetilde{\Phi}$, where

$$\begin{aligned}
\widetilde{\Phi} \;\equiv\; & \frac{\alpha e(y_B\mu_{IG,s}^0 - y_G\mu_{IG,s}^1) + (\alpha - 2\alpha e + e)(y_B\mu_J^0 - y_G\mu_J^1)}{y_G - y_B} \\
& - \frac{\alpha(y_B\mu_{NR,s}^0 - y_G\lambda) + \alpha(y_B\lambda - y_G\mu_{NR,f}^1)}{y_G - y_B} \\
& + \frac{\alpha(\mu_{NR,s}^0 - \lambda) + (1-\alpha)(\lambda - \mu_{NR,f}^1)}{y_G - y_B}
\end{aligned}$$

The above implies that $\widetilde{\Phi}(1,0) = \lambda\left(\alpha\left(-\frac{e}{e(-\lambda)+e+\lambda} - 1\right) + 1\right)$ where $\widetilde{\Phi}(1,0)$ is $\widetilde{\Phi}$ when $y_G = 1$ and $y_B = 0$. It is still the case that four types of equilibria arise, for the sake of the discussion let's fix $e = 3/4$, $\lambda = 1/2$, $V_G = 1$ and $V_{NR} = 1/3$, then use Figure 6 to summarise the findings. The outside option which ensures some funding also in the absence of a rating, results in the CRA being confident even more often than in the case without partial funding.

Lemma 10. *For all parameters values $\hat{\Phi}(1,0) > \widetilde{\Phi}(1,0)$.*



(a) No disclosure model

(b) Contact disclosure model

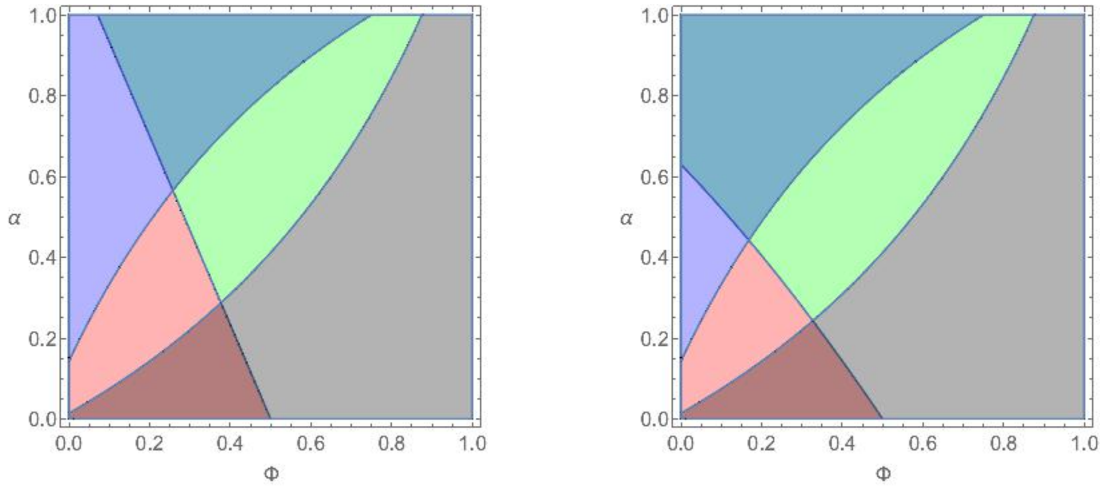
Figure 6: Comparison of equilibrium representation with high cost funding where $e = 3/4$, $\lambda = 1/2$, $V_G = 1$ and $V_{NR} = \frac{1}{3}$.

The intuition goes as follows: when there exists another source of funding, for the entrepreneur to ask for the official report, the probability that the project will be certified as IG, given a certain preliminary, has to be higher.[42] This means that the entrepreneur is less willing to go ahead and pay the fee. Thus the only reasonable thing the CRA can do is to start issuing good preliminary in order to make the entrepreneur more willing to go ahead. Overall the results from the baseline model not only hold, but they are also reinforced by the presence of alternative funding.

4.2 Entrepreneur's Preference for Success

We now relax Assumption 1 stating that the only concern of the entrepreneur is to get funded. In order to have an entrepreneur who prefers success rather than default, let $v < V_G$ be the entrepreneurs own funds. In order to carry out the project v is not sufficient, thus funding from the investors is needed.

[42]Recall that without funding the entrepreneur goes ahead if $q_G > \frac{\phi}{V_G}$ ($q_B > \frac{\phi}{V_G}$) whereas in the current setting he goes ahead if $q_G > \frac{\Phi + V_{NR}}{V_G}$ ($q_B > \frac{\Phi + V_{NR}}{V_G}$).



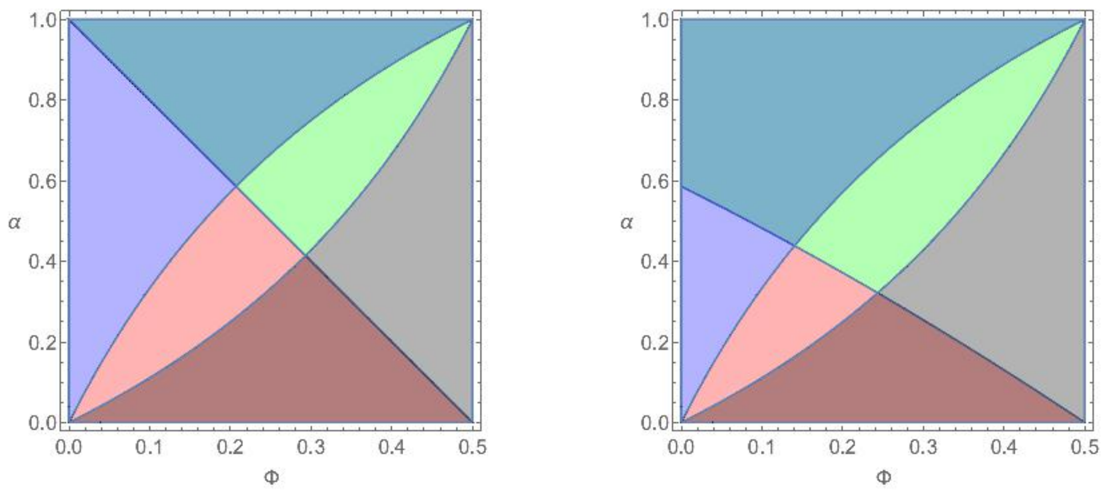
(a) No disclosure model (b) Contact disclosure model

Figure 7: Comparison of equilibrium representation when the entrepreneur has a preference for success. where $e = 3/4$, $\lambda = 1/2$, $V_G = 1$ and $v = \frac{1}{10}$.

The payoff of the entrepreneur can now be written as

$$\begin{cases} {q_G}^{IG} V_G - {q_G}^{J} v > \Phi, & \text{if } P = G \\ {q_B}^{IG} V_G - {q_B}^{J} v > \Phi, & \text{if } P = B \end{cases} \tag{23}$$

where ${q_P}^{IG} + {q_P}^{J} = q_P$. In this setting, with probability ${q_G}^{IG} = \frac{\alpha(e(1-\lambda)x+\lambda)}{\alpha\lambda+(1-\lambda)x}$ (${q_B}^{IG} = \frac{\alpha e(1-\lambda)(1-x)}{-\alpha\lambda+(1-\lambda)(-x)+1}$) the project which received a good preliminary (bad preliminary) gets funded and succeeds and the entrepreneur gets V_G, whereas if the project gets funded and fails the entrepreneur loses its own money v with probability ${q_G}^{J} = \frac{(1-\alpha)(1-e)(1-\lambda)x}{\alpha\lambda+(1-\lambda)x}$ (${q_B}^{J} = \frac{(1-\alpha)((1-e)(1-\lambda)(1-x)+\lambda)}{-\alpha\lambda+(1-\lambda)(-x)+1}$). Under these new preferences it can be shown that all four equilibria still arise and that contact disclosure makes the CRA more overconfident. Let $v = \frac{1}{10}$ be the amount of own funding Figure 7 graphically provides the result. Comparing Figure 7 with Figure 5 a similarity can be picked. This is due to the fact that the change in the preferences of the entrepreneur does not affect the choice tradeoff value according to which the CRA acts. This implies that only equilibrium (i) and (iii) change, the first becoming smaller while the second becoming larger. The main difference is that one of the multiplicity area becomes larger. Once again, overall previous results hold.



(a) Model without disclosure

(b) Model with disclosure

Figure 8: Comparison of equilibrium representation when the CRA is perfectly informed in the official rating stage. Here $\lambda = 1/2$ and $V_G = \frac{1}{2}$.

4.3 Perfect Information in the Official Rating Stage

Assume that instead of being uninformed throughout the game, the CRA becomes informed in the second stage. This means that, regardless of the type of CRA, once the entrepreneur buys the official rating, this will be correct with certainty. The rationale may have to do with the quality of the information provided to the CRA by the entrepreneur after he has paid for a rating.[43]

Corollary 6. *Under the assumption of perfect information in the official rating stage* $\bar{\Phi} = (1-\alpha)\lambda$ *and* $\hat{\Phi} = \frac{\lambda\left(\alpha^2\lambda-2\alpha+1\right)}{1-\alpha\lambda}$.

When $e = 1$, only one type of mistake can occur. It can only happen that a good project is wrongfully stopped but no bad project gets funded. Figure 8 gives a graphical representation of the equilibria.. Under this perfect information assumption, contact disclosure has a positive effect because this requirement results in fewer good projects being dismissed.

Lemma 11. *If there is no noise in the official rating stage, contact disclosure is welfare improving.*

[43]It is not unreasonable to assume that at the preliminary evaluation the information at the CRA's disposal is less precise or less vast than the information the CRA has access to when it is to provide a published rating.

Proof. The probability of stopping good projects in the overconfident-responsive equilibrium is zero while in the conservative responsive equilibrium the same probability is $\alpha(1-\lambda)$ always greater than zero. Hence if the confident responsive equilibrium becomes more prominent with mandatory disclosure, such a requirement results in fewer good projects stopped. $\square$

Concluding Remarks

I develop a model of preliminary ratings and use it to investigate the effect of contact disclosure. The model features a monopolistic CRA which has an incentive to gain reputation among investors and a mass of entrepreneurs who need funding. I characterise the equilibrium under a general framework and then look separately at the case where no evidence of preliminary ratings is available to the market and at what happens when preliminary rating contact has to be disclosed by the CRA.

I show that four types of equilibria can arise. Two of them feature the entrepreneur being responsive by remaining unrated following a bad preliminary or buying the official rating following a good one, while the CRA acts overconfidently in one and conservatively in the other. The tradeoff between the fee and the reputation is such that when α is high the CRA issues a good preliminary rating because the risk of loosing reputation is extremely low whereas when α is low the CRA prefers to issue a bad preliminary rating avoiding to risk reputation.

I find that under contact disclosure, when the entrepreneur is responsive, the equilibrium where the CRA is pushy gains prominence to the expenses of the equilibrium where the CRA is conservative. Thus more projects, even of lower quality, access the official rating stage and can get funded. On the one hand, mandatory contact disclosure reduces the probability of good projects not being financed while on the other hand it opens the ground to the funding of bad projects.

In the light of these findings, the CRA's reputation is negatively affected by the disclosure requirement and this results in a laxer behaviour.

The other two equilibria that I find show an optimistic entrepreneur who buys the official rating regardless of the preliminary and a pessimistic entrepreneur remaining unrated regardless of the preliminary which are shown to be unaffected by the disclosure requirement. In these cases the opinion of the CRA is useless for the entrepreneur and his decision seems to be driven by the magnitude of the impact of the fee on funding and by the likelihood that the project is of high quality. Favourable combinations of the two lead the entrepreneur to continue, unfavourable ones result in a stop.

I also show that the result are robust in extensions where funding of unrated projects is available, where the entrepreneur has a preference for success or when in the second stage of the certification the CRA has access to perfect information.

There are three main implications. First, consistently with a strand of the literature, disclosure is not necessarily welfare improving. Second, disclosure policies and regulation cannot be a one off decision, they require tailoring and timely revisions. Thirdly, for disclosure to be effective, the rating process needs to be as accurate as possible.

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Appendix

The probability at each node

The CRA knows when playing $P = G$ or $P = B$ with which probability she is going to gain reputation. As mentioned, $\pi_{B,s}$ and $\pi_{G,s}$ are respectively the probabilities that a project rated IG succeeds when the uninformed CRA says $P = B$ or when it says $P = G$. For each information set, of all the possible end nodes, only one branch leads to this outcome, therefore

$$\pi_{G,s} = \frac{\alpha e(1-\lambda)xy_G}{(1-\alpha)(1-\lambda)x + \alpha(1-\lambda)x} = \alpha e y_G$$

$$\pi_{B,s} = \frac{\alpha e(1-\lambda)(1-x)y_B}{(1-\alpha)(1-\lambda)(1-x) + \alpha(1-\lambda)(1-x)} = \alpha e y_B$$

Similarly $\pi_{G,J}$ and $\pi_{B,J}$ are the probabilities that a project gets a J rating when the uninformed CRA says either $P = G$ or $P = B$. Once again, for each information set, of the eight official nodes only two are consistent with this outcome, thus

$$\pi_{G,J} = \frac{(1-\alpha)e(1-\lambda)xy_G + \alpha(1-e)(1-\lambda)xy_G}{(1-\alpha)(1-\lambda)x + \alpha(1-\lambda)x}$$

$$\pi_{B,J} = \frac{(1-\alpha)e(1-\lambda)(1-x)y_B + \alpha(1-e)(1-\lambda)(1-x)y_B}{(1-\alpha)(1-\lambda)(1-x) + \alpha(1-\lambda)(1-x)}$$

which simplified are respectively $(\alpha - 2\alpha e + e)y_G$ and $(\alpha - 2\alpha e + e)y_B$. Moreover $\pi_{G,f}$ and $\pi_{B,f}$ are the probabilities that a project rated IG fails when the uninformed CRA says $P = G$ or $P = B$, thus

$$\pi_{G,f} = \frac{(1-\alpha)(1-e)(1-\lambda)xy_G}{(1-\alpha)(1-\lambda)x + \alpha(1-\lambda)x} = (\alpha-1)(e-1)y_G$$

$$\pi_{B,f} = \frac{(1-\alpha)(1-e)(1-\lambda)(1-x)y_B}{(1-\alpha)(1-\lambda)(1-x) + \alpha(1-\lambda)(1-x)} = (\alpha-1)(e-1)y_B$$

Finally, for each information set, of all the possible end nodes, just 2 branches lead to the NR outcome, which means

$$\pi_{G,NR} = \frac{(1-\alpha)(1-\lambda)x\,(1-y_G) + \alpha(1-\lambda)x\,(1-y_G)}{(1-\alpha)(1-\lambda)x + \alpha(1-\lambda)x} = 1 - y_G$$

$$\pi_{B,NR} = \frac{(1-\alpha)(1-\lambda)(1-x)\,(1-y_B) + \alpha(1-\lambda)(1-x)\,(1-y_B)}{(1-\alpha)(1-\lambda)(1-x) + \alpha(1-\lambda)(1-x)} = 1 - y_B$$

The sum of the probabilities with which the CRA gains reputation sums up to one.

$$\alpha e y_B + y_B(\alpha - 2\alpha e + e) + (\alpha-1)(e-1)y_B + 1 - y_B = 1$$

$$\alpha e y_G + y_G(\alpha - 2\alpha e + e) + (\alpha-1)(e-1)y_G + 1 - y_G = 1$$

Proof of Lemma 5

The choice on whether to issue a good ($P = G$) or a bad ($P = B$) preliminary can be summarised as follows. The CRA will issue a good preliminary if

$$\frac{(1-\alpha)\lambda y_B(\alpha - 2\alpha e + e)y_G}{(1-\alpha)\lambda y_B + (1-\lambda)(\alpha - 2\alpha e + e)y_G} + \frac{\alpha e \lambda y_G}{e(1-\lambda) + \lambda} + \beta\,(1-y_G) + \Phi y_G >$$
$$\frac{\alpha e \lambda y_B y_G}{e(1-\lambda)y_B + \lambda y_G} + \frac{(1-\alpha)\lambda y_B(\alpha - 2\alpha e + e)}{(1-\alpha)\lambda + (1-\lambda)(\alpha - 2\alpha e + e)} + \beta\,(1-y_B) + \Phi y_B$$

which can be rewritten as

$$y_G[(\alpha - 2\alpha e + e)\mu_J^1 + \alpha e\mu_{IG,s}^1 - \beta + \Phi] > y_B[\alpha e\mu_{IG,s}^0 + (\alpha - 2\alpha e + e)\mu_J^0 - \beta + \Phi] \quad (24)$$

The RHS of the above inequality is the expected payoff from issuing a positive preliminary. Similarly the LHS is the expected payoff from issuing a negative preliminary which can be simplified to

$$\Phi > \frac{\alpha e(y_B\mu_{IG,s}^0 - y_G\mu_{IG,s}^1) + (\alpha - 2\alpha e + e)(y_B\mu_J^0 - y_G\mu_J^1)}{y_G - y_B} + \beta$$

Proof of Proposition 1

If $y_G = 1$, $y_B = 0$, for the CRA to prefer to issue $P = G$ instead of $P = B$ it has to be that Lemma 5 (i) holds. When the CRA is confident, the ENT faced with $P = G$ will choose to ask for the full report instead of remaining unrated if Lemma 1 holds. By Lemma 3 the ENT faced with $P = B$ would rather not purchase the final rating than ask for it. For the CRA to prefer to issue $P = B$ instead of $P = G$ it has to be the case that Lemma 5 (i) holds with the opposite sign ($\Phi < \Phi^*$). By Lemma 4, the ENT faced with $P = G$ always chooses to get rated whenever the uninformed CRA issue $P = B$, whereas the ENT faced with $P = B$ prefers to stop when Lemma 1 is not satisfied ($q_B < \frac{\Phi}{V_G}$). Lemma 3 rules out equilibria in which the CRA is confident and the ENT faced with $P = B$ prefers to ask for the official rating. Thus, there exist an equilibrium of the type $x = 1$, $y_G = 1$, $y_B = 0$ and an equilibrium of the type $x = 0$, $y_G = 1$, $y_B = 0$.
If $y_G = 0$, $y_B = 0$, the CRA is indifferent between issuing $P = G$ or $P = B$. When the CRA is confident, the ENT faced with $P = G$ chooses not get rated if Lemma 1 is not satisfied ($q_G < \frac{\Phi}{V_G}$), whereas the ENT faced with $P = B$ chooses not to get rated by Lemma 3 . When the CRA is conservative, the ENT faced with $P = G$ always chooses to get rated by Lemma 4, whereas for the ENT faced with $P = B$ to ask for the full report it has to be the case that Lemma 1 holds. Lemma 4 rules out equilibria in which the CRA is conservative and the ENT faced with $P = G$ prefers to remain unrated. Hence there exists an equilibrium of the type $x = 0$, $y_G = 1$, $y_B = 1$ and an equilibrium of the type $x = 0$, $y_G = 1$, $y_B = 1$.

Corollary 1

The choice on whether to issue a good ($P = G$) or a bad ($P = B$), formally (8) > (9) in the model without contact disclosure becomes

$$\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)y_G}{(1-\alpha)\lambda y_B+(1-\lambda)(\alpha-2\alpha e+e)y_G}+\frac{\alpha e\lambda y_G}{e(1-\lambda)+\lambda}+\lambda\left(1-y_G\right)+\Phi y_G >$$
$$\frac{\alpha e\lambda y_B y_G}{e(1-\lambda)y_B+\lambda y_G}+\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)}{(1-\alpha)\lambda+(1-\lambda)(\alpha-2\alpha e+e)}+\lambda\left(1-y_B\right)+\Phi y_B$$

To find the threshold, assume the above holds with equality, leading to

$$\begin{aligned}\bar{\Phi} &= \lambda+\frac{\alpha e\left(\frac{\lambda y_B y_G}{e(1-\lambda)y_B+\lambda y_G}-\frac{\lambda y_G^2}{e(-\lambda)+e+\lambda}\right)}{y_G-y_B}\\ &+\frac{(\alpha-2\alpha e+e)\left(\frac{(\alpha-1)\lambda y_B^2}{-2\alpha\lambda+\alpha+(2\alpha-1)e(\lambda-1)+\lambda}-\frac{(1-\alpha)\lambda y_B y_G}{(1-\alpha)\lambda y_B+(1-\lambda)(\alpha-2\alpha e+e)y_G}\right)}{y_G-y_B}\end{aligned}$$

If $y_G = 1$ and $y_B = 0$ it becomes $\bar{\Phi} = \frac{\lambda(e(\alpha+\lambda-1)-\lambda)}{e(\lambda-1)-\lambda}$.

Corollary 3

The choice on whether to issue a good ($P = G$) or a bad ($P = B$) preliminary can be summarised as follows. The CRA will issue a good preliminary if

$$\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)y_G}{(1-\alpha)\lambda y_B+(1-\lambda)(\alpha-2\alpha e+e)y_G}+$$
$$\frac{\lambda\left(1-y_G\right)\left((\alpha-1)y_B-\alpha y_G+1\right)}{(\alpha\lambda-\lambda)y_B+(-\alpha\lambda+\lambda-1)y_G+1}+\frac{\alpha e\lambda y_G}{e(1-\lambda)+\lambda}+\Phi y_G >$$
$$\frac{\alpha e\lambda y_B y_G}{e(1-\lambda)y_B+\lambda y_G}+\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)}{(1-\alpha)\lambda+(1-\lambda)(\alpha-2\alpha e+e)}+$$
$$\frac{\lambda\left(1-y_B\right)\left((\alpha-1)y_B-\alpha y_G+1\right)}{(\alpha\lambda-1)y_B-\alpha\lambda y_G+1}+\Phi y_B$$

To find the threshold, assume the above holds with equality, leading to

$$\begin{aligned}\hat{\Phi} &= \frac{\alpha e(\frac{\lambda y_G y_B}{e(1-\lambda)y_B+\lambda y_G} - \frac{\lambda y_G^2}{e(-\lambda)+e+\lambda})}{y_G - y_B} \\ &+ \frac{(\alpha - 2\alpha e + e)(\frac{(\alpha-1)\lambda y_B^2}{-2\alpha\lambda+\alpha+(2\alpha-1)e(\lambda-1)+\lambda} - \frac{(1-\alpha)\lambda y_B y_G}{(1-\alpha)\lambda y_B+(1-\lambda)(\alpha-2\alpha e+e)y_G})}{y_G - y_B} \\ &+ \frac{\frac{\lambda((\alpha-1)y_B-\alpha y_G+1)}{(\alpha\lambda-1)y_B-\alpha\lambda y_G+1}(1-y_B) - \frac{\lambda((\alpha-1)y_B-\alpha y_G+1)}{(\alpha\lambda-1)y_B-\alpha\lambda y_G+1}(1-y_G)}{y_G - y_B}\end{aligned}$$

If $y_G = 1$ and $y_B = 0$ it becomes $\hat{\Phi} = \frac{\lambda\left(-\alpha\lambda+e\left(\left(\alpha^2+\alpha-1\right)\lambda-2\alpha+1\right)+\lambda\right)}{(\alpha\lambda-1)(e(\lambda-1)-\lambda)}$.

Proof of Lemma 6

To prove that issuing a bad preliminary leads to a lower reputation payoff in the case of no final report, we have to focus on λ and μ_{NR}^0. For Lemma 6 to be true it has to be the case that

$$\lambda > \frac{\lambda\left(\alpha\left(y_B - y_G\right) - y_B + 1\right)}{\alpha\lambda\left(y_B - y_G\right) - y_B + 1} \quad \text{which implies} \quad 1 > \frac{\alpha\left(y_B - y_G\right) - y_B + 1}{\alpha\lambda\left(y_B - y_G\right) - y_B + 1}$$

If $0 \leq y_B < y_G \leq 1$ then the numerator of the RHS of the inequality is lower than the denominator. This in turn makes the ratio lower than 1.

Proof of Lemma 7

From Proposition 1, Corollary 1 and Corollary 3 we know that

$$\bar{\Phi} = \frac{\lambda(e(\alpha+\lambda-1)-\lambda)}{e(\lambda-1)-\lambda}$$

$$\hat{\Phi} = \frac{\lambda\left(-\alpha\lambda+e\left(\left(\alpha^2+\alpha-1\right)\lambda-2\alpha+1\right)+\lambda\right)}{(\alpha\lambda-1)(e(\lambda-1)-\lambda)}$$

To prove that $\bar{\Phi} > \hat{\Phi}$, let's assume that this is the case, therefore

$$\frac{\lambda(e(\alpha+\lambda-1)-\lambda)}{e(\lambda-1)-\lambda} > \frac{\lambda\left(-\alpha\lambda+e\left(\left(\alpha^2+\alpha-1\right)\lambda-2\alpha+1\right)+\lambda\right)}{(\alpha\lambda-1)(e(\lambda-1)-\lambda)}$$
$$(\alpha\lambda-1)(e(\alpha+\lambda-1)-\lambda) > -\alpha\lambda+e\left(\left(\alpha^2+\alpha-1\right)\lambda-2\alpha+1\right)+\lambda$$

We have divided both sides by $e(\lambda-1)-\lambda$ and multiplied by $(\alpha\lambda-1)$. Both rearrangements require a sign change due to the fact that both are lower than 0. $e(\lambda-1)-\lambda<0$ holds for $\lambda > \frac{e}{e-1}$ which is true $\forall e$. Similarly $(\alpha\lambda-1)<0$ holds for $\alpha\lambda<1$ which is true $\forall \lambda, \alpha$. Simplifying and collecting leads to

$$e\lambda^2-2e\lambda+e-\lambda^2+\lambda>0$$
$$(\lambda-1)(e(\lambda-1)-\lambda)>0$$

which holds for any value of e and λ. Hence $\bar{\Phi}(1,0)$ is greater than $\hat{\Phi}(1,0)$.

Corollary 5

With contact disclosure and availability of partial funding, the choice on whether to issue a good $(P=G)$ or a bad $(P=B)$ preliminary can be summarised as follows. The CRA will issue a good preliminary if

$$\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)y_G}{(1-\lambda)(\alpha-2\alpha e+e)y_G-(\alpha-1)\lambda y_B}+\frac{(\alpha-1)\lambda\left(y_B-1\right)\left(y_G-1\right)}{\lambda y_B-(\lambda-1)y_G-1}+$$
$$\frac{\alpha e\lambda y_G}{e(-\lambda)+e+\lambda}-\alpha\lambda\left(y_G-1\right)+\Phi y_G >$$
$$\frac{\alpha e\lambda y_B y_G}{y_B(e-e\lambda)+\lambda y_G}+\frac{(\alpha-1)\lambda y_B((2\alpha-1)e-\alpha)}{-2\alpha\lambda+\alpha+(2\alpha-1)e(\lambda-1)+\lambda}+$$
$$\frac{\alpha\lambda\left(y_B-1\right)\left(y_G-1\right)}{(\lambda-1)y_B-\lambda y_G+1}+(\alpha-1)\lambda\left(y_B-1\right)+\Phi y_B$$

To find the threshold, assume the above holds with equality, leading to

$$\begin{aligned}\widetilde{\Phi} \;=\;& \frac{\frac{(1-\alpha)\lambda y_B(\alpha-2\alpha e+e)y_G}{(1-\alpha)\lambda y_B+(1-\lambda)(\alpha-2\alpha e+e)y_G}}{y_B-y_G} - \frac{\frac{\alpha e\lambda y_B y_G}{y_B(e-e\lambda)+\lambda y_G}}{y_B-y_G} - \frac{\frac{(\alpha-1)\lambda y_B((2\alpha-1)e-\alpha)}{-2\alpha\lambda+\alpha+(2\alpha-1)e(\lambda-1)+\lambda}}{y_B-y_G} \\ &+\frac{\frac{\alpha e\lambda y_G}{e(-\lambda)+e+\lambda}}{y_B-y_G} + \frac{\frac{(\alpha-1)\lambda(y_B-1)(y_G-1)}{\lambda y_B-(\lambda-1)y_G-1}}{y_B-y_G} - \frac{\frac{\alpha\lambda(y_B-1)(y_G-1)}{(\lambda-1)y_B-\lambda y_G+1}}{y_B-y_G} \\ &-\frac{(\alpha-1)\lambda\left(y_B-1\right)}{y_B-y_G} - \frac{\alpha\lambda\left(y_G-1\right)}{y_B-y_G}\end{aligned}$$

If $y_G = 1$ and $y_B = 0$ it becomes $\widetilde{\Phi} = \lambda\left(\alpha\left(-\frac{e}{e(-\lambda)+e+\lambda} - 1\right) + 1\right)$.

Corollary 6

Without contact disclosure and with perfect information in the official rating stage the choice on whether to issue a good $(P = G)$ or a bad $(P = B)$ preliminary can be summarised as follows. The CRA will issue a good preliminary if

$$\begin{aligned}&\frac{(1-\alpha)^2\lambda y_B y_G}{(1-\alpha)\lambda y_B + (1-\alpha)(1-\lambda)y_G} + \alpha\lambda y_G + \lambda\left(1-y_G\right) + \Phi y_G > \\ &\qquad \frac{\alpha\lambda y_B y_G}{(1-\lambda)y_B + \lambda y_G} + \frac{(1-\alpha)^2\lambda y_B}{(1-\alpha)(1-\lambda)+(1-\alpha)\lambda} + \lambda\left(1-y_B\right) + \Phi y_B\end{aligned}$$

To find the threshold, assume the above holds with equality, leading to

$$\begin{aligned}\bar{\Phi} \;=\;& \frac{-\frac{(1-\alpha)^2\lambda y_B y_G}{(1-\alpha)\lambda y_B+(1-\alpha)(1-\lambda)y_G}}{y_G-y_B} + \frac{\frac{\alpha\lambda y_B y_G}{(1-\lambda)y_B+\lambda y_G}}{y_G-y_B} \\ &+\frac{\frac{(1-\alpha)^2\lambda y_B}{(1-\alpha)(1-\lambda)+(1-\alpha)\lambda}}{y_G-y_B} + \frac{\lambda\left(1-y_B\right)}{y_G-y_B} - \frac{\alpha\lambda y_G - \lambda\left(1-y_G\right)}{y_G-y_B}\end{aligned}$$

if $y_G = 1$ and $y_B = 0$ it becomes $\bar{\Phi} = (1-\alpha)\lambda$ Similarly, with contact disclosure and with perfect information in the official rating stage the choice on whether to issue a good $(P = G)$ or a bad $(P = B)$ preliminary can be summarised as follows. The CRA will issue a good preliminary if

$$\frac{(1-\alpha)^2\lambda y_B y_G}{(1-\alpha)\lambda y_B+(1-\alpha)(1-\lambda)y_G}-\frac{\lambda\left(y_G-1\right)\left(-(\alpha-1)y_B+\alpha y_G-1\right)}{(\alpha-1)\lambda\left(y_G-y_B\right)+y_G-1}+$$
$$\alpha\lambda y_G+\Phi y_G>\frac{\alpha\lambda y_B y_G}{(1-\lambda)y_B+\lambda y_G}+$$
$$\frac{\lambda\left(1-y_B\right)\left((\alpha-1)y_B-\alpha y_G+1\right)}{(\alpha\lambda-1)y_B-\alpha\lambda y_G+1}+\frac{(\alpha-1)^2\lambda y_B}{-2\alpha\lambda+(2\alpha-1)(\lambda-1)+\alpha+\lambda}+\Phi y_B$$

To find the threshold, assume the above holds with equality, leading to

$$\begin{aligned}\hat{\Phi} \quad = \quad & \frac{-\frac{(1-\alpha)^2\lambda y_B y_G}{(1-\alpha)\lambda y_B+(1-\alpha)(1-\lambda)y_G}}{y_G-y_B}+\frac{\frac{\alpha\lambda y_B y_G}{(1-\lambda)y_B+\lambda y_G}}{y_G-y_B}+\frac{\frac{\lambda(1-y_B)((\alpha-1)y_B-\alpha y_G+1)}{(\alpha\lambda-1)y_B-\alpha\lambda y_G+1}}{y_G-y_B}\\ & +\frac{\frac{\lambda(y_G-1)(-(\alpha-1)y_B+\alpha y_G-1)}{(\alpha-1)\lambda(y_G-y_B)+y_G-1}}{y_G-y_B}+\frac{\frac{(\alpha-1)^2\lambda y_B}{-2\alpha\lambda+(2\alpha-1)(\lambda-1)+\alpha+\lambda}}{y_G-y_B}-\frac{\alpha\lambda y_G}{y_G-y_B}\end{aligned}$$

if $y_G=1$ and $y_B=0$ it becomes $\hat{\Phi}=\frac{\lambda\left(\alpha^2\lambda-2\alpha+1\right)}{1-\alpha\lambda}$.