CFCM

CENTRE FOR FINANCE, CREDIT AND MACROECONOMICS

Working Paper 19/01

A theory of outside equity: Financing multiple projects

Spiros Bougheas and Tianxi Wang

Produced By:

Centre for Finance, Credit and Macroeconomics
School of Economics
Sir Clive Granger Building
University of Nottingham
University Park
Nottingham
NG7 2RD

hilary.hughes@nottingham.ac.uk

A Theory of Outside Equity: Financing Multiple Projects*

Spiros Bougheas[†] Tianxi Wang[‡]

January 2019

Abstract

In the financial economics literature debt contracts provide efficient solutions for addressing managerial moral hazard problems. We analyze a model with multiple projects where the manager obtains private information about their quality after the contract with investors is agreed. The likelihood of success of each project depends on both its quality and the level of effort exerted on it by the manager. We find that, depending on the distribution of the quality shock, the optimal financial contract can be either debt or equity.

JEL Classification: G30, D86

Keywords: Outside Equity; Financial Contracts; Principal Agent Model

*Acknowledgements: we would like to thank Paul Mizen, John Moore, Daniel Seidmann, Silvia Sonderegger and seminar participants at the Universities of Loughborough and Nottingham for helpful comments and suggestions. The usual disclaimer applies.

[†]School of Economics, University of Nottingham, University Park, Nottingham NG7 2RD, UK; e-mail: spiros.bouhjeas@nottingham.ac.uk, tel.no: 0044-115-8466108.

[‡]Department of Economics, University of Essex, Colchester, CO4 3SQ, UK. Email: wangt@essex.ac.uk. Fax: +44 (0) 1206 872724.

1. Introduction

In the financial economics literature debt contracts provide efficient solutions for addressing managerial moral hazard problems. For example, Innes (1990) demonstrates that when outside investors are unable to observe the manager's effort and the manager is protected by limited liability the optimal contract requires that the manager is only compensated when the output of the project is above a certain threshold value, which implies that external investors hold a debt claim. Laux (2001) extends Innes (1990) to the case of multiple projects and shows that as long as the returns across projects are not perfectly correlated, the optimal scheme compensates the manager only when all projects succeed, which once more implies that external investors hold a debt claim. Indeed, in both models, when the payoffs of investors are also restricted to be monotonic in the project's output the optimal financial claim is the standard debt contract.

In this paper, we extend Laux (2001) and allow the manager, as an insider, to have better information about the projects under her management than outside investors. In particular, we introduce an interim quality shock that reveals which projects might benefit from managerial effort. The shock is realized after the financial contract is signed and observed only by the manager. We find that depending on the distribution of the quality shock the optimal financial contract can be either debt or equity.[1]

In our model, as in Laux (2001), a manager is managing two projects. Each project can be either type h or type l. The manager by exerting effort can increase the probability of success of a type h project. Type l projects do not benefit from managerial effort. Outside investors cannot observe either the manager's level of effort or project types. When the probability that a project is type h is equal to one, our model is reduced to Laux (2001). Indeed, we show that when that probability is close enough to one, the optimal scheme compensates the manager only when both projects succeed, which leaves the investors

[1] Notice that in our model, at the time when the contract is signed both parties are equally informed. This is in contrast to the hidden information literature where nature chooses the type of agent prior to the signing of the contract. In that literature, it is well konwn that debt, because of its low-information-intensity, is the optimal financial contract (see, for example, Myers and Majluf, 1984).

holding a debt claim, as in Laux (2001). Intuitively, as Laux (2001) and Tirole (2006) have argued, such 'cross-pledging' reduces incentive costs by punishing the manager when only one project succeeds.

Our innovation lies on the observation that this cross-pledging scheme can ruin the manager's incentives when one project is type h and the other is type l. To see this, assume that the probability of success of a type l project is very close to zero. Under the cross-pledging scheme, even if the manager exerts effort on the type h project, the probability that both projects succeed and therefore she is compensated, is still very small. Thus, she has no incentive to exert effort on the type h project. In this case, to provide the manager with incentives to exert effort on the type h project, she should also receive a payoff when only one project succeeds. However, this payoff should not be too large, otherwise, it would ruin her incentives to exert effort on the two projects when both are type h. Thus, we find that when the probability of success of a type l project is sufficiently low, and the investor would like to offer incentives to the manager to always exert effort on a type h project, the optimal scheme offers the manager a proportion of the total output which implies that outside investors hold an equity claim.

However, we also show that providing maximal incentives to the manager is not always optimal. More specifically, we find that when the probability that a project is type h is either too high or too low (in which case the probability that one project is type h and the other is type l is very low) cross-pledging dominates outside equity. The intuition here is that it is too costly to provide incentives to the manager for states of nature that are highly unlikely. In such cases it is better to offer a cross-pledging contact that offers incentives to the manager to exert effort only when both projects are type h.

Relative to the cross-pledging scheme, issuing outside equity maintains the manager's incentives to exert effort when only one project benefits from managerial effort. Thus, our paper re-confirms the commonly held view that issuing outside equity enhances the firm's resilience to negative shocks on its assets, but our innovation is that the benefit of this enhancement is related not to bankruptcy or financial stress, but to agency costs.

Related Literature In their classic paper, Jensen and Meckling (1976) argue that firms choose their ownership structure in order to minimize agency costs related to issue outside equity and debt. Often requiring managers to have a stake in the firm is not sufficient to ensure that they act in the full interest of outside equity holders. Stronger incentives to managers can be provided by having them issuing debt in which case they only hold residual claims on the firm's income streams. However, high debt levels can also be costly as they provide incentives for excessive risk taking. Furthermore, there are other costs related to debt such as those incurred when the firm becomes insolvent and are associated either with its liquidation or reorganization. For example, according to the trade-off theory (e.g. Abel, 2018) bankruptcy costs set a limit to the level of debt that is issued because of the preferential tax treatment of interest payments relative to dividends. In our model these types of costs associated with debt are absent. Nevertheless, we show that there are cases where outside equity provides better incentives to managers.

Myers (2000) demonstrates the optimality of dividend paying outside equity. In a multi-period environment where income streams are not verifiable it is shown that as long as managers pay out a regular dividend, outside equity holders are willing to commit to participate for one additional period. In our work, we demonstrate the optimality of outside equity in single-period financial contracting where incomes (but not effort) are verifiable.

Outside equity can also be part of the optimal design of capital structure when contracts are incomplete because of uncertainty about future actions. In Dewatripont and Tirole (1994) outside equity holders are allocated control rights in those states where the firm performs well, while the holders of debt take control in those states where the firm underperforms. In Berkovitch and Israel (1996) and Fluck (1998) managerial efficiency is achieved by a mix of outside equity and debt and by the contingent allocation of the right to replace the manager. In our model contracting is complete and thus the allocation of decision rights is not an issue.

Outside equity also plays a role as a residual claim when there is an optimal limit to debt financing. For example, when managers have a choice over the size of projects, the optimal capital structure balances the trade-off between underinvestment caused by debt overhang (Myers, 1977) and overinvestment caused by excess cash flow (Jensen, 1986).

The optimality of linear contracts (e.g. equity) has also been considered by Holmström and Milgrom (1987) and Carroll (2015). In Holmström and Milgrom (1987) linear contracts provide incentives to the agent to exert effort when the principal can condition contractual terms on past performance. Our framework is static and therefore no such conditioning is possible. Carroll (2015) demonstrates the optimality of linear contracts when the principal is uncertain about the actions available to the agent, whereas this uncertainty is absent in our model. Furthermore, our paper features a switch between the equity contract and the debt contract as the parameter values change, a switch that is not present in those papers.

2. The Model

We consider a three-date model, $t = 0$, 1 and 2, with an entrepreneur who at date 0 seeks funds from an outside investor to finance two projects. Each project can either succeed or fail. At date 2, if a project succeeds it will return R, while if it fails it will return nothing. The probability of success of a project depends on (a) a binary shock realized at date 1, and (b) the entrepreneur's level of effort. At date 0, it is common knowledge that the shock is identically and independently distributed across the two projects. With probability θ a project is type h, while with probability $1 - \theta$ it is type l. After observing the type of each project, the entrepreneur chooses on how many projects to exert effort. Exerting effort does not affect the probability of success of a type l project which is equal to q. In contrast, for a type h project, exerting effort increases its probability of success from q to p. Exerting effort on a project incurs a nonpecuniary cost c to the entrepreneur. The investor cannot observe either the realized project types or the effort level exerted by the entrepreneur. Let $c_\Delta \equiv c/\,(p - q)$. Suppose that the entrepreneur manages one project of type h. By exerting effort the entrepreneur increases the probability that she receives the payoff specified by the contract from q to p. Then, c_Δ is equal to the minimum payoff the entrepreneur must receive when she is managing only one type h project.

Any contract agreed between the entrepreneur and the outside investor can only be conditioned on the outcomes of the two projects. Let r_i for $i = 0, 1, 2$ denote the repayment to investors when i projects succeed. Then, $w_i = iR - r_i$ equals the compensation to the entrepreneur. Limited liability implies that $r_0 = w_0 = 0$, and

$$0 \leqslant w_1 \leqslant R \quad \text{and} \quad 0 \leqslant w_2 \leqslant 2R. \tag{2.1}$$

We assume that the projects generate sufficient pledgeable income to compensate the investor for his initial investment. When designing the compensation scheme the investor has to consider whether it is optimal to offer incentives to the entrepreneur to exert effort (a) whenever a project is type h regardless the other project's type, or (b) only when both projects are type h. Below we analyze and compare the two cases. Let $m(k, n)$ denote the net expected payoff to the entrepreneur when n projects are type h and she exerts effort on $k \leq n$ projects. Then,

$$\begin{aligned}
m(2,2) &= p^2 w_2 + 2p(1-p)w_1 - 2c; \\
m(1,2) &= pqw_2 + \left(p(1-q) + q(1-p)\right) w_1 - c; \\
m(0,2) &= q^2 w_2 + 2q(1-q)w_1; \\
m(1,1) &= m(1,2); \\
m(0,1) &= m(0,2).
\end{aligned}$$

For example $m(1, 2)$ is equal to the entrepreneur's net payoff when both projects are type h and the entrepreneur chooses to exert effort on only one of them. Then one project succeeds with probability p and the other project with probability q and, thus, the probability that both projects succeed is equal to pq in which case the entrepreneur's compensation is equal to w_2. With probability $p(1-q) + q(1-p)$ only one project succeeds in which case the entrepreneur's compensation is equal to w_1.

2.1. Case 1: The entrepreneur is offered incentives to exert effort on every type h project

In this case the following incentive compatibility constraints must be satisfied:

$$m(2,2) \geqslant m(1,2) \tag{2.2}$$

$$m(2,2) \geqslant m(0,2) \tag{2.3}$$

$$m(1,1) \geqslant m(0,1). \tag{2.4}$$

The incentive compatibility constraint (2.2) requires that when both projects are type h the entrepreneur prefers to exert effort on both of them rather than on only one of them. Constraint (2.3) requires that when both projects are type h the entrepreneur prefers to exert effort on both of them rather than on neither of them. Lastly, constraint (2.4) requires that when only one project is type h the entrepreneur prefers to exert effort on it.

With probability θ a project is type h and the entrepreneur exerts effort on it and thus the project succeeds with probability p. Therefore, in this case, the *ex ante* probability that a project succeeds is give by $p_s \equiv \theta p + (1-\theta)\, q$. Hence, the investor's expected payoff is equal to:

$$p_s^2\,(2R - w_2) + 2p_s\,(1-p_s)\,(R - w_1)\,. \tag{2.5}$$

The investor now solves the following problem:

$$\max_{\{w_1,w_2\}} p_s^2\,(2R - w_2) + 2p_s\,(1-p_s)\,(R - w_1)\,,$$

subject to (2.1), (2.2), (2.3) and (2.4) and the following entrepreneur's participation constraint:

$$p_s^2 w_2 + 2p_s\,(1-p_s)\,w_1 - 2\theta c \geq 0, \tag{2.6}$$

given that the entrepreneur exerts effort on each project with probability θ.

Proposition 1 *Suppose that the investor would like the entrepreneur to exert effort on every type h project. Then the optimal scheme is given by:*

i) $\{w_1, w_2\} = \{c_\Delta, 2c_\Delta\}$ *for* $q < (p-q)\theta$, *and the repayment to investors is equal to* $\{r_0, r_1, r_2\} = \{0, R - c_\Delta, 2(R - c_\Delta)\}$. *As the payoff to investors is linear in project revenues they hold an equity claim.*

ii) $\{w_1, w_2\} = \left\{0, \frac{1}{q}c_\Delta\right\}$ *for* $q > (p-q)\theta$, *and the repayment to investors is equal to* $\{r_0, r_1, r_2\} = \left\{0, R, 2R - \frac{1}{q}c_\Delta\right\}$. *As the payoff to investors is concave in project revenues they hold a debt claim.*

Proof See Appendix 1.

Suppose that the investor would like to offer incentives to the entrepreneur to exert effort whenever a project is type h. Proposition 2 states that cross-pledging is not optimal when the probability of success of a type l project, q, is too small. Cross pledging ruins the entrepreneur's incentive to exert effort on a type h project when the other project is type l. To see this point, consider the case where $q \approx 0$. If the contract compensates the entrepreneur only when both projects succeed she does not have any incentives to exert effort on the type h project given that the increment in the probability that she will receive compensation, $(p-q)q$, is very small (violating (2.4)). This implies that if q is small enough, then the entrepreneur must also be compensated when only one project succeeds. However, there is a limit to how high the compensation can be in that case. If that compensation is more than half the compensation that she would receive if both projects succeed then her incentives to exert effort on both projects when they are type h would be destroyed (violating (2.2)). Thus, the compensation has to be set proportional to the number of successful projects (constraints (2.2) and (2.4) are both binding) and we have an equity contract.

2.2. Case 2: The entrepreneur is offered incentives to exert effort only when both projects are type h

In this case the following incentive compatibility constraint substitutes for (2.4):

$$m(0,1) \geqslant m(1,1) \tag{2.7}$$

With probability θ^2, both projects are type h, the entrepreneur exerts effort on both projects and each project succeeds with probability p. In all the other states of nature a project is either type l or type h but the entrepreneur exerts no effort on it, and hence, it succeeds with probability q. Therefore, *ex ante*, both projects succeed with probability $\theta^2 p^2 + \left(1-\theta^2\right) q^2$, and only one project succeeds with probability $\theta^2 \times 2p(1-p) + \left(1-\theta^2\right) \times 2q(1-q)$, and with the complementary probability neither project succeeds. Therefore, the investor solves:

$$\max_{\{w_1,w_2\}} \left[\theta^2 p^2 + \left(1-\theta^2\right) q^2\right](2R - w_2) + 2\left[\theta^2 p(1-p) + \left(1-\theta^2\right) q(1-q)\right](R - w_1)\,, \tag{2.8}$$

subject to (2.1), (2.2), (2.3), (2.7) and the following entrepreneur's participation constraint:

$$\theta^2\left[p^2 w_2 + 2p(1-p)w_1 - 2c\right] + \left(1-\theta^2\right)\left[q^2 w_2 + 2q(1-q)w_1\right] \geqslant 0. \tag{2.9}$$

The following proposition describes the optimal contract:

Proposition 2 *Suppose that the investor would like the entrepreneur to exert effort only when both projects are type h. Then the entrepreneur's compensation scheme is given by: $\{w_1, w_2\} = \left\{0, \frac{2}{p+q}c_\Delta\right\}$. The repayment to investors is equal to $\{r_0, r_1, r_2\} = \left\{0, R, 2R - \frac{2}{p+q}c_\Delta\right\}$. Given that the investor's payoff is concave in project revenues, she holds a debt claim.*

Proof See Appendix 1.

Here, as in Laux (2001), the optimal scheme features cross-pledging; that is, the entrepreneur does not receive any compensation unless both projects succeed.[2] But in this case the result is due to the supposition that the investor prefers that the entrepreneur exerts effort only when both projects are type h. Next, we compare the investor's payoffs from the two cases.

2.3. The Optimal Contract

Considering Propositions 1 and 2 together, we find that when $q > (p-q)\,\theta$, or equivalently $\theta < q/\,(p-q)$, the optimal financial contract is debt irrespective of whether the manager is offered incentives to exert effort only when both projects are type h or she is offered incentives to always exert effort when a project is type h. However, when $\theta > q/\,(p-q)$, the optimal scheme requires the investor to hold a debt claim in the former case but an equity claim in the latter case. By comparing the investor's payoff between these two cases, we arrive at the main result of the paper:

Theorem 1 *The optimal financial contract:*

i) If $\theta < \frac{q}{p-q}$ *the optimal contract to the investor is always debt;*

ii) if $\theta > \frac{q}{p-q}$ *then*

a) if $\theta(1-\theta)\,(p-q)\,(R-c_{\Delta}) > \frac{pq}{p+q}c_{\Delta}$ *the optimal financial contract is equity and the entrepreneur always exerts effort when a project is type* h*; and*

b) if $\theta(1-\theta)\,(p-q)\,(R-c_{\Delta}) < \frac{pq}{p+q}c_{\Delta}$ *the optimal financial contract is debt and the entrepreneur only exerts effort when both projects are type* h*.*

Proof See Appendix 1.

We know from Proposition 1 that when q is sufficiently low the probability that both projects succeed is also low and therefore a cross-pledging scheme designed to offer incentives to the entrepreneur to exert effort on every type h project does not work. For low

[2] If $R < \frac{2c}{p^2-q^2}$ the payoff to investors will not be monotonic in project revenues and the contract will not be standard debt. Innes (1990) shows how we can still obtain a standard debt contract by imposing an additional monotonicity constraint.

values of q the investor has two options. The first option is to offer an equity contract that preserves the entrepreneur's incentives to exert effort on every type h project. However, Proposition 2 and the theorem have identified another possibility. It might not always be optimal for the entrepreneur to exert effort on every type h project. In particular, when the cost of exerting effort c is high and for either relatively high values or low values of θ, a cross-pledging scheme that offers incentives to the entrepreneur to exert effort only when both projects are type h dominates. The intuition is that it is not cost efficient to provide the entrepreneur with incentives to exert effort in the event when (a) only one project is type h and (b) the likelihood of this event is very small.

Notice that for $\theta = 1$, our model is identical to Laux (2001). In this particular case, the only source of asymmetric information is due to the unobservability of the level of effort exerted by the entrepreneur and cross-pledging is the optimal mechanism for providing incentives. However, cross-pledging is outperformed by outside-equity when q is low, and $2\theta(1-\theta)$, which is equal to the probability that only one project is type h, is sufficiently high. The role of outside equity is to preserve the incentives of the entrepreneur to exert effort when only one project can benefit from such effort.

3. Conclusion

We have demonstrated a novel role for outside equity. In most of the financial economics literature the role of equity has been as a residual claim. This is not surprising given that the main objective has been to explain why debt is so prevalent given that it is a more complex instrument with higher transaction costs than equity. There have been some sporadic attempts to rationalize outside equity for the direct benefits that it provides and this paper falls into that category.

In our model, after projects have been funded, the manager might receive inside information about their prospects in which case she will have to decide whether or not it is worth exerting effort to improve their likelihood of success. As in the classical managerial moral hazard model the likelihood of success of each project depends on the level of effort that the manager will exert on it, which is unobservable by investors. The difference is that in our model only some projects can benefit from the manager's input and projects that do benefit are only revealed to the entrepreneur and only after the funding contract with investors is agreed. There are two main results. We have shown that it is not always optimal to design schemes that provide maximal incentives to the manager. More importantly, we have found that even when it is optimal to provide maximal incentives, in some cases the best way to do so is by having investors hold an equity claim.

Lastly, we consider the impact on our main results of relaxing a couple of the assumptions of our model. As in Laux (2001) our model can be extended to the case where there are more than two projects. In such cases, a debt contract would only compensate the entrepreneur when at least a given number of projects are successful while with an equity contract the compensation would be proportional to the number of successful projects. We have also assumed that project types are independently distributed. Our results suggest that when project types are strongly positively correlated, projects are more likely to be funded by debt, while when types are negatively correlated projects are more likely to be funded by equity.

4. Appendix 1

4.1. Proof of Proposition 1

Together the incentive constraints (2.2) and (2.4) imply (2.3). This is because $m(2,2) \geqslant m(1,2) = m(1,1) \geq m(0,1) = m(0,2)$; where the first inequality is implied by constraint (2.2) and the second inequality by constraint (2.4). We can write the entrepreneur's participation constraint (2.6) as:

$$\begin{array}{rl} & \theta^2\left[p^2 w_2 + 2p(1-p)w_1 - pqw_2 - \left(p(1-q)+q(1-p)\right)w_1 - 2c\right] + \\ \left(2\theta-\theta^2\right) & \left[pqw_2 + \left(p(1-q)+q(1-p)\right)w_1 - q^2 w_2 - 2q(1-q)w_1 - c\right] + \\ & \left[q^2 w_2 + 2q(1-q)w_1\right] \geqslant 0. \end{array}$$

Notice that when (2.2) is binding the expression inside the first square brackets vanishes and when (2.4) binds the expression inside the second square brackets vanishes. We can write (2.2) as

$$pw_2 + (1-2p)w_1 \geqslant c_\Delta$$

and (2.4) as:

$$qw_2 + (1-2q)w_1 \geqslant c_\Delta.$$

At least one of the two constraints must be binding at the optimum; otherwise w_2 can be reduced, which decreases the entrepreneur's compensation and benefits the investor. We are going to show that both constraints are binding in which case the optimal solution is given by $\{w_1, w_2\} = \{c_\Delta, 2c_\Delta\}$.

Suppose that only (2.2) is binding, namely

$$pw_2 + (1-2p)w_1 = c_\Delta.$$

Then (2.4) is satisfied if and only if $qw_2 + (1-2q)w_1 \geq pw_2 + (1-2p)w_1$, namely $2w_1 \geq w_2$. Solving (2.2) for w_2 and substituting the solution in the entrepreneur's expected compensation function, the problem is reduced to

$$\min_{\{w_1,w_2\}} \left(2\theta-\theta^2\right)\left[(p-q)\left(q\frac{c_\Delta-(1-2p)w_1}{p}+(1-2q)w_1\right)-c\right]+\left[q^2\frac{c_\Delta-(1-2p)w_1}{p}+2q(1-q)w_1\right],$$

subject to (2.2).

Differentiating with respect to $w_1$we get $\left(2\theta-\theta^2\right)\left[(p-q)q\left(-\frac{1-2p}{p}+\frac{1-2q}{q}\right)\right]+q^2\left[-\frac{1-2p}{p}+\frac{2(1-q)}{q}\right] >$

0 because $\frac{1-2p}{p} < \frac{1-2q}{q} < \frac{2(1-q)}{q}$. Therefore, the investors would like to increase w_1, and as a result decrease w_2, as much as possible. Hence at the optimum, $2w_1 \geq w_2$ will be binding, that is, (2.4) will also be binding.

Next, consider the case where (2.4) is binding, namely,

$$qw_2 + (1-2q)w_1 = c_\Delta.$$

Then, (2.2) is satisfied if and only if $2w_1 \leq w_2$. Solving (2.4) for w_2 and substituting the solution in the entrepreneur's expected compensation function, the problem is reduced to

$$\min_{\{w_1,w_2\}} \theta^2 \left[(p-q)\left(p\frac{c_\Delta - (1-2q)w_1}{q} + (1-2p)w_1\right) - c\right] + \left[q^2\frac{c_\Delta - (1-2q)w_1}{q} + 2q(1-q)w_1\right],$$

subject to (2.4).

Differentiating with respect to $w_1$we get $\theta^2(p-q)p\left[-\frac{1-2q}{q} + \frac{1-2p}{p}\right] + q^2\left[-\frac{1-2q}{q} + \frac{2(1-q)}{q}\right] = \frac{(p-q)^2}{q}\left(\theta + \frac{q}{p-q}\right)\left(-\theta + \frac{q}{p-q}\right)$. Therefore, we have two cases.

i) If $\theta > \frac{q}{p-q}$ then the derivative is negative and the investor would like to set w_1 as high as possible. As a result, the constraint $2w_1 \leq w_2$, namely (2.2), is binding, which implies that $w_1 = \frac{w_2}{2}$. The last expression together with the binding (2.4) imply that the optimal contract is given by $\{w_1, w_2\} = \{c_\Delta, 2c_\Delta\}$.

ii) If $\theta < \frac{q}{p-q}$ then then the derivative is positive and the investor would like to set w_1 as low as possible, that is $w_1 = 0$ (the manager's limited liability constraint is binding), which together with the binding constraint (2.4) imply that the optimal contract is given by $\{w_1, w_2\} = \left\{0, \frac{1}{q}c_\Delta\right\}$. $\square$

4.2. Proof of Proposition 2

Together the incentive constraints (2.3) and (2.7) imply (2.2). This is because $m(2,2) \geqslant m(0,2) = m(0,1) \geq m(1,1) = m(1,2)$; where the first inequality is implied by constraint

(2.3) and the second inequality by constraint (2.7). Therefore (2.2) is not binding. We can write constraint (2.3) as

$$\frac{p+q}{2} w_2 + (1-(p+q))\, w_1 \geq c_\Delta, \tag{4.1}$$

and constraint (2.7) as

$$q w_2 + (1-2q)\, w_1 \leq c_\Delta. \tag{4.2}$$

We can also write the entrepreneur's participation constraint (2.9) as:

$$\theta^2 \left[p^2 w_2 + 2p(1-p)w_1 - q^2 w_2 - 2q(1-q)w_1 - 2c\right] + q^2 w_2 + 2q(1-q)w_1 \geqslant 0.$$

Constraint (2.3) implies that the expression in the square brackets cannot be negative. It follows that this constraint is never binding.

The investor's problem is then equivalent to one that minimizes the entrepreneur's expected compensation, that is

$$\min_{\{w_1,w_2\}} \left[\theta^2 p^2 + \left(1-\theta^2\right) q^2\right] w_2 + \left[\theta^2 2p(1-p) + \left(1-\theta^2\right) 2q(1-q)\right] w_1$$

subject to (4.1) and (4.2). Any decline in wages tightens constraint (4.1) but relaxes constraint (4.2). Given that $p > q$ the investor would like to reduce w_2 as much as possible. Therefore, at the optimum, constraint (4.1) is binding and we have:

$$\frac{p+q}{2} w_2 + (1-(p+q))\, w_1 = c_\Delta.$$

We can rewrite the objective function as

$$\theta^2 \left[\left(p^2 - q^2\right) w_2 + q^2 w_2 + 2\left(p(1-p) - q(1-q)\right) w_1\right] + q^2 w_2 + 2q(1-q) w_1.$$

The constraint implies that the expression inside the square brackets is equal to $2c$ and thus we can write the objective function as $q\left[qw_2 + 2(1-q)w_1\right] + \theta^2 2c$. Moreover, the constraint implies that

$$w_2 = \frac{2}{p+q}\left(c_\Delta - (1-(p+q))\, w_1\right).$$

Substituting the above in the objective function and differentiating with respect to w_1 we find that the derivative has the same sign as

$$-\frac{2q}{p+q} + 1 < 0.$$

It follows that at the optimum, $w_1 = 0$ and $w_2 = \frac{2}{p+q}c_\Delta$ as long as this contract satisfies the limited liability constraint, namely, $\frac{2}{p+q}c_\Delta \leq 2R$ or $c_\Delta \leq (p+q)\, R$. Otherwise, constraint (4.1), or equivalently (2.3), can never be satisfied and this type of financing is not feasible.

□

4.3. Proof of Theorem 1

Part (i) follows from part (ii) of Proposition 1 and Proposition 2. More specifically we can prove the following result for $\theta < \frac{q}{p-q}$.

Lemma 1 *Optimal debt contract:*

(i) If $2\theta(1-\theta)\left(R - \frac{c_\Delta}{q}(1-pq)\right) - \frac{c_\Delta}{q}p_s^2 > 0$, *the contract is given by* $\{r_0, r_1, r_2\} = \left\{0, R, 2R - \frac{1}{q}c_\Delta\right\}$ *and the entrepreneur always exerts effort when a project is type* h,

(ii) if $2\theta(1-\theta)\left(R - \frac{c_\Delta}{q}(1-pq)\right) - \frac{c_\Delta}{q}p_s^2 < 0$, *the contract is given by* $\{r_0, r_1, r_2\} = \left\{0, R, 2R - \frac{2}{p+q}c_\Delta\right\}$ *and the entrepreneur only exerts effort when both projects are type* h.

Proof We need to compare the investor's expected payoff when the debt contract offered is the one given by part (ii) of Proposition 1 with the corresponding payoff when the debt contract offered is given by Proposition 2.

Consider the revenues generated by the two contracts. After subtracting the revenues generated when the contract offered is that given by Proposition 2 from the revenues generated when the contract offered is given by part (ii) of Proposition 1 and after some simple algebraic manipulation we find that the difference is equal to $2R\theta(1-\theta)(p-q)$. The difference in the revenues arises because the two contacts offer different incentives on exerting effort when only one project is type h.

Next, consider the costs generated by the two contracts. By substituting $w_1 = 0$ and $w_2 = \frac{1}{q}c_\Delta$ in (2.5) we find that when the contract offered is given by part (ii) of Proposition 1, costs are equal to $\frac{1}{q}c_\Delta\left(\theta^2p^2 + 2\theta(1-\theta)pq + (1-\theta)^2q^2\right)$. By substituting $w_1 = 0$ and $w_2 = \frac{2}{p+q}c_\Delta$ in (2.8) we find that when the contract offered is given by Proposition 2, costs are equal to $\frac{2}{p+q}c_\Delta\left(\theta^2p^2 + (1-\theta^2)q^2\right)$. Noticing that $2\theta(1-\theta)pq+(1-\theta)^2q^2 = (1-\theta^2)q^2+2\theta(1-\theta)(p-q)$. and by subtracting the costs generated when the contract offered is that given by Proposition 2 from the costs generated when the contract offered is given by part (ii) of Proposition 1 we find that the difference is equal to $c_\Delta\left(\frac{p-q}{q}\right)\left(\theta^2p^2 + (1-\theta^2)q^2 + 2\theta(1-\theta)\right)$.

Lastly, by subtracting the difference in costs from the difference in revenues we find that

$$\begin{aligned}
& 2R\theta(1-\theta)(p-q) - c_\Delta\left(\frac{p-q}{q}\right)\left(\theta^2p^2 + (1-\theta^2)q^2 + 2\theta(1-\theta)\right) \\
= \;& (p-q)\left(2\theta(1-\theta)\left(R - \frac{c_\Delta}{q}\right) - \frac{c_\Delta}{q}\left(\theta^2p^2 + (1-\theta^2)q^2\right)\right) \\
= \;& (p-q)\left(2\theta(1-\theta)\left(R - \frac{c_\Delta}{q}\right) - \frac{c_\Delta}{q}\left(p_s^2 - 2\theta(1-\theta)pq\right)\right) \\
= \;& (p-q)\left(2\theta(1-\theta)\left(R - \frac{c_\Delta}{q}(1-pq)\right) - \frac{c_\Delta}{q}p_s^2\right)
\end{aligned}$$

which completes the proof. □

To prove part (ii) of the theorem we must compare the investor's expected payoff from offering the debt contract of Proposition 2 with the expected payoff form offering the equity contact in part (i) of Proposition 1.

The entrepreneur's compensation contract implied by the debt contract is given by $\{w_1, w_2\} = \left\{0, \frac{2}{p+q} c_\Delta\right\}$. The investor's payoff is given by

$$\begin{aligned} & \theta^2 \left[p^2 (2R - w_2) + 2p(1-p)(R - w_1)\right] + \left(1 - \theta^2\right) \left[q^2 (2R - w_2) + 2q(1-q)(R - w_1)\right] \\ = & \left[\theta^2 p^2 + \left(1 - \theta^2\right) q^2\right] (2R - w_2) + \left[2p(1-p)\theta^2 + 2q(1-q)\left(1 - \theta^2\right)\right] (R - w_1) \end{aligned}$$

and after we substitute in the entrepreneur's payoffs given by the contract, we get

$$\begin{aligned} V_D &= \left[\theta^2 p^2 + \left(1 - \theta^2\right) q^2\right] \left(2R - \frac{2c_\Delta}{p+q}\right) + \left[2p(1-p)\theta^2 + 2q(1-q)\left(1 - \theta^2\right)\right] R \\ &= 2\left[p\theta^2 + \left(1 - \theta^2\right) q\right] R - 2\left[\theta^2 p^2 + \left(1 - \theta^2\right) q^2\right] \frac{c_\Delta}{p+q}. \end{aligned}$$

The entrepreneur's compensation contract implied by the equity contract is given by $\{w_1, w_2\} = \{c_\Delta, 2c_\Delta\}$. The investor's expected payoff is given by

$$p_s^2 (2R - w_2) + 2p_s (1 - p_s)(R - w_1)$$

and after we substitute the entrepreneur's payoffs given by the contract, we get

$$\begin{aligned} V_E &= p_s^2 (2R - 2c_\Delta) + 2p_s (1 - p_s)(R - c_\Delta) \\ &= 2p_s (R - c_\Delta). \end{aligned}$$

Hence, the equity contract is optimal if and only if

$$
\begin{aligned}
V_E &> V_D \Leftrightarrow \\
2p_s\left(R - c_\Delta\right) &> 2\left[p\theta^2 + \left(1-\theta^2\right)q\right]R - 2\left[\theta^2 p^2 + \left(1-\theta^2\right)q^2\right]\frac{c_\Delta}{p+q} \Leftrightarrow \\
p_s\left(R - c_\Delta\right) &> \left[p\theta^2 + \left(1-\theta^2\right)q\right]R - \left[\theta^2 p^2 + \left(1-\theta^2\right)q^2\right]\frac{c_\Delta}{p+q} \Leftrightarrow \\
\left(p_s - \left[p\theta^2 + \left(1-\theta^2\right)q\right]\right)R &> \left[p_s - \frac{\theta^2 p^2 + \left(1-\theta^2\right)q^2}{p+q}\right]c_\Delta \Leftrightarrow \\
\left(\theta - \theta^2\right)(p-q)R &> \frac{\left(\theta-\theta^2\right)\left(p^2-q^2\right) + pq}{p+q}c_\Delta \\
&= \left(\theta-\theta^2\right)(p-q)\left(R - c_\Delta\right) - \frac{pq}{p+q}c_\Delta
\end{aligned}
$$

which completes the proof of the theorem. □

References

[1] Abel, Andrew. 2018. "Optimal Debt and Profitability in the Trade-Off Theory." *Journal of Finance* 73: 95-143.

[2] Berkovitch, Elazar, and Ronen Israel. 1996. "The Design of Internal Control and Capital Structure." *Review of Financial Studies* 9: 209-40.

[3] Carroll, Gabriel. 2015. "Robustness and Linear Contracts." *American Economic Review* 105: 536-63.

[4] Dewatripont, Mathias, and Jean Tirole. 1994. "A Theory of Debt and Equity: Diversity of Securities and Manager-Shareholder Congruence." *Quarterly Journal of Economics* 109: 1027-54.

[5] Fluck, Zsuzsanna. 1998. "Optimal Financial Contracting: Debt versus Outside Equity." *Review of Financial Studies* 11: 383–418.

[6] Holmström, Bengt, and Paul Milgrom. 1987. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* 55: 303-28.

[7] Innes, Robert. 1990. "Limited Liability and Incentive Contracting with Ex Ante Action Choices." *Journal of Economic Theory* 52: 45-67.

[8] Jensen, Michael. 1986. "Agency Costs of Free Cash Flow, Corporate Finance and Takeovers." *American Economic Review* 76: 323-29.

[9] Laux, Christian. 2001. "Limited-Liability and Incentive Contracting with Multiple Projects." *Rand Journal of Economics* 32: 514-26.

[10] Jensen, Michael, and William Meckling. 1976. "Theory of the Firm: Ownership Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3: 305-60.

[11] Myers, Stuart. 1977. "Determinants of Corporate Borrowing," *Journal of Financial Economics* 5: 147-75.

[12] Myers, Stuart. 2000. "Outside Equity." *Journal of Finance* 55: 1005-37.

[13] Myers, Stuart, and Nicholas Majluf. 1984. "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do not Have." *Journal of Financial Economics* 13: 187-221.

[14] Tirole, Jean. 2006. *The Theory of Corporate Finance.* Princeton University Press: Princeton.