Measuring the fiscal multiplier when plans take time to implement

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Measuring the Fiscal Multiplier when Plans Take Time to Implement

by

Kevin Lee†, James Morley††, Kian Ong††† and Kalvinder Shields††††

Abstract

The paper describes how to measure the fiscal multiplier using budget statements on planned government spending in the current and following years alongside the data on actual outcomes. The multiplier effects can be decomposed to distinguish the effects of ‘policy reactions’ versus ‘policy initiatives’, with the latter shown to be substantially larger than the former in a study of annual US data over 1957-2016. It is noted that the fiscal initiatives undertaken following the events of 2007/8 played an important role in mitigating the recessionary effects of the global financial crisis in the US.

Keywords: Government spending plans, real-time data, fiscal multiplier

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1 Introduction

There has been considerable recent interest on the size of the fiscal spending multiplier prompted by governments’ reactions to the global financial crisis and the constraints imposed on monetary policy by the zero lower bound. In the U.S. for example, the American Recovery and Reinvestment Act of 2009 had, according to the Economic Report of the President, the explicit intention of taking “the largest countercyclical fiscal action in American history” (CEA, 2010, p511–555) and the actual impact of the Act has been studied in detail over the last years.\footnote{See, for example, Taylor (2011) and Ramey (2011a) for an overview.} More generally, several recent studies have employed VAR models to identify spending shocks and to trace out their dynamic effects on output providing a range of views on the size of the multiplier. Ramey (2016) provides a useful summary, observing that the estimated multipliers from this work typically lie in the range [0.6, 1.5]; i.e. positive but with some ambiguity on the role of policy in practice.

The number of studies undertaken to estimate the size of the multiplier, and the relatively wide range of estimates obtained, reflect the difficulties in identifying the effects of exogenous shocks to spending. These difficulties arise in two broad forms. First, an interpretation problem arises in distinguishing the ‘primitive’, exogenous element of spending from that part of spending predictably arising from past macroeconomic decisions or arising as a contemporaneous reaction to shocks to other macroeconomic variables. Further, even within the class of exogenous shocks, it is difficult to distinguish between the effects of unanticipated policy responses (where spending reacts to unanticipated outside events) and the effects of unanticipated policy initiatives encountered when there is an unanticipated but proactive change in policy direction. Blanchard and Perotti [BP]’s (2002) solution to the interpretation problem - identifying spending shocks as the residuals in the spending equation of a VAR explaining spending, tax receipts and output and assuming that spending decisions are made prior to tax and output innovations - has been widely adopted but this clearly cannot in itself distinguish between the potentially very different effects of ‘fire-fighting’ policy responses and of more proactive spending initiatives.

The second class of difficulty arises because the process of planning, decision-making
and implementing government decisions typically takes time. So, for example, an exoge-
nous event (a missile launch by a foreign power or a flood, say) will trigger an immediate policy response from government, changing spending levels from what they would have been in the absence of the event (by increasing military surveillance or emergency relief, say). The government will also incorporate the news about the exogenous event into its plans at the next opportunity, but delays in decision-making (obtaining senate approval for defence build up, or re-housing flood victims, for example) and delays in implementing them (e.g. time taken to draw up contracts and procure missiles and housing) mean that the increased spending may not actually take place for some time. Of course, agents will be aware of the pattern of policy announcement and subsequent delay and will react accordingly, so that output responses may occur before the spending changes are observed in the data. The ‘fiscal foresight’ problem that arises in this situation has been widely acknowledged,\(^2\) and has been addressed explicitly in the measurement of the fiscal multiplier by Forni and Gambetti (2016) [FG] and Caggiano et al (2015), for example, through the use of direct measures of spending expectations obtained from surveys. However, the role of real-time measures of spending and the interplay between spending plans and actual outcomes has only been tangentially considered in the VAR models used to investigate fiscal multipliers.\(^3,4\)

This paper addresses the difficulties in identifying the effects of shocks to government spending by working with real-time measures of actual government spending - published with a one year delay - alongside the announced plans of spending to take place over the coming year and subsequently. We describe a VAR modelling approach that captures the interplay between actual and planned spending in a coherent way and comment on how

\(^2\)Discussion of the impact of anticipation effects on the propagation of fiscal shocks is provided by, among others, Mertens and Ravn (2011), Ramey (2011b) and Leeper et al (2013).
\(^3\)Real-time measures of spending and spending plans have been investigated explicitly in the context of estimating fiscal policy rules. See Beetsma et al (2009), Beetsma and Giuliodori (2010) and Corsetti et al (2012) for good examples, and Cimadomo (2016) for an overview of this literature.
\(^4\)A third class of difficulties in measuring the multiplier arises if the responsiveness of output to fiscal stimuli varies according to the state of the economy; see, for example, Auerbach and Gorodnichenko (2012), Mittnik and Semmler (2012), Fazzari et al. (2014), Caggiano et al. (2015), and Ramey and Zubairy (2016).
impulse response analysis and measures of the multiplier should be adapted to accommo-
date the sequencing of data releases. The identifying structure obtained using real-time
actual and planned spending, and the timing of these, provides insight on the differential
effects of government spending policy responses and policy initiatives. The use of news
on plans circumvents the fiscal foresight problem in the same way that the existing lit-
erate has circumvented the problem using survey data but the sequencing of decisions
highlighted by our analysis provides new insights on the identification schemes used, and
multipliers calculated, previously. The approach is applied to US data over 1957-2015,
including the recent period when interest rates have been at their zero lower bound. We
find the multiplier to be at the lower end of the range of estimates found in the literature,
at around 0.25 overall, but find that the multiplier for spending arising from policy initia-
tives are considerably higher. Despite the relatively small multiplier estimate, an exercise
estimating the effects of the 2009 policy initiatives shows the policy had a substantial mit-
igating effect on output during times of crisis with output with the accumulated benefit
of the measures estimated to be around 8% of gdp between 2009-2016.

2 Measuring the Fiscal Multiplier with Real-Time Data

Our modelling approach exploits the information contained in the inter-play between
three real-time measures of (the logarithm of) government spending: \( t g_{t-1} \), the actual
level of government spending made during the year \( t - 1 \) as documented in the budget
published at the beginning of the fiscal year \( t \); \( t g_t \), the level of spending planned to take
place in the current year \( t \) as published in the budget at the beginning of the year; and
\( t g_{t+1} \) the level of spending planned to take place in the following year, \( t + 1 \), as published
in the budget at the beginning of year \( t \). Importantly, the measure of actual spending
here is published with a one year delay reflecting the fact that the accurate measurement
of macroeconomic variables takes time; see Croushore (2011) for a general review and
Cimadomo (2016) on the specifics of measurement in fiscal data. Our assumption in what
follows is that measurement issues are resolved within one year so that \( t g_{t-1} \) provides an
accurate measure of actual output in \( t - 1 \) and is not subsequently revised (so that \( t g_{t-1} =
T g_{t-1} \), the measure of \( g_{t-1} \) in the most up-to-date, ‘final’ data vintage observed at \( T \)).
Importantly, we can be sure that our delayed measure of actual spending is consistent with
the measures of planned spending since they are taken from the same budget statement.

Our approach is based on a VAR model of the three series. This provides a characterisation of the dynamic responses of the series to new information, capturing the effects of exogenous events on spending outcomes and plans and the interactions between these as they play out over time. The model can be embedded within a broader VAR model, including output measures, to estimate the fiscal multiplier. We shall argue that the multiplier can be estimated more precisely using the three spending variables than is the case when only the actual spending series is used and, under reasonable assumptions on the timing of spending decisions, the multiplier effects can be decomposed in terms of the effects of different types of news on outcomes and plans made in real time.

2.1 Interpreting shocks to government spending

The three series \( tg_{t-1}, tg_t \) and \( tg_{t+1} \) evolve in different ways in reaction to exogenous events. An exogenous event occurring during the fiscal year between the budget at the beginning of \( t - 1 \) and the budget at the beginning of \( t \) triggers a reaction from government, changing spending outcomes and decisions from what they would have been in the absence of the event. Some part of this reaction might be achieved by drawing on contingency funds or by shifting existing planned resources from one function to another but usually the government will spend more or less than it had planned to do. The unexpected exogenous event implies an ‘implementation error’ then that drives a wedge between the original plan for spending in \( t - 1, t_{-1}g_{t-1} \), and the outcome \( tg_{t-1} \):

\[
tg_{t-1} - t_{-1}g_{t-1} = f_1(\Omega_{t-1}) + \varepsilon_{1t} \tag{1}
\]

where \( \varepsilon_{1t} \) is the implementation error and \( f_1(\Omega_{t-1}) \) captures any element of unplanned spending that is systematically related to known information at \( t - 1, \Omega_{t-1} \) because of information rigidities or deliberate misinformation say.\(^5\)

\(^5\)If plans reflect a full-information rational expectation of current period spending, there would be no systematic content to unplanned spending. If plans are influenced by information rigidities or noisy information as discussed in Coibion and Gorodnichenko (2011, 2012), or if - irrationally - unplanned
The government will incorporate the news about the exogenous event into its plans at the next opportunity so that we can write

\[ t g_t - t g_{t-1} = f_2(\Omega_{t-1}) + \varepsilon_{2t} \]  \hspace{1cm} (2)

\[ t g_{t+1} - t g_t = f_3(\Omega_{t-1}) + \varepsilon_{3t} \]  \hspace{1cm} (3)

where \( \varepsilon_{2t} \) and \( \varepsilon_{3t} \) are innovations to planned current and future government spending respectively, each correlated with \( \varepsilon_{1t} \) to the extent that plans accommodate the effects of the previous period’s policy response. The terms \( f_2(\Omega_{t-1}) \) and \( f_3(\Omega_{t-1}) \) capture the systematic influence of lagged information, including that from lagged plans and previous deviations of outcomes from plans, propagated over time because of the delays in making and implementing spending decisions.

Taking (1), (2) and (3) together, we have three equations in \( t g_{t-1}, t g_t \) and \( t g_{t+1} \) driven by lagged information and three interrelated shocks, \( \varepsilon_{1t}, \varepsilon_{2t} \) and \( \varepsilon_{3t} \). The above motivation suggests \( t g_{t-1} \) is determined first and without reference to the way in which subsequent plans might adjust to unplanned spending today. In this case, \( \varepsilon_{1t} \) is readily identified from the innovations to unplanned spending capturing the policy response to the unanticipated event. The two variables \( t g_t \) and \( t g_{t+1} \) are driven by the implementation shocks and by joint innovations to this year’s and next year’s plans capturing unanticipated policy initiatives. If we write \( \varepsilon_{2t} = \rho_{21} \varepsilon_{1t} + v_{2t} \) and \( \varepsilon_{3t} = \rho_{31} \varepsilon_{1t} + \rho_{32} \varepsilon_{2t} + v_{3t} \), then \( v_{2t} \) is the ‘initial fiscal shock’ (abstracting from the effects of the implementation error at \( t - 1 \)) and \( v_{3t} \) is the ‘deferred fiscal shock’, abstracting from implementation shocks and from the initial fiscal shock. These latter shocks are defined by timing assumptions and can be usefully interpreted as capturing the effects of unanticipated government spending initiatives, as distinct from the unanticipated spending responses of the implementation shocks.

spending is larger at times of greater growth, for example, then unplanned spending will be related to past spending \( f(\Omega_{t-1}) \).
2.2 The VAR modelling framework

Expressions (1), (2) and (3) can motivate a VAR model in which unplanned spending and current and future planned spending are modelled in a first-order VAR of the form:

\[
G_t = \begin{pmatrix} t g_{t-1} - t-1 g_{t-1} \\ t g_t - t g_{t-1} \\ t g_{t+1} - t g_t \end{pmatrix} = A \begin{pmatrix} t-1 g_{t-2} - t-2 g_{t-2} \\ t-1 g_{t-1} - t-1 g_{t-2} \\ t-1 g_t - t-1 g_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \tag{4}
\]

\[
= AG_{t-1} + \varepsilon_t
\]

where \( A \) is a \( 3 \times 3 \) matrix of parameters, \( AG_{t-1} \) captures the systematic element of \( G_t \) and \( \varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})' \) contains the shocks to unplanned and current and future planned spending. In terms of modelling, this form has the advantage that it implicitly builds in sensible time series properties for the three series. Specifically, if actual government spending is integrated of order 1 and if planned spending cannot permanently deviate from actual spending, then the three series \( t g_{t-1}, t g_t \) and \( t g_{t+1} \) will each have a unit root and will be driven by the same stochastic trend. The three series in \( G_t \) can all be written in terms of actual spending growth and deviations of actual from planned spending, and so they can be treated as stationary series appropriately captured by a VAR. Further, as shown in the Appendix, the model in (4) can be rewritten as a cointegrating VAR, tying the three levels series together in the long run, or as the following VAR in levels:

\[
Z_t = \begin{pmatrix} t g_{t-1} \\ t g_t \\ t g_{t+1} \end{pmatrix} = B_1 \begin{pmatrix} t-1 g_{t-2} \\ t-1 g_{t-1} \\ t-1 g_t \end{pmatrix} + B_2 \begin{pmatrix} t-2 g_{t-3} \\ t-2 g_{t-2} \\ t-2 g_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \tag{5}
\]

\[
= B_1 Z_{t-1} + B_2 Z_{t-2} + e_t
\]

where the \( B \)'s are transformations of the parameters in \( A \) (restricted to retain the cointegrating properties of (4)) and where \( e_t = (\varepsilon_{1t}, \varepsilon_{1t} + \varepsilon_{2t}, \varepsilon_{1t} + \varepsilon_{2t} + \varepsilon_{3t})' \) - i.e. an accumulation of the errors in (4). It is possible to obtain consistent estimates of the parameters of the model at (5) working with an unrestricted VAR in levels even if the variables in \( Z_t \) are \( I(1) \). But estimates of this unrestricted VAR will not take into account the structure
imposed by the assumption that actual and planned spending are cointegrated and move
together one-for-one in the long run. Consequently, the parameters obtained from the
unrestricted VAR in levels - and associated impulse responses and multipliers - will be
less precisely estimated than those obtained using (4).

It is worth emphasising at this stage that the innovations of $e_t$ are simply combined
values of the errors in $\varepsilon_t$ and that these errors can in turn be written as linear functions of
the $\varepsilon_{1t}$, $v_{2t}$ and $v_{3t}$, as discussed above. For this reason, and with details relegated to the
Appendix, we can write $e_t = P \nu_t$ where $\nu_t = (\varepsilon_{1t}, v_{2t}, v_{3t})'$ and the VAR in (5) can be
equivalently expressed in terms of the orthogonalised ‘implementation error’ and ‘initial’
and ‘deferred’ fiscal shocks.\footnote{We note that here, when $e_t = P \nu_t$, identification of $\nu_t$ from $e_t$ requires nine relations to describe
the elements of $P$. Matching the variance-covariance terms provides six relationships and the timing
assumptions (that $t g_{t-1}$ depends on $\varepsilon_{1t}$ but not $v_{2t}$ or $v_{3t}$ and that $t g_t$ depends on $\varepsilon_{1t}$ and $v_{2t}$ but not
$v_{3t}$) provides the other three.}

2.3 Measuring the effects of shocks to time-$t$ government spending

The effects of news on $Z_t$ can be seen best by writing (5) in its Moving Average form

$$Z_t = e_t + C_1 e_{t-1} + C_2 e_{t-2} + C_3 e_{t-3} + \ldots$$

$$= C(L) e_t$$

where $C(L) = (I - B_1 L - B_2 L^2)^{-1}$ is the polynomial in the lag operator $L$.\footnote{So the parameters of the $C$’s are functions of the $B$’s with
$C_0 = I$, $C_1 = B_1$, $C_2 = C_1 B_1 + C_0 B_2$, $C_3 = C_2 B_1 + C_1 B_2$, $C_4 = C_3 B_1 + C_2 B_2$, and so on}

where $C(L)^{-1} = \mathcal{C}(1)$ describes the effect of news on actual government spending as $t g_{t-1}$ is contained
within $Z_t$ but, given the delay in publication, it describes the effects on spending in $t - 1$
rather than $t$. If our interest is the effect of news on time-$t$ government spending, we
should use (5) as a forecasting model noting that the one-step-ahead forecast of actual
spending at $t$ based on information at $t$ is $E[ t_{t+1} g_{t} | \Omega_{t} ]$ which is contained within $E[
Z_{t+1} | \Omega_{t} ]$. Writing (6) at $t + 1$ and taking expectations based on $\Omega_{t}$ we have

$$E[Z_{t+1} | \Omega_{t}] = \mathcal{C}(L) \tilde{e}_t$$

$$= C(L) e_t$$

(6)
where $\bar{e}_t = E[Z_{t+1} | \Omega_t] - E[Z_{t+1} | \Omega_{t-1}] = B_1 e_t$ describes the news arriving at $t$ on actual spending in $t$ and $\tilde{C}(L) = (C(L) - I)B_1^{-1}$.

The time-profile of the effect of shocks on government spending will be captured by an impulse response function but the impulse responses should be calculated to take into account the fact that decisions take time to implement, describing the effect of the news arriving at time $t$ on the one-step-ahead forecast of time-$t$ government spending. Writing $E[Z_{t+1} | \Omega_t] = \tilde{Z}_t$ for notational convenience, and following Koop et al. (1996), a generalised impulse response (GIR) analysis applied to the three-variable VAR in (7) shows the effect on $\tilde{Z}_t$ of a specified shock $d_t$ compared to the outcome where no shock occurs:

$$IR(t, s, d_t) = E[\tilde{Z}_{t+s} | e_t = d_t, \Omega_t] - E[\tilde{Z}_{t+s} | e_t = 0, \Omega_t] = \tilde{C}_s d_t, \quad s = 0, 1, 2, \ldots$$

This GIR considers the effect of a ‘system-wide’ shock to the three-variables taking into account the correlations that are typically observed to occur between shocks. Here, writing the variance/covariance matrix of $e_t$ as $V[e_t] = \Phi = [\phi_{ij}]$, and noting that $V[\bar{e}_t] = B_1 V[e_t] B_1' = \bar{\Phi} = [\bar{\phi}_{ij}]$, a candidate for $d_t$ in (8) is $\frac{1}{\phi_{11}}(\phi_{11}, \phi_{12}, \phi_{13})'$; this means that (8) shows the dynamic response of the three series to a system-wide shock that causes $E[\zeta_{t+1} | \Omega_t]$ - the time-$t$ forecast of actual government spending at $t$ - to rise by one standard deviation on impact.\(^8\)

### 2.4 Comparing (8) with impulse responses found in the literature

The impulse response functions of (8) differ from those typically found in the literature in a number of important ways. In particular, many of the responses found in the literature are based on the time series analysis of a single measure of actual government spending, expressed in terms of the orthogonalised shocks. Since the impulse responses are equivalent, those for the shocks in $\nu_t$ provide a decomposition of the impulse response in (8).
usually the final vintage measure $\tau g_t$. While the inclusion of revenue and output variables in a VAR will improve the model’s characterisation of spending dynamics - as, of course, it will in the three variable system described above - a univariate model of actual outcomes will struggle to capture the sophistication of the interplay between plans and outcomes described above. If the true data generating process is described by the three-variable VAR in (5), the first row of the model explains actual spending and, as is well known, a univariate specification can be derived for any individual series embedded within a multivariate ARMA model. In this case, the single variable $t_{-1}g_t$ would have an ARMA(4,2) specification. In the absence of systematic revisions, the difference between the first-release measure and the final vintage measure is random and unpredictable on the basis of information available at $t$, so that the univariate specification for the final vintage series will also have this sophisticated specification. This means that any estimated univariate model based only on final vintage data is sure to involve a fierce simplifying approximation given the sample lengths of available data.

More significantly in terms of an economic interpretation, the univariate specification will be non-fundamental and the associated shocks will consist of a complicated amalgam of the underlying structural shocks. Hence it will be impossible to identify the separate effects of the $\varepsilon_{1t}$, $\varepsilon_{2t}$ or $\varepsilon_{3t}$, let alone the orthogonalised $\nu_{2t}$ and $\nu_{3t}$, and any derived impulse responses (and associated fiscal multipliers) could be misleading. FG make this point, arguing for the inclusion in the analysis of direct measures of expected future government spending - as provided in the Survey of Professional Forecasters reported by the Federal Reserve Bank of Philadelphia - alongside actual spending outcomes. By placing actual spending before expected future spending in the VAR, FG then distinguish between what they call a ‘surprise shock’ and a ‘news shock’ assuming that the surprise shock effects spending immediately while the news shock does not. This identification scheme helps avoid the problem of non-fundamentalness but, seen from the perspective of our modelling framework, the use of final vintage measures means FG assume that there are no implementation errors and no deviations between time-$t$ plans and final outcomes

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9 See Hamilton (1994), p106, for example.

10 Auerbach and Gorodnichenko (2012) make the same important point.
the impulse responses of (8) and those found in the literature relates to the treatment of long-run trends and unit roots in these macroeconomic series. This is particularly important where the focus is on measuring the fiscal multiplier. In this case, the VAR analysis of the three government spending variables will be embedded within a larger system including output (and potentially government receipts too). For example, in what we refer to as the ‘LMOS model’ below, we add in terms involving receipts and output as follows:

\[
\begin{pmatrix}
\tau_{t-1} - \tau_{t-2} \\
\tau_{t-1} - \tau_{t-2} \\
y_{t-1} - y_{t-2} \\
y_t - y_{t-1} \\
y_{t+1} - y_t \\
\end{pmatrix} = \mathbf{A}
\begin{pmatrix}
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-1} \\
\end{pmatrix} + \mathbf{\alpha}
\begin{pmatrix}
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-2} \\
t_{t-1} - t_{t-1} \\
\end{pmatrix} + \begin{pmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t} \\
\varepsilon_{3t} \\
\varepsilon_{4t} \\
\varepsilon_{5t} \\
\end{pmatrix}
\] 

(9)

The measures \(\tau_{t-1}\) and \(y_{t-1}\) are, respectively, the (logarithms of the) actual level of government receipts received and the actual level of output observed during year \(t – 1\) as reported in the fiscal budget published at the beginning of year \(t\). The variables acknowledge that, in practice, accurate measurements of receipts and output are achieved with a delay and the timing also ensures that they are consistent with the published figures on actual and planned spending. The model explains the growth in receipts and in output, as well as the growths in spending, in terms of their various lags and also in terms of the lagged ratios of spending-to-output and receipts-to-output on the grounds that these may exert equilibrating pressures on the series (captured by the \(5 \times 2\) matrix of feedback coefficients \(\mathbf{\alpha}\)). As before, (9) can be rewritten in the levels form at (6) where now \(\mathbf{Z}_t = (t_{t-1}, \tau_{t-1}, y_{t-1}, t_t, t_{t+1})'\) and \(\mathbf{e}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{1t} + \varepsilon_{4t}, \varepsilon_{1t} + \varepsilon_{4t} + \varepsilon_{5t})'\).

The model of (9) again builds in the assumption that government spending is \(I(1)\) and...
that the gap between plans and outcomes is stationary. But it also assumes that receipts and output are $I(1)$ and that the ratios of spending-to-output and receipts-to-output are stationary too. The stationarity of the spending-to-output ratio reflects the constancy of one of the ‘Great Ratios’ whereby social and political pressures keep government activity as a proportion of total activity broadly constant over time. If this is the case, the assumed stationarity of the receipts-to-output ratio reflects the idea that the government also runs a broadly balanced budget over time. The empirical validity of these assumptions are testable but embedding them in the model of (9) ensures that spending and receipts are cointegrated with output, each in a one-to-one relationship, and that all five of the series in $Z_t$ are driven by a single stochastic trend. The fiscal multiplier is typically calculated as the ratio of the accumulated effect on output of a shock to government spending relative to the accumulated effect of the shock on government spending. Since both series are $I(1)$, shocks to the system will cause both spending and output to be permanently higher than in the absence of the shock, potentially resulting in an infinite or zero multiplier in an unrestricted VAR model depending on whether output rises by more than spending at the infinite horizon or vice versa. Incorporating a constant spending-to-output ratio in the long run in (9) ensures that the series move together at long horizons and that measures of the multiplier are finite.\footnote{Most models in the literature work with a VAR in levels, accommodating the possibility of a unit root in principle, but treating the variables as stationary in levels in practice. A finite multiplier is obtained in those models because the responses of spending and output to shocks both fall to zero in the long run (although the measures are also often based on truncating the accumulated effects at some arbitrary finite horizon if convergence to zero is too protracted).}

The use of data on actual spending and spending plans means the effects of spending responses can still be distinguished from spending initiatives in this extended model but the identifying assumptions discussed earlier need updating. Specifically, in the five-variable system of (9) we can write $e_t = P \nu_t$ for five economically-meaningful orthogonal shocks $\nu_t = (\nu_{1t}, \nu_{2t}, \nu_{3t}, \nu_{4t}, \nu_{5t})'$. Identification of the $\nu_t$ from the $e_t$ requires 25 relations based on a priori information to explain the elements of the $5 \times 5$ matrix $P$. Fifteen of these relations are provided by matching the variance-covariance of the $e_t$ with those of the $P \nu_t$. A further four are provided by the long-run relations embedded within (9) if

\[11\]
we assume that $v_{5t}$, say, is a ‘productivity shock’ representing the single stochastic trend that drives the long-run changes in the variables. Finally, the timing assumptions that $t\gamma_{t-1}$ is determined before $t\tau_{t-1}$, that both are determined before $t\gamma_t$ and that this is determined before $t\gamma_{t+1}$ provides us with six further restrictions: $v_{2t}$ does not effect $t\gamma_{t-1}$; $v_{3t}$ and $v_{4t}$ do not influence $t\gamma_{t-1}$ and $t\tau_{t-1}$; and $v_{4t}$ does not influence $t\gamma_t$. These restrictions allow us to interpret $v_{1t}$ and $v_{2t}$ as the implementation shocks to spending and receipts respectively, and $v_{3t}$ and $v_{4t}$ as the initial and deferred fiscal shocks as before.

We do not impose any restrictions on the timing of the effects of the persistent shocks, $v_{5t}$, allowing them to impact on any of our five variables. This is an important contrast to models where series are ‘de-trended’ to achieve stationarity prior to analysis since these transformations will typically introduce serial dependencies in the data that undermine or contradict the timing assumptions used to identify the effects of the other shocks.

In summary, our treatment of the unit root properties of the series means we have to take this aspect of the modelling seriously, taking into account the permanent effects of productivity shocks within the model when calculating impulse responses and fiscal multipliers. But the insights on the use of the data when plans take time to implement - analysing the effects of shocks to the one-step-ahead forecasts of $t\gamma_{t-1}$ and $t\gamma_{t-1}$ and separating out the effects of government responses from government initiatives - remain relevant in this more complicated context.

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\(^{12}\)To see this, note that the long-run effect of shocks to the levels of the variables in $Z_t$, according to the representation at (6), are given by

$$C(1)e_t = C(1)P e_t$$

and that, since there is just one stochastic trend, $C(1)$ takes the form $(1, 1, 1, 1, 1)' \times (k_1, k_2, k_3, k_4, k_5)$. Then, following a similar argument to that of Blanchard and Quah (1989), the assumption that none of $v_{1t}$, $v_{2t}$, $v_{3t}$ or $v_{4t}$ have a long-run effect on our variables provides four restrictions on the parameters of $P$.

\(^{13}\)The assumption that fiscal policy has only transitory effects on output is typical in the literature. The assumption means that the fiscal multiplier will be understated to the extent that fiscal policy has permanent effects - if it increases productivity directly through public investment in infrastructure, say, or indirectly through hysteresis effects, for example - as these effects are interpreted as ‘productivity’ shocks.
Our empirical work uses annual measures of government spending obtained from successive budget papers of the US Government for each fiscal year from 1941-2016. We also use annual measures of (the logarithm of) actual government receipts as observed in real time, denoted $r_{t-1}$, and the (logarithm of the) actual output, denoted by $y_{t-1}$. Final vintage measures of government spending, $g_{t-1}$, receipts, $\tau_{t-1}$, and output, $y_{t-1}$, where $T = 2016$, are also used in a comparison to benchmark models found in the literature. All magnitudes are in real terms and are described in more detail in the Data Appendix.

Figure 1 plots the three real time measures of government spending $g_{t-1}$, $g_t$ and $g_{t+1}$ taken from the budget paper at time $t$ and Table 1 reports the mean and standard deviations of the growth of each of these series. Plots go back to 1941 although we shall focus on the period 1957-2016 in our analysis to avoid the effects of WWII and the Korean War. The plots show that both of the planned series ($g_t$ and $g_{t+1}$) typically anticipate and track actual government spending ($g_{t-1}$) quite closely. This is illustrated well during the years 1941-49, say, where the three series are close to simple one-year displacements of each other. But there are periods where the series diverge from each other with the divergence perhaps most apparent from the onset of the financial crisis in 2008. Here, the plots show the very substantial planned in-year increase in government spending in 2009 - and the lower, but still high, planned spending in 2010 - associated with the American Recovery and Reinvestment Act of 2009. They also show that these plans did indeed translate into increased spending, according to the outturn spending figures reported in subsequent years, although not at the levels that had been originally announced. Table 1 shows that, across the sample as a whole, the growth rate of actual spending was 2.8% per annum while current and future plans were typically slightly lower at 2.7% and 2.6% respectively. Plans were also more volatile on average than actual outcomes, but this is driven mainly by the very large increases announced in the 2009 budget.

Figure 2 provides plots of the government spending to output ratio and the government receipts to output ratio. The plots show some extreme outcomes during the WWII years 1941-45 and the post-Korean War years 1954-57, but the more remarkable feature of these
series is the relative constancy of the ratios over the last sixty years. These average at 0.21 and 0.18 for the spending and receipts ratios respectively but with standard deviation of just 0.015 and 0.017, the data reflect strong political and social equilibrating pressures to achieve broadly balanced budgets and to maintain a relatively constant ‘Great Ratio’ fixing the size of government relative to the economy as a whole.

3.1 The real-time model

A preliminary data analysis showed that the three alternative real time measures of government spending, real time receipts and real time output are all I(1). The growth variables, in actual spending, receipts and output and in planned spending, are all stationary and the ratios \((t \gamma_{t-1} - t \gamma_{t-1})\) and \((t \tau_{t-1} - t \gamma_{t-1})\) are stationary too.\(^{14}\) This ensures that the modelling framework set out in (9) is appropriate although, in practice, the model is estimated including two dummy variables, \(d_{2009,t}\) and \(d_{2010,t}\) which take the value 1 in 2009 and 2010 respectively and zero otherwise. These dummies capture the extreme innovations in the five series following the global financial crisis while effectively insulating the estimated regression coefficients from the effects of these extreme values.\(^{15}\)

Figure 3 highlights some of the complicated dynamics captured by the model, and the importance of properly defining the impulse of interest, by showing generalised impulse responses (GIRs) of the system to two illustrative shocks: a straightforward shock to the system in Figure 3a, and a shock to the one-step-ahead forecast in Figure 3b. Figure 3a shows the ’standard’ GIR of a system-wide shock scaled to cause \(t \gamma_{t-1}\) to rise by 1% on impact. The clear picture is that effects of the shock take a long while to play out, with \(t \gamma_{t-1}\) and other spending measures falling to their long-run position only some ten or eleven years later. Importantly though, as shown in Table 2, there are some strong correlations among the shocks of \(e_t\), so that a typical system-wide shock will also include

\(^{14}\)We conducted standard ADF tests and details are available on request.

\(^{15}\)A likelihood ratio test of the exclusion of these dummies takes the value 71.53 (with a p-value of 1.00), showing how important it is to accommodate the effects of the global financial crisis in this way. On the other hand, the equivalent tests of the exclusion of dummies for 2008 and 2011 take values of 1.38 and 3.67 (with p-values of 0.00 and 0.39 respectively), showing that the inclusion of the 2009 and 2010 dummies is sufficient adjustment of the baseline model.
innovations to other variables taking place contemporaneously to the shock to $t-1g_t$. The GIRs of Figure 3a take these into account and show output falling by 0.06% on impact and by 0.14% in steady-state, with spending falling to match the reduction in output in the long-term to re-establish the spending-to-output Great Ratio. In contrast, Figure 3b illustrates the effect of a different type of system-wide shock to our estimated model, tracing the effects of a shock to $E[t+1g_t | \Omega_t]$ taking into account the innovations to the other variables in $E[Z_{t+1} | \Omega_t]$ that are typically observed. The GIR of Figure 3b again shows the protracted influence from the spending shock, again taking some ten years to reach steady-state, but the contemporaneous innovations and long-term effects on output are now small but positive, with spending and output ultimately just 0.01% higher than they would have been in the absence of the shock.

The pattern of responses in the ‘standard’ GIR takes no account of the fact that decisions take time to implement and reflect the outcome of an unanticipated adverse event taking place during $t-1$ but after time $t-1$ plans are set. The event serves to reduce output on impact and to instigate a positive spending response in $t-1$, capturing the effects of automatic stabilisers and other policy responses. The GIRs of Figure 3b are the more appropriate responses to consider when considering the effects of time-$t$ news on planned time-$t$ government spending. As shown at (7), innovations to the one-step-ahead forecast series are captured by $\tilde{e}_t = B_1 e_t$ and the focus on the effect of a $\tilde{e}_{1t}$ shock shifts attention from the backward-looking policy responses to the effect of more forward-looking fiscal policy initiatives\(^\text{16}\).\footnote{If there were full-information rational expectations (FIRE), then $t g_{t-1} = t-1g_{t-1} + e_{1t}$ and $b_{14} = 1$ in $B_1$ so that $\tilde{e}_{1t}$ would be strongly influenced by $e_{4t}$ - i.e. news on $t g_t$. In the event, the estimated $\tilde{b}_{14} = 0.7$, while $\tilde{b}_{11} = 0.3$ and $\tilde{b}_{15} = 0.2$. So FIRE appears not to hold but $\tilde{e}_{1t}$ is nevertheless dominated by $e_{4t}$ and $e_{5t}$ rather than $e_{1t}$.}

Figures 4 and 5 pursue this idea further and show the outcome of decomposing the effect of the $\tilde{e}_t$ shock to $E[t+1g_t | \Omega_t]$ into the more economically-meaningful shocks described earlier where $e_t = P v_t$ for five orthogonal shocks $v_t = (v_{1t}, v_{2t}, v_{3t}, v_{4t}, v_{5t})'$, identified on the basis of our timing assumptions and the assumption that fiscal policy does not have long-run effects. Here then, $v_{1t}$ is explicitly identified as the spending
implementation shock, \( v_{3t} \) is the initial fiscal shock, \( v_{4t} \) is the deferred fiscal shock and all of these are assumed orthogonal to the productivity shock \( v_{5t} \) which captures the influence of the single stochastic trend with long-run consequences. The implementation-, initial fiscal- and deferred fiscal shocks obtained in this way are shown in Figure 4, with large increases in current planned spending observed in 1967 and, especially, 2009, but with offsetting plans to reduce spending in 2010. Figure 5 shows how the effects of these orthogonalised shocks would play out over time according to our model, decomposing the impulse response of the \( \hat{\varepsilon}_t \) shock that causes \( E_{t+1} g_t | \Omega_t \) to rise by 1\% shown in Figure 3 into the Orthogonalised Impulse Responses (OIR) relating to the \( v_{1t}, v_{3t} \) and \( v_{4t} \). This shows that the forward-looking policy initiatives captured by \( v_{3t} \) are actually quantitatively more important than the backward-looking policy responses captured by \( v_{1t} \), with both contributing to the protracted decade-long macroeconomic response to policy shocks.

3.2 Estimates of the fiscal multiplier

Of course, the primary interest here is to see how these dynamic responses to innovations translate into measures of the fiscal multiplier and how these compare with previous measures found in the literature. This is shown in Figure 6, where our estimated impulse responses are plotted alongside the equivalent estimates obtained following the methodologies employed in measuring the multiplier in FG and BP. Recall that, in BP, the analysis focuses on a three-variable VAR using final vintage measures of spending, receipts and output. In FG, the BP series are supplemented with the SPF survey expectations of one-year-ahead spending which we substitute here by the planned spending variable \( \hat{n}_t \) taken from the budget statements.

Figure 6 shows the GIR responses of (final vintage measures of) spending and output to a system-wide shock that causes spending to rise by 1\% on impact obtained from these two models, set alongside the impulse responses obtained from our model in response to an \( \hat{\varepsilon}_t \) shock reproduced from Figure 3b. In terms of government spending, the FG and BP responses are similar to those obtained in our model over the early years, with all three spending responses dropping to around 50\% of their original value after four or five years.
result is that the multiplier for the policy initiatives is 0.39, almost twice the size of that obtained considering the effects of shocks to policy responses and policy initiatives taken together.\footnote{The estimate is also of a similar magnitude to that obtained following the approach of FG. As noted earlier, our model simplifies to that of FG if time-$t$ plans are reflected in the final vintage data and \textit{there are no implementation errors}.} The innovations we have termed ‘implementation shocks’ therefore raise spending but are actually counter-productive as far as the output response is concerned. As we have discussed, the implementation shocks are easily interpreted in terms of the backward-looking policy reaction to adverse macroeconomic conditions and it is to be expected that the effects of such a within-year response to an unexpected crisis will be different to the effects of a forward-looking spending plan. But these results show that measures of the multiplier will understate the potential impact of fiscal policy if they fail to distinguish between the different types of policy. Of course, this can only be achieved through the use of real time data on actual and planned expenditures.

Figure 8 illustrates the macroeconomic significance of \textit{planned} fiscal policy by showing the effects of the counter-cyclical policy initiatives pursued following the financial crisis as captured by our model. The diagram focuses on potential output measures, derived by tracing through the effects of our estimated permanent shocks, $v_{st}$, on output and abstracting from the transitory government spending shocks. The diagram shows (the final vintage measure of ) actual output $y_t$ plus (i) the current and forecasted potential output level as would have been estimated prior to the crisis, in 2008; (ii) the current and forecasted potential output level as estimated after the crisis, in 2011; and (iii) the current and forecasted total output, taking account of the government spending shocks as well as the permanent shocks, again as estimated in 2011. The diagram captures the very significant permanent loss of output resulting from the crisis, with actual (and projected potential) output some 11.5\% lower in 2016 than it would have been in the absence of the crisis. But it shows also the very substantial offsetting effects of the planned fiscal expansion of 2009/10, with the accumulated benefit of the measures estimated to be around 8\% of gdp between 2009-2016.
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\textsuperscript{18}The estimate is also of a similar magnitude to that obtained following the approach of FG. As noted earlier, our model simplifies to that of FG if time-\( t \) plans are reflected in the final vintage data and \textit{there are no implementation errors}.\textsuperscript{19}
4 Concluding Comments

This paper uses data on spending outcomes and on spending plans to take into account that spending decisions take time to implement. The data helps draw the distinction between fiscal policy responses to adverse events, where policy is dominated by offsetting policy reactions and automatic stabilisers, and the effects of proactive and planned fiscal policy expenditure. The explicit treatment of the timing of expenditure - and the use of direct measures of planned future spending - also helps overcome the fiscal foresight problem that arises in measuring the multiplier when using only the most up-to-date actual data. Our analysis shows that fiscal policy shocks can have very protracted effects on output, lasting up to a decade, even in a model that builds in the assumption that spending does not have a permanent effect on output levels. The analysis shows too that it is important to distinguish between the effects of backward-looking policy spend (in the form of automatic stabilisers and ‘fire-fighting’ implementation errors) and the effects of more forward-looking, proactive spending captured in fiscal plans. Our analysis shows estimated multipliers for proactive planned spending are much larger than the multipliers estimated based on spending taken as a whole and the experience of 2008-2011 shows planned, proactive fiscal actions can have a substantial mitigating effect on output during times of crisis.
Table 1: Actual, Current Planned and Future Planned Spending Growth, 1957 – 2015

<table>
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<tr>
<th></th>
<th>Actual $t-1$</th>
<th>Current Plan $t$</th>
<th>Future Plan $t+1$</th>
<th>Final Vintage $t+2$</th>
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<tr>
<td>Mean</td>
<td>.0282</td>
<td>.0272</td>
<td>.0263</td>
<td>.0294</td>
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<tr>
<td>SD</td>
<td>.0397</td>
<td>.0537</td>
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<td>Min</td>
<td>-.0478</td>
<td>-.0687</td>
<td>-.0592</td>
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<tr>
<td>Max</td>
<td>.1439</td>
<td>.2790</td>
<td>.1696</td>
<td>.1630</td>
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Notes: The growth series are aligned according to their reference periods. Summary statistics refer to the mean, standard deviation, minimum and maximum values, respectively.

Table 2: Correlations between the Shocks from the Estimated LMOS Model

<table>
<thead>
<tr>
<th></th>
<th>$e_{1t}$</th>
<th>$e_{2t}$</th>
<th>$e_{3t}$</th>
<th>$e_{4t}$</th>
<th>$e_{5t}$</th>
<th>$\tilde{e}_{1t}$</th>
<th>$\tilde{e}_{2t}$</th>
<th>$\tilde{e}_{3t}$</th>
<th>$\tilde{e}_{4t}$</th>
<th>$\tilde{e}_{5t}$</th>
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<tr>
<td>$e_{1t}$</td>
<td>1.000</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$e_{2t}$</td>
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<td>1.000</td>
<td>-</td>
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<td>$e_{3t}$</td>
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<td>1.000</td>
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<td>$e_{4t}$</td>
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<td>-0.231</td>
<td>-0.243</td>
<td>1.000</td>
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<tr>
<td>$e_{5t}$</td>
<td>0.357</td>
<td>-0.109</td>
<td>-0.071</td>
<td>0.810</td>
<td>1.000</td>
<td>-</td>
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<table>
<thead>
<tr>
<th></th>
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<th>$\tilde{e}_{2t}$</th>
<th>$\tilde{e}_{3t}$</th>
<th>$\tilde{e}_{4t}$</th>
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<td>$\tilde{e}_{1t}$</td>
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<td>0.014</td>
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<td>0.949</td>
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<td>$\tilde{e}_{3t}$</td>
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<td>0.465</td>
<td>0.978</td>
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<td>-0.014</td>
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<td>$\tilde{e}_{4t}$</td>
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<td>0.820</td>
<td>0.973</td>
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<tr>
<td>$\tilde{e}_{5t}$</td>
<td>0.488</td>
<td>-0.038</td>
<td>0.024</td>
<td>0.804</td>
<td>0.983</td>
</tr>
</tbody>
</table>

Notes: The shocks relate to model (9) estimated using US data 1957-2016.
Figure 1: Actual ($tG_{t-1}$), Current Planned ($tG_t$) and Future Planned ($tG_{t+1}$) Government Spending for the US: 1941 - 2015
Figure 2: Ratios of Actual Spending to Output ($G_{t-1}/Y_{t-1}$) and Actual Receipts to Output ($\tau_{t-1}/Y_{t-1}$)
Figure 3a: LMOS Model: GIR of a unit $t_{g_{t-1}}$ ('e') shock
Figure 3b: LMOS Model: GIR of a unit ('e-tilde') $t+1g_t$
Figure 4: LMOS Orthogonal Shocks: Implementation Shock ($v_{1t}$), Initial Fiscal Shock ($v_{3t}$) and Deferred Fiscal Shock ($v_{4t}$) (with 2009, 2010 dummies)
Figure 5: LMOS Model: Orthogonal Impulse Response Functions:
Dynamic effects of 'implementation shocks', 'initial fiscal shocks' and 'deferred fiscal shocks' on forecasted actual spending at time $t$, $t+1g_t$.
Figure 6: Impulse Response Functions of a unit shock to forecasted actual spending, $t+1g_t$, and output, $t+1y_t$ (for LMOS) and to final vintage actual spending, $Tg_t$, and output, $Ty_t$, (for FG and BP).
Figure 7a: Fiscal Multipliers for LMOS, FG and BP Modelling Frameworks

- LMOS Total Fiscal Multiplier
- FG Fiscal Multiplier
- BP Fiscal Multiplier
Figure 7b: LMOS Total Fiscal Multiplier relative to LMOS Fiscal Multiplier abstracting from 'Implementation Shocks' ($v_{1t}$)

LMOS Total Fiscal Multiplier: $(v_1 + v_3 + v_4)$ shock

LMOS 'Initiatives' Fiscal Multiplier: $(v_3 + v_4)$ shock
Figure 8: Actual, Forecast and Potential Output, estimated at 2008 and 2011

- **Actual Output:** $t+1Y_t$
- **Potential Output estimated at 2008**
- **Forecast Output estimated at 2011**
- **Potential Output estimated at 2011**
5 Appendix

The model in (4) can be rewritten as a cointegrating VAR tying the three levels series together in the long run. To see this, write \( G_t = M_0 Z_t + M_1 Z_{t-1} \) where \( z_t = (t g_{t-1}, t g_t, t g_{t+1})' \) and

\[
M_0 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \quad \text{and} \quad M_1 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

Then (4) can be written as a VAR in levels as in (5):

\[
Z_t = B_1 Z_{t-1} + B_2 Z_{t-2} + e_t
\]

where \( B_1 = M_0^{-1}(AM_0 - M_1) \), \( B_2 = M_0^{-1}AM_1 \) and \( e_t = M_0^{-1}e_t = (\varepsilon_{1t}, \varepsilon_{1t} + \varepsilon_{2t}, \varepsilon_{1t} + \varepsilon_{2t} + \varepsilon_{3t}) \). Writing (5) as a VAR in differences, we obtain

\[
\Delta Z_t = (I - B_1)\Delta Z_{t-1} - (I - B_1 - B_2)z_{t-2} + e_t
\]

where \( \Pi = \alpha \beta' \) and \( \beta \) takes the form \( \beta' = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \).

Recall that we defined the ‘implementation shocks’, ‘initial fiscal shocks’ and ‘deferred fiscal shocks’ by writing \( \varepsilon_{2t} = \rho_{21} \varepsilon_{1t} + \nu_{2t} \) and \( \varepsilon_{3t} = \rho_{31} \varepsilon_{1t} + \rho_{32} \varepsilon_{2t} + \nu_{3t} \). Hence, we have

\[
\begin{bmatrix} 1 & 0 & 0 \\ -\rho_{21} & 1 & 0 \\ -\rho_{31} & -\rho_{32} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} = \begin{bmatrix} \varepsilon_{1t} \\ \nu_{2t} \\ \nu_{3t} \end{bmatrix}
\]

i.e.

\[
R e_t = \nu_t
\]

and

\[
\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \rho_{21} & 1 & 0 \\ \rho_{21} \rho_{32} + \rho_{31} & \rho_{32} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \nu_{2t} \\ \nu_{3t} \end{bmatrix}
\]

i.e.

\[
e_t = M_0^{-1}R^{-1} \nu_t = P \nu_t
\]
where $\mathbf{P} = \mathbf{M}_0^{-1}\mathbf{R}^{-1}$. In practice, the parameters of $\mathbf{R}$, and hence $\mathbf{P}$, are readily obtained by regressions involving the residuals from the estimated equations of (4).
6 Data Appendix

We source the real-time budget data from the official publication *Budget of The United States Government* issued by the Office of Management and Budget (OMB) and accessed via https://fraser.stlouisfed.org/title/54 and https://www.gpo.gov/fdsys/browse/collection.action?collection=

The data is produced annually and relates to a fiscal year that differs from a calendar year. Prior to 1977, the fiscal year ran from the third quarter to the second quarter but has run from the fourth quarter to the third quarter since 1977. The publication produced in the fourth quarter of year \( t-1 \) is dated as year \( t \) in the analysis, providing measures of \( t_{gt-1}, t_{gt}, \) etc.

The analysis is conducted on data on real outlays, real receipts and real outputs measured in billions of 2009 dollars, obtained from successive OMB Budgets. The first data vintage for both outlays and receipts is 1941 and the first data vintage for outputs is 1966, providing output figures from 1940. The final vintage of data available at the time of writing for all three series is 2016. When data is available in nominal terms only (e.g. outlay data to 1976), it is converted to a 2009 dollar value using the final-vintage GDP deflator.

The measures of outlays, receipts and outputs each underwent one definitional change between 1941 and 2016: the measures of outlays and receipts changed from the administrative budget to the unified budget in 1968; and the published measure of output changed from GNP to GDP in 1993.
References


